Time-Consistent Fiscal Guarantee for Monetary Stability

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- textbook Samuelson (1958)/Sims (2013) model of fiat money
- discretionary policy=f*(portfolio choice)

1. Model

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OLG Model: consumption-saving problem

- Discrete time: $t \in \{0, 1, ...\}$
- Overlapping generations of agents living for two periods.
- Representative agent born at time t maximizes:

$$U_t \equiv \log C_{t,y} + \log C_{t+1,o}$$

subject to:

young :
$$C_{t,y} + \frac{M_t}{P_t} + S_t + T_{t,y} = W$$

old :
$$C_{t,o} = \frac{M_{t-1}}{P_t} + \theta S_{t-1} + T_{t,o}$$

where:

- individual endowment W, lump sum taxes/transfers $T_{t,y}$, $T_{t,o}$;
- ▶ agents choose consumption *C* and composition of savings:
- either in real cash holdings M/P
- or in freely available storage S with a return $\theta < 1$
- At date 0, $M_{-1} = \overline{M}$.

At date-t, the authority's objective is:

$$\log C_{y,t} + \log C_{o,t} + \lambda \log G_t,$$

 G_t : government expenditures, and $\lambda > 0$. Its budget constraint is:

$$T_{t,y} + rac{M_{g,t-1}}{P_t} = rac{M_{g,t}}{P_t} + T_{t,o} + G_t$$

with $M_{g,t} + M_t = \overline{M}$.

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 - at the core of time-consistency

2. Benchmark: No Policy

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Optimal choices of agents

No policy benchmark: $\mathcal{P}_t = (0, 0, 0, 0).$ Savings

$$D_t \equiv S_t + \frac{M_t}{P_t} = \frac{W}{2}$$

for any expected return (property of log-utility)

$$\rho_t = \frac{\theta S_t + M_t / P_{t+1}}{D_t}$$

Portfolio allocation:

$$\begin{split} \frac{M_t}{P_t} &= D_t \quad \text{and} \ S_t = 0 \qquad \text{if} \qquad \Pi_{t+1} < \frac{1}{\theta}, \\ \frac{M_t}{P_t} &+ S_t = D_t \qquad \text{if} \qquad \Pi_{t+1} = \frac{1}{\theta}, \\ S_t &= D_t \quad \text{and} \ \frac{M_t}{P_t} = 0 \qquad \text{if} \qquad \Pi_{t+1} > \frac{1}{\theta}, \end{split}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the inflation rate from time t to time t+1.

No policy leads to indeterminacy



Figure : Equilibria without policy intervention for $\theta = 0.9$, W = 2 and $\overline{M} = 1$.

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At any t, an optimal policy is a $\mathcal{P}_t^* = (T_{y,t}^*, M_{g,t}^*, G_t^*, 0)$ that solves:

$$\max_{\mathcal{P}_t, G_t} \left\{ \log C_{y,t} + \log C_{o,t} + \lambda \log G_t \right\},\$$

subject to

$$T_{y,t} + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + G_t$$

taking into account agents' decision process on consumption:

$$C_{y,t} = \frac{M_t}{P_t} + S_t = \frac{W - T_{y,t}}{2}$$
$$C_{o,t} = \frac{M_{t-1}}{P_t} + \theta S_{t-1}$$

and market clearing conditions, with $S_0 = 0$ and $M_0 \leq \overline{M}$.

WLoG: no transfers to old.

We can rewrite the problem of the authority as

$$\max_{\mathcal{P}_{t},G_{t}} \left\{ \log \underbrace{\left(\mathcal{W} - G_{t} - \frac{M_{t-1}}{P_{t}} - S_{t} \right)}_{=C_{y,t}} + \log \underbrace{\left(\frac{M_{t-1}}{P_{t}} + \theta S_{t-1} \right)}_{=C_{o,t}} + \lambda \log G_{t} \right\}$$

whose solution is

$$\begin{cases} G_t = \lambda C_{y,t}, \ P_t = \frac{(2+\lambda)M_{t-1}}{W - (1+\lambda)\theta S_{t-1} - S_t} & \text{with} \quad C_{y,t} \ge C_{o,t} \\ G_t = \lambda C_{y,t}, \ P_t \to \infty & \text{otherwise.} \end{cases}$$

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If no private money demand \rightarrow incentive to infinite deflation \rightarrow autarky is not an equilibrium



Figure : Uniqueness with optimal policy for $\theta = 0.9$, W = 2, $\overline{M} = 1$ and $\lambda \to 0$.

Monetary equilibrium

A single equilibrium:

(i) no inflation $\Pi_t = 1$,

(ii) real value of money:

$$rac{ar{M}}{P_t} = rac{M_t}{P_t} = rac{W}{2+\lambda} \ \, ext{and} \ \, S_t = 0,$$

(iii) no public open market interventions:

$$T_{y,t} = G_t = \frac{\lambda}{2+\lambda}W,$$

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for each $t \geq 1$.

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for each $t \geq 1$.

- \blacktriangleright \neq Fiscal theory of the price level:
 - No surplus in equilibrium: $T_{y,t} = G_t$
 - ▶ No money purchase in equilibrium: $M_{g,t} = M_{g,t-1}$
 - Money = bubble \rightarrow no-fundamental dividend

At any t, an optimal policy is a $\mathcal{P}_t^* = (\overline{T}, M_{g,t}^*, G_t^*, 0)$ that solves:

$$\max_{M_{g,t},G_t} \left\{ \log C_{y,t} + \log C_{o,t} + \lambda \log G_t \right\},\,$$

subject to

$$\bar{T} + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + G_t$$

taking into account agents' decision process on consumption:

$$C_{y,t} = \frac{M_t}{P_t} + S_t = \frac{W - \overline{T}}{2} \text{ and } C_{o,t} = \frac{M_{t-1}}{P_t} + \theta S_{t-1}$$

and market clearing conditions, with $S_0 = 0$ and $M_0 \leq \overline{M}$.

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and market clearing conditions, with $S_0 = 0$ and $M_0 \leq \overline{M}$.

The authority has real endowment, but cannot raise taxes in response to a change in private savings!

We can then rewrite the problem of the authority as

$$\max_{P_t} \left\{ \log \frac{W - \bar{T}}{2} + \log \underbrace{\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right)}_{=C_{o,t}} + \lambda \log \underbrace{\left(\frac{W + \bar{T}}{2} - \frac{M_{t-1}}{P_t} - S_t\right)}_{=G_t} \right\}$$

whose solution is

$$\begin{cases} P_t = \frac{2(1+\lambda)M_{t-1}}{W+\overline{T}-2\lambda\theta S_{t-1}-2S_t} & \text{with} \quad \lambda C_{o,t} \leq G_t \\ P_t \to \infty & \text{otherwise.} \end{cases}$$

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The authority trades-off public and old's cons. \rightarrow **it produces** inflation!

But inflation fixed by arbitrage \rightarrow less storage is needed for the same inflation rate \rightarrow at same money and storage can steadily coexist If no private money demand \rightarrow infinite inflation could be possible \rightarrow autarky can be an equilibrium



Figure : Uniqueness with fixed taxes for $\theta = 0.9$, W = 2, $\overline{M} = 1$ and $\lambda = 0.05$.

The monetary equilibrium?

In this equilibrium:

$$\begin{array}{rcl} \frac{M_t}{P_t} &=& \frac{M_0}{P^*} = \frac{W-\bar{T}}{2}, \mbox{ for any } t \ge 1, \\ \Pi_t &=& 1+\lambda, \mbox{ for any } t > 1, \\ \mathcal{S}_t &=& 0, \mbox{ for any } t > 1. \end{array}$$

- \blacktriangleright \neq previous monetary equilibrium:
 - Consumption not equalized across generation.
 - Seigniorage in equilibrium.
- ► Exist only when 1 + λ ≤ θ⁻¹. Otherwise: storage strictly preferred to money.

Multiplicity without fiscal power



Figure : Multiplicity: A=autarky, M/S=asymptotic storage, M=pure monetary

5. Conclusion

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Conclusion

A new way to think about the uniqueness of the monetary equilibrium.

Monetary stability relies on the **active but off-equilibrium** role of an authority with fiscal power.

Fiscal power is needed to let agents trust that money will not be used to implicitly tax instead!

Thanks

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No policy leads to indeterminacy **back**



Figure : Equilibria without policy intervention for $\theta = 0.9$, W = 2 and $\overline{M} = 1$.

Appendix: Fluctuations in Endowment

Optimal policy reaction

- ▶ What happens when stochstic increases in endowment makes $\Pi_t = W_t / W_{t+1} > \theta^{-1}$?
- We build our solution on two elements:
 - First, in a solution where $S_t > 0$ we have that

$$\Pi_{t} = \frac{W_{t} + \theta S_{t-1} - 3S_{t}}{W_{t+1} - \theta S_{t} - S_{t+1}} = \theta^{-1}$$

Second, whenever S_t = 0 instead imposes

$$\Pi_t = \frac{W_t + \theta S_{t-1}}{W_{t+1} - S_{t+1}} < \theta^{-1}$$

▶ Backward Implication: Suppose $S_t > 0$, $S_{t+1} = 0$ and $S_{t+2} = 0$. Having $W_t = W$ at all times implies $S_{t-1} > 0$ which in turn implies $S_{t-2} > 0$ and so on.

Optimal policy reaction

Consider $W_1 = W + \epsilon$. The solution is a number *n* of periods of use of storage such that

$$S_{T-n} = \frac{n\theta^{n}\varepsilon + \left(n\theta^{n} - \frac{1-\theta^{n}}{1-\theta}\right)W}{(1+n)\theta^{n}} \ge 0 \qquad \text{for } n = T-1$$
$$S_{T-n} = \frac{n\theta^{n}\theta S_{T-n-1} + \left(n\theta^{n} - \frac{1-\theta^{n}}{1-\theta}\right)W}{(1+n)\theta^{n}} \ge 0 \quad \text{for } 0 \le n \le T-2$$

and

$$heta S_{\mathcal{T}-1} < rac{1- heta}{ heta} W$$
 and $S_{\mathcal{T}} = 0$

Optimal policy reaction: $w_3 = 0.2$, $\theta = 0.99$



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Optimal policy reaction: $w_3 = 0.5$, $\theta = 0.99$



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Optimal policy reaction: $w_3 = 1$, $\theta = 0.99$



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Optimal policy reaction: $w_3 = 1$, $\theta = 0.98$



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