Bank Recapitalizations, Credit Supply, and the Transmission of Monetary Policy*

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– PRELIMINARY DRAFT, PLEASE DO NOT CIRCULATE –

Abstract

We integrate a banking sector in a standard New-Keynesian DSGE model, and examine how government policies to recapitalize banks after a crisis affect the supply of credit and the transmission of monetary policy. We examine two types of recapitalizations: immediate and delayed ones. In the steady state, both policies cause the banking sector to charge inefficiently low lending rates, which leads to an inefficiently large capital stock. Raising bank equity requirements reduces this dynamic inefficiency and thereby increases social welfare. After a crisis, a delay in recapitalizations causes the banking sector to suffer from debt-overhang. This debt-overhang leads to inefficiently high lending rates, which reduces the supply of credit and weakens the transmission of monetary policy to inflation (the transmission to output is largely unchanged). The welfare effect of raising bank equity requirements may then become negative. Overall, our analysis shows that bank recapitalization policies may have considerable macro-economic implications.

Keywords: bank recapitalizations, credit supply, monetary policy transmission, bank equity requirements, NK-DSGE models.
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1 Introduction

Almost one decade after the financial crisis of 2008-09, the recovery of bank credit supply in the euro area remains sluggish despite ongoing monetary accommodation. At the same time, bank credit supply in the U.S. has recovered much stronger. One major difference between the U.S. and the euro area was the policy response to undercapitalized banks. Following the crisis, U.S. authorities intervened rather swiftly to recapitalize the banking sector. By contrast, the recapitalization of the banking sector in the euro area was delayed by the sovereign debt crisis and suffered from limited coordination at the European level.[1] While it seems plausible that different bank recapitalization policies may lead to different economic outcomes, macro-economic models typically do not put these policies at the center stage. We fill this gap in the literature by developing a macro-economic model to analyze how immediate versus delayed bank recapitalizations after a crisis affect the supply of credit and the transmission of monetary policy.

We augment a parsimonious New-Keynesian DSGE framework with a banking sector that issues equity and deposits to households and makes loans to capital producers.[2] The key friction that we build into the banking sector is a recapitalization that is received from the government if a negative productivity shock causes the income on loans to be insufficient to fully repay the deposits (the recapitalization is financed with a lump sum tax on the household). We refer to this difference between loan income and deposits as a *shortfall*. The government may either recapitalize the banking sector immediately after a shortfall, or with a delay of one period. Both recapitalization policies ensure that the depositors will always be fully repaid, so that the bank never defaults. In case of a delayed recapitalization, the profits made by the banking sector after a shortfall reduce the size of the recapitalization that will be received in the next period. Whether the government responds to a shortfall with an immediate recapitalization or with a delayed one is determined exogenously.[3]

We solve the model numerically and show that under both types of recapitalization policies, the banking sector in the steady state charges inefficiently low lending rates. These low lending rates reflect that the expected value of a future recapitalization effectively constitutes a subsidy, which

[1] In 2009, the largest U.S. banks were required by regulators to participate in the supervisory capital assessment program. As a part of this program, banks had to participate in a stress test to evaluate the adequacy of their capital buffers, and those banks that failed the test were forced to recapitalize. At the start of the European banking union in 2014, European banks were subject to a similar exercise as the European Central Bank published the results of its asset quality review.

[2] The structure of our model is similar to that of Smets and Wouters (2007), although we abstain from most of the real and nominal rigidities. To analyze monetary policy, we retain price rigidities and persistence of the monetary policy interest rate.

[3] We focus on the timing of recapitalizations, immediate or delayed, and do not analyze other aspects such as their motivation or design. In practice, governments may choose to recapitalize large banks in distress because it considers them to be too-big-to-fail (e.g., O'hara and Shaw, 1990), or it may recapitalize smaller banks in distress because they are considered to be with too-many-to-fail (e.g., Acharya and Yorulmazer, 2007). Mariathasan and Merrouche (2012) find that recapitalizations are more successful if they are designed to increase common equity and are sufficiently large. Phillippon and Schnabel (2009) suggest to add warrants and conditions that limit moral hazard.

the (competitive) banking sector passes on to its borrowers. Both recapitalization policies therefore lead to over-lending and an inefficiently high capital stock. The effects of government safety nets on bank behavior have received ample attention in the literature, see, for example, Merton (1977), Kareken and Wallace (1978), Dam and Koetter (2012), Farhi and Tirole (2012) and Admati et al. (2013). Our model shows that lending rates decline by more if the banking sector expects to receive an immediate instead of a delayed recapitalization after a shortfall (as the former constitutes a larger subsidy). The extent of over-lending is also larger when expected future shortfalls are larger, which is the case when bank equity requirements are lower and when factor productivity is more volatile.

The main contribution of the paper is to show that during the period in between a shortfall and a recapitalization, the banking sector effectively suffers from debt-overhang. In its classic form, debt-overhang describes the problem where a firm under-invests because the income on new investments is at least partially appropriated by its pre-existing debtholders instead of by its equityholders (Myers, 1977). In the context of the banking sector, the literature shows that pre-existing debt may render undercapitalized banks reluctant to issue new equity and may distort their lending decisions (e.g., Hanson et al., 2011, Thakor, 2014, Bahaj and Malherbe, 2016, Occhino, 2017 and Admati et al., 2018). In practice, however, banking sectors are less likely to suffer from debt-overhang in the traditional sense, as most bank debt is of short-maturity so that pre-existing debt claims are relatively small. Still, our model shows that a debt-overhang problem may arise during the period in between a shortfall and a recapitalization. The reason is that part of the income on new lending is effectively appropriated by the government, as this income reduces the expected value of the recapitalization that will be received in the next period. During the period in between a shortfall and a recapitalization, the banking sector therefore charges inefficiently high lending rates, which implies a reduction in the supply of credit.

The debt-overhang in the banking sector during the period in between a shortfall and a recapitalization also affects the transmission of monetary policy to lending rates.[4] An increase in the policy rate causes the banking sector to increase its lending rate more than one-for-one.[5] The reason is that a part of the higher interest income on loans will be appropriated by the government and therefore cannot be used to cover the higher interest expenses on deposits. This effect results in a spread between lending rates and deposit rates, as in Goodfriend and McCallum (2007), Gerali et al. (2010), Gertler and Karadi (2011) and Curdia and Woodford (2016). The result is a weakened transmission of changes in the policy rate to inflation (and a largely unchanged transmission

[4]Monetary policy may affect bank lending rates through its impact on reserves (e.g., Bernanke and Blinder, 1988 and Kashyap and Stein, 1994), on equity (e.g., Van den Heuvel, 2002), and on risk-taking (Borio and Zhu, 2012). We do not model these channels, but follow the literature by letting the central bank directly set the interest rate on bank deposits. See Beck, Colciago and Pfajfar (2014) for a review of DSGE models that explore the role financial intermediaries in monetary policy transmission.

[5]Gambacorta and Shin (2018) show empirically that the lending behavior of weakly capitalized banks responds more strongly to monetary policy.

to output). This weaker transmission reflects that an increase in the policy rate, say, leads to a larger increase in bank lending rates (the marginal cost of capital), and thereby exerts more upward pressure on inflation. The net effect on inflation remains negative due to the decline in wages (the marginal cost of labor), but less so than when the banking sector would not have suffered from debt-overhang.

A key property of our model is that higher bank equity requirements increase social welfare (by reducing dynamic inefficiency), even though equity requirements are privately costly for the banking sector.[6] We illustrate this property by numerically analyzing the transition dynamics associated with raising bank equity requirements (see also Meh and Moran, 2010, Angelini et al., 2014, and Clerc at al., 2015). In the steady state, higher equity requirements reduce the probability of future shortfalls and thereby reduce the expected value of future recapitalizations. Raising equity requirements therefore causes the banking sector to increase its lending rates, which reduces investment and output. Lifetime utility increases, however, as consumption and leisure increase in the short run before they arrive at their lower steady state values. The positive effect on lifetime utility is smaller, and may even be negative, when equity requirements are raised during the period in between a shortfall and a recapitalization. During this period the banking sector charges inefficiently high lending rates, which is aggravated by an increase in equity requirements. Hence, the welfare effects of increases in bank equity requirements may be modified by the timing of bank recapitalizations.

Our model is part of a broader class of DSGE models that focuses on how financial frictions interact with macroeconomic fluctuations, which builds on seminal contributions by Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). The incorporation of financial intermediaries in DSGE models is more recent, with key contributions by, amongst others, Goodfriend and McCallum (2007), Gerali et al. (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Brunnermeier and Sannikov (2014). A typical source of amplification and propagation in such models is that the banking sector cannot issue outside equity, but has to slowly accumulate equity through retaining earnings. We drop this assumption by allowing for outside equity, which ensures that the banking sector adds no dynamics to the model other than through the role of government recapitalization policies. This approach enables us to isolate the effect of recapitalization policies on the supply of bank credit and on the transmission of monetary policy.

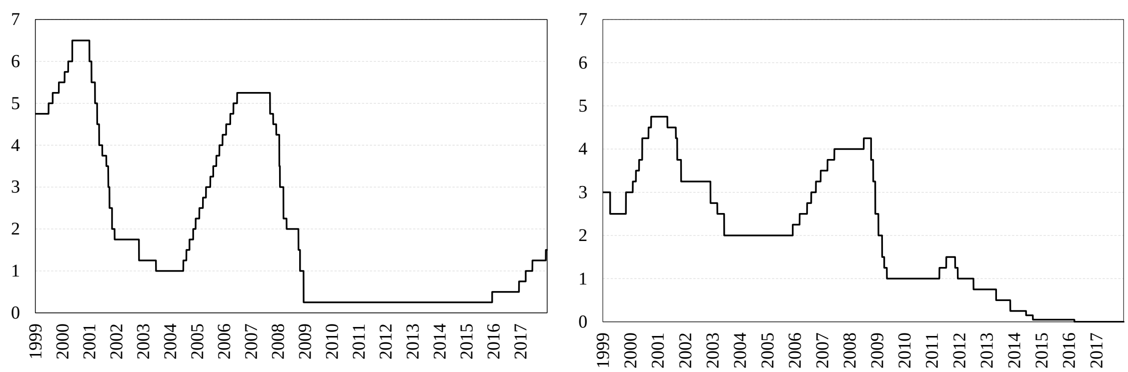
The remainder of the paper is organized as follows. Section 2 discusses some stylized facts that motivate our analysis. The section thereafter models a banking sector without frictions, and then extends this benchmark with bank recapitalizations by the government. Section 4 calibrates the model, the section thereafter analyzes its properties, and the final section concludes. Appendix A describes the standard New-Keynesian DSGE framework into which we embed the banking sector,

[6] Admati et al. (2013) emphasize the need to distinguish between the private and social costs of bank equity requirements.

Appendix B summarizes the entire model, Appendix C derives the steady state, and Appendix D provides some auxiliary derivations.

2 Empirical background

In response to the banking crisis of 2007-08, central banks aggressively reduced their monetary policy interest rates. Figure 1 illustrates that between September 2007 and the end of 2009, the U.S. Federal Reserve System (Fed) reduced its target interest rate by as much as five percentage points, until it arrived at the zero lower bound. About one year later, the European Central Bank (ECB) started to reduce its target rate as well, which fell by 3 percentage points in less than one year. The European sovereign debt crisis of 2010 and the subsequent recession prompted a further decline in the ECB's interest rate, until it arrived at the zero lower bound by the end of 2014. The ECB then kept its interest rate at this low level until the end of 2017, while the Fed gradually started to increase its interest rate from the end of 2015 onwards.



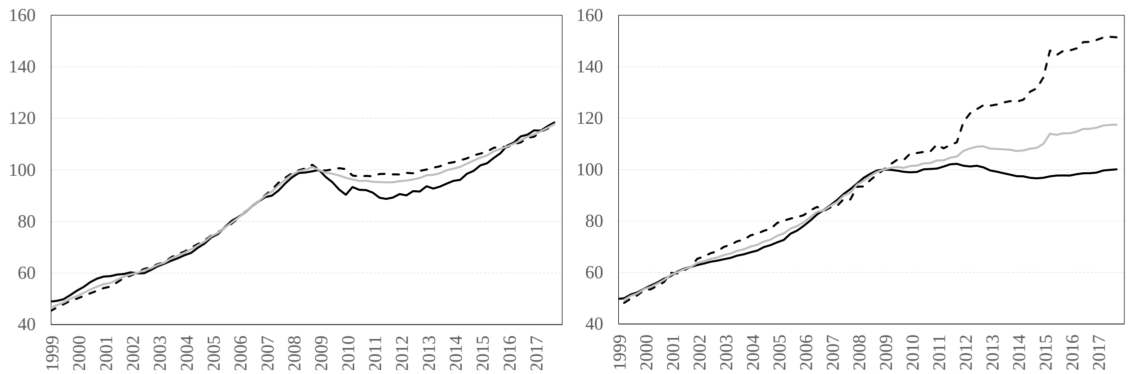
(a) United States

(b) Euro area

Figure 1: Monetary policy interest rates in the U.S. and EMU

Note: the left panel displays the official target interest rate set by the United States Federal Reserve System, the right panel displays the target interest rate set by the European System of Central Banks. Source: Fed and ECB.

One channel through which a reduction in monetary policy interest rates stimulates economic activity, is through its impact on the funding costs of banks. A decline in their funding costs enables banks to lower their lending rates, which increases the supply of credit to the real economy and stimulates investment. Figure 2 illustrates how bank credit supplied to the non-financial private sector developed around the crisis. Comparing both panels shows that both in the U.S. and in the euro area, the stock of bank credit reached a local maximum by the end of 2008. Until then, it had grown at virtually the same pace in both regions, at an annual rate of about 7.5 percent on average. This pattern abruptly changed once the crisis hit, with the growth of bank credit in the euro area falling back to zero while the growth of bank credit in the U.S. turned sharply negative. The growth of bank credit in the U.S. turned positive again by mid-2012, amounting to a bit less then 2 percent on average since the end of 2008, while the growth of bank credit in the euro area remained equal to zero. As such, the growth of bank credit in both regions was relatively small compared to the growth of non-bank credit, with the difference being particularly large in the euro area. Hence, despite historically low monetary policy interest rates, the banking crisis triggered a large slowdown in bank credit growth, especially in the euro area.



(a) United States

(b) Euro area

Figure 2: Bank and non-bank credit supply in the U.S. and EMU

Note: the lines display the stock of credit to the non-financial private sector supplied by banks (solid black lines) non-banks (dashed black lines) and the aggregate of both (gray line). All series are based on data in local currencies, and are normalized to equal 100 by the end of 2008. Source: BIS Local Credit Statistics.

In addition to triggering a decline in monetary policy interest rates and in the supply of bank credit, the crisis had a large negative impact on the capitalization of the banking sector. Figure 3 illustrates this impact by focusing on the ratio of bank equity to total assets. The figure shows that bank capitalization in terms of the the book value of equity, which is the object of regulatory minimum requirements for bank capitalization, deteriorated somewhat in 2008 and gradually improved thereafter.[7] By contrast, bank capitalization based on the market value of equity fell much more during the crisis, and especially in the euro area is yet to recover. Since the end of 2008, the capitalization of euro area banks in terms of market values has been consistently below the capitalization in terms of book values, while prior to the crisis this was the other way around. For banks in the U.S., capitalization in terms of market values started to improve after 2008, and has since 2013 been above capitalization in terms of book values. Still, in both regions, capitalization

[7] During these years, bank regulators adopted the Basel III reforms which raised minimum equity requirements.

in terms of market values has not yet returned to pre-crisis levels (as highlighted by Sarin and Summers, 2016). The next section connects this empirical observation with those in the previous two figures, by developing a model to analyze how the recapitalization of the banking sector after a crisis affects the supply of credit and the transmission of monetary policy.

(a) United States

(b) Euro area

Figure 3: Bank capitalization in the U.S. and EMU

Note: the solid lines display the market value of bank equity divided by total assets, and the dashed lines display the book value of equity divided by total assets. The market value of equity is obtained by multiplying the book value of equity with the market-to-book ratio. Source: BIS (2018) and Thomson Reuters Eikon.

3 Model

We integrate a banking sector in a standard New-Keynesian DSGE model with sticky prices and capital accumulation (this standard framework is described in Appendix A). Section 3.1 develops the benchmark version of the banking sector, which consists of a representative bank that operates without frictions. The model with the benchmark banking sector therefore has the same properties as the standard DSGE framework without banks. Section 3.2 augments the benchmark version of the banking sector with recapitalizations provided by the government, which the banking sector may receive either immediately after suffering large losses, or with a delay.

3.1 The banking sector without frictions

The banking sector without frictions consists of a representative bank that intermediates between the household and the capital producing firm (in DSGE models without banks, the household typically invests in the capital producer directly). The bank finances itself with deposits D_t and equity E_t from the household, which have expected returns equal to R_t^D and R_t^E.[8] The bank uses these funds to make loans L_t to the capital producer, against nominal lending rate R_t^L. Taking the returns on deposits and equity as given, the bank maximizes excess profits:

$$\max_{L_t, D_t, E_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi^B_{t+1+\tau} \right). \tag{1}$$

where $\Lambda_{t+\tau} \equiv \beta^\tau \lambda_{t+\tau}/\lambda_t$ is the stochastic discount factor of the household (which is the owner of the bank, see Appendix A). The excess profit of the bank is defined as:

$$\Pi^B_{t+1} \equiv \frac{R_t^L}{\pi_{t+1}} L_t - \frac{R_t^D}{\pi_{t+1}} D_t - \frac{R_t^E}{\pi_{t+1}} E_t + \Pi^K_{t+1}, \tag{2}$$

where inflation $\pi_t \equiv P_t/P_{t-1}$ is defined as the change in the price level P_t. As the bank is the only financier of the capital producer, the excess profits of the capital producer Π_t^K are appropriated by the bank as well. This way, losses incurred by the capital producer directly affect the excess profit of the bank.[9] The bank maximizes its excess profit subject to the balance sheet identity:

$$L_t \equiv D_t + E_t, \tag{3}$$

which may alternatively be interpreted as a production function for bank loans. In addition, the bank is subject to a regulatory minimum equity requirement:

$$E_t = \kappa L_t, \tag{4}$$

where κ is exogenously determined by the bank regulator. We simplify the analysis by assuming that the equity requirement holds with equality.[10] Next, substituting the equity requirement, the balance sheet identity, and the profit function in the objective function, we obtain:

$$\max_{L_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\frac{R^L_{t+\tau}}{\pi_{t+1+\tau}} L_{t+\tau} - \frac{R^D_{t+\tau}}{\pi_{t+1+\tau}} (1-\kappa) L_{t+\tau} - \frac{R^E_{t+\tau}}{\pi_{t+1+\tau}} \kappa L_{t+\tau} + \Pi^K_{t+1+\tau} \right). \tag{5}$$

[8] The expected return on equity, also known as the cost of equity, is equal to the expected stream of dividend payments and capital gains on the equity of the bank. The distinction between dividend payments and capital gains is not important for our analysis. Implicitly, dividends at the end of time t are equal to shareholder value at the end of t minus shareholder value at the start of $t+1$. A negative value implies that the bank issues additional equity.

[9] The bank also receives any positive excess profits from the capital producer, which effectively converts the loan contract into an equity claim. This simplification is harmless for our purposes, as we focus on the effect of loan losses on bank behavior and do not study the capital structure of the firm.

[10] The frictionless bank is indifferent about its share of equity funding, as equity and deposits are perfect substitutes from the perspective of the household. Introducing a binding equity requirement therefore does not affect the dynamics of the model. By contrast, a bank that may receive a recapitalization from the government prefers its share of equity funding to be as small as possible, as we show in the next section. The minimum equity requirement will therefore hold with equality.

Taking the derivative with respect to the choice variable L_t yields the first-order condition:

$$R_t^L = (1-\kappa)R_t^D + \kappa R_t^E, \tag{6}$$

where we used the fact that $\partial\,\mathbb{E}_t\left(\Pi_{t+1}^K\right)/\partial L_t = 0$. This condition states that the lending rate of the bank is equal to its weighted average cost of funds. As a result, the price of a bank loan is equal to the marginal cost of producing it.

3.2 Banking sector recapitalizations

We now extend the benchmark version of the banking sector with a recapitalization provided by the government (auxiliary derivations for this section can be found in Appendix D). The bank is recapitalized if the excess profits that it receives from the capital producer are sufficiently negative to reduce the value of its deposits. A key variable in this respect is the amount by which the claims of depositors exceed the asset value of the bank. We refer to this amount as the shortfall:

$$\begin{aligned} S_{t+1} &\equiv \max\left(0; \frac{R_t^D}{\pi_{t+1}}D_t - \frac{R_t^L}{\pi_{t+1}}L_t - \Pi_{t+1}^K\right), \\ &= \max\left(0; \bar{\omega}_t - \omega_{t+1}\right)\frac{R_t^L}{\pi_{t+1}}L_t, \end{aligned} \tag{7}$$

and define $\bar{\omega}_t \equiv (1-\kappa)\frac{R_t^D}{R_t^L}$ and $\omega_{t+1} \equiv \frac{R_{t+1}^K-\delta}{\mathbb{E}_t\left(R_{t+1}^K\right)-\delta}$, so that $\mathbb{E}_t\left(\omega_{t+1}\right) = 1$. The threshold $\bar{\omega}_t$ mainly depends on the bank equity requirement, while the stochastic variable ω_{t+1} is driven by (productivity) shocks that affect the excess profits of the capital producer. We assume that these shocks imply that ω_{t+1} is normally distributed with standard deviation σ_ω. Hence, the expression shows that a shortfall S_{t+1} can occur if the return on capital R_{t+1}^K is below expectation, and that such a shortfall will be larger when the equity requirement κ is lower. For the special case where $\kappa = 1$, shortfalls are eliminated, and the banking sector below is identical to the frictionless banking sector above. The model then has the same properties as a standard New-Keynesian DSGE model without banks.

3.2.1 An immediate recapitalization

If a shortfall occurs, the bank receives a recapitalization from the government. In practice, such recapitalizations have taken the form of injecting new equity, purchasing or guaranteeing troubled assets, or bank nationalizations. These transactions have in common that they involve a transfer from the government to the bank, either explicitly or implicitly, to increase the value of its equity. We therefore model a recapitalization as a transfer from the government to the bank, and assume that this transfer is financed by a lump sum tax on the household. This section models the case where the recapitalization takes place immediately after a shortfall, while the next section models the case where the recapitalization takes place with a delay.

An immediate recapitalization implies that the bank receives a transfer from the government directly when it experiences a shortfall. The transfer at time $t+1$ is equal to the size of the shortfall S_{t+1}, so that the expected stream of excess profits is:

$$\begin{aligned}
& \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^{B} + S_{t+1+\tau}\right) \\
= \quad & \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^{B} + \int_0^{\bar{\omega}_{t+\tau}} \left(\bar{\omega}_{t+\tau} - \omega_{t+1+\tau}\right) f(\omega_{t+1+\tau}) d\omega_{t+1+\tau} \frac{R_{t+\tau}^{L}}{\pi_{t+1+\tau}} L_{t+\tau}\right), \quad (8)
\end{aligned}$$

where $f(\cdot)$ is the probability density function of a normal distribution with mean one and standard deviation σ_ω. Taking the derivative of the expected stream of excess profits with respect to L_t yields the first-order condition:

$$R_t^L = \frac{(1-\kappa)R_t^D + \kappa R_t^E}{1+\Gamma(\bar{\omega}_t)}, \tag{9}$$

where we define $\Gamma(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \left(\bar{\omega}_t - \omega_{t+1}\right) f(\omega_{t+1+\tau}) d\omega_{t+1} > 0$. This term indicates the size of the recapitalization that the bank expects to receive in the next period, expressed as a percentage of its current loan portfolio. The first-order condition shows that a bank charges a lower lending rate if it expects to be recapitalized by the government after experiencing a shortfall. A convenient way to calculate the magnitude of the effect on the lending rate is to use:

$$\Gamma(\bar{\omega}_t) \quad = \quad \bar{\omega}_t - 1 - \sigma_\omega \frac{f(0) - f(\bar{\omega}_t)}{F(\bar{\omega}_t) - F(0)}, \tag{10}$$

where $F(\cdot)$ is the cumulative density function of a normal distribution with mean one and standard deviation σ_ω.

3.2.2 A delayed recapitalization

Bank recapitalizations are politically unpopular, for one reason because they impose a burden on taxpayers. Governments may therefore delay recapitalizations after a shortfall, hoping that the banking sector will recovers by itself. We model a delayed recapitalization as a transfer to the bank one period after it experienced a shortfall. This transfer ensures that the depositors will always be fully repaid, so that the bank never defaults. While the size of the immediate recapitalization is equal to the shortfall, the size of the delayed recapitalization is equal to the shortfall (plus interest) minus any profits that the bank has made since the shortfall occurred. The transfer at time $t+1$

equals:

$$
\begin{aligned}
& \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t - \max\left(0; \Pi_{t+1}^K + \frac{R_t^L}{\pi_{t+1}} L_t - \frac{R_t^D}{\pi_{t+1}} D_t\right)\right) \\
= \quad & \max\left(0, \bar{\omega}_t + \hat{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t - S_{t+1}, \qquad (11)
\end{aligned}
$$

where the threshold $\hat{\omega}_t \equiv (S_t/L_t)\,\frac{R_t^D}{R_t^L}$ is a function of the shortfall in the previous period t. If there was no shortfall during this period the threshold is equal to zero. The expected stream of excess profits equals:

$$
\begin{aligned}
& \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^B - S_{t+1+\tau} + \max\left(0, \bar{\omega}_{t+\tau} + \hat{\omega}_{t+\tau} - \omega_{t+1+\tau}\right) \frac{R_{t+\tau}^L}{\pi_{t+1+\tau}} L_{t+\tau}\right) \\
= \quad & \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^B - S_{t+1+\tau}\right) \\
+ \quad & \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\int_0^{\bar{\omega}_{t+\tau}+\hat{\omega}_{t+\tau}} \left(\left(\bar{\omega}_{t+\tau} + \hat{\omega}_{t+\tau} - \omega_{t+1+\tau}\right) \frac{R_{t+\tau}^L}{\pi_{t+1+\tau}} L_{t+\tau}\right) f(\omega_{t+1+\tau}) d\omega_{t+1+\tau}\right). \qquad (12)
\end{aligned}
$$

Taking the derivative with respect to L_t yields the first-order condition:

$$
R_t^L = \frac{(1-\kappa) R_t^D + \kappa R_t^E}{1 + F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right) \Gamma\left(\bar{\omega}_t\right) + \Gamma\left(\hat{\omega}_t\right)}, \qquad (13)
$$

where $F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right)$ is the probability that if the bank experiences a shortfall in the next period, a recapitalization in the period thereafter will still be necessary. This will be the case if the bank does not make sufficient profits during the period following the shortfall to recover on its own. Furthermore, we define $\Gamma(\hat{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\bar{\omega}_t+\hat{\omega}_t} \left(\bar{\omega}_t - \omega_{t+1}\right) f(\omega_{t+1+\tau}) d\omega_{t+1} \leq 0$, which negatively depends on $\hat{\omega}_t$ and therefore is smaller when there was a larger shortfall in the previous period.

Comparing the first-order condition to the one in (9) shows that relative to an immediate recapitalization, a delayed recapitalization drives up the lending rate in two ways. First, if the bank has not experienced a shortfall, the anticipation of a delayed recapitalization leads to a smaller decline in the lending rate than the anticipation of an immediate recapitalization. The reason is that $F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right) < 1$, and may even equal zero if the profits of the bank after a shortfall will with certainty be sufficient to render a delayed recapitalization unnecessary. Second, $\Gamma(\hat{\omega}_t) < 0$ if the bank has experienced a shortfall (while $\Gamma(\hat{\omega}_t) = 0$ otherwise), which also causes the lending rate to increase. The reason is that during the period in between a shortfall and a recapitalization, the income on loans reduces the size of the delayed recapitalization. This income is thereby partially appropriated by the government rather than by the shareholders of the bank, for which the bank compensates by charging a higher lending rate. A convenient way to calculate the magnitude of the resulting effect on the lending rate is to use:

$$\Gamma(\hat{\omega}_t) \quad = \quad \bar{\omega}_t - 1 - \sigma_\omega \frac{f\left(0\right) - f\left(\bar{\omega}_t + \hat{\omega}_t\right)}{F\left(\bar{\omega}_t + \hat{\omega}_t\right) - F\left(0\right)} - \Gamma(\bar{\omega}_t). \tag{14}$$

4 Calibration

Appendix B summarizes the model that results when integrating the banking block into the macroeconomic framework. Table 1 describes the calibration of the parameters of the model. Focusing on the banking sector, we calibrate the equity requirement at $\kappa = 0.04$. This choice reflects that the international standard for bank regulation as agreed in the Basel III Accord requires banks to maintain an equity buffer of at least three percent of total assets. This number reflects the 'leverage-ratio' requirement, but banks may need to maintain a larger equity buffer if they are systemically important or if their assets are relatively risky. Moreover, as a safe margin, banks tend to keep their equity ratios somewhat above the regulatory minimum. Calibrating the equity requirement at four percent therefore seems to be a reasonable choice.

By calibrating the household discount factor at $\beta = 0.99$, the frictionless version of the banking sector charges a lending rate of about one percent per quarter (so that the annual lending rate is about four percent). Given the four percent equity requirement, the bank experiences a shortfall if the return on loans turns out to be smaller than minus four percent. We calibrate the standard deviation of the return on loans at $\sigma_\omega = 0.02$, so that a return on loans of minus four percent is $(-0.04 - 0.01)/0.02 = 2.5$ standard deviations away from the mean return. Under the assumption that the return on loans is normally distributed, this calibration implies that the banking sector experiences a shortfall once in every 40 years. With the recent banking crisis fresh in mind, this seems a conservative estimate.

5 Results

To illustrate the dynamics of the model, we first focus on the version with the frictionless banking sector. The dynamics of this model are the same as those of the New-Keynesian DSGE model without a bank that is described in Appendix A. Figure 4 shows the impulse response functions that are associated with a one percent decrease in total factor productivity. This decrease in productivity leads to a decline in output. Inflation increases on impact, as firms need more capital and labor to produce a unit of output and therefore raise their prices. The central bank responds to this increase in inflation by raising the nominal monetary policy interest rate. This monetary contraction leads to an increase in the nominal bank lending rate, but the real lending rate declines due to the higher inflation. The real returns on bank equity and deposits therefore decline as well

Table 1: Calibration of the model

Parameter	Description	Value
β	Household discount factor	0.99
σ	Rate of inter-temporal substitution	1
φ	Inverse of the labor supply elasticity	2
χ	Weight of labor in the utility function	15
κ	Bank equity requirement	0.04
σ_ω	Standard deviation of the return on bank loans	0.02
α	Share of capital in the production function	0.3
ρ^Z	Autoregressive coefficient for productivity shocks	0.67
δ	Capital depreciation rate	0.025
θ	Final good substitution elasticity	6
ξ	Share of firms that cannot re-optimize their price	0.75
γ	Degree of price indexation	0
π^*	Steady state inflation rate	1
ϕ^R	Smoothing coefficient in the interest rate rule	0.9
ϕ^P	Response to inflation in the interest rate rule	1.5

(both are identical to the real lending rate when the banking sector is frictionless), which reduces the amount of equity and deposit savings by the households. The lower productivity also lowers life-time income and causes consumption to fall, so that firms reduce investment and hire less labor. As a result of the latter, wages fall as well.

5.1 Monetary policy transmission

We now turn out attention to the model in which the banking sector may receive a recapitalization from the government (as described in Section 3.2), and analyze the effect of such recapitalizations on the transmission of monetary policy. To this end we first focus on the derivative of the nominal bank lending rate with respect to the nominal monetary policy rate:

$$\frac{\partial R_t^L}{\partial R_t^D} = \frac{1}{1 + F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right)\Gamma\left(\bar{\omega}_t\right) + \Gamma\left(\hat{\omega}_t\right)} > 0. \tag{15}$$

This derivative depends negatively on $\Gamma\left(\bar{\omega}_t\right)$, on $F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right)$ and on $\Gamma\left(\hat{\omega}_t\right)$. As $\Gamma\left(\bar{\omega}_t\right) \geq 0$ and as $F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right) \geq 0$, the response of the lending rate to a change in the policy rate is smaller when recapitalizations are larger and when recapitalizations are expected to be immediate instead of delayed. By contrast, as $\Gamma\left(\hat{\omega}_t\right) \leq 0$, the response of the lending rate to a change in the policy rate is larger during the period in between a shortfall and a delayed recapitalization. Hence, the transmission of monetary policy to bank lending rates is stronger during this period.

A stronger transmission of monetary policy to nominal bank lending rates weakens the transmission of monetary policy to inflation. A policy rate increase, for example, reduces inflation by reducing aggregate demand and thereby reducing wages. Part of this deflationary effect is offset, however, by the fact that a policy rate increase also raises the cost of capital, which firms pass on to their customers by increasing their prices. As the capital share in production is typically calibrated to be substantially smaller than the labor share, the overall effect of an increase in the policy rate on inflation is negative. When changes in the policy rate have a larger effect on the bank lending rate, however, this offsetting effect becomes stronger, which weakens the overall effect of changes in the policy rate on inflation.

A stronger transmission of monetary policy to nominal bank lending rates leaves the transmission to output largely unaffected. The reason is that the larger response of the nominal lending rate does not translate into a larger response of the real lending rate, due to the weaker transmission of monetary policy to inflation. An increase in the nominal monetary policy rate, for example, leads to a larger increase in the nominal lending rate but to a smaller decrease in inflation. The net effect of monetary policy on the real lending rates is therefore largely unaffected.

Figure 5 illustrates the transmission of monetary policy in three versions of the model (we calibrate $S/L = 0.01$ in $\hat{\omega} \equiv \frac{S/L}{\beta R^L}$ in the debt-overhang version of the model). The first row describe the situation before a shortfall when recapitalizations are immediate, the second row describes the situation before a shortfall when recapitalizations are delayed, and the last row describes the situation after a shortfall but before the bank recapitalization. The panels show how a negative productivity shock affects the output gap, the inflation gap, and the monetary policy interest rate (for comparison, the panels also report the response when the banking sector is frictionless). In most models, these three variables are the main ingredients of the monetary policy rule.

The top row in the figure focuses on the situation before a shortfall when recapitalizations are immediate. A negative productivity shock leads to a relatively small increase in the policy rate and a relatively small increase in the inflation gap. This observation reflects that the transmission to inflation is stronger than in the model with the frictionless version of the banking sector. This stronger transmission causes the central bank to raise the policy rate by less than in the benchmark model (in our calibration the central bank responds only to inflation and not to output), so that the output gap declines by a relatively small amount. Hence, the stronger transmission of monetary policy to inflation alleviates the trade-off between output and inflation stabilization after a negative productivity shock.

The middle row in the figure shows that when recapitalizations are delayed, the responses to a negative productivity shock are virtually the same as in the model with the frictionless banking sector. Although the degree of similarity also reflects the calibration of the model, the responses illustrate that monetary transmission is less affected by the anticipation of a delayed recapitalization than by the anticipation of an immediate one.

The bottom row reports the responses for the period in between a shortfall and a delayed recapitalization. During the period between a shortfall and a recapitalization, a negative productivity shock induces a relatively large change in the policy rate, the output gap, and the inflation gap. This pattern reflects that the monetary transmission to inflation is weaker, so that stabilizing inflation requires a larger increase in the policy rate. As this larger increase in the policy rate amplifies the decline in output, the weaker transmission of monetary policy to inflation aggravates the trade-off between output and inflation stabilization after a negative productivity shock.

5.2 Bank equity regulation

A key property of the model is that bank equity requirements increase social welfare, even though they are privately costly for the banking sector. The reason is that a higher equity requirement reduces the size of expected future shortfalls, and thereby reduces the magnitude of bank recapitalizations by the government. The expectation of such recapitalizations cause banks to charge inefficiently low lending rates, which leads to an inefficiently large capital stock so that the equilibrium is dynamically inefficient. Table 2 shows that the capital stock is particularly large if recapitalizations are expected to be provided immediately after a shortfall, but is also larger than optimal if recapitalizations are provided with a delay. By contrast, during the period in between a shortfall and a recapitalization, the steady state capital stock is smaller than optimal as banks charge inefficiently high lending rates.

Table 2: Comparison of steady states

	Y^*	K^*	R^{L*}
Frictionless banking sector	-	-	4.0%
Before shorfall with immediate recapitalization	19.7%	58.3%	0.6%
Before shortfall with delayed recapitalization	0.3%	0.9%	3.9%
In between shortfall and recapitalization	-0.1%	-0.2%	4.1%

Note: Y^* and K^* are expressed in percentage differences from the steady state of the model with the frictionless banking sector.

To illustrate that raising bank equity requirements may increase welfare, we analyze how the economy responds to a permanent increase in the bank equity requirement. In the version of the model with a frictionless banking sector, such an increase leads to an increase in bank equity and a decrease in bank deposits and leaves all other variables unchanged. This result confirms that the frictionless banking sector does not violate the Modigliani-Miller (1958) conditions and capital structure is irrelevant. This property of the model disappears, however, when the model is augmented with government recapitalization policies.

Figure 6 displays the response of the model variables to a permanent 0.5 percentage point increase in the bank equity requirement (i.e., from 4 to 4.5 percent). The transition paths to this new steady state are reported for the situation before a shortfall when recapitalizations are delayed, and for the situation where the banking sector is in between a shortfall and a delayed recapitalization (the transition paths associated with an immediate recapitalization are not reported, as these are merely an amplified version of the paths associated with the situation before a shortfall when recapitalizations are delayed). In both cases, an increase in the equity requirement leads to a higher real lending rate as the magnitude of expected future recapitalizations declines. This reduces investment and initially raises consumption, but as the capital stock starts to shrink the economy ultimately becomes smaller and consumption declines as well. As the figure illustrates, the impact of an increase in equity requirements on the economy is larger during the period in between a shortfall and a delayed recapitalization.

Figure 7 shows that despite the negative effect on the size of the economy, raising bank equity requirements initially increases household utility. This initial increase in utility reflects the initial increase in consumption and leisure. The positive initial effect is larger when equity requirements are raised in between a shortfall and a recapitalization, as the initial decline in investment (and therefore the initial increase in consumption) is particularly large under these circumstances. Ultimately, however, household utility in both models ends up at a permanently lower level.

The effect of an increase in equity requirements on lifetime utility is positive when this increase takes place before a shortfall, as the higher requirements improve the dynamic efficiency of the equilibrium. Raising equity requirements enables households to increase their lifetime utility, as they increase their consumption and need to save less to maintain the capital stock. By contrast, raising equity requirements during the period in between a shortfall and a recapitalization reduces lifetime utility, as bank lending rates are already inefficiently high and the capital stock is inefficiently small (if S/L were calibrated at a smaller value than 0.01, the impact of an increase in equity requirements on lifetime utility could still be positive). The under-lending and under-investment in this situation are aggravated by the increase in equity requirements. Hence, the positive effect on lifetime utility of an increase in bank equity requirements may be reversed if such an increase takes place when the banking sector has not yet been recapitalized after a shortfall.

6 Concluding remarks

We integrated a banking sector in a parsimonious New-Keynesian DSGE model to examine how government policies to recapitalize banks affect the supply of credit and the transmission of monetary policy. We examined two types of recapitalizations after a crisis: immediate and delayed ones. In the steady state, both policies constitute a subsidy for the banking sector that causes banks to charge inefficiently low lending rates. The dynamic inefficiency that results can be mitigated by means of regulatory bank equity requirements, which are privately costly for banks but improve social welfare. During the period in between a crisis and a recapitalization, the banking sector effectively suffers from debt-overhang. The reason is that part of the income on new loans is appropriated by the government, as this income reduces the expected value of the recapitalization that the bank will receive in the next period. Banks therefore charge inefficiently high lending rates during this period, which reduces the supply of credit and weakens the transmission of monetary policy to inflation (the transmission to output remains largely unchanged). Raising bank equity requirements under such circumstances may actually lower social welfare, as this aggravates the problem of inefficiently high lending rates and low investment levels.

Overall, our analysis implies that delaying banking sector recapitalizations after a crisis may hamper the recovery of economic activity in three distinct ways. First, in between a crisis and a recapitalization, the banking sector effectively suffers from debt-overhang and charges inefficiently high lending rates. These high lending rates imply a reduction of credit supply and depress economic activity. Second, during the period until recapitalization, the transmission of monetary policy to inflation is weakened. The central bank therefore needs to implement larger changes in the policy rate in order to stabilize inflation, which may be difficult to achieve if the policy rate is constrained by the zero lower bound. Third, policy makers typically use the aftermath of a banking crisis as an opportunity to raise bank equity requirements, but such an increase has a particularly large negative effect on the economy when banks have not yet been recapitalized. These effects illustrate that bank recapitalization policies may have considerable macro-economic implications in the aftermath of a banking crisis, as well as during normal times.

Figure 4: Response to a negative productivity shock when the banking sector is frictionless

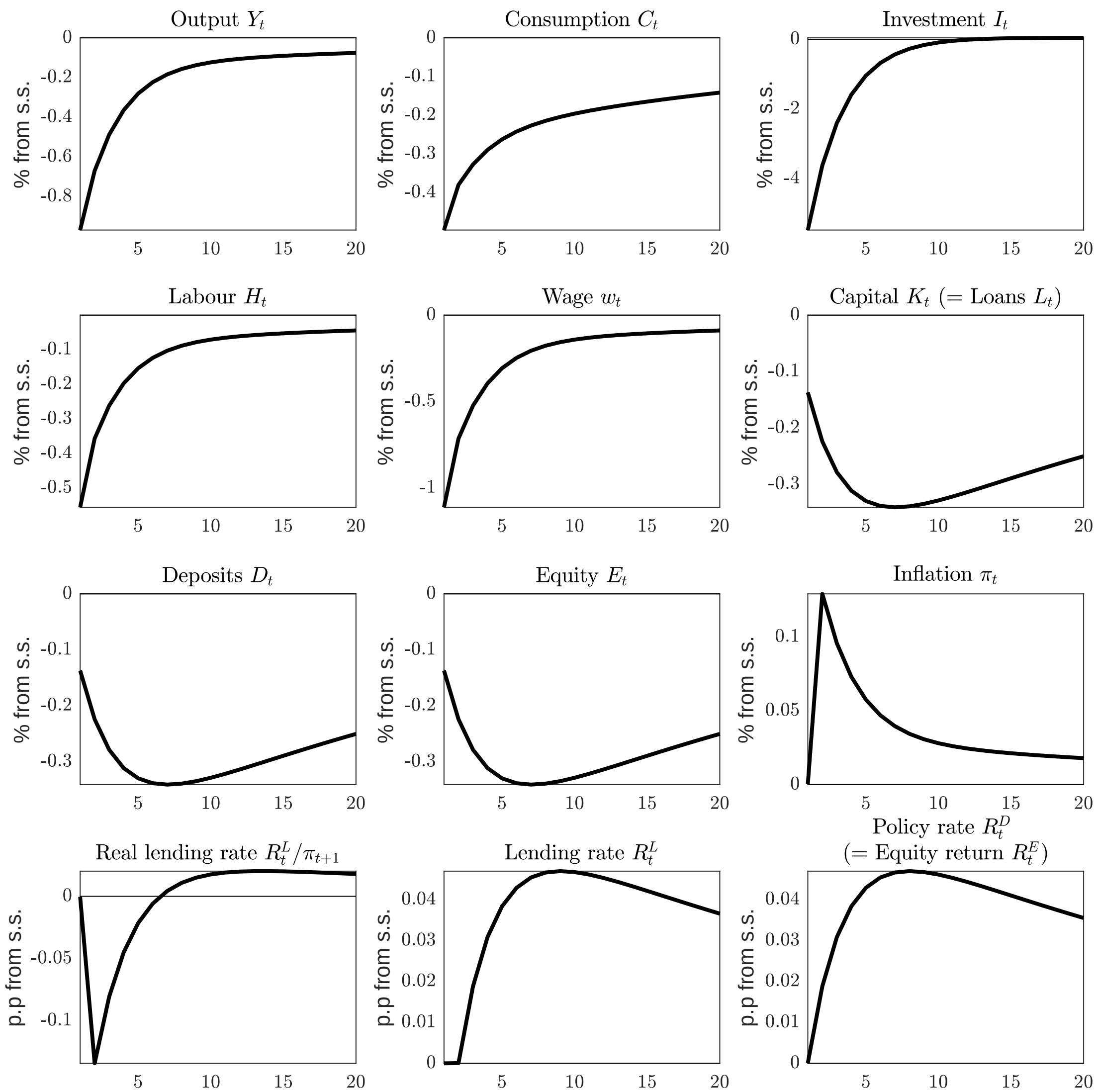
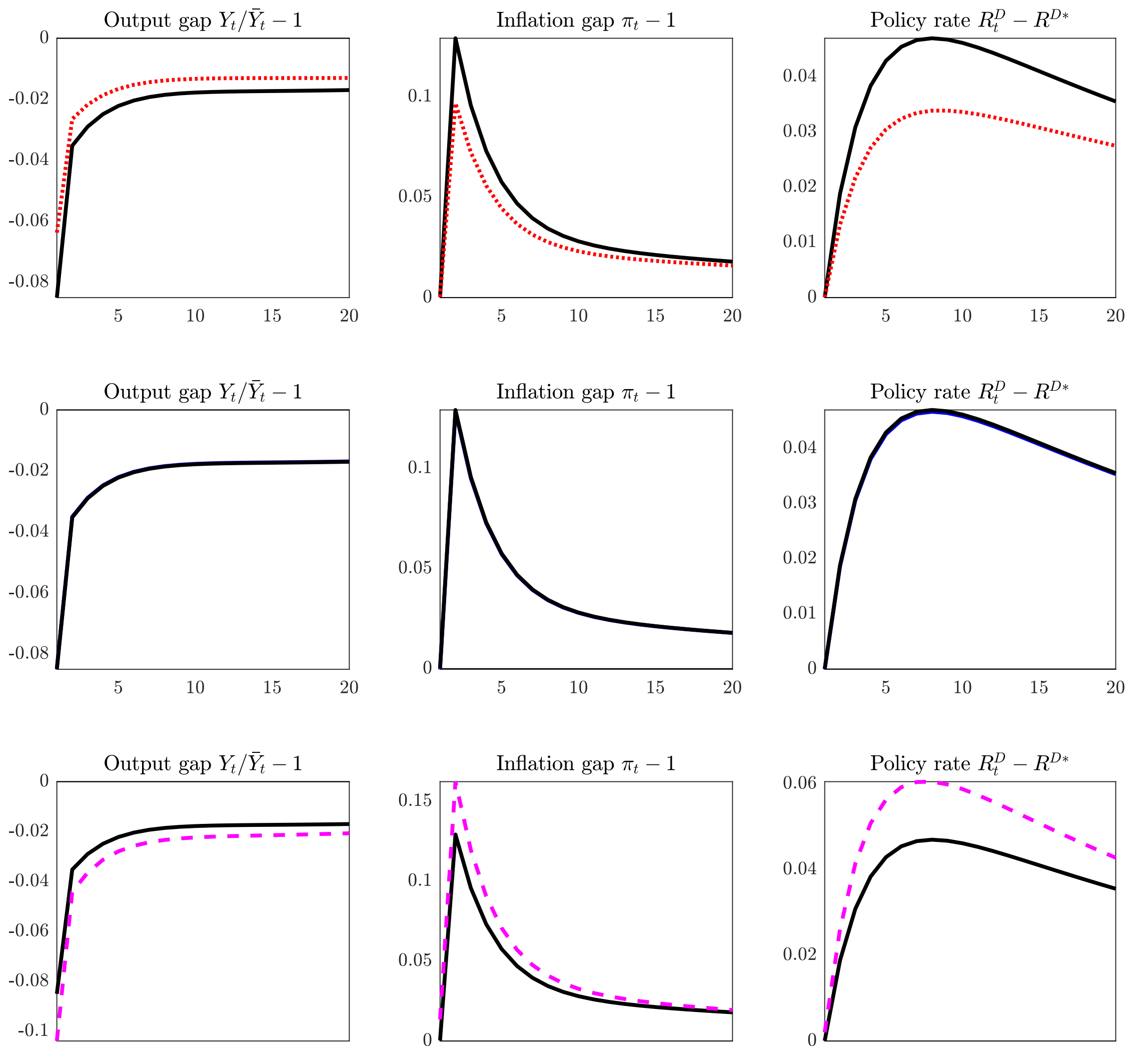
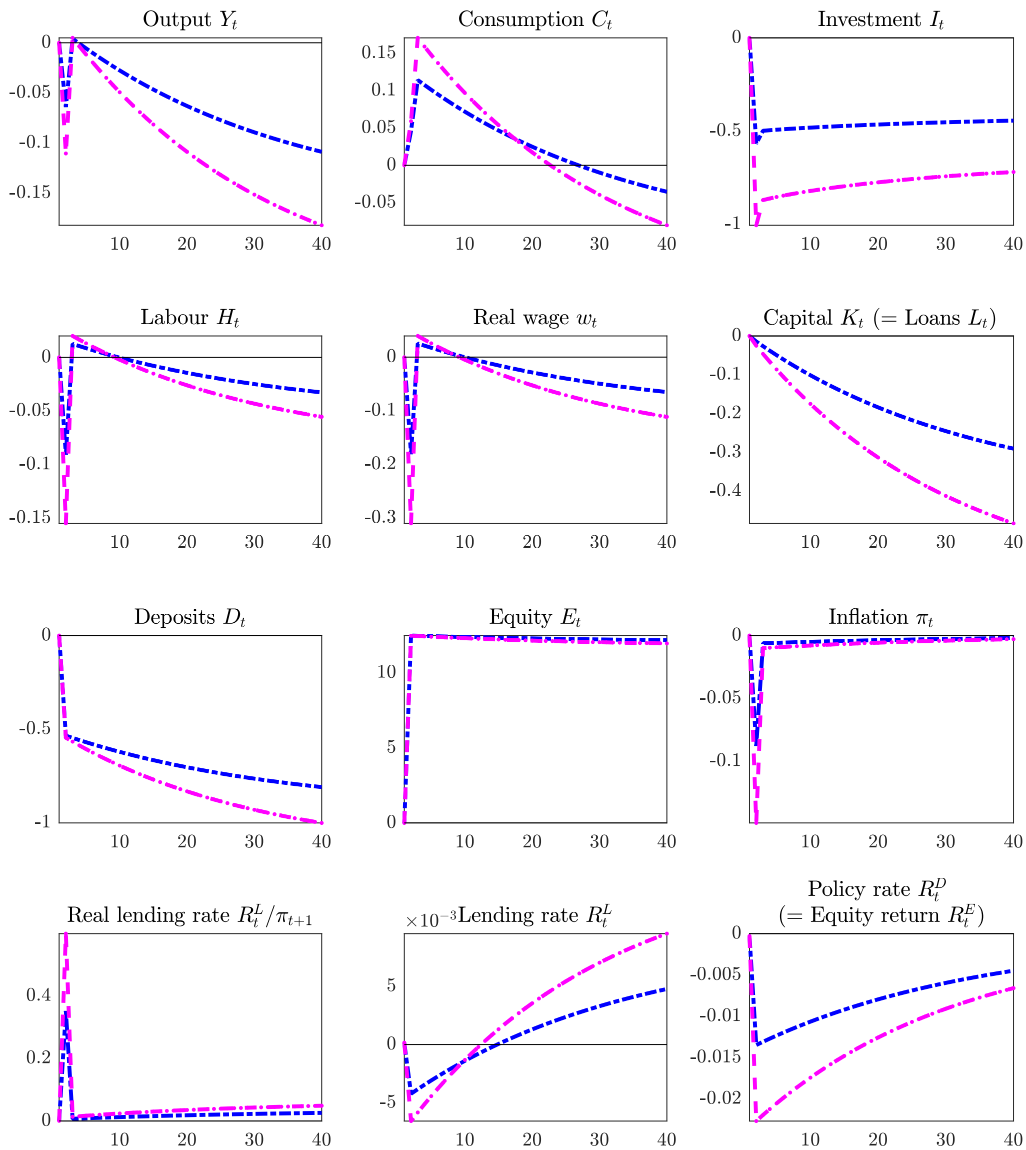


Figure 5: Monetary policy transmission after a negative productivity shock



Note: the black line in each of the rows reflects the response for the case where the banking sector is frictionless without recapitalizations. The top row focuses on the case before a shortfall when recapitalizations are immediate, the middle row focuses on the the case before a shortfall when recapitalizations are immediate, and the bottom row focuses on the case between a shortfall and a delayed recapitalization. In each of these four cases, to facilitate comparing the different panels, the monetary policy rule is calibrated such that the effect of the productivity shock on inflation is zero on impact. For the frictionless banking sector this implies a response to inflation of $\phi^P = 1.397$, while we calibrated ϕ^P at 1.308, 1.397, and 1.440 for the cases in the top, middle, and bottom row. Hence, ϕ^P is smaller when the transmission of monetary policy to inflation is stronger.

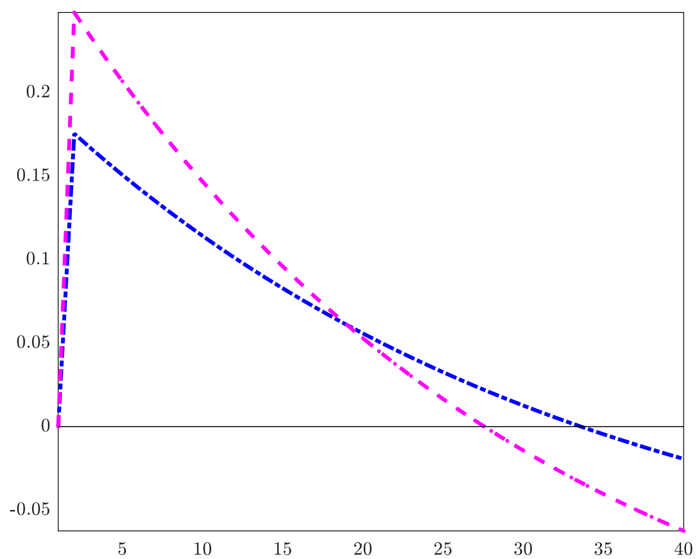
Figure 6: Response to a permanent increase in the bank equity requirement of 0.5 p.p.



Note: the blue line describes the case before a shortfall when recapitalizations are delayed and the pink line describes the case in between a shortfall and a delayed recapitalization.

[h]

Figure 7: Utility after an increase in the bank equity requirement by 0.5 p.p.



Note: the blue line describes the case before a shortfall when recapitalizations are delayed and the pink line describes the case in between a shortfall and a delayed recapitalization.

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Appendix A: A standard New Keynesian DSGE model

The household

The representative household maximizes its expected lifetime utility U_t:

$$U_t \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(u_{t+\tau} \right), \tag{16}$$

where β is the discount factor. The period utility function takes the following form:

$$u_t \equiv \frac{1}{1-\sigma} \left(C_t - \frac{\chi {H_t}^{1+\varphi}}{1+\varphi} \right)^{1-\sigma} \tag{17}$$

where C_t is consumption, H_t is the number of hours worked, $\sigma > 0$ is the rate of inter-temporal substitution, $\varphi > 0$ is the inverse of the labor supply elasticity, and $\chi > 0$ is the weight of labor in the utility function. The household can save in bank deposits D_t and in bank equity E_t, so that its budget constraint equals:

$$C_t + D_t + E_t = w_t H_t + \frac{R_{t-1}^{D}}{\pi_t} D_{t-1} + \frac{R_{t-1}^{E}}{\pi_t} E_{t-1} + \Pi_t, \tag{18}$$

where w_t denotes the real wage in the perfectly competitive labour market, R_t^D and R_t^E denote the returns on bank equity and bank deposits, and $\pi_t = P_t / P_{t-1}$ denotes inflation as a function of the price level P_t. The household in addition receives lump sum transfers $\Pi_t = \Pi_t^I + \Pi_t^F + \Pi_t^B + \Pi_t^G$, which consist of excess profits from the intermediate goods producing firms, the final goods producing firm, the bank, and transfers from the government (the excess profits from the capital producing firm enter in the profit function of the bank). Maximizing lifetime utility subject to the budget constraint yields the first-order conditions with respect to C_t, H_t, D_t, E_t:

$$\left(C_t - \frac{\chi {H_t}^{1+\varphi}}{1+\varphi} \right)^{-\sigma} = \lambda_t, \tag{19}$$

$$\chi {H_t}^{\varphi} = w_t, \tag{20}$$

$$\beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^D}{\pi_{t+1}} \right) = 1, \tag{21}$$

$$\beta \, \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^E}{\pi_{t+1}} \right) = 1, \tag{22}$$

where λ_t denotes the Lagrange multiplier for the budget constraint.

The firm

The household owns all firms in the economy. Firms therefore maximize the present value of their future profits discounted by the household discount factor. To preserve tractability we split the firm in three sub-firms. First we describe the capital producing firm, which is perfectly competitive. Next, we describe the final goods producing firm, which is perfectly competitive as well. Consequently, both firms have zero expected excess profits: $\mathbb{E}_t(\Pi^K_{t+1}) = 0$ and $\mathbb{E}_t(\Pi^F_{t+1}) = 0$. Finally, we describe the intermediate goods producing firms. These firms are monopolistically competitive and can set prices above marginal costs to maximize excess profits $\mathbb{E}_t(\Pi^I_{t+1}) > 0$. However, as they cannot update their prices every period, prices in the model are sticky.

The capital producing firm

The capital producing firm produces capital K_t, and supplies this capital to the intermediate good producing firms for a rental rate r^K_t. The capital producing firm must decide today how much capital it wants to supply in the next period. It finances this amount with a loan L_t from the bank, which it pays using the rental payments received during the next period. The capital producing firm maximizes its expected excess profits:

$$\max_{K_t, L_t} \mathbb{E}_t \left(\Pi^K_{t+1}\right). \tag{23}$$

Excess profits are equal to:

$$\Pi^K_{t+1} \equiv (1 + r^K_{t+1})K_t - \delta K_t - \frac{R^L_t}{\pi_{t+1}} L_t, \tag{24}$$

where δ denotes the percentage depreciation of the capital stock, so that investment is defined as:

$$I_t \equiv K_t - (1 - \delta)K_{t-1}. \tag{25}$$

The balance sheet identity of the capital producing firm reads:

$$K_t \equiv L_t, \tag{26}$$

which can be substituted into the profit function. Taking the derivative with respect to K_t then yields the first-order condition:

$$\frac{R^L_t}{\mathbb{E}_t(\pi_{t+1})} = \mathbb{E}_t \left(R^K_{t+1}\right) - \delta, \tag{27}$$

where $R^K_t \equiv 1 + r^K_t$. As a result, expected excess profits are equal to zero: $\mathbb{E}_t(\Pi^K_{t+1}) = 0$.

The final goods producing firm

The final goods producing firm is perfectly competitive. It combines a continuum of differentiated intermediate goods $Y_t(j)$ produced by intermediate firm $j \in [0,1]$ into a final good denoted by Y_t, which it then sells to the household. As there are no inter-temporal effects the profit function is static and equals:

$$\Pi_t^F \equiv Y_t - \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj, \tag{28}$$

where $P_t(j)$ is the price of the j^{th} intermediate input and P_t is the price for which the final good is sold. The firm maximizes these profits subject to the production technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{29}$$

where $\frac{\theta-1}{\theta}$ reflects the steady-state mark-up of the intermediate goods producing firms. Substituting this expression into the profit function and calculating the first-order condition with respect to $Y_t(j)$ yields:

$$Y_t(j)^{-\frac{1}{\theta}} = \frac{P_t(j)}{P_t} \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{-\frac{1}{\theta-1}}. \tag{30}$$

Raising both sides of this expression to the power $-\theta$ and substituting the expression for the production technology gives the demand curve for intermediary good $Y_t(j)$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t. \tag{31}$$

Substituting this expression in the profit function yields the aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{32}$$

where we used that $\Pi_t^F = 0$ because of perfect competition.

The intermediate goods producing firms

The intermediate goods producing firms use capital supplied by the capital producing firm and labor supplied by the household. Each intermediate goods producing firm j is monopolistically competitive, and uses these inputs to produce intermediate good $Y_t(j)$. For convenience, we split the profit maximization problem of each firm in two parts. First, the firm determines its optimal ratio of labor demand H_t to capital demand $K_t^d = K_{t-1}$, by minimizing the total costs to produce an intermediate good amount $Y_t(j)$. The solution to this cost minimization problem provides the marginal cost of the intermediate goods producing firms. Then, in a second step, intermediate goods producing firms maximize their profits by setting the optimal price.

Cost minimization

The first step involves minimizing total costs:

$$\min_{K_t^d(j), H_t(j)} w_t H_t(j) + r_t^K K_t^d(j), \tag{33}$$

subject to the production technology:

$$Y_t(j) = Z_t H_t(j)^{1-\alpha} K_t^d(j)^{\alpha}, \tag{34}$$

which is a standard Cobb-Douglas production function. Productivity Z_t is common across all firms and follows an autoregressive process: $\log(Z_t) = \rho^Z \log(Z_{t-1}) + \varepsilon_t^Z$, with autoregression parameter ρ^Z and where ε_t^Z is an i.i.d. Gaussian shock. We denote the Lagrangian multiplier associated with the production technology constraint by mc_t, which can be interpreted as the marginal cost of the intermediate goods producing firms. Taking the derivative of the Lagrangian with respect to $K_t^d(j)$ and $H_t(j)$ then yields the first-order conditions:

$$mc_t Z_t \alpha H_t(j)^{1-\alpha} K_t^d(j)^{\alpha-1} = r_t^K, \tag{35}$$

$$mc_t Z_t (1-\alpha) H_t(j)^{-\alpha} K_t^d(j)^{\alpha} = w_t. \tag{36}$$

Combining both first-order conditions gives the optimal ratio of labour to capital as a function of their respective costs:

$$\frac{\alpha}{1-\alpha} \frac{H_t(j)}{K_t^d(j)} = \frac{r_t^K}{w_t}. \tag{37}$$

Substituting the first-order conditions in the production function and rewriting the result shows that marginal costs equal:

$$mc_t = \frac{1}{Z_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^K}{\alpha} \right)^{\alpha}. \tag{38}$$

Price setting

In the second step, intermediate goods producing firms determine their price. In each time period a firm can re-optimize its price with probability $1 - \xi < 1$. We define $\tilde{P}_t$ as the price optimally chosen at time t by firms that re-optimize their price. Those firms that cannot re-optimize adjust their price by the inflation rate from the previous period:

$$\hat{P}_t(j) = \hat{P}_{t-1}(j)\pi_{t-1}^{\gamma}, \tag{39}$$

where γ represents the degree of indexation. Using this expression we define $P_{t,t+\tau}$ (which does not depend on the index j) as the level at time $t+\tau$ of a price that was last re-optimized at time t:

$$P_{t,t+\tau} \equiv \tilde{P}_t \prod_{s=1}^{\tau} \pi_{t-1+s}^{\gamma} = \tilde{P}_t \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}. \tag{40}$$

Firms that are allowed to re-optimize their price maximize the discounted value of expected profits. A firm that is allowed to re-optimize its price therefore faces the following optimization problem:

$$\max_{\tilde{P}_t(j)} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t(j)}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma} - mc_{t+\tau}\right) Y_{t+\tau}(j)\right]. \tag{41}$$

where $Y_{t+\tau}(j)$ is the demand by the final goods producing firm for intermediate good j with price $P_{t,t+\tau}$. The demand in period $t+\tau$ follows from (31) and is given by:

$$Y_{t+\tau}(j) = \left(\frac{P_{t,t+\tau}(j)}{P_{t+\tau}}\right)^{-\theta} Y_{t+\tau} = \left(\frac{\tilde{P}_t(j)}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}\right)^{-\theta} Y_{t+\tau}. \tag{42}$$

Substituting the demand expression in the maximization problem and rewriting yields:

$$\max_{\tilde{P}_t(j)} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\left(\frac{\tilde{P}_t(j)}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}\right)^{1-\theta} - \left(\frac{\tilde{P}_t(j)}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}\right)^{-\theta} mc_{t+\tau}\right) Y_{t+\tau}\right]. \tag{43}$$

Maximizing with respect to $\tilde{P}_t(j)$ and multiplying the result by $\tilde{P}_t$ gives:

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}\right)^{1-\theta} Y_{t+\tau}\right] \tag{44}$$

$$= \frac{\theta}{\theta-1} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}}\right)^{\gamma}\right)^{-\theta} mc_{t+\tau} Y_{t+\tau}\right], \tag{45}$$

where we dropped the firm index j as all firms are identical at this point. Next we define:

$$F_t^1 \equiv \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} Y_{t+\tau} \right], \tag{46}$$

$$F_t^2 \equiv \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}} \right)^{\gamma} \right)^{-\theta} mc_{t+\tau} Y_{t+\tau} \right], \tag{47}$$

so that the first-order constraint in (45) can be rewritten as:

$$F_t^1 = \frac{\theta}{\theta - 1} F_t^2. \tag{48}$$

Define $\tilde{\pi}_t \equiv \frac{\tilde{P}_t}{P_t}$ allows us to write F_t^1 in recursive form as:

$$\begin{aligned}
F_t^1 &\equiv \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} Y_{t+\tau} \right], \\
&= \left(\frac{\tilde{P}_t}{P_t} \right)^{1-\theta} Y_t + \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+\tau}} \left(\frac{P_{t-1+\tau}}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} Y_{t+\tau} \right], \\
&= \tilde{\pi}_t^{1-\theta} Y_t + \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{1+\tau} \frac{\lambda_{t+1+\tau}}{\lambda_t} \left(\frac{\tilde{P}_t}{P_{t+1+\tau}} \left(\frac{P_{t+\tau}}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} Y_{t+1+\tau} \right], \\
&= \tilde{\pi}_t^{1-\theta} Y_t + \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{1+\tau} \frac{\lambda_{t+1+\tau}}{\lambda_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{P}_{t+1}}{P_{t+1+\tau}} \frac{\tilde{P}_t}{\tilde{P}_{t+1}} \left(\frac{P_{t+\tau}}{P_t} \frac{P_t}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} Y_{t+1+\tau} \right], \\
&= \tilde{\pi}_t^{1-\theta} Y_t + \mathbb{E}_t \left[\beta\xi \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{P}_t}{\tilde{P}_{t+1}} \left(\frac{P_t}{P_{t-1}} \right)^{\gamma} \right)^{1-\theta} \mathbb{E}_{t+1} \left[\sum_{\tau=0}^{\infty} (\beta\xi)^{\tau} \frac{\lambda_{t+1+\tau}}{\lambda_{t+1}} \left(\frac{\tilde{P}_{t+1}}{P_{t+1+\tau}} \left(\frac{P_{t+\tau}}{P_t} \right)^{\gamma} \right)^{1-\theta} Y_{t+1+\tau} \right] \right], \\
&= \tilde{\pi}_t^{1-\theta} Y_t + \mathbb{E}_t \left[\beta\xi \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\gamma}}{\tilde{\pi}_{t+1}} \right)^{1-\theta} F_{t+1}^1 \right].
\end{aligned} \tag{49}$$

Using the same reasoning, we write F_t^2 in recursive form as:

$$F_t^2 = \tilde{\pi}_t^{-\theta} mc_t Y_t + \mathbb{E}_t \left[\beta\xi \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\gamma}}{\tilde{\pi}_{t+1}} \right)^{-\theta} F_{t+1}^2 \right]. \tag{50}$$

The price index (32) can be rewritten as:

$$P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj. \tag{51}$$

Splitting between firms that cannot re-optimize their price and therefore update their price according the indexation rule and firms that can optimize their price yields:

$$P_t^{1-\theta} = (1-\xi)\tilde{P}_t^{1-\theta} + \xi \left(P_{t-1}\pi_{t-1}^{\gamma}\right)^{1-\theta}. \tag{52}$$

We divide by $P_t^{1-\theta}$ to get rid of the potentially non-stationary P_t variable and obtain:

$$1 = (1-\xi)\tilde{\pi}_t^{1-\theta} + \xi \left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\theta}. \tag{53}$$

The government and central bank

There is no fiscal policy, except for the fact that the government may recapitalize the bank. As this is a zero sum game between the bank and the government, excess profits in the banking sector plus transfers from the government to the household are always equal to the excess profits of the frictionless bank:

$$\Pi_t^B + \Pi_t^G = \Pi_t^B, \tag{54}$$

which helps to simplify the household budget constraint. Furthermore, monetary policy involves the central bank setting the nominal interest rate on bank deposits by responding to inflation according to a Taylor rule:

$$\frac{R_t^D}{R^{D^*}} = \left(\frac{R_{t-1}^D}{R^{D^*}}\right)^{\phi^R} \left(\left(\frac{\pi_t}{\pi^*}\right)^{\phi^P}\right)^{1-\phi^R}, \tag{55}$$

where R^{D^*} is the steady state deposit rate and π^* is steady state inflation.

Market clearing

The supply of each intermediary goods producing firm j must equal the demand from the final goods producing firm:

$$Y_t(j) = Z_t H_t(j)^{1-\alpha} K_t^d(j)^{\alpha} = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t. \tag{56}$$

Integrating over all intermediary firms and denoting the total supply of intermediary goods by Y_t^I yields the market clearing condition for the intermediary goods market (the final goods market clears by Walras' law):

$$Y_t^I \equiv \int_0^1 Y_t(j)dj = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} djY_t = s_t Y_t, \tag{57}$$

where s_t can be written recursively as:

$$\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} dj, \\
&= (1-\xi)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\theta} + (1-\xi)\,\xi\left(\frac{\tilde{P}_{t-1}\pi_{t-1}^{\gamma}}{P_t}\right)^{-\theta} + (1-\xi)\,\xi^2\left(\frac{\tilde{P}_{t-2}\pi_{t-1}^{\gamma}\pi_{t-2}^{\gamma}}{P_t}\right)^{-\theta} + ..., \\
&= (1-\xi)\tilde{\pi}_t^{-\theta} + \xi\left(\frac{P_{t-1}\pi_{t-1}^{\gamma}}{P_t}\right)^{-\theta}\left[(1-\xi)\left(\frac{\tilde{P}_{t-1}}{P_{t-1}}\right)^{-\theta} + (1-\xi)\,\xi\left(\frac{\tilde{P}_{t-2}\pi_{t-2}^{\gamma}}{P_{t-1}}\right)^{-\theta} + ...\right], \\
&= (1-\xi)\tilde{\pi}_t^{-\theta} + \xi\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{-\theta} s_{t-1}.
\end{aligned} \tag{58}$$

Defining $H_t \equiv \int_0^1 H_t(j)dj$ and $K_t^d \equiv \int_0^1 K_t^d(j)dj$ allows us to rewrite total profits of the intermediary goods producing firms as the value of the total output minus the compensation for labor and capital:

$$\Pi_t^I = Y_t - w_t H_t - r_t^K K_t^d. \tag{59}$$

Substituting the excess profits of the intermediary goods producing firm, the final goods producing firm, and the bank (using Π_t^B, while setting $\Pi_t^G = 0$) into the household budget constraint yields:

$$C_t + I_t = Y_t, \tag{60}$$

which verifies that aggregate demand is equal to aggregate supply.

Appendix B: Model summary

The model is summarized by the following expressions:

The household

$$C_t + D_t + E_t = w_t H_t + \frac{R_{t-1}^D}{\pi_t} D_{t-1} + \frac{R_{t-1}^E}{\pi_t} E_{t-1} + \Pi_t, \tag{61}$$

$$\Pi_t = \Pi_t^I + \Pi_t^F + \Pi_t^B, \tag{62}$$

$$\left(C_t - \frac{\chi {H_t}^{1+\varphi}}{1+\varphi} \right)^{-\sigma} = \lambda_t, \tag{63}$$

$$\chi {H_t}^{\varphi} = w_t, \tag{64}$$

$$\beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^D}{\pi_{t+1}} \right) = 1, \tag{65}$$

$$\beta\, \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^E}{\pi_{t+1}} \right) = 1. \tag{66}$$

The bank

$$\Pi_t^B \equiv \frac{R_{t-1}^L}{\pi_t} L_{t-1} - \frac{R_{t-1}^D}{\pi_t} D_{t-1} - \frac{R_{t-1}^E}{\pi_t} E_{t-1} + \Pi_t^K, \tag{67}$$

$$L_t \equiv D_t + E_t, \tag{68}$$

$$E_t \equiv \kappa L_t, \tag{69}$$

$$R_t^L = \frac{(1-\kappa) R_t^D + \kappa R_t^E}{1 + F\left(\bar{\omega}_{t+1} + \hat{\omega}_{t+1}\right) \Gamma\left(\bar{\omega}_t\right) + \Gamma\left(\hat{\omega}_t\right)} \tag{70}$$

$$\Gamma(\bar{\omega}_t) = \bar{\omega}_t - 1 - \sigma_\omega \frac{f\left(0\right) - f\left(\bar{\omega}_t\right)}{F\left(\bar{\omega}_t\right) - F\left(0\right)}, \tag{71}$$

$$\Gamma(\hat{\omega}_t) = \bar{\omega}_t - 1 - \sigma_\omega \frac{f\left(0\right) - f\left(\bar{\omega}_t + \hat{\omega}_t\right)}{F\left(\bar{\omega}_t + \hat{\omega}_t\right) - F\left(0\right)} - \Gamma(\bar{\omega}_t), \tag{72}$$

$$\bar{\omega}_t \equiv (1-\kappa)\frac{R_t^D}{R_t^L} \tag{73}$$

$$\hat{\omega}_t \equiv (S_t/L_t)\,\frac{R_t^D}{R_t^L} \tag{74}$$

$$S_t \equiv \max\left(0; \bar{\omega}_{t-1} - \omega_t\right)\frac{R_{t-1}^L}{\pi_t}L_{t-1}, \tag{75}$$

$$\omega_t \equiv \frac{R_t^K - \delta}{\mathbb{E}_{t-1}\left(R_t^K\right) - \delta} \tag{76}$$

The capital goods producing firm

$$\Pi_t^K \equiv (1+r_t^K)K_{t-1} - \delta K_{t-1} - \frac{R_{t-1}^L}{\pi_t}L_{t-1}, \tag{77}$$

$$K_t \equiv L_t, \tag{78}$$

$$I_t \equiv K_t - (1-\delta)K_{t-1}, \tag{79}$$

$$\frac{R_t^L}{\mathbb{E}_t(\pi_{t+1})} = \mathbb{E}_t\left(R_{t+1}^K\right) - \delta. \tag{80}$$

$$R_t^K = 1 + r_t^K, \tag{81}$$

The intermediary goods producing firms

$$Y_t^I = Z_t {H_t}^{1-\alpha} {K_t^d}^{\alpha}, \tag{82}$$

$$K_t^d = K_{t-1}, \tag{83}$$

$$\log(Z_t) = \rho^Z \log(Z_{t-1}) + \varepsilon_t^Z, \tag{84}$$

$$mc_t = \frac{1}{Z_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}, \tag{85}$$

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{H_t}{K_{t-1}}\right) = \frac{r_t^K}{w_t}, \tag{86}$$

$$\Pi_t^I = Y_t - w_t H_t - r_t^K K_t^d, \tag{87}$$

$$F_t^1 = \tilde{\pi}_t^{1-\theta} Y_t + \mathbb{E}_t\left[\beta\xi\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^{\gamma}}{\tilde{\pi}_{t+1}}\right)^{1-\theta} F_{t+1}^1\right], \tag{88}$$

$$F_t^2 = \tilde{\pi}_t^{-\theta} mc_t Y_t + \mathbb{E}_t\left[\beta\xi\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^{\gamma}}{\tilde{\pi}_{t+1}}\right)^{-\theta} F_{t+1}^2\right], \tag{89}$$

$$F_t^1 = \frac{\theta}{\theta-1} F_t^2, \tag{90}$$

$$1 = (1-\xi)\tilde{\pi}_t^{1-\theta} + \xi\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\theta}. \tag{91}$$

The final goods producing firm

$$\Pi_t^F = 0. \tag{92}$$

The government and central bank

$$\Pi_t^G = 0, \tag{93}$$

$$\frac{R_t^D}{R^{D^*}} = \left(\frac{R_{t-1}^D}{R^{D^*}}\right)^{\phi^R}\left(\left(\frac{\pi_t}{\pi^*}\right)^{\phi^P}\right)^{1-\phi^R}. \tag{94}$$

Market clearing

$$Y_t^I = s_t Y_t, \tag{95}$$

$$s_t = (1-\xi)\tilde{\pi}_t^{-\theta} + \xi \left(\frac{\pi_{t-1}^{\gamma}}{\pi_t} \right)^{-\theta} s_{t-1}. \tag{96}$$

Appendix C: Steady state

In the steady state, the expressions in Appendix B simplify to:

The household

$$C = wH + \left(R^D - 1\right) D + \left(R^E - 1\right) E + \Pi, \tag{97}$$

$$\Pi = \Pi^I + \Pi^F + \Pi^B, \tag{98}$$

$$\left(C - \frac{\chi H^{1+\varphi}}{1+\varphi} \right)^{-\sigma} = \lambda, \tag{99}$$

$$\chi H^{\varphi} = w, \tag{100}$$

$$R^D = \frac{1}{\beta}, \tag{101}$$

$$R^E = \frac{1}{\beta}. \tag{102}$$

The bank

$$\Pi^B = R^L L - R^D D - R^E E + \Pi^K, \tag{103}$$

$$L = D + E, \tag{104}$$

$$E = \kappa L, \tag{105}$$

$$R^L = \frac{(1-\kappa)R^D + \kappa R^E}{1 + F\left(\bar{\omega} + \hat{\omega}\right)\Gamma\left(\bar{\omega}\right) + \Gamma\left(\hat{\omega}\right)} \tag{106}$$

$$\Gamma(\bar{\omega}) = \bar{\omega} - 1 - \sigma_\omega \frac{f\left(0\right) - f\left(\bar{\omega}\right)}{F\left(\bar{\omega}\right) - F\left(0\right)}, \tag{107}$$

$$\Gamma(\hat{\omega}) = \bar{\omega} - 1 - \sigma_\omega \frac{f\left(0\right) - f\left(\bar{\omega} + \hat{\omega}\right)}{F\left(\bar{\omega} + \hat{\omega}\right) - F\left(0\right)} - \Gamma(\bar{\omega}), \tag{108}$$

$$\bar{\omega} = (1-\kappa)\frac{R^D}{R^L} \tag{109}$$

$$\hat{\omega} = \left(S/L\right)\frac{R^D}{R^L} \tag{110}$$

$$S = \max\left(0; \bar{\omega} - \omega\right) R^L L, \tag{111}$$

$$\omega = 1 \tag{112}$$

The capital goods producing firm

$$\Pi^K = (1 + r^K - \delta)K - R^L L, \tag{113}$$

$$K = L, \tag{114}$$

$$I = \delta K, \tag{115}$$

$$R^L = R^K - \delta. \tag{116}$$

$$R^K = 1 + r^K \tag{117}$$

The intermediary goods producing firms

$$Y^I = ZH^{1-\alpha}K^{d\alpha}, \tag{118}$$

$$K^d = K, \tag{119}$$

$$Z = 1, \tag{120}$$

$$mc = \frac{1}{Z}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K}{\alpha}\right)^{\alpha}, \tag{121}$$

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{H}{K}\right) = \frac{r^K}{w}, \tag{122}$$

$$\Pi^I = Y - wH - r^K K^d, \tag{123}$$

$$F^1 = \frac{Y}{1-\beta\xi}, \tag{124}$$

$$F^2 = \frac{mcY}{1-\beta\xi}, \tag{125}$$

$$F^1 = \frac{\theta}{\theta-1}F^2, \tag{126}$$

$$1 = 1. \tag{127}$$

The final goods producing firm

$$\Pi^F = 0. \tag{128}$$

The government and central bank

$$\Pi^G = 0, \tag{129}$$

$$1 = 1, \tag{130}$$

Market clearing

$$Y^I = sY, \tag{131}$$

$$s = 1. \tag{132}$$

Solving the steady state

Given that:

$$R^E = R^D = \frac{1}{\beta}, \tag{133}$$

we can write:

$$\bar{\omega} \equiv \frac{1-\kappa}{\beta R^L}, \tag{134}$$

and:

$$\hat{\omega} \equiv \frac{S/L}{\beta R^L}. \tag{135}$$

When we focus on a steady state without a shortfall it follows that $S/L = 0$, while otherwise we calibrate $S/L = 0.01$. Using these ingredients, the bank lending rate equals:

$$R^L = \frac{1/\beta}{1 + F\left(\bar{\omega} + \hat{\omega}\right)\Gamma\left(\bar{\omega}\right) + \Gamma\left(\hat{\omega}\right)} \tag{136}$$

which we solve for R^L numerically. Given this value for R^L we obtain $1 + r^K = R^K = R^L$. Noting that $mc = (\theta - 1)/\theta$, we use the value for r^K in:

$$\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K}{\alpha}\right)^{\alpha} = \frac{\theta - 1}{\theta}, \tag{137}$$

to obtain the value of w. Together with the calibrated value $H = \bar{H}$ we then use:

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{H}{K}\right) = \frac{r^K}{w}, \tag{138}$$

to obtain the value of K. The rest of the model can be solved recursively.

Appendix D: Auxiliary derivations for the banking sector

The result in expression (7) is derived as:

$$\begin{aligned}
S_{t+1} &\equiv \max\left(0; \frac{R_t^D}{\pi_{t+1}} D_t - \frac{R_t^L}{\pi_{t+1}} L_t - \Pi_{t+1}^K\right), \\
&= \max\left(0; (1-\kappa)\frac{R_t^D}{R_t^L} - 1 - \frac{\Pi_{t+1}^K / L_t}{R_t^L/\pi_{t+1}}\right)\frac{R_t^L}{\pi_{t+1}} L_t, \\
&= \max\left(0; (1-\kappa)\frac{R_t^D}{R_t^L} - \frac{R_{t+1}^K - \delta}{R_t^L/\pi_{t+1}}\right)\frac{R_t^L}{\pi_{t+1}} L_t, \\
&= \max\left(0; \bar{\omega}_t - \omega_{t+1}\right)\frac{R_t^L}{\pi_{t+1}} L_t,
\end{aligned} \tag{139}$$

where we get from the first line to the second line by using the balance sheet identity in (3) and the equity requirement in (4), while factoring out $\frac{R_t^L}{\pi_{t+1}} L_t$. The third line follows from the description of the capital producing firm in Appendix A, which shows that $\Pi_t^K \equiv R_t^K K_{t-1} - \delta K_{t-1} - \frac{R_{t-1}^L}{\pi_t} L_{t-1}$ and $K_t = L_t$. The last line follows from defining $\bar{\omega}_t \equiv (1-\kappa)\frac{R_t^D}{R_t^L}$ and $\omega_{t+1} \equiv \frac{R_{t+1}^K - \delta}{R_t^L/\pi_{t+1}} = \frac{R_{t+1}^K - \delta}{\mathbb{E}_t\left(R_{t+1}^K\right) - \delta}$.

The result in expression (10) is derived as:

$$\begin{aligned}
\Gamma(\bar{\omega}_t) &\equiv \int_0^{\bar{\omega}_t} (\bar{\omega}_t - \omega_{t+1}) f(\omega_{t+1}) d\omega_{t+1}, \\
&= \mathbb{E}_t\left(\bar{\omega}_t - \omega_{t+1} | 0 < \omega_{t+1} < \bar{\omega}_t\right), \\
&= \bar{\omega}_t - \mathbb{E}_t\left(\omega_{t+1} | 0 < \omega_{t+1} < \bar{\omega}_t\right), \\
&= \bar{\omega}_t - \mathbb{E}_t\left(\omega_{t+1}\right) - \sigma_\omega \frac{f(0) - f(\bar{\omega}_t)}{F(\bar{\omega}_t) - F(0)}, \\
&= \bar{\omega}_t - 1 - \sigma_\omega \frac{f(0) - f(\bar{\omega}_t)}{F(\bar{\omega}_t) - F(0)},
\end{aligned} \tag{140}$$

where the fourth line uses the fact that $\mathbb{E}_t\left(\omega_{t+1} | 0 < \omega_{t+1} < \bar{\omega}_t\right)$ is the expectation of a truncated normal distribution $\mathcal{N}(1, \sigma_\omega)$ that is bounded from below by 0 and bounded from above by $\bar{\omega}_t$.

The result in expression (11) follows from:

$$
\begin{aligned}
& \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t - \max\left(0; \Pi_{t+1}^K + \frac{R_t^L}{\pi_{t+1}} L_t - \frac{R_t^D}{\pi_{t+1}} D_t\right)\right) \\
= \quad & \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t - \max\left(0; \omega_{t+1} - \bar{\omega}_t\right) \frac{R_t^L}{\pi_{t+1}} L_t\right), \\
= \quad & \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t + \min\left(0; \bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t\right) + S_{t+1} - S_{t+1}, \\
= \quad & \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t + \min\left(0; \bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t\right) + \max\left(0; \bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t - S_{t+1}, \\
= \quad & \max\left(\max\left(0; \bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t, \frac{R_t^D}{\pi_{t+1}} S_t + \left(\bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t\right) - S_{t+1}, \\
= \quad & \max\left(0, \frac{R_t^D}{\pi_{t+1}} S_t + \left(\bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t\right) - S_{t+1}, \\
= \quad & \max\left(0, \bar{\omega}_t + \hat{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t - S_{t+1},
\end{aligned} \tag{141}
$$

where the second line follows from using the negative of the definition of the shortfall in (7). The third line uses $-\max\left(0; \omega_{t+1} - \bar{\omega}_t\right) = \min\left(0; \bar{\omega}_t - \omega_{t+1}\right)$, and adds and subtracts the shortfall S_{t+1}. The fourth line uses the definition of the shortfall in (7). The expression in the fifth line is obtained by factoring $\max\left(0; \bar{\omega}_t - \omega_{t+1}\right) \frac{R_t^L}{\pi_{t+1}} L_t$ in the first maximization operator and noting that $\max(0, \bar{\omega}_t - \omega_{t+1}) + \min(0, \bar{\omega}_t - \omega_{t+1}) = \bar{\omega}_t - \omega_{t+1}$. The sixth line follows from evaluating the fifth line for $S_t = 0$ and also for $S_t > 0$, and observing that the result in both cases can be written as the sixth line. The last line defines $\hat{\omega}_t \equiv \mathbb{E}_t \left(\frac{R_t^D}{\pi_{t+1}} S_t / \frac{R_t^L}{\pi_{t+1}} L_t\right) = \left(S_t / L_t\right) \frac{R_t^D}{R_t^L}$.

The result in expression (13) follows from first observing that:

$$
\begin{aligned}
& \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^B - S_{t+1+\tau}\right) + \\
& \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\int_0^{\bar{\omega}_{t+\tau} + \hat{\omega}_{t+\tau}} \left(\left(\bar{\omega}_{t+\tau} + \hat{\omega}_{t+\tau} - \omega_{t+1+\tau}\right) \frac{R_{t+\tau}^L}{\pi_{t+1+\tau}} L_{t+\tau}\right) f(\omega_{t+1+\tau}) d\omega_{t+1+\tau}\right), \\
= \quad & \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\Pi_{t+1+\tau}^B - S_{t+1+\tau}\right) + \\
& \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t+1+\tau} \left(\int_0^{\bar{\omega}_{t+\tau} + \hat{\omega}_{t+\tau}} \left(\left(\bar{\omega}_{t+\tau} - \omega_{t+1+\tau}\right) \frac{R_{t+\tau}^L}{\pi_{t+1+\tau}} L_{t+\tau} + \frac{R_{t+\tau}^D}{\pi_{t+1+\tau}} S_{t+\tau}\right) f(\omega_{t+1+\tau}) d\omega_{t+1+\tau}\right),
\end{aligned} \tag{142}
$$

and then taking the derivative with respect to L_t using Leibniz's integral rule. This derivative equals:

$$\begin{aligned}
&\Lambda_{t+1}\frac{\partial\Pi^B_{t+1}}{\partial L_t}-\Lambda_{t+1}\frac{\partial S_{t+1}}{\partial L_t}+\Lambda_{t+1}\int_0^{\bar{\omega}_t+\hat{\omega}_t}(\bar{\omega}_t-\omega_{t+1})\frac{R^L_t}{\pi_{t+1}}f(\omega_{t+1})d\omega_{t+1}\\
&+\Lambda_{t+1}\left[(\bar{\omega}_t-\bar{\omega}_t-\hat{\omega}_t)\frac{R^L_t}{\pi_{t+1}}L_t+\frac{R^D_t}{\pi_{t+1}}S_t\right]f(\bar{\omega}_t+\hat{\omega}_t)\frac{\partial(\bar{\omega}_t+\hat{\omega}_t)}{\partial L_t}\\
&+\Lambda_{t+2}\int_0^{\bar{\omega}_{t+1}+\hat{\omega}_{t+1}}\left(\frac{R^D_{t+1}}{\pi_{t+2}}\frac{\partial S_{t+1}}{\partial L_t}\right)f(\omega_{t+2})d\omega_{t+2}\\
&+\Lambda_{t+2}\left[(\bar{\omega}_{t+1}-\bar{\omega}_{t+1}-\hat{\omega}_{t+1})\frac{R^L_{t+1}}{\pi_{t+2}}L_{t+1}+\frac{R^D_{t+1}}{\pi_{t+2}}S_{t+1}\right]\times f(\bar{\omega}_{t+1}+\hat{\omega}_{t+1})\frac{\partial(\bar{\omega}_{t+1}+\hat{\omega}_{t+1})}{\partial L_t},
\end{aligned} \tag{143}$$

where the expressions in the second and fourth line are equal to zero as the terms between square brackets are zero. Noticing that $\frac{\partial S_{t+1}}{\partial L_t}=\Gamma\left(\bar{\omega}_t\right)\frac{R^L_t}{\pi_{t+1}}$ and using $\Lambda_{t+2}\frac{R^D_{t+1}}{\pi_{t+2}}=\Lambda_{t+1}$ gives:

$$\begin{aligned}
&\Lambda_{t+1}\frac{\partial\Pi^B_{t+1}}{\partial L_t}-\Lambda_{t+1}\Gamma\left(\bar{\omega}_t\right)\frac{R^L_t}{\pi_{t+1}}+\Lambda_{t+1}\int_0^{\bar{\omega}_t+\hat{\omega}_t}(\bar{\omega}_t-\omega_{t+1})\frac{R^L_t}{\pi_{t+1}}f(\omega_{t+1})d\omega_{t+1}\\
&+\Lambda_{t+1}\int_0^{\bar{\omega}_{t+1}+\hat{\omega}_{t+1}}\Gamma\left(\bar{\omega}_t\right)\frac{R^L_t}{\pi_{t+1}}f(\omega_{t+2})d\omega_{t+2},
\end{aligned} \tag{144}$$

which can be simplified as:

$$\begin{aligned}
&\Lambda_{t+1}\frac{\partial\Pi^B_{t+1}}{\partial L_t}+\Lambda_{t+1}\left(\int_0^{\bar{\omega}_t+\hat{\omega}_t}(\bar{\omega}_t-\omega_{t+1})f(\omega_{t+1})d\omega_{t+1}-\Gamma\left(\bar{\omega}_t\right)\right)\frac{R^L_t}{\pi_{t+1}}\\
&+\Lambda_{t+1}F\left(\bar{\omega}_{t+1}+\hat{\omega}_{t+1}\right)\Gamma\left(\bar{\omega}_t\right)\frac{R^L_t}{\pi_{t+1}},
\end{aligned} \tag{145}$$

where in the second line we used $\int_0^{\bar{\omega}_{t+1}+\hat{\omega}_{t+1}}f(\omega_{t+2})d\omega_{t+2}=\int_0^{\bar{\omega}_{t+1}+\hat{\omega}_{t+1}}f(\omega_{t+1})d\omega_{t+1}=F\left(\bar{\omega}_{t+1}+\hat{\omega}_{t+1}\right)$. Defining $\Gamma(\hat{\omega}_t)\equiv\int_{\bar{\omega}_t}^{\bar{\omega}_t+\hat{\omega}_t}(\bar{\omega}_t-\omega_{t+1})f(\omega_{t+1})d\omega_{t+1}=\int_0^{\bar{\omega}_t+\hat{\omega}_t}(\bar{\omega}_t-\omega_{t+1})f(\omega_{t+1})d\omega_{t+1}-\Gamma(\bar{\omega}_t)$, solving $\frac{\partial\Pi^B_{t+1}}{\partial L_t}$, setting the resulting expression equal to zero and rearranging then gives:

$$R^L_t=\frac{(1-\kappa)R^D_t+\kappa R^E_t}{1+F\left(\bar{\omega}_{t+1}+\hat{\omega}_{t+1}\right)\Gamma\left(\bar{\omega}_t\right)+\Gamma\left(\hat{\omega}_t\right)}. \tag{146}$$