THE RISE AND FALL OF THE NATURAL INTEREST RATE

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The natural interest rate (r^*)

The real interest rate consistent with full employment & no nominal rigidities *(Woodford, 2003)*

- **A relevant concept for the conduct of monetary policy:**

It serves as optimal target

The central bank should set the nominal interest rate in order to close the real interest rate gap (r - r^*), thereby closing the output gap and stabilizing inflation

It gauges the stance of monetary policy

- contractionary if $r>r^*$
- expansionary if $r<r^*$

- **Being r^* not directly observable, it has to be inferred from the data**

Available estimates suggest that r^* stands at historically low (or possibly negative) levels

However, the conventional view is that estimates of r^* are very imprecise

Question 1: why so large uncertainty in r^*?

We dig into the mechanics of the workhorse tool to estimate *r**

Holston, Laubach, & Williams (2017) model, hereafter **HLW**

2-equations model inspired on the New Keynesian framework

Phillips curve	**IS curve**
inflation depends on output gap	*output gap depends on interest rate gap ($r - r^*$)*

Large uncertainty arises in 2 cases: when either the IS or the Phillips curve is flat

Data contain no info about the unobserved states of the model

The model fails to meet the observability condition (Kalman, 1960)

These cases are empirically relevant (more the rule than the exception)

Question 2: how to precisely measure *r**?

Observation:
the HLW model treats the observed real interest rate as exogenous

Hence the dynamic properties of interest rate gap & output gap are unspecified

Nothing guarantees the stationarity of both gaps!

We consider an augmented HLW model to make both gaps stationary

The extra-equation is a **local level** specification for the observed real rate

$$\underbrace{r_t}_{observed\ real\ rate} = \underbrace{r_t^*}_{natural\ rate\ I(1)} + \underbrace{\tilde{r}_t}_{interest\ rate\ gap\ I(0)}$$

The model identifies *r even with flat IS & Phillips curves**

Interestingly, the univariate local level model can also identify *r**

Cons: it says nothing about drivers of *r**

since it exploits data on interest rate only

Pros: it always precisely estimate *r**

since it always meets observability, so it is robust when data imply flat IS & Phillips curves

International evidence on *r**

We collect historical data (yearly frequency 1891-2016) for 17 advanced economies

Data likely to produce flat IS and Phillips curves

(due to breaks & low frequency)

We use the data as testing ground for the local level model

We document

- a general decline of r* since the start of XX century until the 1960's
- a subsequent **rise and fall**, peaking around the end of the 1980's

What has driven the rise & fall of the natural interest rate?

Estimate a Panel ECM which exploits panel variation on key determinants of *r** (productivity growth, demographics, risk)

The evolving **demographic composition** can explain this rise & fall

Since the 1960's	**Once baby boom ends**
rise of young baby boomers → *r rises**	**young share falls due to ageing → *r** falls**

Road map

1. **Why is the uncertainty on *r** so large?**
    - **Uncertainty in the HLW model**
    - **Observability in the HLW model**

2. **How to precisely estimate *r**?**
    - **The augmented HLW & the local level model**
    - **International evidence on *r***

3. **Conclusions**

The Holston, Laubach, & Williams (2017) model

Two key equations inspired by the New Keynesian framework

Phillips curve

$$\pi_t = \alpha_\pi \pi_{t-1} + (1-\alpha_\pi)\pi_{t-2,4} + \kappa \tilde{y}_{t-1} + \varepsilon_t^\pi$$

inflation

output gap

$$\tilde{y}_t \equiv y_t - y_t^*$$

potential output

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}$$

trend growth

$$g_t = g_{t-1} + \varepsilon_t^g$$

IS curve

$$\tilde{y}_t = \alpha_{y,1}\tilde{y}_{t-1} + \alpha_{y,2}\tilde{y}_{t-2} - \gamma(r_{t-1} - r_{t-1}^*) + \varepsilon_t^{\tilde{y}}$$

real interest rate

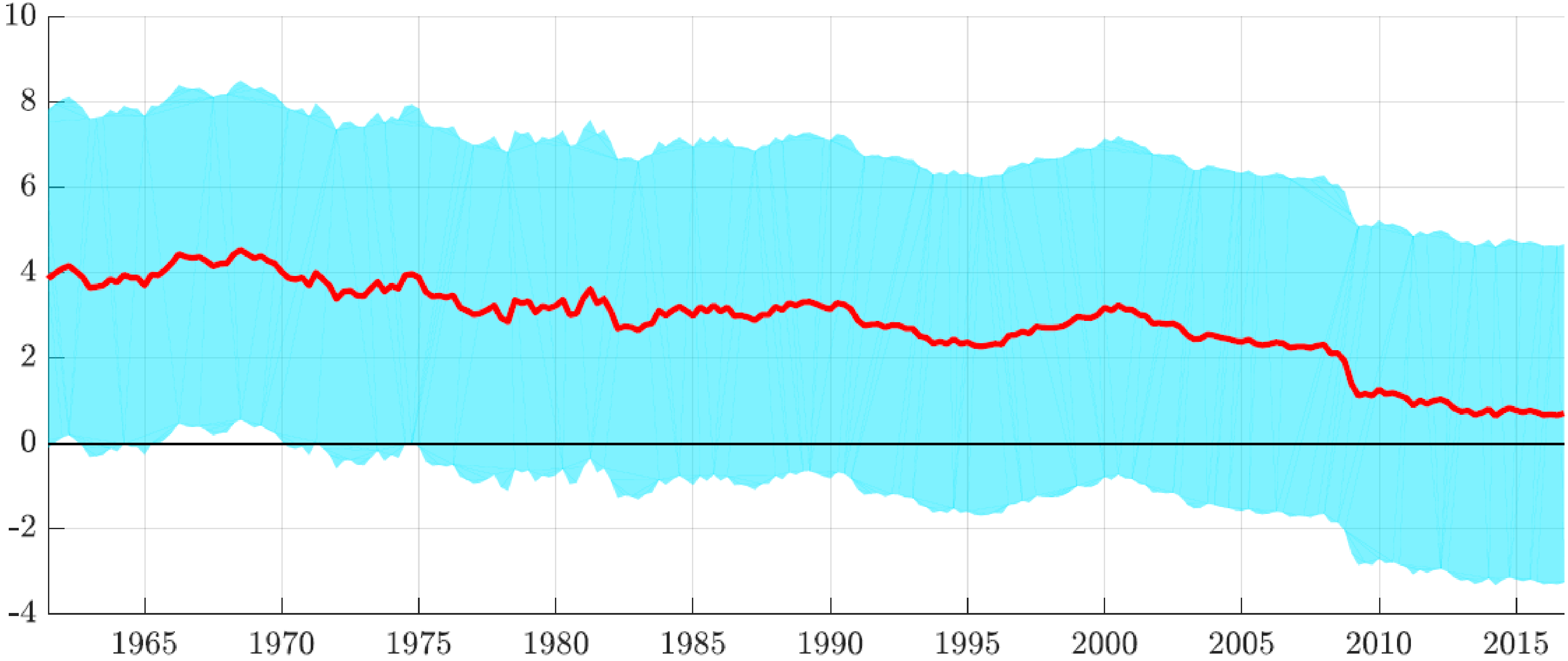
natural rate

$$r_t^* = 4g_t + z_t$$

unobserved factors unrelated to growth

$$z_t = z_{t-1} + \varepsilon_t^z$$

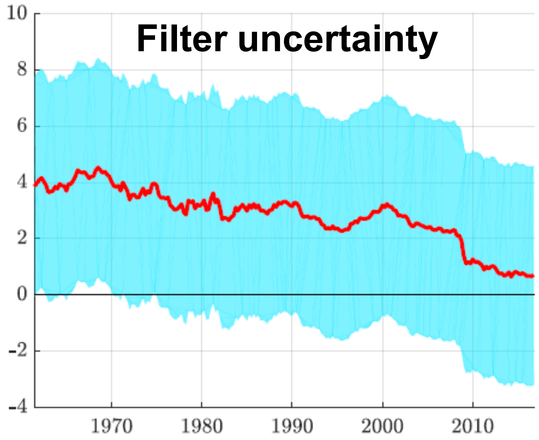
High uncertainty in estimated U.S. *r** by HLW model

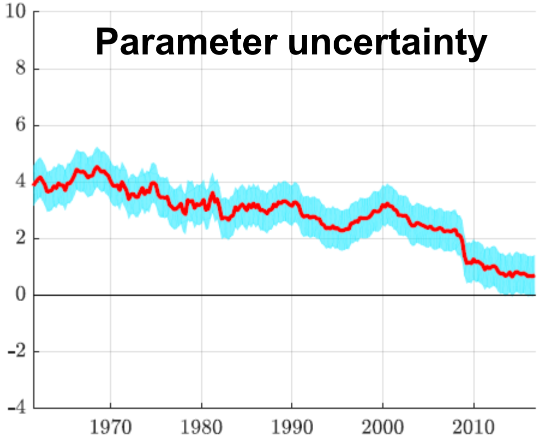


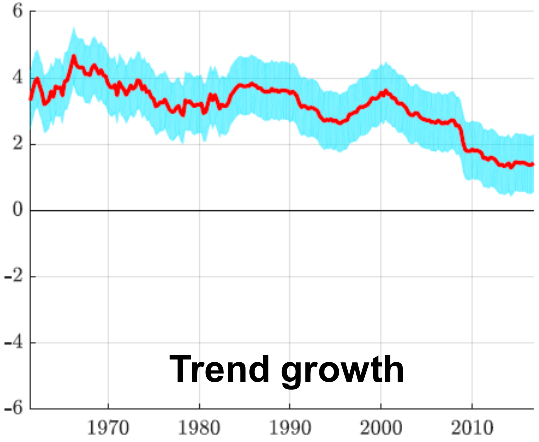
Notes: 1961Q2:2016Q3, one-sided filter with 90% bands (both parameter and filter uncertainty)

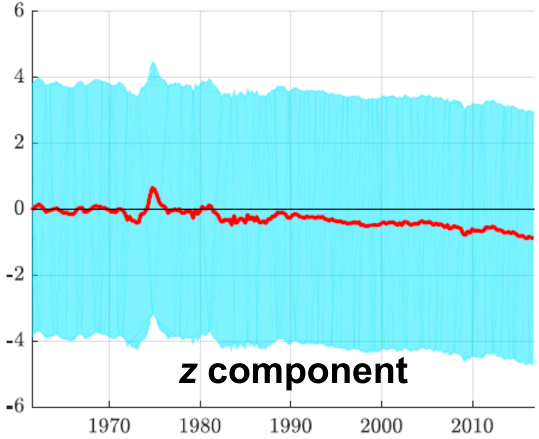
Why so large uncertainty?

Large uncertainty is mostly due to filter uncertainty...









... and the large filter uncertainty stems from the *z* component

Observability in the HLW model

Measurement equation

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1-\alpha_y & 1+4\gamma & \gamma \\ -\kappa & 0 & 0 \end{bmatrix}}_{\mathbf{Z}} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \alpha_y & 0 & 0 & -\gamma \\ \kappa & \alpha_\pi & 1-\alpha_\pi & 0 \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \pi_{t-2|4} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^{\tilde{y}}_t + \varepsilon^{y^*}_t \\ \varepsilon^{\pi}_t \end{bmatrix}$$

Transition equation

$$\begin{bmatrix} y^*_t \\ g_t \\ z_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} y^*_{t-1} \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^{y^*}_t \\ \varepsilon^{g}_t \\ \varepsilon^{z}_t \end{bmatrix}$$

Which conditions allow for recovering the state vector from the data?

Observability

The rank of the observability matrix equals the number of unobserved states

(Harvey, 1989)

$$\text{Rank} \begin{bmatrix} \mathbf{Z} \\ \mathbf{ZT} \\ \mathbf{ZT}^2 \\ \vdots \\ \mathbf{ZT}^{s-1} \end{bmatrix} = s$$

Observability in the HLW model

Given that the HLW model features three unobserved states, the observability matrix reads

$$\mathbf{O} = \begin{bmatrix} \mathbf{Z} \\ \mathbf{ZT} \\ \mathbf{ZT}^2 \end{bmatrix} = \begin{array}{ccc} \underline{y_t^*} & \underline{g_t} & \underline{z_t} \\ \end{array} \begin{bmatrix} 1-\alpha_y & 1+4\gamma & \gamma \\ -\kappa & 0 & 0 \\ 1-\alpha_y & 2+4\gamma-\alpha_y & \gamma \\ -\kappa & -\kappa & 0 \\ 1-\alpha_y & 3+4\gamma-2\alpha_y & \gamma \\ -\kappa & -2\kappa & 0 \end{bmatrix}$$

This matrix is rank deficient when the IS and/or the Phillips curves are flat

Flat IS curve $\gamma = 0$

$$\mathbf{O} = \begin{bmatrix} 1-\alpha_y & 1 & 0 \\ -\kappa & 0 & 0 \\ 1-\alpha_y & 2-\alpha_y & 0 \\ -\kappa & -\kappa & 0 \\ 1-\alpha_y & 3-2\alpha_y & 0 \\ -\kappa & -2\kappa & 0 \end{bmatrix}$$

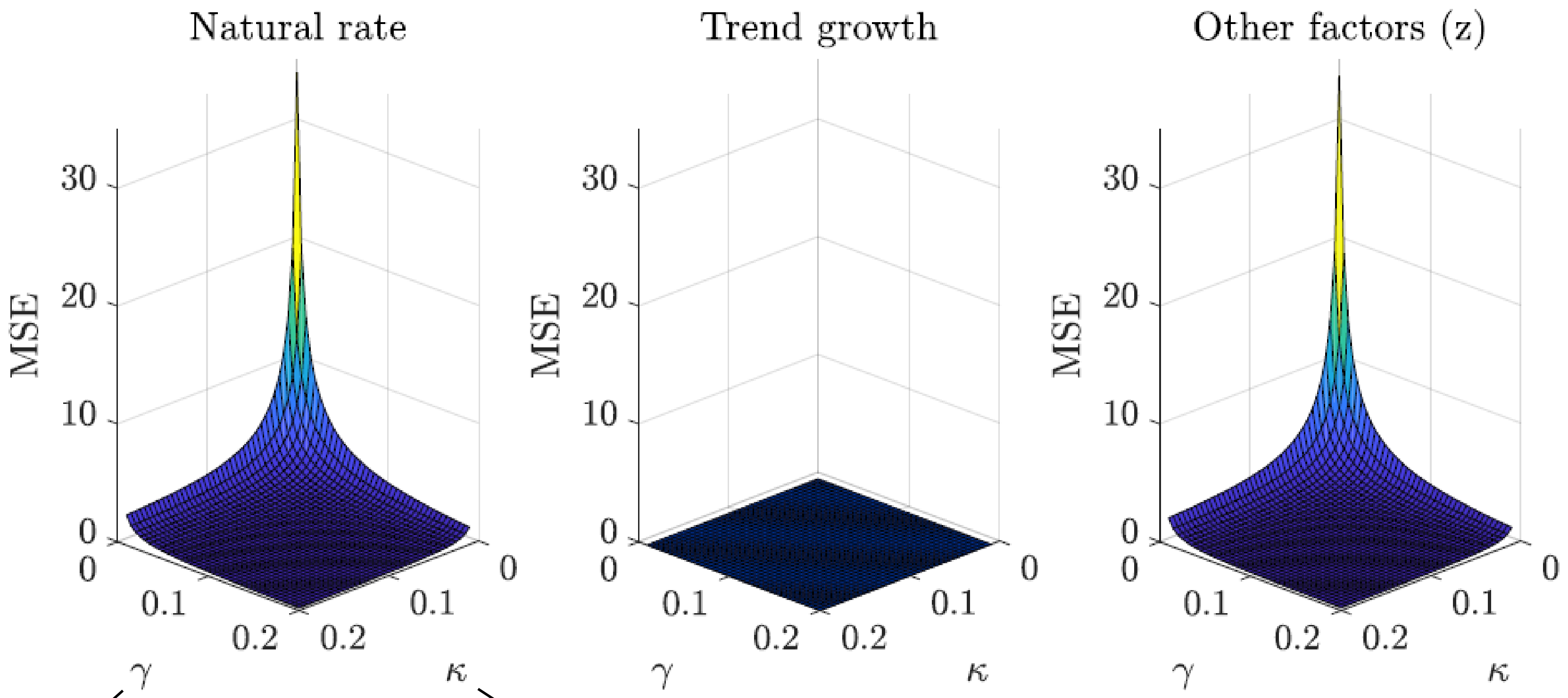
Cannot identify the *z* process

Flat Phillips curve $\kappa = 0$

$$\mathbf{O} = \begin{bmatrix} 1-\alpha_y & 1+4\gamma & \gamma \\ 0 & 0 & 0 \\ 1-\alpha_y & 2+4\gamma-\alpha_y & \gamma \\ 0 & 0 & 0 \\ 1-\alpha_y & 3+4\gamma-2\alpha_y & \gamma \\ 0 & 0 & 0 \end{bmatrix}$$

Cannot separately identify *z* and *y**

Filter uncertainty of HLW model & slopes of IS, Phillips curves

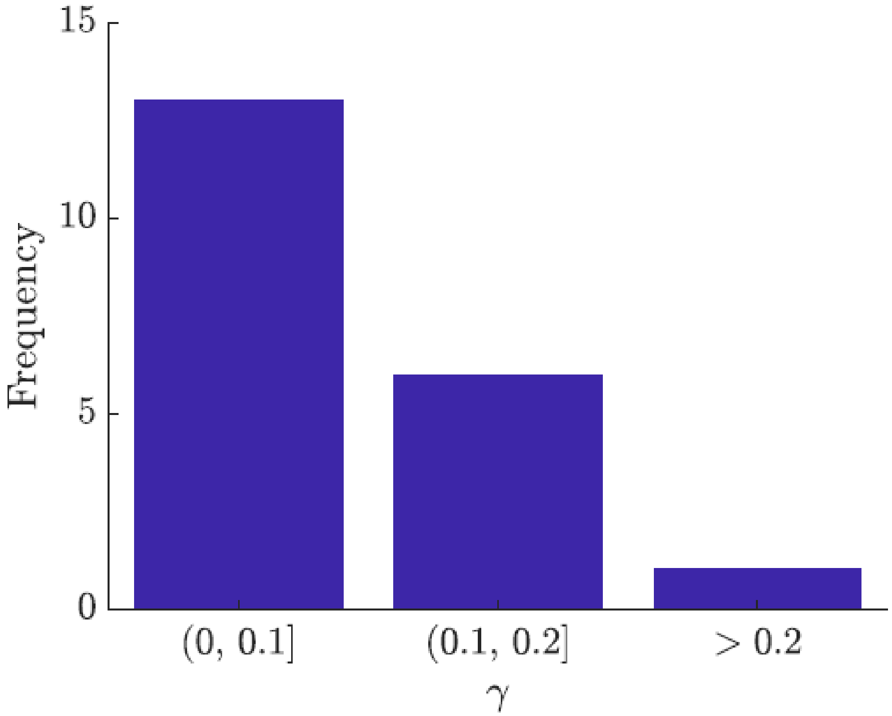


Slope of IS curve

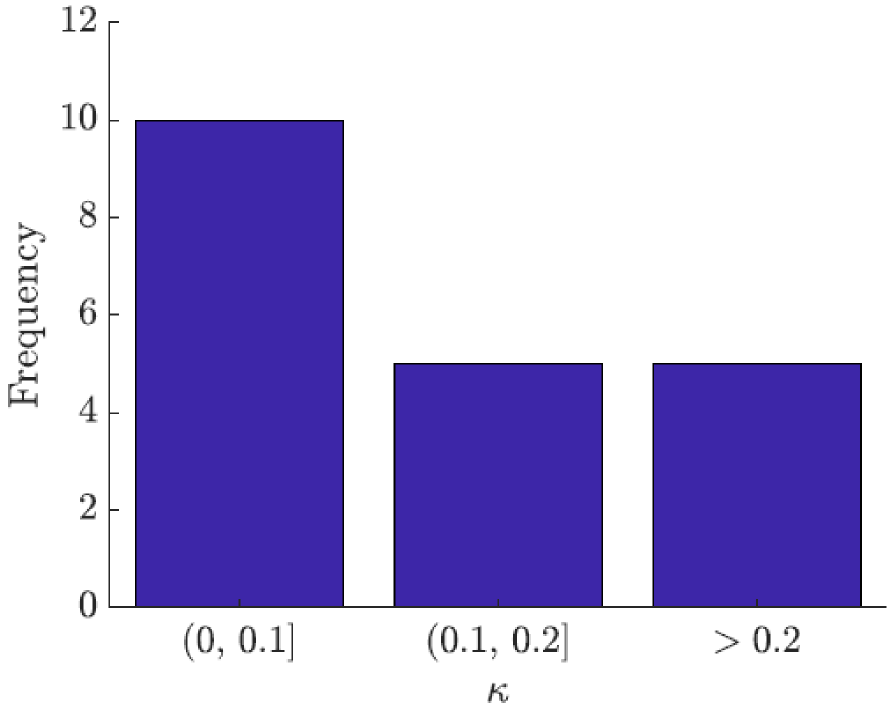
Slope of Phillips curve

IS and Phillips curves are generally flat

Steepness of IS and Phillips curves: estimates in the literature



Slope of IS curve



Slope of Phillips curve

Road map

1. Why is the uncertainty on *r** so large?
   - Uncertainty in the HLW model
   - Observability in the HLW model

2. How to precisely estimate *r**?
   - The augmented HLW & the local level model
   - International evidence on *r**

3. Conclusions

The augmented HLW & the local level model

The HLW model treats the observed real interest rate as exogenous

$$\alpha_y(L)\tilde{y}_t = -\gamma(r_{t-1} - r^*_{t-1}) + \varepsilon^{\tilde{y}}_t$$

Hence the dynamic properties of both interest rate gap & output gap are unspecified

(gaps may be nonstationary!)

We consider an augmented HLW model to make both gaps stationary

The extra-equation is a **local level** specification for the observed real rate

$$\underbrace{r_t}_{\textit{observed real rate}} = \underbrace{r^*_t}_{\textit{natural rate } I(1)} + \underbrace{\tilde{r}_t}_{\textit{interest rate gap } I(0)}$$

The model identifies *r**

even with flat IS & Phillips curves

Interestingly, the univariate local level model can also identify *r**

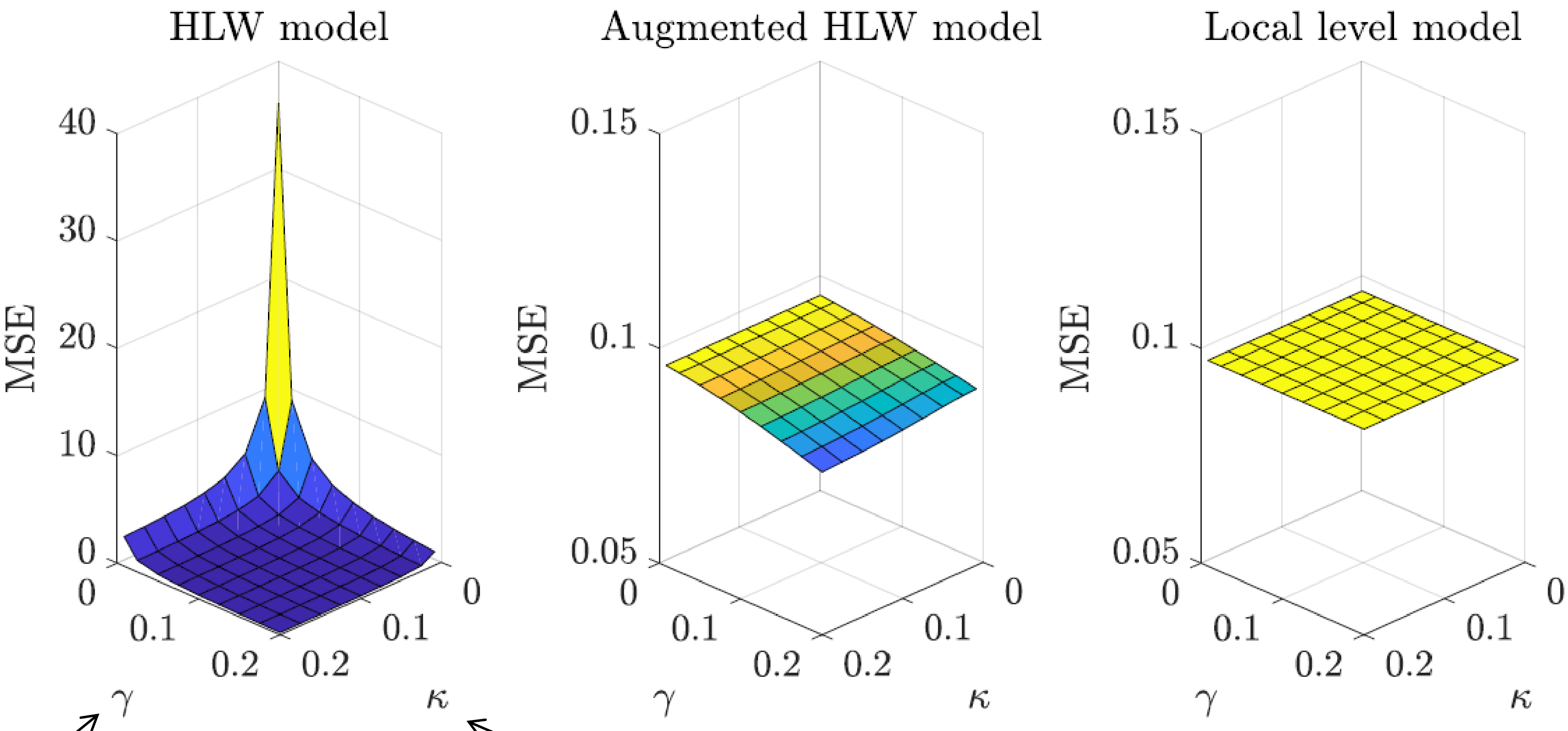
Cons: it says nothing about drivers of *r**

since it exploits data on interest rate only

Pros: it always precisely estimate *r**

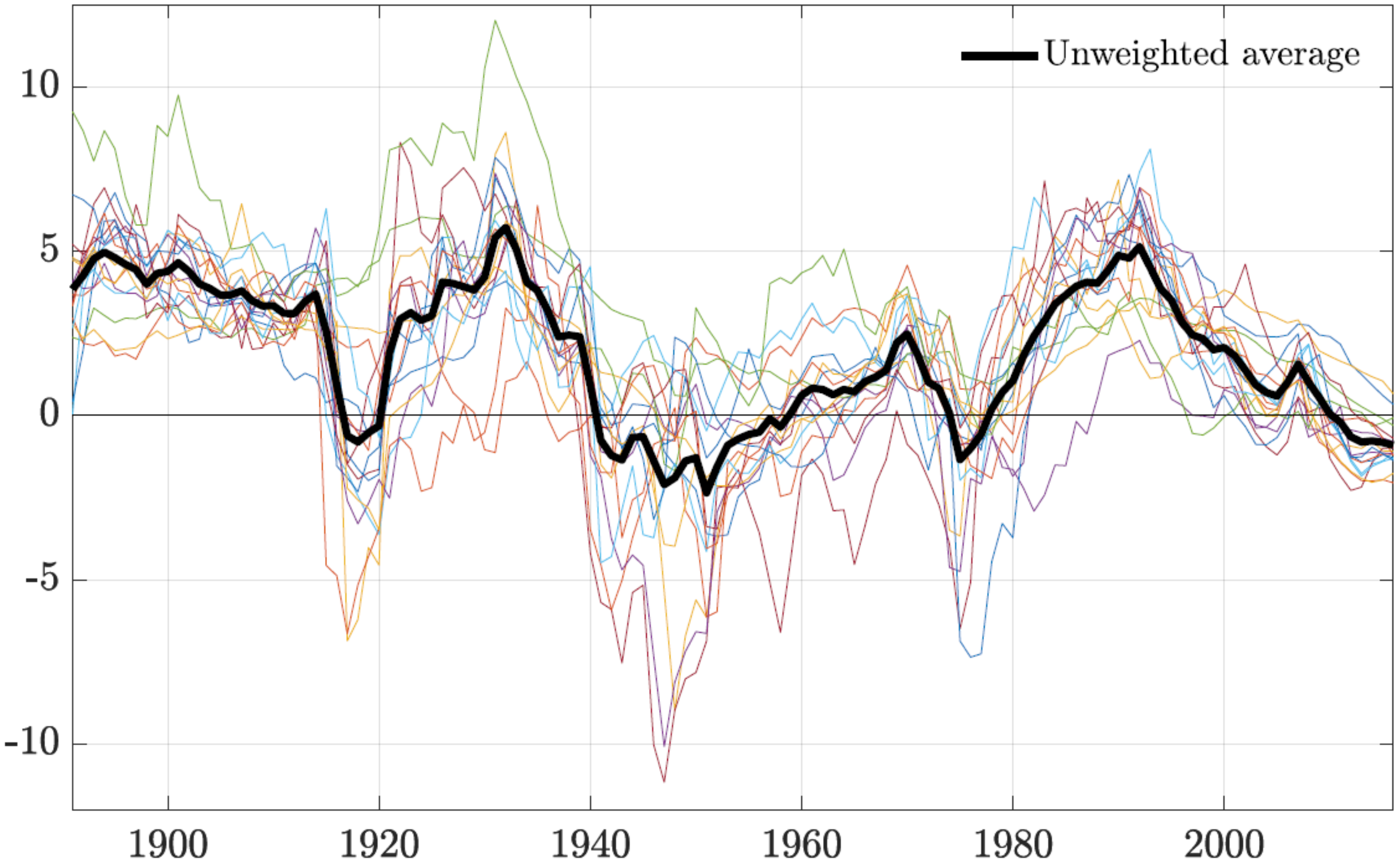
since it always meets observability

Filter uncertainty of *r** across models

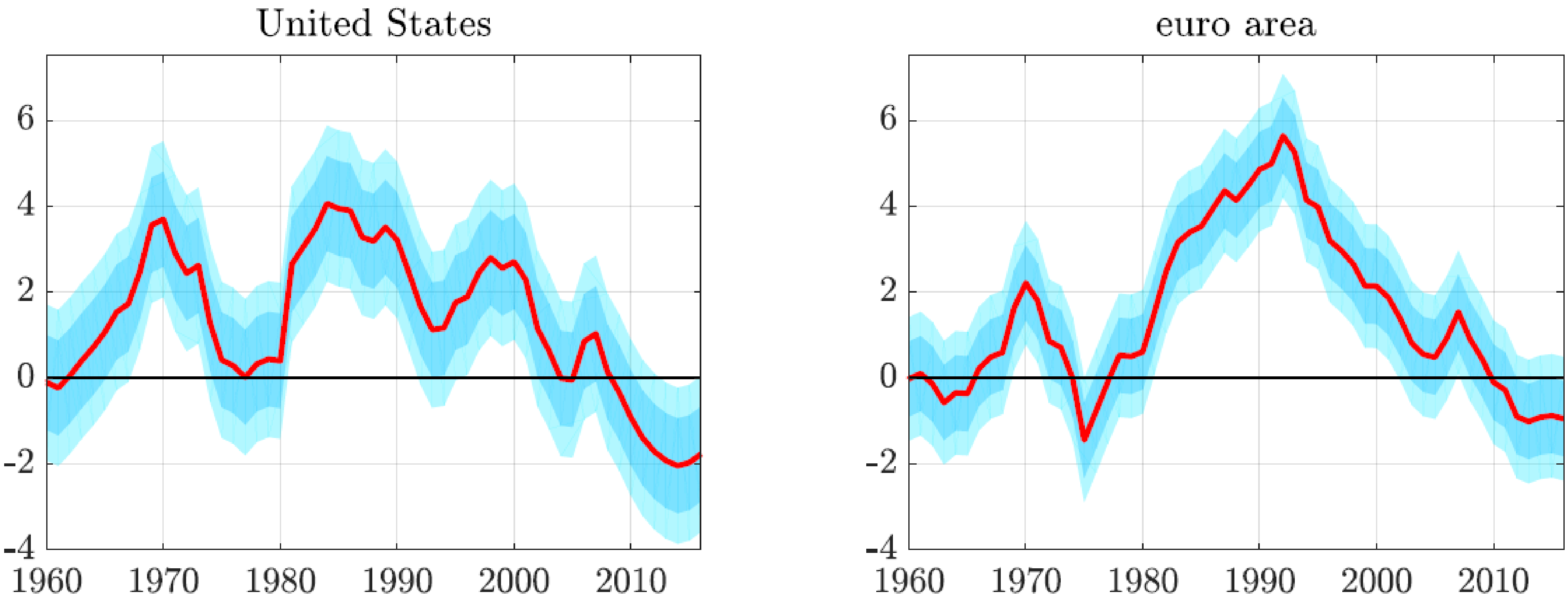


Slope of IS curve **Slope of Phillips curve**

International evidence on estimated r^* by the local level model

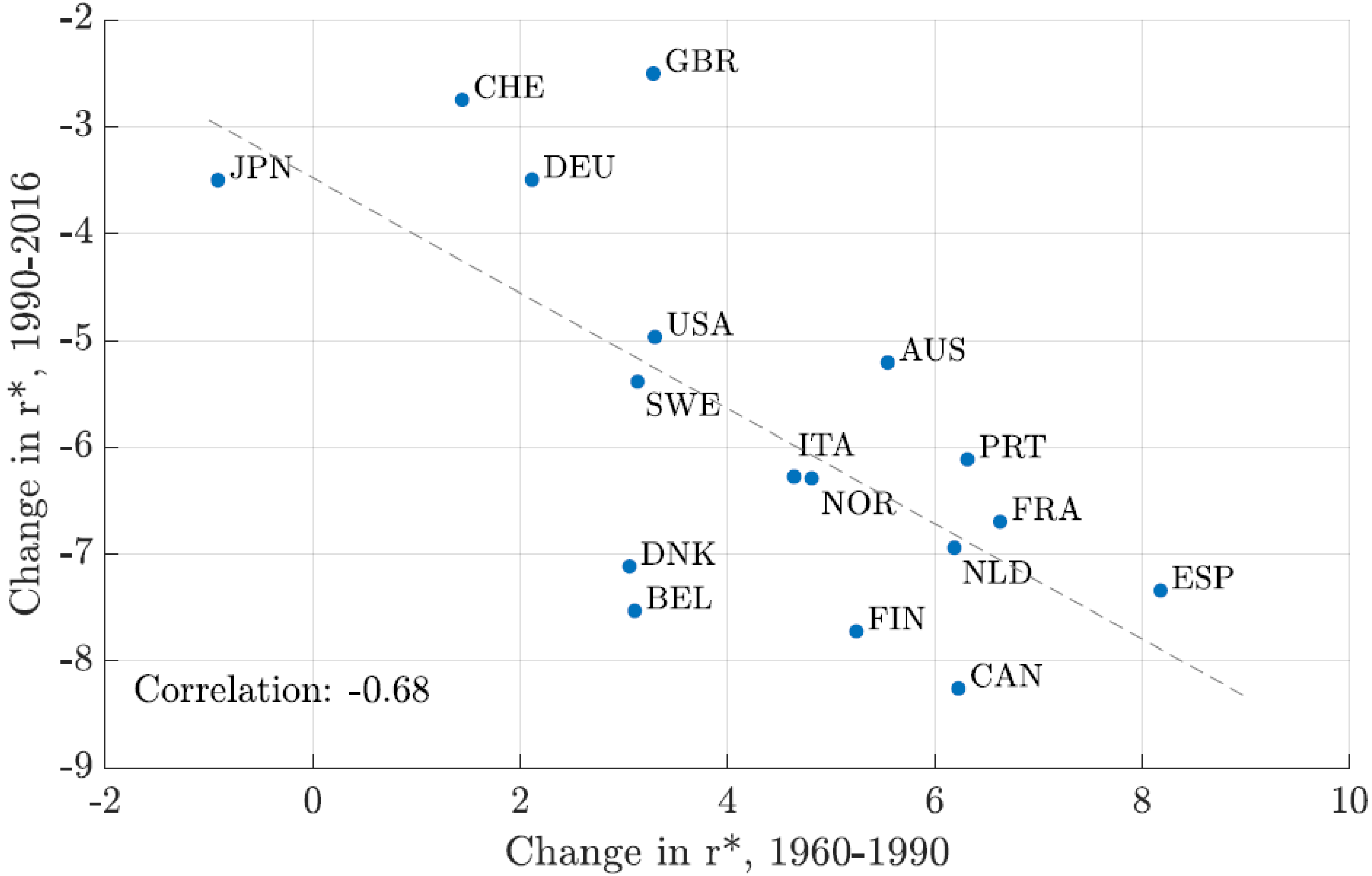


U.S. and euro area natural rates



Notes: median estimates with 68% and 90% bands (both parameter and filter uncertainty)

What has driven the rise and fall in *r**?



The Panel Error Correction Model

The *r of the local level model is silent about its drivers**

We consider an alternative but complementary approach by estimating a Panel ECM

$$\Delta r_{i,t} = \alpha\left[r_{i,t-1} - \beta' X_{i,t-1}\right] + \gamma' \Delta X_{i,t} + \varepsilon_{i,t}$$

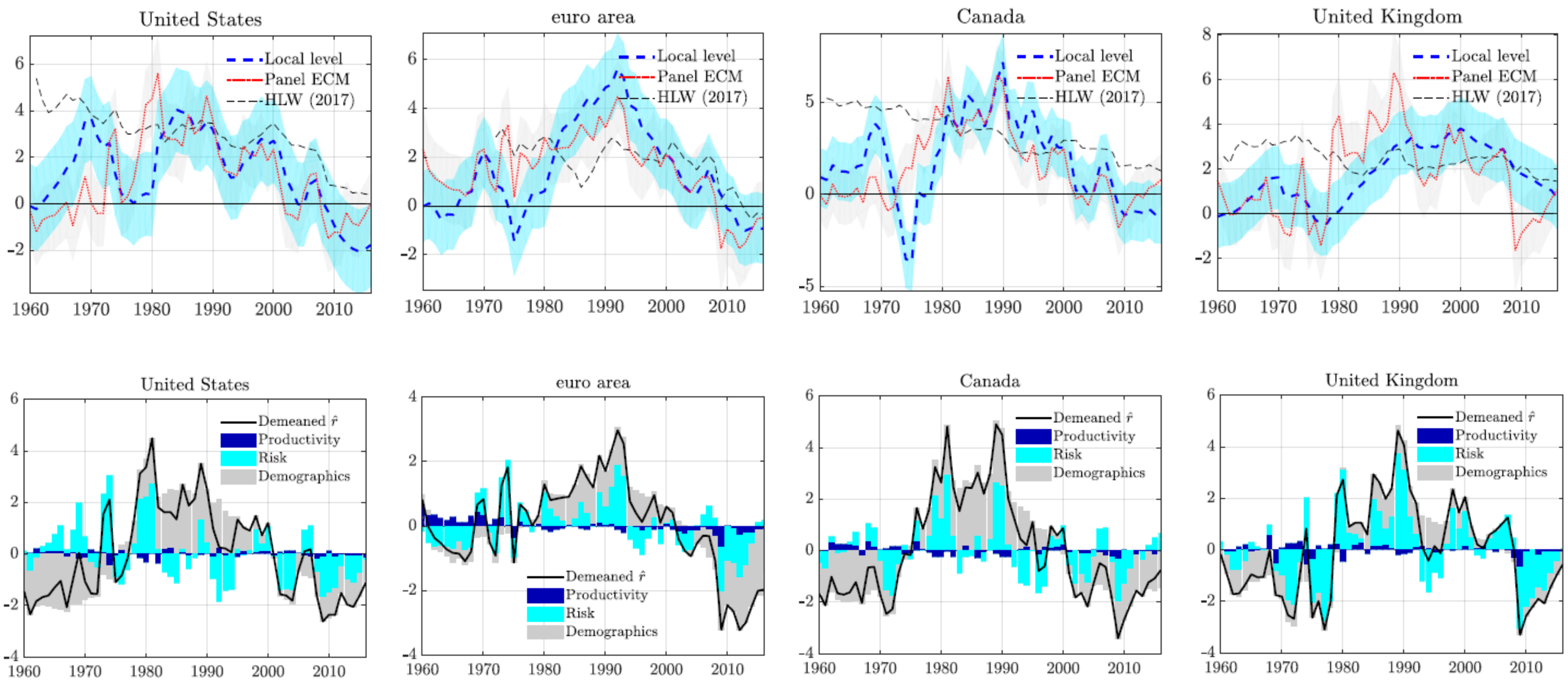
real interest rate

(annual data, 1960-2016)

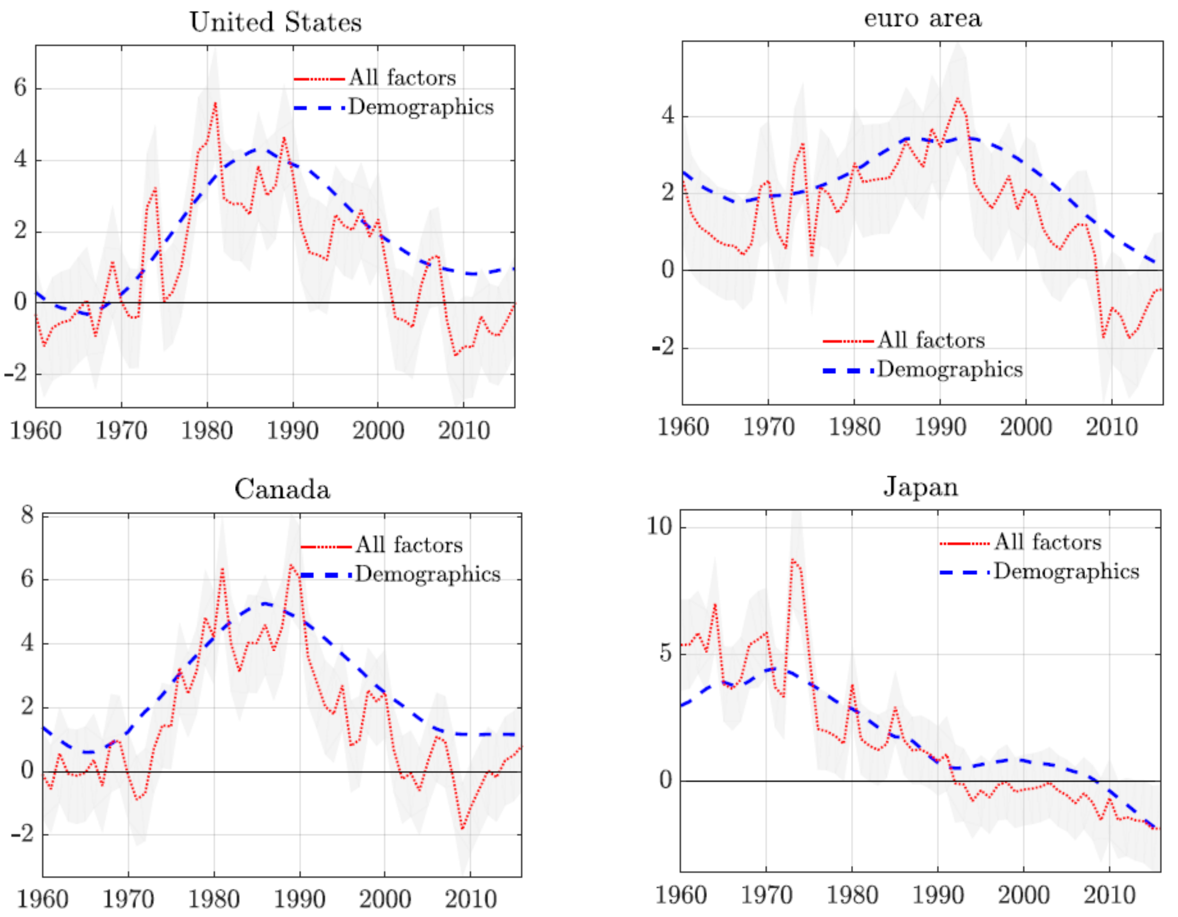
indicators of potential determinants of *r*

- productivity (TFP) growth
- demographics (young share in population)
- risk (term spread)

Estimated *r** by Panel ECM and contributing factors



The role of demographics



Conclusions

Why is the uncertainty on *r** so large?

The precision of the HLW model dramatically drops with flat IS and/or Phillips curves

These cases appear to be more the rule than the exception

How to precisely estimate *r**?

Augmented HLW model

which guarantees stationarity of rate & output gaps

Local level model

on the observed real interest rate

Using historical panel data we show a rise and fall of *r**

*r** rises since the 1960's and peaks around the end of the 1980's

The evolving demographic composition can explain part of this rise & fall

Thank you very much for your attention!