Firm and Worker Dynamics in an Aging Labor Market

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Banca d’Italia
**Figure I: Share of labor force aged 40+ & dynamism**

- Log dynamism (normalized in 1986)
- Fraction of labor force aged 40 and older

- EU
- JJ

- Aging
- Firm dynamics
- Worker dynamics
- Growth
- Longer time series
- Shift-share
**BACKGROUND**

**Figure I: Share of labor force aged 40+ & dynamism**

![Graph showing the share of labor force aged 40+ and dynamism over time. The graph includes lines for different regions and years, with a focus on the aging population and firm dynamics.]
1. Reduced-form empirical assessment

- Exploit predictable variation in aging across US states

Aging predicts 40–50% of declines & negative growth effect.

Structural assessment:

- Theory that links firm dynamism, worker dynamism, & growth to aging
- 40–50% of declines in firm & worker dynamism and 1 percentage point in annual economic growth
- Half due to equilibrium effects
What is the impact of aging on dynamism?

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2. **Structural assessment**
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   - 40–50% of declines in firm & worker dynamism and $-\frac{1}{4}$ percentage point in annual economic growth
   - Half due to equilibrium effects
Related literature

Growth in frictional labor markets

- Bean & Pissarides (’93); Aghion & Howitt (’94); Mortensen & Pissarides (’98); Postel-Vinay (’98); Hornstein et al. (’07); Michau (’13)

- My contribution: Endogenous growth & on-the-job search

Declining dynamism, secular stagnation

- Gordon (’12); Hyatt & Spletzer (’13); Davis & Haltiwanger (’14); Fernald (’14); Malloy et al (’14); Karahan et al (’16); Hsieh & Klenow (’17)

- My contribution: Structural framework & quant. assessment of aging
1. **Cross-state Evidence of the Impact of Aging**

2. **A Job Ladder with Creative Destruction**

3. **Structural Estimate of the Impact of Aging**
   - Life-cycle firm & worker dynamics
   - Impact of aging
   - Decomposing the mechanism
Cross-state Evidence on the Impact of Aging
State-fixed effect framework

- State-year data on dynamism & age composition 1978–2014

- Regress dynamism on share 40–64, controlling for state + year

\[
\log (y_{st}) = \alpha \log (\text{share}^{40-64}_{st}) + \xi_s + \xi_t + X_{st}\beta + \varepsilon_{st}
\]

- Standard errors clustered at state and year
STATE-FIXED EFFECT FRAMEWORK

- State-year data on dynamism & age composition 1978–2014

- Regress dynamism on share 40–64, controlling for state + year

\[
\log (y_{st}^a) = \alpha \log (\text{share}_{st}^{40-64}) + \xi_s + \xi_t + X_{st}\beta + \xi_a + \varepsilon_{st}^a
\]

- Standard errors clustered at state and year
Identifying variation

- Variation in timing & magnitude of aging across states

Figure II: Fraction aged 40–64 in four selected states
**Issue:** Mobility across states in response to dynamism
**Endogenous Mobility**

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- **Differential** mobility in response to *temporary* variation
**Endogenous Mobility**

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- **Differential** mobility in response to temporary variation

**Instrument current age composition with**

1. 10-year lagged age composition

   - Only effect on dynamism through current age composition
   - Strong explanatory power on current age composition
**Issue**: Mobility across states in response to dynamism

- **Differential mobility in response to temporary variation**

**Instrument current age composition with**

1. 10-year lagged age composition
   - Only effect on dynamism through current age composition
   - Strong explanatory power on current age composition

2. Birth rates 40–64 years earlier
   - Only effect on dynamism through current age composition
   - Decent explanatory power on current age composition
## Firm Dynamism

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>IV I</td>
<td>IV II</td>
<td>OLS</td>
<td>IV I</td>
<td>IV II</td>
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<td><strong>Labor force</strong></td>
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<td>Job reallocation</td>
<td>-0.448***</td>
<td>-0.527***</td>
<td>-1.183***</td>
<td>-0.518***</td>
<td>-0.539***</td>
<td>-0.978***</td>
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<td></td>
<td>(0.127)</td>
<td>(0.191)</td>
<td>(0.256)</td>
<td>(0.124)</td>
<td>(0.186)</td>
<td>(0.205)</td>
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<tr>
<td>Turnover</td>
<td>-0.630***</td>
<td>-0.961***</td>
<td>-1.573***</td>
<td>-0.774***</td>
<td>-0.984***</td>
<td>-1.300***</td>
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<td></td>
<td>(0.203)</td>
<td>(0.268)</td>
<td>(0.458)</td>
<td>(0.202)</td>
<td>(0.256)</td>
<td>(0.374)</td>
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<tr>
<td>Entry</td>
<td>-0.668***</td>
<td>-0.999***</td>
<td>-1.374***</td>
<td>-0.753***</td>
<td>-1.022***</td>
<td>-1.136***</td>
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<td></td>
<td>(0.189)</td>
<td>(0.247)</td>
<td>(0.498)</td>
<td>(0.188)</td>
<td>(0.245)</td>
<td>(0.409)</td>
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<tr>
<td>Exit</td>
<td>-0.600**</td>
<td>-0.940***</td>
<td>-1.753***</td>
<td>-0.809***</td>
<td>-0.962***</td>
<td>-1.449***</td>
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<td>(0.243)</td>
<td>(0.322)</td>
<td>(0.480)</td>
<td>(0.239)</td>
<td>(0.304)</td>
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<td><strong>Working age population</strong></td>
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<tr>
<td>Turnover</td>
<td>-0.764***</td>
<td>-1.266***</td>
<td>-1.680***</td>
<td>-0.923***</td>
<td>-1.296***</td>
<td>-1.411***</td>
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<td></td>
<td>(0.230)</td>
<td>(0.302)</td>
<td>(0.455)</td>
<td>(0.223)</td>
<td>(0.299)</td>
<td>(0.394)</td>
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<tr>
<td>Entry</td>
<td>-0.827***</td>
<td>-1.361***</td>
<td>-1.455***</td>
<td>-0.932***</td>
<td>-1.393***</td>
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<td>(0.195)</td>
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<td>(0.440)</td>
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<td>-1.795***</td>
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<td>(0.298)</td>
<td>(0.355)</td>
<td>(0.519)</td>
<td>(0.283)</td>
<td>(0.339)</td>
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## Worker Dynamism and Growth

### Panel C: Worker Dynamics

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<tr>
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<th>Labor Force</th>
<th>Working Age Population</th>
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<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>IV I</td>
</tr>
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<td>EU</td>
<td>-0.439***</td>
<td>-0.924**</td>
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<td></td>
<td>(0.145)</td>
<td>(0.375)</td>
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<tr>
<td>JJ</td>
<td>-0.477*</td>
<td>-0.113</td>
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<td></td>
<td>(0.229)</td>
<td>(0.732)</td>
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<td>UE</td>
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<td></td>
<td>(0.126)</td>
<td>(0.273)</td>
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</table>

### Panel D: Growth in GDP per Worker

<table>
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<th>Growth</th>
<th>OLS</th>
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<th>IV II</th>
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</thead>
<tbody>
<tr>
<td>Growth</td>
<td>-0.066</td>
<td>-0.090**</td>
<td>-0.137**</td>
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<tr>
<td></td>
<td>(0.046)</td>
<td>(0.040)</td>
<td>(0.061)</td>
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<tr>
<td></td>
<td>-0.063</td>
<td>-0.092**</td>
<td>-0.115**</td>
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<tr>
<td></td>
<td>(0.043)</td>
<td>(0.039)</td>
<td>(0.047)</td>
</tr>
</tbody>
</table>
PREDICTED IMPACT OF AGING

(A) Turnover rate

(B) Entry rate

(C) EU hazard

(D) UE hazard

Log change in annual hazard rate

Log change in monthly hazard rate


Predicted
Actual

Predicted
Actual

Predicted
Actual

Predicted
Actual
A Job Ladder Model with Creative Destruction
Key ingredients

○ **Job ladder**: Ranking of firms that workers gradually climb

○ **Entrepreneurial choice**

○ **Creative destruction**: Entrants push out incumbents
Demographics & preferences

- **Agents**: Unit mass of individuals, \( a = 1, \ldots, A \)
  - Move to the next age at rate \( \kappa(a) \)
  - Oldest age group dies at rate \( \kappa(A) \) and is replaced by offspring

- **Preferences**: Risk-neutral and altruistic w.r.t. offspring

\[
\mathbb{E}_t \int_t^\infty \exp \left( -\tilde{\rho}(\tau - t) \right) \left[ C(\tau) + \tilde{B}(\tau) \right] d\tau
\]

where \( \tilde{B}(\tau) = B(\tau) \) if unemployed; zero o.w.
○ **Multiworker firms**: Idiosyncratic productivity $\tilde{z}$

$$d\tilde{z}(t) = \mu_o dt + \sigma dW(t)$$

○ **Production**: At match level, $y(z, x) = e^{\tilde{z}} \times x$

  ○ $x$ = quality of match; starts at $x = 1$

  ○ Jumps to $x_b$ or $x_g$, $x_b < 1 < x_g$, with equal prob at rate $\psi$

  ○ Worker flows $>>$ job flows
Two sources of growth:

1. Growth of incumbents at exogenous rate $\mu_o$

2. Selection of firms at endogenous rate $\mu$

$\implies$ Total growth rate $\mu_e = \mu_o + \mu$

Transformation: $z = \tilde{z} - \tilde{z}(t)$ etc.

- Incumbents fall behind at rate of obsolescence, $\mu = \mu_e - \mu_o$
INDIVIDUAL’S PROBLEM

WHEN TO SWITCH EMPLOYER & BECOME ENTREPRENEUR

○ Job finding rate $\lambda$ from both U & E

○ Entrepreneurship opportunities at rate $\gamma$
  ○ Entry cost $c \sim \Omega(a)$ and has to quit job (if employed)
  ○ Draws productivity from innovation distribution $\phi(z)$
  ○ Sells idea to MF and returns to labor market as unemployed

○ Wage setting following Cahuc et al (2006)
\[ \rho V (z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V (z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V (z, x_u, a)}{\partial z^2} + \]

\[ \text{obsolescence} \]

\[ + \kappa(a) \left[ \tilde{V} (z, x_u, a + 1) - V (z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V} (z, x_i, a) - V (z, x_u, a) \right] + \]

\[ \text{individual ages} \]

\[ \text{match quality is revealed} \]

\[ + \lambda \beta \int_0^\infty \left\{ V (z', x_u, a) - V (z, x_u, a) \right\}^+ dF(z') + \gamma \int_\tilde{c}^{\bar{c}} \left\{ E + U (a) - V (z, x_u, a) - c \right\}^+ d\Omega(c; a) \]

\[ \text{new job offer} \]

\[ \text{Entrepreneurship opportunity} \]
Value of match & decision rules

\[
\rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \\
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\]

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\text{new job offer}
\]

\[
\text{Entrepreneurship opportunity}
\]
\[ \rho V (z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V (z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V (z, x_u, a)}{\partial z^2} + \]

obsolescence

\[ + \kappa(a) \left[ \tilde{V} (z, x_u, a + 1) - V (z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V} (z, x_i, a) - V(z, x_u, a) \right] + \]

individual ages

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new job offer

Entrepreneurship opportunity

---

**Known quality**

**Unemployment**

**Entrepreneurship**
\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

\[ + \kappa(a) \left[ \tilde{V}(z, x_u, a + 1) - V(z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V}(z, x_i, a) - V(z, x_u, a) \right] + \]

\[ + \lambda \beta \int_{0}^{\infty} \{ V(z', x_u, a) - V(z, x_u, a) \}^+ dF(z') + \gamma \int_{\bar{c}}^{\tilde{c}} \{ E + U(a) - V(z, x_u, a) - c \}^+ d\Omega(c; a) \]
Value of match & decision rules

\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

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Known quality
Unemployment
Entrepreneurship

Entrepreneurship opportunity

new job offer
Value of match & decision rules

\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

obsolescence

\[ + \kappa(a) \left[ \tilde{V}(z, x_u, a + 1) - V(z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V}(z, x_i, a) - V(z, x_u, a) \right] + \]

individual ages

match quality is revealed

\[ + \lambda \beta \int_0^\infty \left\{ V(z', x_u, a) - V(z, x_u, a) \right\}^+ dF(z') + \gamma \int_{\bar{c}}^\infty \left\{ E + U(a) - V(z, x_u, a) - c \right\}^+ d\Omega(c; a) \]

new job offer

Entrepreneurship opportunity
\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

\[
\text{obsolescence} + \kappa(a) \left[ \tilde{V}(z, x_u, a + 1) - V(z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V}(z, x_i, a) - V(z, x_u, a) \right] + \]

\[
\text{individual ages} \quad \text{match quality is revealed} \quad \text{new job offer} + \lambda \beta \int_0^\infty \{ V(z', x_u, a) - V(z, x_u, a) \}^+ dF(z') + \gamma \int_{\bar{c}}^\infty \{ E + U(a) - V(z, x_u, a) - c \}^+ d\Omega(c; a) \]

\[ \text{Entrepreneurship opportunity} \]
\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

obsolescence

\[ + \kappa(a) \left[ \tilde{V}(z, x_u, a + 1) - V(z, x_u, a) \right] + \psi \sum_{i} \pi(x_i) \left[ \tilde{V}(z, x_i, a) - V(z, x_u, a) \right] + \]

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new job offer

Entrepreneurship opportunity
Value of match & decision rules

\[ \rho V(z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V(z, x_u, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V(z, x_u, a)}{\partial z^2} + \]

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\[ + \kappa(a) \left[ \tilde{V}(z, x_u, a + 1) - V(z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V}(z, x_i, a) - V(z, x_u, a) \right] + \]

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\[ \text{new job offer} \]

\[ \text{Entrepreneurship opportunity} \]

○ JJ MOBILITY: \( V(z(z, x, a), x_u, a) = V(z, x, a) \)
\[ \rho V (z, x_u, a) = y(z, x_u) - \mu \times \frac{\partial V (z, x_u, a)}{\partial z} + \mu \times \frac{\partial^2 V (z, x_u, a)}{\partial z^2} + \]
\[ \text{obsolescence} \]
\[ + \kappa(a) \left[ \tilde{V} (z, x_u, a + 1) - V (z, x_u, a) \right] + \psi \sum_i \pi(x_i) \left[ \tilde{V} (z, x_i, a) - V (z, x_u, a) \right] + \]
\[ \text{idividual ages} \]
\[ + \lambda \beta \int_0^\infty \{ V (z', x_u, a) - V (z, x_u, a) \}^+ dF(z') + \gamma \int_{\bar{c}} \{ E + U(a) - V (z, x_u, a) - c \}^+ d\Omega(c; a) \]
\[ \text{new job offer} \]

- **JJ Mobility:** \( V (z(z, x, a), x_u, a) = V (z, x, a) \)
- **Entrep. Entry:** \( \bar{c}(z, x, a) + V (z, x, a) = E + U(a) \)
Firm’s problem

Post vacancies \( v \) subject to cost \( C(v) = r + c(v) \)

- \( c(v) \) is strictly convex flow cost per vacancy
- \( r \) is fixed cost associated with employing a unit of capital

\[ \implies \text{Stop paying} \Rightarrow \text{exit} \]

\[
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) \right] + \right.
\]

\[ + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \left[ - c(v) \right] \left\} - \frac{\sigma^2}{2} J''(z) \right. \]

- \( \textbf{Vacancy policy}: \ v(p) \) defined by FOC

\[ \mu J'(z) - r - \text{fixed cost} - \text{drift in } z + \text{shocks to } z \]
CLOSING THE MODEL

HOUSEHOLDS OWN FIRMS THROUGH MUTUAL FUND

- Avoids age of founder as state (Romer, 1990)
- Rents out $K$ capital to firms in competitive market
  $\implies$ Factor in fixed supply $\implies$ Creative destruction

LABOR MARKET: Cobb-Douglas matching function, $m = \chi V^\alpha$
CHARACTERIZING BEHAVIOR & THE EQUILIBRIUM

Prop. 1 (Mismatch and worker dynamism)
(a) Better matched individuals are less likely to move
(b) A better matched labor market discourages vacancy creation

Prop. 2 (Mismatch and entrepreneurship)
(a) Better matched individuals are less entrepreneurial
(b) A better matched labor market discourages entrepreneurship

Prop. 3 (Amplification)
Rate of obsolescence increases in the aggregate entry rate, \( \mu = \frac{e}{\zeta} \)

Less entry \( \implies \) Lower rate of obsol. \( \implies \) Less mismatch \( \implies \) Less entry
Amplification

Aging

↓

Entry

Worker pairings

↑

Growth

Michael Bloomberg  Aging and JJ  Aging and entry
Amplification

Aging
Amplification

Aging

Worker pairing with firms
Amplification

Aging  →  Entry

Worker

Pairing with firms
AMPLIFICATION

Aging

Worker pairing with firms

→ Entry

↑ Pairing with firms
Amplification

Aging \rightarrow \downarrow \text{Entry} \leftarrow \uparrow \text{Worker pairing with firms} \rightarrow \downarrow \text{Growth}

Michael Bloomberg Aging and JJ Aging and entry
Structural Estimate of the Impact of Aging
AGING EXPERIMENT WITHIN THE MODEL

○ **TARGET**: salient features of aggregate firm & worker dynamism in BDS + SIPP in 2012–2014

○ **VALIDATION**
  1. Life-cycle firm dynamics
  2. Life-cycle worker dynamics
  3. Link between worker and firm dynamics

○ **CHANGE AGE COMPOSITION TO 1986**
  ○ Reduce rate at which old individuals exit
  ○ Evaluate impact on dynamism holding everything else constant
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More on UE/JJ  
Graphs  
Decker et al. (2017)’s empirical facts  
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## The decline in dynamism

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Aging has had negative growth effect

**Table III: Impact of aging on growth & unemployment**

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Aging has had negative growth effect but positive level effect

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Table IV: Log change in level of output, model

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Direct & indirect effects

Figure IV: Employment distribution over firm productivity

- Young individuals in young economy
- Old individuals in young economy
- Old individuals in old economy
**Direct & indirect effects**

**Figure IV: Employment distribution over firm productivity**

- Young individuals in young economy
- Old individuals in young economy
- Old individuals in old economy

- Log firm productivity
- Fraction of workforce

Match quality
Equilibrium effects account for half of declines

\[
\text{Hazard} = \sum_a \text{share}_a \int_y \text{Decision}_a(y) \times d\text{Employment}_a(y)
\]

Age conditional rate

Table V: Decomposing the change in the JJ & entry hazard

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Details
Shift share
Life-cycle I
Life-cycle II
Emp. by size

26
Equilibrium effects account for half of declines

\[ \text{Hazard} = \sum_{a} \text{share}_a \int \text{Decision}_a(y) \times d\text{Employment}_a(y) \]

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Details Shift share Life-cycle I Life-cycle II Emp. by size 26
Equilibrium effects account for half of declines

\[ \text{Hazard} = \sum_a \text{share}_a \int \frac{\text{Decision}_a(y) \times d\text{Employment}_a(y)}{y} \]

Age conditional rate

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Total effect

Details Shift share Life-cycle I Life-cycle II Emp. by size
**Equilibrium effects account for half of declines**

$$\text{Hazard} = \sum_a \text{share}_a \int \text{Decision}_a(y) \times d\text{Employment}_a(y)$$

**Age conditional rate**

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How much does entry fall with mismatch?

Figure V: Distribution of old individuals & entry policy

- Young economy
- Old economy
- Policy

Fraction of workforce vs. Log firm productivity

Entry rate vs. Log firm productivity
1. Age-segregated labor markets

2. No aging of potential entrepreneurs

3. Approximate transition dynamics

4. Income dynamics
Aging explains

1. **40–50% of declines** in entry, exit, incumbent job reallocation, EU and JJ mobility; modest fall in UE

2. $-\frac{1}{4}$ percentage points decline in growth

3. Half due to equilibrium effects

**Policy:** Regulation/taxation or immigration?
1. Aging typically accounts for at most half of declines
   - Labor supply (Karahan et al, 2016)
   - Licensing (Kleiner and Krueger, 2013)
   - Training requirements (Cairo, 2013)
   - EPL (Autor et al., 2007)

2. Anecdotal evidence that aging has reduced dynamism & growth in other countries
   - A rigorous cross-country analysis is missing
Thank you
A longer perspective

Figure VI: Share 40 and older and EU hazard

Fraction of labor force aged 40 and older

Share older

EU hazard

Back to motivation  Back to conclusion
**Age Composition**

**(A) Age Distribution (Labor Force)**

Age distribution of the labor force.

**(B) Share 40+ (Labor Force)**

Share of the labor force aged 40+.

**(C) Age Distribution (Working Age)**

Age distribution of the working age population.

**(D) Share 40+ (Working Age)**

Share of the working age population aged 40+.

*Motivation*
Firm dynamics definitions

Data

- Annual data on firms and establishments covering private sector

Definitions

- Job creation: $JC_t = \sum_i (size_{it} - size_{it-1})^+$
- Job destruction: $JD_t = \sum_i (-(size_{it} - size_{it-1}))^+$

$$JC_t + JD_t = JC_{t}^{inc} + JD_{t}^{inc} + JC_{t}^{entry} + JD_{t}^{exit}$$

Job reallocation $t = \text{Inc job reallocation}_t + \text{Estabs. turnover}_t$
Figure VIII: Establishment reallocation rates

Motivation
Exit/entry
By firm age
By industry
ENTRY AND EXIT

(A) ESTABLISHMENT

(B) ESTABLISHMENT (UNWEIGHTED)

(C) FIRM

(D) FIRM (UNWEIGHTED)

Appendix

Motivation
(A) Establishment exit rate

(B) Incumbent job reallocation

(C) Firm exit rate
Dynamics by industry

(A) Turnover

(B) Job reallocation

(c) Turnover

(d) Job reallocation

Appendix Motivation
Worker mobility definitions

Data

- CPS (1978–2015)
- BLS (1948–2015)

Definitions

- $EU_{it} = \text{employed in month } t, \text{ unemployed in } t + 1$
- $UE_{it} = \text{unemployed in } t, \text{ employed in } t + 1$
- $JJ_{it} = \text{employed in } t, \text{ different main employer in } t + 1$
Worker dynamics

- Large fall in EU & JJ hazard
- Little evidence of secular decline in UE hazard

(A) EU hazard
(B) JJ hazard
(C) UE hazard
LN AND NL FLOWS

- Declines in the hazard of moving in and out of the labor force

### Appendix

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<td>2018</td>
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**Motivation**
- Annual growth in real GDP per worker slowed from 2.6% in 1984–1988 to 1.7% in 2012–2016

**Figure XIV: Annual HP-filtered growth rate**

- Annual growth rate
- 1977: 0.02
- 1987: 0.04
- 1997: 0.02
- 2007: 0.00
- 2017: -0.02
- 2018: -0.04

- Per worker
- Per hour
Karahan, Pugsley and Sahin (2016)

- Labor supply growth explains $\frac{1}{4}$ of fall in start-up rate
- No change in incumbent life-cycle dynamics

Two key differences

1. "Quality"/composition of labor force rather than quantity
2. Partly different set of outcomes: Worker dynamics, incumbent dynamics and growth
Denote by $rate_{a}^{late}$ age-conditional mobility rates in 2012–2014.

Denote by $share_{a}^{p}$ the share of the labor force in age $a$ in period $p$.

Direct effect = change due to shift in age composition under fixed age-conditional mobility rates.

$$rate^{direct} = \sum_{a} rate_{a}^{late} \left[ share_{a}^{early} - share_{a}^{late} \right]$$
## Shift-share

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<td>CPS</td>
<td>0.251</td>
<td>0.199</td>
<td>0.221</td>
<td>0.196</td>
<td>13.7</td>
<td>1.9</td>
<td>13.9</td>
</tr>
<tr>
<td><strong>Panel D: Entry to entrepreneurship</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>156.5</td>
<td>109.9</td>
<td>100</td>
<td>100</td>
<td>56.5</td>
<td>9.9</td>
<td>17.5</td>
</tr>
<tr>
<td>Opportunistic</td>
<td>156.5</td>
<td>110.0</td>
<td>100</td>
<td>100</td>
<td>56.5</td>
<td>10.0</td>
<td>17.7</td>
</tr>
<tr>
<td>Expect to grow</td>
<td>156.5</td>
<td>110.8</td>
<td>100</td>
<td>100</td>
<td>56.5</td>
<td>10.8</td>
<td>19.2</td>
</tr>
</tbody>
</table>
Michael Bloomberg

- Partner at Solomon Brothers, laid off in 1981 (at age 39)
- Started financial service company Bloomberg LP
- Current net worth: $47.8bn

- **Would he have started Bloomberg if he had not been laid off?**
  - Walt Disney, JK Rowling, Thomas Edison, Mark Cuban, Oprah Winfrey, Sallie Krawcheck, Bernie Marcus and Arthur Blank...
Figure XV: Entrepreneurship entry by age

<table>
<thead>
<tr>
<th>Age</th>
<th>Probability of starting a business</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-24</td>
<td>.04</td>
</tr>
<tr>
<td>25-34</td>
<td>.08</td>
</tr>
<tr>
<td>35-44</td>
<td>.12</td>
</tr>
<tr>
<td>45-54</td>
<td>.16</td>
</tr>
<tr>
<td>55+</td>
<td>0</td>
</tr>
</tbody>
</table>

- **Baseline**
- **Opportunistic**
- **Expects to grow**
(A) **Cover owners’ salary**

(B) **Hired at least one employee**

(C) **Share surviving**

(D) **Log firm size**
Search and matching

- Individuals may be either employed or unemployed

\[ \text{Search with the same efficiency (normalized to one)} \]

If firms post \( \bar{v} \) vacancies, total number of matches equals \( \chi \bar{v} \alpha \)

Denote by \( \lambda \) rate at which individuals meet with open vacancies, \( q \) rate at which vacancy contacts individuals

\[ \lambda = \chi \bar{v} \alpha, \quad q = \chi \bar{v} \alpha - 1 \]
Search and matching

- Individuals may be either employed or unemployed

- Search with the same efficiency (normalized to one)
Search and matching

- Individuals may be either employed or unemployed
- Search with the same efficiency (normalized to one)
- If firms post $\bar{v}$ vacancies, total number of matches equals $\chi \bar{v}^\alpha$
Search and matching

- Individuals may be either employed or unemployed
- Search with the same efficiency (normalized to one)
- If firms post $\bar{v}$ vacancies, total number of matches equals $\chi \bar{v}^\alpha$
- Denote by $\lambda$ rate at which individuals meet with open vacancies, $q$ rate at which vacancy contacts individuals

$$
\lambda = \chi \bar{v}^\alpha, \quad q = \chi \bar{v}^{\alpha-1}
$$
APPENDIX C
Wage setting


- **Unemployed**: Outside value plus $\beta$ of surplus

- **Employed (I)**: Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus $\beta$ of differential

- **Employed (II)**: Poacher with higher valuation
  - Switch to poacher, get current match plus $\beta$ of differential

$\Rightarrow$ Renegotiation when one party has credible threat
Wage setting


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$\Rightarrow$ Renegotiation when one party has credible threat
**Offer matching framework of Cahuc et al (2006)**

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WAGE SETTING

OFFER MATCHING FRAMEWORK OF CAHUC ET AL (2006)

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- **EMPLOYED (II)**: Poacher with higher valuation
  - Switch to poacher, get current match plus $\beta$ of differential

⇒ Renegotiation when one party has credible threat
Balanced growth

- On the BGP, \( Z(t) \) and \( \tilde{r}(t) \) grow at endogenous rate \( \mu \), while incumbent firm productivity in expectation does not change.
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by

\[
\begin{align*}
1. z &= \log\left(\frac{Z(t)}{\bar{Z}(t)}\right) \quad \text{normalized log firm productivity} \\
2. \bar{r} &= \text{the normalized price of a marketing specialist} \\
3. \phi(z) &= \text{the normalized innovation distribution} \\
4. \rho &= \tilde{\rho} - \mu \quad \text{the effective discount rate} \quad \Rightarrow \\
& \text{Incumbent firm productivity drifts at } -\mu \text{ while } \bar{r} \text{ is constant.}
\end{align*}
\]
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

- Study transformed economy in which $Z(t)$ and $\tilde{r}(t)$ do not grow.

- Normalize by $Z(t)$ and denote by
  1. $z = \log(Z(t)/Z(t))$ normalized log firm productivity,
Balanced growth

- On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change.

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  1. $z = \log(Z(t)/Z(t))$ normalized log firm productivity
  2. $r$ the normalized price of a marketing specialist
**Balanced growth**

○ On the BGP, $Z(t)$ and $\tilde{r}(t)$ grow at endogenous rate $\mu$, while incumbent firm productivity in expectation does not change

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  3. $\phi(z)$ the normalized innovation distribution
  4. $\rho = \tilde{\rho} - \mu$ the effective discount rate

$\implies$ Incumbent firm productivity drifts at $-\mu$ while $r$ is constant.
\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \]

Aging
Value of unemployment

\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_{0}^{\infty} \{V(z, x_u, a) - U(a)\}^+ dF(z) \]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)

Aging

Job offer
Value of unemployment

\[
\rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_0^\infty \{V(z, x_u, a) - U(a)\}^+ dF(z)
\]

- An individual meets firm with productivity \( z \) at rate \( \lambda f(z) \)
  - Initial match productivity is unknown, \( x = x_u \)
Value of unemployment

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**Value of unemployment**

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  - Initial match productivity is unknown, \( x = x_u \)
  - Gets \( \beta \) of difference between value of match, \( V(z, x_u, a) \), and \( U(a) \)
- Opportunity to start business at rate \( \gamma(a) \)
\[ \rho U(a) = b + \kappa(a) [U(a + 1) - U(a)] + \lambda \beta \int_{0}^{\infty} \{V(z, x_u, a) - U(a)\}^+ dF(z) + \gamma(a) \int_{\bar{c}}^{\tilde{c}} \{E - c\}^+ d\Omega(c) \]

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- Opportunity to start business at rate \( \gamma(a) \)
  - Associated entry cost \( c \) drawn from \( \Omega \)
  - \( E \) denotes expected value of entrepreneurship
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- Decision rules: \( z_u(x_u, a) \)
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- Opportunity to start business at rate \( \gamma(a) \)
  - Associated entry cost \( c \) drawn from \( \Omega \)
  - \( E \) denotes expected value of entrepreneurship

- Decision rules: \( z_u(x_u, a) \) and \( \bar{c}_u \)
\[
\rho V (z, x, a) = e^z - \mu \frac{\partial V (z, x, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V (z, x, a)}{\partial z^2} + \\
\text{drift in } z \quad \text{shocks to } z \\
+ \kappa(a) [\max \{V (z, x, a + 1), U(a + 1)\} - V (z, x, a)] + \\
\text{individual ages} \\
+ \lambda \beta \int_0^\infty \max \{V (z', x_u, a) - V (z, x, a), 0\} dF(z') + \\
\text{new job offer} \\
+ \gamma(a) \int_{\bar{c}}^{\bar{c}} \max \{E - c - V (z, x, a) + U(a), 0\} d\Omega(c) + \\
\text{entrepreneurship opportunity}
\]
An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$. 
An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$

She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea
Value of entrepreneurship

○ An individual who enters entrepreneurship draws an initial productivity $z$ from $\Phi$

○ She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea

○ Hence the expected value of entry equals

$$E = \int_{0}^{\infty} J(z) d\Phi(z)$$
Denote by $J(z)$ the value of hiring to a firm

$$
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_{a} \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) \right] + 
\right. 
+ (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right\} - c(v) \left. 
\right\} - r - \mu J'(z) + \frac{\sigma^2}{2} J''(z)
$$

- value from meeting unemployed individual
- value from meeting employed individual
- fixed cost
- drift in $z$
- shocks to $z$
Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( u(a) \{ V(z, x_u, a) - U(a) \}^+ \right) + \right. \right.$$

$$+ (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \left. \right] - c(v) \right\} - \frac{r}{2} \mu J'(z) + \frac{\sigma^2}{2} J''(z)$$

Post vacancies $v$ subject to $c(v)$
Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1-\beta)q \left[ \sum_{a} u(a) \{ V(z, x_u, a) - U(a) \}^+ \right] + \right.$$  

$$+ (1-u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right\} - c(v) \right\} - r \text{ fixed cost} - \mu J'(z) + \sigma^2 J''(z)$$

Post vacancies $v$ subject to $c(v)$

Has to pay fixed cost $r$ to remain in hiring market
Denote by $J(z)$ the value of hiring to a firm

$$
\rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( \frac{u(a) \{ V(z, x_u, a) - U(a) \}}{\text{value from meeting unemployed individual}} \right) + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right] - c(v) \right\} - r \text{ \text{fixed cost}} - \mu J'(z) + \sigma^2 J''(z) \text{ \text{shocks to z}}
$$

Post vacancies $v$ subject to $c(v)$

Has to pay fixed cost $r$ to remain in hiring market

**Decision rules:** vacancy policy, $v(z)$, and exit threshold, $z$
Vacancy policy

\[
v(z) = \left\{ \frac{(1 - \beta)q}{c_v} \left[ \sum_a u(a) \{V(z, x_u, a) - U(a)\}^+ + (1 - u) \int \{V(z, x_u, a) - V(z', x, a)\}^+ dG(z', x, a) \right] \right\}^{1/\eta}
\]

Individual is unemployed

Individual is employed
Vacancy policy

\[ v(z) = \left\{ \begin{array}{ll}
\frac{(1 - \beta)q}{c_v} \left[ \sum_a u(a) \{ V(z, x_u, a) - U(a) \}^+ + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right] \\
\end{array} \right\}^{1/\eta}
\]

Less labor market mismatch \( \implies \) less vacancy creation

1. Larger share of individuals are employed
Vacancy policy

\[ v(z) = \left\{ \frac{1 - \beta}{c_v} \left[ \sum_a u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+ + (1 - u) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ dG(z', x, a) \right] \right\}^{1/\eta} \]

**Less labor market mismatch \implies less vacancy creation**

1. Larger share of individuals are employed

2. Employed individuals are less mismatched
Value functions \( \{U, V, E, J\} \); policies \( \{\bar{c}_u, z_u(x, a), z(z, x, a), \bar{c}(z, x, a)\} \); policies \( \{z, v(z)\} \); numbers \( \{r, e, \mu, \bar{v}, \lambda, q\} \); and distributions \( \{h(z), f(z), u(a), g(z, x, a)\} \); such that

1. Value and policy functions of unemployed, match and recruiting firm solve the respective problems

2. The aggregate entry rate \( e \) is consistent with individual behavior

3. The growth rate \( \mu \) is consistent with the entry rate

4. Aggregate vacancies \( \bar{v} \) are consistent with firm behavior and the finding rates are \( \lambda = \chi \bar{v}^{\alpha}, q = \chi \bar{v}^{\alpha-1} \)

5. Distributions solve respective KFE and are stationary
1. $h(z)$ denotes the pdf of recruiting firms

2. $f(z)$ denotes the *vacancy-weighted* pdf of recruiting firms

3. $u(a)$ denotes the mass of unemployed individuals of age $a$

4. $g(z, x, a)$ denotes the pdf of employed individuals

For all densities, upper case letters denote the corresponding cdf
The distribution of recruiting firms, $h$, solves the KFE

$$0 = \mu h'(z) + \frac{\sigma^2}{2} h''(z) + e\zeta \exp(-\zeta z), \quad z > 0$$  \hspace{1cm} (1)

subject to,

$$h(0) = 0, \quad \int_0^\infty h(z) \, dz = 1, \quad e = \frac{\sigma^2}{2} h'(0)$$  \hspace{1cm} (2)

where $e$ is the aggregate entry rate

○ Last condition can be seen by integrating (1) from 0 to $\infty$, which gives

$$0 = -\mu h(0) - \frac{\sigma^2}{2} h'(0) + e,$$

and imposing $h(0) = 0$.
(1) is a second-order ordinary differential equation with solution,

\[ h(z) = e^{\mu - \frac{\sigma^2}{2} \zeta} \left[ \exp(-\zeta z) - \exp\left(-\frac{2\mu}{\sigma^2} z\right) \right] \]  (3)
(1) is a second-order ordinary differential equation with solution,

$$h(z) = \frac{e}{\mu - \frac{\sigma^2}{2} \zeta} \left[ \exp(-\zeta z) - \exp \left( -\frac{2\mu}{\sigma^2} z \right) \right]$$

where the growth rate of the economy is a function of the aggregate entry rate of entrepreneurs,

$$\mu = \frac{e}{\zeta}$$

(4)
The vacancy-weighted distribution of firms, $f(z)$, equals the density of recruiting firms at $z$ times the amount of vacancies they post,

$$f(z) = \frac{v(z)h(z)}{\bar{v}} \quad (5)$$

where $v(z)$ is the firm’s optimal vacancy policy and

$$\bar{v} = \int_{0}^{\infty} v(\tilde{z}) \, dh(\tilde{z})$$
Distribution of employment

On the BGP, \( g(z, x, a) \) satisfies the KFE

\[
0 = \mu \frac{\partial g(z, x, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 g(z, x, a)}{\partial z^2} + \lambda \frac{u(a)}{1 - u} f(z) \mathbb{1} \{x = x_u\} \mathbb{1} \{z > z^u(x_u, a)\} + \]

\[
\text{inflow from unemployment}
\]

\[
+ \kappa(a - 1) \mathbb{1} \{z > z^u(x_u, a)\} g(z, x, a - 1) - \kappa(a) g(z, x, a) + \]

\[
\text{inflow from aging}
\]

\[
+ \lambda f(z) \mathbb{1} \{x = x_u\} \int \mathbb{1} \{z > z^e(z', x', a)\} G(dz', dx', a) - \lambda \left[1 - F\left(z^e(z, x, a)\right)\right] g(z, x, a) + \]

\[
\text{inflow from lower rungs in job ladder}
\]

\[
+ \psi \mathbb{1} \{z > z^u(x, a)\} \pi(x) g(z, x_u, a) - \psi \mathbb{1} \{x = x_u\} g(z, x, a) - \gamma(a) g(z, x, a) \Omega\left(e^e(z, x, a)\right) + \]

\[
\text{outflow from learning}
\]

\[
\text{outflow from learning}
\]

\[
\text{outflow to entrepreneurship}
\]

with \( \pi(x_u) = 0 \) and \( g(z, x, 0) \equiv 0, \forall z, x \), subject to workers exiting at the boundary so that the density is zero and the pdf integrates to one.
The mass of unemployed of each age group, \( u(a) \), satisfies,

\[
0 = -\lambda \left[ 1 - F\left( z^u(x_u, a) \right) \right] u(a) + \left( 1 - u(a) \right) \sum_x \frac{\sigma^2}{2} \frac{\partial g(z^u(x, a), x, a)}{\partial z} + \]

\[
+ (1 - u(a)) \psi \pi(x_b) G\left( z^u(x_b, a), x_u, a \right) + 1 \{ a = 1 \} \kappa(A) -
\]

\[
- \kappa(a) u(a) + \kappa(a - 1) \left[ u(a - 1) + (1 - u) \sum_x G\left( z^u(x, a), x, a - 1 \right) \right] +
\]

\[
+ (1 - u(a)) \gamma(a) \int \Omega \left( \tilde{\omega}^e(z, x, a) \right) G(dz, dx, a)
\]

\[
\text{outflow to employment} \quad \text{individuals drifting below the threshold}
\]

\[
\text{individuals jumping below the threshold due to learning} \quad \text{newborn}
\]

\[
\text{outflow from aging} \quad \text{inflow from aging}
\]

\[
\text{entry to entrepreneurship}
\]

with the convention that \( u(0) = 0 \)
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] dG(z, x, a) \]
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F (z^e (z, x, a))] dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F (z^e (z, x, a))] d\hat{G}(z, x|a) \quad (8) \]
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \quad (8) \]

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched.
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] dG(z, x, a) \]

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1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation
4 EFFECTS OF AGING ON JJ HAZARD

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] \, dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left( 1 - \frac{u(a)}{m(a)} \right) \times \lambda \times \int [1 - F(z^e(z, x, a))] \, d\hat{G}(z, x | a) \]  \hspace{1cm} (8)

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation

3. \( F \) may change as firms change their vacancy posting decisions
4 Effects of Aging on JJ Hazard

\[ JJ = \lambda \int [1 - F(z^e(z, x, a))] \, dG(z, x, a) \]

\[ = \frac{1}{1 - u} \sum_a m(a) \left(1 - \frac{u(a)}{m(a)}\right) \times \lambda \times \int [1 - F(z^e(z, x, a))] \, d\hat{G}(z, x|a) \quad (8) \]

1. Changing \( m(a) \) will affect the aggregate JJ hazard since older individuals typically are better matched.

2. \( \lambda \) may change as firms respond to the changed economic environment by adjusting vacancy creation.

3. \( F \) may change as firms change their vacancy posting decisions.

4. Aging may give rise to changes in age-conditional labor market mismatch, \( \hat{G}(z, x|a) \)
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

(9)
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e (z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e (z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \] (9)
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \] (9)

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter
3 Effects of Aging on Entry Rate

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \gamma(a) dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]  

(9)

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect \( \bar{c}^e(z, x, a) \) (\( \bar{c}^u \)) as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting
3 EFFECTS OF AGING ON ENTRY RATE

\[ e = \frac{1}{M} \left\{ (1 - u) \int \Omega [\bar{c}^e(z, x, a)] \, \gamma(a) \, dG(z, x, a) + \Omega (\bar{c}^u) \sum_a u(a) \gamma(a) \right\} \]

\[ = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega [\bar{c}^e(z, x, a)] \, d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \quad (9) \]

1. Changing \( m(a) \) will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect \( \bar{c}^e(z, x, a) \) (\( \bar{c}^u \)) as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting

3. Age conditional labor market mismatch—\( \hat{G}(z, x|a) \) and \( u(a)/m(a) \)—may change through equilibrium effects
Appendix D
**Strategy**

**Target**: Salient features of *aggregate* firm & worker dynamism in BDS + SIPP in 2012–2014

- Monthly frequency
- 3 age groups
- Pre-set a few parameters to standard values
- Remaining parameters internally
# Calibrated values

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_v ) Cost of vacancy creation</td>
<td>Aggregate UE (2005–07)</td>
<td>( 4.5 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \pi(x_b) ) ( P(\text{match is low productive}) )</td>
<td>Aggregate EU</td>
<td>0.5</td>
</tr>
<tr>
<td>( x_g ) Productivity of high prod. match</td>
<td>Aggregate JJ</td>
<td>1.3</td>
</tr>
<tr>
<td>( \psi ) Rate of learning</td>
<td>Timing of decline in JJ with tenure</td>
<td>0.043</td>
</tr>
<tr>
<td>( b ) Flow value of unemployment</td>
<td>Indifference at margin</td>
<td>1.09</td>
</tr>
</tbody>
</table>

## Panel B: Entrepreneurship

<table>
<thead>
<tr>
<th>Description</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta ) Innovation distribution</td>
<td>Growth due to selection</td>
<td>20</td>
</tr>
<tr>
<td>( \gamma(a) ) Entrepreneurship opportunity</td>
<td>Entry rate and entry rate by age</td>
<td>([4.2; 4.5; 2.1] \times 10^{-3})</td>
</tr>
<tr>
<td>( C ) Dispersion in entry cost</td>
<td>Decline in entry with tenure</td>
<td>72</td>
</tr>
</tbody>
</table>

## Panel C: Firms

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta ) Curvature of vacancy creation</td>
<td>Size distribution of entrants</td>
<td>2</td>
</tr>
<tr>
<td>( \sigma ) Shocks to productivity</td>
<td>Size distribution</td>
<td>( 7 \times 10^{-3} )</td>
</tr>
<tr>
<td>( d ) Exit shock for firms</td>
<td>Average exit rate</td>
<td>( 3.8 \times 10^{-4} )</td>
</tr>
<tr>
<td>( K ) Capital</td>
<td>Average firm size</td>
<td>0.13</td>
</tr>
</tbody>
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</table>

### Panel A: Labor market mobility

### Panel B: Entrepreneurship

| $\zeta$      | Innovation distribution | Growth due to selection | 20 |
| $\gamma(a)$  | Entrepreneurship opportunity | Entry rate and entry rate by age | $[4.2; 4.5; 2.1] \times 10^{-3}$ |
| $C$          | Dispersion in entry cost | Decline in entry with tenure | 72 |

### Panel C: Firms

| $\eta$       | Curvature of vacancy creation | Size distribution of entrants | 2 |
| $\sigma$     | Shocks to productivity | Size distribution | $7 \times 10^{-3}$ |
| $d$          | Exit shock for firms | Average exit rate | $3.8 \times 10^{-4}$ |
| $K$          | Capital | Average firm size | 0.13 |
Calibrated values

- $C \Rightarrow$ Elasticity of entry to net value

**Figure XVII: Tenure profile of entrepreneurship entry hazard**

Tenure distribution

Hazard vs. Tenure (months)

0

0.001

0.002

0.003

0.004

0

24

48

72

96

120

Tenure (months)

Model

Data

Model

Data
## Calibrated values

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- $\eta \Rightarrow$ Elasticity of vacancy creation to net value

(A) Emp. share by size, entrants

(B) Emp. share by size
## Calibrated values

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</table>
Success I: Life cycle firm dynamics

- Calibration targets aggregate firm size and exit rate

⇒ Captures well life-cycle firm dynamics

(A) Firm size

(B) Exit rate

(C) Incumbent job realloc.
Success II: Life cycle labor market mobility

- Calibration targets aggregate JJ & EU hazard

⇒ SUPPORTS JOB LADDER AND LEARNING MECHANISMS

(A) EU

(B) JJ

<table>
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</table>
Success III: Linking firms and workers

- Matches hire & separation rates as function of firm growth

⇒ Supports joint model of firm & worker dynamics

(A) Data

(B) Model

Hires/separations by origin/destination by firm age  Average worker age by firm age  Back
**Standard values**

**Table VI: Pre-set parameter values**

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ Discount rate</td>
<td>Annual interest rate of 4%</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\chi$ Matching efficiency</td>
<td>Normalization</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha$ Elasticity of matching function</td>
<td>Petrongolo and Pissarides (2001)</td>
<td>0.7</td>
</tr>
<tr>
<td>$\beta$ Bargaining power</td>
<td>Bagger et al (2014)</td>
<td>0.3</td>
</tr>
</tbody>
</table>
### Calibration targets—Individuals

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Calibration targets—individuals

- ↑ share of low-productive matches $\implies$ ↑ EU hazard

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**Calibration targets—individuals**

- ↑ share of low-productive matches  \( \implies \) ↑ EU hazard

- ↑ \( x_g \)  \( \implies \) ↑ opportunity cost of JJ mobility  \( \implies \) ↓ JJ hazard

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</tr>
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Calibration targets—individuals

- ↑ share of low-productive matches  \(\implies\) ↑ EU hazard

- ↑ \(x_g\)  \(\implies\) ↑ opportunity cost of JJ mobility  \(\implies\) ↓ JJ hazard

- ↑ \(\psi\)  \(\implies\) learning is faster  \(\implies\) JJ falls quickly with tenure

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## Calibration targets—entrepreneurs

- \( \gamma(a) \): Entrepreneurship opportunity
- \( \zeta \): Innovation distribution
- \( C \): Dispersion in entry cost

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### Calibration targets—entrepreneurs

Calibration targets are used to adjust model parameters to match observed data. In this context, the entries are related to entrepreneurship and innovation distribution.

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Opportunity cost is positively correlated with tenure and hence decline in entry with tenure informs.
Calibration targets—entrepreneurs

- $\Omega \sim U(-C, C)$

- $\uparrow C \implies \downarrow$ change in entry for given change in value of entry

- Opportunity cost is positively correlated with tenure and hence decline in entry with tenure informs $C$

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Calibration targets—firms

- $\eta \uparrow \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size

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Calibration targets—firms

- $\eta \uparrow \Rightarrow$ more costly to hire many workers $\Rightarrow$ less dispersion in initial firm size

- Introduce small probability of firm death, $d$, that is independent of firm productivity

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- $\eta \uparrow \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size

- Introduce small probability of firm death, $d$, that is independent of firm productivity

- $\sigma \uparrow \implies \uparrow$ dispersion in steady-state firm productivity $\implies \uparrow$ dispersion in steady-state firm size

<table>
<thead>
<tr>
<th>Description</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Curvature of vacancy creation</td>
<td>Size distribution of entrants</td>
</tr>
<tr>
<td>$d$</td>
<td>Exit shock for firms</td>
<td>Average exit rate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Shocks to productivity</td>
<td>Size distribution</td>
</tr>
<tr>
<td>$K$</td>
<td>Capital</td>
<td>Average firm size</td>
</tr>
</tbody>
</table>
Worker mobility by tenure

Figure XX: Worker mobility by tenure

(A) JJ

(B) EU
Figure XXII: UE hazard by age

Monthly hazard rate by age category and data source:
- CPS
- SIPP
- Model

Age categories:
- 16-24
- 25-34
- 35-44
- 45-54
- 55+

Back
Model matches well average wages by tenure $\Rightarrow$ confidence in $\beta$

**Figure XXIII: Wage by tenure**
Figure XXIV: Average wage by firm age

Average wage by firm age

Log average wage

EIN age

Model

Data

Back
Figure XXV: Exit rate by firm size
Employment share by firm age

Figure XXVI: Employment shares

![Graph showing employment share by firm age]
Figure XXVII: Average wage by firm size
Hires and separations by origin and destination

(A) Hires by origin

(B) Separations by destination

(C) Share of hires from other firms

(D) Net poaching
Average worker age by firm age

Figure XXIX: Average worker age by firm age
Appendix E
Details on change in age composition

Change the age composition of the economy to 1986 and evaluate its impact on dynamism

- Increase the rate at which older individuals exit the market, $\kappa(3)$
  1. Increases the share of young people
  2. Shortens the time individuals expect to remain in the market

- The retirement age has not changed suggesting that individuals did not expect to spend less time in the market in the 1980s

$\Rightarrow$ Use original $\kappa(3)$ in value functions and new $\kappa(3)$ when computing individual transitions
○ Target change in share of older => Understates somewhat fall in the share of young

**Table VII:** Share of individuals in each age group by period

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Early</td>
<td></td>
<td>Late</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>0.492</td>
<td>0.434</td>
<td>0.356</td>
<td>0.339</td>
<td>-0.136</td>
<td>-0.095</td>
</tr>
<tr>
<td>Middle aged</td>
<td>0.231</td>
<td>0.289</td>
<td>0.208</td>
<td>0.226</td>
<td>-0.023</td>
<td>-0.063</td>
</tr>
<tr>
<td>Older</td>
<td>0.277</td>
<td>0.277</td>
<td>0.436</td>
<td>0.436</td>
<td>0.159</td>
<td>0.158</td>
</tr>
</tbody>
</table>

*Note: Empirical moments corresponds to the share of the labor force age 16–34 (young), 35–44 (middle aged) and 45+ (older) in 1986 and 2015 from the BLS.*
EE versus UE

- Two opposing effects on vacancy creation

- Firms post fewer vacancies conditional on productivity

- Slower turnover rate shifts distribution of firms out

- Only modest decline in $\lambda$

- In contrast, the less dynamic economy implies that

  - Employment has shifted up the ranks of firms

  - A higher share of matches has learned its productivity

  - Less likely individual accepts job offer

  - JJ hazard falls over and above the decline in $\lambda$
JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity

⇒ Less likely individual accepts job offer

⇒ JJ hazard falls over and above the decline in $\lambda$
Two opposing effects on vacancy creation

1. Firms post fewer vacancies conditional on productivity
2. Slower turnover rate shifts distribution of firms out
Two opposing effects on vacancy creation

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2. Slower turnover rate shifts distribution of firms out

⇒ Only modest decline in $\lambda$
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In contrast, the less dynamic economy implies that
JJ VERSUS UE

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  \[ \Rightarrow \] Only modest decline in \( \lambda \)

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JJ VERSUS UE

○ Two opposing effects on vacancy creation
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  2. Slower turnover rate shifts distribution of firms out
     \[ \Rightarrow \] Only modest decline in \( \lambda \)

○ In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity
     \[ \Rightarrow \] Less likely individual accepts job offer
**J J versus U E**

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

  $\Rightarrow$ Only modest decline in $\lambda$

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity

  $\Rightarrow$ Less likely individual accepts job offer

  $\Rightarrow$ J J hazard falls over and above the decline in $\lambda$
**Figure XXX: Change in vacancy policy and firm distribution**

(A) **Vacancy creation**  

(B) **Firm productivity distribution**
Decker et al. (2017)

1. The fall in job reallocation is not due to a more benign economic environment

2. Older firms adjust employment less in response to productivity shocks

3. Employment has shifted towards older firms, accounting for some of the decline in the passthrough

4. The response has fallen within firm age groups
Explaining the empirical patterns

1. No change in variance of shocks

2. Lower passthrough of older firms as equilibrium outcome
   - Employment change to productivity shock is linked to #ranks
   - Log distance between ranks is larger further up the ladder
   - Shock moves firm fewer ranks at top => smaller employment response
   - Older, surviving firms are on average further up the ladder

3. Aging results in shift of employment towards older firms

4. Employment has also shifted up the ladder within age groups
Table VIII: Passthrough from productivity to employment innovations

<table>
<thead>
<tr>
<th></th>
<th>(1) All firms</th>
<th>(2) Young firms</th>
<th>(3) Mature firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ TFP</td>
<td>3.504***</td>
<td>5.604***</td>
<td>2.394***</td>
</tr>
<tr>
<td>Late period × Δ TFP</td>
<td>-0.566***</td>
<td>-0.212***</td>
<td>-0.177***</td>
</tr>
</tbody>
</table>

Note: Young firms are <5 years, mature firms ≥ 5 years. Outcome variable is annual change in log firm size. Independent variable is annual change in log firm productivity. Weighted by employment.

⇒ Declines driven by weaker passthrough
Table IX: Decomposition of change in log output

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.014</td>
<td>0.044</td>
<td>0.004</td>
<td>0.055</td>
<td>-0.040</td>
</tr>
<tr>
<td>composition</td>
<td>Firm productivity</td>
<td>Match productivity</td>
<td>Net output</td>
<td>Discounted net output</td>
<td></td>
</tr>
</tbody>
</table>
### Table IX: Decomposition of change in log output

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<tr>
<th>(1)</th>
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<td>0.044</td>
<td>0.004</td>
<td>0.055</td>
<td>-0.040</td>
</tr>
</tbody>
</table>
## Decomposition of level difference

Table IX: Decomposition of change in log output

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<thead>
<tr>
<th>(1) Age composition</th>
<th>(2) Firm productivity</th>
<th>(3) Match productivity</th>
<th>(4) Net output</th>
<th>(5) Discounted net output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014</td>
<td>0.044</td>
<td>0.004</td>
<td>0.055</td>
<td>-0.040</td>
</tr>
</tbody>
</table>
\[
e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\}
\]

\[
JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (z^e(z, x, a))] d\hat{G}(z, x|a)
\]
Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]

**Table X: Decomposing the change in the JJ and entry hazard**

<table>
<thead>
<tr>
<th></th>
<th>(1) Entry hazard</th>
<th>(2) % of total</th>
<th>(3) JJ hazard</th>
<th>(4) % of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct: ( m(a) )</td>
<td>10.5</td>
<td>47.5</td>
<td>7.0</td>
<td>53.6</td>
</tr>
</tbody>
</table>
\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left( 1 - \frac{u(a)}{m(a)} \right) \int \Omega \left[ \tilde{c}^e(z, x, a) \right] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\tilde{c}^u) \right\} \]

\[ J J = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (z^e(z, x, a))] d\hat{G}(z, x|a) \]

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<td></td>
<td></td>
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\[ e = \sum_{a} m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega \left[ \hat{c}^e(z, x, a) \right] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega \left( \tilde{c}^u \right) \right\} \]

\[ JJ = \sum_{a} m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(\hat{z}^e(z, x, a))] d\hat{G}(z, x|a) \]

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<td>47.5</td>
<td>7.0</td>
<td>53.6</td>
</tr>
<tr>
<td>Policy: ( \tilde{c}^e(z, x, a)/\lambda [1 - F(\hat{z}^e(z, x, a))] )</td>
<td><strong>1.2</strong></td>
<td>5.4</td>
<td><strong>-17.3</strong></td>
<td><strong>-133</strong></td>
</tr>
</tbody>
</table>
Decomposition

\[ e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega [\bar{c}^e(z, x, a)] d\hat{G}(z, x|a) + \frac{u(a)}{m(a)} \Omega (\bar{c}^u) \right\} \]

\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F(z^e(z, x, a))] d\hat{G}(z, x|a) \]

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\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (\bar{z}^e(z, x, a))] d\hat{G}(z, x|a) \]

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</tr>
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<td>53.6</td>
</tr>
<tr>
<td>Policy: \bar{c}^e(z, x, a)/\lambda [1 - F (\bar{z}^e(z, x, a))]</td>
<td>1.2</td>
<td>5.4</td>
<td>-17.3</td>
<td>-133</td>
</tr>
<tr>
<td>Mismatch: \hat{G}(z, x</td>
<td>a)</td>
<td>10.4</td>
<td>47.2</td>
<td>23.3</td>
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<td>5.4</td>
<td>-17.3</td>
<td>-133</td>
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<tr>
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<td>a) )</td>
<td>10.4</td>
<td>47.2</td>
<td><strong>23.3</strong></td>
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\[ JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1 - u} \times \lambda \times \int [1 - F (z^e(z, x, a))] d\hat{G}(z, x|a) \]

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<th>JJ hazard</th>
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</thead>
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</tr>
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<td>10.5</td>
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</tr>
<tr>
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<td>1.2</td>
<td>5.4</td>
</tr>
<tr>
<td>Mismatch: (\hat{G}(z, x</td>
<td>a))</td>
<td>10.4</td>
</tr>
<tr>
<td>Total</td>
<td>22.2</td>
<td>100</td>
</tr>
</tbody>
</table>
Shift in match productivity

Figure XXXII: Share with high match productivity

- Young
- Middle aged
- Old

Share high productive

- Old economy
- Young economy
How much does JJ fall with mismatch?

Figure XXXIII: Distribution of older individuals and JJ hazard
Figure XXXIV: Tenure profile of JJ mobility

⇒ Large equilibrium effects are not hardwired
**Figure XXXV: Tenure distribution**

- **Share**
  - 0.05
  - 0.1
  - 0.15
  - 0.2
  - 0.25

- **Tenure (years)**
  - 0
  - 10
  - 20
  - 30

- **Data, late**
- **Data, early**
- **Model, late**
- **Model, early**
Aging explains key changes in life-cycle firm dynamics

- Employment has shifted substantially towards older firms
- Exit has fallen the most for old firms
- Age conditional firm size has declined

(A) Employment share

(B) Exit rate

(C) Average firm size
Change in life-cycle worker dynamics

- Aging in model replicates patterns across states
- Relatively larger effect on mobility rates late in careers

(A) EU

(B) JJ

(C) UE

![Graphs showing change in life-cycle worker dynamics for EU, JJ, and UE]
### Shift-share analysis

**Table XI:** Shift share analysis with firm and worker age

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: Firm dynamics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exit</td>
<td>-0.008</td>
<td>-0.003</td>
</tr>
<tr>
<td>Direct effect</td>
<td>-0.008</td>
<td>-0.004</td>
</tr>
<tr>
<td>% of total</td>
<td>96.8</td>
<td>142.4</td>
</tr>
<tr>
<td>Incumbent</td>
<td>-0.045</td>
<td>-0.024</td>
</tr>
<tr>
<td>Direct effect</td>
<td>-0.010</td>
<td>-0.018</td>
</tr>
<tr>
<td>% of total</td>
<td>22.7</td>
<td>74.4</td>
</tr>
<tr>
<td><strong>Panel B: Worker dynamics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU</td>
<td>-0.003</td>
<td>-0.001</td>
</tr>
<tr>
<td>Direct effect</td>
<td>-0.001</td>
<td>-0.000</td>
</tr>
<tr>
<td>% of total</td>
<td>20.7</td>
<td>34.4</td>
</tr>
<tr>
<td>JJ</td>
<td>-0.005</td>
<td>-0.002</td>
</tr>
<tr>
<td>Direct effect</td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td>% of total</td>
<td>40.8</td>
<td>51.7</td>
</tr>
</tbody>
</table>
Aging generates modest shift of employment to larger firms in line with the data over this period.
No aging of potential hires

\[ \rho J(z) = \max_{v \geq 0} \left\{ v(1 - \beta)q \left[ \sum_a \left( \tilde{m}(a) \frac{u(a)}{m(a)} \{ V(z, x_u, a) - U(a) \}^+ \right) + \right. \right. \]

\[ + \sum_a \left\{ \tilde{m}(a) \left( 1 - \frac{u(a)}{m(a)} \right) \int \{ V(z, x_u, a) - V(z', x, a) \}^+ d\hat{G}(z', x|a) \right\} - c(v) \right\} - \]

\[ - r - \mu J'(z) + \frac{\sigma^2}{2} J''(z) \]

- Hold firms’ expectations of age composition fixed at original age composition, \( \tilde{m}(a) \)

\[ \implies \text{No change in age-composition externality} \]
## No aging of potential hires

# Table XII: No direct congestion externality due to aging

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<tbody>
<tr>
<td>Baseline</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>No aging of hires</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Share</td>
<td></td>
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</tbody>
</table>

### Panel A: Firm dynamics
- **Entry rate**
  - Baseline: -0.012
  - No aging of hires: -0.008
  - Share: 72
- **Job reallocation**
  - Baseline: -0.039
  - No aging of hires: -0.031
  - Share: 80

### Panel B: Worker dynamics
- **EU hazard**
  - Baseline: -0.001
  - No aging of hires: -0.001
  - Share: 87
- **JJ hazard**
  - Baseline: -0.002
  - No aging of hires: -0.002
  - Share: 72

### Panel C: Growth
- **Growth per worker**
  - Baseline: -0.26
  - No aging of hires: -0.18
  - Share: 69
○ Adjust \( \gamma(a) \) to have no direct effect through aging entrepreneurs

### Table XIII: No aging of potential entrepreneurs

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
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<tr>
<td><strong>No aging of entrep.</strong></td>
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<tr>
<td><strong>Share</strong></td>
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</tr>
</tbody>
</table>

**Panel A: Firm dynamics**

- **Entry rate**  
  - Baseline: -0.012
  - No aging of entrep.: -0.003
  - Share: 27

- **Job reallocation**  
  - Baseline: -0.039
  - No aging of entrep.: -0.009
  - Share: 22

**Panel B: Worker dynamics**

- **EU hazard**  
  - Baseline: -0.001
  - No aging of entrep.: -0.001
  - Share: 61

- **JJ hazard**  
  - Baseline: -0.002
  - No aging of entrep.: -0.002
  - Share: 65

**Panel C: Growth**

- **Growth per worker**  
  - Baseline: -0.26
  - No aging of entrep.: -0.11
  - Share: 42
Transition dynamics

- Start with 1986 BGP
- Adjust $\kappa(3)$ and decision rules to 2014 BGP starting in 1990
- Relatively fast convergence of entry rate
- Level effect outweighs growth effect initially

(A) Share older

(B) Entry rate

(C) Output per worker
Discussion of transition

- Would want to eventually solve for full transition path

- Difficulty is that sequence of distributions $G(z, x, a; t)$ becomes a state

- Well known issue in search models—cannot boil down problem to shooting only an interest rate or average wage
## Inequality and Income Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th></th>
<th>Old</th>
<th></th>
<th>Change</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
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<tr>
<td><strong>Panel A: Inequality</strong></td>
<td></td>
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<tr>
<td>St.d of productivity</td>
<td>0.35</td>
<td>0.13</td>
<td>0.42</td>
<td>0.14</td>
<td>0.07</td>
<td>0.01</td>
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<tr>
<td>Variance of firm pay</td>
<td>0.40</td>
<td>0.45</td>
<td>0.48</td>
<td>0.46</td>
<td>0.08</td>
<td>0.02</td>
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<tr>
<td><strong>Panel B: Annual income innovations</strong></td>
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<tr>
<td>St.d of innovations</td>
<td>0.55</td>
<td>0.54</td>
<td>0.51</td>
<td>0.52</td>
<td>-0.04</td>
<td>-0.02</td>
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<tr>
<td>Skewness</td>
<td>-0.21</td>
<td>-0.25</td>
<td>-0.31</td>
<td>-0.32</td>
<td>-0.10</td>
<td>-0.07</td>
</tr>
</tbody>
</table>
(A) **Standard deviation**

![Graph showing standard deviation of annual income innovations from 1980 to 2010](image)

- Standard deviation values: 0.45, 0.50, 0.55, 0.60

(B) **Skewness**

![Graph showing skewness of annual income innovations from 1980 to 2010](image)

- Skewness values: -1, -0.75, -0.5, -0.25, 0
Data

- Demographic data from the March CPS and Census Bureau’s Intercensal Censi projections

- Establishment and firm dynamics from the BDS

- Merged CPS monthly files for worker mobility rates

- State real GDP per worker from state private sector GDP (BEA), regional CPIs (BLS), and private sector employment (BDS)
FOUR OTHER PROMINENT CHANGES

1. Increasing gender and racial diversity
   - Estimated coefficients on share female and non-white are in most cases not statistically significant
   - Typically predict a small *increase* in dynamism
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   - Share college is associated with higher dynamics
   - Hence also predicts an *increase* in dynamism
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3. Increasing educational attainment
   ◦ Share college is associated with higher dynamics
   ◦ Hence also predicts an *increase* in dynamism

4. Slowdown in labor supply growth
   ◦ Confirming Karahan et al. (2016), labor supply growth is positively correlated with entry
   ◦ But does not alter conclusion regarding the importance of the *age composition*
Figure XLI: Share female, non-white and with a college degree

(A) Diversity

(B) College
(A) $\Delta$ labor force

(B) $\Delta$ working age population

(C) Entry (lf)

(D) Entry (wp)