Quantifying the Welfare and Equality Effects of Carbon-Tax Policy

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Abstract

Previous research presents policymakers instituting a revenue-neutral carbon tax with a dismal trade-off between welfare and equality. In particular, policymakers must choose between a policy that minimizes average welfare cost, but exacerbates inequality (i.e. recycling carbon tax revenues through a reduction in pre-existing distortionary taxes) versus a policy that has higher welfare costs but does not exacerbate inequality (i.e. providing lump-sum rebates). We eliminate this dismal choice when we broaden the set of rebate-instruments policymakers can use for the carbon-tax revenue and account for the impact of age and income heterogeneity on welfare. In particular, rebating the carbon-tax revenue through both a reduction in the capital tax rate and an increase in the progressivity of the labor income tax minimizes the average welfare cost and does not increase inequality.

Keywords: Carbon tax; overlapping generations; inequality
JEL codes: E62; H21; H23

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1 Introduction

There is widespread support among economists for establishing a price on carbon emissions to mitigate climate change (Stern (2008)). Outside of the economics literature, however, carbon taxes face staunch resistance. In France, the recent "Yellow Vest" riots were sparked by push-back against a proposed environmental tax on gasoline. In the U.S., a carbon tax proposal on the 2018 ballot in left-leaning Washington State was rejected by voters. Much of the public resistance to carbon taxes stems from concerns surrounding the policy's costs. By taxing carbon, the price of energy derived from fossil fuels will increase, imposing a direct financial burden on households. The concern is not only that the resulting welfare costs will be large, but also that these costs will be distributed unevenly. In particular, it is often thought that the burden of a carbon tax will be relatively larger for low income households who devote a larger share of their total expenditure towards energy (Metcalf (1999), Grainger and Kolstad (2010)).

In contrast to these concerns, previous studies demonstrate that a carbon tax policy need not be regressive. Ultimately, the magnitude and distribution of the costs imposed by a carbon tax policy depends critically on (A) how the resulting tax revenues are used, and (B) the resulting general equilibrium impacts.[1] Notably, general equilibrium analyses of revenue-neutral carbon tax policies consistently find that returning carbon tax revenue to households through uniform, lump-sum rebates would result in a progressive outcome, with the lowest income households experiencing sizable welfare gains (e.g., Williams et al. (2015), Fried et al. (2018)). In practice, the lump sum rebate strategy is receiving a great deal of support among policymakers. Canada's recently adopted climate policy returns revenues to households through lump sum payments. Similarly, the Carbon Dividend proposal put forward by the Climate Leadership Council (CLC) – and publicly supported by 27 Nobel Laureates, all the living former Chairs of the Federal Reserve Board, and 15 former Chairs of the President's Council of Economic Advisers – calls for the U.S. federal government

[1]For example, see Fullerton and Heutal (2007), Dinan and Rogers (2002), Metcalf (2007), Parry (2004), and Parry and Williams (2010).

to institute a carbon tax and return the revenue “directly to U.S. citizens through equal lump-sum rebates.”[2]

Unfortunately, while uniform rebates can achieve what is perhaps a more politically palatable distributional outcome, the aggregate welfare costs of such a policy are thought to be quite large (e.g., Parry (1995), Goulder (1995), de Mooij and Bovenberg (1998), Bovenberg (1999)). Instead, to minimize the aggregate welfare costs, it is well established that revenues from a carbon tax should be used to offset pre-existing, distortionary taxes (i.e. the labor or capital income tax) – an approach which ultimately exacerbates the regressivity of a carbon tax (e.g., Williams et al. (2015), Fried et al. (2018)). In sum, the existing literature paints a bleak picture in terms of designing a politically feasible carbon tax. Policymakers seemingly face a choice between a policy that minimizes average welfare cost, but exacerbates inequality (i.e. recycling carbon tax revenues through a reduction in existing taxes) versus a policy that does not exacerbate inequality but has far higher welfare costs (i.e. providing lump-sum rebates).

In this paper, we highlight that this stark trade-off between equality and welfare has been overstated. While the existing literature has focused on a small set of blunt approaches for recycling carbon tax revenues – i.e. lump-sum rebates, capital tax reductions, or labor tax reductions – in practice, policymakers can use any number of different approaches for recycling carbon tax revenues. When we consider a richer set of revenue recycling options, we find that the revenue-neutral carbon-tax policy that minimizes welfare costs simultaneously reduces inequality.

To explore the welfare consequences of a wide range of potential revenue-neutral carbon tax policies, we use a quantitative overlapping generations (OLG) model that builds on the analysis conducted in Fried et al. (2018). Importantly, the model incorporates three types of heterogeneity that are crucial to quantifying the distributional impacts of a carbon tax. First, we model an individual’s entire life cycle, leading to heterogeneity across age-cohorts. Second, we include idiosyncratic shocks to labor-income, which produce a full

[2]See opinion piece, “Economists Statements’ Statement on Carbon Dividends,” January 16th, 2019 *Wall Street Journal*.

income distribution within each age cohort. Third, our model uses Stone-Geary preferences to generate different expenditures on energy as a fraction of total expenditures for each income group. Consistent with the empirical evidence, low income households use a higher fraction of their expenditures for energy, implying that the carbon tax by itself is regressive. The inclusion of these three types of heterogeneity makes our model uniquely well suited for studying the distributional impacts of carbon tax policies.

We use the model to explore the long-run welfare impacts of a continuum of revenue-neutral carbon-tax policies. We consider convex combinations of the following rebate options for the carbon-tax revenue: (i) reduce the level of the labor income tax rate, (ii) reduce the capital income tax rate, (iii) provide lump-sum rebates that can vary with household income, and (iv) increase the progressivity of the labor income tax. Thus, our analysis features a continuum of different rebate-instruments, instead of just the three instruments used in previous literature.

Our results reveal a number of important patterns. First, consistent with the existing literature, we find that if policymakers are restricted to return the carbon tax revenue using only one rebate instrument, then the welfare costs are minimized by reducing a pre-existing, distortionary tax. In particular, we find that using carbon tax revenues to exclusively reduce the capital tax rate results in the lowest welfare costs for the average household. However, these welfare costs are not evenly distributed. Ultimately, the carbon tax coupled with a capital tax rebate policy is regressive, with the highest income individuals experiencing the smallest welfare cost.

Next, we examine the potential to counter-act the regressive effects of the capital-tax rebate by combining it with a progressive rebate. We quantify the welfare cost for the capital-tax rebate combined with the income-dependent lump-sum transfer and/or an increase in labor-tax progressivity. We find the capital-tax rebate combined with an increase in the progressivity of the labor tax minimizes the average welfare cost and also reduces inequality. Thus, in contrast to the existing literature, our results imply that the policy that minimizes the welfare costs actually decreases inequality, eliminating the welfare-equality trade-off.

To understand why this trade-off disappears, we decompose the welfare effect into three components: (1) increases in equality (distribution component), (2) smoother profiles of average consumption and labor over the life cycle (age component), and (3) increases in the size of the economy (level component).

Relative to the simple capital-tax rebate policy, the age and distribution components of welfare are larger under the combined policy, driving the additional welfare gains. When the policymaker diverts some revenue to increase the progressivity of the labor-tax, there is less revenue to rebate through a reduction in the capital-tax. As a result, the capital-tax rate is higher under the combined policy, implying that the after-tax interest rate is lower. The lower after-tax interest rate incentivizes individuals to consume more early in life, flattening their consumption profiles and generating welfare gains. Furthermore, the higher level of equality under the combined policy raises expected lifetime consumption, increasing ex-ante welfare for an individual before she has any indication of her lifetime consumption.

Expanding the set of rebate instruments to include increases in the labor-tax progressivity is crucial to eliminating the trade-off between welfare and equality. Relative to the lump-sum rebate, the increase in labor-tax progressivity is a less-costly tool to achieve a given equality target. Since individuals receive the lump-sum rebate in every period of life, including retirement, it reduces their need to save for retirement, crowding out capital. In contrast, individuals only receive the rebate from the increase in labor-tax progressivity during their working years, and thus it does not (directly) reduce incentives to save for retirement. We find that if we prohibit the policymaker from changing the progressivity of the labor-tax, then she is again faced with a choice between a policy that minimizes average welfare cost and a policy that does not exacerbate inequality.

In sum, many of the recent carbon-tax policy proposals seek to return the carbon-tax revenue through lump-sum rebates to achieve an equality or distributional target. For example, the US Climate Leadership Council proposal suggests that "all the proceeds from this carbon fee would be returned to the American people on an equal and quarterly basis via dividend checks." (CLC, 2019). Similarly, the Canadian Citizens' Climate Policy proposal states that “Equal, twice-yearly dividends from carbon fees paid to each Canadian person will equitably recycle the revenue obtained from carbon fees.”(CCL, 2019). Like these proposals suggest, we find that it is possible to use the lump-sum to fully offset the regressive effects of a carbon tax policy. However, in the end, our results reveal that it would be far more effective to achieve the same distributional outcome by altering the progressivity of the labor-income tax.

2 Model and calibration

We analyze a quantitative general equilibrium overlapping generations model that incorporates within-cohort income heterogeneity. The main structure of the model is similar to Fried et al. (2018), with two substantial modifications. First, we change the production-side of the economy to incorporate endogenous energy production (as in the Appendix to Fried et al. (2018)). Second, we update the progressive labor-tax function to be consistent with more recent estimates of the US tax code.

2.1 Demographics

The model incorporates many overlapping generations of individuals of different ages. Agents enter the model when they start working, which we approximate with a real world age of 20, and can live to a maximum age of J. A continuum of new individuals is born each period and the relative size of the newborn cohort grows at a constant rate, n. Agents make labor-supply and savings decisions throughout their working years, until they are forced to retire at exogenously-determined age, j^r. After retirement, individuals finance consumption from social security payments and accumulated assets.

Lifetime length is uncertain and mortality risk varies over the lifetime. Parameter ψ_j denotes the probability an individual lives to age $j+1$ conditional on being alive at age j. All individuals who live to age J die with probability one the following period, i.e. $\psi_J = 0$. Since individuals are not certain how long they will live, they may die with positive asset holdings. In this case, we treat the assets as accidental bequests and redistribute them lump-sum across all living individuals during period t in the form of transfers, T_t^a.

2.2 Individuals

We model an income distribution within each age cohort. We generate this income heterogeneity though differences in labor productivity, which directly affects earnings. In period t, at age j, individual i earns labor income $y_{i,j,t}^h \equiv w_t \cdot \mu_{i,j,t} \cdot h_{i,j,t}$, where w_t is the market wage-rate during period t, $h_{i,j,t}$ denotes hours worked, and $\mu_{i,j,t}$ is the individual's idiosyncratic productivity. Following Kaplan (2012), the log of an individual's idiosyncratic productivity consists of four additively separable components,

$$\log \mu_{i,j,t} = \epsilon_j + \xi_i + \nu_{i,j,t} + \theta_{i,j,t}. \tag{1}$$

Component ϵ_j governs age-specific human capital and evolves over the life cycle in a predetermined manner. Component $\xi_i \sim NID(0, \sigma_\xi^2)$ is an individual-specific fixed effect (i.e. ability) that is observed when an individual enters the model and is constant for an individual over the life cycle. Component $\theta_{i,j,t} \sim NID(0, \sigma_\theta^2)$ is an idiosyncratic transitory shock to productivity received every period, and $\nu_{i,j,t}$ is an idiosyncratic persistent shock to productivity, which follows a first-order autoregressive process:

$$\nu_{i,j,t} = \rho \nu_{i,j-1,t-1} + \psi_{i,j,t} \text{ with } \psi_{i,j,t} \sim NID(0, \sigma_\nu^2) \text{ and } \nu_{i,20,t} = 0. \tag{2}$$

The different ability types, as well as the initial realization of the transitory shock, generate an income distribution within the cohort of 20-year-old entrants to the model. The different realizations of the persistent shock over the lifetime cause the within-cohort variation to grow with age. The age-specific human capital generates variation in the average labor productivity of individuals of different ages.

We assume that individuals cannot insure against idiosyncratic productivity shocks by trading explicit insurance contracts. Moreover, we assume that there are no annuity markets to insure against mortality risk. However, individuals are able to partially self insure against labor-income risk by purchasing risk-free assets, $a_{i,j,t}$, that have a pre-tax rate of return, r_t. Agents are not allowed to borrow; we require that asset holdings are always positive, $a_{i,j,t} \geq 0$.

Agents directly consume carbon-emitting energy, such as electricity or gasoline, in addition to a generic consumption good. Empirically, lower income households devote a larger share of their budgets to energy, implying that the direct impact of the carbon tax, prior to any revenue recycling, is likely to be regressive (Metcalf (2007), Mathur and Metcalf (2009)). To ensure that our model captures this negative relationship between income and energy consumption shares, we assume that all individuals must consume a minimum amount of energy, $\bar{e}$, and that individuals derive no utility from the energy consumed up to this subsistence level.

An individual is endowed with one unit of productive time per period that can be divided between hours and leisure. Agents have time-separable preferences over a consumption-energy composite, $\tilde{c}_{i,j,t}$, and hours, $h_{i,j,t}$. The utility function is given by

$$U(\tilde{c}_{i,j,t}, h_{i,j,t}) = \frac{\tilde{c}_{i,j,t}^{1-\theta_1}}{1-\theta_1} - \chi \frac{h_{i,j,t}^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}} \tag{3}$$

where $\tilde{c}_{i,j,t} = c_{i,j,t}^{\gamma}(e_{i,j,t}^{c} - \bar{e})^{1-\gamma}$. Variables $c_{i,j,t}$ and $e_{i,j,t}^{c}$ denote the consumption of the generic consumption good and energy, respectively, by individual i at age j in period t. Parameter θ_1 is the coefficient of relative risk aversion and parameter θ_2 is the Frisch elasticity of labor supply. Parameter χ determines the dis-utility of hours.

2.3 Firms

Perfectly competitive firms produce the final good, Y, from capital, K^y, efficiency labor, N^y, and carbon-emitting energy, E^y. Following Golosov et al. (2014), the production technology is Cobb-Douglas between the three inputs,

$$Y_t = A_t^y (K_t^y)^{\alpha_y} (N_t^y)^{1-\alpha_y-\zeta} (E_t^y)^{\zeta}. \tag{4}$$

Parameters α_y and ζ denote capital share and energy share, respectively, in the production of the final good. Parameter A^y denotes total factor productivity. The final good is the numeraire and can be used for consumption and investment.

Energy is produced competitively from capital, K^e and efficiency labor, N^e according to the Cobb-Douglas production technology,

$$E_t = A_t^e (K_t^e)^{\alpha_e} (N_t^e)^{1-\alpha_e}. \tag{5}$$

Parameter α_e denotes capital's share in the production of energy.

2.4 Government

The government runs a pay-as-you-go Social Security system. To finance Social Security benefits, the government institutes a flat payroll τ_t^s, on labor income, $y_{i,j,t}^h$, up to a taxable maximum, $y^{h,max}$. Agents do not pay any taxes on labor income greater than $y^{h,max}$. Thus, social security tax payments by individual i, age j, in year t equal

$$T_{i,j,t}^s(y_{i,j,t}^h) = \tau^s \min(y_{i,j,t}^h,\, y^{h,max}) \tag{6}$$

Consistent with US tax law, half of the payroll taxes are withheld from labor income by the employer and the other half are paid directly by the employee. The payroll tax rate is set such that the Social Security system has a balanced budget in every period.

The government pays Social Security benefits, S_t, to all individuals that are retired. Each individual receives a constant payment each period, which is independent of the specific individual's lifetime earnings (Conesa and Krueger 2006). Specifically, the Social Security benefit equals fraction η of the average labor earnings of all working individuals in period t,

$$S_t = \eta \times \frac{\sum_{j<j^r}[\min(y^h_{ijt},\, y^{h,max})\Phi(x)]}{\sum_{j<j^r}\Phi(x)}. \tag{7}$$

The earnings that enter into the calculation in equation (7) are capped at the taxable maximum, $y^{h,max}$. Function $\Phi(x)$ denotes the distribution of individuals over the state space, x. The denominator in equation (7) thus corresponds to the mass of working-age individuals.

The government taxes capital income, labor income and energy (e.g., a carbon tax), to finance an exogenous level of unproductive government spending, G. The government taxes each individual's capital income, $y^k_{i,j,t}$, according to a constant marginal tax rate, τ^k. An individual's capital income is the return on her assets plus the return on any assets she receives as accidental bequests, $y^k_{i,j,t} \equiv r_t(a_{i,j,t} + T^a_t)$.

The government taxes labor income according to a progressive tax schedule. A working individual's taxable labor income is her labor income, $y^h_{i,j,t}$, net of her employer's contribution to Social Security which is not taxable under U.S. tax law. Thus, $\tilde{y}^h_{i,j,t} \equiv y^h_{i,j,t} - T^s(y^h_{i,j,t})/2$, where $T^s(y^h_{i,j,t})/2$ is the employer's Social Security contribution. We use a two-parameter functional form from the quantitative public finance literature (Benabou (2002), Heathcote et al. (2017) and Guner et al. (2014)) to model total labor income taxes for an individual with labor income $\tilde{y}^h_{i,j,t}$,

$$T^h(\tilde{y}^h_{i,j,t}) = \left[1 - \lambda_1 \left(\frac{\tilde{y}^h_{i,j,t}}{\bar{\tilde{y}}^h_t}\right)^{-\lambda_2}\right] \tilde{y}^h_{i,j,t}. \tag{8}$$

Variable $\bar{\tilde{y}}^h_t$ is the mean value of labor income in the economy. Parameter λ_1 determines the level of the labor-tax and parameter λ_2 governs the curvature the tax function. An increase in λ_1 increases the after-tax labor-income of all individuals by the same percentage, $1/\lambda_1$. Hence, changes in λ_1 only affect the level of the labor-tax rate; they have no impact on the distribution of after-tax income across individuals.[3]

[3]Specifically, the Gini coefficient calculated from after-tax labor income is independent of the value of λ_1.

In contrast, changes in λ_2 do affect the distribution of after-tax labor income. If $\lambda_2 = 0$, then T^h corresponds to a flat-tax on labor-income. Increases in λ_2 reduce the average tax rate for low-income households and increase the average tax rate for high-income households, reducing the inequality in the distribution of after-tax labor income. Thus, higher values of λ_2 imply a more progressive labor-tax schedule.

The carbon tax, τ^c, prices the externality, carbon. Thus, the government applies the tax per unit of energy consumed, raising the price of energy from p^e to $p^e + \tau^c$.[4] In the quantitative experiments, we will consider simulations in which the government returns the carbon-tax revenue to the households through transfers, T^c.

2.5 Definition of a Stationary Competitive Equilibrium

In this section, we define a stationary competitive equilibrium. In the long-run steady state, the factor prices, tax parameters, and aggregate macroeconomic variables will be constant. The individual state variables, x, are asset holdings, a, idiosyncratic labor productivity, μ, and age j. In addition, we signify an individual's chosen level of capital savings in the subsequent period as a'. We suppress the i, j, and t subscripts throughout the stationary equilibrium definition. The summations are taken over the distribution of individuals over the state space, x.

Given Social Security benefits, S, government expenditures, G, demographic parameters, $\{n, \Psi_j\}$, a sequence of age-specific human capital, $\{\epsilon_j\}_{j=20}^{j^r-1}$, a labor-tax function, $T^h : \mathbb{R}_+ \to \mathbb{R}_+$, a capital-tax rate, τ^k, a carbon-tax rate, τ^c, transfers from the climate policy, T^c, an energy price, p^e, a utility function $U : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, factor prices, $\{w, r, p^e\}$, and capital depreciation rate δ, a stationary competitive equilibrium consists of individuals' decisions rules, $\{c, h, e^c, a'\}$, firms' production plans, $\{(E^y), (K^y), (N^y), K_e, N_e\}$, transfers from accidental bequests T^a, a social security tax rate, τ^s, and the distribution of individuals, $\Phi(x)$, such that the following holds:

[4]Given that fossil fuel combustion accounts for over 80 percent of GHG emissions, a carbon tax behaves much like a tax on energy. This, of course, abstracts from substitution between fossil fuel energy sources with varying carbon intensities that could occur with a carbon tax.

1. Given prices, policies, transfers, benefits, and ν that follows equation (2) the individual maximizes equation (3) subject to:

$$c + (p^e + \tau^c)e^c + a' = \\ \mu h w - T^s + (1 + r(1 - \tau^k))(a + T^a) - T^h\big(\mu h w(1 - .5T^s)\big) + T^c \text{ for } j < j^r \quad (9)$$

$$c + (p^e + \tau^c)e^c + a' = S + (1 + r(1 - \tau^k))(a + T^a) + T^c \text{ for } j \geq j^r$$

$$c \geq 0,\ e^c \geq 0,\ 0 \leq h \leq 1,\ a \geq 0,\ a_{20} = 0$$

2. Final good firms' demands for K^y, N^y, and E^y satisfy:

$$r = \alpha_y A^y (K^y)^{\alpha_y - 1}(N^y)^{1-\alpha_y-\zeta}(E^y)^{\zeta} - \delta \quad (10)$$

$$w = (1 - \alpha_y - \zeta)A^y(K^y)^{\alpha_y}(N^y)^{-\alpha_y-\zeta}(E^y)^{\zeta} \quad (11)$$

$$p^e + \tau^c = \zeta A^y (K^y)^{\alpha_y}(N^y)^{1-\alpha_y-\zeta}(E^y)^{\zeta-1} \quad (12)$$

3. Energy firms' demands for K^e and N^e satisfy:

$$r = \alpha_e A^e (K^e)^{\alpha_y - 1}(N^e)^{1-\alpha_y} - \delta \quad (13)$$

$$w = (1 - \alpha_e)A^e(K^e)^{\alpha_e}(N^e)^{-\alpha_e} \quad (14)$$

4. The Social Security tax satisfies:

$$\tau^s = \frac{S \sum_{j \geq j^r} \Phi(x)}{\sum_{j < j^r}[\min(y^h_{ijt},\ y^{h,max})\Phi(x)]} \quad (15)$$

5. Transfers from accidental bequests satisfy:

$$T^a = \sum (1 - \Psi) a' \Phi(x) \tag{16}$$

6. The government budget balances:

$$G = \sum \left[\tau^k r(a + T^a) + T^h \big(\mu h w - T^s/2\big) + \tau^c e^c\right] \Phi(x) + \tau^c E^y - T^c \tag{17}$$

7. Markets clear:

$$K^y + K^e = \sum a\Phi(x), \quad N^y + N^e = \sum \mu h \Phi(x), \quad E^y + \sum e^c \Phi(x) = E \tag{18}$$

$$\sum (c + p^e e^c + a')\Phi(x) + G = Y + p^e E - p^e E^y + (1 - \delta)K \tag{19}$$

8. The distribution of $\Phi(x)$ is stationary. That is, the law of motion for the distribution of individuals over the state space satisfies $\Phi(x) = Q_\Phi \Phi(x)$ where Q_Φ is the one-period recursive operator on the distribution.

3 Calibration

Following standard procedure, the calibration has two steps. First, we choose one set of parameters directly from the data and existing literature. Then, given these directly calibrated parameters, we choose the remaining parameters so that a set of moments in the model match their empirical values in the data. Table 1 reports the values of the parameters we take directly from the data and Table 2 reports the values of the parameters we pin down with the method-of-moments procedure. The majority of the calibration is similar to Fried et al. (2018).

Table 1: Parameter Values: Direct Calibration

Parameter	**Value**	**Source**
Demographics		
Surv. Prob: Ψ_j	Bell and Miller (2002)	Data
Pop. Growth: n	1.1%	Data
Firm Parameters		
Final Good Capital Share: α_y	0.3	Golosov et al. (2014)
Energy Share: ζ	0.04	Golosov et al. (2014)
Energy Capital Share: α_e	0.597	Barrage (2018)
Final Good Productivity: A^y	1	Normalization
Energy Productivity: A^e	1	Normalization
Productivity Parameters		
Persistence shock variance: σ_ν^2	0.017	Kaplan (2012)
Persistence: ρ	0.958	Kaplan (2012)
Permanent shock variance: σ_ξ^2	0.065	Kaplan (2012)
Transitory shock variance: σ_θ^2	0.081	Kaplan (2012)
Preference Parameters		
Risk Aversion: θ_1	2	Conesa et al. (2009)
Frisch Elasticity: θ_2	0.5	Kaplan (2012)
Government Parameters		
Labor Tax Function: λ_1	0.832	Clears market
Labor Tax Function: λ_2	0.053	Guner et al. (2014)
Capital Tax Rate: τ^k	0.36	Trabandt and Uhlig (2011)
Replacement rate: η	0.5	Conesa and Krueger (2006)
SS Payroll Tax: τ^s	0.1096	Clears market

Table 2: Parameter Values: Method of Moments

Parameter	**Value**	**Main Target**
Firm Parameters		
Depreciation: δ	0.083	$\frac{I}{Y} = 0.255$
Preference Parameters		
Conditional Discount: β	1.0002	$\frac{K}{Y} = 2.7$
Disutility of Labor: χ	74.9	Avg. $h_{i,j} = 0.333$
Subsistence Energy: $\bar{e}$	0.00474	$\frac{[(p^e e^c)/(p^e e^c+c)]_{top}}{[(p^e e^c)/(p^e e^c+c)]_{bottom}} = 0.9256$
Consumption Energy Share: $1-\gamma$	0.069	$E^c/(E^c+E^p) = 0.183$
Government Parameters		
Government Spending: G	0.12	$\frac{G}{Y} = 0.155$
SS max income: $y^{h,max}$	0.96	2.42×(avg labor income)

3.1 Production

We normalize the total factor productivities in both energy and final-good production to unity, $A^e = A^y = 1$. Following Barrage (2018), we set capital's share in energy production equal to 0.597. Following Golosov et al. (2014), we choose capital's share in the production of output equal to 0.3 and fossil energy's share in the production output equal to 0.04.

3.2 Preferences

We choose the discount rate, β, to match the U.S. capital-output ratio of 2.7 (Conesa et al. 2009). We choose χ such that individuals spend an average of one third of their time endowment working. Following Conesa et al. (2009), we set the coefficient of relative risk aversion (θ_1) equal to 2 and consistent with Kaplan (2012), we set the Frisch elasticity (θ_2) equal to 0.5.

Energy subsistence parameter, $\bar{e}$, governs how energy budget share changes with income. Using data from the five most recent years of the Consumer Expenditures Survey (CEX) (2013-2017), we find that energy budget share in the top half of the income distribution is 35.275 percent smaller than energy budget share in the bottom half of the income distribution. However, the variance in expenditures in the CEX is larger than in our model. We adjust for this difference in variance and choose $\bar{e}$ to target an energy-share difference between the top and bottom halves of the expenditure distribution of 7.44 percent.[5]

Parameter $1 - \gamma$ is fossil energy's share in the consumption-energy composite, $\tilde{c}$. All else constant, an increase in γ reduces energy's share in the consumption-energy composite and thus decreases the individual's demand for energy. We pin down γ to match the ratio of total energy demand by individuals, $E^c \equiv \sum e^c \Phi(x)$, relative to total energy produced in the economy, E.

[5] In particular, the percent difference in total expenditures between the top and bottom half of the distribution is 256.11 percent in the CEX and only 54 percent in our model. The key reason for the smaller differential in total expenditures in our model is that the productivity shocks are assumed to be log normal. This distributional assumption, while standard in the literature, results in our model failing to capture the extreme top tail of the income distribution. We adjust the energy-share difference so that $\frac{54}{256.11} = \frac{7.44}{-35.275}$. We normalize the CEX data by the square root of family size in all of the calculations.

We calculate the empirical value of E^c/E from data on total primary energy consumption from the Energy Information Administration (EIA). Total fossil energy consumption, E, equals total primary energy consumption of coal, oil, and natural gas reported in EIA Table 1.1. Total fossil energy consumption by individuals, E^c, equals total primary consumption of coal, oil, natural gas by the residential sector (see EIA Table 2.2).[6] The average empirical value of E^c/E over the most recent five years of data, 2013-2017, equals 0.183. Thus, we choose γ so that household energy consumption is 18.3 percent of total energy consumption.

3.3 Government policy

We calibrate the curvature parameter of the labor-tax function from the estimates in Guner et al. (2014): $\lambda_2 = 0.053$. We choose the level parameter to clear the government budget constraint: $\lambda_1 = 0.83$. These parameters imply that a household with the mean labor income faces an average labor-tax rate of approximately 16.7 percent and a marginal labor-tax rate of 21.2 percent. The average and marginal labor-tax rates increase with the household's labor income. For example, a household with twice the mean labor-income faces an average labor-tax rate of 19.7 percent and a marginal labor-tax rate of 24.0 percent.

We determine government consumption, G, so that it equals 15.5 percent of output, the average value in the U.S data (Peterman and Sager 2018). We set the tax rate on capital income, τ^k, to 36 percent based on estimates in Kaplan (2012), Nakajima (2010) and Trabandt and Uhlig (2011). We set the replacement rate on Social Security, η, equal to 50 percent of average labor income (Conesa and Krueger 2006). Following Huggett and Carlos Parra (2010), we choose the taxable maximum, $y^{h,max}$, so that it equals 2.42 times average labor income.

We analyze a carbon tax set at \$40 dollars per ton of CO_2, the value proposed by the Climate Leadership Council (CLC, 2019). To calibrate the size of the tax in the model, we calculate the empirical value of the tax as a fraction of the price of a fossil energy composite

[6]The EIA data report residential energy consumption of coal, oil, natural gas and electricity. To convert residential electricity consumption to primary energy consumption of coal, oil, and natural gas, we calculate household electricity use relative to total electricity use (see EIA Table 7.6). We multiply this fraction the total amounts of coal, oil, and natural gas used in the electricity sector (see EIA Table 2.6).

of coal, oil, and natural gas. We take the average of this fraction over the previous five years, 2013-2017. We calculate the price of this energy composite averaging over the price of each type of energy in each year, and weighting by the relative consumption in each year. Similarly, we calculate the carbon emitted from the energy composite by averaging over the carbon intensity of each type of energy in each year, and weighting by the relative consumption in each year. This process implies that a \$40 per ton carbon tax equals 49 percent of our composite fossil energy price.[7]

4 Computational experiments

We study the long-run welfare and distributional implications of revenue-neutral carbon tax policies. We analyze different policies that combine a carbon tax set at \$40 per ton of CO_2 with one or more rebate instruments to return all of the tax revenue back to the households. Our approach is to compare the welfare and distributional outcomes in a counterfactual steady state with a carbon-tax policy in place to the corresponding outcomes in the baseline steady state with no carbon-tax policy.

In each counterfactual simulation of carbon-tax policy, we adjust the Social Security benefits so that the purchasing power is unchanged from the baseline. In the simulations, the carbon tax raises the price of the energy-good which reduces the relative price of the numeraire. Since Social Security benefits are denominated in terms of the numeraire, the purchasing power of the Social Security benefits falls from its value in the baseline. In practice, the U.S. government adjusts Social Security payments each year to ensure that the purchasing power remains constant. Consistent with this policy, we adjust the Social Security payment in each simulation to ensure that the retiree can buy the same bundle of energy and non-energy goods as she could in the baseline steady state. Specifically, Social Security payments in each simulation equal Social Security payments in the baseline times $\frac{c^e(p^e+\tau^c)}{c^e p^e + c}$ where c^e and c are the baseline values of energy and non-energy consumption, respectively.

[7]Data on the carbon intensity, energy prices, and energy consumption are from the EIA.

We adjust the Social Security tax to ensure that the Social Security budget balances.[8]

4.1 Rebate policies

We analyze combinations of the following four rebate instruments: (i) a reduction in the capital-tax rate, τ^k, (ii) a reduction in the level of the labor-tax rate, λ_1 (iii) an increase in the progressivity of the labor-tax rate, λ_2, and (iv) an income-dependent lump-sum transfer.

The increase in the progressivity of the labor-tax schedule is designed to mimic a change in the US tax code in which the government reduces in the average labor-income tax rate for the lower-income households (perhaps by changing the deduction schedule) and does not change in the average labor-income tax rate for higher-income households. However, increasing curvature parameter, λ_2, in the labor-tax function in equation (8) lowers the average labor tax rate for low-income individuals but raises the average labor tax-rate for high-income individuals. This change alone would not constitute a rebate because the tax rate increases for a fraction of the population. Therefore, we require that the average labor tax rate cannot increase from the baseline for any level of labor income. Specifically, the labor tax rate for an individual with taxable labor income, $\tilde{y}^h_{i,j,t}$, is

$$\min\left[1-\lambda_1\left(\frac{\tilde{y}^h_{i,j,t}}{\bar{\tilde{y}}^h_t}\right)^{-\lambda_2'},\ 1-\lambda_1\left(\frac{\tilde{y}^h_{i,j,t}}{\bar{\tilde{y}}^h_t}\right)^{-\lambda_2}\right].$$

Parameters λ_1 and λ_2 are the baseline values of the level and curvature parameters in Table 1, and λ_2' is the value of the curvature parameter in the counterfactual simulation. Note that at high levels of progressivity, the average labor tax rate is negative for lower-income households, implying that these households receive a labor transfer instead of paying a labor tax.

The standard lump-sum rebate returns the revenue to the households through equal lump-sum transfers. We consider a more general case in which we allow the size of the transfer to

[8] To calculate labor taxes in each counterfactual simulation, we keep the value of average taxable labor income, $\bar{\tilde{y}}^h$ fixed at its value in the baseline.

decrease linearly with the household's income. Specifically, the size of the lump-sum rebate for household i, age j with income y_{ij} is,

$$T^c_{ij} = \max\left[\Upsilon_1 + \Upsilon_2 y_{ij},\ 0\right]. \tag{20}$$

Parameter Υ_1 is the intercept for the lump-sum rebate function and parameter Υ_2 is the slope. If $\Upsilon_2 = 0$, then equation (20) generates an equal lump-sum rebate across all individuals, as in the previous literature. We are particularly interested in regions of the parameter space in which $\Upsilon_2 < 0$, so that the size of the rebate decreases with the individual's income. We bound the lump-sum rebate function below by zero to avoid raising taxes on any individual.

Both the increase in the labor-tax progressivity and the lump-sum rebate are progressive rebates; the size of the rebate relative to consumption is larger for lower-income individuals. However, an important difference between these progressive options is that the lump-sum rebate is a function of the individual's total income (capital and labor) while the rebate from the change in labor-tax progressivity depends only on the individual's labor income. This different structure implies that individuals only receive the rebate from the change in labor-tax progressivity during their working years, while individuals receive the lump-sum rebate during their working years and during retirement. All else constant, the extra transfer during retirement under the lump-sum rebate reduces individuals' need to save for retirement, crowding out capital.

4.2 Metrics

We construct measures of equality and welfare to evaluate the long-run steady state implications of the different carbon tax policies. Consistent with much of the double-dividend literature, we specifically examine the non-environmental welfare consequences of the carbon tax policies.

We calculate the Gini coefficient for lifetime non-environmental welfare to measure the level of equality under each policy, as in Fried et al. (2018). We define the Gini coefficient, $\mathcal{G}$, as

$$\mathcal{G} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} |x_i - x_j|}{2N^2 \bar{x}}, \tag{21}$$

where x_i represents lifetime welfare of individual i, $\bar{x}$ is the mean of lifetime welfare, and N is the total number of individuals in the economy. The Gini coefficient ranges between zero and one with zero implying perfect equality and one implying perfect inequality.

We use the consumption equivalent variation (CEV) to measure the aggregate welfare impacts of the different carbon-tax policies. The CEV measures the uniform percentage change in an individual's expected consumption that is required to make her indifferent – prior to observing her idiosyncratic ability, productivity, and mortality shocks – between the baseline steady state and the steady state under the carbon tax policy. Importantly, this welfare measure is ex-ante, or behind the veil of ignorance, in that it depends on the individual's expected lifetime consumption before any information about the individual is revealed. This ex-ante measure implies that an increase in equality raises welfare because it reduces the uncertainty over lifetime consumption for an individual behind the veil of ignorance.

Following Peterman and Sager (2018), we decompose the aggregate welfare impact into level, age, and distribution components. The level component measures changes in aggregate welfare that result from changes in the aggregate, economy-wide, values of hours worked and consumption of the generic and energy good. For example, suppose the carbon-tax policy increases the size of the economy, leading to more economy-wide consumption.[9] The effect of the increase in consumption on aggregate welfare is the level component. The level component would equal the total CEV in a model without income and age heterogeneity (i.e., an infinitely-lived representative individual).

The age component refers to changes in aggregate welfare that result from changes in the shape of the life cycle profiles of average consumption and hours worked, holding the

[9] See Peterman and Sager (2018) for a full description of how these effects are calculated.

aggregate level of consumption and hours constant. For example, suppose that the carbon-tax policy flattens the consumption profile across ages, implying that an individual's consumption is allocated more evenly over her lifetime. The welfare benefit from the flatter consumption-profile, holding the aggregate level of consumption constant, is the age component.

Finally, the distribution component refers to changes in aggregate welfare that result from changes in the allocation of consumption and hours within an age cohort. For example, suppose that the carbon tax policy increases the variance in consumption within an age-cohort. The welfare cost of this increase in variance is the distribution component.

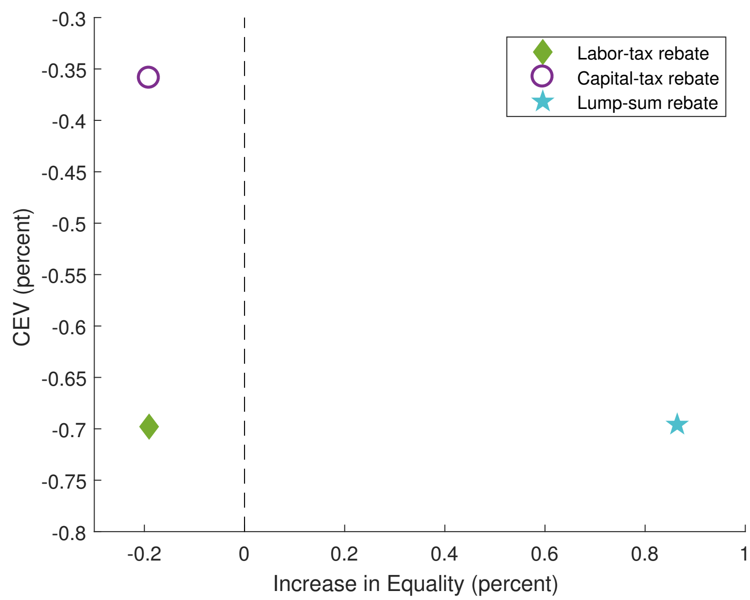
In sum, we decompose the welfare effects of a carbon-tax policy into changes in the economy wide level of consumption and hours (level component), changes in the average allocations of consumption and hours by age (age component) and changes in the distributions (around constant mean values) of consumption and hours within an age cohort (distribution component). Comparing the level component with the age and distribution components allows us to quantify the importance of changes in age and income heterogenity for aggregate welfare outcomes. Appendix A reports the formula for each CEV component.

4.3 Quantitative results

Previous work has focused on three simple rebate instruments: (1) reduction in the capital tax rate, (2) reduction in the labor tax rate, and (3) equal lump-sum transfers. Using these three instruments, researchers often find that there is a trade-off between a policy that minimizes average welfare cost but exacerbates inequality versus a policy that does not worsen inequality, but has higher average welfare cost. To demonstrate this trade-off in our model, Figure 1 plots the welfare and distributional implications of the three simple policies. The x-axis in Figure 1 is the percent increase in equality, measured as the percent decrease in the Gini coefficient from its baseline value. The y-axis is the CEV under each policy.

The CEV is always negative, implying that all three policies reduce welfare relative to the baseline. The capital-tax rate minimizes this welfare cost; it has the highest CEV of all three simple policies. However, the capital tax rebate also increases inequality; the purple open circle is to the left of the dashed line, implying that the Gini coefficient is higher under the capital tax rebate then in the baseline. The lump-sum rebate is the only policy that does not increase inequality; the blue star is to the right of the dashed line. But the welfare cost of this more progressive policy is approximately double that of the capital-tax rebate. The labor-tax rebate is inferior on both dimensions; the increase in inequality is similar to the capital-tax rebate and the welfare cost is similar to the lump-sum rebate. Thus, when policymakers are restricted to these three rebate-instruments, they face a stark trade-off between a carbon-tax policy that minimizes welfare cost (capital-tax rebate) and one that does not exacerbate inequality (lump-sum rebate). For the purpose of this paper, we take the welfare and equality effects of these simple policies as given. We refer the reader to Fried et al. (2018) for detailed explanations of each effect.

Figure 1: Welfare and Equality Effects of Simple Carbon-Tax Policies



This trade-off between welfare and equality disappears when policymakers can use the broader set of rebate instruments described in Section 4.1. In addition to the simple rebates plotted in Figure 1, this set includes the modified lump-sum rebate that varies with income, changes to the progressivity of the labor-tax, and any convex combination of these rebate instruments. To quantify the welfare and equality effects of carbon tax policy using this broader set of rebate instruments, Figure 2 plots a welfare-equality frontier. Specifically, for each level of equality, we calculate the carbon-tax policy that minimizes welfare cost. The x-axis in Figure 2 is the percent increase in equality, measured again as the percent decrease in the Gini coefficient from its baseline value. The y-axis plots the CEV for the carbon-tax policy that achieves the level of equality on the x-axis, and minimizes welfare cost. For comparison, the purple open circle, green diamond, and blue star in Figure 2 plot the simple policies from Figure 1. The most desirable policies are those in the upper right corner of Figure 2, with the smallest welfare cost and largest increase in equality.

Figure 2: Welfare-Equality Frontier

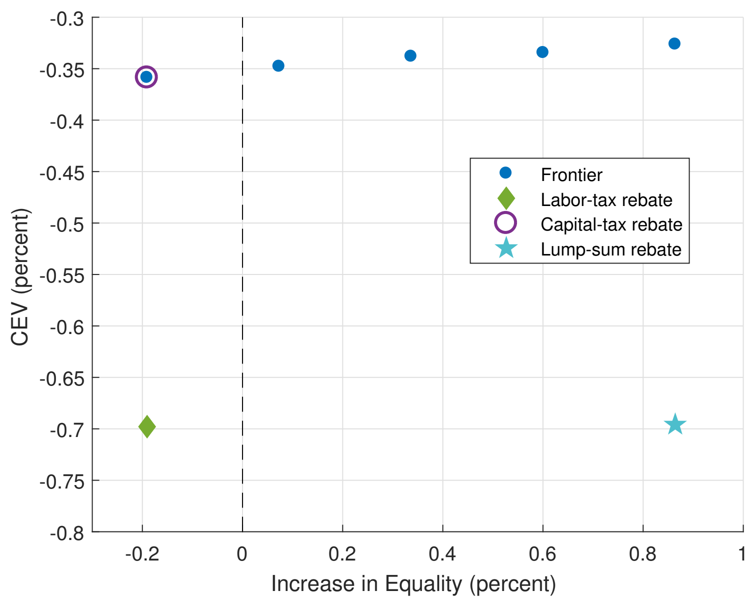
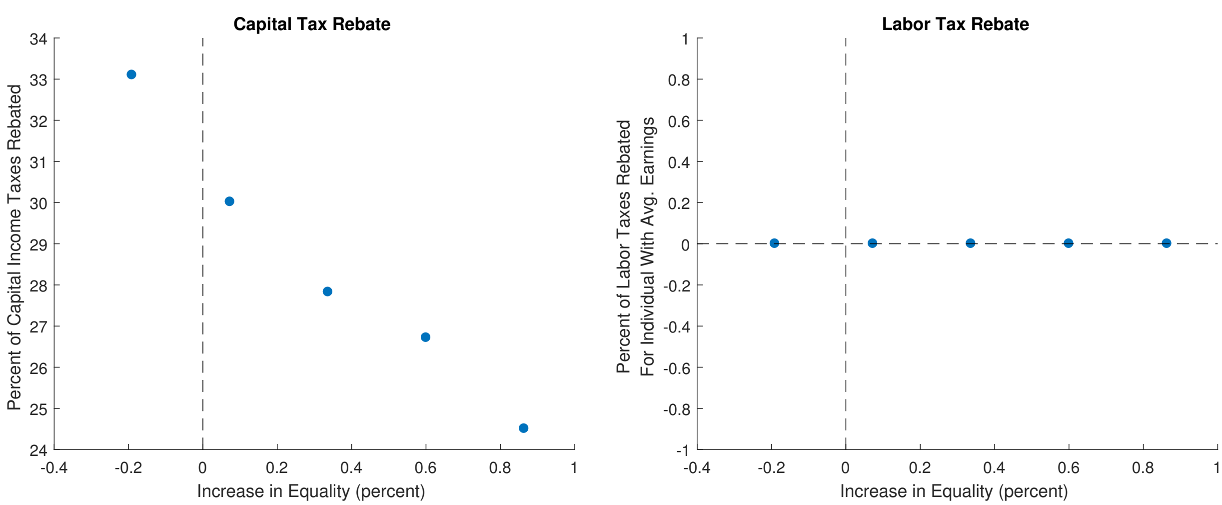


Figure 2 reveals the surprising result that the policy that minimizes the welfare cost does not increase inequality. Unlike in Figure 1, the policies with the highest CEVs in Figure 2 lie to the right of the dashed line, instead of to the left. Furthermore, the frontier in Figure 2 is slightly upward sloping, implying that the policies that achieve higher equality, are actually slightly less costly then their more regressive counterparts. Thus, using a broader set of policy instruments implies that the policymaker can simultaneously minimize welfare cost and decrease inequality.

The carbon-tax policies that achieve the frontier points are combinations of the rebate instruments discussed in Section 4.1. Figure 3 shows the capital and labor-tax rebates at each point on the frontier. We measure the capital tax rebate as the percent of capital income taxes returned to each individual. For example, at the left-most point on the frontier, each individual pays 34 percent less in capital-income taxes than in the baseline. Similarly, we measure the labor tax-rebate as the percent of labor income taxes returned to an individual with the average earnings.

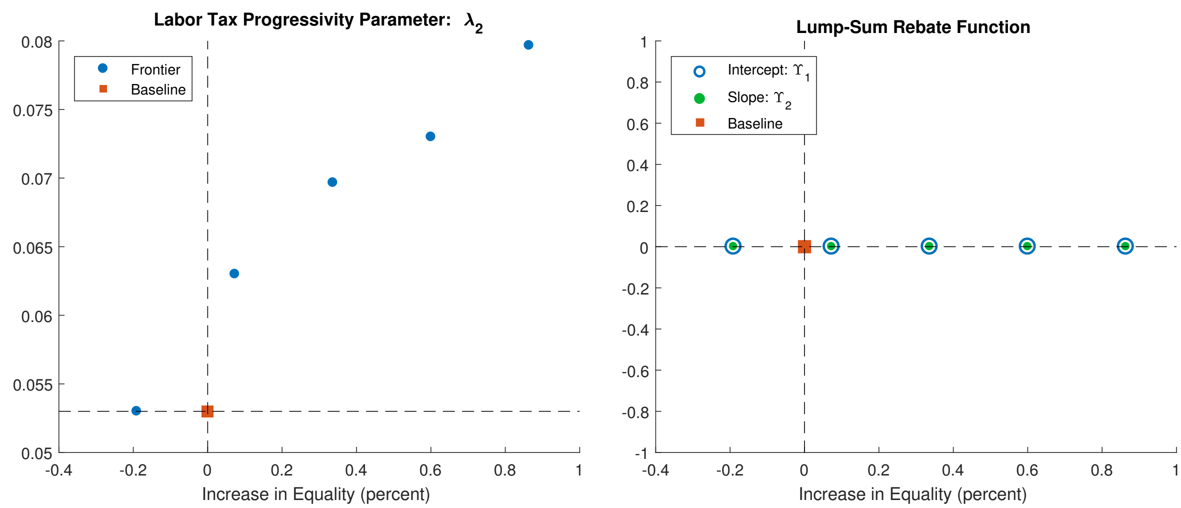
Every frontier policy includes a capital-tax rebate and none of the frontier policies include a labor-tax rebate. The result that reductions in the capital tax minimize the welfare cost is not surprising, since in isolation, the CEV under the capital-tax rebate is considerably larger than the CEV under the labor-tax rebate (see Figure 1). The capital tax has a larger distortionary cost then the labor tax in the calibrated model, and as a result, the policymaker achieves the largest welfare gains from reducing the capital-tax rate.

Figure 3: Capital and Labor Tax Rebates Along the Frontier



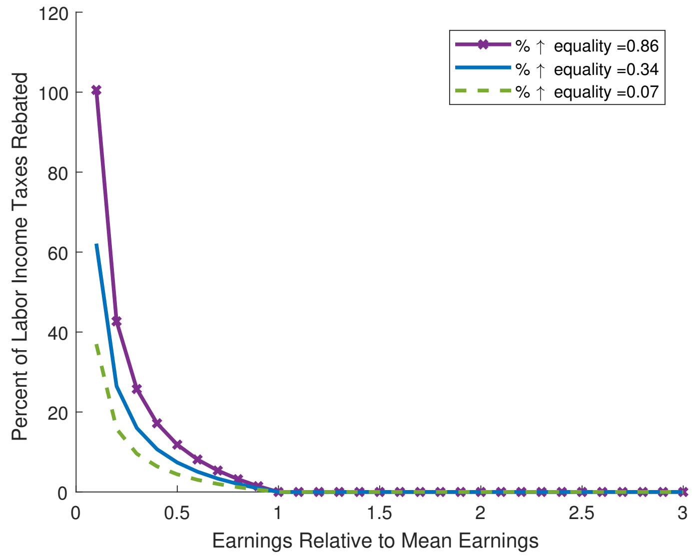
The simple policies in Figure 1 demonstrate that the carbon tax combined with only a capital-tax rebate is regressive. To achieve the improvements in equality along the frontier, the policymaker must combine the capital-tax rebate with a progressive rebate. Figure 4 plots the labor-tax progressivity parameter, λ_2 (left), and intercept and slope of the lump-sum rebate function (right). The orange squares plot baseline values of the parameters in each panel. Figure 4 reveals that the policymaker always chooses to achieve the equality target by increasing the progressivity of the labor tax. Along the frontier, the curvature parameter, λ_2, is always above its baseline value and the lump-sum rebate always equals its baseline value of zero.

Figure 4: Progressive Rebates Along on the Frontier



To interpret the increase in the labor-tax progressivity along the frontier, Figure 5 plots the percent of labor-income taxes returned to the individual as a function of the individual's labor income. The three lines correspond to different points on the frontier; the purple line is the right-most point with the largest increase in equality; the dashed green line is the second from the left-most point on the frontier, and the solid blue line is in the middle. The increase in labor-tax progressivity is designed so that the individuals with the lowest labor income receive the largest rebate and individuals with labor-income greater than the mean do not receive a rebate. Comparing the three three different lines reveals that the size of the rebate increases with the equality target, as the parameter values plotted in Figure 4 suggest. For individuals with very low labor income, the size of the rebate can exceed the tax payments. In this case, the rebate also includes a direct transfer to low labor-income individuals.

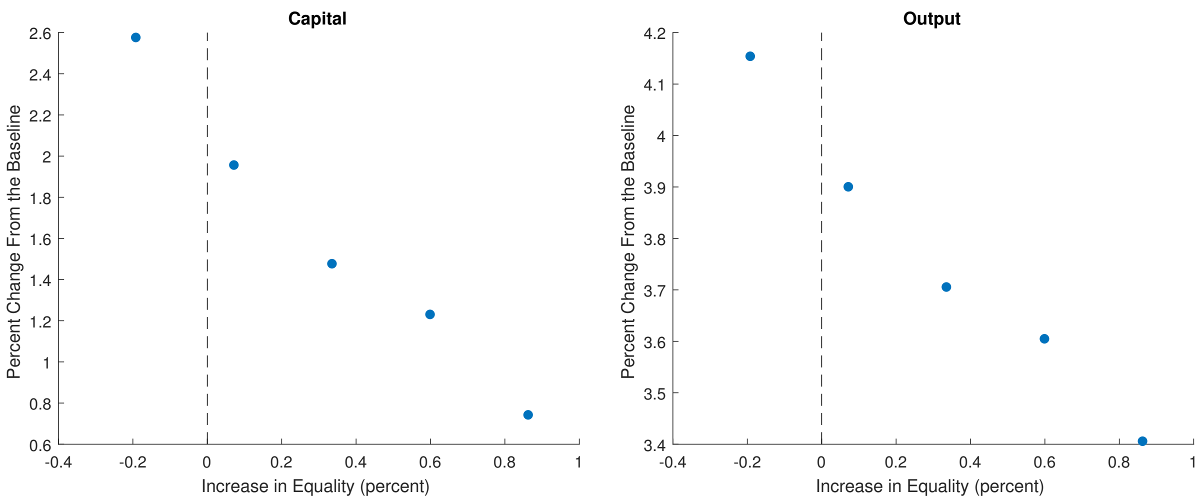
Figure 5: Rebate From Increase in Labor Tax Progressivity



In sum, the policies along the frontier combine the capital-tax rebate with an increase in the progressivity of the labor-tax (left panels of Figures 3 and 4). The left-most point on the frontier corresponds to the capital-tax rebate in Figure 1. Moving from left to right along the frontier, the policy-maker decreases the amount of revenue she returns to individuals through the capital-tax rebate (the capital-tax rebate in Figure 3 falls) and instead returns that revenue through larger progressive labor-tax rebates (the curvature parameter in Figure 4 rises). The surprising result is that rebating more of the revenue through an increase in the progressivity of the labor-income tax and less though a reduction in the capital tax actually reduces the average welfare cost of the policy.

This result is surprising for two reasons. First, increasing equality with the simple policies plotted in Figure 1 was extremely costly; the welfare cost of the progressive lump-sum rebate is approximately double that of the regressive capital-tax rebate. Second, the decrease in the capital-tax rebate along the frontier reduces the return to saving. As a result, both capital and output fall as equality increases. Figure 6 plots the percent change in these variables at each point on the frontier. Under the capital-tax rebate, the left-most point on the frontier, aggregate capital is 2.6 percent above its' value in the baseline, but at the right-most point on the frontier, aggregate capital is only 0.6 percent above its value in the baseline.

Figure 6: Capital and Output Along the Frontier

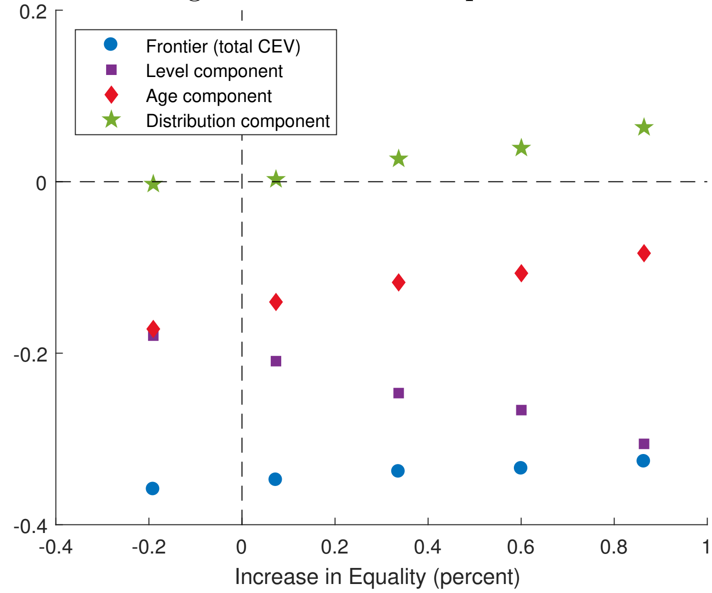


In the next two sections, we explain why we find this upward sloping frontier. First, we decompose the CEV into level, age, and distribution components to better understand which factors produce the upward slope. Second, we show that the lifecycle features of our model imply that it is much less costly to offset the regressivity of the tax by increasing the progressivity of the labor tax, then by using lump-sum rebates.

4.3.1 CEV Decomposition

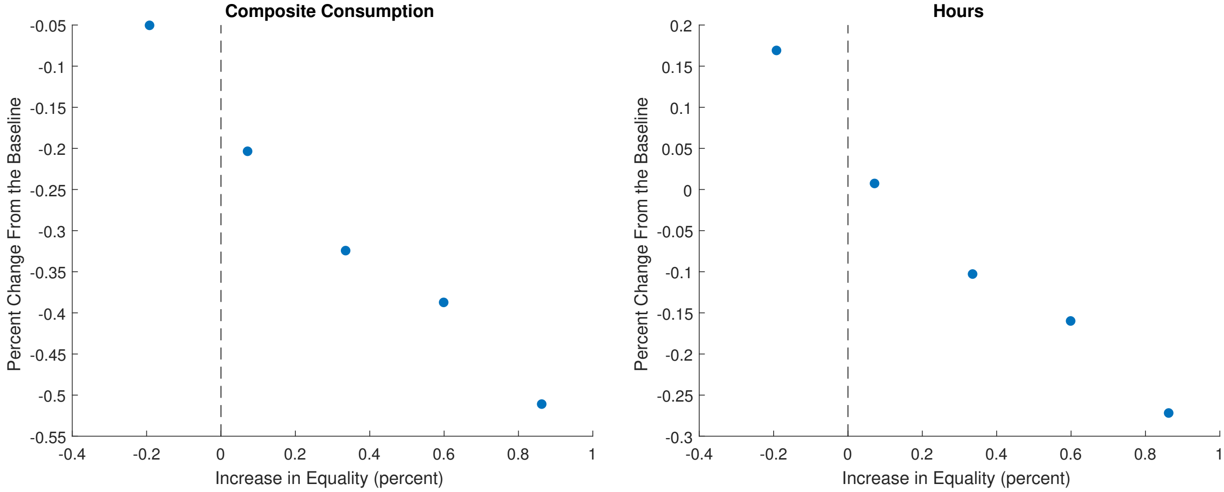
Aggregate welfare includes the contributions of changes in the level of consumption and hours (level component), changes in the shape of the consumption and hours profiles over the lifecycle (age component) and changes in the distribution of consumption and hours within an age cohort (distribution component). To quantify these different impacts, Figure 7 plots the level (purple square) age (red diamond) and distribution (green star) components of the total CEV for each policy along the frontier. For comparison, the blue dots plot the welfare-equality frontier which is equal to the total CEV.

Figure 7: CEV Decomposition



The level component of the CEV captures the impact of a change in average consumption and average hours. Figure 8, plots these variables along the frontier. Consistent with the declines in capital and output, both consumption and hours fall along the frontier. The fall in aggregate consumption reduces welfare while the fall in hours raises welfare because individuals experience disutility from working. The welfare effect of the fall in consumption dominates, causing the level component to slope downwards. Thus, abstracting from the welfare effects of changes in age and income heterogeneity produces a trade-off between welfare and equality; the policy that minimizes the level component of welfare also increases inequality.

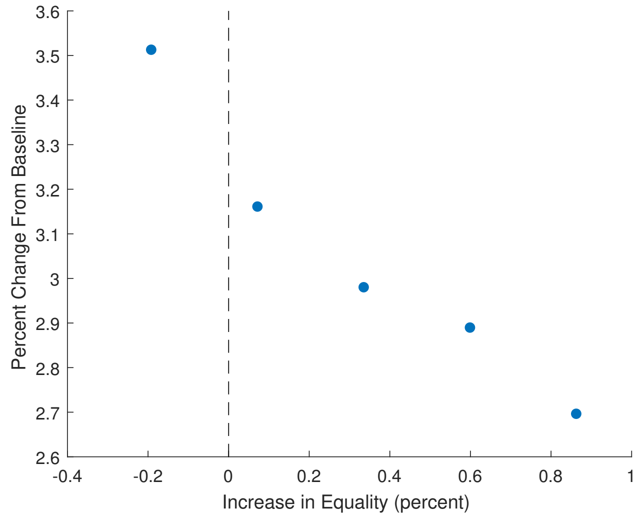
Figure 8: Composite Consumption and Hours Along the Frontier



Both the distribution and age components of total CEV are upward sloping. Combined, these two components dominate the downward-sloping level component, causing the total CEV to be slightly upward sloping along the frontier. The distribution component is upward sloping because an increase in equality reallocates consumption from high-income individuals with low marginal utilities of consumption to low-income individuals with high marginal utilities of consumption, raising average welfare. Furthermore, our ex-ante measure of welfare implies that all individuals value this increase in equality because it increases expected lifetime welfare and reduces uncertainty over lifetime welfare from behind the veil of ignorance.

The age component is upward sloping along the frontier because the after-tax interest rate falls as equality increases (Figure 9). The fall in the after-tax interest rate leads individuals to shift consumption to earlier in life, effectively flattening their consumption profiles. A flatter consumption profile brings the individual closer to perfectly smooth life cycle consumption, raising welfare.

Figure 9: After-Tax Interest Rate



4.3.2 Increase in Labor Tax Progressivity vs. Lump-Sum Rebate

The ability to rebate the carbon tax revenue through an increase in the progressivity of the labor tax is crucial for the upward-sloping frontier in Figure 2. To demonstrate the importance of this rebate option, we calculate a restricted welfare-equality frontier in which the policymaker cannot change the progressivity of the labor tax. Along this restricted frontier, the lump-sum rebate function is the policymaker's only tool to increase equality. The green open circles in Figure 10 plot the restricted welfare-equality frontier when the policymaker cannot change the progressivity of the labor tax. For comparison, the blue dots plot the unrestricted welfare-equality frontier from Figure 2 when the policymaker can use the full set of rebate instruments.

In contrast to the unrestricted frontier, the restricted frontier is downward sloping, implying that the policy that minimizes the welfare cost also increases inequality, as was the case with the simple policies plotted in Figure 1. Thus, consistent with the results in previous literature, we find that there is a stark trade-off between welfare and equality when policymakers can only use lump-sum rebates to increase the progressivity of the carbon-tax policy.

Figure 10: Restricted and Unrestricted Welfare-Equality Frontiers

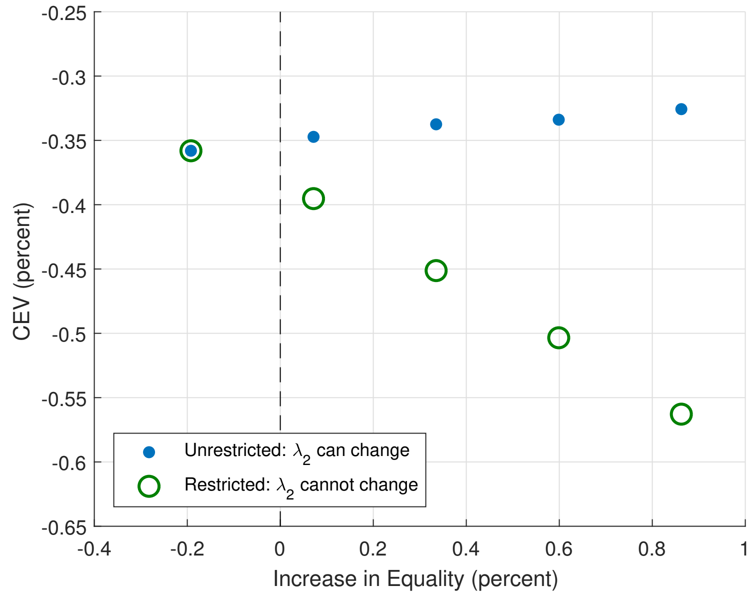
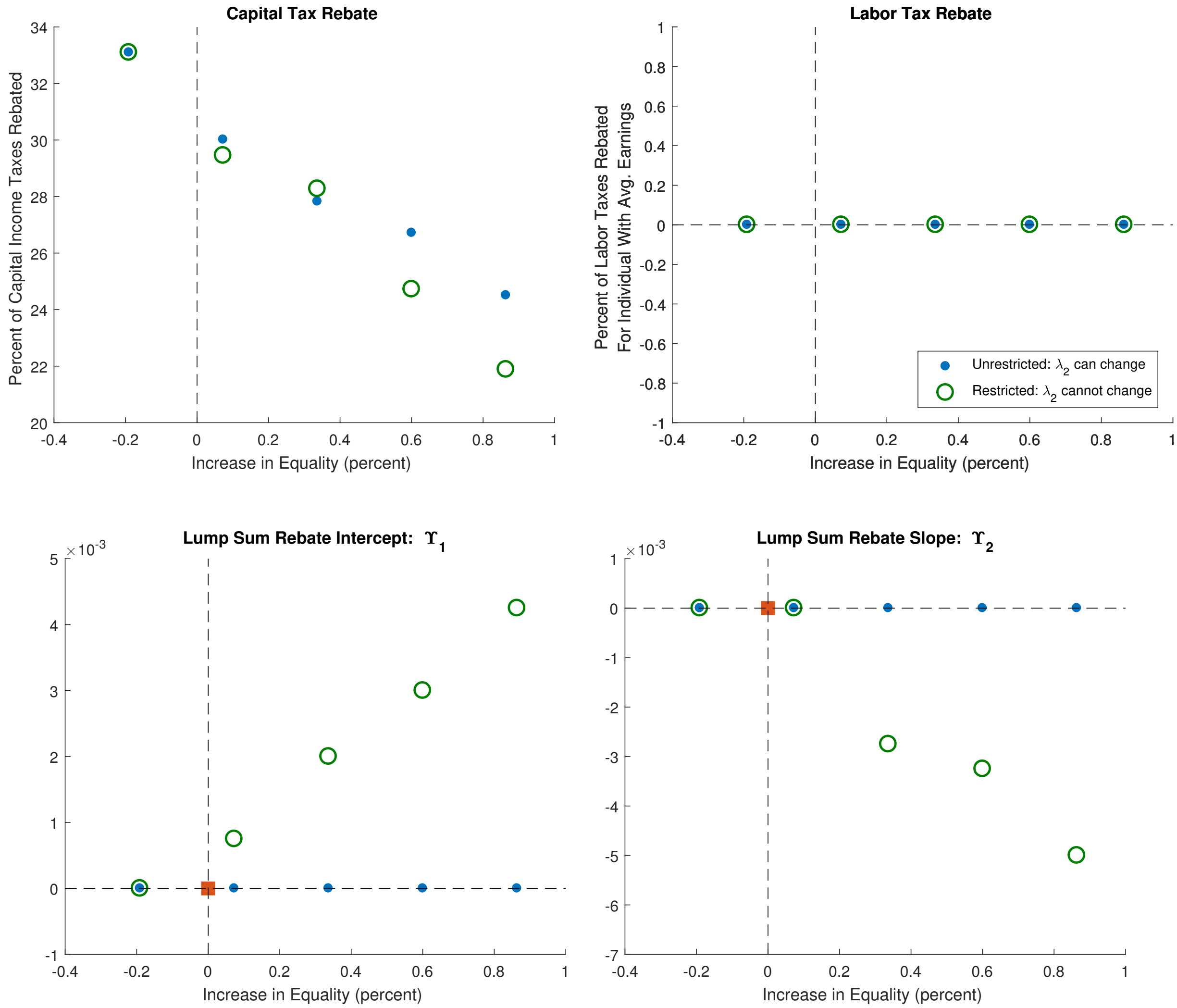


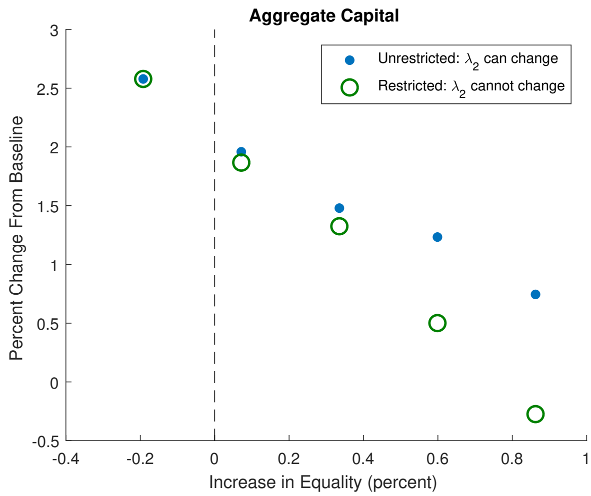
Figure 11 plots the rebate instruments on the restricted (open green circles) and unrestricted (blue dots) frontiers. Like the unrestricted frontier, the restricted frontier relies only on the capital-tax rebate, the labor-tax rebate always equals zero (top two panels of Figure 11). Instead of relying on changes in the progressivity of the labor-tax rebate to achieve a give level of equality, the policymaker on the restricted frontier increases the level of the lump-sum rebate function and decreases its slope (bottom two panels of Figure 11).

Figure 11: Rebate Instruments on the Restricted and Unrestricted Frontiers



The restricted frontier lies below the unrestricted frontier, implying that the lump-sum rebate is more costly than the increase in labor-tax progressivity. The cost difference stems from the rebates' different impacts on individuals savings' behavior over the lifecycle. Importantly, individuals receive the lump-sum rebate in every year of life, including retirement. As a result, the lump-sum rebate directly reduces an individual's need to save for retirement, crowding out capital. In contrast, since individuals only receive the labor-tax rebate during their working years, it does not crowd out as much capital and thus is less costly. Figure 12 plots the percent change in capital from the baseline at each point on the restricted and unrestricted frontiers. Capital is always lower along the restricted frontier.

Figure 12: Percent Change Capital Along the Restricted and Unrestricted Frontiers



Our results imply that policymakers should combine the capital-tax rebate with an increase in the labor-tax progressivity to minimize the welfare cost of the policy and without increasing inequality. However, an important concern is that the rebate from increase in labor-tax progressivity might not reach working-age people who are unemployed. If these people are also lower income, then this would reduce the effectiveness of the rebate as a tool for increasing equality. Our model abstracts from unemployment. However, the annual time period implies that as long as an individual (or head of household) has at least some labor income over the course of the year, then she will receive a labor-tax rebate. Furthermore, very low income individuals receive a transfer from the labor-tax function, instead of paying a tax. Under the increase in labor-tax progressivity, individuals who are unemployed most of the year, and thus have very low labor income, would be compensated for the effects of the carbon tax through a larger transfer.

5 Conclusion

Overall, the classic rebate instruments (capital-tax, labor-tax, and lump-sum) imply that policymakers must choose between a policy that minimizes welfare cost but increases inequality or a policy that does not increase inequality, but has a higher welfare cost. However, we find that incorporating a broader set of rebate instruments and allowing for combinations of different instruments eliminates this stark choice between welfare and equality. The set of carbon tax policies that minimize welfare cost without increasing inequality return the revenue to the households through both a capital-tax rebate and an increase in the progressivity of the labor-tax function. Previous studies typically focus on the lump-sum rebate as the progressive rebate instrument. However, we find that allowing policymakers to change to the progressivity of the labor-tax function is a more powerful tool to reduce inequality than the standard lump-sum rebate.

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A CEV Decomposition

Define the CEV between two steady states, denoted by a, and, b, as the percent increase in expected consumption an individual would need in every period in steady state a so that she is indifferent between steady state a and steady state b. More formally, the CEV between steady states a and b solves the following equation

$$\int_0^1 \sum_{j=1}^{J} \beta^{j-1} \Psi_j U((1+\Delta_{CEV})\tilde{c_j}^a, h_j^a) d\Phi(x^a) = \int_0^1 \sum_{j=1}^{J} \beta^{j-1} \Psi_j U(\tilde{c_j}^b, h_j^b) d\Phi(x^b), \tag{22}$$

where superscripts a and b denote the values in steady states a and b respectively and $\Psi_j \equiv \prod_{j=1}^{j-1} \psi_j$. Hence, positive values of the CEV imply that individuals are better off in steady state b and negative values imply that they are better off in steady state a. Following Peterman and Sager (2018), we divide the CEV into level, age, and distribution components,

$$(1+\Delta_{CEV}) = (1+\Delta_{level})(1+\Delta_{age})(1+\Delta_{dist}). \tag{23}$$

Since utility depends on both consumption and hours, we further divide each CEV component into a consumption part and an hours part,

$$(1+\Delta_{CEV}) = [(1+\Delta_{C_{level}})(1+\Delta_{H_{level}})][(1+\Delta_{C_{age}})(1+\Delta_{H_{age}})][(1+\Delta_{C_{dist}})(1+\Delta_{H_{dist}})].$$

We provide the definition and intuition for each component below. See Peterman and Sager (2018) for a complete proof that the decomposition holds.

The level-consumption component measures the effect of a change in the average level of consumption between the two steady states,

$$1 + \Delta_{C_{level}} = \frac{\bar{c}^b}{\bar{c}^a},$$

where $\bar{c}^a$ and $\bar{c}^b$ denote average consumption in steady states a and b, respectively. The level component is equivalent to the total CEV in a model without age and income heterogeneity (e.g., an infinitely lived representative individual).

The age-consumption component measures the welfare effects of changes in an individual's profile of consumption over the life-cycle, controlling for any changes in the average level of consumption (the level component). Mathematically, the age-component equals,

$$1 + \Delta_{C_{age}} = \frac{\left(\sum_{j=1}^{J} \beta^{j-1}\Psi_j(\bar{c}_j^b)^{1-\theta_1}\right)^{\frac{1}{1-\theta_1}} / \bar{c}^b}{\left(\sum_{j=1}^{J} \beta^{j-1}\Psi_j(\bar{c}_j^a)^{1-\theta_1}\right)^{\frac{1}{1-\theta_1}} / \bar{c}^a} \tag{24}$$

where $\bar{c}_j^a$ and $\bar{c}_j^b$ denote the average consumption at age j in steady states a and b respectively. Thus, the numerator in equation (24) denotes the expected lifetime welfare, measured in terms of consumption-equivalence, for an average individual in steady state b, normalized by the level of consumption in steady state b. The denominator is the analogous value for steady state a.

Finally, the distribution effect measures the welfare effects of changes in the variance of consumption within an age cohort, holding constant the average level of consumption at each age. The distribution component equals,

$$1 + \Delta_{C_{dist}} = \left[\frac{\left(\int_0^1 \sum_{j=1}^{J} \beta^{j-1}\Psi_j(\tilde{c}_j{}^b)^{1-\theta_1} d\Phi(x^b)\right) / \left(\sum_{j=1}^{J} \beta^{j-1}\Psi_j(\bar{c}_j{}^b)^{1-\theta_1}\right)}{\left(\int_0^1 \sum_{j=1}^{J} \beta^{j-1}\Psi_j(\tilde{c}_j{}^a)^{1-\theta_1} d\Phi(x^a)\right) / \left(\sum_{j=1}^{J} \beta^{j-1}\Psi_j(\bar{c}_j{}^a)^{1-\theta_1}\right)}\right]^{\frac{1}{1-\theta_1}}. \tag{25}$$

The left term in the numerator equals the consumption component of expected lifetime welfare for an individual in steady state b. We divide this value by the consumption component of expected lifetime welfare for an individual with the average consumption profile in steady state b. This division removes the welfare effects of the age and level components. What remains is the distribution component. The denominator is the equivalent expression for steady state a.

The intuition for the hours components of CEV is same as for the consumption-components. However, the expressions are more complex because we measure the utility effects of changes in hours in terms of consumption, instead of hours. The hours pieces of the level, age, and distribution CEV components are,

$$1+\Delta_{H_{level}} = \left[1+\left(1-\frac{\bar{h}^b}{\bar{h}^a}\right)^{\frac{1}{1+\frac{1}{\theta_2}}} \times \left(\frac{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \chi \left(\frac{({h_j}^a)^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}}\right) d\Phi(x^a)}{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \left(\frac{(\tilde{c_j}^b)^{1-\theta_1}}{1-\theta_1}\right) d\Phi(x^b)}\right)\right]^{\frac{1}{1-\theta_1}}$$

$$1+\Delta_{H_{age}} = \left[1+\left(\left(\frac{\bar{h}^b}{\bar{h}^a}\right)^{\frac{1}{1+\frac{1}{\theta_2}}} - \frac{\sum_{j=1}^J \beta^{j-1}\Psi_j(\bar{h}_j^b)^{1+\frac{1}{\theta_2}}}{\sum_{j=1}^J \beta^{j-1}\Psi_j(\bar{h}_j^a)^{1+\frac{1}{\theta_2}}}\right) \times \left(\frac{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \chi \left(\frac{({h_j}^a)^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}}\right) d\Phi(x^a)}{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \left(\frac{(\tilde{c_j}^b)^{1-\theta_1}}{1-\theta_1}\right) d\Phi(x^b)}\right)\right]^{\frac{1}{1-\theta_1}}$$

$$1+\Delta_{H_{dist}} = \left[1+\left(\frac{\sum_{j=1}^J \beta^{j-1}\Psi_j(\bar{h}_j^b)^{1+\frac{1}{\theta_2}}}{\sum_{j=1}^J \beta^{j-1}\Psi_j(\bar{h}_j^a)^{1+\frac{1}{\theta_2}}} - \frac{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j({h_j}^b)^{1+\frac{1}{\theta_2}} d\Phi(x^b)}{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j(\bar{h}_j^a)^{1+\frac{1}{\theta_2}} d\Phi(x^a)}\right) \times \right.$$
$$\left.\left(\frac{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \chi \left(\frac{({h_j}^a)^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}}\right) d\Phi(x^a)}{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \left(\frac{(\tilde{c_j}^b)^{1-\theta_1}}{1-\theta_1}\right) d\Phi(x^b)}\right)\right]^{\frac{1}{1-\theta_1}}$$

The term,

$$\frac{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \chi \left(\frac{({h_j}^a)^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}}\right) d\Phi(x^a)}{\int_0^1 \sum_{j=1}^J \beta^{j-1}\Psi_j \left(\frac{(\tilde{c_j}^b)^{1-\theta_1}}{1-\theta_1}\right) d\Phi(x^b)}$$

converts the welfare effect of a change in hours into consumption units.