

Spillovers in a Monetary Union with Endogenous Fiscal Limits

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Preliminary and incomplete. Please do not cite without the authors' permission

February 28, 2019

Abstract

In this paper we study fiscal policy effects and fiscal spillovers for countries in a monetary union with different levels of public debt. We develop a two-country euro area DSGE model, calibrated to match the characteristics of Spain and Germany, with an endogenously determined fiscal limit a la [Bi \(2012\)](#) that shapes the responses of the risk premium on public debt. Policy shocks change the market's expectation about future primary surplus, producing a direct effect on the sovereign risk premium and macroeconomic responses of the economy. We find that a fiscal consolidation, when combined with endogenous responses of the fiscal limit, may become expansionary in the long run. Then we analyze how different risk sharing mechanisms within a monetary union affect the fiscal limit and the macroeconomic performance.

JEL codes: E31, E62, H30

1 Introduction

The global financial and economic crisis left a legacy of historically high levels of public debt in advanced economies, at a scale unseen during modern peace time. Keeping public debt at high levels, however, is a source of vulnerability in itself, particularly given the arising fiscal and economic pressures from ageing. A high public debt burden is even more problematic in a monetary union like the euro area (EA), as monetary policy focuses on the EA aggregate while fiscal policies remain at national level. As shown in [Figure 1](#), although the average debt to GDP ratio stays high at 90%, a great dispersion exists across countries. In fact we have a group of countries with reasonably low debt levels, standing at around or below 60%, including Germany (64%) and The Netherlands (56%), while another group is

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characterized by high debt levels, around or above 100% of GDP, including France (97%), Italy (132%) and Spain (98%). In those highly indebted countries, borrowing costs have increased sharply, which undermine their solvency. Moreover, risks to debt sustainability in a high-debt Member State, like the ones recently experienced in Italy in relation with the change of government, can entail risks to the stabilization of the monetary union as a whole, while cross-country spillovers of disorderly default can threaten the very existence of the EA.

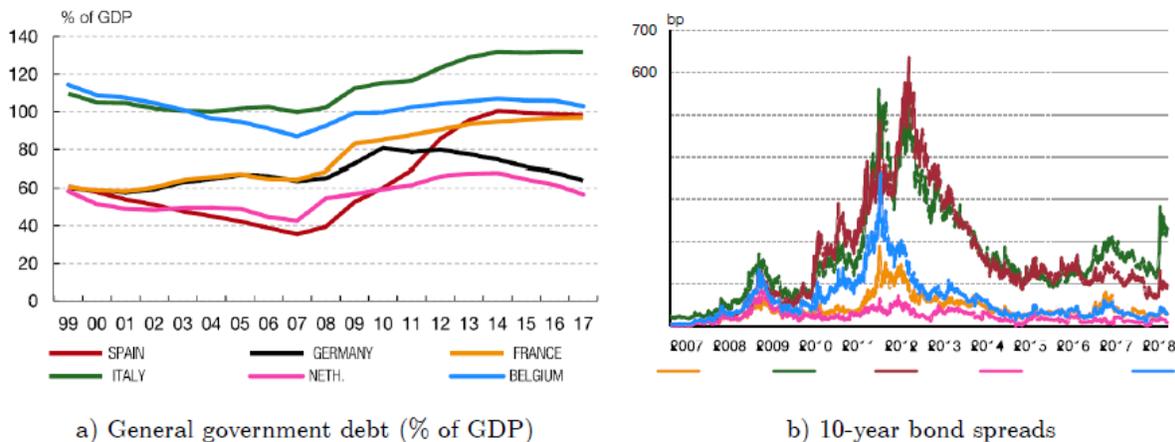


Figure 1: Evolution of government debt and spreads in the euro area

In response to the crisis, European authorities embarked on a thorough process of reforming the economic governance of the EU and the EA. It aimed to strengthen first the resilience of sovereigns and then of banks to shocks, by both reducing their individual risk potential and increasing risk sharing. Significant progress has been achieved, but risk reduction has been favored over risk sharing.

This paper aims to contribute to exploring the interactions among high public debt-GDP levels, country-specific fiscal policy, and fiscal spillovers within a monetary union. To do this, we extend an otherwise standard DSGE model of a two-country monetary union along the lines of [Benigno and Benigno \(2006\)](#), modified to allow for endogenously determined fiscal limits. In particular, we allow for the possibility of (partial) government default, so that a haircut is applied whenever one of the country members of the monetary union hits its “fiscal limit.” This fiscal limit, following [Bi \(2012\)](#), is defined as the expected discounted sum of

maximum primary surplus that can be generated in the future. Thus, this limit depends upon both the state of the home and foreign economies, and the political constraints on revenue collection capacity. Given exogenous fluctuations in productivity, government spending and political risk, this fiscal limit is stochastic and represented by a distribution of the maximum debt-GDP levels that can be supported. Therefore, investors may demand risk premia on government debt before reaching the fiscal limit, generating a nonlinear relationship between sovereign risk premia and the level of government debt. The high-debt country (home), as it approaches its fiscal limit, pays a higher default risk premium on its public debt. The low-debt country (foreign), however, is far away from its fiscal limit and hence pays the risk free rate.

The simulated fiscal limit is dynamic, depending on the underlying macroeconomic fundamentals and fiscal policy in both countries. Policy decisions affect the fiscal limit distribution and the sovereign risk premium. A fiscal consolidation in the home country, even if temporary, implies an enhancement in public finances and an immediate improvement on its debt sustainability prospects in the short and medium run. As a result, lower government spending raises fiscal surplus and the fiscal limit. A fiscal expansion in the foreign country, on the other hand, lowers the home country's debt sustainability, as the central bank tightens its policy rate in response to the inflationary pressures coming from the foreign country. The endogenous fiscal limits in a two-country monetary union creates fiscal spillovers that imply quite different macroeconomic performance for a country with high debt, in comparison to an economy operating well away from its fiscal limit, generating a powerful sovereign risk premium channel.

With endogenous fiscal limit distributions, the model is used to analyze three fiscal issues often debated among European countries: fiscal consolidation in the high-debt country, fiscal expansion in the low-debt country, and policy coordination in the two countries. We show that in the high-debt country, after a fiscal consolidation through spending cuts the risk premium returns to its steady-state level faster, contributing to generate expansionary effects in the long run. A fiscal consolidation today implies higher future primary surpluses,

shifting the fiscal limit distribution immediately to the right and reducing on impact the probability of default for all debt levels, which then go back slowly towards its steady state level as the transitory reduction in government spending is unwound. This effect on the risk premium is small but persistent in time. In our simulations a 1% reduction in government spending reduces risk premium on impact by 1.5 basis points (bp), reaching a maximum of 4 bp after 5 years, but it is still 2 bp lower after 10 years. The lower financing costs reduce significantly the cost of consolidating and in the long run it may even change its sign, becoming expansionary, consistent with the empirical findings in [Giavazzi and Pagano \(1990\)](#), [Alesina *et al.* \(1998\)](#), and [Alesina and Ardagna \(2010\)](#).

The results of expansionary fiscal consolidation in the long run depend crucially on the endogenous response of fiscal limits to policy shocks. [Corsetti *et al.* \(2013\)](#) and [Corsetti *et al.* \(2014\)](#) introduce sovereign default risk by assuming the *ex ante* probability of default as an increasing function of sovereign indebtedness, which we refer as “exogenous fiscal limit.” Although their specification captures the uncertainty that surrounds default decisions, the default probability is independent of current domestic and foreign fiscal stances. Therefore, in their model, a fiscal consolidation does not trigger an immediate fall in the risk premium upon the policy implementation, but instead the risk premium is reduced slowly as debt falls. According to our simulations the sovereign risk premium channel can be economically sizable. The 10-year cumulative fiscal multiplier after a discretionary government spending cut in a model without sovereign default is 0.27 (i.e.: a 1\$ reduction in government spending reduces output by 0.27\$), it changes its sign, becoming expansionary (-0.2) when default is introduced through exogenous fiscal limit, while the fiscal consolidation becomes more expansionary (-0.50), when the fiscal limit is endogeneized. When monetary policy is constrained by the Zero Lower Bound (ZLB) on nominal interest rates, the expansionary fiscal consolidation is counteracted by an increase in the real interest rate.

We then analyze the spillover effects on a high debt country from the fiscal decisions of the other monetary union member, characterized by low debt, as well as the result of fiscal policy coordination in the union. A fiscal expansion in the foreign country leads to inflation,

and given its large size, increases the nominal interest rate in the Euro Area and real interest rates in both countries. Lower demand and higher real rates in the home country worsen its fiscal sustainability and raise the risk premium. As debt accumulates faster, high tax rates further depress output in the home country. We find that these negative spillover effects are weakened when nominal interest rates reach the ZLB.

Finally, we explore how these results are affected by some of the risk sharing mechanisms available within a monetary union and currently under discussion in the policy debate. In particular, we extend the baseline model to allow for cross border purchases of sovereign debt (in the baseline version we assume total home bias in sovereign debt), the introduction of a Eurobond or the creation of a fiscal capacity at the EA level to smooth the effects of asymmetric shocks. [This section is to be completed]

Our paper is related to several studies that assess sovereign risk premia and fiscal sustainability. [Daniel and Shiamptanis \(2012\)](#) assume government debt is constrained by an ad-hoc fiscal limit to assess fiscal crisis probabilities in the context of monetary union. [Polito and Wickens \(2015\)](#) present a model-based measure of sovereign credit ratings for EU countries by estimating the probability that the debt to GDP ratio will exceed a given limit or threshold at any time over a given time horizon. [Uribe and Yue \(2006\)](#), [Garcia-Cicco *et al.* \(2010\)](#), [Corsetti *et al.* \(2013\)](#), and [Corsetti *et al.* \(2014\)](#) consider an exogenous risk premium by assuming that the sovereign risk premium is monotonically increasing in the level of government debt. Our paper constructs model-consistent fiscal limits that account for interactions between economic policy and the sovereign risk premium. Our analysis is also related to papers that study cross-border spillovers from fiscal stimulus, such as [Corsetti *et al.* \(2010\)](#), [Arce *et al.* \(2016\)](#), and [Blanchard *et al.* \(2017\)](#). These works find that fiscal adjustment instruments, structural reforms, and monetary policy all matter for the magnitude of fiscal spillovers in the Euro Area, but they do not incorporate default risk. Our paper is not meant to add to the theory of sovereign default, as in [Eaton and Gersovitz \(1981\)](#), [Arellano \(2008\)](#) and [Mendoza and Yue \(2012\)](#). Our model retains the DSGE framework convenient for incorporating several economic and policy shocks and conducting fiscal experiments without

explicitly modeling the strategic default decision.

2 Model

We use a two-country New Keynesian model to analyze monetary unions along the lines of [Benigno and Benigno \(2006\)](#), modified only by allowing government debt to be risky. Specifically, the monetary union consists of two countries, home and foreign, each inhabited by a continuum of households, with parameter s determining the relative size of the home country. The foreign variables are defined with an asterisk. Households in each country of the union supply labor to imperfectly competitive intermediate goods producing firms. Facing costly [Rotemberg \(1982\)](#)-style price adjustment, these firms do not completely adjust prices in the face of shocks. Moreover, we assume that households' labor and profit income is taxed. The distortionary taxes influence households' labor supply decisions, which in turn affect firms' marginal costs and pricing decisions.

We then further extend this model to allow for the possibility of (partial) government default: a haircut is applied whenever one of the 2 country members of the monetary union hits its "fiscal limit". This fiscal limit is defined as the present value of maximum future primary surpluses, which depends upon both the state of the home and foreign economies and the political constraints on taxing at the peak of Laffer curve. Given exogenous fluctuations in productivity, government spending and political risk, the fiscal limit is stochastic and investors may demand risk premia on government debt before reaching the fiscal limit. The resulting debt dynamics may imply quite different macroeconomic performance in comparison to an economy operating well away from its fiscal limit. Since in reality, the fiscal positions of union members tend to be very different, so that one country is always far away from its fiscal limit, for simplicity we assume that in that case there is no fiscal limit.

Finally, we start by assuming total home bias in domestic government debt. This assumption will be relaxed in [section 6](#), where we study the impact of different risk sharing mechanisms within a monetary union [TBC].

In the rest of the section we will describe the model for the home country and only mention the rest of the union when there is an asymmetry.

2.1 Households

The home country is populated by a large number of households indexed by $h \in [0, s)$, while those living in the foreign country are indexed by $f \in [s, 1]$. Preferences are given by:

$$\max_{c_t, B_t, n_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} \right] \quad (1)$$

where β is the households' subjective discount factor, c_t is consumption and n_t the households' labor supply. The inverse of intertemporal elasticity of substitution, σ , measures relative risk aversion. The parameter φ governs the Frisch elasticity of labor supply. The household receives nominal wages W_t and monopoly profits Υ_t from the firm, both of which are taxed at the rate τ_t , and lump-sum transfers, z_t , from the government. The household maximizes utility subject to the budget constraint,

$$P_t c_t + \frac{B_t}{R_t} = (1 - \delta_t) B_{t-1} + (1 - \tau_t)(W_t n_t + P_{H,t} \Upsilon_t) + P_{H,t} z_t \quad (2)$$

where P_t is the CPI and $P_{H,t}$ is the PPI.

The government debt in the home country is subject to default risk. The default decisions depend on a realized effective fiscal limit, \mathcal{B}_t^H , drawn from a fiscal limit distribution $\mathcal{B}^H(\mathcal{S}_t)$, conditional on the state \mathcal{S}_t . Specifically,

$$\delta_t = \begin{cases} 0 & \text{if } b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if } b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t) \end{cases} \quad (3)$$

where $b_{t-1} = \frac{B_{t-1}}{P_t}$ is the real government debt. If the real value of debt at the beginning of period t , b_{t-1} , exceeds the effective fiscal limit, $\mathcal{B}^H(\mathcal{S}_t)$, then the government partially defaults and outstanding debt at the beginning of period t becomes $(1 - \delta) b_{t-1}$, otherwise it repays in full amount with $\delta_t = 0$. The simulation of $\mathcal{B}^H(\mathcal{S}_t)$ is to be described in Section

4.1.

Government debt in the home country, therefore, pays a risky yield of R_t . Optimization conditions for households in the home country are:

$$n_t^\varphi = \lambda_t(1 - \tau_t)w_t \quad (4)$$

$$\lambda_t = \beta R_t E_t \frac{(1 - \delta_{t+1})\lambda_{t+1}}{\pi_{t+1}} \quad (5)$$

where $\lambda_t = c_t^{-\sigma}$, $\pi_{t+1} = \frac{P_{t+1}}{P_t}$, and $w_t = \frac{W_t}{P_t}$ is the real wage.

The households' optimization problem must also satisfy the following transversality condition:

$$\lim_{j \rightarrow \infty} E_t \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} (1 - \delta_{t+j+1}) b_{t+j} = 0 \quad (6)$$

In the case of the foreign country government, we assume the level of debt is well below its fiscal limit. Thus the foreign government will never default on its debt and the foreign bonds (B_t^*) pay the risk-free rate (R_t^f). In this case we have the standard intertemporal Euler equation:

$$\lambda_t^* = \beta R_t^f E_t \frac{\lambda_{t+1}^*}{\pi_{t+1}^*} \quad (7)$$

where $\lambda_t^* = c_t^{*-\sigma}$, $\pi_{t+1}^* = \frac{P_{t+1}^*}{P_t^*}$. Using both Euler equations in the two countries ((5) and (7)), we can derive an arbitrage condition linking the real exchange rate, $RE R_t = \frac{P_t^*}{P_t}$, to differences in nominal interest rates and consumption levels

$$RE R_t = \Gamma_0 \left(\frac{c_t^*}{c_t} \right)^{-\sigma} \frac{R_{t-1}^f}{R_{t-1} (1 - \delta_t)} \quad (8)$$

where $\Gamma_0 = RE R_0 \left(\frac{c_0}{c_0^*} \right)^{-\sigma} \frac{R_0^f}{R_0 (1 - \delta_1)} = 1$, is a constant including only initial conditions for asset holdings and interest rates, which we assume equal to 1 to simplify the analysis.

2.2 Final consumption goods

Households consume the following basket of final goods produced at home, $c_{H,t}$, and abroad, $c_{F,t}$,

$$c_t = \left(\frac{c_{H,t}}{\eta} \right)^\eta \left(\frac{c_{F,t}}{1-\eta} \right)^{1-\eta} \quad (9)$$

where η represents the preference by home consumers for goods produced at home, we say it exists home bias in consumption when $\eta > \frac{1}{2}$. The demand for final goods produced at home and abroad and the home consumer price index are

$$c_{H,t} = \eta \left(\frac{P_{F,t}}{P_{H,t}} \right)^{1-\eta} c_t = \eta TOT_t^{1-\eta} c_t \quad (10)$$

$$c_{F,t} = (1-\eta) \left(\frac{P_{F,t}}{P_{H,t}} \right)^{-\eta} c_t = (1-\eta) TOT_t^{-\eta} c_t \quad (11)$$

$$P_t = P_{H,t}^\eta P_{F,t}^{1-\eta} \quad (12)$$

where $TOT_t = P_{F,t}/P_{H,t}$ represents the relative terms of trade.

2.3 Final intermediate goods

Differentiated Intermediate goods produced at home $y_{H,t}(h)$ are bundled together into final home intermediate goods $y_{H,t}$, according to the following technology:

$$y_{H,t} = \left[\left(\frac{1}{s} \right)^{\frac{1}{\theta}} \int_0^s y_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} \quad (13)$$

where θ represents the elasticity of substitution between different good-varieties, equal across regions, and $\frac{\theta}{\theta-1}$ is the price mark-up. These final intermediate goods can be used to produce final home or foreign consumption goods ($c_{H,t}(h)$ or $c_{H,t}^*(h)$) and home public spending (g_t). Cost minimization on the part of final goods producers results in the following demand curve for the intermediate home good, $y_{H,t}(h)$, and the corresponding home producer price index, $P_{H,t}$,

$$y_{H,t}(h) = \frac{1}{s} \left(\frac{p_{H,t}(h)}{P_{H,t}} \right)^{-\theta} y_{H,t} \quad (14)$$

$$P_{H,t} = \left[\frac{1}{s} \int_0^s p_{H,t}(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}. \quad (15)$$

2.4 Intermediate goods production

Intermediate goods producers adopt a linear production technology, $y_t(h) = a_t n_t(h)$, with real marginal costs, $mc_t(h) = \frac{P_t}{P_{H,t}} \frac{w_t}{a_t}$ ¹. These firms enjoy some monopoly power in producing a differentiated product and therefore face a downward sloping demand curve, but are also subject to [Rotemberg \(1982\)](#) quadratic-adjustment costs in changing prices. That is, in each period, firms pay a cost proportional in real terms to aggregate real income $pac_t(i) = \frac{\psi}{2} \left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)} - 1 \right)^2 y_t$ to be able to change their prices and this penalizes large price changes in excess of steady state inflation rates. The dynamic problem of firm h is:

$$\max_{n_t(h), P_{H,t}(h)} E_t \sum_{t=\tau}^{\infty} \beta^t \frac{\lambda_t}{\lambda_\tau} \left[\frac{P_{H,t}(h)}{P_{H,t}} y_t(h) - mc_t y_t(h) - \frac{\psi}{2} \left(\frac{P_{H,t}(h)}{\pi P_{H,t-1}(h)} - 1 \right)^2 y_t \right] \quad (16)$$

$$s.t. \quad : \quad y_t(h) = a_t n_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\theta} y_t \quad (17)$$

The first order condition after imposing symmetry across firms is

$$(1 - \theta) + \theta mc_t - \psi \left(\frac{\pi_{H,t}}{\pi} - 1 \right) \frac{\pi_{H,t}}{\pi} + \psi \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\frac{\pi_{H,t+1}}{\pi} - 1 \right) \frac{\pi_{H,t+1}}{\pi_{t+1} \pi} \frac{y_{t+1}}{y_t} \right] = 0 \quad (18)$$

which represents the home non-linear New Keynesian Phillips curve (NKPC) under Rotemberg pricing.

2.5 Government

The government (of each country member of the union) finances exogenous lump-sum transfers to households (z_t) and unproductive purchases (g_t) by collecting tax revenue and issuing one-period bonds (B_t). The tax revenue is raised through a distortionary time varying tax

¹Note that we have defined the real wage in terms of the CPI ($w_t = \frac{W_t}{P_t}$), while the real marginal cost is defined in terms of domestic PPI ($mc_t(h) = \frac{MC_t(h)}{P_{H,t}}$)

rate (τ_t) on labor income. It faces the following budget constraint:

$$\frac{B_t}{R_t} + \tau_t \left[1 - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] P_{H,t} y_t = (1 - \delta_t) B_{t-1} + P_{H,t} (g_t + z_t) \quad (19)$$

Note that in the case of the home country, where default may happen, the relevant stock of debt is the one net of default ($(1 - \delta_t) B_{t-1}$). Initially, we assume only domestic households may purchase domestic government bonds, that is, there is total home bias on domestic debt, but we will relax this assumption in section X below. In our model, default is costless in the sense that the defaulting government is neither forced to reform its policies by dramatically reducing deficits, nor is it locked out of credit markets for some period. The government's budget constraint can be rewritten in real terms:

$$\frac{b_t}{R_t} + \frac{\tau_t \left[1 - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] y_t - g_t - z}{TOT_t^{1-\eta}} = \frac{(1 - \delta_t) b_{t-1}}{\pi_t} \quad (20)$$

where $b_t = B_t/P_t$ is real government debt.

We assume the government follows a simple tax rule

$$\tau_t = \tau + \gamma_b (b_{t-1} - b) \quad (21)$$

where $\gamma_b > 0$ is the tax adjustment parameter, so that a larger γ_b means that the government is more willing to retire debt by raising the tax rate. We assume for now that transfers are constant $z_t = z$, $z_t^* = z^*$ and government purchases follow an AR(1) process

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \quad (22)$$

where g is the steady state government purchase at home.

2.6 Monetary policy

Finally, we assume the Central Bank of the Monetary Union sets the gross nominal interest rate according to:

$$R_t = \begin{cases} R + \alpha_\pi(\pi_{MU,t} - \pi_{MU}) + \alpha_y(y_{MU,t} - y_{MU}), & \text{if } s_t^R = 1 \\ 1, & \text{if } s_t^R = 2 \end{cases} \quad (23)$$

where α_π and α_y are the policy responses to inflation and the output. In terms of the evolution of union wide inflation, $\pi_{MU,t} = s\pi_t + (1-s)\pi_t^*$, and union wide output, $y_{MU,t} = sy_t + (1-s)y_t^*$. The monetary policy regime index s_t^R evolves according to the transition matrix

$$\begin{pmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{pmatrix}.$$

In a normal monetary policy regime ($s_t^R = 1$), the Central Bank obeys the Taylor principle and in a zero lower bound regime ($s_t^R = 2$), the Central Bank exogenously pegs the gross nominal interest rate at 1. Thus, all ZLB events are due to exogenous changes in s_t^R , and the switches between the two monetary policy regimes are similar to large exogenous shocks.²

2.7 Union-wide demand and market clearing

Union-wide demand for home goods, y_t^D , comes from the producers of home and foreign final consumption goods ($c_{H,t}$, $c_{H,t}^*$), home government spending (assuming absolute home bias in government spending $g_{H,t} = g_t$) and to pay for price adjustment costs

$$y_t^D(h) = sc_{H,t}(h) + sg_{H,t}(h) + (1-s)c_{H,t}^*(h) + \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 y_t \quad (24)$$

²We impose the ZLB by exogenous regime switches in monetary policy rules to minimize the number of state variables in solving the nonlinear model. In section 5, the qualitative responses of the real interest rate from fiscal shocks are similar between exogenous and endogenous ZLB events.

and substituting the demands from (14) above we get³

$$y_t^D(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{TOT_t^{1-\eta}}{s} c_{MU,t}^H + g_t \right) + \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 y_t \quad (25)$$

where we define union-wide private consumption of home produced goods as $c_{MU,t}^H = \eta c_t + \eta^* \frac{1-s}{s} c_t^*$.

The real exchange rate, the ratio of relative consumption price levels, can be expressed as the ratio of the home and foreign producer prices

$$RER_t = \frac{P_t^*}{P_t} = \left(\frac{P_{F,t}}{P_{H,t}} \right)^{\eta-\eta^*} = TOT_t^{\eta-\eta^*} \quad (26)$$

To derive the equilibrium in the goods market in the home country we equate the demand for each intermediate good producer of the home product, equation (25), with its production function $y_t^D(h) = y_t(h)$ and aggregate across all home intermediate firms $\int_0^s y_t(h) dh$ to get

$$an_t \left[1 - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] = TOT_t^{1-\eta} c_{MU,t}^H + g_t \quad (27)$$

where we have defined home aggregate labour as $n_t = \int_0^s n_t(h) dh$.

3 Calibration

The model is calibrated at a quarterly frequency. In general, the home country is calibrated using data for Spain and the foreign country using data for Germany. However, there are a number of parameters which are common across areas. The household discount rate is 0.99. Preference over consumption are logarithmic, so $\sigma = 1$. The inverse of Frisch elasticity of labor supply $\varphi = 1$. The productivity level at the steady state are normalized to 1.

Parameterizations of the shock processes for a_t and g_t follow the empirical evidence available for the euro area and Spain and Germany. For instance, [Batini *et al.* \(2018\)](#) and [Gadatsch *et al.* \(2015\)](#) estimate a monetary union with Spain and Germany as members and

³We assume the law of one price holds: (i.e.: the price of variety $h(f)$ of the home (foreign) good is equal at home and abroad).

find estimates for the technological processes around $\rho^a = \rho^{a^*} = 0.9$ and $\sigma^a = \sigma^{a^*} = 0.01$, while for the government spending processes they get estimates around $\rho^g = \rho^{g^*} = 0.9$ and $\sigma^g = \sigma^{g^*} = 0.01$. In addition, these numbers are in line with the theoretical literature (see [Schmitt-Grohe and Uribe \(2007\)](#)). The price elasticity of demand, θ , is assumed to be 11, indicating a steady state markup of 10 percent. The Rotemberg adjustment parameter, ψ , is 116.5, which implies that 25 percent of the firms reoptimize each quarter (see [lvarez *et al.* \(2006\)](#) and [Vermeulen *et al.* \(2012\)](#) for evidence on CPI and PPI firms' price setting in the euro area). The steady state inflation rate is 1 and the Taylor rule parameter is assumed to be 2.5.

The fiscal parameters are calibrated to match Spain and German data since the creation of the euro area (1999-2016). In steady state, government purchases are 18.3 and 18.7 percent of GDP, respectively, and the tax rates are 0.3005 and 0.3425. The steady state debt-to-GDP ratio, is 0.6 for both countries, and the model implied lump-sum transfers are 9.4 and 13.08 percent of GDP. The tax adjustment parameter in the fiscal rule γ_b is calibrated to 0.3. The two countries are assumed to have the same degree of home bias, $\eta = 0.63$ and $\eta^* = 0.37$, calibrated from Euro Area's import share. We calibrate the size of the home country by comparing the nominal GDP of the Euro Area periphery (Spain & Italy) vs core (Germany & France), and $s = 0.36$.⁴ Finally, we use a relatively small default rate to underscore that even small rates can generate quantitatively important effects. The default haircut, δ , is assumed to 0.07, implying a 28% annual default rate. [Table 1](#) summarizes the parameter values.

4 Fiscal limit distribution

In this section, we first define the fiscal limit and then simulate the unconditional distribution for the home country (i.e., the distribution with an initial state at the steady state).

To see how current economic shocks can affect a fiscal limit distribution, state-dependent

⁴Thus, the relative size of the domestic economy ($s = 0.36$) is meant to encompass a broader group of countries in the union with comparable debt sustainability problems, so that fiscal responses in this group of countries exert some meaningful effects on the monetary union as a whole.

parameters	values	
β	0.99	the discount factor
θ	11	elasticity of substitution
ψ	116.5	Rotemberg adjustment parameter
α_π	2.5	Taylor rule parameter to inflation
γ_b	0.04	tax response parameter to changes in debt
s	0.36	share of home country
η	0.63	home country bias in home goods
η^*	0.37	foreign country bias in home goods
δ	0.07	quarterly haircut on debt if default occurs
b/y	0.6	steady state debt to output ratio (home)
b^*/b^*	0.6	steady state debt to output ratio (foreign)
g/y	0.183	steady state gov spending to output ratio (home)
g^*/y^*	0.187	steady state gov spending to output ratio (foreign)
τ	0.3005	steady state income tax rate (home)
τ^*	0.3425	steady state income tax rate (foreign)
a, a^*	1	steady state technology
ρ^g, ρ^{g^*}	0.9	AR(1) coefficient in government spending rules
σ_g, σ_{g^*}	0.01	standard deviation of government spending shock
p_1	0.9917	regime-switching parameter for the normal monetary policy regime
p_2	0.65	regime-switching parameter for the ZLB regime

Table 1: Parameter calibration

distributions are also simulated conditional on government spending and monetary policy regimes.

4.1 Simulating fiscal limit distribution

Fiscal limits are defined, following Bi (2012), as the present value of maximum future primary surpluses over an infinite horizon. When the tax revenue reaches its maximum level, the expected present value of future primary surpluses is maximized, given the level of government expenditures and transfers. Government expenditures, monetary policy, and institutional quality vary with the stochastic shocks hitting the economy, generating a distribution for the maximum debt level that a government is able to service.

Since the fiscal limits are the maximum level of debt that can be supported without default, when simulating fiscal limits, $\delta_t = 0$ for all t . Derive the intertemporal government budget constraint given the real government budget constraint, (20), the Euler equation, (5),

and the transversality condition, (6),

$$b_{t-1} = \pi_t E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{T_{t+j} - g_{t+j} - z}{TOT_{t+j}^{1-\eta}} \quad (28)$$

Following Bi *et al.* (2016), fiscal limits are simulated based on (28), but all the variables are computed under $\tau_{t+i} = \tau^{\max}$, the maximum income tax rate a government is willing and able to impose.⁵

$$\mathcal{B}^H(\mathcal{S}_t) = \beta_t^p \pi^{\max}(\mathcal{S}_t) E_t \sum_{j=0}^{\infty} \beta^j \frac{1}{(TOT^{\max}(\mathcal{S}_{t+j}))^{1-\eta}} \frac{\lambda^{\max}(\mathcal{S}_{t+j})}{\lambda^{\max}(\mathcal{S}_t)} (\mathcal{T}^{\max}(\mathcal{S}_{t+j}) - g_{t+j} - z) \quad (29)$$

The fiscal limits simulated are state-dependent and uncertain, conditional on an initial state of the economy $\mathcal{S}_t = \{g_t, g_t^*, TOT_{t-1}, s_t^R\}$. This notion of fiscal limit also captures the private sector's perception of the limit, as it uses the stochastic discount factor evaluated at the maximum tax rate, $\beta^j \frac{\lambda^{\max}(\mathcal{S}_{t+j})}{\lambda^{\max}(\mathcal{S}_t)}$, and allows for a stochastic political risk (β_t^p) that follows an AR(1) process,

$$\ln \frac{\beta_t^p}{\beta^p} = \rho^{\beta^p} \ln \frac{\beta_{t-1}^p}{\beta^p} + \varepsilon_t^{\beta^p}, \quad \varepsilon_t^{\beta^p} \sim N(0, (\sigma^{\beta^p})^2) \quad (30)$$

Lower β_t^p indicates higher political risk and hence lower fiscal limits. It can be interpreted as either that the policymakers have a shorter planning horizon than the private sector, or that agents place probability mass on both the maximum surpluses and zero surpluses (Bi *et al.*, 2018)

In the data, risk premia in several European countries start to increase even at lower levels of debt. Setting $\beta^p < 1$ and $\varepsilon_t^{\beta^p} \sim N(0, (\sigma^{\beta^p})^2)$ serves to shift down the mean and increase the dispersion of the fiscal limit, which generates plausible movements in risk premia as observed in the data. In particular, we calibrate the political risk in the home country by

⁵Another way to quantify the fiscal limit is the Laffer curve. Bi (2012) derives the peak of the Laffer curve analytically in a real business cycle model. In a nominal model, Bi *et al.* (2018) assume that the Central Bank is able to set the inflation rate equal to its objective, which allows for a simple solution for the main variables determining the maximum of the Laffer curve. However, in a monetary union setting the aggregate inflation at its target does not guarantee that each country's inflation is also equal to its target, and thus it does not allow for an analytical solution of the Laffer Curve. We set $\tau^{\max} = 0.435$, the max statutory rate in Spain (European Commission, 2018).⁶ Let the superscript max indicates variables computed under $\tau_t = \tau^{\max}$. The maximum sustainable level of debt at t in the home country, defined as a fiscal limit distribution, is given by:

using an indicator about the current political situation derived from a Spanish nation-wide sociological survey (see Gil *et al.* (2017)) to get $\beta^p = 0.37$, $\rho^{\beta^p} = 0.96$, and $\sigma^{\beta^p} = 0.13$.

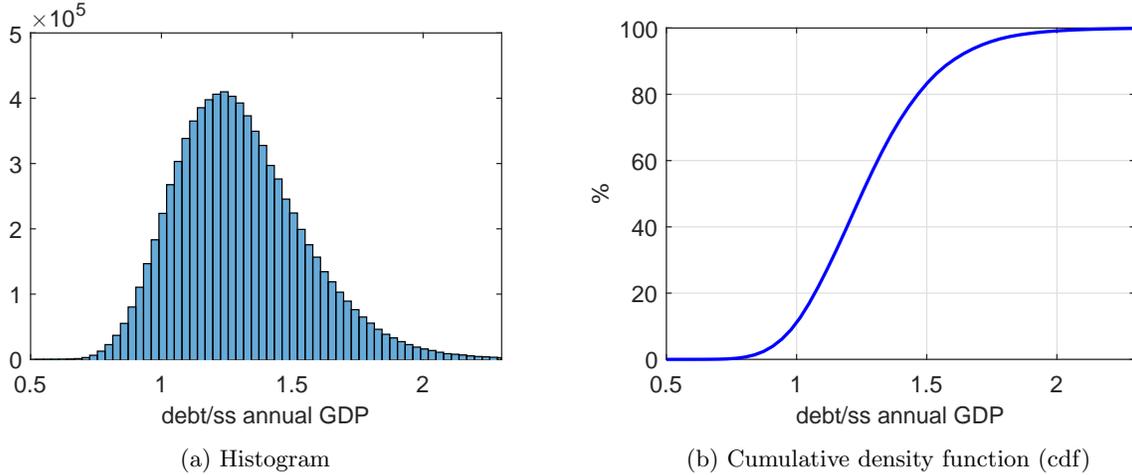


Figure 2: Distribution of unconditional fiscal limit computed from expression for \mathcal{B}^H in (29)

We simulate the distributions of fiscal limits using Markov Chain Monte Carlo method, which is described in Appendix B.1. As shown by (29), each draw of a fiscal limit from the distribution is conditional on the current state, \mathcal{S}_t , and particular sequences of realized shocks in the Markov Chain Monte Carlo simulations. As a baseline, Figure 2 plots the histogram of the simulated fiscal limits for the home country and the corresponding cumulative density function (cdf), starting from the steady state and a normal monetary policy regime ($\mathcal{S}_t = \{g, g^*, TOT, 1\}$), which we define as the unconditional fiscal limit. The x-axis plots fiscal limits in the ratio of government debt to steady-state annual GDP. The histogram in the left panel indicates that the fiscal limit is centered around a debt to GDP ratio of 128% ($\overline{\mathcal{B}^H} = 1.28$) with a standard deviation ($\sigma^{\mathcal{B}^H}$) of about 0.25.⁷ The cumulative distribution function (cdf) of the home fiscal limit, in the right panel shows that the probability of the home government defaulting on its debt is nil for debt levels close to 80% of GDP, while it converges to 1 for debt levels above 200% of GDP. In between those values, the probability of default gradually increases as debt accumulates.

Although default does not occur when simulating fiscal limits, recall the default mecha-

⁷The histogram has a slightly longer right tail. This asymmetry is due to the effect of the stochastic process estimated for the political factor, which is bounded above zero and has a fairly large standard deviation. The simulated distribution of the fiscal limit without the political factor is symmetric.

nism in (3), which makes the role of fiscal limits in default decisions clear,

$$\delta_t = \begin{cases} 0 & \text{if } b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if } b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t) \end{cases}$$

The choice $\mathcal{B}^H(\mathcal{S}_t)$, is uncertain and depends on fiscal policy, monetary policy, and political considerations. The fiscal limit defined here describes the stochastic upper bound on how much debt a government is willing and able to service given the economic and political constraints. A large literature adopts strategic sovereign default approach in which an optimizing government accounts for some economic costs in making default decisions (Aguiar and Gopinath, 2006; Arellano, 2008; Yue, 2010). Rather than making the default decision a strategic choice, we opt to treat the intrinsically political decision as a random draw from the distribution of fiscal limits. Another literature incorporates default risk by exogenously specifying fiscal limits. Daniel and Shiamptanis (2012) assume government debt is constrained by an ad-hoc fiscal limit to study a small open economy in a monetary union under alternative fiscal policy responses to a fiscal crisis. Corsetti *et al.* (2013) and Batini *et al.* (2018) propose a model where the euro area periphery government is faced with a fiscal limit following beta distribution calibrated using data for Greece. The fiscal limits in this paper capture uncertainty in default decisions, and the model consistent approach is also able to generate endogenous responses of fiscal limits to economic disturbances, which we will discuss in the next section.

4.2 Fiscal and monetary policy

In line with the definition of fiscal limit proposed by Bi (2012), the state of the economy can have a significant impact upon the default probability in the home country. A shock in fiscal or monetary variables changes household's perception on debt sustainability and thus can shift the fiscal limit. Figure 3 compares the state-dependent distributions at different government spending levels in the home country while keeping the other states at the steady

state. In particular, the black solid line in the left panel represents the cdf of the home's fiscal limit when all the states are at their steady state values ($\mathcal{S}_t = \{g, g^*, TOT, 1\}$), while the blue dashed (red dotted dashed) lines represent the cdf when government spending in the home country is 4.5% below (above) the steady state value.⁸ To show the difference in default probabilities, the right panel plots the change in the home government's default probability relative to the steady state when government spending in the home country takes values away from the steady state.

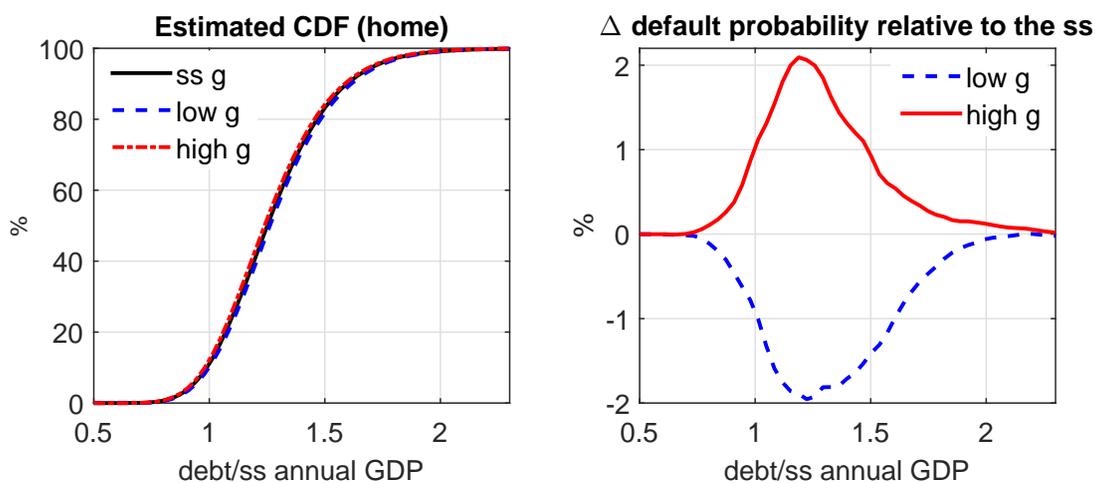


Figure 3: State-dependent distributions of fiscal limits: government spending in the home country

A 4.5% increase in home's government spending (red dotted dashed line) increases aggregate demand and generates more tax revenues. On the other hand, the fiscal expansion also rises its public deficit today and worsens the sustainability perspectives of home government's finances, shifting its fiscal limit to the left (left panel) and increasing its default probability for debt levels between 80-200% of GDP (right panel). The marginal changes in the default probability from government spending shocks in the home country is the largest when debt to steady-state annual GDP reaches 120%, where the slope of the estimated cdf is the steepest. Similarly, a 4.5% fiscal consolidation improves debt sustainability and decreases the default probability. The maximum impact of a 4.5% increase (reduction) in government spending raises (lowers) the default probability by 2 percentage, that is from 38% to 40% (to 36%).

⁸The black solid line is the unconditional fiscal limit as shown in Figure 2b.

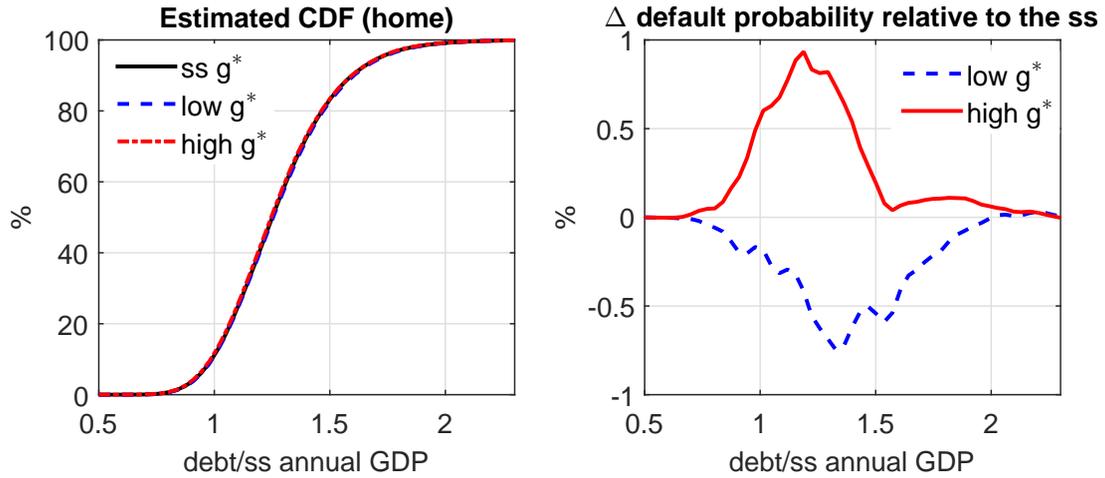


Figure 4: State-dependent distributions of fiscal limits: government spending in the home country

When the increase in government spending occurs instead in the foreign country of the monetary union, the model generates significant spillover effects on the home country's fiscal limit. Figure 4 shows the distribution of home country's fiscal limit in response to government spending shocks in the foreign country, while keeping the other state variables at the steady state. A higher level of foreign government spending increases its output and inflation. As monetary policy follows the Taylor rule ($s_t^R = 1$), given the greater size of the foreign country, fiscal expansions lead to higher nominal interest rate, which also raises the real interest rate in the home country, thus depressing the home country's demand and increasing its financing cost of debt. This monetary channel worsen the perspectives of home country's public finances, reducing its fiscal limit (shifts the cdf to the left). On the contrary, the trade channel will expand the fiscal limit in the home country, since the increase in foreign's activity will stimulate home's exports and output. In net terms, the monetary channel dominates and an increase in foreign's government spending will generate a negative spillover and reduce home's fiscal limit. In quantitative terms, the negative spillover effect of a fiscal expansion abroad on home's probability of default on its public debt is about half the size of the direct effect of its own fiscal expansion.

When the ZLB is binding the distribution of fiscal limits in response to home government spending shock is similar but with smaller changes in the default probability. In our model,

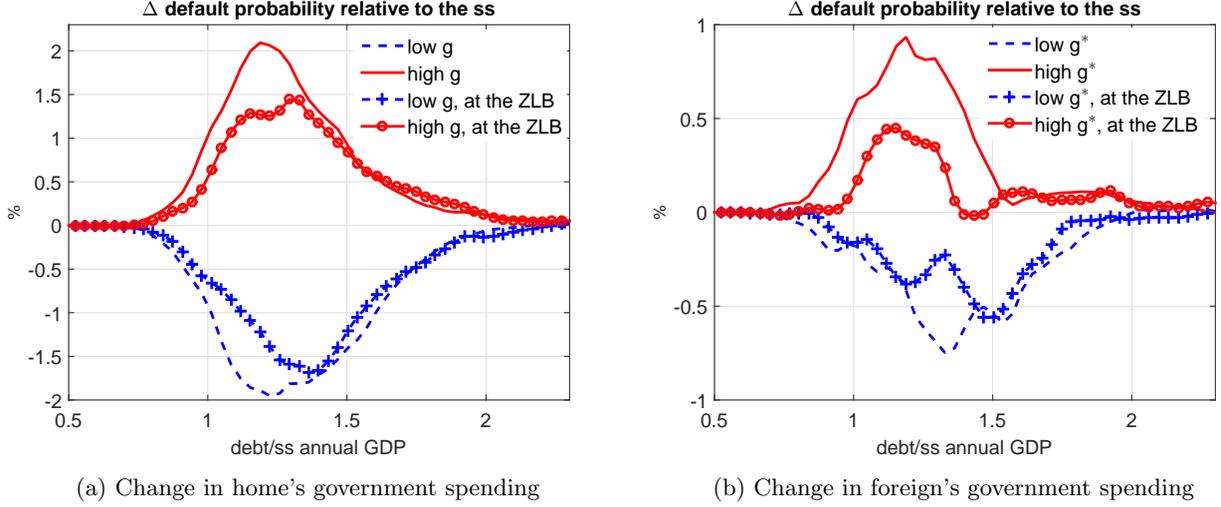


Figure 5: State-dependent distribution of fiscal limits under the ZLB

a high government spending in the home country increases the real interest in a normal monetary policy regime, lowering demand and worsening fiscal sustainability. A binding ZLB, on the other hand, counteracts the spending effects on real interest rates. As nominal interest rate is constrained, an inflation expectation from persistent government spending increase can lower the real interest rate and mitigate the changes to the default probability due to fiscal policy changes. The same pattern shows up in home fiscal consolidation and foreign fiscal expansion (consolidation).

Therefore, in our model the endogenous fiscal limit is relevant when the home country has a debt level above 80% of GDP, where shocks in fiscal or monetary variables produce significant changes in the sustainability of home's public finances. As shown in Figure 1 above debt to GDP ratio is currently in Spain, France and Belgium around 100%, in Italy is above 130% while in Germany is close to 60%. This confirms our modelling choice to have the home country, representing a high debt country with the possibility of default, while the foreign country does not.

5 Fiscal policy in a monetary union

We now undertake several fiscal policy exercises. We concentrate on fiscal issues since this is very much in the current policy debate in the monetary union and it is precisely the economic policy where endogenizing the fiscal limit ought to be more relevant. First, we analyze the long-term process of public deleveraging that is required for high-debt countries to converge back to the steady state. We study how the speed of convergence and the fiscal instrument on which it is based determines its cost. Second, we look at how short-run discretionary policy along this process of convergence is different with high debt. In particular, we will show the effect on the economy of a transitory fiscal consolidation in a member of the euro area with high debt, a transitory fiscal expansion in a member with low debt, and a fiscal policy coordination between both countries, combining the two previous exercises. These cases are analyzed under two alternative assumptions about the monetary policy space in the union depending on whether Taylor rule is operative or the economy is temporarily stuck at the zero lower bound so that the nominal interest rate cannot be reduced any further. As we shall discuss, these alternative monetary regimes yield different policy results. Throughout the section, we highlight the endogenous risk premium channel by comparing three high-debt economies: no sovereign default risk, sovereign risk determined by unconditional fiscal limits, and sovereign risk determined by conditional fiscal limits.

5.1 Long-run fiscal consolidation at home

One of the current main challenges in the euro area is for high debt countries to converge back towards more sustainable debt levels. In fact, the Growth and Stability Pact sets a limit of 60% of GDP for public debt, beyond which the debt rule is active. Given the high level of government debt in many countries, this implies a long-term process of consolidation, which could take several decades, and which will affect the area as a whole. This process is the focus of this section.

As shown in the previous section, the endogenous fiscal limit becomes relevant when debt

over GDP is around 100%, but the steady state debt in both members of the euro area is 60% of GDP. Therefore, the policy relevant exercises have to be based on an scenario with high debt in the home country, while the foreign country is at its steady state. In particular, to achieve this we make the initial stock of home debt (b_{t-1}) to a level of 100% of GDP at the beginning of the scenario and let the fiscal and monetary rules bring the economy back to its steady state.⁹

As shown by the solid lines in Figure 6, this process is characterized by a long and costly fiscal consolidation, instrumented through an significant increase in labor tax rates (as implied by the fiscal rule), to slowly reduce debt, which will take several decades to reach its steady state level. The consolidation process is worsened by the increase in the home debt's default probability and its risk premium, which increases the interest burden. This has a great impact on the economy, with a significant fall in the level of output and consumption for all that period and a rise in CPI inflation, which worsens the terms of trade and further deprives activity. The fiscal adjustment in the high-debt economy also spills over to the rest of the euro area, where foreign output falls persistently.

The long term convergence back towards the 60% level of debt will be different depending on the intensity of the consolidation process, which in our model is controlled by the parameter of debt on the fiscal rule. A more frontloaded consolidation process (higher γ , dashed lines in figure 6) in our model reduces its long-term cost in terms of GDP accumulated loss. The reason is that although the initial impact on the risk premium is the same, as the quicker consolidation is implemented agents reduce the risk premium and this improves the economy. This is despite the fact that the stronger initial consolidation increases inflation on impact by more, which makes the monetary authority to react more strongly, but this is quickly undone once the reduction in risk premium improves the economy.

⁹This approach provides a good approximation to the true high debt scenario, since although in our model TOT_{t-1} is also a state variable, endogenizing it would have only a negligible effect.

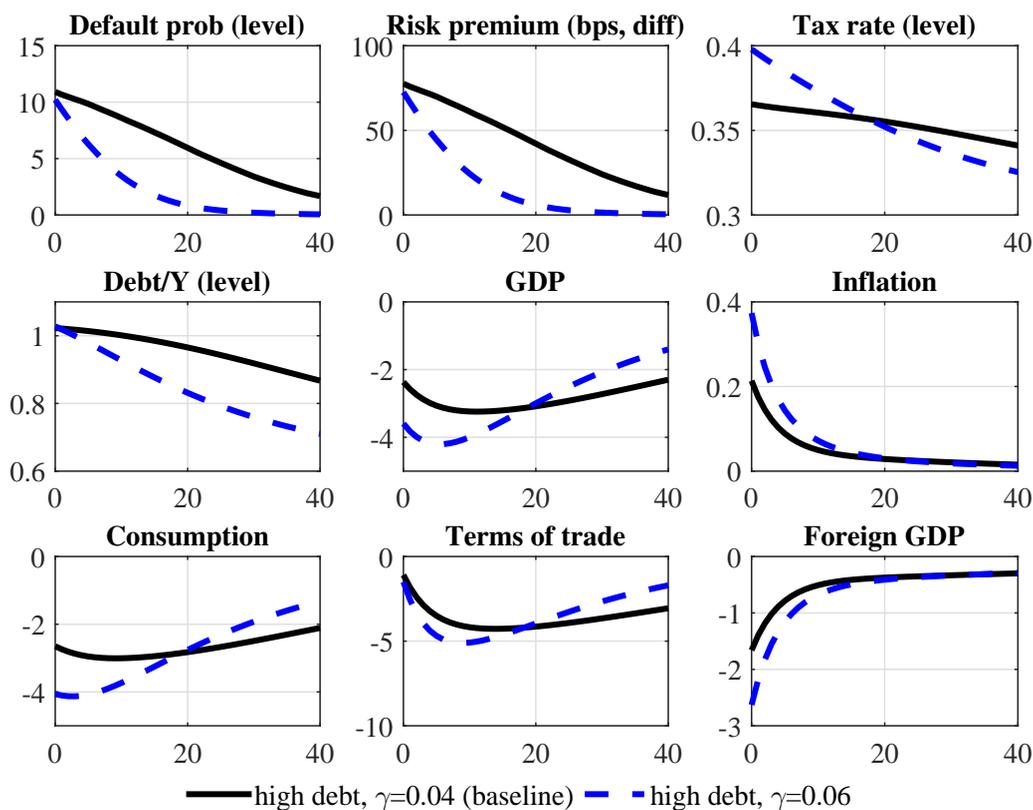


Figure 6: **Convergence back to the steady state from a high debt scenario at home.** The responses (for those without a parenthesis) are plotted as the differences in percent of stochastic steady-state levels. The default probability, the risk premium, the tax rate, and the debt-to-annual GDP ratio are level differences in percent of stochastic steady-state levels.

5.2 Discretionary fiscal consolidation at home

To see how government indebtedness matters for discretionary fiscal consolidation effects in the home country, we examine an exogenous government consumption cut in different states of debt. Before the spending cut, the high-debt state at $t = 0$ is simulated by the same method in the previous section and the low-debt state is the stochastic steady state. Given these initial states, a 1% negative government spending shock is injected at $t = 0$.

Solid lines in Figure 7 show the macroeconomic responses to a 1% transitory government spending cut in a high-debt member of a monetary union. Discretionary fiscal consolidation reduces output and inflation on impact due to lower demand for domestic goods. Following the Taylor rule, the real interest rate falls. On the fiscal side, the lower spending reduces the deficit and starts a slow process of debt reduction, while at the same time, the fiscal rule sets

a slightly lower tax rate. The public deficit reduction in the short term leads to a slight improvement of the expected medium and long-term sustainability of public finances and to an increase in the home's fiscal limit (see Figure 3). This has an immediate effect on home government's default probability, which falls on impact and during the first 4 years by half a percentage point, and on the risk premium on home government bonds, which falls by almost 4 basis points. Both of these effects generate a persistent reduction in home government's cost of financing, which further contributes to the improvement of public finances and to debt reduction. After 1 year output starts to recover, thanks to the improvement in the terms of trade due to lower domestic inflation and to the reduction in the home's cost of financing, together with gradual reduction in the tax rate. In addition, the shock produces a positive spillover to the rest of the union, mainly through two channels. On the one hand, the fall in activity and inflation at home will push down slightly the ECB's nominal interest rate and foster economic activity in the rest of the union. On the other the increase in home's consumption will foster exports from the rest of the union. This leads to a small reduction in foreign debt.

As one would expect, our non-linear model is capable of showing that the benefits from fiscal consolidation are greater when an economy is in a high debt situation, than when its public finances are in good shape. The blue dashed lines of Figure 7 depicts the effect of the same cut in Home's government spending but starting from a low level of debt (60% of GDP, the stochastic steady state). In this case, Home's government finances are in much better shape and therefore very far from the fiscal limit, so the risk premium is very small and insensitive to small changes in debt sustainability. Therefore, in this scenario the fiscal consolidation cannot improve the probability of default, which is almost nil, and thus does not reduce the risk premium. Although in this case the initial recession is slightly milder, the recovery does not benefit from the lower interest rates and is therefore much weaker, achieving a smaller reduction in public debt. The same is true of the spillover effect to the rest of the union.

In order to study more closely the role of the risk premium in the transmission channel

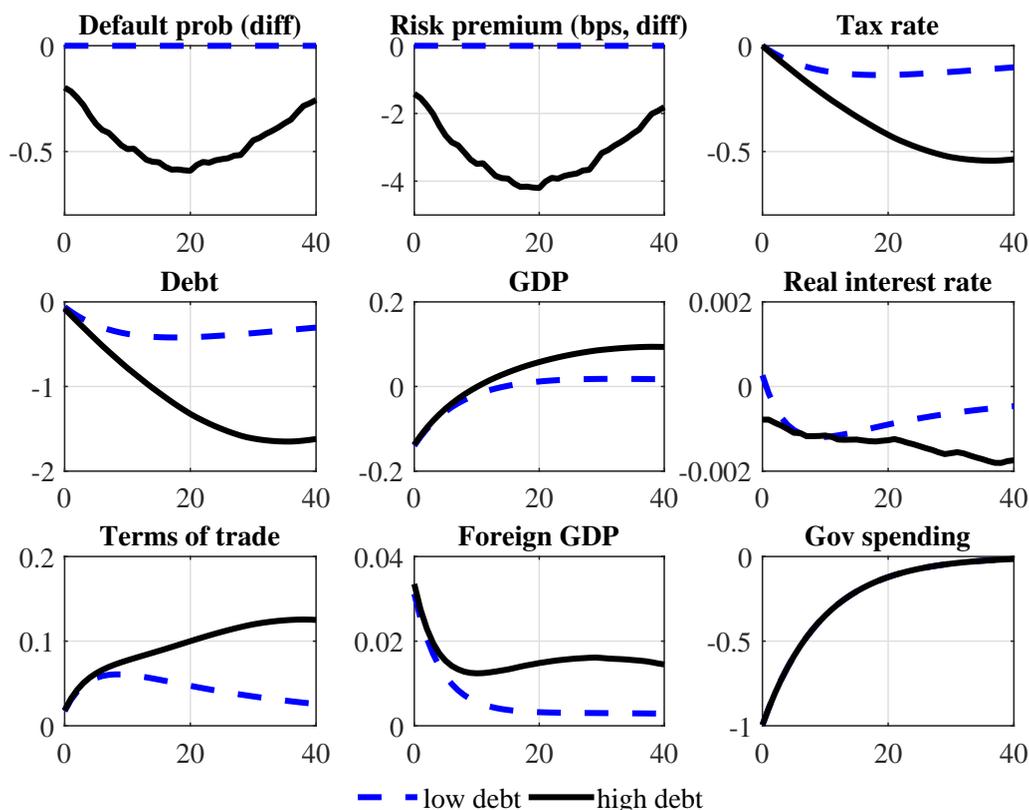


Figure 7: **Transitory government spending cut at Home: low debt vs. high debt.** The responses (for those without a parenthesis) are plotted as the differences in percent of stochastic steady-state levels between the paths with and without a 1% government spending cut. The default probability is the level difference in percent and risk premium is the level difference in basis point between the paths with and without the shock.

of fiscal shocks. In Figure 8, we compare our model with two alternative ones previously used in the literature: red dashed-dotted lines depict the effect of a fiscal consolidation at home in a standard model without default and risk premium, while the blue dashed lines show the effect on a model with an exogenous fiscal limit, in which there is the possibility of home government default and thus it pays a risk premium, but it does not change with the evolution of the economy. The literature often models the cumulative density function of default probability as a logistic function (see Davig *et al.* (2010), Corsetti *et al.* (2013), Corsetti *et al.* (2014) and Batini *et al.* (2018)). Although the fiscal limit in this paper is model-based, we can map the simulated unconditional fiscal limit (as shown in Figure 2b) to a logistic function, which we use as an exogenous fiscal limit.

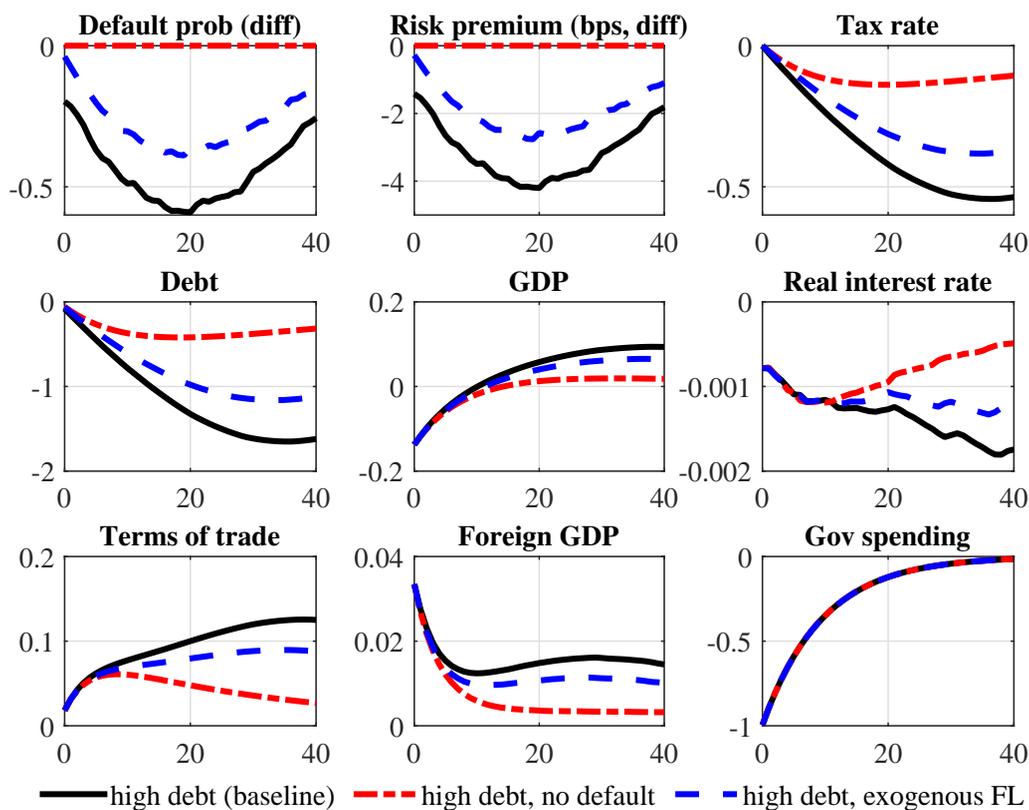


Figure 8: **Transitory government spending cut at Home: The effect of endogenizing risk premium.** See Figure 7 for units of y-axes.

In a model without risk premium, reducing public debt does not affect the cost of financing and therefore, the impulse responses look very similar to the ones under an scenario of low debt. In the model with exogenous fiscal limits, the fiscal consolidation reduces debt and the risk premium slowly since the government only improves its long term sustainability prospects as it slowly moves down the fiscal limit cdf in Figure 2b. In our model, the initial improvement in public finances moves the whole fiscal limit bettering immediately the home government debt’s sustainability, making the risk premium and the risky nominal interest rate jump down on impact and converge slower towards its steady state. This shortens the initial recession and increases significantly the strength of the recovery, pushing debt further down. In addition, the more powerful risk premium channel in our model increases significantly the positive spillover effect to the rest of the union.

Figure 9 compares our baseline simulation, with an alternative monetary policy regime

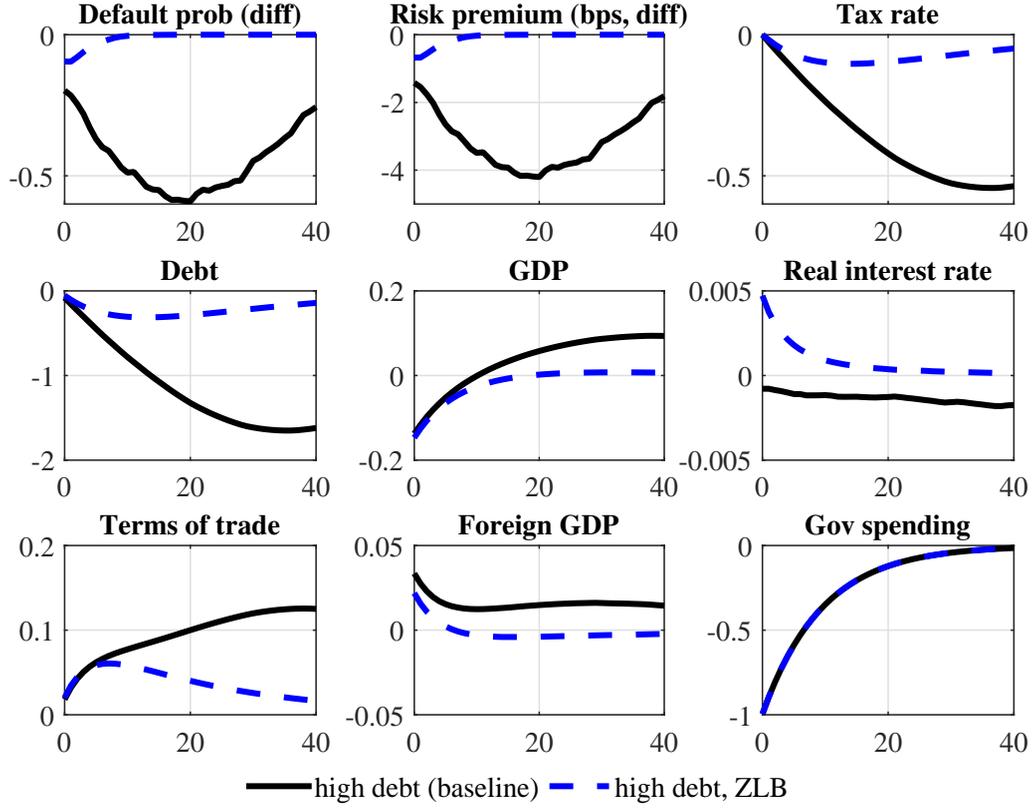


Figure 9: **Transitory government spending cut at Home: The effect of binding ZLB.** See Figure 7 for units of y-axes.

at the ZLB. A persistent fiscal consolidation generates deflation expectation, but when the nominal interest rate is constrained at zero, the real interest rate increases. Higher real interest rates lower demand and mitigates the endogenous risk premium channel explained before.

Table 2 reports the cumulative government spending multipliers for output in the home country, foreign country, and the Euro area for various models, computed as

$$\frac{\sum_{i=1}^k r_{t+i-1}^{-1} \Delta x_{t+i-1}}{\sum_{i=1}^k r_{t+i-1}^{-1} \Delta g_{t+i-1}}, \quad x \in \{y, y^*, y_{MU}\}, \quad (31)$$

where Δ denotes level changes relative to a path without government spending changes. To keep the comparison consistent among different models, use $r_t = \beta^{-1}$ as the real interest rate.

Multiplier	Periphery (home)			Spillover to the core (foreign)			Euro area		
	$\frac{PV(\Delta y)}{PV(\Delta g)}$			$\frac{PV(\Delta y^*)}{PV(\Delta g)}$			$\frac{PV(\Delta y_{MU})}{PV(\Delta g)}$		
Models	impact	1 yr	10 yr	impact	1 yr	10 yr	impact	1 yr	10 yr
Baseline	0.75	0.69	-0.50	-0.14	-0.13	-0.24	0.18	0.17	-0.34
No default	0.75	0.70	0.27	-0.14	-0.12	-0.11	0.18	0.17	0.02
Exogenous FL	0.75	0.70	-0.20	-0.14	-0.12	-0.19	0.18	0.17	-0.20
ZLB	0.80	0.75	0.45	-0.09	-0.07	0.01	0.23	0.22	0.17

Table 2: Output multipliers from a discretionary government spending cut in the home country

The fiscal multiplier, measuring the reduction in output due to a 1 euro fall in government spending, is similar the first year after the shock (see rows 1 and 4 of the top panel in Table 2) without or with default risk, but the difference can be sizable in the long run. The 10-year cumulative fiscal multiplier after a discretionary government spending cut in a model without sovereign default is 0.27, it changes its sign, becoming expansionary (-0.2) when default is introduced through exogenous fiscal limit, while the fiscal consolidation becomes more expansionary (-0.50), when the fiscal limit is endogenized.

5.3 Discretionary fiscal expansion abroad

In normal times, when the ZLB is not binding, the home economy will not benefit from a foreign fiscal expansion, even if we do not consider the constraint posed by the fiscal limit. The red dashed-dotted lines plot the effects of foreign fiscal expansion without default. The increase in foreign government spending increases the union-wide inflation rate and the real interest rate. When we take into account the fragility of home public finances, either in the form of an exogenous or an endogenous fiscal limit, things get worse for the home country. In the former case, the recession raises the debt to GDP ratio increasing the probability of default and, along with it, the risk premium. If we also endogenize the effect of higher interest rates on future surpluses, the fiscal limit itself falls and the risk premium increases even further. In both cases, but more so in the latter, the initial temporary recession at home worsens as time goes by due to a long lasting risk premium. When the ZLB is binding, the changes in the default probability and risk premium become minimal, close to the model without default risk.

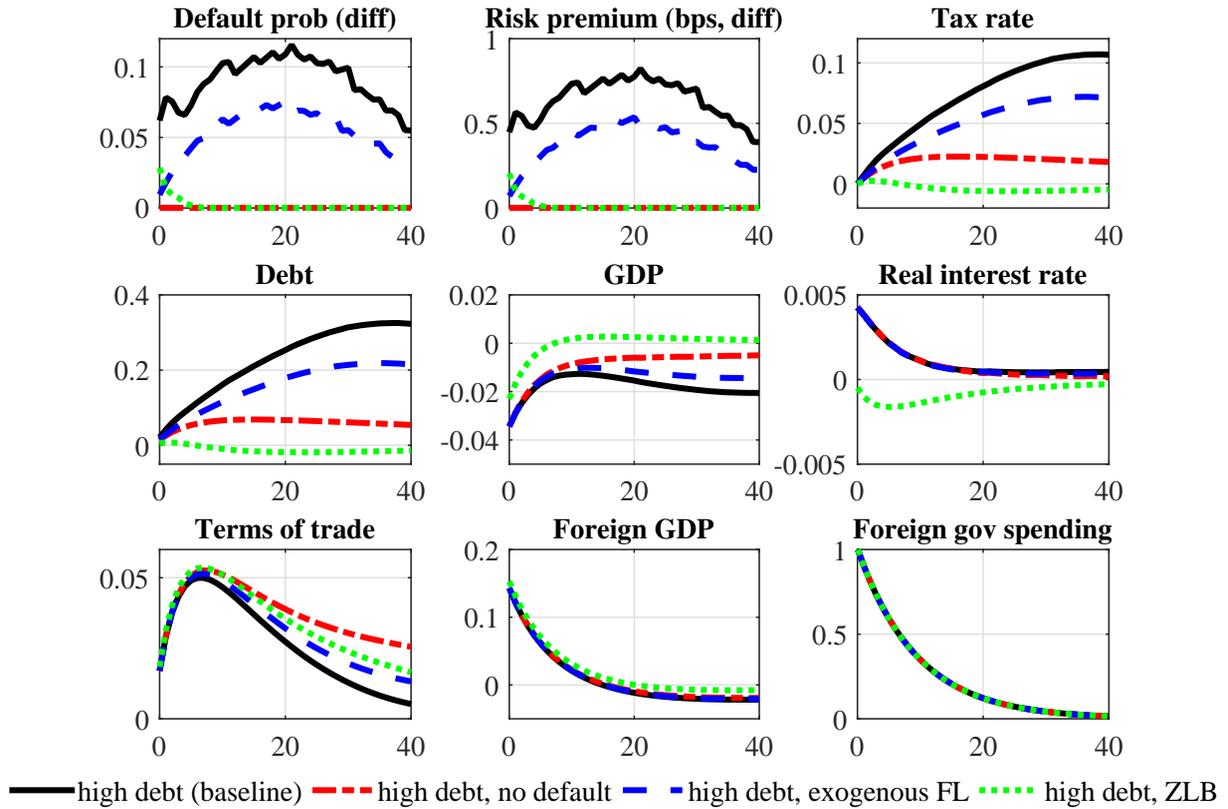


Figure 10: **Transitory fiscal expansion abroad.** See Figure 7 for units of y-axes.

5.4 Discretionary fiscal coordination

In Figure 11, we show the effect of policy coordination across both areas of the union, with a consolidation in the high debt Home country and an expansion in the rest. In this case the impulse response is dominated by the effects of the fiscal consolidation at home, while the rest of the union experiences a significant boom.

However, this results change significantly when the economy is at the ZLB for nominal interest rates. In this case, the fiscal expansion in the rest of the union leads to a boom in that country, but the ECB's nominal interest rate remains constant at the ZLB, therefore the high-debt Home country benefits from the increase in exports, without paying more to service its debt.

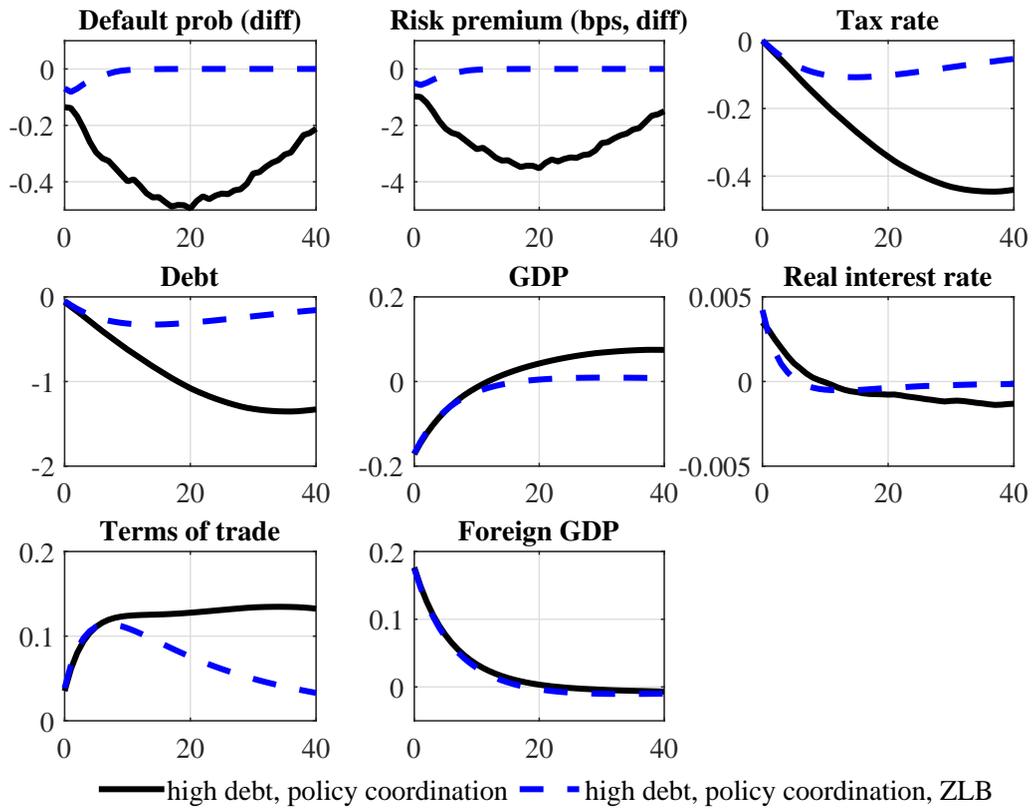


Figure 11: Discretionary fiscal coordination: consolidation at home and expansion abroad. See Figure 7 for units of y-axes.

6 Fiscal risk sharing in a monetary union

[TBC]

7 Conclusion

[TBC]

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Appendix A Equilibrium

The equilibrium consists of 36 equations (16 for the home country, 15 for the foreign country and 5 are common), to solve for 37 variables ($\lambda_t, c_t, n_t, w_t, \tau_t, \pi_t, \pi_{H,t}, mc_t, y_t, b_t, T_t, c_{MU,t}^H, \Upsilon_t, \delta_t, a_t, g_t$ for the home country, $\lambda_t^*, c_t^*, n_t^*, w_t^*, \tau_t^*, \pi_t^*, \pi_{F,t}, mc_t^*, y_t^*, b_t^*, T_t^*, c_{MU,t}^F, \Upsilon_t^*, a_t^*, g_t^*$ for the foreign country and $6 = TOT_t, RER_t, \pi_{MU,t}, y_{MU,t}, R_t^f, R_t$).

Home equations

$$\lambda_t = c_t^{-\sigma} \quad (\text{A.1})$$

$$n_t^\varphi = \lambda_t(1 - \tau_t)w_t \quad (\text{A.2})$$

Since we have default, we need both the Euler equation under default and under no default to evaluate the expectation about future defaults

$$\lambda_t = \beta R_t^f E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \quad (\text{A.3})$$

$$\lambda_t = \beta R_t E_t \frac{(1 - \delta_{t+1})\lambda_{t+1}}{\pi_{t+1}} \quad (\text{A.4})$$

$$\delta_t = \begin{cases} 0 & \text{if } b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if } b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t) \end{cases} \quad (\text{A.5})$$

$$\pi_t = \pi_{H,t}^\eta \pi_{F,t}^{1-\eta} \quad (\text{A.6})$$

$$mc_t = TOT_t^{1-\eta} \frac{w_t}{a_t} \quad (\text{A.7})$$

$$\psi \left(\frac{\pi_{H,t}}{\pi} - 1 \right) \frac{\pi_{H,t}}{\pi} = (1 - \theta) + \theta mc_t + \psi \beta E_t \left(\frac{c_t}{c_{t+1}} \left[\frac{\pi_{H,t+1}}{\pi} - 1 \right] \left(\frac{y_{t+1} \pi_{H,t+1}}{y_t \pi \pi_{t+1}} \right) \right) \quad (\text{A.8})$$

$$\left[1 - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] y_t = TOT_t^{1-\eta} c_{MU,t}^H + g_t \quad (\text{A.9})$$

$$c_{MU,t}^H = \eta c_t + \eta^* \frac{1-s}{s} c_t^* \quad (\text{A.10})$$

$$y_t = a_t n_t \quad (\text{A.11})$$

$$\Upsilon_t = \left[1 - mc_t - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] y_t \quad (\text{A.12})$$

$$\frac{b_t}{R_t} + \frac{T_t - g_t - z}{TOT_t^{1-\eta}} = \frac{(1 - \delta_t) b_{t-1}}{\pi_t} \quad (\text{A.13})$$

$$T_t = \left[1 - \frac{\psi}{2} \left(\frac{\pi_{H,t}}{\pi} - 1 \right)^2 \right] \tau_t y_t \quad (\text{A.14})$$

$$\tau_t = \tau + \gamma_b (b_{t-1} - b) \quad (\text{A.15})$$

$$\ln \frac{a_t}{a} = \rho^a \ln \frac{a_{t-1}}{a} + \varepsilon_t^a \quad (\text{A.16})$$

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \quad (\text{A.17})$$

Foreign equations

$$\lambda_t^* = (c_t^*)^{-\sigma} \quad (\text{A.18})$$

$$(n_t^*)^\varphi = \lambda_t^* (1 - \tau_t^*) w_t^* \quad (\text{A.19})$$

In the full model, if there is no default in either country, the risky and risk free rates are identical and we can use the RER_t expression for foreign consumption. If instead home country sovereign debt is subject to default, while foreign country's sovereign debt is not, then the rate of return in foreign is equal to the risk-free rate, but we have to use the foreign Euler equation to solve the model.

$$\text{no default in the model (both home and foreign): } \left(\frac{c_t}{c_t^*} \right)^\sigma = RER_t = TOT_t^{\eta-\eta^*} \quad (\text{A.20})$$

$$\text{default in home but no default in foreign: } \lambda_t^* = \beta R_t^f E_t \frac{\lambda_{t+1}^*}{\pi_{t+1}^*} \quad (\text{A.21})$$

$$\pi_t^* = \pi_{H,t}^{\eta^*} \pi_{F,t}^{1-\eta^*} \quad (\text{A.22})$$

$$mc_t^* = TOT_t^{-\eta^*} \frac{w_t^*}{a_t^*} \quad (\text{A.23})$$

$$\psi \left(\frac{\pi_{F,t}}{\pi^*} - 1 \right) \frac{\pi_{F,t}}{\pi^*} = (1 - \theta) + \theta mc_t^* + \psi \beta E_t \left(\frac{c_t^*}{c_{t+1}^*} \left(\frac{\pi_{F,t+1}}{\pi^*} - 1 \right) \frac{y_{t+1}^* \pi_{F,t+1}}{y_t^* \pi_{t+1}^* \pi^*} \right) \quad (\text{A.24})$$

$$\left[1 - \frac{\psi}{2} \left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] y_t^* = TOT_t^{-\eta^*} c_{MU,t}^F + g_t^* \quad (\text{A.25})$$

$$c_{MU,t}^F = \frac{s(1-\eta)}{1-s} c_t + (1-\eta^*) c_t^* \quad (\text{A.26})$$

$$y_t^* = a_t^* n_t^* \quad (\text{A.27})$$

$$\Upsilon_t^* = \left[1 - m c_t^* - \frac{\psi}{2} \left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] y_t^* \quad (\text{A.28})$$

$$\frac{b_t^*}{R_t^f} + TOT_t^{\eta^*} (T_t^* - g_t^* - z^*) = \frac{b_{t-1}^*}{\pi_t^*} \quad (\text{A.29})$$

$$T_t^* = \left[1 - \frac{\psi}{2} \left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] \tau_t^* y_t^* \quad (\text{A.30})$$

$$\tau_t^* = \tau^* + \gamma_b (b_{t-1}^* - b^*) \quad (\text{A.31})$$

$$\ln \frac{a_t^*}{a^*} = \rho^a \ln \frac{a_{t-1}^*}{a^*} + \varepsilon_t^{a^*} \quad (\text{A.32})$$

$$\ln \frac{g_t^*}{g^*} = \rho^g \ln \frac{g_{t-1}^*}{g^*} + \varepsilon_t^{g^*} \quad (\text{A.33})$$

Union wide equations

$$TOT_t = TOT_{t-1} \frac{\pi_{F,t}}{\pi_{H,t}} \quad (\text{A.34})$$

$$R_t = \begin{cases} R + \alpha_\pi (\pi_{MU,t} - \pi_{MU}) + \alpha_y (y_{MU,t} - y_{MU}), & \text{if } s_t^R = 1 \\ 1, & \text{if } s_t^R = 2 \end{cases} \quad (\text{A.35})$$

$$y_{MU,t} = s y_t + (1-s) y_t^* \quad (\text{A.36})$$

$$\pi_{MU,t} = s \pi_t + (1-s) \pi_t^* \quad (\text{A.37})$$

$$RER_t = TOT_t^{\eta-\eta^*} \quad (\text{A.38})$$

Appendix B The Numerical Solution Method

Appendix B.1 Solving the fiscal limit

This appendix describes procedures in simulating fiscal limit distributions. First, derive the expression of the fiscal limit as (29). Second, replace the fiscal rules (A.15) and (A.31) with the maximum tax rate τ^{\max} and $\tau^{*\max}$ and solve the full model (without default) non-linearly evaluated at the maximum tax rate.

When solving the nonlinear model without default, the state space is $S_t = \{g_t, g_t^*, TOT_{t-1}, s_t^R\}$, depending on the number of exogenous shocks we consider to build the fiscal limit. Since the model without default has three expectation terms in equations (Euler equation of home country (A.3)¹⁰, Phillips curve of home (A.8) and foreign (A.24) we need three decision rules. It can be shown that these three equations can be written as a function of only three variables: n_t , $\pi_{H,t}$ and $\pi_{F,t}$. Therefore, the decision rule for labor in home country is $n_t^{\max} = f^n(S_t)$, the rule for inflation of the home goods is $\pi_{H,t}^{\max} = f^{\pi_H}(S_t)$, and the rule for inflation of the foreign goods is $\pi_{F,t}^{\max} = f^{\pi_F}(S_t)$.

From the converged rules for $f^n(\cdot)$, $f^{\pi_H}(\cdot)$, and $f^{\pi_F}(\cdot)$, we derive the rules for the remaining variables determining the fiscal limit $T_t^{\max} = f^T(S_t)$, $T_t^{*\max} = f^{T^*}(S_t)$, $\lambda_t^{\max} = f^\lambda(S_t)$, $\lambda_t^{*\max} = f^{\lambda^*}(S_t)$, and $TOT_t^{\max} = f^{TOT}(S_t)$, which are consistent with the optimization conditions from the household's and the firms' problems.

To solve the model we proceed as follows:

1. Define the grid points by discretizing the state space (over the 4 dimensions). Make initial guesses for f_0^n , $f_0^{\pi_H}$, and $f_0^{\pi_F}$ over the state space.
2. Under the maximum tax rates $(\tau^{\max}, \tau^{*\max})$, at each grid point, solve the nonlinear model using the given rules f_{i-1}^n , $f_{i-1}^{\pi_H}$, and $f_{i-1}^{\pi_F}$, and obtain the updated rules f_i^n , $f_i^{\pi_H}$, and $f_i^{\pi_F}$. Specifically:

- (a) derive π_t and TOT_t in terms of $\pi_{H,t}$ and $\pi_{F,t}$ using (A.6) and (A.34). Derive y_t in

¹⁰We do not have an expectation term in the foreign Euler equation because in the model without risk we can use instead equation (A.21).

terms of a_t and n_t using (A.11).

- (b) Compute $c_{MU,t}^H$ from (A.9). Given (A.21) and (A.10), we have $c_t = \frac{c_{MU,t}^H}{\eta + \eta^* \frac{1-s}{s} TOT_t^{\eta^* - \eta}}$ and $c_t^* = c_t TOT_t^{\eta^* - \eta}$. Then $\lambda_t = (c_t)^{-\sigma}$ and $\lambda_t^* = (c_t^*)^{-\sigma}$.
- (c) Compute w_t , mc_t , and T_t using (A.2), (A.7), and (A.14).
- (d) From (A.25), (A.27), (A.30), (A.19), and (A.23), we can derive $c_{MU,t}^F$, y_t^* , N_t^* , T_t^* , w_t^* , and mc_t^* .
- (e) Derive π_t^* using (A.22). Given π_t , π_t^* , y_t , y_t^* , and (A.36), (A.37), obtain the nominal interest rate R_t from equation (A.35).¹¹
- (f) Use linear interpolation to obtain $f_{i-1}^n(S_{t+1})$, $f_{i-1}^{\pi_H}(S_{t+1})$, and $f_{i-1}^{\pi_F}(S_{t+1})$, where the state vector is $S_{t+1} = (g_{t+1}, g_{t+1}^*, TOT_t, s_{t+1}^R)$. This is necessary because the policy function at time t is a mapping from a value of the state variables on the grid points $(g_t, g_t^*, TOT_{t-1}, s_t^R)$ to endogenous variables $n_t, \pi_{H,t}, \pi_{F,t}$, but the policy function at time $t+1$ that evaluates $n_{t+1}, \pi_{H,t+1}, \pi_{F,t+1}$ may correspond to a value of the state variables in between two grid points, and therefore, to calculate it we have to linearly interpolate those two points. Then, follow the above steps to solve λ_{t+1} , λ_{t+1}^* , y_{t+1} , y_{t+1}^* , π_{t+1} , π_{t+1}^* .
- (g) Update the decision rules f_i^n , $f_i^{\pi_H}$, and $f_i^{\pi_F}$, using (A.3), (A.8), and (A.24). The integral in expectation terms is evaluated using numerical quadrature.

3. Check convergence of the decision rules. If $|f_i^n - f_{i-1}^n|$, $|f_i^{\pi_H} - f_{i-1}^{\pi_H}|$, or $|f_i^{\pi_F} - f_{i-1}^{\pi_F}|$ is above the desired tolerance (set to $1e-6$), go back to step 2. Otherwise, f_i^n , $f_i^{\pi_H}$, and $f_i^{\pi_F}$ are the decision rules.
4. Use the converged rules— f^n , f^{π_H} , and f^{π_F} —to compute the decision rules for f_i^T , f_i^{T*} , f_i^λ , $f_i^{\lambda*}$, and f_i^{TOT} .

Since the maximum tax rate is quite far away from the average tax rate, we may need to solve the non-linear model increasing the tax rate gradually from the calibration until we reach the maximum level.

¹¹When considering the ZLB if $R_t < 1$, use 1 as the nominal interest rate.

Using the maximum tax revenue $f^{\mathcal{T}}(\cdot)$, $f^{\mathcal{T}^*}(\cdot)$, $f^{\lambda}(\cdot)$, $f^{\lambda^*}(\cdot)$, and $f^{TOT}(\cdot)$, the distribution of fiscal limits is obtained using Markov Chain Monte Carlo simulations. Now, since we want to obtain the whole distribution, we evaluate expressions (29) without taking expectations. To proceed,

1. For each simulation j , we randomly draw sequences of the exogenous shocks for government spending shocks in the home country ($\varepsilon_{t+i}^{g^*,j}$), and government spending shocks in the foreign country ($\varepsilon_{t+i}^{g^*,j}$) for 1000 periods, $i = \{1, 2, 3, \dots, 1000\}$, conditional on the starting state $S_t = \{g_t, g_t^*, TOT_{t-1}, s_t^R\}$. If the simulation starts from the steady state, we call it the unconditional fiscal limit, otherwise it is the conditional one. At each period i , we obtain $T_{t+i}^{max,j}$, $T_{t+i}^{*max,j}$, $\lambda_{t+i}^{max,j}$, $\lambda_{t+i}^{*max,j}$, and $TOT_{t+i}^{max,j}$, ($i = 1, \dots, 1000$) by interpolating on the decision rules $f^{\mathcal{T}}(\cdot)$, $f^{\mathcal{T}^*}(\cdot)$, $f^{\lambda}(\cdot)$, $f^{\lambda^*}(\cdot)$, and $f^{TOT}(\cdot)$. This is necessary because the policy function is a mapping from a value of the state variables on the grid points $(a_t, g_t, a_t^*, g_t^*, TOT_{t-1})$ to endogenous variables \mathcal{T}_t , \mathcal{T}_t^* , λ_t , λ_t^* , and TOT_t , but following the stochastic processes, the realizations of the state variables may fall in between two grid points, and therefore, to calculate it we have to linearly interpolate those two points. Then, the fiscal limit for simulation j is computed (without taking expectations), conditional on S_t and particular sequences of shocks,
2. Repeat the simulation 50,000 times ($j = \{1, \dots, 50,000\}$) to have $\{B^{max,j}(S_t)\}_{j=1}^{50000}$ and $\{B^{*max,j}(S_t)\}_{j=1}^{50000}$, which form the distribution of $B^H(S_t)$.

Appendix B.2 Solving full model

When solving the nonlinear model with default, the state space is $S_t = \{(1 - \delta_t)b_{t-1}, b_{t-1}^*, g_t, g_t^*, TOT_{t-1}, s_t^R\}$

In this model there are 5 expectation terms in equations (Euler equation of home country (A.3) and (A.4), Euler equation of the foreign country (A.21)¹², Phillips curve of home (A.8) and foreign (A.24)) and thus we need five decision rules.¹³ It can be shown that these five

¹²Since we have assumed that the foreign country is Germany, we do not need to have default in that economy, and we have replaced the Foreign Euler equation with (A.21).

¹³If we consider two countries which are potentially close to their fiscal limits, then we would need to include the Foreign Euler equation and the interest rate on foreign sovereign bonds (R_t^*) and we would have an additional expectation term and an additional decision rule for n_t^* .

equations can be written as a function of only five variables: b_t , c_t , c_t^* , $\pi_{H,t}$ and $\pi_{F,t}$. Define the decision rules for the end-of-period home government bond as $b_t = f^b(S_t)$, consumption in home country as $c_t = f^c(S_t)$, consumption in foreign country as $c_t^* = f^{c^*}(S_t)$, inflation of the home goods is $\pi_{H,t} = f^{\pi_H}(S_t)$, and inflation of the foreign goods is $\pi_{F,t} = f^{\pi_F}(S_t)$. The decision rules are solved as follows:

1. Define the grid points by discretizing the state space (over the 6 dimensions). Make initial guesses for f_0^b , f_0^c , $f_0^{c^*}$, $f_0^{\pi_H}$, and $f_0^{\pi_F}$ over the state space.
2. At each grid point, solve the nonlinear model and obtain the updated rules f_i^b , f_i^c , $f_i^{c^*}$, $f_i^{\pi_H}$, and $f_i^{\pi_F}$ using the given rules f_{i-1}^b , f_{i-1}^c , $f_{i-1}^{c^*}$, $f_{i-1}^{\pi_H}$, and $f_{i-1}^{\pi_F}$:
 - (a) Derive π_t and TOT_t in terms of $\pi_{H,t}$ and $\pi_{F,t}$ using (A.6) and (A.34). Derive RER and τ_t using (A.38) and (A.15).
 - (b) Compute $c_{MU,t}^H$ from (A.10). Then y_t can be obtained from (A.9), and n_t is given by (A.11).
 - (c) Compute w_t , mc_t , and T_t using (A.2), (A.7), and (A.14).
 - (d) From (A.25), (A.27), (A.31), (A.30), (A.19), and (A.23), we can derive $c_{MU,t}^F$, y_t^* , N_t^* , τ_t^* , T_t^* , w_t^* , and mc_t^* .
 - (e) Derive π_t^* using (A.22). Given s_t^R , π_t , π_t^* , y_t , y_t^* , and (A.36), (A.37), obtain the nominal risk free interest rate R_t^f from equation (A.35).
 - (f) Compute b_t^* using (A.29) and the risky rate R_t using (A.13).
 - (g) Use linear interpolation to obtain $f_{i-1}^b(S_{t+1})$, $f_{i-1}^c(S_{t+1})$, $f_{i-1}^{c^*}(S_{t+1})$, $f_{i-1}^{\pi_H}(S_{t+1})$ and $f_{i-1}^{\pi_F}(S_{t+1})$, where $S_{t+1} = ((1 - \delta_{t+1})b_t, b_t^*, g_{t+1}, g_{t+1}^*, TOT_t)$. Then follow the above steps to solve λ_{t+1} , λ_{t+1}^* , y_{t+1} , y_{t+1}^* , π_{t+1} , π_{t+1}^* .
 - (h) Update the decision rules f_i^b , f_i^c , $f_i^{c^*}$, $f_i^{\pi_H}$, and $f_i^{\pi_F}$, using (A.3), (A.4), (A.8), (A.21), and (A.24).
3. Check convergence of the decision rules. If $|f_i^b - f_{i-1}^b|$, or $|f_i^c - f_{i-1}^c|$, or $|f_i^{c^*} - f_{i-1}^{c^*}|$, or $|f_i^{\pi_H} - f_{i-1}^{\pi_H}|$, or $|f_i^{\pi_F} - f_{i-1}^{\pi_F}|$ are above the desired tolerance (set to $1e - 6$), go back to

step 2; otherwise, f_i^b , f_i^c , $f_i^{c^*}$, $f_i^{\pi_H}$, and $f_i^{\pi_F}$ are the decision rules.