Weaving the Property Graph of Company Ownerships
We build graphs of company networks, to:

1. reveal power
   1. finding controllers
   2. studying the structure of Italian market
   3. studying dispersion of control
   4. global shareholding analysis

2. detect collusion and do forensics
   1. support AML
   2. detecting ultimate beneficial owners

3. evaluate risks
4. model propagations (e.g., of shocks)
5. guarantee compliance
6. perform enhanced due diligence
7. understand complex foreign shareholder structures
8. know real cash flows
• Who takes **decisions**?
• Who’s the ultimate **beneficial owner**?
• Is there **collusion**?
• How does **risk** propagate?
• What are the **real cash flows**?
Ownership and Control

- **Integrated ownership** is about direct and indirect, owners of a company
  - it can be seen in terms of cash flow rights

- **Control** is about voting power
  - of any direct and indirect owner of a company
Companies A and B have both **direct** (shares) and **indirect** ownership of company C.

Ownership of C is comparable when:
- A has 51% and B has 49% of C.
- A has 20% and 41% of C, and B has 49% of C.

Control is determined by decision power and ownership structure:
- A has 100% decision power and controls C when A has 51% and B has 49% of C.
- A has 10% decision power and does not control C when A has 10% and 41% of C, and B has 49% of C.
Integrated Ownership

- Traditional systems only store the first-level (the closest) shareholders for a specific company

```
+-------------------+     +-------------------+
| Mr Johns          |     | Mr Cedric         |
+-------------------+     +-------------------+
        | 15 %    |      | 5 %               |
+-------------------+     +-------------------+
| Lac               |     | Rupa              |
+-------------------+     +-------------------+
        | 40 %    |      | 30 %              |
+-------------------+     +-------------------+
| Lex               |     |                  |
```
Integrated Ownership: the basic math

- Indirect ownership

Mr Johns

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<th>20%</th>
<th>Lac</th>
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<th>30%</th>
<th>Rupa</th>
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| 90% | Lex |

Mr Johns

Lac

Rupa

Lex

0.2

0.2 x 0.3 = 0.06

0.06 x 0.9 = 0.054
Integrated Ownership: the basic math

- Parallel ownership

Mr. Johns

Lac

Rupa

Lex

\[ 0.2 \times 0.95 = 0.19 \]
\[ 0.3 \times 1.0 = 0.3 \]
\[ 0.19 + 0.3 = 0.49 \]
Integrated Ownership: the basic math

• Cycles (direct)

Mr Johns

- 20%

Lac

- 30%

- 20%

Lex

- 50%

Mr Ben

- 20%

\[
(0.2 \times 0.3) / (1 - 0.5) = 0.12
\]

\[
0.2 / (1-0.5) = 0.4
\]
Integrated Ownership: the basic math

- Cycles (indirect)

\[
1 \times 0.9 \times 0.6 = 0.54 \\
0.3 / (1 - 0.54) = 0.65
\]
Integrated Ownership: the basic math

- Cycles (nested)

Mr. Johns

Lac

Mr. Ben

Lex

30%

30%

70%

50%
Integrated Ownership: real cases

- Cycles (nested)

Mr Cedric → Poor → Rex → Felder → Lac → Poor
The weight of a path $P$ in an ownership graph is $w(P) = \prod_{(p_i, p_j) \in P} w(p_i, p_j)$.

An $\epsilon$-Baldone path $P$ from $s$ to $t$ is a sequence $[s, p_1, \ldots, p_n, t]$ such that $s \neq p_i$ for $i = 1, \ldots, n$ and $w(P) > \epsilon$ with $\epsilon \in \mathbb{R}^+$.

The $\epsilon$-Baldone ownership of a company $s$ on a company $t$ in an ownership graph $G$ is a function $\mathcal{O}_\epsilon^G(s, t) : (s, t) \rightarrow \mathbb{R}$ defined as $\sum_B w(P_i)$ where $B$ is the set of all possible $\epsilon$-Baldone paths from $s$ to $t$.

The Baldone ownership of a company $s$ on a company $t$ in an ownership graph $G$ is a function $\mathcal{O}^G(s, t) : (s, t) \rightarrow \mathbb{R}$ defined as $\lim_{\epsilon \rightarrow 0} \mathcal{O}_\epsilon(s, t)$ where $B$ is the set of all possible $\epsilon$-Baldone paths from $s$ to $t$. 

Problem complexity

Computing “all-to-all” Baldone ownerships can be solved in polynomial time in the number of companies. Conjecture: $n^y, y \in [2,3]$

Our approaches:

- **Closed-form expression** (let’s see how it works)
- **Pure Reasoning** (for approximated but efficient results)
- **Ad-hoc algorithm** (ongoing, for top-level performance)
Integrated Ownership: closed form

Adjacency matrix

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Integrated Ownership: closed form

\[ A_{ik} = \sum_j A_{ij} \times A_{jk} \]

Squared Adjacency Matrix
Integrated Ownership: **closed form**

\[ A_{ik} = \sum_j A_{ij} \times A_{jk} \]

**Cubed Adjacency Matrix**
Integrated Ownership: closed form

\[ A_{ik} = \sum_j A_{ij} \times A_{jk} \]

Cubed Adjacency Matrix
Integrated Ownership: closed form

\[ \sum_{i=1}^{N-1} A^i \]
\[ \hat{W}_{st} = W_{st} + \sum_{k \neq s} \hat{W}_{sk} W_{kt} \]

which can be manipulated into:

\[ \hat{W} = (I - \text{diag}(\hat{W})) W + \hat{WW} \]

and solved as:

\[ \hat{W} = \text{diag}(V)^{-1} VW \quad \text{with} \quad V = (I - W)^{-1} \]
In summary

• Ownership problem characterization
  • Theoretical study (e.g., complexity analysis)
  • Novel algorithms to compute all-to-all Baldone ownerships
  • Efficient and fully transparent ownership model

• Construction of the Italian company graph
  • all Italian companies, all links, all shareholders
    • 4.059M nodes, 3.960M edges, ~4M SCC, ~600K WCC
  • family links between shareholders

• Data available soon + basic AI tools for many applications …
Conclusions

1. reveal **power**
   1. finding **controllers**
   2. studying the **structure** of Italian market
   3. studyng **dispersion** of control
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2. detect **collusion** and do **forensics**
   1. support **AML**
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Open discussion