

# Natural Unemployment and Activity Rates: Flow-based Determinants and Implications for Price Dynamics\*

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## Abstract

Motivated by the magnitude and cyclicity of transitions into and out of the labor force, we jointly estimate natural unemployment and participation rates through a forward-looking Phillips curve informed by structural labor market flows and demographic trends. We find that the estimated reaction of inflation to the participation gap is twice as large as the one to the unemployment gap, and that the participation margin accounts for a relevant share of total slack. Moreover, by exploiting a far reaching and unexpected pension reform, we study the effects of a sudden expansion in labor supply that was not directly related to unemployment. The reform triggered a marked reduction in employment to inactivity transitions of the elderly, delivering an increase in natural participation but not in natural unemployment. Thus, failing to account for changes in natural participation would have produced a positive bias in inflation forecasts in the years after the reform.

*Keywords:* Labor market flows, Labor supply, Demographic trends, Phillips curve, Inflation

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# 1 Introduction

The assessment of labor market slack is crucial for the understanding of an economy’s cyclical position and its price dynamics. There is a long tradition in the macro literature that considers the unemployment rate as the statistic conveying all relevant information on slack. At the same time, it is well-known that direct flows into and out of the labor force are larger than the ones between employment and unemployment by an order of magnitude [Blanchard and Diamond, 1990]. For instance, in the US, over the 1990-2020 period an average of 11.4 million individuals moved each month either into or out of activity, as opposed to 4 million between employment and unemployment. The same is true in the main European countries.<sup>1</sup> Moreover, it has recently been documented that, besides being very large, these flows account for a relevant share of the cyclical variation in the unemployment and participation rates in the US [Elsby et al., 2015] and other countries. Hence, several recent contributions have analyzed the potential role of the participation margin for price dynamics by constructing new measures of search intensity that consider the whole population and go beyond the official unemployment rate [Abraham et al., 2020].

However, to the best of our knowledge, none of the existing papers takes into account that movements into and out of the labor force can affect price dynamics without necessarily being related to the number of job seekers or to search intensity. For example, whenever workers quit employment and move directly to inactivity, labor supply decreases with no impact on labor market tightness until firms eventually post new vacancies. Nevertheless, effects on prices could well emerge, for example if the bargaining position of exiting workers differs from that of the remaining ones.

In this paper, we try to fill this gap by developing a unified framework for the derivation of the natural unemployment and participation rates, and by assessing their role for price dynamics through an augmented version of the Phillips curve. We apply our framework to Italian data, a choice motivated by two main reasons. First, Italy is one of the countries where fluctuations into and out of the labor force are largest compared to those between employment and unemployment Eurostat [2020]. Second, we leverage a large and unexpected pension reform taking place in 2012 to study how an exogenous shift in participation – unrelated to unemployment or job search intensity – is captured by our Phillips curve framework and helps interpreting price dynamics in recent years.

Our estimation framework builds on Crump et al. [2019] and consists of two main steps. First, using the flow-based model of unemployment dynamics proposed by Shimer [2012],<sup>2</sup> we estimate structural participation and unemployment rates in six demographic cells defined by gender and age. Our estimates are based on the steady state unemployment and participation rates consistent with the trend components of labor market flows between employment, unemployment and inactivity in each demographic cell. We then aggregate

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<sup>1</sup>According to harmonized data recently made available Eurostat [2020], the ratio between the sum of quarterly gross flows into and out of activity over the flows between employment and unemployment during the 2010-2020 period was equal to 1.5 in Spain, 2 in France, 2.5 in the United Kingdom and a staggering 4.6 in Italy.

<sup>2</sup>For an application of the Shimer [2012] approach to Italy see Sestito [1988] and Rosolia [2014].

the cell-specific rates to obtain the overall structural unemployment and participation rates. These rates are determined by purely structural factors such as changes in the demographic composition and in the labour market environment (preferences, institutions, matching technology). In our second step, we use the estimated structural rates as anchors in the estimation of the natural unemployment and participation rates through a forward looking Phillips Curve.

We leverage the Crump et al. [2019] approach, that enriches the standard techniques in several directions. First, by analyzing the structural components of flows into and out of each labor market state over time, we are able to identify the determinants of the trends in the structural rates and to better interpret such changes. This makes our framework naturally well-suited to perform policy evaluations and to derive policy implications. Second, since the analysis is carried out within demographic cells, we are able to characterize the structural and cyclical patterns in the flows that are specific to each segment of the population. As such, this allows us to quantify the contribution of the rapidly changing demographic structure on the unemployment and participation rates. Third, by allowing for both an unemployment and a participation gap in the estimation of the Phillips curve, we construct a comprehensive measure of slack and are able to separately distinguish the effect of the two components. This is a distinctive feature of our work, and constitutes the main innovation of our framework. In this respect, it is important to notice that the same underlying structural flows jointly determine the unemployment and the participation rates, giving discipline to the exercise.

We find that the structural unemployment rate in Italy exhibited little fluctuations throughout the period 1984–2018, whereas the structural participation rate steeply increased by about 6 percentage points in the same period. This suggests that, even though they are determined by the same underlying flows, these rates feature a very different response to cyclical and structural shocks. Our framework allows us to dig deeper into the factors shaping these dynamics. We show that most of the rise in the structural activity rate was due to the increasing participation of older workers (55-64); in turn, this can be traced back to a marked decline in the flows from employment to inactivity, very likely linked to a number of pension reforms that took place in Italy during the time period of our analysis. Moreover, in a projection exercise, we find that the continuation of the positive trends in participation experienced in the recent past by the elderly will be key to balance the negative effects of population ageing on the number of active individuals.

We then turn to the Phillips curve estimation to assess the role of the participation gap in determining labor market slack and in shaping price dynamics. We find that, on average, the participation gap accounts for about 30% of total slack and it is less cyclical than the unemployment gap. Also, the participation gap provides additional information on price dynamics: the estimated reaction of inflation to the participation gap is two times larger than the corresponding one on the unemployment gap. Moreover, in a horse race comparison, we find that a Phillips curve model augmented with both the unemployment and the participation gap outperforms otherwise identical models including only the unemployment or the employment gap in one-quarter ahead inflation forecasts. The participation gap thus explains part of the missing inflation in recent years.

To provide further evidence on the effects of participation on price dynamics, we exploit our rich framework to evaluate the effects of a far reaching pension reform that took place in Italy in 2012 (the Fornero reform). The reform, which significantly increased the minimum retirement age, was unexpected and swiftly implemented (Carta et al. [2020] and Carta and De Philippis [2020]), leaving no room for firms and workers to adapt in advance turnover and labor supply decisions. We first evaluate the impact of the reform by deriving counterfactual structural participation and unemployment rates and comparing our best forecast for the 2011-2015 period based on the pre-reform trends with new estimates using all the post reform data up to 2015. We find that the reform increased structural participation of older individuals (55-64) - through a sharp contraction of the employment to inactivity transitions - and had negligible effects on the participation of the other age classes, implying a 0.7 p.p increase in the aggregate structural activity rate. Importantly, the reform had virtually no impact on the structural unemployment rate. Finally, we use our Phillips curve model to produce inflation forecasts by conditioning only on the estimated counterfactual paths of the structural rates. We find a large effect of the pension reform on the natural participation rate (+0.7 p.p.) and no noticeable effect on the natural unemployment rate, determining an overall increase in potential labor input by 1.1%. Taken together, our results show that there is no clear trade-off between increasing participation of the elderly and unemployment: we find that the increase in retirement age augmented potential output without affecting the natural unemployment rate. An expansion of the natural participation rate also implies that, following the reform, a given observed employment rate is associated with lower inflation pressures. Failing to account for changes in natural participation would have thus resulted in a positive bias of inflation forecasts after the reform.

Our work relates to the large literature on the flow-based analysis of labor market dynamics (Choi et al. [2015], Crump et al. [2019], Elsby et al. [2019], Fujita and Ramey [2009], Gomes [2012], Petrongolo and Pissarides [2008], Shimer [2012], among others). In particular, we extend the framework of Crump et al. [2019] by explicitly taking into account the participation margin. We also contribute to the literature that studies the role of labor supply fluctuations on cyclical dynamics (Strand and Dernburg [1964], Garibaldi and Wasmer [2005], Pries and Rogerson [2009], King [2011], Kudlyak and Schwartzman [2012], Elsby et al. [2015], Krusell et al. [2017, 2020], Lalé [2013]) and as a determinant of labor market slack (Aaronson et al. [2014], Abraham et al. [2020], Faberman et al. [2020], Hornstein et al. [2020], among others). Relative to these papers, we are the first to gauge the relative importance of the unemployment and the participation gaps in the context of the Phillips curve estimation, exploiting a consistent and comprehensive framework. Moreover, by showing that the inclusion of the participation gap significantly improves the inflation forecasting accuracy, we provide an additional explanation to the failure of the standard Phillips curve in accounting for inflation dynamics in recent years (Ball and Mazumder [2011], Bobeica and Jarociński [2019]). We hence complement the literature proposing alternative measures of labour market slack (Abraham et al. [2020], Bell and Blanchflower [2013], Gordon [2013]).

On the theoretical side, models considering the participation margin within a tradi-

tional New Keynesian framework include Campolmi and Gnocchi [2016], Erceg and Levin [2014] and Galí et al. [2011]. We view our work as complementary to these papers, as none of them jointly estimates the natural unemployment and participation rates embedded in the model-based Phillips curve. Finally, our projections based on expected demographic trends and the policy evaluation of a pension reform are informative for the debate about the macroeconomic effects of ageing (see for instance Acemoglu and Restrepo, 2017, Barnichon and Mesters, 2018, Engbom, 2019, Feyrer, 2007), as we estimate the effects a policy increasing retirement age on the level of the natural participation rate.

The rest of the paper is organized as follows. Section 2 defines the main quantities at study; Section 3 presents the methodology to obtain structural activity and unemployment and outlines the results. Section 4 sets up a Phillips curve model to estimate natural participation and unemployment rates and to understand their role for price dynamics; using such framework, Section 5 analyzes the impact of the 2012 pension reform on structural and natural rates. Finally, Section 6 concludes.

## 2 Definitions

Following Crump et al. [2019], we distinguish two concepts that have often been used interchangeably in the previous literature: i) the *structural* (or trend) unemployment and participation rates, derived by extracting the trend components from labor market flows and evaluating them at the steady state; they are determined by purely structural factors like changes in the demographic composition and in the labor market environment (preferences, institutions, matching technology), and ii) the *natural* unemployment and participation rates, that is the rates coherent with constant inflation, estimated within the context of a Phillips curve framework that uses price and wage dynamics to infer the degree of slack in the economy.

As in Crump et al. [2019], we adopt the following decomposition of the unemployment rate:

$$u_t = \bar{u}_t + \underbrace{(u_t - u_t^*)}_{x_t^u} + \underbrace{(u_t^* - \bar{u}_t)}_{z_t^u}, \quad (1)$$

where  $u_t$  is the actual unemployment rate;  $\bar{u}_t$  is the structural (or trend) unemployment rate,  $u_t^*$  is the natural unemployment rate and  $x_t^u = (u_t - u_t^*)$  is the unemployment gap.  $z_t^u = (u_t^* - \bar{u}_t)$  is the gap between the natural and the structural unemployment rates. While the structural unemployment rate tracks the evolution of unemployment due to structural forces, the natural unemployment rate connects the real and the nominal side of the economy. We use  $\bar{u}$  to discipline  $u^*$ , as we assume that  $u^*$  converges to  $\bar{u}$  in the long-run; in the short-run, monetary policy shocks or temporary changes in price or wage setting may affect  $u^*$  without having an impact on  $\bar{u}$ . For example, the introduction of an ex-post wage indexation mechanism in an economy with accelerating inflation would temporarily drive  $u^*$  up but would have no effect on structural unemployment.

We introduce a similar decomposition also for the participation rate:

$$p_t = \bar{p}_t + \underbrace{(p_t - p_t^*)}_{x_t^p} + \underbrace{(p_t^* - \bar{p}_t)}_{z_t^p}, \quad (2)$$

where  $p_t$  is the actual participation rate;  $\bar{p}_t$  is the structural (or trend) participation rate;  $p_t^*$  is the natural participation rate, consistent with constant inflation.  $x_t^p = (p_t - p_t^*)$  is the participation gap; for a given unemployment gap, this variable conveys information on the additional degree of slack in the economy.  $z_t^p = (p_t^* - \bar{p}_t)$  is the gap between the natural and the structural participation rates. Again, to discipline the estimation of  $p_t^*$ , we assume that  $p_t^*$  will converge to  $\bar{p}_t$  in the long-run (i.e. that  $z_t^p$  will converge to zero), while in the short-run monetary policy shocks or temporary changes in price or wage setting may determine a positive or negative gap  $z_t^p$ .

### 3 Structural unemployment and activity rates

In this section we describe the methodology used to estimate the structural unemployment and activity rates, which we then use as anchors for the estimation of the natural rates described in Section 4. We then briefly discuss how we project these rates in the future. Finally, we present and discuss our main estimates.

#### 3.1 Estimation

The estimation procedure involves four steps (for a full description with all the technical details see Section A of the Appendix). First, we use the Italian Labour Force Survey micro data to estimate labor market flows between the three labor market states (employment  $E$ , unemployment  $U$  and inactivity  $N$ ) over the 1984-2018 period for six demographic groups defined by three age classes (15–34, 35–54 and 55–64) and gender.<sup>3</sup> Following the existing literature (for instance Shimer, 2012, Elsby et al., 2015 and Barnichon and Mesters, 2018), we perform two important adjustments to the measured flows: i) we make them consistent with the dynamics of the stocks (*margin error correction* - MEC) and ii) we derive continuous-time flow rates from discrete time transition probabilities, in order to account for the possibility of multiple transitions taking place within the observation window (*temporal aggregation correction* - TAC). In this way, we obtain six underlying hazard rates for each group  $g$ :

$$\{f_{g,t}^{NU}, f_{g,t}^{NE}, f_{g,t}^{EU}, f_{g,t}^{EN}, f_{g,t}^{UE}, f_{g,t}^{UN}\}_{t=1984,q1}^{2018q4},$$

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<sup>3</sup>The choice of the starting and ending point is dictated by data availability. Instead, the choice of the age groups is dictated by patterns of participation, which are increasing over the age 15–34, substantially flat between 35 and 54, and progressively declining in the region 55–64 (Figures C.1 and C.2). Throughout the paper, we will therefore estimate aggregate rates for the population 15–64. We believe this to be a reasonable choice, as the overwhelming majority of the changes in the participation patterns of the population occurred before age 64 (see again Figures C.1 and C.2).

where  $f_{g,t}^{XY}$  is the transition rate between labor market state  $X$  and  $Y$  for demographic group  $g$  at time  $t$ .

Second, following Tasci [2012], we decompose each labour market flow (in each demographic cell) into a stochastic trend and a stationary cyclical component, using an unobserved component model which takes into account their joint dynamics with real log GDP (see Section A of the Appendix for details). The outcome of this second step are the trend components of the flow rates ( $\bar{f}_{g,t}^{XY}$ ), which represent the building blocks of the structural unemployment and activity rates computed in the next step.

Third, to compute the structural rates of unemployment and participation, we rely on the notion of flow-consistent rates (as in Shimer, 2012). Let  $U_{g,t}$ ,  $E_{g,t}$  and  $N_{g,t}$  be the relevant stocks of unemployment, employment and inactive population for demographic group  $g$  at time  $t$ . The evolution of the stocks over time depends on the hazard rates through the following differential equations:

$$\dot{U}_{g,t} = f_{g,t}^{EU} E_{g,t} + f_{g,t}^{NU} N_{g,t} - (f_{g,t}^{UE} + f_{g,t}^{UN}) U_{g,t}, \quad (3)$$

$$\dot{E}_{g,t} = f_{g,t}^{UE} U_{g,t} + f_{g,t}^{NE} N_{g,t} - (f_{g,t}^{EU} + f_{g,t}^{EN}) E_{g,t}, \quad (4)$$

$$\dot{N}_{g,t} = f_{g,t}^{UN} U_{g,t} + f_{g,t}^{EN} E_{g,t} - (f_{g,t}^{NU} + f_{g,t}^{NE}) N_{g,t}. \quad (5)$$

Under the assumption of constant transition rates, we can use (3), (4) and (5) to solve for the steady-state levels of  $U_g^*$ ,  $E_g^*$  and  $N_g^*$ , by setting  $\dot{U}_{g,t} = \dot{E}_{g,t} = \dot{N}_{g,t} = 0$ . In order to obtain the trend unemployment and participation rates, we evaluate equations (3)-(5) in steady state plugging in the estimated trend components of the flows,  $\bar{f}_{g,t}^{XY}$  (as in Tasci [2012] and Crump et al. [2019]). Let us then formally define the structural unemployment and participation rates as:

$$\bar{u}_{g,t} = \frac{\bar{U}_{g,t}^*}{\bar{U}_{g,t}^* + \bar{E}_{g,t}^*}, \quad \bar{p}_{g,t} = \frac{\bar{U}_{g,t}^* + \bar{E}_{g,t}^*}{\bar{U}_{g,t}^* + \bar{E}_{g,t}^* + \bar{N}_{g,t}^*}.$$

Plugging in the equilibrium values, and using the fact that total population is normalized to 1, we can solve for the structural unemployment rate  $\bar{u}_{g,t}$  and participation rate  $\bar{p}_{g,t}$  of each group  $g$  as a function of the structural rates  $\bar{f}_{g,t}^{XY}$ .<sup>4</sup>

Forth, we aggregate structural unemployment and activity rates using a weighted average of the group-specific ones. In particular, for the aggregation of the group-specific participation rates, the weight of group  $g$  at time  $t$  is represented by its share in the total population at a given point in time. Let us denote this population weight as  $\omega_{g,t}^p$ , such that  $\sum_g \omega_{g,t}^p = 1 \forall t$ . Hence, the aggregate structural participation rate is computed using

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<sup>4</sup>Note that in this setting we are assuming that the population of each group is constant over time in steady state. This is a standard assumption in the literature (which, for the papers that do not explicitly consider inactivity, translates into assuming that the active population is constant over time). Barnichon and Mesters [2018] discuss the possible bias generated by a changing within group population in the US. They show that it is negligible, since flows tend to be much larger than population growth in the US. We find a similar result for Italy: for instance, on average, for all demographic groups, flows from inactivity to employment and vice versa contribute about 10 times more than changes in population to variations in the stock of employed individuals. Indeed, flows are more than 10 times larger on average than the deviation between the growth rates of the population of employed individuals and that of the overall population.

$\omega_{g,t}^p$  as weights:

$$\bar{p}_t = \sum_g \omega_{g,t}^p \bar{p}_{g,t}. \quad (6)$$

The aggregation of group-specific unemployment rates is slightly more involved. In this case the weights are a combination of the group-specific weights in the total population (the  $\omega_{g,t}^p$  defined above) and their incidence in the active population. Therefore, the aggregate structural unemployment is calculated as follows:

$$\bar{u}_t = \sum_g \omega_{g,t}^p \underbrace{\frac{\bar{p}_{g,t}}{\bar{p}_t}}_{\bar{\omega}_{g,t}^u} \bar{u}_{g,t}, \quad (7)$$

where  $\bar{\omega}_{g,t}^u$  can be interpreted as the structural weight in labor force of group  $g$  at time  $t$ . It represents the share of structural active population (that is, the active population identified by the structural rates) accounted for by the specific group  $g$ . For instance, for a given share of the elderly in the population ( $\omega_{g,t}^p$ ), their weight on the structural unemployment rate will typically be smaller because their structural participation rate is lower than the average one.

## 3.2 Results

### Structural unemployment rate

We plot the estimated series of the aggregate structural unemployment rate in Figure 1. Notice that the structural unemployment rate was essentially flat between 1984 and 1995, to then assume a slow but continuous downward trend until 2008, when it started to rise again until 2015. Since then, the structural unemployment rate appears stable. Overall, its time series smooths out the large oscillations of the actual unemployment rate over the business cycle. In 2018q4, the aggregate structural unemployment rate is estimated to be at 9.2%. The negative gap between the aggregate unemployment rate and the actual one since the sovereign debt crisis is the result of the joint contribution of all demographic groups (see Figure C.3), meaning that for all groups the actual unemployment rate lies above its structural level.

In order to understand what generates the dynamics between 1984 and 2018, Figure 2 shows the evolution of the two components of the aggregate structural unemployment rate: i) the weights of each demographic group, and ii) the group-specific structural unemployment rates.<sup>5</sup> The downward trend started in 1995 was brought about by the fast change in the structure of active population, with the 15–34 age group losing share in favour of the group 35–54, characterized by a substantially lower level of structural unemployment. From 2000 onward, the negative trend was further boosted by the increase in the share of the age group 55–64, also characterized by low trend unemployment. At the same time, the rapid increase in the structural unemployment rate of the youngest groups represented a counteracting force, resulting in a slight increase of the aggregate

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<sup>5</sup>Figure C.4 in the Appendix displays these group-specific structural rates together with the actual unemployment rate in each group.



rate after 2008. From 2015, due to the flattening of the structural unemployment of the youngest groups, the aggregate rate appears stable.

## Structural participation rate

As for the structural participation rate, Figure 3 reveals that it constantly increased since 1995, reaching the level of 66.3%, in 2018q4. Overall, it followed closely the evolution of the observed participation rate; however, the structural component kept increasing despite the flattening of the actual rate between 2000 and 2010. Indeed, our filtering technique ascribes the decrease in the actual participation rate of prime-age men and of the youth during that period mainly to cyclical conditions, while identifying the increase in the participation rate of the elderly as structural (see Figure C.5).

Figure 4 analyzes the determinants of its evolution over time. It reveals that the increase was initially driven by the rising weight on the population of prime age individuals, characterized by higher activity rates; from the early 2000s it was instead mostly driven by the strong growth in the structural activity rate of the eldest groups, together with their increasing weight in the population.

To dig into the causes of these strong trends in the group-specific structural activity rates, we decompose them into the underlying flows. We divide the six flows into three groups (exit: EN, UN; entry: NE, NU; churn: EU, UE)<sup>6</sup>, and let only one of them vary over time, fixing the others at their level in 2009q1.<sup>7</sup> To fix ideas, when we let vary only the exit margin (EN and UN), we construct a counterfactual series of trend activity, i.e. the one generated by movements in the exit margin only. We find that different forces have driven the trends for the different demographic groups (Figure 5). For the youth, the observed reduction in trend activity was almost entirely due to a slower entry. For prime age individuals, both the entry and the exit margins played a relevant role: while for men they almost exactly offset each other throughout the period, for women they both contributed to the increasing participation, especially the entry margin. Finally, the exit margin clearly drove almost all the increase in the activity rate of the elderly, with an additional push coming from the entry margin. We focus our attention on the role of the exit margin for these groups and distinguish the contribution of the EN and the UN flow (Figure 6), concluding that the increase was entirely accounted for by the effect of the EN flow.

Overall, the aggregate dynamics in the structural activity rate reflected primarily the unprecedented increase in the structural activity of the elderly, which – on the basis of the results of our decomposition – can be traced back to the strong decline in their EN flows. Part of the change in the behavior of these groups of workers was arguably due to a number of pension reforms, starting in the 2000’s. In Section 5, we focus our attention on the Fornero reform, that took place in 2012, and study its effect on both the structural and the natural rates.

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<sup>6</sup>We follow the same groups definition of Elsbey et al. [2019].

<sup>7</sup>This resembles the decomposition performed by Shimer [2012]. The only difference is that we fix the non-varying rates to their level at a specific point of time, instead of their average.

## Projections of the structural rates in the future

Finally, we exploit the disaggregated nature of our estimates – and in particular the availability of structural unemployment and activity rates for each demographic cell – to project these rates in the future and isolate the role of the changing demographic structure. As for the evolution of the population weights  $\omega_{g,t}^p$ , we use the demographic projections provided by Eurostat [2020]; instead, for projecting the group-specific structural rates, we construct two different scenarios. The first assumes that each group-specific trend in the structural flows  $\bar{f}_{g,t}^{XY}$  will continue in the next years according to similar dynamics as those registered in the last years (baseline projection); the second hypothesizes that the structural flow rates  $\bar{f}_{g,t}^{XY}$  would remain constant at their estimated level in 2018, instead of changing according to past dynamics (see Section A of the Appendix for more details). While the sudden break in all trends is a strong assumption, we view this as a useful and instructive exercise, as it allows to separately identify the effects of the ongoing demographic change.

Figure 7 shows that the structural unemployment rate is expected to continue on the same flat trend observed in the last years according to both scenarios. When we assume that the group specific flows will continue behaving in a way which is similar to what observed in the most recent years (baseline projection), the aggregate unemployment rate remains flat because the same underlying forces will continue to operate and offset each other: the modest positive trends in the structural unemployment of the groups aged below 55 will be compensated by the increasing weight of the elderly, characterized by a low and decreasing level of unemployment (see the dashed lines of Figure 2). In other words, it turns out that the trends in structural unemployment rates go in opposite directions for the different groups, almost exactly counterbalancing each other. As a result, turning to our second scenario, the projections for the aggregate structural unemployment rate are virtually unaffected by fixing the trend flow rates to the 2018 level.

Regarding the structural participation rate, the dashed lines of Figure 8 reveal instead that the assumptions on the future dynamics of the group-specific trend rates crucially affect the behavior of the aggregate rate. Indeed, if the trends in the participation rates were to suddenly stop in 2018, the structural aggregate participation rate is predicted to quickly decline, driven by population ageing. This stands in stark contrast with the baseline projections, according to which the participation rate would have a tendency to increase in the next years, because the decreasing weight of prime age individuals (highly attached to the labor market) will be more than compensated by the increasing participation of the elderly (see the dashed lines in Figure 4).

This exercise shows the importance of investigating the behavior of the participation rate by disentangling the contribution of different flows and different demographic groups: at this level of disaggregation it is thus easier to make assumptions about future developments, which turn out to have very different impact on projections.

## 4 Natural unemployment and activity rates

We now turn to the estimation of natural unemployment and activity rates. Our estimation framework leverages an augmented version of the standard Phillips curve that exploits the information contained in the structural rates estimated in the previous section.

### 4.1 The augmented Phillips Curve

In the standard Phillips curve framework, one of the building blocks of New Keynesian models, price and wage inflation are negatively related to the economic slack, that can be measured by the unemployment gap, i.e. the difference between the observed unemployment rate and the natural unemployment rate ( $u_t^*$ ). The natural unemployment rate is thus defined as the level of unemployment for which price inflation remains stable in absence of supply shocks. The standard model, however, neglects the participation margin, which is likely to provide additional information on labor market slack. Indeed, for a given unemployment gap, a participation rate that is below its natural level could be an indication that the economy is running below potential, as some inactive workers could switch to activity.

While some theoretical papers consider the participation margin within a traditional New Keynesian model (Campolmi and Gnocchi, 2016, Erceg and Levin, 2014 and Galí et al., 2011), they do not estimate the model-based Phillips curve. Here we move one step further by proposing an augmented Phillips curve model where price and wage inflation are also related to the participation gap, defined as the difference between the observed participation rate and the unobserved level  $p_t^*$ , consistent with constant price inflation.

Differently from Erceg and Levin [2014], who use institutional projections of the unemployment and participation rates as proxies for  $u^*$  and  $p^*$ , we aim at estimating the unobserved natural unemployment and participation rates. Hence, we extend the approach of Crump et al. [2019], using a rich state-space model that allows us to jointly estimate the Phillips curve, the unemployment gap and the participation gap by making standard assumptions on the data generating process. The estimation also exploits the information from labor market flows using the structural unemployment and participation rates previously estimated ( $\bar{u}_t, \bar{p}_t$ ) as an anchor for  $u^*$  and  $p^*$ . Furthermore, our setup builds on the insights of a recent literature highlighting the importance of inflation expectations to explain price dynamics (Ball and Mazumder [2019], Coibion and Gorodnichenko [2015]) and incorporates them to pin down the inflation trend.<sup>8</sup>

#### 4.1.1 Model specification

We present here only the main equations of the model (the full specification is discussed in the Appendix, section B). We estimate a generalized version of the Phillips Curve that

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<sup>8</sup>Although it is known that the textbook Phillips Curve model failed to account for the missing disinflation during the Great Financial Crisis and for the missing inflation over the ensuing recovery (Ball and Mazumder, 2011, Bobeica and Jarociński, 2019), recent contributions show that some refinements of this standard tool considerably improve its performance also in the last years. For instance, Coibion and Gorodnichenko [2015] and Ball and Mazumder [2019] argue that the puzzling inflation dynamics can be explained within the context of the Phillips curve framework when inflation expectations are properly taken into account.

can be derived from a New Keynesian model like the one presented in Galí [2011]. Consistent with the theoretical insights of the model, we expect inflation to depend negatively on current and future unemployment gaps, i.e. the difference between the observed unemployment rate and the unobserved natural unemployment rate ( $u_t^*$ ). Recall that we denote the unemployment gap at time  $t$  by  $x_t^u = u_t - u_t^*$ . We augment the standard specification by allowing inflation to depend also on the participation gap  $x_t^p = p_t - p_t^*$ , where  $p^*$  is the natural participation rate.<sup>9</sup>

As Crump et al. [2019], we assume that inflation expectations follow a time-varying trend ( $\pi_t^*$ ), pinned down using short and long-run inflation expectations derived from Consensus Forecasts. This trend represents the stable inflation path when both the unemployment and the participation gaps are closed. Formally, we estimate the following equation, where the dependent variable is the price inflation gap, that is the difference between realized price inflation and the estimated trend:

$$\pi_t - \pi_t^* = \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u E_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u + \kappa^p E_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^p - \beta \frac{1 - \rho_a}{1 - \beta \rho_a} \Delta a_t, \quad (8)$$

where  $\gamma$  captures inflation inertia, and  $\kappa^u$  and  $\kappa^p$  denote the reaction of inflation to the present discounted value of future unemployment and participation gaps, respectively. Notice that we expect inflation to depend negatively on unemployment gaps and positively on participation gaps.  $\epsilon_t^{\pi^*}$  indicates shocks to the inflation trend and hence to long-term inflation expectations which effectively identify it; such shock can thus be interpreted as a measure of (de)anchoring of professional forecasters' expectations to a given inflation target.  $\Delta(a_t)$  follows an AR(1) process and represents supply factors (e.g. productivity) which affect price inflation beyond the demand ones captured by the unemployment gap. We further assume that the inflation trend follows a random walk and the unemployment and participation gaps are represented by AR(2) processes. Moreover, the rational expectations hypothesis implies that short and long-term inflation expectations are consistent with the forward iteration of eq. (8) with a margin of error.

We can estimate the unemployment and the participation gaps recalling the decomposition of the realized unemployment and participation rates introduced in Section 2:

$$u_t = x_t^u + z_t^u + \bar{u}_t \quad (9)$$

$$p_t = x_t^p + z_t^p + \bar{p}_t \quad (10)$$

where  $z_t^u = u_t^* - \bar{u}_t$  is the deviation of the natural unemployment rate from the structural unemployment rate and  $z_t^p = p_t^* - \bar{p}_t$  is the deviation of the natural participation rate from the structural participation rate. We assume that both  $z_t^u$  and  $z_t^p$  follow an AR(1)

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<sup>9</sup>Erceg and Levin [2014] provide a theoretical underpinning for the inclusion of the participation margin in the Phillips curve, introducing a broad definition of employment gap that could be rewritten as follows:  $e - e^* = (1 - u)(p - p^*) - p(u - u^*)$ . To be consistent with such definition, in practice we build scaled measures of the unemployment and participation gap:  $x^u = p(u - u^*)$  and  $x^p = (1 - u)(p - p^*)$ . However, to ease the interpretation we mostly report figures and graphs for the non-scaled variables where not else specified.

process:

$$z_t^u = \rho_{z^u} z_{t-1}^u + \sigma_{z^u, \varsigma} \sigma_\varsigma \epsilon_t^{z^u} \quad (11)$$

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p, \varsigma} \sigma_\varsigma \epsilon_t^{z^p}. \quad (12)$$

Equations (11-12) imply that the  $u^*$  and  $p^*$  converge to their respective structural rates  $\bar{u}_t$  and  $\bar{p}_t$  in the long-run; however, in the short-run, deviations are allowed with degrees of persistence  $\rho_{z^u}$  and  $\rho_{z^p}$ . Since  $\sigma_\varsigma$  represents the volatility of inflation due to supply shocks<sup>10</sup>,  $\sigma_{z^u, \varsigma}$  and  $\sigma_{z^p, \varsigma}$  can be interpreted as the signal-to-noise ratios, that is the volatility of the unobserved states  $u_t^*$  and  $p_t^*$  relative to inflation. Notice that, from the perspective of the Phillips curve model, the trend unemployment and participation rates are exogenous observable inputs; for this reason, the shocks moving  $z_t^u$  and  $z_t^p$  are fully reflected in the natural unemployment and participation rates. Equations (8) to (12) together with the others described in Appendix B allow us to jointly estimate the parameters of the Phillips curve,  $u^*$  and  $p^*$ . More specifically, the observable variables are inflation, the unemployment rate, the participation rate, and inflation expectations. The unobserved state variables, which are estimated together with the model parameters through the Kalman filter, include the natural unemployment and participation rates, the inflation trend and the proxy for supply-type inflation pressures.

In order to use all the available information, our baseline model further includes three wage measures: wage per hour in the private sector, wage per equivalent unit of labor in the private sector and negotiated wages. Following Crump et al. [2019], we assume that wage and price inflation are tied by the following relationship:  $\pi_t^w = \pi_t + \Delta a_t$ . We further consider that real wages grow at rate  $g_w$ . We thus add to the model three measurement equations, one for each wage variable:

$$\pi_t^{w^j} = \delta_j (g_w + \pi_t + \Delta a_t) + oe_t^{w^j} \quad \text{with} \quad j = 1, \dots, 3$$

where  $\pi_t^{w^j}$  denotes the growth rate of the  $j$ -th nominal wage measure,  $g_w$  is the constant mean growth rate of real wages and  $oe_t^{w^j}$  is an i.i.d. normally distributed measurement error. The unemployment and the participation gaps have the same impact on both wages and prices which both help to identify the Phillips curve coefficients. Each wage measure is linked to the others through the scale factor  $\delta_j$ :  $\pi_t^{w^j} = \delta_j \pi_t^{w^1}$ , with  $\delta_1$  normalized to 1.

We estimate the model with Bayesian techniques using Italian data over the period 1996Q1-2018Q4.<sup>11</sup> We use inflation expectations 2 and 6 quarters ahead.<sup>12</sup>

Finally, when projecting  $u^*$  out-of-sample, we set to zero the shocks in equations (11) and determine the evolution of  $u^*$  combining equations (9) and (11), and using the projections of  $\bar{u}$  (trend unemployment) obtained in the previous Section. The same considerations apply for projecting  $p^*$ .

<sup>10</sup>If we define  $\varsigma_t = -\beta \frac{1-\rho_a}{1-\beta\rho_a} \Delta a_t = \rho_a \varsigma_{t-1} + \sigma_\varsigma \epsilon_t^\varsigma$ , inflation is affected by shocks of volatility  $\sigma_\varsigma$ .

<sup>11</sup>The choice of the sample period is motivated by data availability, since national accounts are released from 1995 onwards.

<sup>12</sup>Consensus Forecast is available from 1989 onwards.

## 4.2 Results

In what follows we report results based on the structural unemployment and participation rates presented in Section 3, estimated with Kalman filter techniques.<sup>13</sup> The priors and the posterior estimates of the model parameters are described in Table 1 and compared to those obtained from an analogous Phillips curve including only the unemployment gap.<sup>14</sup> The inflation process displays a moderate degree of inertia (the median estimate of  $\gamma$  is 0.25). The deviation of  $u^*$  and  $p^*$  from their respective long-run trends are highly auto-correlated, as  $\rho_{z^u}$  and  $\rho_{z^p}$  are very close to 1. The median estimate of the signal-to-noise ratio is substantially higher for  $p^*$  (the median estimate for  $\sigma_{z^p, \varsigma}$  is 0.57) than for the  $u^*$  (the median estimate of  $\sigma_{z^u, \varsigma}$  is 0.23) implying larger deviations of  $p^*$  from its long-run trend.

Let us now turn to the most interesting parameters, those capturing the reaction of prices and wages to the unemployment and participation gaps.  $\kappa^u$ , the coefficient on the discounted sum of future unemployment gaps, is relatively small (the median is 0.018); however, the implied overall reaction to the current and lagged unemployment gap ( $K^u$ ) is substantial (the median is 0.176), in line with the estimates of Eser et al. [2020] for the Euro area.<sup>15</sup> Interestingly, the median estimates of  $\kappa^u$  and  $K^u$  in the augmented model are somewhat higher than those obtained with a Phillips curve including only the unemployment gap, but less precisely estimated due to greater model complexity. The estimated median reaction of inflation to the participation gap ( $\kappa^p$  and  $K^p$ ) is stronger than the impact of the unemployment gap: the overall coefficient on current and lagged participation gap ( $K^p$ ) is more than two times larger than the corresponding one on the unemployment gap. This suggests that the participation gap is a very relevant margin for explaining inflation dynamics. To corroborate this intuition, we perform the historical decomposition of the estimated inflation gap (the dependent variable of the Phillips curve). Figure 9 shows that the participation gap (purple bars) provides a sizable contribution to inflation dynamics and, together with the unemployment gap, it explains the lion share of the inflation gap for most of the sample period.

Figures 10 and 11 plot the natural unemployment and participation rates estimated through the baseline Phillips curve regression including both the unemployment and the participation gap.<sup>16</sup> At the end of the sample period, the median  $u^*$  is estimated slightly above 9%, implying an unemployment gap of approximate 1.5 percentage points, while the participation gap is almost nihil. While  $u^*$  is very smooth and well anchored to the trend unemployment rate,  $p^*$  follows more closely the observed participation rate.<sup>17</sup> As a result,

<sup>13</sup>When the trend unemployment and participation rates are estimated using the HP filter, they display much higher volatility, which is transmitted also to the estimates of the natural unemployment and participation rates. Results are available upon request.

<sup>14</sup>See also Figures C.7-C.8 in the Appendix.

<sup>15</sup> $K^u$  and  $K^p$  denote the overall coefficients on current and lagged unemployment and participation gaps, respectively:  $K^u = \kappa^u(\omega_{\pi,1}^u + \omega_{\pi,2}^u)$ ,  $K^p = \kappa^p(\omega_{\pi,1}^p + \omega_{\pi,2}^p)$  (see equation (B.8) in the Appendix).

<sup>16</sup>Appendix Figure C.9 further shows that the inflation trend is precisely estimated and follows very closely the long-term inflation expectations.

<sup>17</sup>One possible explanation is related to the assumption that it takes time for the natural rates to converge to their structural values: while the structural unemployment rate has remained constant in the last decades, the structural participation rate has increased substantially starting from the end of the nineties. It may therefore still take time for  $p^*$  to fully incorporate such changes, while  $u^*$  is already around the structural unemployment

the unemployment gap is more volatile than the participation gap (Figure 12, where the participation gap is reported on a reverse scale so that positive values signal a slack labor market, likewise the unemployment gap). On average, the participation margin accounts for about 30% of total slack and correlates negatively with the unemployment gap (-0.4), indicating that the two margins generally reinforce each other. Focusing on the recent years, we estimate a negative and significant participation gap since 2013, which closed only in 2018. This may look surprising in view of the fast increase in the participation rate over the same period (almost 2 pp., more than what realized in the previous 10 years). However, as Figure 11 makes clear, the rise in participation reflected only partially the strong increase in  $\bar{p}$  and  $p^*$ . Further inspection reveals that the recent rise in participation was driven by the reduction in the exit rate of the elderly induced by a pension reform enacted in 2012 (see next Section). The participation rates of the youngsters and prime-age men, however, hovered around cyclically low levels (Figure C.5), thus signalling the presence of more slack than what suggested by aggregate figures. Our framework is able to detect these changes in a parsimonious way through the estimation of the natural participation rate.

The results of our baseline augmented Phillips curve can be compared to those derived from alternative, simpler models. First of all, we consider a Phillips curve regression where inflation positively depends on the employment gap, defined as the difference between the observed employment rate  $e_t$  and the unobserved estimated value  $e_t^*$ . Second, we consider the standard model including only the unemployment gap. On the one hand, if the participation margin has no explanatory power on inflation dynamics we expect the three specifications to yield similar results and our baseline model to underperform in terms of estimation precision due to its greater complexity. On the other hand, if the participation margin is relevant but its effect is akin to unemployment, the employment gap would summarize all the relevant information. To compare these different models we combine the unemployment and participation gaps obtained in our baseline setup to obtain the corresponding employment gap<sup>18</sup>:  $e - e^* = (1 - u)(p - p^*) - p(u - u^*)$ . Results are shown in Figure 13, where the measures of employment gaps are divided by the actual participation rate to make them comparable with the unemployment gap. The gap estimated under our baseline augmented model lies fairly close to the unemployment gap obtained under a standard specification, but also displays some notable differences. Focusing on the most recent period, the first one anticipates the worsening of labor market conditions after the Great Recession with the negative employment gap closing already in late 2009 rather than in 2013. Moreover, the peak level recorded in 2015 was one third (1 percentage point) larger according to our preferred measure of slack compared to the estimates based solely on unemployment. These considerations are amplified when looking at the results of the model including only the employment gap: the gap turns positive already at the beginning of 2009 and the degree of slack during the double-dip recession is considerably higher.

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rate level, since it has remained constant in the last few years.

<sup>18</sup>See Erceg and Levin [2014] for a similar decomposition.

### 4.3 Inflation forecasts

To discriminate among these different specifications, we ask which estimated measure of slack yields the most accurate inflation forecasts. This exercise is similar in spirit to Jarociński and Lenza [2018], who rank several variants of their model according to the precision of their out-of-sample inflation forecasts. We thus estimate each specification of the Phillips curve on the first part of the sample (1996Q1–2011Q4) and evaluate the out-of-sample forecasting performance on the last seven years (2012Q1–2018Q4). We use an expanding window, meaning that the model is fully re-estimated every time we add a new observation. Our baseline augmented model which explicitly takes into account the participation gap clearly outperforms the other two specifications for one-quarter ahead inflation forecasts (Figure 14).<sup>19</sup> The negative participation gap estimated over the final part of the sample adds to the traditional measure of slack, thus contributing to explain the missing inflation puzzle.

The inflation forecasts produced by the regressions based either on the employment or the unemployment gap lie mostly outside the 68% confidence bands around the median projections obtained through the augmented Phillips curve and fail to explain the low inflation period ensuing the sovereign debt crisis. Notice that the model including only the employment gap has the lowest accuracy despite its estimate of a much larger amount of slack (blue dashed line in Figure 13): this happens because the reaction of inflation to the employment gap is poorly estimated and its median is very low. To conclude, this exercise shows that the participation margin plays an autonomous and relevant role in explaining price dynamics: explicitly taking it into account in a Phillips curve regression could change our assessment of the labor market outlook and help to predict inflation.

## 5 The effects of an unexpected pension reform

Relative to other estimation methods, our framework has two main advantages in terms of policy evaluation. First, given that we directly estimate the structural component of labor market flows, our approach is naturally well-suited to evaluate the impact of labor market reforms. Second, by allowing for non-participation, we can study the interplay between participation and unemployment and the overall effects on potential employment.

In this Section, we use our framework to evaluate the impact on potential employment of the Fornero reform, a far reaching and unexpected pension reform that took place in Italy at the height of the sovereign debt crisis (in 2012), motivated by the need of limiting pension expenditures.<sup>20</sup> It consisted of a slight reduction in the pension transfer and a substantial increase in the statutory retirement age (the maximum delay was equal to seven years for certain workers' categories). In our context, such a reform would increase the structural participation rate through a change in the exit margin, due to both the decrease in the outside option (pension income) and the increase in the minimum statutory age for retirement. Our goal is to quantify the macro effect of this reform on natural

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<sup>19</sup>The same ranking applies also longer forecasting horizons, with a significantly better forecasting performance of the augmented Phillips curve.

<sup>20</sup>For an overview of the reform see [Carta et al., 2020] and [Carta and De Philippis, 2020].



participation and unemployment rates, and to assess its implications for price dynamics. The richness of our approach, that relies on detailed micro-level data and that adds a third labor market state –inactivity– provides the ideal setting to estimate these effects. In the research design, we leverage the fact that the reform was unexpected and implemented a few days after its announcement; as a consequence its effects were magnified, as firms’ turnover and workers’ labor supply decisions were not affected in the years preceding its implementation. We use this element in the evaluation of the impact of the reform, by deriving our best forecast of natural and structural rates for the 2011-2015 period based on the pre-reform trends in labor market flows. In order to do that, we first repeat the estimation outlined in Sections 3 and 4 using only pre-reform data (until 2011) and we project structural and natural rates until 2015.<sup>21</sup> Concerning the structural rates, we project them using our baseline projection method, i.e. assuming that in the absence of the reform each group specific rate would have behaved similarly to what observed in the years before the reform. We denote as  $\{\bar{u}_t^{PRE}\}_{t=2012}^{2015}$  and  $\{\bar{p}_t^{PRE}\}_{t=2012}^{2015}$  the paths of the structural unemployment and structural participation in place prior to the reform; we interpret them as the counterfactual evolution of the Italian economy throughout the period 2012–2015 absent the reform. Then, we estimate again the structural rates using the data until 2015, i.e. including the post reform period, and we obtain  $\{\bar{u}_t^{POST}\}_{t=2012}^{2015}$  and  $\{\bar{p}_t^{POST}\}_{t=2012}^{2015}$ ; we normalize their levels in 2011 to be equal to those estimated in the pre-reform period.<sup>22</sup> Finally, we compute the effect of the reform as the simple difference between the POST and the PRE-series.

According to our estimates, the reform was successful in increasing the structural participation of the older groups, by about 3 p.p. and 1.p.p. for males and females aged 55–64 respectively, and by 1 p.p. for females aged 35-54 (see Figure C.10). Moreover, we find that the impact of the reform on structural unemployment was negligible (Figure C.11). Overall, these effects imply that the reform increased the aggregate structural participation rate by about 0.7 p.p. in 2015, while the trend unemployment rate remained roughly unchanged, as shown in Figures 15 and 16. Reflecting the differential impact across groups, the total effect on aggregate participation was mainly due to the 55-64 groups (that explains about two thirds of the overall variation), with almost all the rest accounted for by females aged 35–54 (see Figure C.12).<sup>23</sup> We conclude that the Fornero reform had a substantial impact on structural participation, with a negligible effect on the structural unemployment rate.

Finally, we further investigate whether the Phillips curve model detects any change in the level of the natural unemployment and participation rates before and after the

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<sup>21</sup>The reform was passed in December 2011 and started to have effect in January 2012. We assume that the reform produced (most of) its effects by 2015, as the average delay in retirement age for 55+ workers was less than 3 years (Carta et al. [2020]). Enlarging the time window may be problematic as we would likely be capturing other shocks not related to the reform.

<sup>22</sup>As we want to avoid that our filtering technique changes the estimation for the period prior to the reform, for each subgroup we define an adjustment factor  $adj_g^x = \bar{x}_{g,2011q4}^{PRE} - \bar{x}_{g,2011q4}^{POST*}$ , and construct our final POST-series accordingly:  $\bar{x}_{g,t}^{POST} = \bar{x}_{g,t}^{POST*} + adj_g^x$  for  $x \in \{u, p\}$  and  $t \in [2012, 2015]$ , where  $x_{g,t}^{POST*}$  refers to the non-adjusted series.

<sup>23</sup>This result is consistent with Carta and De Philippis [2020], who look at whether the reform affected also labour supply of middle-aged individuals and find a positive and significant effect.

implementation of the pension reform. In order to do that, we estimate the model on the pre-reform period (1996Q1–2011Q4) and then project the model forward conditioning only on the estimated path of the structural unemployment and participation rates. Hence, we do not use any data of the post-reform period, which most likely is contaminated by other shocks, except for the estimated structural rates which have been purged from cyclical components. In line with our results on the structural rates, we find negligible effects of the pension reform on  $u^*$  (Figure 17) and more substantial effects on  $p^*$  (Figure 18). According to our median estimates, in 2015 the natural participation rate would have been 0.7 pp lower absent the Fornero reform. This implies, everything else equal, that a given observed participation rate is now associated to lower inflation pressures due to the reform. This explains why the relatively strong increase in the participation rate since 2012 did not have a positive impact on wage growth: the even larger rise in  $p^*$  driven by the pension reform resulted in a negative participation gap, thus contributing to subdued inflation. Overall, our estimates indicate that the reform increased potential employment by 1.1%.

## 6 Conclusions

Motivated by the magnitude and cyclicity of the transitions into and out of the labor force, we investigate the role of the participation margin for the measurement of labor market slack and for price dynamics. By building on Crump et al. [2019]’s approach, we thus propose a unified framework for the estimation of natural unemployment and participation rates. We do so by estimating a forward-looking Phillips curve augmented with the participation margin and informed by labor market flows between employment, unemployment and inactivity among different demographic groups.

We focus on Italy for two main reasons. First, Italy is one of the countries where fluctuations into and out of the labor force are largest compared to those between employment and unemployment. Second, a far reaching and unexpected pension reform taking place in 2012 dramatically increased elderly workers’ labor supply by decreasing their flows into inactivity; this provides an ideal setting to tease out the implications for the measurement of labour market slack of an exogenous shift in labor supply unrelated to unemployment or job search intensity.

Our results show that the inclusion of the participation margin is crucial for the measurement of overall labour market slack and for a better understanding of inflation dynamics. Based on our estimates of the Phillips curve, we find that the participation margin accounts for about 30% of total slack in the labor market and that the estimated reaction of inflation to the participation gap is twice as large as the corresponding one on the unemployment gap. Moreover, taking into account the participation margin improves inflation forecasts and contributes to explain the recent low inflation regime.

By analyzing the effects of the pension reform, we find that the large increase in participation (over 2 p.p.) of older individuals, determined by the change in the statutory retirement age, was not accompanied by any change in structural activity in younger age classes or on structural unemployment. Overall, the natural unemployment rate was

unaffected, while the natural participation rate increased by 0.7 p.p., determining a 1.1% increase in potential employment. Failing to account for changes in natural participation would have thus led to underestimate the degree of economic slack after the reform and produced a positive bias in inflation forecasts. Taken together, these results show that the inclusion of the labor force participation margin in the standard Phillips curve model helps explaining price dynamics. Our estimates indicate that, for a given unemployment gap, deviations of the participation rate from its natural level do have an impact on inflation. Going forward, investigating the structural mechanisms through which the participation gap generates price pressures looks like a promising avenue for future research.

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## Tables and Figures

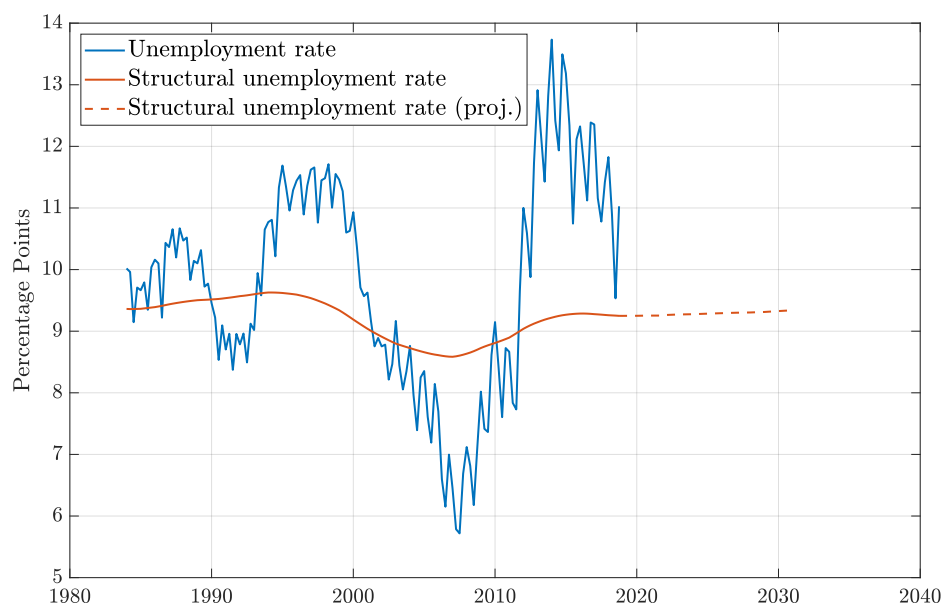


Figure 1: Aggregate Structural Unemployment Rate.

**Note:** The figure plots the quarterly unemployment rate of the population 15–64 (blue line) and the estimated trend unemployment rate (red line) for the years between 1984q1-2018q4. The dashed line represents the projection of the trend unemployment rate until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the the same dynamics observed between 2015 and 2018.



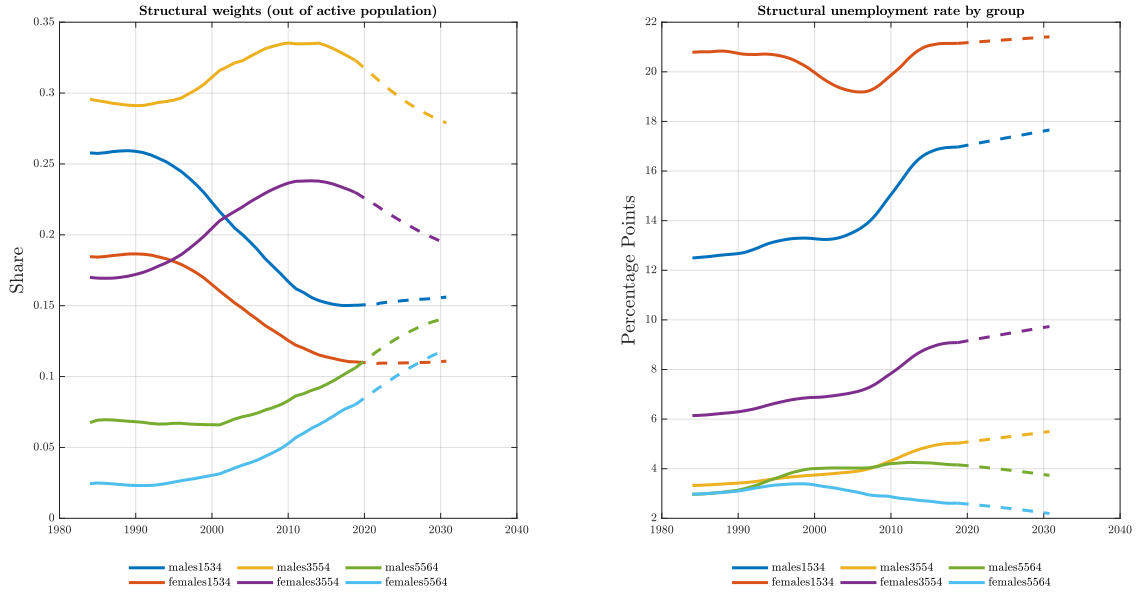


Figure 2: Determinants of Aggregate Structural Unemployment.

**Note:** The left panel plots the weights  $\bar{\omega}_{g,t}^u$  of each demographic group  $g$ . These weights are equal to the product between the weight in the population of each group  $g$  ( $\omega_{g,t}^p$ ) and the ratio between the group-specific and the aggregate trend participation rate ( $\frac{\bar{p}_{g,t}}{\bar{p}_t}$ ). The right panel displays the trend unemployment rate of each subgroup  $g$ . The dashed lines represent projections until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the same dynamics observed between 2015 and 2018.

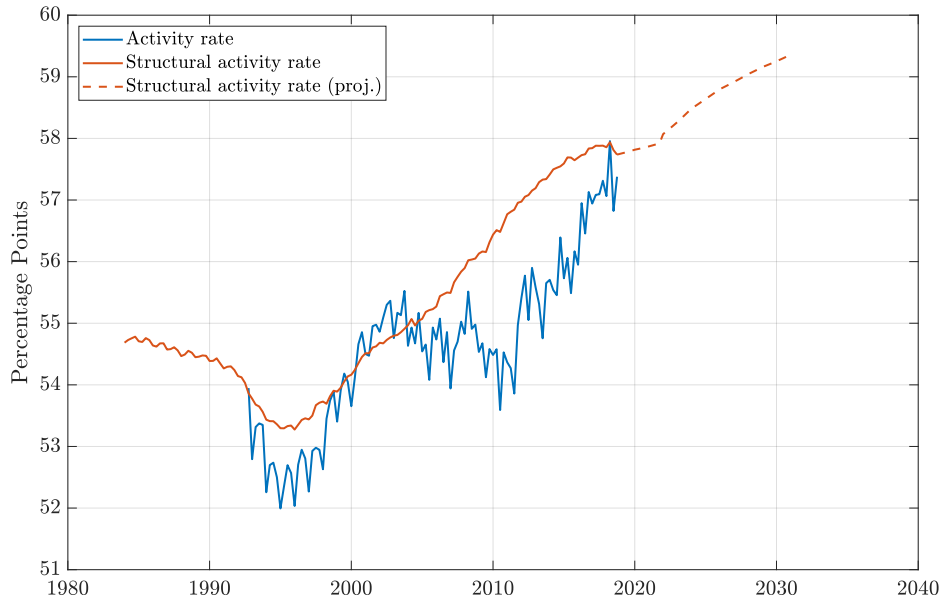


Figure 3: Aggregate Structural Participation Rate.

**Note:** The figure plots the quarterly activity rate of the population 15–64 (blue line) and the estimated trend activity rate (red line) for the years between 1984q1-2018q4. The dashed line represents the projection of the trend activity rate until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the same dynamics observed between 2015 and 2018.

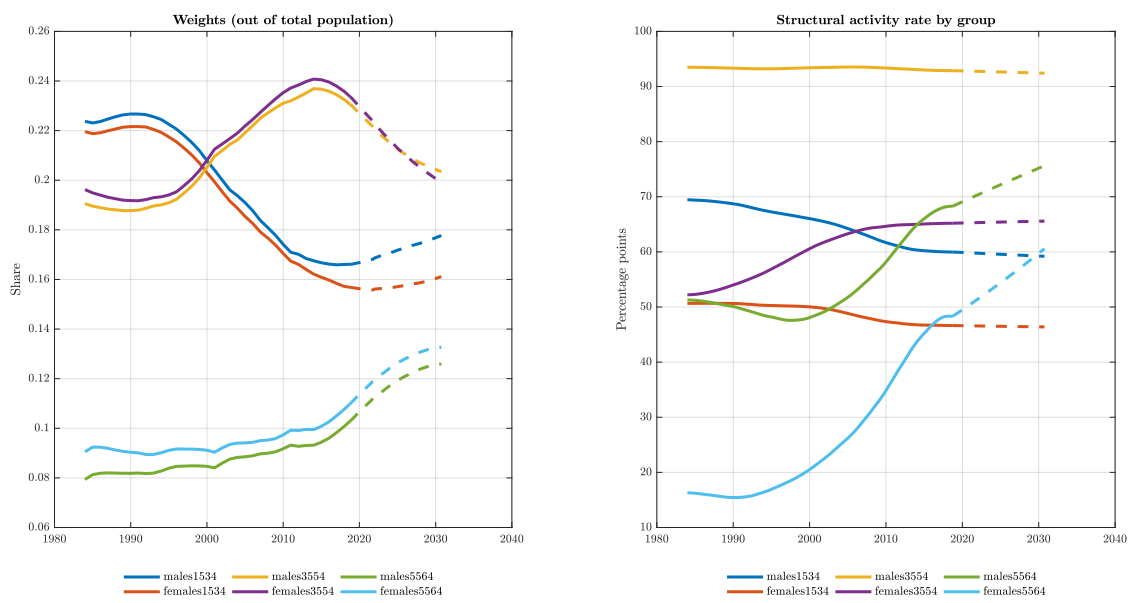


Figure 4: Determinants of Aggregate Structural Participation.

**Note:** The left panel plots the population weights  $\omega_{g,t}^p$  of each demographic group  $g$ . The right panel displays the trend activity rate of each subgroup  $g$ . The dashed lines represent projections until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the same dynamics observed between 2015 and 2018.

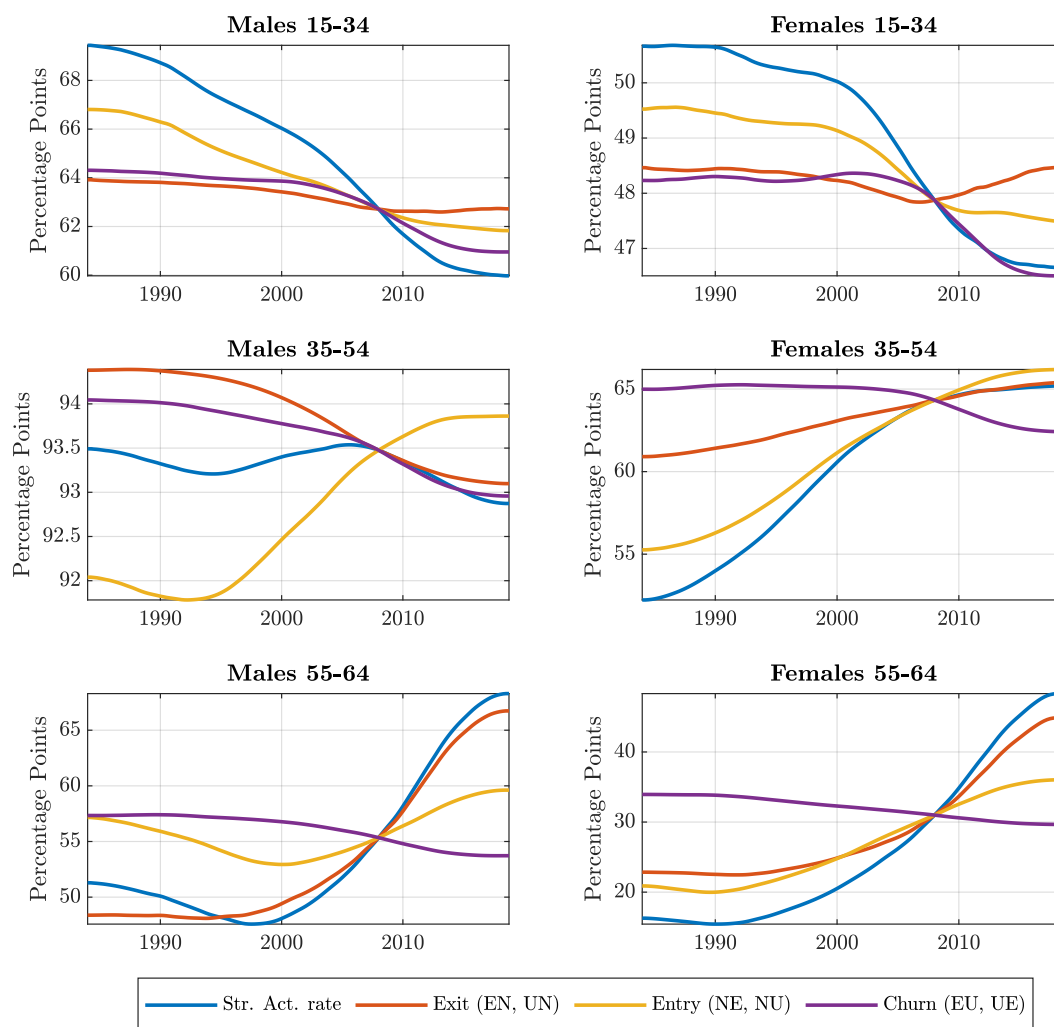


Figure 5: Decomposition of group-specific structural activity rates.

**Note:** The figure plots the group-specific estimated trend activity rates (blue line) for the years between 1984q1-2018q4, and the counterfactual series in which we let vary only a subset of flows at the time (exit: EN, UN; entry: NE, NU; churn: EU, UE), fixing the other flows at their level in 2009q1.

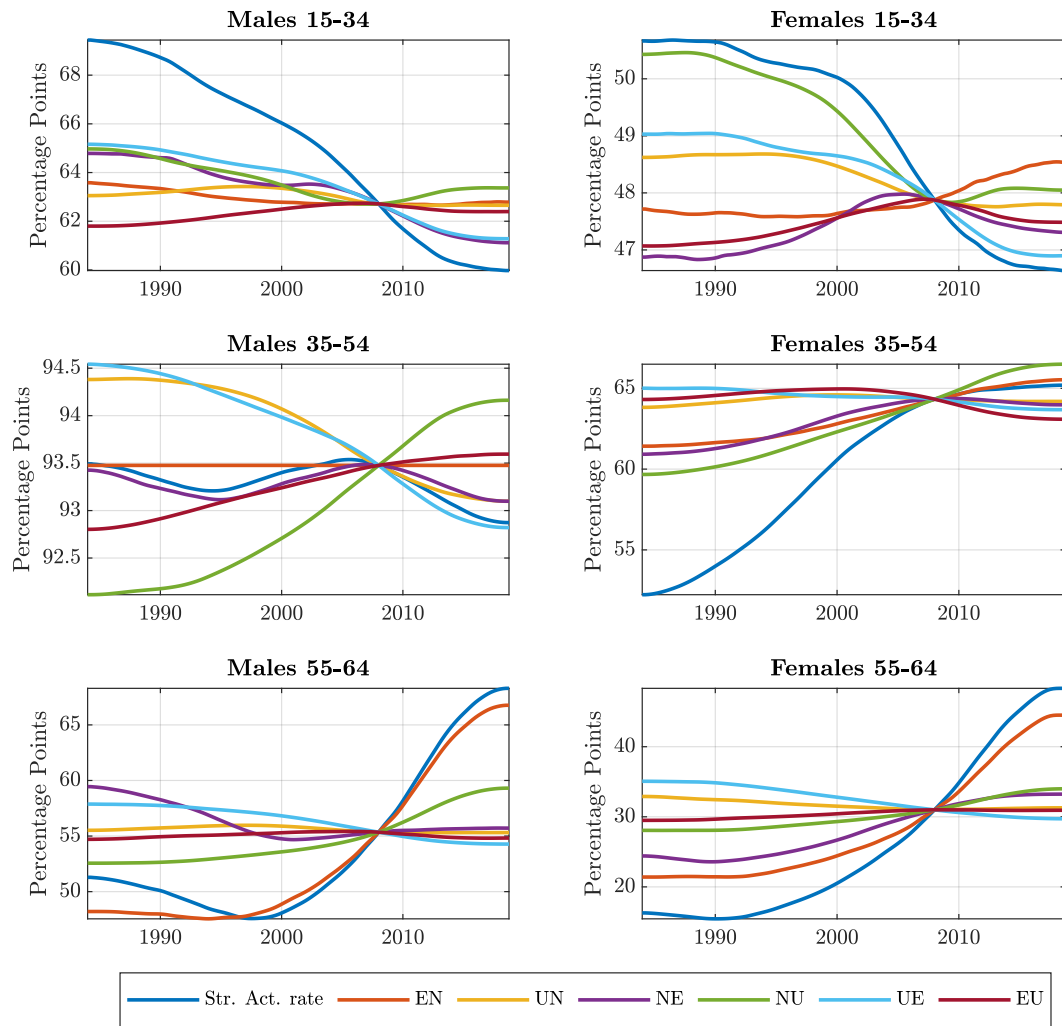


Figure 6: Decomposition of group-specific structural activity rates.

**Note:** The figure plots the group-specific estimated trend activity rates (blue line) for the years between 1984q1-2018q4, and the counterfactual series in which we let vary only a flow at the time, fixing the other flows at their level in 2009q1.

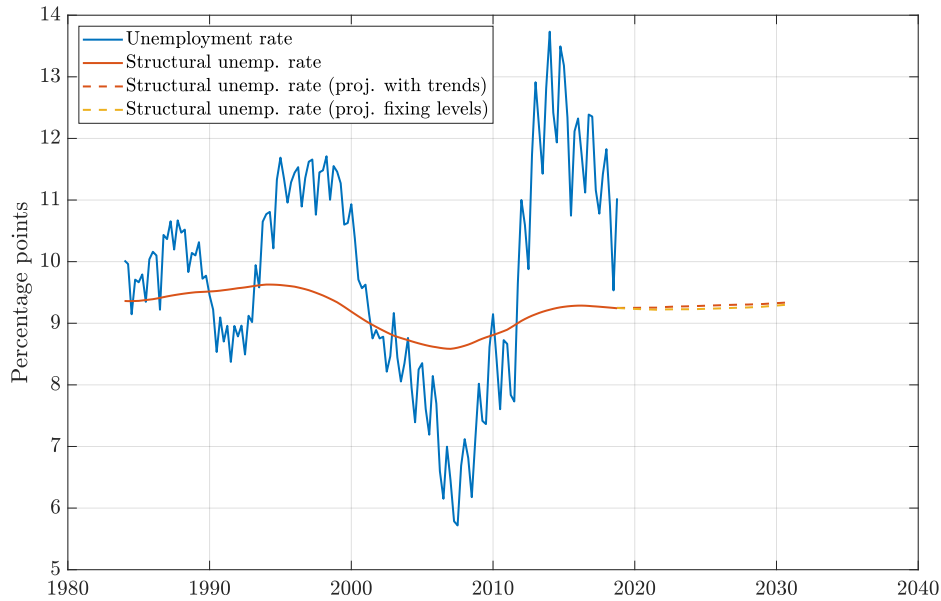


Figure 7: Aggregate Structural Unemployment Rate, alternative projections.

**Note:** The figure plots the quarterly unemployment rate of the population 15–64 (blue line) and the estimated trend unemployment rate (red line) for the years between 1984q1-2018q4. The dashed line represents the projection of the trend unemployment rate until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the same dynamics observed between 2015 and 2018 (red line), or that they will remain constant at their level in 2018 (yellow line).

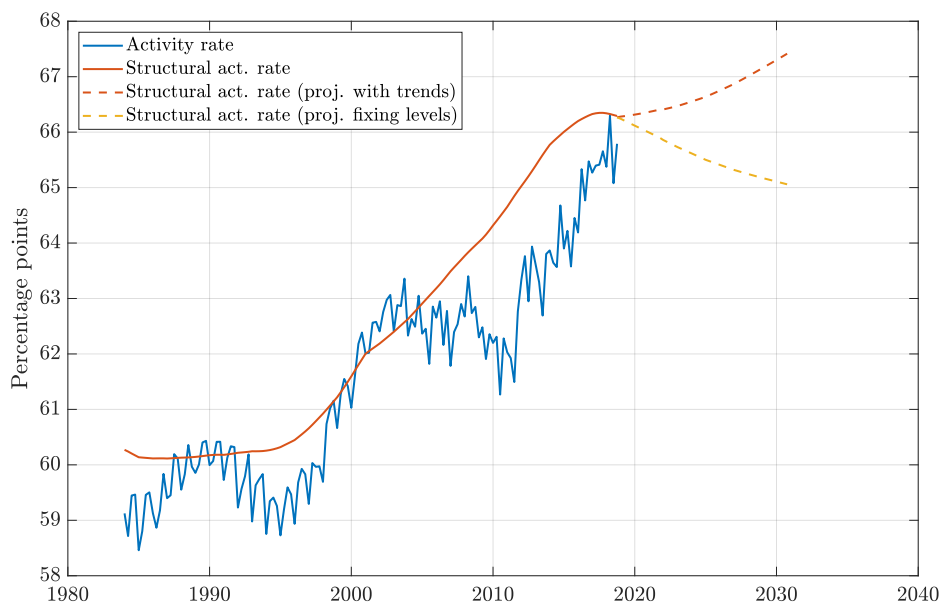


Figure 8: Aggregate Structural Participation Rate, alternative projections.

**Note:** The figure plots the quarterly activity rate of the population 15–64 (blue line) and the estimated trend activity rate (red line) for the years between 1984q1-2018q4. The dashed lines represent projections of the trend activity rate until 2030, based on Eurostat demographic projections and assuming that the long term trends in labor market flows will follow the same dynamics observed between 2015 and 2018 (red line), or that they will remain constant at their level in 2018 (yellow line).

Table 1: Parameter estimates

	Prior			Posterior (UGAP + PGAP)			Posterior (UGAP only)		
	Dist.	Mean	Std	Median	5%	95%	Median	5%	95%
$a_{x^u,1}$	Gamma	1.25	0.200	1.63	1.48	1.72	1.71	1.53	1.79
$a_{x^u,2}$	Normal	0.000	1.00	-0.669	-0.754	-0.517	-0.740	-0.823	-0.552
$\theta_w^u$	Beta	0.750	0.100	0.877	0.784	0.921	0.904	0.834	0.940
$\gamma$	Beta	0.500	0.265	0.255	0.162	0.335	0.250	0.169	0.347
$\rho_{z^u}$	Beta	0.950	0.035	0.956	0.902	0.988	0.959	0.895	0.987
$\rho_a$	Beta	0.500	0.200	0.731	0.655	0.813	0.544	0.445	0.621
$\sigma_{x^u}^2$	InvGamma	0.050	0.000	0.020	0.012	0.028	0.027	0.011	0.056
$\sigma_\zeta^2$	InvGamma	1.00	0.000	0.049	0.027	0.083	0.172	0.134	0.227
$\sigma_{\pi^*}^2$	InvGamma	0.112	0.000	0.094	0.045	0.155	0.035	0.022	0.056
$\sigma_{z^u,\varsigma}$	InvGamma	0.150	0.050	0.230	0.127	0.383	0.322	0.175	0.459
$g_w$	Normal	0.400	0.050	0.377	0.315	0.456	0.376	0.317	0.438
$\kappa^u$	–	–	–	0.018	0.007	0.061	0.011	0.004	0.035
$K^u$	–	–	–	0.176	0.095	0.327	0.112	0.061	0.182
$a_{x^p,1}$	Gamma	1.25	0.200	1.34	1.15	1.45	–	–	–
$a_{x^p,2}$	Normal	0.000	1.00	-0.528	-0.651	-0.350	–	–	–
$\theta_w^p$	Beta	0.750	0.100	0.626	0.498	0.901	–	–	–
$\rho_{z^p}$	Beta	0.950	0.035	0.978	0.953	0.992	–	–	–
$\sigma_{x^p}^2$	InvGamma	0.050	0.000	0.050	0.035	0.068	–	–	–
$\sigma_{z^p,\varsigma}^p$	InvGamma	0.150	0.050	0.570	0.211	1.13	–	–	–
$\kappa^p$	–	–	–	0.226	0.012	0.511	–	–	–
$K^p$	–	–	–	0.456	0.127	0.791	–	–	–

**Note:** The table displays the prior and the posterior estimates of the model parameters under two specifications: a Phillips curve that includes both the unemployment and the participation gap (UGAP+PGAP) and a standard Phillips curve that only includes the unemployment gap (UGAP).  $\kappa^u$  is derived from  $\theta_w^u$  as follows:  $\kappa^u = (1 - \theta_w^u)(1 - \beta\theta_w^u) / \theta_w^u$ . An analogous relation holds between  $\kappa^p$  and  $\theta_w^p$ .  $K^u$  and  $K^p$  denote the overall coefficients on current and lagged unemployment and participation gaps, respectively:  $K^u = \kappa^u(\omega_{\pi,1}^u + \omega_{\pi,2}^u)$ ,  $K^p = \kappa^p(\omega_{\pi,1}^p + \omega_{\pi,2}^p)$  (see equation (B.8) in the Appendix).

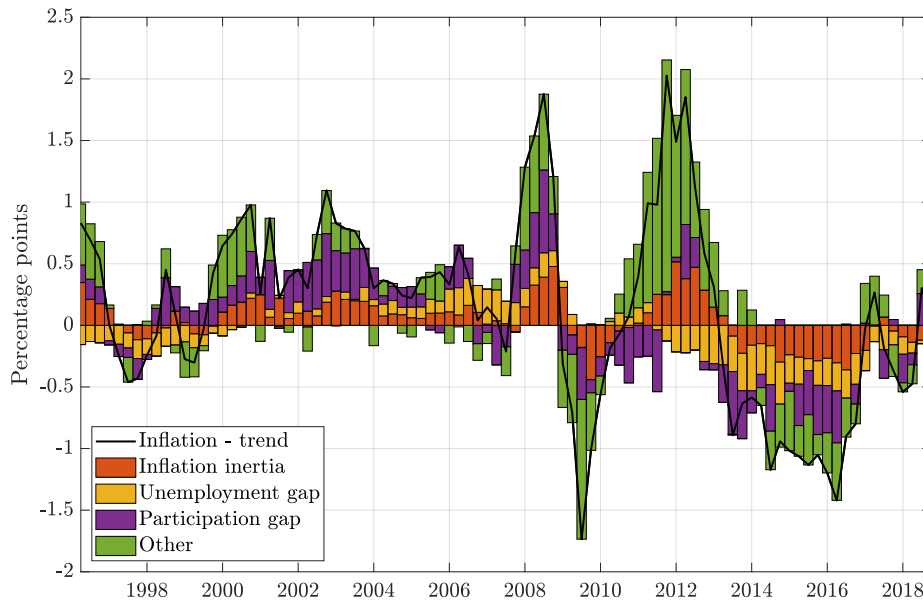


Figure 9: Historical decomposition of the inflation gap

**Note:** The solid line represents the historical evolution of median inflation gap (realized inflation - estimated trend inflation) and the colored bars the median contributions of the factors included in the baseline Phillips curve model. The model is estimated over the period 1996Q1–2018Q4.

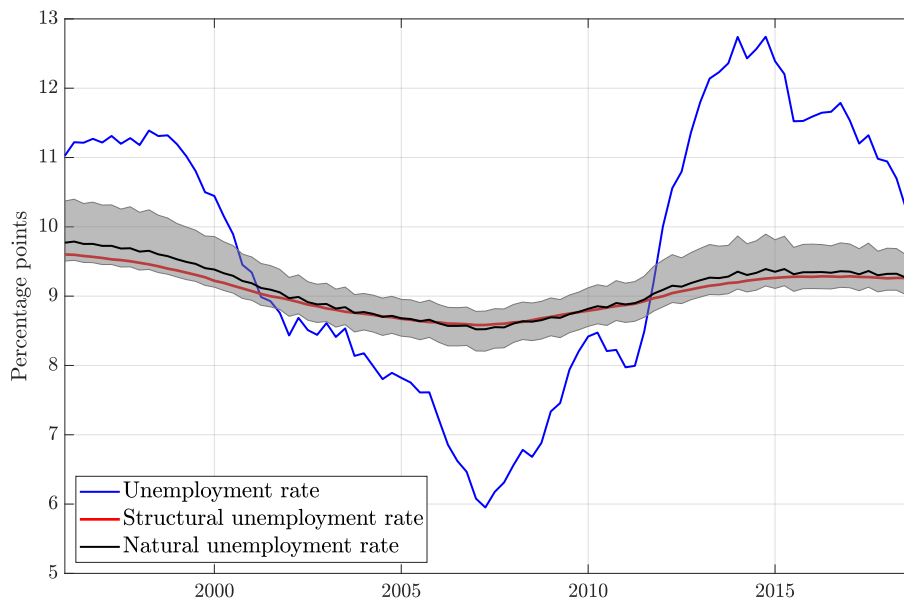


Figure 10:  $u^*$  estimated through the augmented Phillips curve model (UGAP + PGAP)

**Note:** Shading denotes the 68% coverage interval. The model is estimated over the period 1996Q1–2018Q4.

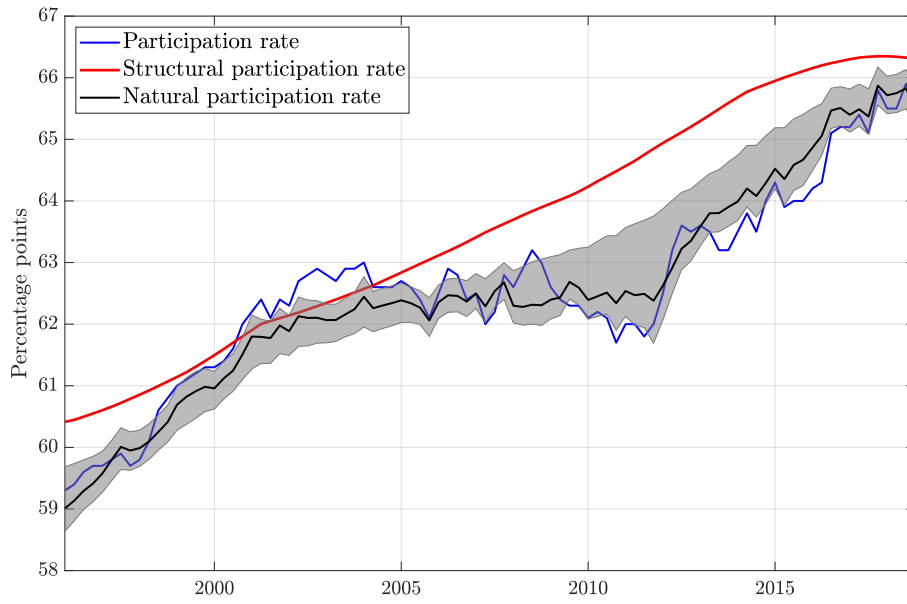


Figure 11:  $p^*$  estimated through the augmented Phillips curve model (UGAP + PGAP)

**Note:** Shading denotes the 68% coverage interval. The model is estimated over the period 1996Q1–2018Q4.

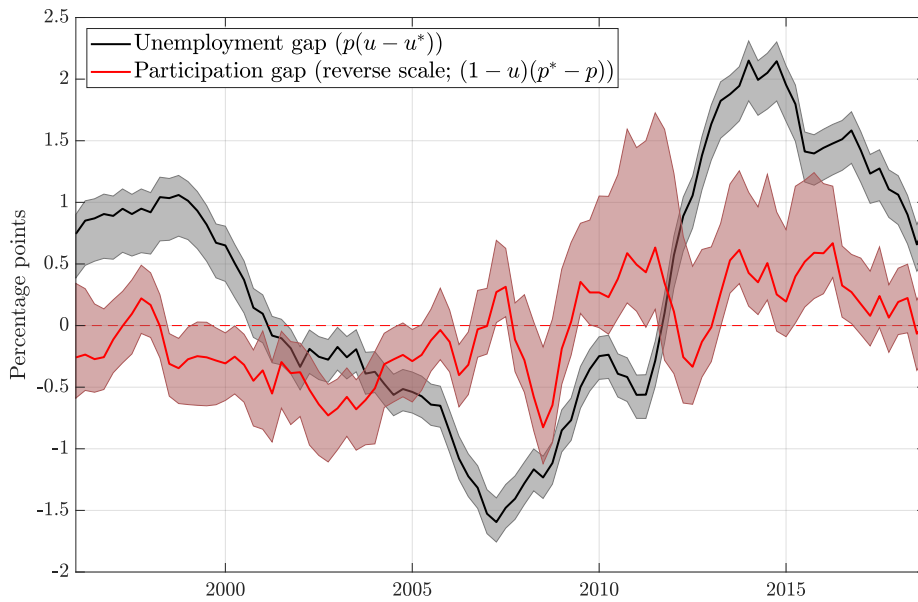


Figure 12: Unemployment and participation gaps estimated through the augmented Phillips curve model

**Note:** Shading denotes the 68% coverage interval. The model is estimated over the period 1996Q1–2018Q4. Positive unemployment and participation gaps denote a slack labor market.



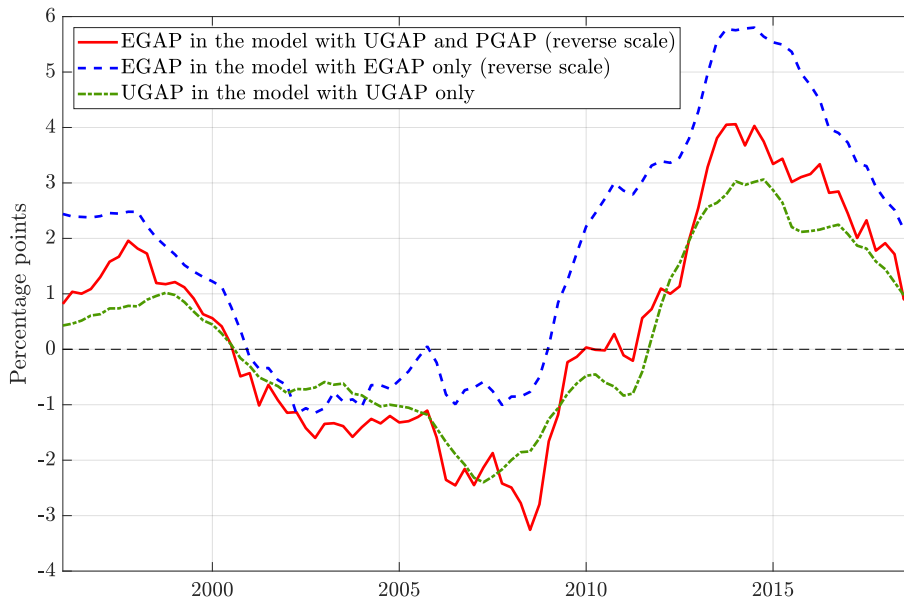


Figure 13: Employment and unemployment gap estimates in different models

**Note:** The red solid line represents the median employment gap estimated through the baseline augmented Phillips curve including both the unemployment and the participation gap. The blue dashed line refers to the model including only a combined employment gap. The green line represents the median unemployment gap obtained through a Phillips curve including only the unemployment gap. The employment gaps in the first and second models are represented on a reverse scale (positive values indicate a slack labor market) and are divided by the actual participation rate to make them comparable with the UGAP derived from the third model.

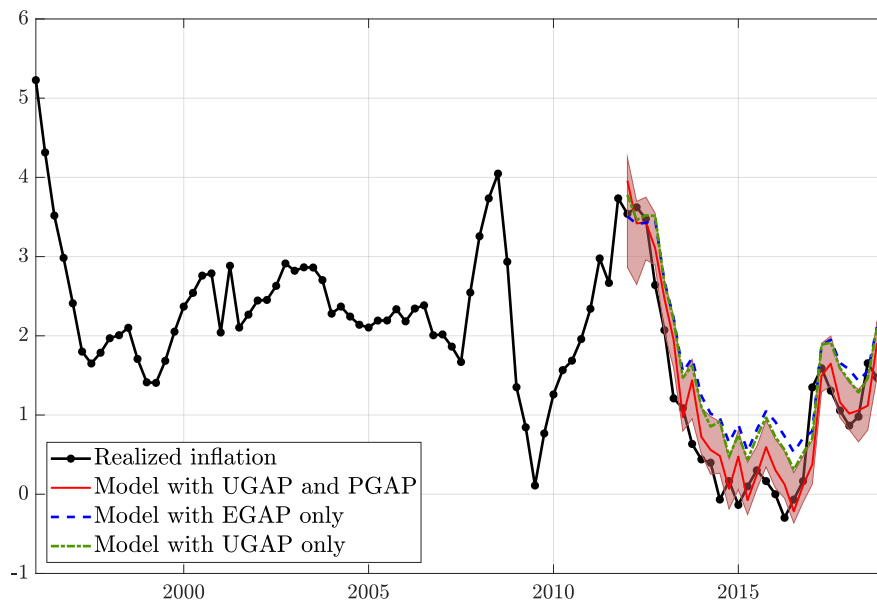


Figure 14: One quarter-ahead inflation forecasts according to different models

**Note:** The red solid red line represents the median inflation forecasts obtained from the baseline Phillips curve including both the unemployment and the participation gap; shading denotes the 68% coverage. The dashed blue line refers to the model including only a combined employment gap. The green line represents the median inflation forecasts obtained through a Phillips curve including only the unemployment gap. The estimation period is 1996Q1-2011Q4 and the forecasting period runs from 2012Q1 until 2018Q4; all the models are estimated recursively.

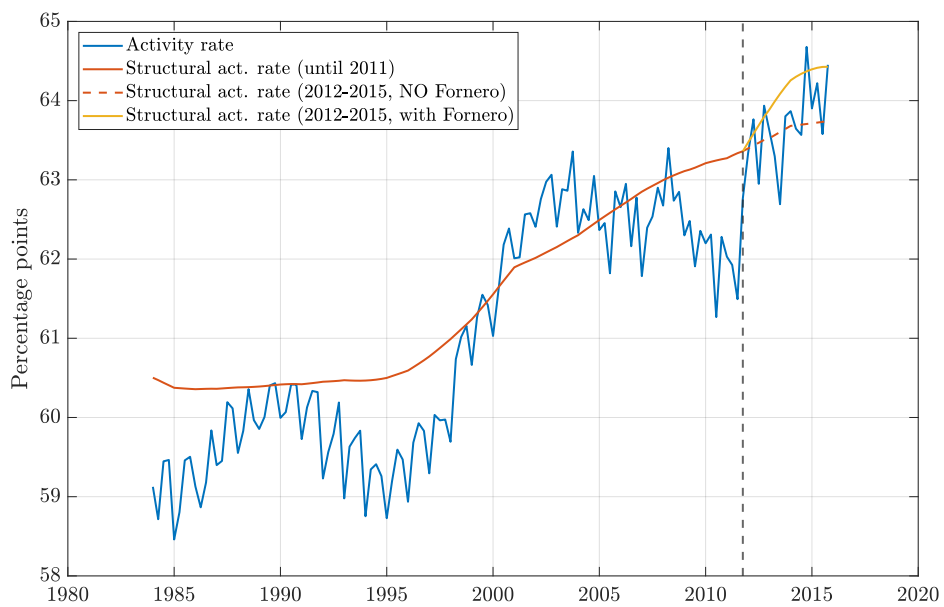


Figure 15: Effect of the Fornero reform on Structural Activity.

**Note:** The figure plots the quarterly activity rate of the population 15–64 (blue line), the estimated trend activity rate without (red line) and with the reform (yellow line), for the years between 1984q1–2015q4 (see Section 5). The dashed red line represents projections of the trend activity rate until 2015, on the basis of the estimates available in 2011. The red vertical line represents 2011q4, the end of the period prior to the reform.

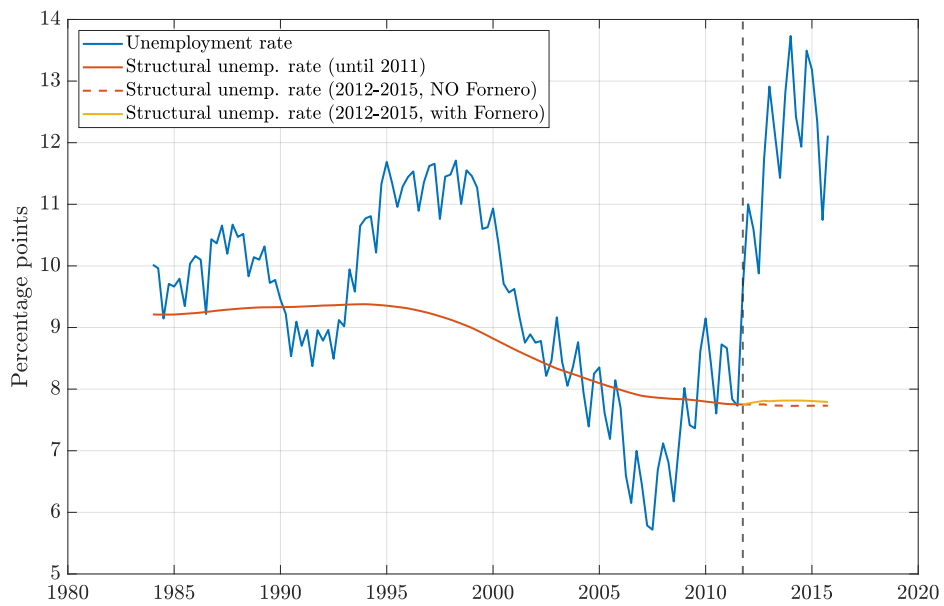


Figure 16: Effect of the Fornero reform on Structural Unemployment.

**Note:** The figure plots the quarterly unemployment rate of the population 15–64 (blue line), the estimated trend unemployment rate without (red line) and with the reform (yellow line), for the years between 1984q1-2015q4 (see Section 5). The dashed red line represents projections of the trend unemployment rate until 2015, on the basis of the estimates available in 2011. The red vertical line represents 2011q4, the end of the period prior to the reform.

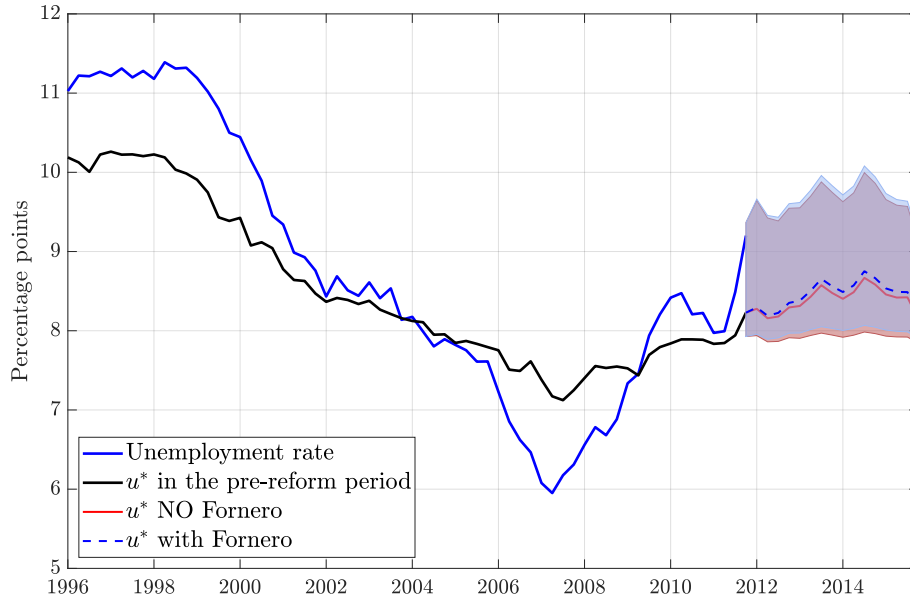


Figure 17: Effect of the Fornero reform on  $u^*$

**Note:** The figure plots the quarterly unemployment rate (blue line), the estimated natural unemployment rate ( $u^*$ ) in the pre-reform period (1996Q1-2011Q4) and the projected  $u^*$  in the post-reform period (2012Q1:2015Q4) conditional on trend unemployment rate without the reform (red line) and with the reform (blue-dashed line); see Section 5. Shading denotes the 68% coverage interval.

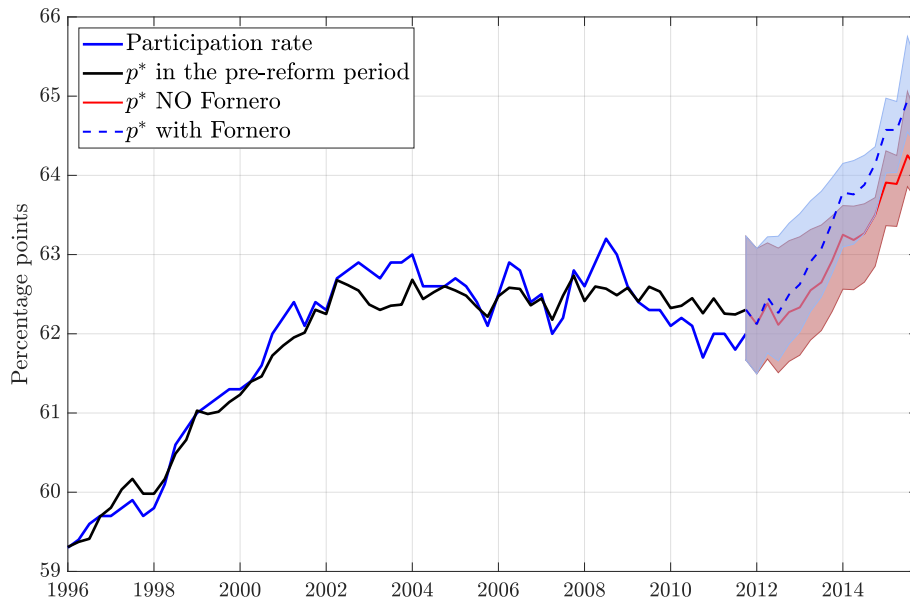


Figure 18: Effect of the Fornero reform on  $p^*$

**Note:** The figure plots the quarterly participation rate (blue line), the estimated natural participation rate ( $p^*$ ) in the pre-reform period (1996Q1-2011Q4) and the projected  $p^*$  in the post-reform period (2012Q1:2015Q4) conditional on trend unemployment rate without the reform (red line) and with the reform (blue-dashed line); see Section 5. Shading denotes the 68% coverage interval.

# A The estimation of structural unemployment and activity rates

## Estimating the group-specific labor market transition rates

We define six demographic groups along three age classes (15–34, 35–54 and 55–64) and gender. In a given period an individual can either be employed, unemployed or inactive. We define as  $\lambda_{g,t}^{XY}$  the probability that an individual, who belongs to demographic group  $g$ , is observed in status  $X$  at time  $t$  and is then observed in status  $Y$  at time  $t + 1$ .

Hence, for each group  $g$ , we construct quarterly time series of the six transition probabilities:

$$\{\lambda_{g,t}^{NU}, \lambda_{g,t}^{NE}, \lambda_{g,t}^{EU}, \lambda_{g,t}^{EN}, \lambda_{g,t}^{UE}, \lambda_{g,t}^{UN}\}_{t=1984,q1}^{2018q4},$$

where  $E$  stands for employment,  $U$  for unemployment and  $N$  for inactivity.

From 2004 onward, the construction of these series relies on panel data from the Labor Force Survey, which allows us to observe the same individual for consecutive quarters. In this way, we are able to directly construct transition probabilities by using the multiple individual observations. For the period prior to 2004, when the panel component is not available, we are able to extend the series back until 1984 relying on: (i) the observed changes in the relevant stocks of unemployment  $U_{g,t}$ , employment  $E_{g,t}$ , inactivity  $N_{g,t}$ ; (ii) the changes in the stocks of short-term unemployment  $U_{g,t}^{st}$  constructed using the retrospective question on unemployment duration, as in Shimer [2012]; (iii) the changes in the stock of short-term employment  $E_{g,t}^{st}$ , constructed using the retrospective question on employment duration; and (iv) the flows from unemployment towards inactivity, estimated exploiting a question on the reason for being inactive to elicit information on the nature of the transition towards inactivity.<sup>24</sup>

Following the existing literature (for instance Shimer, 2012, Elsby et al., 2015 and Barnichon and Mesters, 2018), We perform two important adjustments to the measured flows between the employment, unemployment and inactivity.

First, we make them consistent with the dynamics of the stocks (*margin error correction - MEC*). The inconsistency between transition probabilities estimated through individual-level data and the observed dynamics of the stocks is a well-known issue and may be due to a variety of reasons, mainly related to the turnover of individuals in the sample (for instance, people leaving the sample because of migration, death or not-responding or, conversely, new people entering the sample). In this respect we closely follow the correction proposed by Elsby et al. [2015] and Barnichon and Mesters [2018]. Their proposed solution consists in finding the minimum adjustment to flows which makes them fully consistent with observed changes in the stocks. They show that this adjustment takes a very simple analytical form (see Appendix A.2 of Elsby et al. [2015] for details).

Second, we derive continuous-time flow rates from discrete time transition probabilities, in order to account for the possibility of multiple transitions taking place within the

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<sup>24</sup>Notice that flows between inactivity and unemployment should be taken with caution, given that the distinction between these two labor market states can be difficult for some groups of workers [Brandolini et al., 2006].

observation window (*temporal aggregation correction* - TAC). Due to the discrete time nature (quarterly, in our case) of our observations, the gross flows are in fact a series of snapshots of an individual's labor market status. Upon observing an individual in state  $X$  at time  $t$  and state  $Y$  at time  $t + 1$ , we classify that as a movement from  $X$  to  $Y$  in our transition matrix. In practice, that individual may have taken multiple transitions within the period, or could have transited to  $Y$  through state  $Z$ . This means that the gross transition probabilities provide estimates which may be not accurate of the actual underlying flows. By construction, this procedure misses some transitions and incorrectly includes others. A solution to this problem has been found by Shimer [2012], who shows that it is possible to back out the underlying flows in continuous time using a simple eigendecomposition of the Markov transition matrix in discrete time (see Appendix A.3 of Elsby et al. [2015] for details). By applying the TAC, we get the underlying hazard rates for each group  $g$ :

$$\{f_{g,t}^{NU}, f_{g,t}^{NE}, f_{g,t}^{EU}, f_{g,t}^{EN}, f_{g,t}^{UE}, f_{g,t}^{UN}\}_{t=1984,q1}^{2018q4},$$

where  $f_{g,t}^{XY}$  is the corresponding hazard rate, derived from the discrete-time probability  $\lambda_{g,t}^{XY}$ .

### Separating the trend from the cyclical component of the transition rates

Following Tasci [2012], we use an unobserved component model in which real log GDP  $y_t$  and the six labor market flows can be decomposed into a stochastic trend and a stationary cyclical component.<sup>25</sup> These components are not observed by the econometrician. Our model reads as follows:

$$y_t = \bar{y}_t + y_t^c,$$

$$f_{g,t}^j = \bar{f}_{g,t}^j + f_{g,t}^{j,c},$$

with the latter equation holding for each flow  $j \in \{NU, NE, EU, EN, UE, UN\}$  and each group  $g$ . The trend components are denoted by  $\bar{y}_t$  and  $\bar{f}_{g,t}^j$ , while  $y_t^c$  and  $f_{g,t}^{j,c}$  capture the cyclical movements. In turn, the stochastic trends are assumed to follow a random walk:

$$\bar{y}_t = \bar{y}_{t-1} + \epsilon_t^y,$$

$$\bar{f}_{g,t}^j = \bar{f}_{g,t-1}^j + \epsilon_{g,t}^j.$$

Instead, the cyclical component of log GDP follows an AR(2) process and affects the cyclical component of the flows as follows:

$$y_t^c = \phi_1 y_{t-1}^c + \phi_2 y_{t-2}^c + \epsilon_t^{y^c},$$

$$f_{g,t}^{j,c} = \rho_{1,g}^j y_t^c + \rho_{2,g}^j y_{t-1}^c + \epsilon_{g,t}^{j,c}.$$

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<sup>25</sup>We also replicate the estimation of trends using a standard HP(1600) filter. Results are compared to those obtained in the baseline exercise in Figure C.13. Relative to our baseline structural rates, those obtained via HP filter appear excessively volatile, following closely the actual series.

All error terms are assumed to be drawn from Normal distributions with zero mean. Notice that all trend flows are jointly estimated but we do not allow for interactions between them to avoid overparameterization. For the same reason, we estimate the trends separately for each group  $g$ .

Fixing a group  $g$ , the statistical model to be estimated can be written in state space form. The measurement equation is given by:

$$\begin{bmatrix} y_t \\ f_{g,t}^{NU} \\ f_{g,t}^{NE} \\ f_{g,t}^{EU} \\ f_{g,t}^{EN} \\ f_{g,t}^{UE} \\ f_{g,t}^{UN} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_1^{NU} & \rho_2^{NU} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_1^{NE} & \rho_2^{NE} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \rho_1^{EU} & \rho_2^{EU} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \rho_1^{EN} & \rho_2^{EN} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \rho_1^{UE} & \rho_2^{UE} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \rho_1^{UN} & \rho_2^{UN} & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ y_t^c \\ y_{t-1}^c \\ \bar{f}_{g,t}^{NU} \\ \bar{f}_{g,t}^{NE} \\ \bar{f}_{g,t}^{EU} \\ \bar{f}_{g,t}^{EN} \\ \bar{f}_{g,t}^{UE} \\ \bar{f}_{g,t}^{UN} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{g,t}^{NUc} \\ \epsilon_{g,t}^{NEc} \\ \epsilon_{g,t}^{EUc} \\ \epsilon_{g,t}^{ENc} \\ \epsilon_{g,t}^{UEc} \\ \epsilon_{g,t}^{UNc} \end{bmatrix}. \quad (\text{A.1})$$

Instead, the transition equation reads as follows:

$$\begin{bmatrix} \bar{y}_t \\ y_t^c \\ y_{t-1}^c \\ \bar{f}_{g,t}^{NU} \\ \bar{f}_{g,t}^{NE} \\ \bar{f}_{g,t}^{EU} \\ \bar{f}_{g,t}^{EN} \\ \bar{f}_{g,t}^{UE} \\ \bar{f}_{g,t}^{UN} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ y_{t-1}^c \\ y_{t-2}^c \\ \bar{f}_{g,t-1}^{NU} \\ \bar{f}_{g,t-1}^{NE} \\ \bar{f}_{g,t-1}^{EU} \\ \bar{f}_{g,t-1}^{EN} \\ \bar{f}_{g,t-1}^{UE} \\ \bar{f}_{g,t-1}^{UN} \end{bmatrix} + \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^{yc} \\ 0 \\ \epsilon_{g,t}^{NU} \\ \epsilon_{g,t}^{NE} \\ \epsilon_{g,t}^{EU} \\ \epsilon_{g,t}^{EN} \\ \epsilon_{g,t}^{UE} \\ \epsilon_{g,t}^{UN} \end{bmatrix}. \quad (\text{A.2})$$

The unobserved components, along with the variance of the error terms, are estimated with the Kalman filter via Maximum Likelihood.<sup>26</sup> The filtered rates, as well as the raw time series of the hazard rates constructed in step 1, can be found in the Appendix (Figure C.15 to C.20).

The outcome of this second step are the structural components of the flow rates, which represent the building blocks of the structural unemployment and activity rate computed as carefully described in Section 3.1 of the paper.

## Projections of the structural rates to 2030

We project our rates in the future to assess what may be the implications of the changing demographic structure of the population for the aggregate structural rates (as in Section

<sup>26</sup>We impose the following assumptions on the ratio between the variance of the trend and cyclical components:  $\sigma^y = \frac{\sigma_{\epsilon^y}}{\sigma_{\epsilon^{yc}}} = 0.1$ ;  $\sigma^{EN} = \frac{\sigma_{\epsilon^{EN}}}{\sigma_{\epsilon^{ENc}}} = 0.0045$ ,  $\sigma^j = \frac{\sigma_{\epsilon^j}}{\sigma_{\epsilon^{jc}}} = 0.001$  for the other five flows, for all demographic groups. In a robustness check, we find that the aggregate estimates are fundamentally robust to deviations from these baseline values (see Figure C.14 in the Appendix).



3.2). Moreover, in Section 5, we use the projections based on the pre-2012 period only to assess what would have been the evolution of the structural rates in the absence of the far reaching pension reform implemented in 2012 (Fornero reform).

In this section we describe with more details how we computed out two scenarios for the projection of each trend  $\bar{f}_{g,t}^{XY}$  (for each demographic group  $g$ ).

**Baseline projections.** A good starting point is to assume that group-specific trends in the structural flows will continue in the next years according to similar dynamics as those registered in the last years. For the sake of concreteness, we estimate trends of the group-specific flows in the square root of time<sup>27</sup> for the period 2015–2018, and use these coefficients to project the flows in the period 2019–2030. We then compute the aggregate projected structural unemployment (activity) rate as a weighted average of the projected structural unemployment (activity) rates of each subgroup.

**Alternative projections.** We also calculate alternative projections under the hypothesis that the structural flow rates will remain at their estimated level in 2018, instead of changing according to past dynamics. This alternative scenario may be interpreted as one in which a sudden reversal in labor market policy (concerning for instance pension reforms, especially for the participation rate) totally halts the trends observed in the last years.

## B Derivation of the natural unemployment and participation rates

Let us start by considering the standard price Phillips curve derived from the simple model in [Crump et al., 2019], which only depends on the unemployment gap:

$$\pi_t = -\kappa^u x_t^u + \beta \mathbb{E}_t \pi_{t+1} + \beta \mathbb{E}_t (\Delta a_{t+1} - \Delta a_t), \quad (\text{B.1})$$

where the exogenous process  $a_t$  capture (log) productivity and mark-up shocks to firms. Assuming rational expectations, we can iterate forward equation (B) to obtain:

$$\pi_t = -\kappa^u \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u + \beta \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (\Delta a_{T+1} - \Delta a_T) \quad (\text{B.2})$$

The process  $\Delta a_t$  is assumed to be AR(1):  $\Delta a_t = \rho_a \Delta a_{t-1} + \sigma_a \varepsilon_t^a$ . Hence,

$$\mathbb{E}_t (\Delta a_{t+1} - \Delta a_t) = -(1 - \rho_a) \Delta a_t$$

Replacing the previous expression in equation (B.3), we obtain:

$$\pi_t = -\kappa^u \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u - \beta(1 - \rho_a) \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Delta a_T \quad (\text{B.3})$$

---

<sup>27</sup>This is done in order to induce some degree of concavity in the future dynamics (to avoid explosive trends). Moreover, in order to avoid that the hazard rates  $\bar{f}_{g,t}^{EN}$  attain unrealistically low levels, we set a lower bound to 0.01 in 2018q4, reaching 0.008 in 2030q3.

By considering that  $\mathbb{E}_t \Delta a_T = \rho_a^{T-t} \Delta a_t$  and  $\sum_{T=t}^{\infty} (\beta \rho_a)^{T-t} = \frac{1}{1-\beta \rho_a}$ , we get:

$$\pi_t = -\kappa^u \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u - \beta \frac{1-\rho_a}{1-\beta \rho_a} \Delta a_t \quad (\text{B.4})$$

Let us further consider the existence of an exogenous inflation trend  $\pi_t^*$  which follows a RW:

$$\pi_t^* = \pi_{t-1}^* + \sigma_{\pi^*} \epsilon_t^{\pi^*} \quad (\text{B.5})$$

We assume that equation (B.4) holds for the inflation gap, that is the deviation between realized inflation  $\pi_t$  and  $\pi_t^*$ . Furthermore, the inflation gap displays some inertia, captured by the parameter  $\gamma$ :

$$\pi_t - \pi_t^* = \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u - \beta \frac{1-\rho_a}{1-\beta \rho_a} \Delta a_t \quad (\text{B.6})$$

By augmenting the model to take into account the effects of participation rate on inflation we finally get equation (8) in the main text:

$$\pi_t - \pi_t^* = \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^u + \kappa^p \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} x_T^p - \beta \frac{1-\rho_a}{1-\beta \rho_a} \Delta a_t \quad (\text{B.7})$$

We further assume that the the unemployment and the participation gaps follow an AR(2) process:

$$\begin{aligned} x_t^u &= a_{x^u,1} x_{t-1}^u + a_{x^u,2} x_{t-2}^u + \sigma_{x^u} \epsilon_t^{x^u} \\ x_t^p &= a_{x^p,1} x_{t-1}^p + a_{x^p,2} x_{t-2}^p + \sigma_{x^p} \epsilon_t^{x^p} \end{aligned}$$

and we define

$$\varsigma_t = -\beta \frac{1-\rho_a}{1-\beta \rho_a} \Delta a_t = \rho_a \varsigma_{t-1} + \sigma_{\varsigma} \epsilon_t^{\varsigma}$$

Hence we can rewrite equation (B.7) as:

$$\pi_t - \pi_t^* = \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u \omega_{\pi,1}^u x_t^u - \kappa^u \omega_{\pi,2}^u x_{t-1}^u + \kappa^p \omega_{\pi,1}^p x_t^p + \kappa^p \omega_{\pi,2}^p x_{t-1}^p + \rho_a \varsigma_{t-1} + \sigma_{\varsigma} \epsilon_t^{\varsigma} \quad (\text{B.8})$$

where  $\omega_{\pi,1}^u = (1 - \beta(a_{x^u,1} + \beta a_{x^u,2}))^{-1}$ ,  $\omega_{\pi,2}^u = \beta a_{x^u,2} \omega_{\pi,1}^u$ ,  $\omega_{\pi,1}^p = (1 - \beta(a_{x^p,1} + \beta a_{x^p,2}))^{-1}$  and  $\omega_{\pi,2}^p = \beta a_{x^p,2} \omega_{\pi,1}^p$ .

Theoretically, the relationship between price and wage inflation is:

$$\pi_t^w = \pi_t + \Delta a_t$$

Hence wage inflation can be expressed as:

$$\pi_t^w = \pi_t^* + \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u \omega_{\pi,1}^u x_t^u - \kappa^u \omega_{\pi,2}^u x_{t-1}^u + \kappa^p \omega_{\pi,1}^p x_t^p + \kappa^p \omega_{\pi,2}^p x_{t-1}^p - \frac{1-\beta}{\beta(1-\rho_a)} \varsigma_t$$

We further assume that wage inflation has its own specific growth rate  $g_w$ , so that the

final equation becomes:

$$\begin{aligned}\pi_t^w = & g_w + \pi_t^* + \gamma (\pi_{t-1} - \pi_{t-1}^*) - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} - \kappa^u \omega_{\pi,1}^u x_t^u - \kappa^u \omega_{\pi,2}^u x_{t-1}^u \\ & + \kappa^p \omega_{\pi,1}^p x_t^p + \kappa^p \omega_{\pi,2}^p x_{t-1}^p - \frac{1-\beta}{\beta(1-\rho_a)} \varsigma_t + oe_t^w\end{aligned}\quad (\text{B.9})$$

where  $oe_t^w$  denotes a normally distributed measurement error. Notice that, by plugging eq. (B.8) into (B.9) we obtain an alternative expression for wage inflation:

$$\pi_t^w = g_w + (\pi_t - \pi_t^*) + \pi_t^* - \frac{1-\beta\rho_a}{\beta(1-\rho_a)} \varsigma_t + oe_t^w$$

## B.1 State-space model

The model described above can be cast into a state-space form of the following type:

$$\mathbf{y}_t = \mathbf{M}_y \alpha_t + \mathbf{H} \mathbf{s}_t \quad (\text{B.10})$$

$$\mathbf{s}_t = \mathbf{F} \mathbf{s}_{t-1} + \mathbf{G} \epsilon_t \quad (\text{B.11})$$

where eq. (B.10) is the measurement equation and eq. (B.11) is the transition equation.

For simplicity, we start from the model including only price inflation.  $\mathbf{y}_t$  is a vector of  $n_y = 5$  elements collecting the observed variables:  $\mathbf{y}_t = [u_t, p_t, \pi_t, \mathbb{E}_t \pi_{t+2Q}, \mathbb{E}_t \pi_{t+6Q}]$ , that is the unemployment rate, the participation rate, inflation, and short and medium-term inflation expectations.  $\alpha_t$  is a vector of exogenous variables:  $\alpha_t = [\bar{u}_t, \bar{p}_t]$ .  $\mathbf{s}_t$  is a vector of  $n_s = 11$  elements collecting the state variables:  $\mathbf{s}_t = [\pi_t - \pi_t^*, x_t^u, x_{t-1}^u, x_t^p, x_{t-1}^p, \varsigma_t, \pi_t^*, z_t^u, z_t^p, oe^{6Q}, oe^{2Q}]$ .  $x_t^u$  denotes the unemployment gap ( $x_t^u = u_t - u_t^*$ ),  $x_t^p$  the participation gap ( $x_t^p = p_t - p_t^*$ ),  $\varsigma_t = -\beta \frac{1-\rho_a}{1-\beta\rho_a} \Delta a_t$  is a transformation of the productivity shock,  $\pi_t^*$  is the inflation trend,  $z_t^u$  the difference between the natural and the structural unemployment rates ( $u_t^* - \bar{u}_t$ ),  $z_t^p$  the difference between the natural and the structural participation rates ( $p_t^* - \bar{p}_t$ ).  $oe^{6Q}$  and  $oe^{2Q}$  represent the observation errors on the medium and short-term inflation expectations, respectively.  $\epsilon_t$  is a vector of shocks of dimension  $n_\epsilon = 8$ :  $\epsilon_t = [\epsilon_t^{x^u}, \epsilon_t^{x^p}, \epsilon_t^\varsigma, \epsilon_t^{\pi^*}, \epsilon_t^{z^u}, \epsilon_t^{z^p}, \epsilon_t^{oe^{6Q}}, \epsilon_t^{oe^{2Q}}]$ .

$$H_{n_y \times n_s} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ F_{1,1}^{2Q} & F_{1,2}^{2Q} & F_{1,3}^{2Q} & F_{1,4}^{2Q} & F_{1,5}^{2Q} & F_{1,6}^{2Q} & 1 & 0 & 0 & 0 & 1 \\ F_{1,1}^{6Q} & F_{1,2}^{6Q} & F_{1,3}^{6Q} & F_{1,4}^{6Q} & F_{1,5}^{6Q} & F_{1,6}^{6Q} & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

where  $F_{i,j}^A$  is the element of row  $i$  and column  $j$  of matrix  $F^A$ , as defined below.

$$\mathbf{M}_y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$$

$$F_{n_s \times n_s} = \begin{bmatrix} \gamma & \xi_1 & \xi_2 & \xi_3 & \xi_4 & \rho_a & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{x^u,1} & a_{x^u,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{x^p,1} & a_{x^p,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{z^u} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{z^p} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where  $\xi_1 = \kappa^u (\omega_{\pi,1}^u a_{x^u,1} + \omega_{\pi,2}^u)$ ;  $\xi_2 = \kappa^u \omega_{\pi,1}^u a_{x^u,2}$ ;  $\xi_3 = \kappa^p (\omega_{\pi,1}^p a_{x^p,1} + \omega_{\pi,2}^p)$  and  $\xi_4 = \kappa^p \omega_{\pi,1}^p a_{x^p,2}$ .

$$G_{n_s \times n_\epsilon} = \begin{bmatrix} -\kappa^u \omega_{\pi,1}^u \sigma_{x^u} & \kappa^p \omega_{\pi,1}^p \sigma_{x^p} & \sigma_\varsigma & -\gamma \sigma_{\pi^*} & 0 & 0 & 0 & 0 \\ \sigma_{x^u} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{x^p} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\varsigma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\pi^*} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{z^u, \varsigma} \sigma_\varsigma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z^p, \varsigma} \sigma_\varsigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{oe^{6Q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{oe^{2Q}} \end{bmatrix}$$

We can rewrite the system in non matricial form.

The measurement equations are

$$\begin{aligned} u_t &= x_t^u + z_t^u + \bar{u}_t \\ p_t &= x_t^p + z_t^p + \bar{u}_t \\ \pi_t &= (\pi_t - \pi_t^*) + \pi_t^* \\ \mathbb{E}_t \pi_{t+2Q} &= \pi_t^* + l'_\pi \underbrace{\left[ \frac{1}{2} \sum_{j=1}^{j=2} \tilde{F}^j \right]}_{F^{12Q}} \tilde{\mathbf{y}}_t + oe^{2Q} \\ \mathbb{E}_t \pi_{t+6Q} &= \pi_t^* + l'_\pi \underbrace{\left[ \frac{1}{6} \sum_{j=1}^{j=6} \tilde{F}^j \right]}_{F^{6Q}} \tilde{\mathbf{y}}_t + oe^{6Q} \end{aligned}$$

with  $\tilde{\mathbf{y}}_t = [\pi_t - \pi_t^*, x_t^u, x_{t-1}^u, x_t^p, x_{t-1}^p, \varsigma_t]$ ,  $l'_\pi = [1, 0, 0, 0, 0]$  is a selection vector of the

inflation equation and  $\tilde{F}$  denotes the first six rows and columns of matrix  $F$ .

When adding information from wages, we extend the model with one or more measurement equations, depending on the number of wage measures included:

$$\pi_t^{w^j} = \delta_j \left[ g_w + (\pi_t - \pi_t^*) + \pi_t^* - \frac{1 - \beta \rho_a}{\beta(1 - \rho_a)} \varsigma_t \right] + oe_t^{w^j} \quad \text{with} \quad j = 1, 2, 3$$

where  $\delta_j$  is fixed and equal to one and  $oe_t^{w^j}$  is the measurement error of the  $j$ -th wage measure.

The transition equations read as follows:

$$\begin{aligned} \pi_t - \pi_t^* &= \gamma (\pi_{t-1} - \pi_{t-1}^*) - \kappa^u (\omega_{\pi,1}^u a_{x^u,1} + \omega_{\pi,2}^u) x_{t-1}^u - \kappa^u \omega_{\pi,1}^u a_{x^u,2} x_{t-2}^u + \\ &\quad \kappa^p (\omega_{\pi,1}^p a_{x^p,1} + \omega_{\pi,2}^p) x_{t-1}^p + \kappa^p \omega_{\pi,1}^p a_{x^p,2} x_{t-2}^p + \rho_a \varsigma_{t-1} \\ &\quad - \kappa^u \omega_{\pi,1}^u \sigma_{x^u} \epsilon_t^{x^u} + \kappa^p \omega_{\pi,1}^p \sigma_{x^p} \epsilon_t^{x^p} + \sigma_\varsigma \epsilon_t^\varsigma - \gamma \sigma_{\pi^*} \epsilon_t^{\pi^*} \\ x_t^u &= a_{x^u,1} x_{t-1}^u + a_{x^u,2} x_{t-2}^u + \sigma_{x^u} \epsilon_t^{x^u} \\ x_{t-1}^u &= x_{t-1}^u \\ x_t^p &= a_{x^p,1} x_{t-1}^p + a_{x^p,2} x_{t-2}^p + \sigma_{x^p} \epsilon_t^{x^p} \\ x_{t-1}^p &= x_{t-1}^p \\ \varsigma_t &= \rho_a \varsigma_{t-1} + \sigma_\varsigma \epsilon_t^\varsigma \\ \pi_t^* &= \pi_{t-1}^* + \sigma_{\pi^*} \epsilon_t^{\pi^*} \\ z_t^u &= \rho_{z^u} z_{t-1}^u + \sigma_{z^u, \varsigma} \sigma_\varsigma \epsilon_t^{z^u} \\ z_t^p &= \rho_{z^p} z_{t-1}^p + \sigma_{z^p, \varsigma} \sigma_\varsigma \epsilon_t^{z^p} \\ oe_t^{6Q} &= \sigma_{oe^{6Q}} \epsilon_t^{oe^{6Q}} \\ oe_t^{2Q} &= \sigma_{oe^{2Q}} \epsilon_t^{oe^{2Q}} \end{aligned}$$

## C Additional Figures

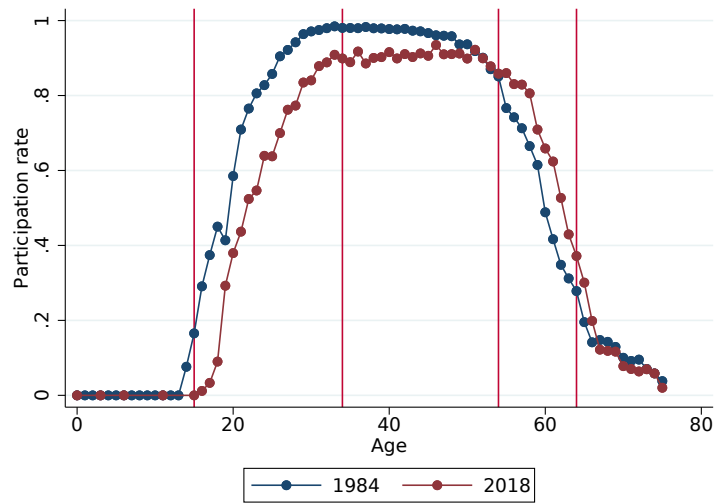


Figure C.1: Participation rate by age (males) - Comparison between 1984 and 2018.

**Note:** The figure plots the participation rate of males by age in 1984 (blue line) and 2018 (red line). The red vertical lines denote ages 15, 34, 55 and 64.

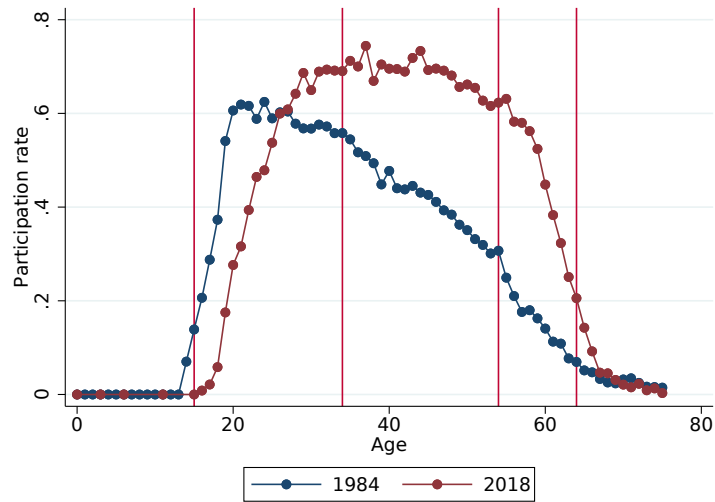


Figure C.2: Participation rate by age (females) - Comparison between 1984 and 2018.

**Note:** The figure plots the participation rate of females by age in 1984 (blue line) and 2018 (red line). The red vertical lines denote ages 15, 34, 55 and 64.

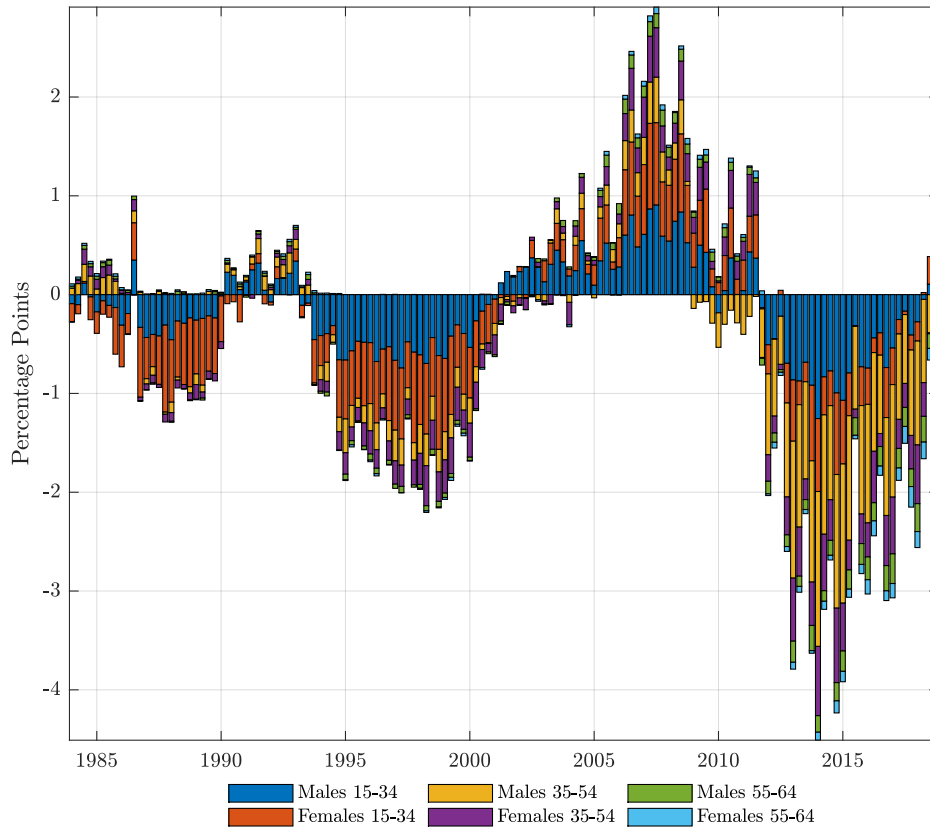


Figure C.3: Decomposition of aggregate structural unemployment gap.

**Note:** The bars represent the contribution of each demographic group to the overall gap between the aggregate trend unemployment rate and the actual one, for the years 1984q1–2018q4. The contribution of each group equals the difference between the trend unemployment rate (weighted by ratio between the group-specific and the aggregate structural participation rate) and the actual unemployment rate (weighted by the ratio between the group-specific and the aggregate participation rate) multiplied by the corresponding population weight (see Equation 7).

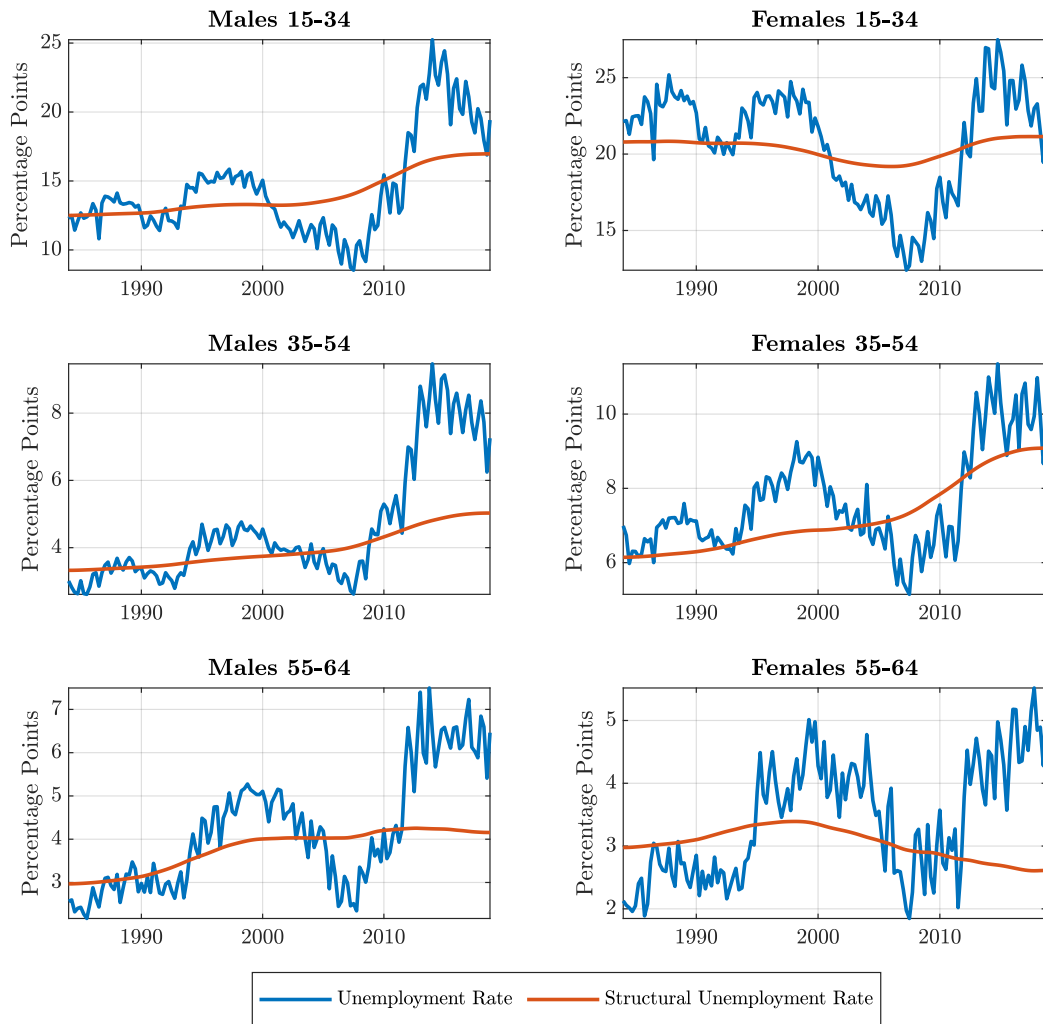


Figure C.4: Structural unemployment rate, by demographic group.

**Note:** The figure plots the trend unemployment rate (red line) and the actual unemployment rate (blue line) by demographic subgroup, for the years 1984q1–2018q4. The trend rates are obtained via Kalman filter, as explained in Appendix A.



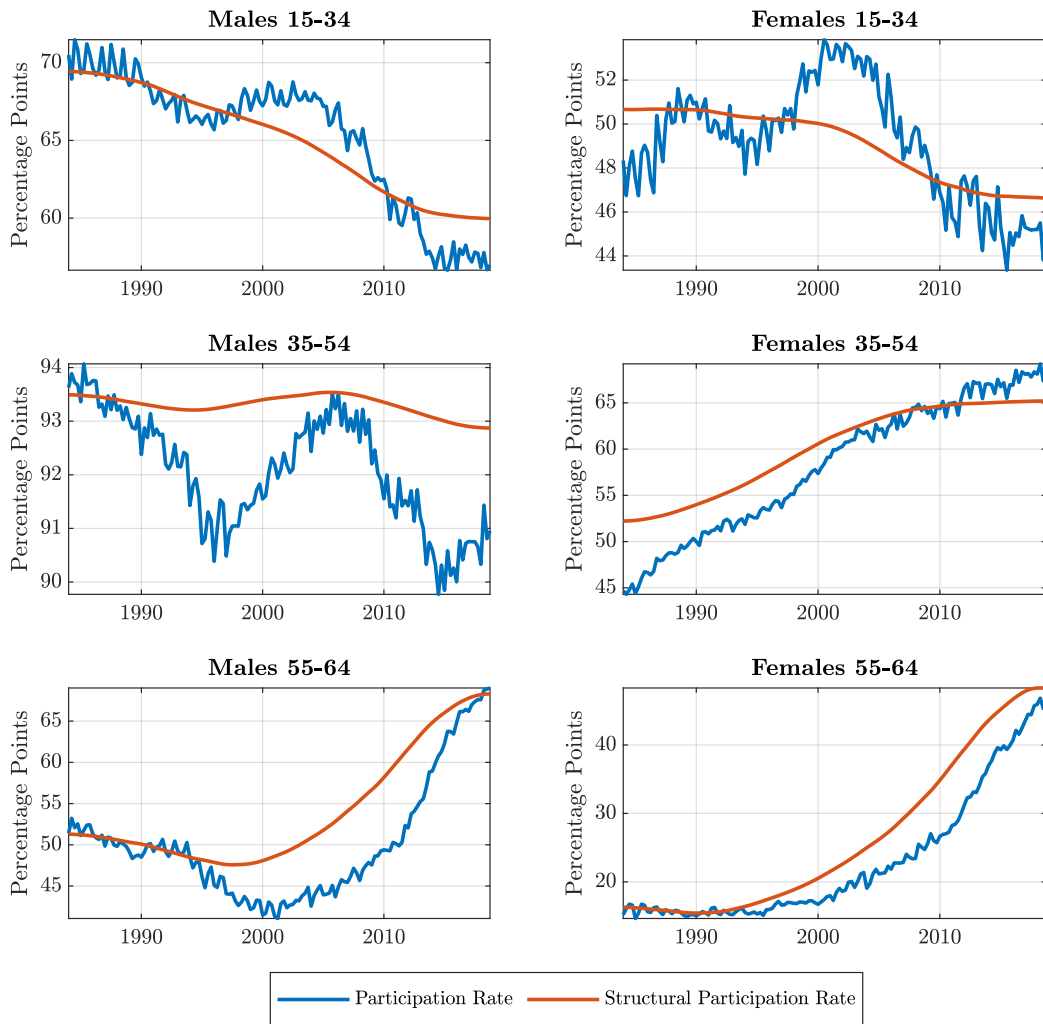


Figure C.5: Structural participation rate, by demographic group.

**Note:** The figure plots the trend participation rate (red line) and the actual participation rate (blue line) by demographic subgroup, for the years 1984q1–2018q4. The trend rates are obtained via Kalman filter, as explained in Appendix A.

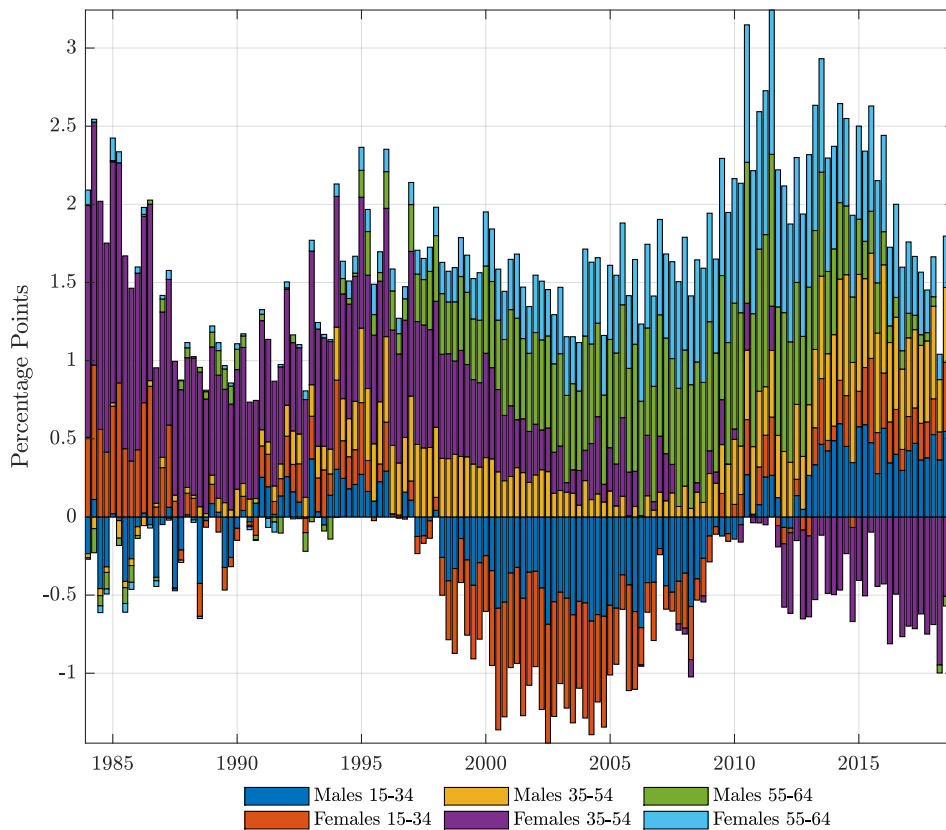


Figure C.6: Decomposition of aggregate structural participation gap.

**Note:** The bars represent the contribution of each demographic group to the overall gap between the aggregate trend participation rate and the actual one, for the years 1984q1–2018q4. The contribution of each group equals the difference between the trend participation rate and the actual participation rate multiplied by the corresponding population weight (see Equation 6).

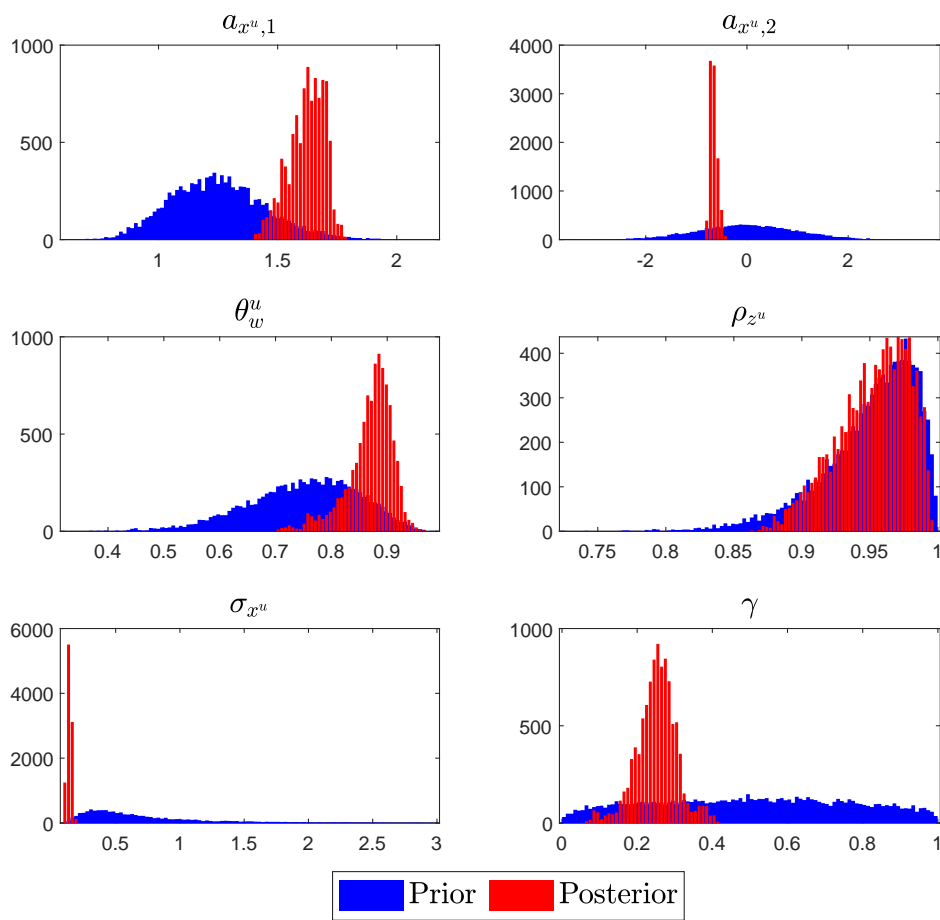


Figure C.7: Prior and posterior distributions (1)

**Note:** The figure represents prior and posterior distributions of the main parameters of the baseline Phillips curve model including both unemployment and participation gaps.

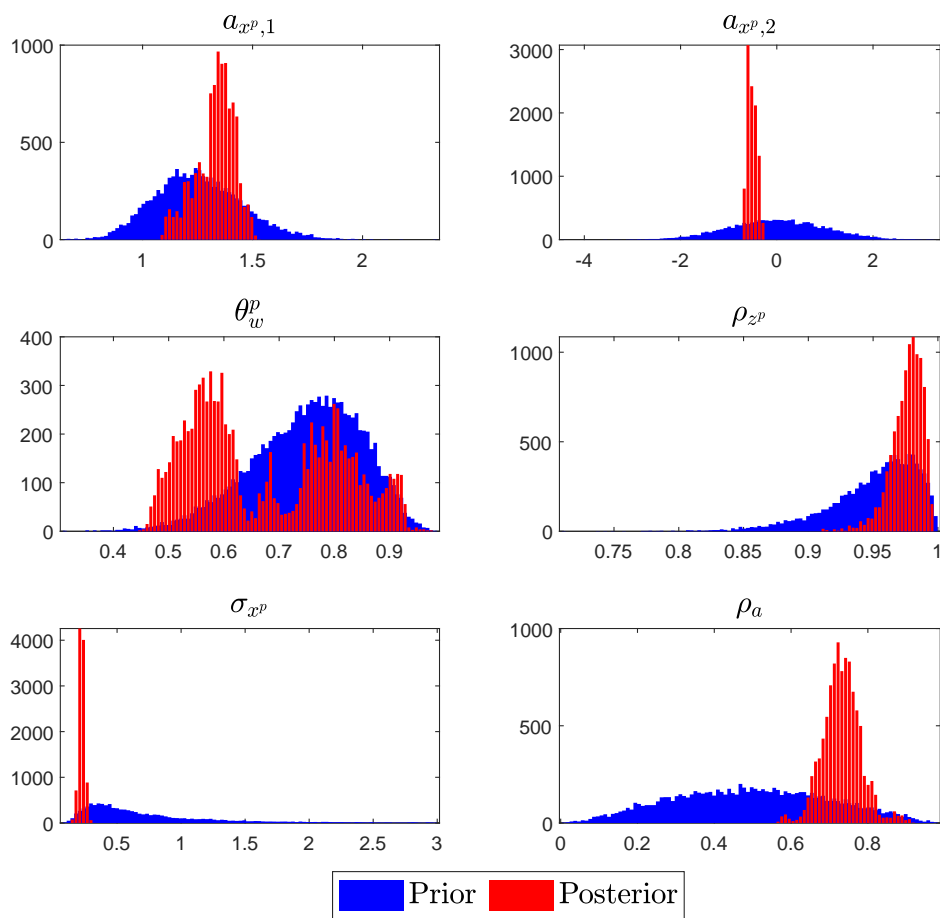


Figure C.8: Prior and posterior distributions (2)

**Note:** The figure represents prior and posterior distributions of the main parameters of the baseline Phillips curve model including both unemployment and participation gaps.

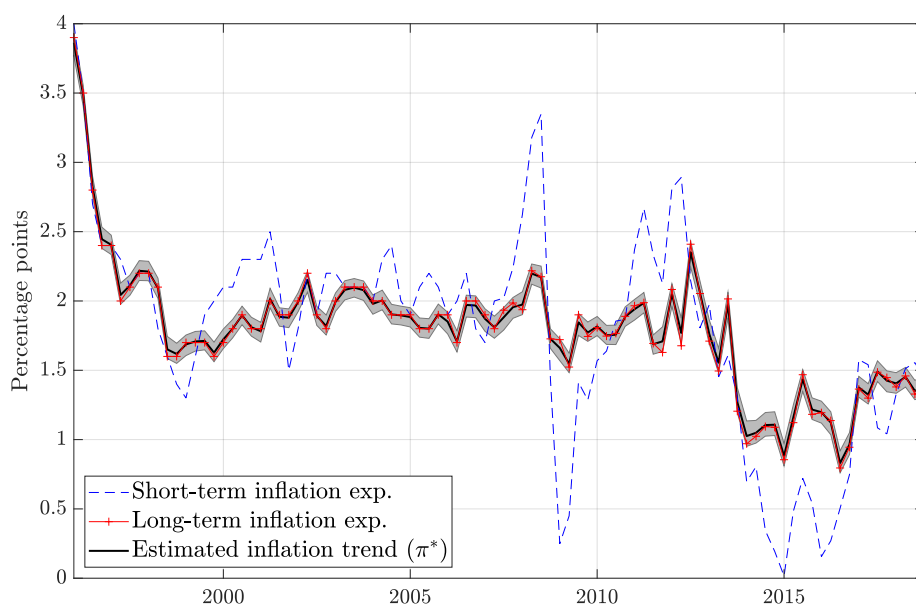


Figure C.9: Inflation expectations and estimated inflation trend in the augmented Phillips curve model (UGAP + PGAP)

**Note:** Shading denotes the 68% coverage interval.

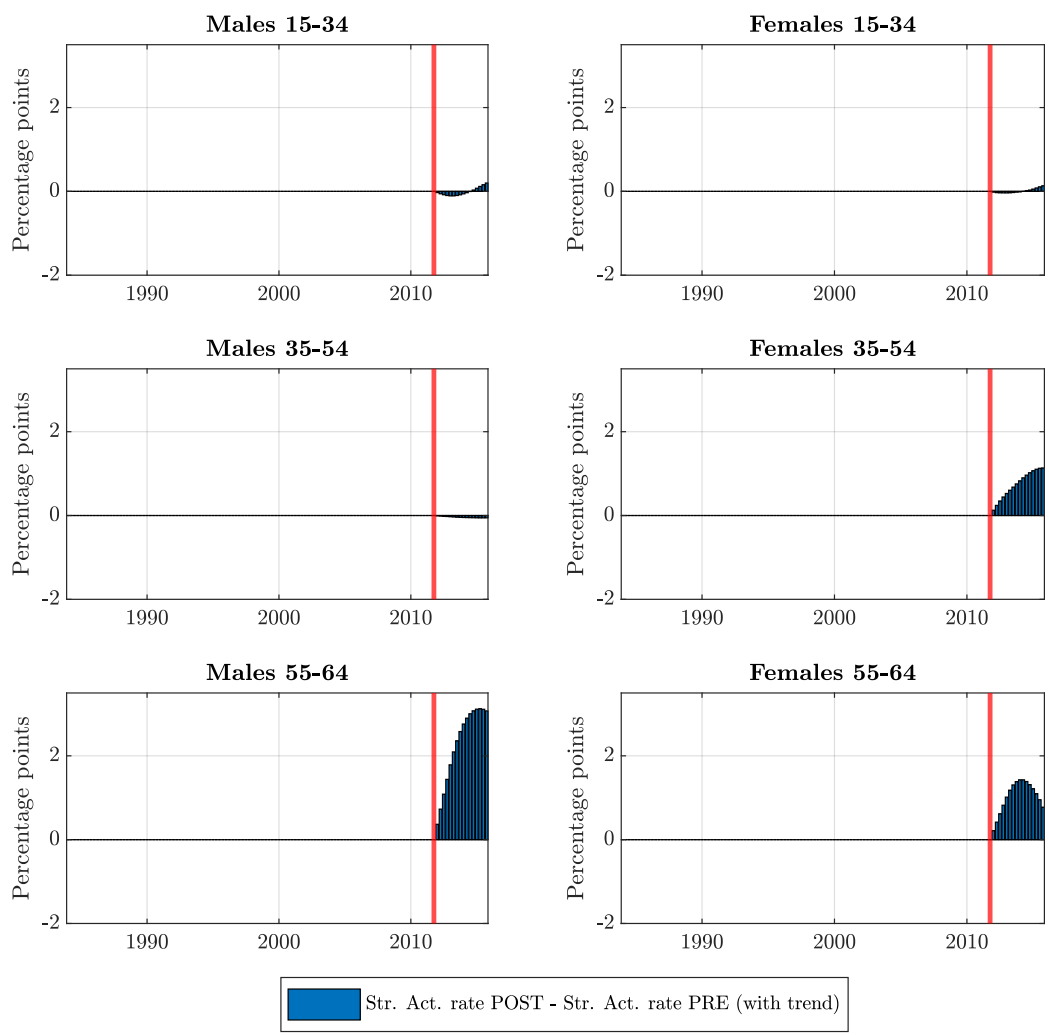


Figure C.10: Effect of the Fornero reform on the group-specific structural activity.

**Note:** The bars indicate the difference between  $\bar{p}_t^{POST}$  and  $\bar{p}_t^{PRE}$ , for  $t \in [2012, 2015]$ , as defined in Section 5. The red line indicates 2011q4 (the end of the period prior to the reform).

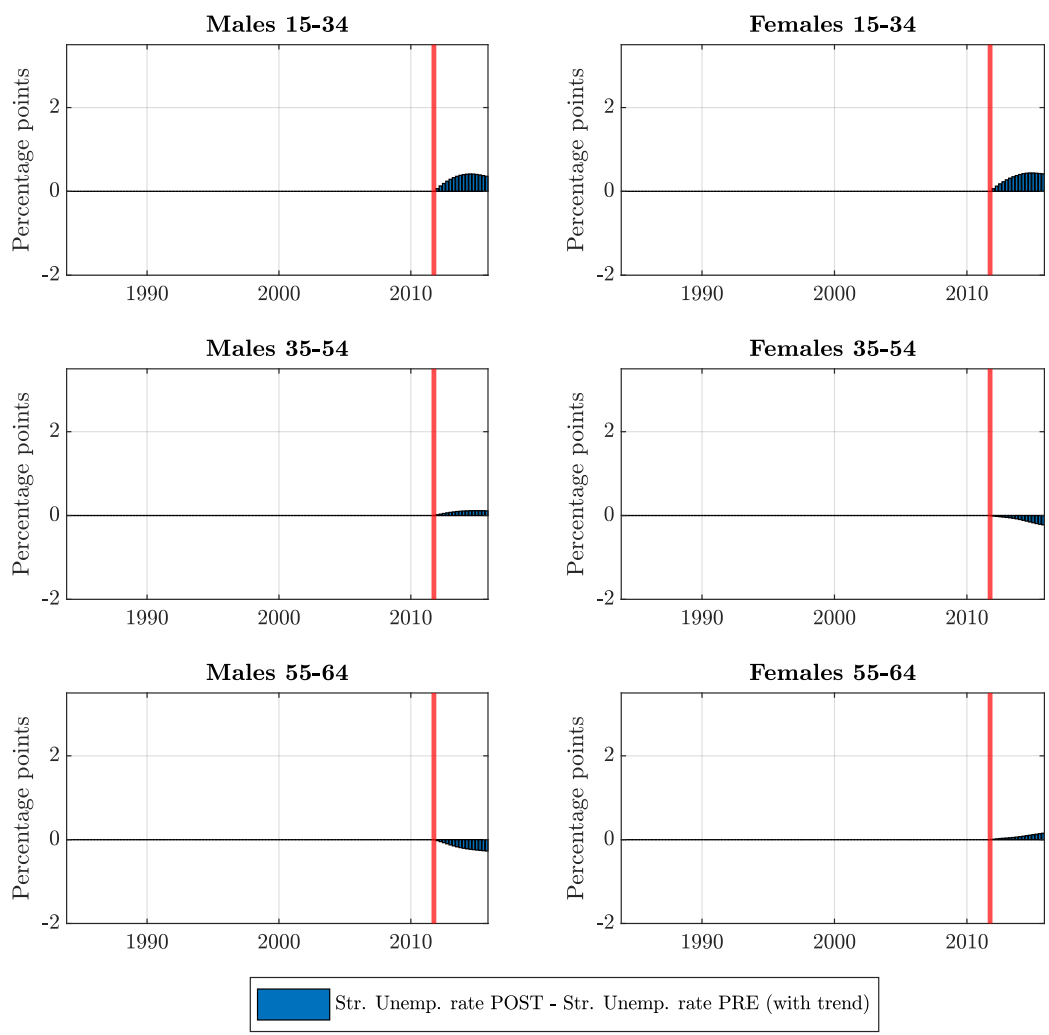


Figure C.11: Effect of the Fornero reform on the group-specific structural unemployment.

**Note:** The bars indicate the difference between  $\bar{u}_t^{POST}$  and  $\bar{u}_t^{PRE}$ , for  $t \in [2012, 2015]$ , as defined in Section 5. The red line indicates 2011q4 (the end of the period prior to the reform).

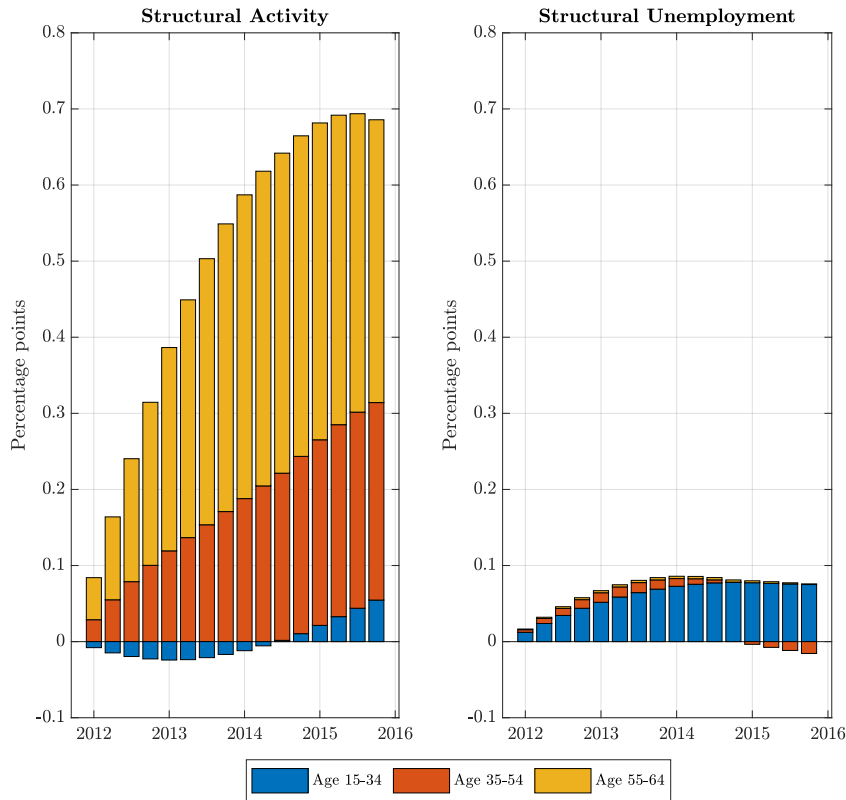


Figure C.12: Decomposition of the aggregate effect of the Fornero reform on structural activity and structural unemployment.

**Note:** The bars indicate the contribution to the aggregate effect on the structural activity (left chart) and structural unemployment (right chart), by age groups. The contributions depend on both the weight in the total population and on the incidence of each group in the active pool of workers (for unemployment).

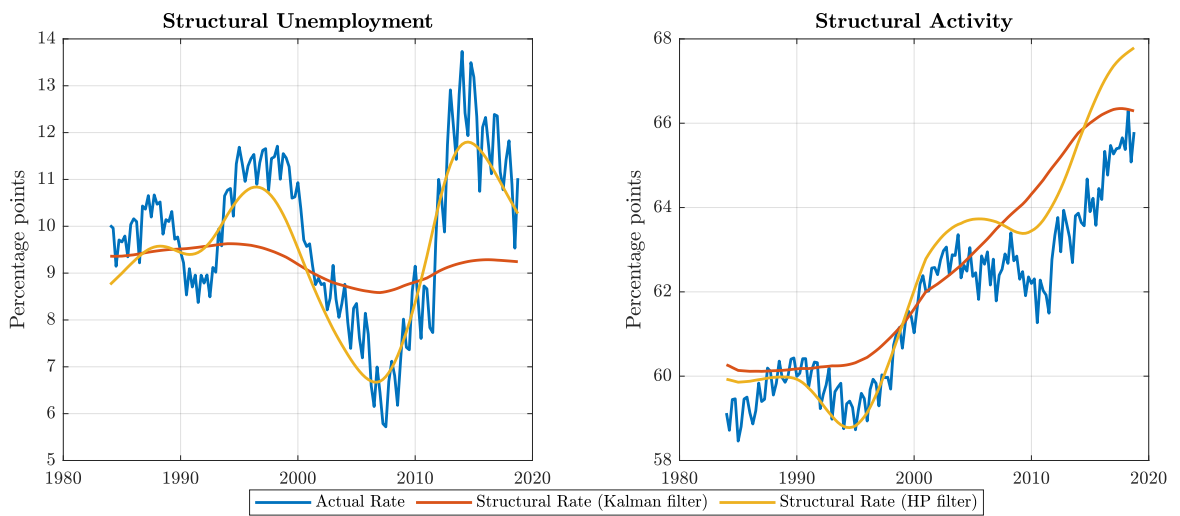


Figure C.13: Aggregate Structural Unemployment and Participation Rate, comparison among filters and population definitions.

**Note:** The figure plots the unemployment and activity rate of the population 15–64, along with the estimated trend rates (Kalman filter, red line; HP filter, yellow line) for the years between 1984q1–2018q4.



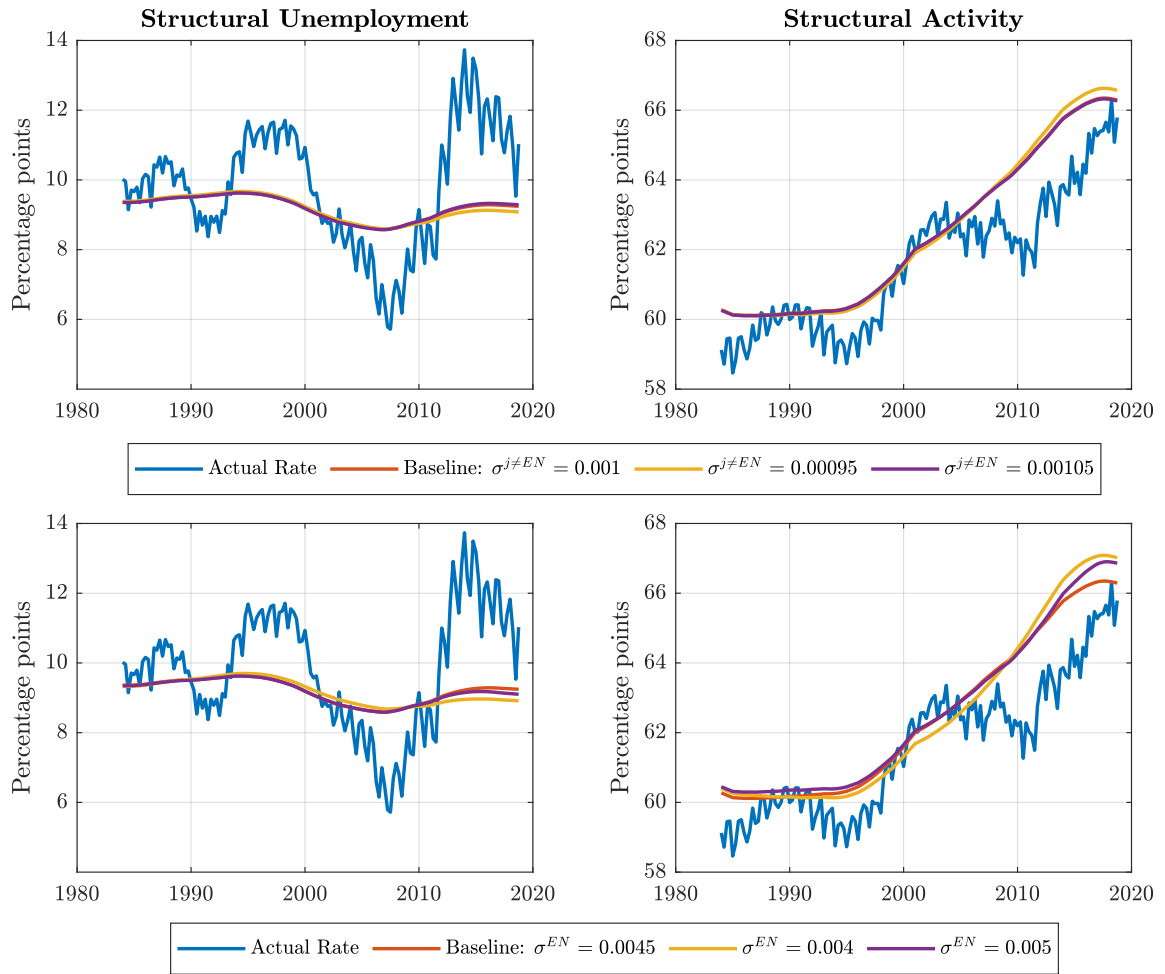


Figure C.14: Aggregate Structural Unemployment and Participation Rate, robustness checks.

**Note:** The figure plots the unemployment (left charts) and activity rate (right charts) of the population 15–64, along with the estimated trend rates (Kalman filter) for the years between 1984q1–2018q4, under different assumptions: i) we vary the ratio between the variance of trend to cycle (for all flows except the EN one,  $\sigma^{j \neq EN}$ ), in the upper charts; ii) we vary the ratio between the variance of trend to cycle for the EN flow ( $\sigma^{EN}$ ), in the lower charts.

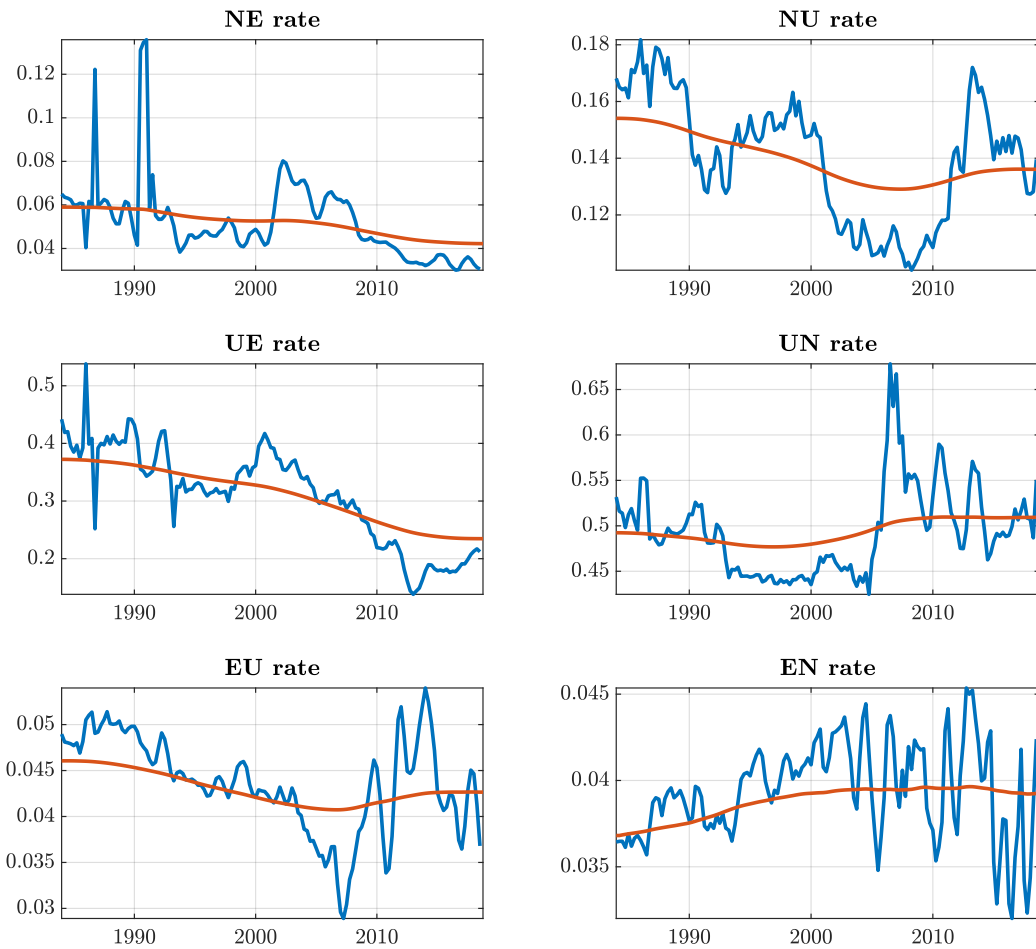


Figure C.15: Labor Market Flows (hazard rates) and trend - Males 15–34.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.

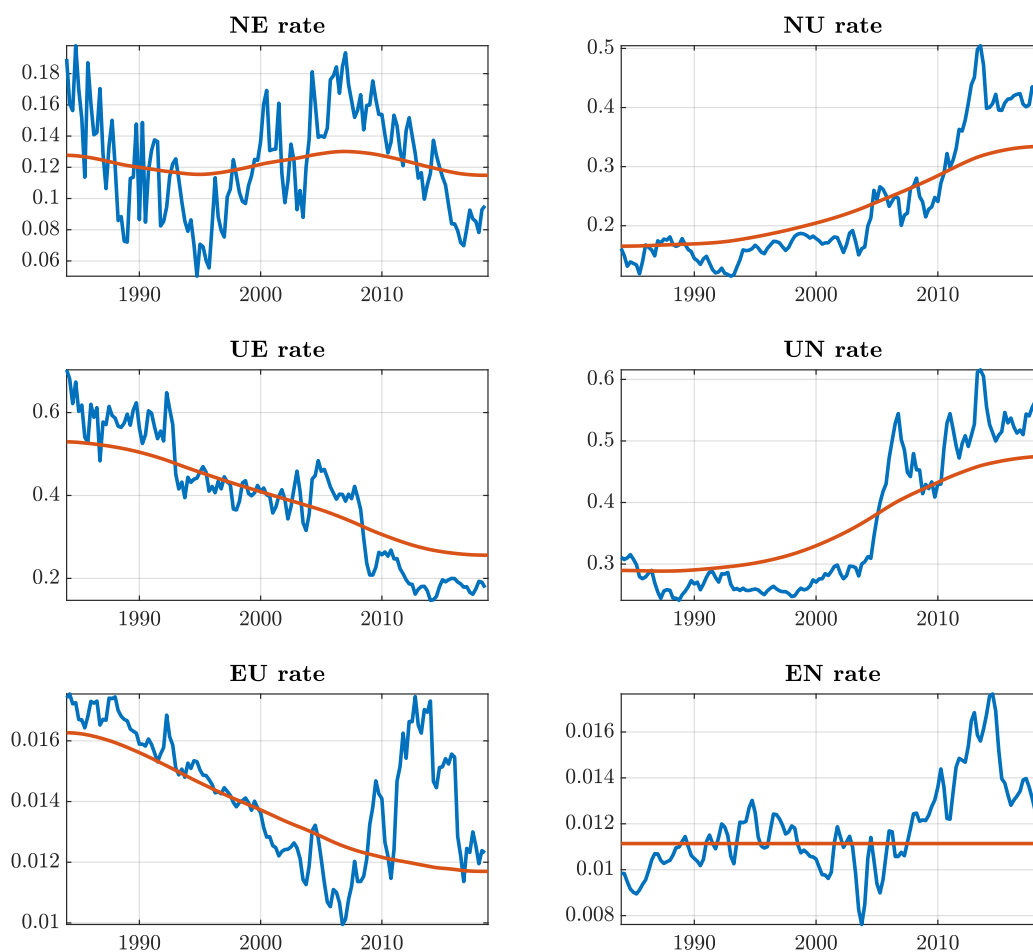


Figure C.16: Labor Market Flows (hazard rates) and trends - Males 35–54.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.

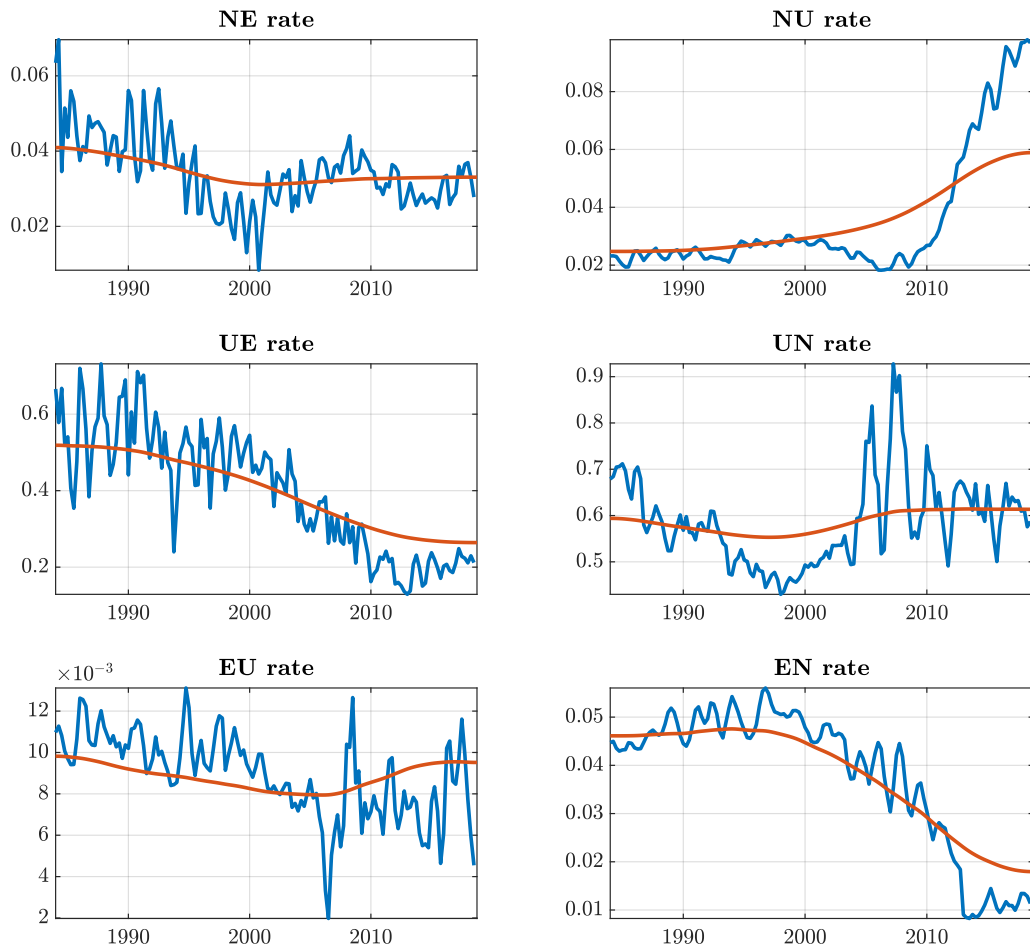


Figure C.17: Labor Market Flows (hazard rates) and trends - Males 55–64.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.

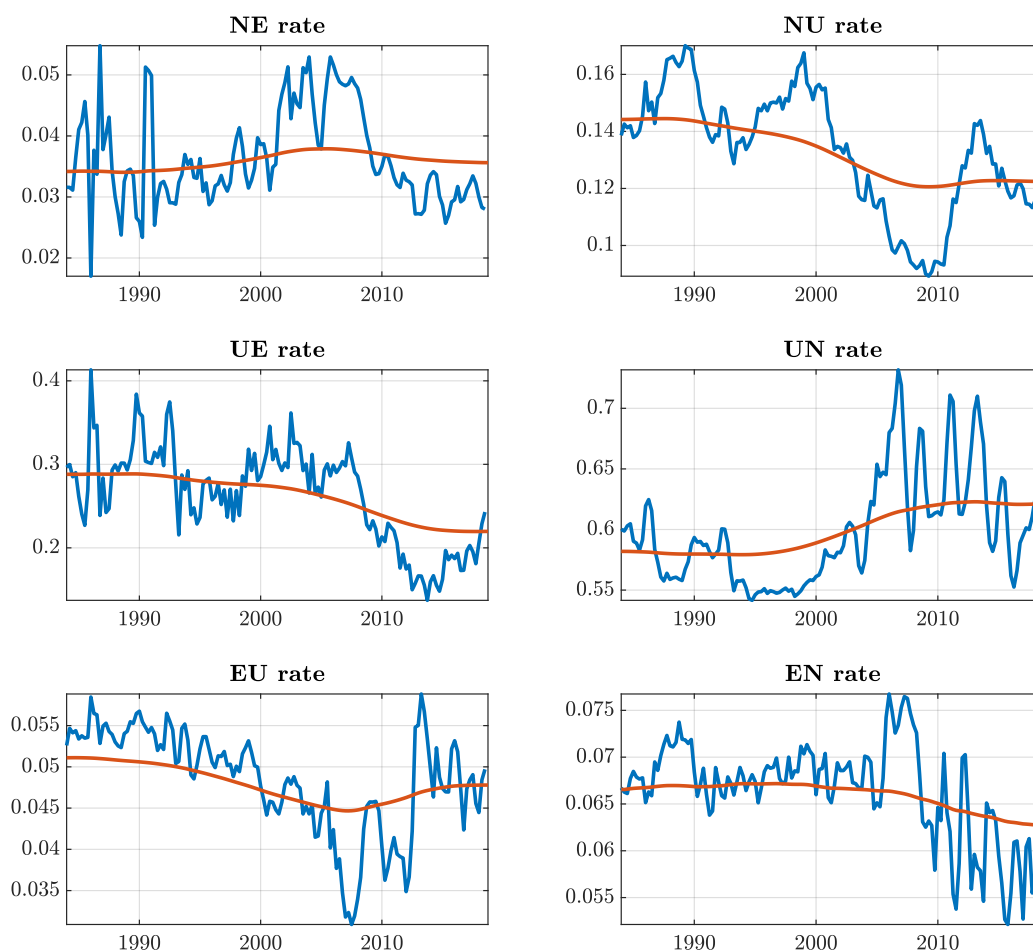


Figure C.18: Labor Market Flows (hazard rates) and trends - Females 15–34.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.

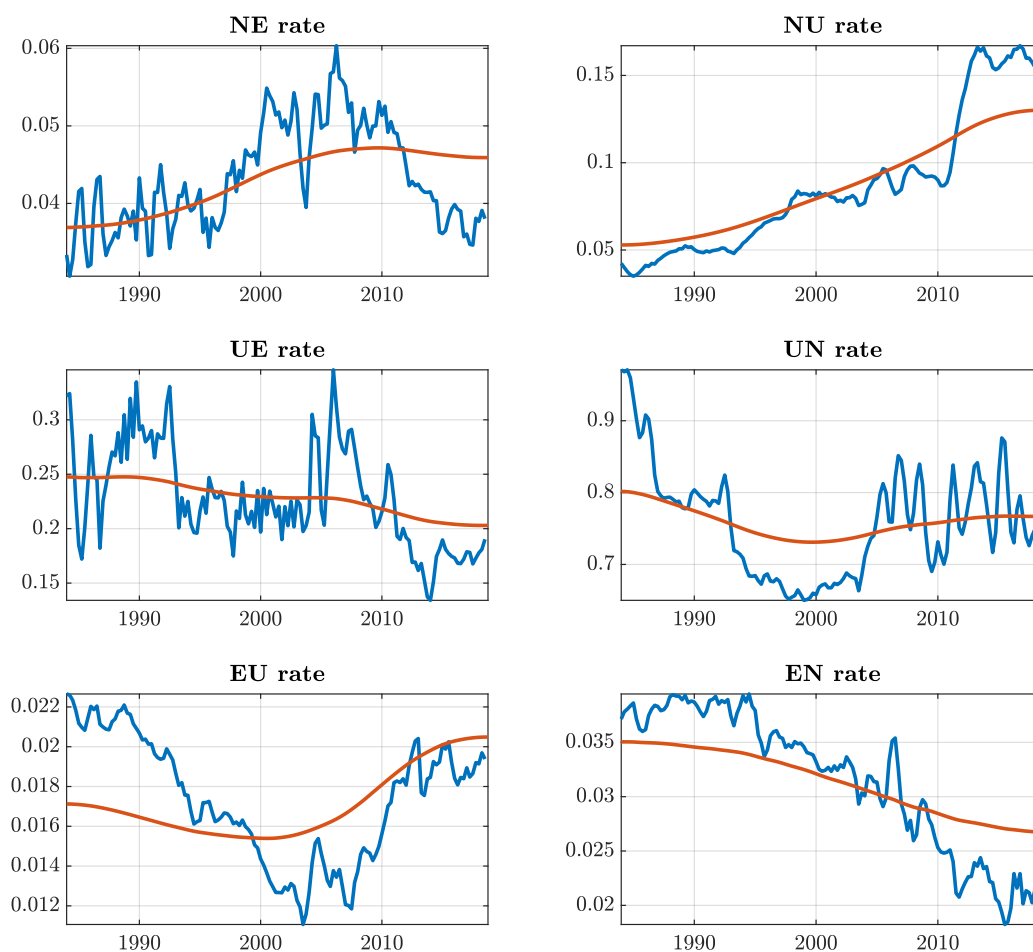


Figure C.19: Labor Market Flows (hazard rates) and trends - Females 35–54.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.

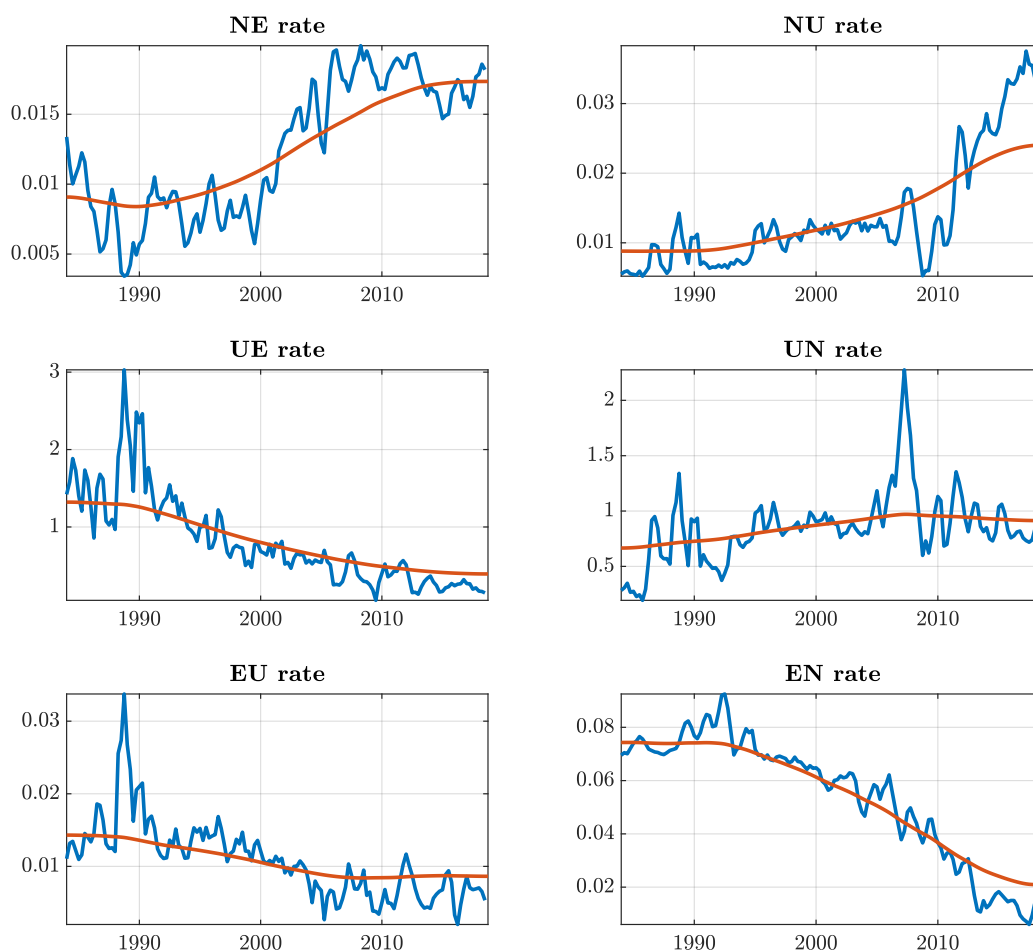


Figure C.20: Labor Market Flows (hazard rates) and trends - Females 55–64.

**Note:** The figure plots the hazard rates (blue line) and the trend hazard rates (red line), for the years 1984q1–2018q4. The hazard rates are obtained exploiting microdata and the trend rates are obtained via Kalman filter, as explained in Appendix A.