Benefits of Gradualism or Costs of Inaction? Monetary Policy in Times of Uncertainty

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Abstract

Should monetary policy be more aggressive or more cautious when facing uncertainty on the relationship between macroeconomic variables? This paper's answer is: "it depends"on the degree of persistence of the shocks that hit the economy. The paper studies optimal monetary policy in a basic (two-equation) forward looking New-Keynesian (NK) framework with random parameters. It relaxes the assumption of full central bank information in two ways: by allowing for uncertainty on the model parameters and by assuming asymmetric information. While the private sector observes the realizations of the random process of the parameters as they occur, the central bank observes them with a one period delay. Compared to the problem with full information, the monetary authority must solve the Bayesian decision problem of minimizing the expected stream of future welfare losses integrating over its prior probability distribution of the unknown parameters. The paper proposes a general method to account for uncertainty on any subset of parameters of the model. As an application, it focuses on two cases: uncertainty on the natural rate of interest and on the slope of the Phillips curve.

Keywords: Optimal monetary policy; Parameter uncertainty; Asymmetric information; Natural rate of interest; Slope of the Phillips curve.

JEL Classification: E31; E32; E52.

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We are grateful to Martin Ellison, Gaetano Gaballo, Andrea Gerali, Stefan Laseen, Stefano Neri, Massimiliano Pisani, Alessandro Secchi, Luigi Federico Signorini, Ignazio Visco, Mirko Wiederholt and two anonymous referees for useful comments and fruitful discussions. The views expressed herein are those of the authors only and do not reflect those of Banca d'Italia or the Eurosystem.

Notably, we now appreciate that policy decisions under uncertainty must take into account a range of possible scenarios about the state or structure of the economy, and those policy decisions may look quite different from those that would be optimal under certainty.

— Chairman Ben S. Bernanke, *October 19, 2007*

1 Introduction

Dealing with uncertainty about the true state of the economy is one of the main challenges faced by central banks when conducting monetary policy. Macroeconomic models, which are used to assess their policy action, are simplifications of reality, necessarily incomplete and surrounded by great uncertainty. Beyond the uncertainty about the state and the fundamental shocks that drive the dynamics of the economy, there is also uncertainty about the value of the deep parameters defining the fundamental economic relationships, such as the slope of the Phillips Curve (PC) or the level of the natural rate of interest in the intertemporal investment-saving (IS) equation.

Optimal monetary policy crucially depends on the model parameters. While, under the assumption that the central bank has perfect knowledge of the parameters of the model, the optimal design of monetary policy is well understood, in the more realistic setup of parameter uncertainty it is still an open subject of debate on the agenda of both policymakers and academic researchers. Failing to properly account for parameter uncertainty when designing optimal monetary policy would lead to a sub-optimal allocation of resources.

This paper studies optimal monetary policy in the simplest (two-equation) forward looking New-Keynesian (NK) framework with parameter uncertainty. It relaxes the assumption of full central bank information in two ways: allowing for uncertainty on model parameters (which are assumed to be time-varying) and allowing for asymmetric information. It addresses the question of how monetary policy should be optimally designed, when the central bank faces uncertainty on the model parameters. It explains under which circumstances the central bank should be more cautious, more aggressive or behave as in the full information environment (certainty equivalence).

In a seminal work, Brainard (1967) advanced an explanation of the common intuition that uncertainty about the impact of monetary policy should require a more cautious policy. His result suggests that in response to exogenous shocks to the economy it is often optimal for policymakers to change their instrument by less than would be optimal if all parameters were perfectly known. This result has been dubbed the "Brainard conservatism principle" and has found wide acceptance, on both theoretical and empirical grounds. Recently, however, there has been a growing body of literature challenging this principle. Using a backward-looking model of the economy, Söderström (2002) has argued that uncertainty about inflation persistence may lead policymakers to pursue a more aggressive policy. Kimura and Kurozumi (2007) have analyzed the same type of parameter uncertainty in a hybrid model that incorporates persistence by allowing for a fraction of firms that use a backward-looking rule to set prices and a fraction that is perfectly rational and forward-looking. They have found that an optimal policy calls for a stronger response of interest rates to demand shocks.[1]

The reason provided in these works for overturning the Brainard principle is that the backward-looking components in the model amplify the impact that current variability in inflation has on future inflation. In this case "it will pay to make sure current inflation is very stable by reacting more aggressively to shocks" (Walsh, 2004).

[1] In the literature there are two distinct approaches to describe the policymaker's action against uncertainty: the Bayesian and the minmax (or robust control) frameworks. In the Bayesian approach, the policymaker is assumed to have a prior belief of the distribution of the parameters and it minimizes the expected loss based on this parameter distribution. In the minmax approach, instead, the policymaker gives up considering all possibilities and conducts the policy that works reasonably well in the worst possible case. Brainard (1967), Söderström (2002), and Kimura and Kurozumi (2007) have adopted the Bayesian approach. An example of the second approach, instead, is Giannoni (2002), that has introduced uncertainty for the several structural parameters of the forward-looking model. He concludes that the robust rule for the model involves a stronger response of the interest rate, again compared to the case in the absence of uncertainty.

Practitioners have often shown skepticism for these results. In the words of Blinder (1999) "Brainard's result was never far from my mind when I occupied the Vice Chairman's office at the Federal Reserve [...] Still, I find these new anti-Brainard results both puzzling and troubling. Though my confidence in the conclusion has been shaken by recent research, my gut still tells me that Brainard was right in practice." In the same vein, in the words of Praet (2018) "Gradualism is not a doctrine, but a pragmatic approach that is generally suitable to situations characterized by significant uncertainty about the impact of available policy instruments. A more aggressive monetary policy response, however, is warranted when there is clear evidence of heightened risks to price stability, i.e. when it is established that the degree of inflation persistence is likely to be high and risks disanchoring inflation expectations."

Compared to the existing literature, this paper is the first to provide a complete analytical characterization of the solution in a forward looking New-Keynesian model with parameter uncertainty. So far, an analytical characterization of the solution was available only for backward-looking models (Söderström 2002). For forward looking models (ie. models with rational expectations), instead, the literature has computed the optimal policy, however no closed-form solution was available (such as in Kimura and Kurozumi (2007)). Quite to the contrary, we provide a general method for analytically compute the equilibrium and we are thus able to perform comparative static analyses for output, inflation and interest rate.

The paper introduces uncertainty by letting a subset of the parameters vary over time as a stochastic process. Moreover, while all agents perfectly observe the contemporaneous shocks hitting the economy, it assumes that information on parameters that characterize households' preferences and firms' price setting behaviour and productivity is asymmetric: while the private sector fully observes realizations of the random process of those parameters as they occur, the central bank (the planner) observes them with a period of delay. This assumption is in contrast wity the literature on optimal monetary policy with imperfect information, where it is (implicitly) assumed that both private agents and the central bank have the same (imperfect) information set. Quite to the contrary, here we assume that private agents are better informed about the parameters driving their economic choices. We deem such assumption reasonable, since households and firms are likely to have more information on their preferences, productivity process and pricing strategy than the central bank. Compared to the problem with full information, the planner must solve the Bayesian decision problem of minimizing the expected stream of future welfare losses integrating over his prior probability distribution of the unknown parameters. The paper proposes a general method to account for uncertainty on any subset of parameters of the model. As an application, it focuses on two parameters: the natural rate of interest and the slope of the PC. The most popular micro-foundation of the NK model, as exposed for example in Woodford (2003), Galí (2008) and Clarida et al. (1999), connects these two parameters to some of the deep parameters that govern the behavior of the private sector: the persistence of shock to the technological progress and the frequency of price adjustment. This justifies the nature of the asymmetric information assumption.

The paper analytically characterizes the solution of the planner's problem and the dynamics of the economy. It solves for the equilibrium dynamics, generating from the interaction of entities with different information sets: the planner's and the private sector's ones. The planner minimizes its objective function integrating over parameter uncertainty and taking into account, as a constraint, the rational expectations equilibrium of the private sector, which is in turn a function of the policy rule chosen by the planner and of the current realization of parameters. As a consequence, since current inflation and output gap are equilibrium objects determined by the interaction of private agents and the planner, they will be determined by the expectation of the planner on the current state of parameters. In other words, current inflation and output gap are functions of unknown parameters, for the planner, thus random variables with known distribution. It turns out that the problem can be solved sequentially, by first characterizing the planner's problem in terms of "expected" current inflation and output gap and advancing some guesses on the covariances between endogenous variables and time-varying parameters. This allows to derive the optimal monetary policy, to plug it into the rational expectations equilibrium of the private sector, to derive closed forms for the solution (exclusively in terms of shocks) and finally to compute exact covariances that verify the initial guesses.

The paper shows two results. First, when the planner is uncertain about the persistence of technology shocks, thus about the exact value of the natural rate of interest, the certainty equivalence principle holds: optimal monetary policy, expressed as function of central bank's observables (expected current inflation and output gap), should not react more aggressively or more cautiously to exogenous shocks relative to the full information case. Even with uncertainty and asymmetric information, the coefficients describing the optimal central-bank response to the bank's estimates of the predetermined variables characterizing the state of the economy are independent of the nature of the bank's partial information. The rationale for this result is that the persistence of technology shocks enters the problem additively, without multiplying any endogenous variable, so that its uncertainty can be integrated out without affecting the rest of the problem. Ex-post forecast errors on the parameter affect the volatility of output and inflation in the same direction, so that the central bank does not face a trade-off between the stabilization of output and inflation, and therefore does not change the optimal monetary policy with respect to the perfect information case.

Second, when uncertainty concerns the slope of the PC, it is optimal for the planner to modify the parameters of its policy, expressed as function of central bank's observables, relative to the full information case. In this case, therefore, the certainty equivalence principle does not hold anymore. As the slope of the PC does not enter the problem additively, but rather multiplied by the output gap, its uncertainty cannot be simply integrated out because its variance and covariances with the endogenous variables matter for the problem. In this case, the optimal monetary policy can be either more cautious or more aggressive than in the full information case, depending on the degree of persistence of the cost-push shock. For low levels of persistence, optimal policy should become more cautious. On the contrary, for high levels of persistence (beyond a threshold that is function of the parameters of the model) the opposite becomes true: optimal policy should become more aggressive than in the full information case, and the degree of aggressiveness should increase as uncertainty become larger.

The intuition for this fairly general result is the following. Since the policymaker is risk-averse, uncertainty about the slope of the PC reduces his welfare. The larger the uncertainty, the higher the probability that a more aggressive monetary policy response to shocks may move inflation and output away from targets and the larger the costs in terms of welfare. Therefore, the Brainard principle applies and being more cautious in response to cost-push shocks than under perfect information reduces its welfare loss; however, since agents in the model are perfectly rational, they anticipate that monetary policy inaction implies a deviation of current inflation and output gap from their target not only today but also in the future. Since current endogenous variables in the model depend also on expectations, the effects of inaction on the deviation of inflation and output from targets is amplified. The more persistent are the shocks, the larger will be the effect of inaction on expectations and the cost in terms of welfare. There is a threshold value for the persistence of the cost-push shock at which gains and losses from being cautious offset each other. Beyond that threshold, losses from inaction would outweigh current gains, so that it pays for monetary policy to be more aggressive. Below that threshold, it pays to be more cautious.

The paper is organized as follows. Section 2 presents the general setup in which the central bank operates. Section 3 considers the optimal policy under discretion and perfect information, as described, for example, in Galí (2008). The section shows the equilibrium for inflation, output gap and interest rate. In Section 4 we study the optimal policy under discretion when the central bank is uncertain about the natural rate process and the slope of the Phillips curve. Section 5 provides a characterization of the solution. Section 6 and 7 compare the equilibrium under perfect information and the one under uncertainty and asymmetric information. In particular it focuses on the optimal interest rate reaction to exogenous shocks in the two setups and addresses the question of whether uncertainty calls for gradualism or stronger action. A welfare analysis provides some insights on the main findings. Section 8 concludes.

2 The general setup

Much of the theoretical analysis on monetary policy of the last decades has been conducted within the New Keynesian paradigm reviewed in Clarida et al. (1999) and Woodford (2003). Therefore, we build our exercise taking the linearized reduced form of a dynamic general equilibrium model with temporary nominal price rigidities as a primitive. This way of proceeding is in line with most of the papers in the literature building upon the "two equations linearized New-Keynesian framework" (see *ex multis* Aoki 2003 and Orphanides 2003). In such framework, aggregate demand is summarized by the following intertemporal *IS curve*[2]:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right) \tag{1}$$

where x_t is the output gap, measured as the log deviation of actual output from potential output; i_t is the short-term nominal interest rate and is taken to be the instrument for monetary policy; $E_t x_{t+1}$and $E_t \pi_{t+1}$ are the level of output gap and inflation expected by private agents for period $t+1$, given the information at time t; σ is the inverse of the rate of in-

[2]The IS relationship approximates the Euler equation characterizing optimal aggregate consumption choices and the parameter σ can be interpreted as the rate of intertemporal substitution.

tertemporal substitution; finally, r_t^n, is the natural rate of interest, which positively depends on changes in technology, a_t:[3]

$$r_t^n = \rho + \sigma\psi\left(E_t a_{t+1} - a_t\right). \tag{2}$$

The aggregate supply curve (AS) is modelled as an expectations-augmented *Phillips curve*[4]:

$$\pi_t = \beta E_t \pi_{t+1} + k_t x_t + u_t \tag{3}$$

where $\beta \equiv \frac{1}{1+\rho}$. We assume that $\{u_t\}$ follows an AR(1) stochastic process

$$u_t = \varphi^u u_{t-1} + \zeta_{u,t} \tag{4}$$

with $E\left(\zeta_{u,t}\right) = 0$ and $E\left(\zeta_{u,t}^2\right) = \sigma_u^2 > 0$.

Differently from standard New Keynesian models with constant coefficients, we allow for two parameters of the model to change over time. First we assume that

$$k_t = k + \nu_{k,t} \tag{5}$$

with $E\left(\nu_{k,t}\right) = 0$ and $E\left(\nu_{k,t}^2\right) = \sigma_k^2 > 0$. We also assume that $\nu_{k,t}$ is uncorrelated with the other shocks.

Second, we assume that $\{a_t\}$ follows a Random Coefficient Autoregressive processes $RCA\left(1\right)$

$$a_t = \varphi_{t-1}^a a_{t-1} + \zeta_{a,t} \tag{6}$$

[3] ρ is the discount rate and ψ is a convolution of parameters that characterize households' preferences and firms' production function. For a complete characterization of the parameters see Galí (2008).

[4] The AS relation approximates aggregate pricing emerging from monopolistically competitive firms' optimal behaviour in Calvo's model of staggered prices.

with

$$\varphi_t^a = \varphi^a a + \nu_{a,t} \tag{7}$$

and $\{(\zeta_{a,t}, \nu_{a,t})\}$ are iid random vectors with $E(\zeta_{a,t}) = E(\nu_{a,t}) = 0$, $E\left(\zeta_{a,t}^2\right) = \sigma_a^2 > 0$, $E\left(\nu_{a,t}^2\right) = \sigma_{\phi a}^2 > 0$ and covariances are 0 for all t. Assuming that

$$(\varphi^a)^2 + \sigma_{\phi a}^2 < 1$$

the processes are stationary.[5]

In order to complete the model, it is necessary to specify how the interest rate is chosen and the information set of agents and policymakers when they take decisions. Concerning the former, we consider the nominal interest rate as the policy instrument and model it by means of a reaction function that minimizes intertemporal deviations of inflation from target and output from potential. Concerning the information set, we will start with the case of perfect information and no uncertainty, as a benchmark, and move afterwards to the case of uncertainty and asymmetric information.

2.1 Optimal discretionary monetary policy under perfect information

To illustrate the mechanics of the model, we first solve the model under no uncertainty on $\varphi_t^a$and k_t. This implies that $\sigma_k^2 = \sigma_{\phi a}^2 = 0$.

The policy problem consists in choosing the time path for the instrument i_t to engineer a contingent plan for the target variables π_t and x_t that maximizes a given objective function

[5]The RCA process was first proposed by (Anděl, 1976), who also studied its properties. It is a non-linear time series model, with fat-tailed stationary distribution. For a detailed study of this type of process and for a proof of the necessary and sufficient conditions for ensuring stationarity of the process see Nicholls and Quinn (1982).

for the policymaker, which we assume to be as follows:[6]

$$\underset{\{i_t,x_t,\pi_t\}_{t=0}^{\infty}}{Max} - E_0\left[\sum_{t=0}^{\infty}[\beta/2]^t\left[\pi_t^2+\lambda x_t^2\right]\right] \tag{8}$$

subject to the constrain that a given interest rate policy $\{i\}_t$ induces at all t the Walrasian equilibrium:

$$\begin{aligned} x_t &= E_t x_{t+1} + \frac{1}{\sigma}\left[i_t - E_t\pi_{t+1} - \rho + \sigma\psi\left(1-\varphi^a\right)a_t\right] \\ \pi_t &= \beta E_t\pi_{t+1} + k_t x_t + u_t \end{aligned}$$

The dynamics of the economy are driven by realizations of the exogenous stochastic process of shocks. The First Order Conditions (FOCs) imply the well known relation of inflation and output gap under discretion

$$\lambda x_t = -k\pi_t \tag{9}$$

and the Minimum State Variable (MSV) solution under rational expectations is

$$\pi_t = \frac{\lambda}{k^2+\lambda\left(1-\beta\varphi^u\right)}u_t \tag{10}$$

$$x_t = -\frac{k}{k^2+\lambda\left(1-\beta\varphi^u\right)}u_t \tag{11}$$

$$i_t = \rho + \frac{k\sigma\left(1-\varphi^u\right)+\lambda\varphi^u}{k^2+\lambda\left(1-\beta\varphi^u\right)}u_t - \sigma\psi\left(1-\varphi^a\right)a_t \tag{12}$$

Equations (10), (11) and (12) show that, under the optimal discretionary policy and per-

[6] We consider the relative weight to output gap, λ, as an exogenous policy parameter, as it is often done in the literature. An alternative approach is to obtain it as the result of a general equilibrium problem. In this case λ would depend on representative consumer preferences and firms' price setting behaviour. The planner's uncertainty about parameters describing households and firms behaviour would, in this case, translate into uncertainty about its own policy objective. We let for future research the analysis of optimal monetary policy under such an assumption.

fect information, the central bank is able to completely offset the aggregate effects of changes to the natural rate in the IS curve, but not those generated by cost-push shocks. As a consequence, output gap and inflation deviate from the targets in proportion to the current realization of the cost-push shock.

3 Optimal discretionary policy under imperfect information and asymmetric information

We now assume that the planner has a coarser information set than that of agents: while both can observe realizations of $\zeta_{a,t}$ in period t, only agents can observe (thus condition expectations and actions on) realizations of $\theta_t = \{\varphi^a, k\}_t$, which the planner observes only with a one period lag. In other words, at time t the planner observes θ_{t-1} and $\zeta_{a,t}$ (thus a_t) and announces i_t; agents, instead, observe also θ_t and choose x_t and π_t.[7] We acknowledge that alternative assumptions on the information structure could be envisaged. One could indeed overturn the assumption, positing that the central bank has more information than private agents (Svensson and Woodford 2004) or assuming that both the central bank and the other agents in the economy have imperfect information (Lorenzoni 2010). However, we decided to pursue this path, as it seems reasonable to assume that private agents are better informed (in a representative agent model) about the deep parameters driving their choices and expectations.

The operator E_t defines agents' expectations and $\hat{E}_t$ those of the planner. The two are connected by the identity:

[7]Since asymmetric information involves only information about parameters at time t, the predictions of private agents and the planner about future inflation and output will coincide. See the minimum state variable solution of the problem with asymmetric information as described by Propositions 1 and 2.

$$E_t[\cdot] = \hat{E}_t[\cdot|\,\theta_t] \quad \text{for all } t \text{ and } \theta_t.$$

Let the vector $z_t = \{a_t, u_t\}$ be the state of the system in period t, sufficient to characterize its dynamics.[8] A solution $\{x, \pi, i\}_t$ of the planner's problem is (at each t) a set of functions: of the state of the system z_t and the state of parameter uncertainty θ_t:[9]

$$\{x_t, \pi_t, i_t\} = \{x(z_t, \theta_t), \pi(z_t, \theta_t), i(z_t, \theta_t)\}$$

While agents' choices, and therefore the dynamics of the economy, are conditional on θ_t, those of the planner cannot be, as he only observes θ_{t-1} when forming expectations and choosing i_t. Even though describing the dynamics of the system requires that, at any t, the planner's problem, expectations and choices must be formulated as functions of all states $\{z_t, \theta_t\}$, they are not truly conditional on θ_t. It follows that, while at time t agents fully observe "contemporaneous" variables of the system $\{i_t, x_t, \pi_t\}$, the planner can only observe expectations of them (unconditional to θ_t).

Let's define a set of new variables, describing the planner's expectation of contemporaneous variables:

$$\hat{x}_t = \hat{x}(z_t, \theta_t) = \hat{E}_t x_t = \int x(z_t, \theta_t)\, dF(\theta_t)$$

$$\hat{\pi}_t = \hat{\pi}(z_t, \theta_t) = \hat{E}_t \pi_t = \int \pi(z_t, \theta_t)\, dF(\theta_t)$$

$$\hat{\imath}_t = \hat{\imath}(z_t, \theta_t) = \hat{E}_t i_t = \int i(z_t, \theta_t)\, dF(\theta_t).$$

Clearly, while $E_t x_t = x_t$, this is not true for the planner's expectations: $\hat{E}_t x_t \neq x_t$. More-

[8] Note that, given z_{t-1}, observing the vector z_t is equivalent to obseving ε_t and vice-versa.

[9] In general, the functions may be time varying:

$$\{i_t, x_t, \pi_t\} = \{i_t(z_t, \varepsilon_t, \theta_t), x_t(z_t, \varepsilon_t, \theta_t), \pi_t(z_t, \varepsilon_t, \theta_t)\}.$$

When the problem is recursive, under specific conditions, they are time invariant: $i_t() = i()$, $x_t() = x()$ and $\pi_t() = \pi()$, for all t.

over, while in general $x(z_t, \theta_1) \neq x(z_t, \theta_2)$ for arbitrary θ_1 and θ_2, it is true that $\hat{x}(z_t, \theta_1) = \hat{x}(z_t, \theta_2)$ for all θ_1 and θ_2. The same is also true for π, $\hat{\pi}$, i and $\hat{\imath}$. For pure notational convenience, we also define $\hat{a}_t = a_t$ and $\hat{u}_t = u_t$ which restates that the planner fully observes the contemporaneous exogenous states z_t, just like agents. Note however that in general $\hat{a}_{t+1}|\hat{a}_t$ is a random variable with a distribution different from that of $a_{t+1}|a_t$, because the latter is conditioned on θ_t while the former is not.

Characterizing the system requires characterizing the joint dynamics of

$$\{x, \pi, i, \hat{x}, \hat{\pi}, \hat{\imath}\}_t$$

for a given exogenous realization of $\{z, \theta\}_t$.

The FOCs of the problem can be characterized as the FOCs for a minimum of the unconstrained Lagrangian:

$$\underset{\{i_t, x_t, \pi_t \phi_{1t}, \phi_{2t}\}_{t=0}^{\infty}}{Min} \hat{E}_0 \begin{bmatrix} \sum_{t=0}^{\infty} \left[\frac{[1+\rho]^{-t}}{2} \left[\pi_t^2 + \lambda x_t^2 \right] \right] \\ ... + 2\phi_{1t} \left[x_t - E_t x_{t+1} + \frac{1}{\sigma} \left[i_t - E_t \pi_{t+1} - \rho - \sigma\psi \left[E_t a_{t+1} - a_t \right] \right] \right] \\ ... + 2\phi_{2t} \left[\pi_t - \beta E_t \pi_{t+1} - k_t x_t - u_t \right] \end{bmatrix}$$

where $a_{t+1} = \varphi_t^a a_t + \zeta_{a,t+1}$, $\varphi_t^a = \varphi^a + \nu_{a,t}$ and $k_t = k + \nu_{k,t}$.

Necessary conditions for a minimum are that:

(a) For each t and each state $\{z_t, \theta_t\}$:

$$\lambda \hat{E}_t x_t + \hat{E}_t k_t \hat{E}_t \pi_t + \widehat{cov}_t (k_t, \pi_t) = 0 \tag{13}$$

Note that, at any period t, this equation is only truly a distinct equation at each state $\{z_t, \varepsilon_t\}$.

(b) For each t and each state $\{z_t, \theta_t\}$:

$$\begin{aligned}
\hat{E}_t\left[x_t - E_t x_{t+1} + \frac{1}{\sigma}\left[i_t - E_t\pi_{t+1} - \rho - \sigma\psi\left[E_t a_{t+1} - a_t\right]\right]\right] &= 0 \\
\hat{E}_t\left[\pi_t - \beta E_t\pi_{t+1} - k_t x_t - u_t\right] &= 0
\end{aligned}$$

using the fact that $\hat{E}_t E_t = \hat{E}_t$ and the law of iterated expectations $\hat{E}_t = \hat{E}_t\hat{E}_{t+1}$this implies :

$$\hat{x}_t = \hat{E}_t\hat{x}_{t+1} - \frac{1}{\sigma}\left[\hat{\imath}_t - \hat{E}_t\hat{\pi}_{t+1} - \rho - \sigma\psi\left[\hat{E}_t a_{t+1} - a_t\right]\right] \quad (14)$$

$$\hat{\pi}_t = \beta\hat{E}_t\hat{\pi}_{t+1} + \hat{x}_t\hat{E}_t k_t + \widehat{cov}_t\left(k_t, x_t\right) + u_t \quad (15)$$

(c) For each t and each state $\{z_t, \theta_t\}$ and using the law of iterated expectations $\hat{E}_t = \hat{E}_t\hat{E}_{t+1}$:

$$\hat{E}_t\hat{a}_{t+1} = \varphi^a a_t \text{ and } \hat{E}_t k_t = k \quad (16)$$

Subtituting (16) into (14) and (15) we obtain:

$$\hat{x}_t = \hat{E}_t\hat{x}_{t+1} - \frac{1}{\sigma}\left[\hat{\imath}_t - \hat{E}_t\hat{\pi}_{t+1} - \rho + \sigma\psi\left(1 - \varphi_a\right)a_t\right] \quad (17)$$

$$\hat{\pi}_t = \beta\hat{E}_t\hat{\pi}_{t+1} + k\hat{x}_t + \widehat{cov}_t\left(k_t, x_t\right) + \hat{u}_t. \quad (18)$$

Equations (13), (17) and (18) are a closed system that can be solved for the joint dynamics of $\{\hat{x}, \hat{\pi}, \hat{\imath}\}_t$. Given an interest rate policy $\{i\}_t$, equations (1), (2) and (3) would define the joint dynamics of $\{x, \pi, i\}_t$. A unique interest rate policy $\{i\}_t$ can be pinned down by the solution of the system of equations along with an "informational feasibility" constraint: the requirement that, for all t, the planner's policy $i_t = i\left(z_t, \theta_t\right)$ must be constant in θ_t. This amounts to requiring that:

$$i_t = i\left(z_t, \theta_t\right) = \bar{i}\left(z_t\right) \text{ for all } t \text{ and } \left\{z_t, \theta_t\right\} \tag{19}$$

where $\bar{i}\left(z_t\right)$ is a different constant for each t and $\{z_t\}$. Averaging (19) over θ_t implies:

$$i_t = \hat{\imath}_t \text{ for all } t \text{ and } \left\{z_t, \theta_t\right\} \tag{20}$$

so that equation (20) closes, along with (1), (2) and (3), the characterization of the dynamic system $\{x, \pi, i\}_t$.

The dynamic system $\{\hat{x}, \hat{\pi}, \hat{\imath}, x, \pi, i\}_t$ is characterized by six equations:

$$\begin{aligned}
\hat{x}_t &= \hat{E}_t \hat{x}_{t+1} - \frac{1}{\sigma}\left[\hat{\imath}_t - \hat{E}_t \hat{\pi}_{t+1} - \rho + \sigma\psi\left(1 - \varphi_a\right) a_t\right] \\
\hat{\pi}_t &= \beta \hat{E}_t \hat{\pi}_{t+1} + k\hat{x}_t + \widehat{cov}_t\left(k_t, x_t\right) + \hat{u}_t \\
\lambda \hat{x}_t &= -k\hat{\pi}_t - \widehat{cov}_t\left(k_t, \pi_t\right) \\
x_t &= E_t x_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} - \rho + \sigma\psi\left(1 - \varphi_t^a\right) a_t\right] \\
\pi_t &= \beta E_t \pi_{t+1} + k_t x_t + u_t \\
i_t &= \hat{\imath}_t
\end{aligned}$$

where $u_t = \varphi^u u_{t-1} + \zeta_{u,t}$ and $a_t = \varphi_{t-1}^a a_{t-1} + \zeta_{a,t}$. As the expectation operators are different in the two sub-blocks, the system can be solved recursively: first we solve for the three equations for $\{\hat{x}, \hat{\pi}, \hat{\imath}\}_t$, then substituting the solution for $\hat{\imath}_t$ the second block for $\{x, \pi, i\}_t$. In order to solve for the first block we need to make some guesses about the covariances

$$\widehat{cov}_t\left(k_t, x_t\right) \text{ and } \widehat{cov}_t\left(k_t, \pi_t\right)$$

that will be verified once we have solved also for the second block of equations.

We will now provide a characterization of the equilibrium, by considering separately the case of imperfect information on the natural rate of interest and the one on the slope of the Phillips curve.

Proposition 1 *Under optimal discretionary policy, when the Central bank is uncertain about the persistence of the technology shock (i.e. the parameter determining the evolution of the natural rate of interest), certainty equivalence holds. Ex-post the deviation of the output gap, x_t, and inflation, π_t, from those obtained under perfect information is increasing in the difference between the actual parameter, φ_t^a, from its unconditional mean, φ^a.*

The minimum state variable (MSV) solution writes[10]:

$$\hat{x}_t = -\frac{k}{\lambda + k^2 - \beta\lambda\varphi^u} u_t \tag{21}$$

$$\hat{\pi}_t = \frac{\lambda}{\lambda + k^2 - \beta\lambda\varphi^u} u_t \tag{22}$$

$$i_t = \rho + \hat{\imath}_t = \frac{k\sigma + \lambda\varphi^u - k\sigma\varphi_u}{\lambda + k^2 - \beta\lambda\varphi^u} u_t - \sigma\psi\left(1 - \varphi^a\right) a_t \tag{23}$$

$$x_t = -\psi\left(\varphi^a - \varphi_t^a\right) a_t - \frac{k}{k^2 + \lambda\left(1 - \beta\varphi^u\right)} u_t \tag{24}$$

$$\pi_t = -\psi k\left(\varphi^a - \varphi_t^a\right) a_t + \frac{\lambda}{k^2 + \lambda\left(1 - \beta\varphi^u\right)} u_t. \tag{25}$$

Comparing equations (12) and (23) we can observe that, when the central bank has imperfect information on the persistence of the technological shock, the optimal response to the observable state variables at time t, a_t and u_t, shouldn't be more aggressive nor more cautious relative to the full information case. In other words, even with uncertainty and asymmetric information, the coefficients describing the optimal central-bank response to the bank's estimates of the predetermined variables characterizing the state of the economy

[10] Proofs of all the propositions in the text and the procedure used to obtain the MSV are available upon request.

are independent of the nature of the bank's partial information. The rationale for this result is that the persistence of technology shocks enters the problem additively so that its uncertainty can be integrated out without affecting the rest of the problem. This does not mean that ex-post the Central bank is able to provide the same allocation as under perfect information. Comparing (10), (11), (24) and (25) we see that the difference between the realized inflation and output gap under perfect and imperfect information will depend on the distance between the true parameter, φ_t^a, and its unconditional mean, φ^a.

The next Proposition focuses on uncertainty about the slope of the Phillips curve.

Proposition 2 *Under optimal discretionary policy, when the Central bank is uncertain about the elasticity of inflation to the output gap, (i.e. the slope of the Phillips curve), certainty equivalence does not hold. Ex-post the deviation of the output gap, x_t, and inflation, π_t, from those obtained under perfect information is increasing in the uncertainty on the slope of the Philllips curve.*

The MSV solution of the problem writes:

$$\hat{x}_t = -\frac{k}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\varphi^u - \beta\varphi^u\sigma_k^2} u_t \tag{26}$$

$$\hat{\pi}_t = \frac{\sigma_k^2 + \lambda}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\varphi^u - \beta\varphi^u\sigma_k^2} u_t \tag{27}$$

$$\hat{\imath}_t = \rho + \frac{k\sigma + \varphi^u\sigma_k^2 + \lambda\varphi^u - k\sigma\varphi^u}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\varphi^u - \beta\varphi^u\sigma_k^2} u_t - \sigma\psi\left(1 - \varphi^a\right) a_t \tag{28}$$

$$x_t = -\frac{k}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\varphi^u - \beta\varphi^u\sigma_k^2} u_t \tag{29}$$

$$\pi_t = \frac{k^2 + \sigma_k^2 + \lambda - kk_t}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\varphi^u - \beta\varphi^u\sigma_k^2} u_t \tag{30}$$

Comparing equations (12) and (28) we can observe that, when the central bank has imperfect information on the slope of the Phillips curve, the optimal response to the observable state variables at time t, a_t is just as under perfect information; the response to the cost-push shock, instead, will depend on the degree of uncertainty about the slope of the Phillips curve, σ_k^2. In this case, it is optimal for the planner to violate the certainty-equivalence principle and to modify the parameters of its optimal policy, relative to the full information case. As the slope of the PC does not enter the problem additively, but rather multiplied by some endogenous variables, the covariance $\widehat{cov}_t\left(k_t, \pi_t\right)$ that enters into the first order condition (18) is not equal to 0. This modifies the optimal response of the Central bank, as uncertainty cannot be simply integrated out. This implies also that ex-post x_t and π_t will differ from those under perfect information. In particular, the larger the uncertainty about the slope of the Phillips curve, the larger the difference between the allocations and price dynamics obtained under perfect and imperfect information.

The following Proposition characterizes the optimal interest rate response as a function of the uncertainty on the slope of the Phillips curve and the persistence of the cost-push shock. Let us first define a useful object:

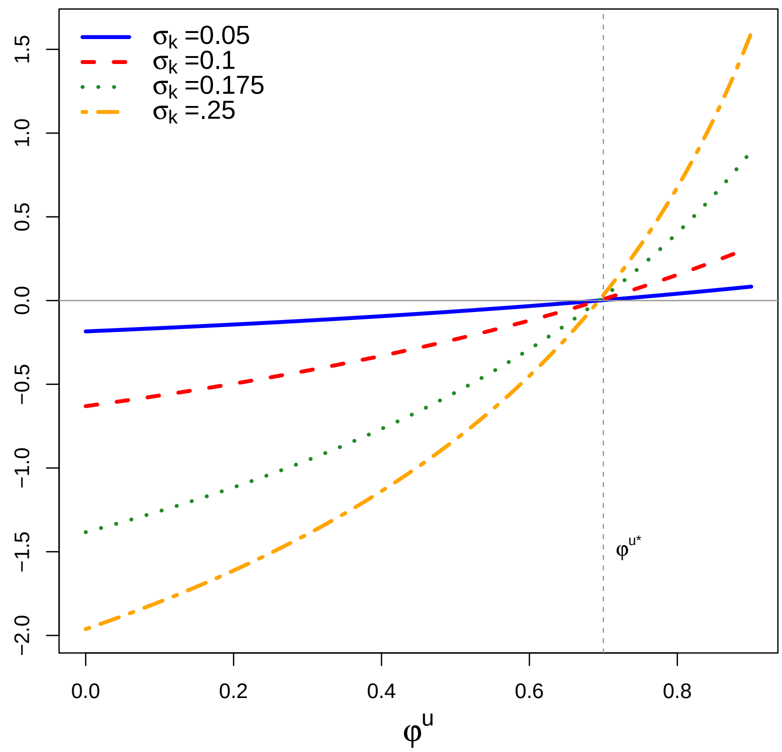
$$\varphi^{u*} = 1 + \frac{k}{2\sigma} - \left(\frac{k\left(k+4\sigma\right)}{4\sigma^2}\right)^{1/2}. \tag{31}$$

Proposition 3 *When the Central bank is uncertain about the slope of the Phillips curve, optimal discretionary monetary policy can be either more cautious or more aggressive than in the perfect information case, depending on the degree of persistence of the cost-push shock. For low levels of persistence, $0 < \varphi^u < \varphi^{u*}$, optimal policy should be more cautious. On the contrary, for high levels of persistence, $\varphi^{u*} < \varphi^u < 1$, the opposite becomes true: optimal policy should be more aggressive than under perfect information (and the higher the degree of uncertainty the more aggressive the policy should be).*

The intuition is the following. Since the policymaker is risk-averse, uncertainty about the slope of the PC reduces his welfare. In particular, the larger the uncertainty, the higher the probability that a more aggressive monetary policy response to shocks may move inflation and output away from targets and the larger the costs in terms of welfare. However, since agents in the model are perfectly rational, they know that monetary policy inaction implies a deviation of current inflation and output gap from their target not only today but also in the future; and the larger the persistence of the cost-push shock, the larger would be the cumulated deviation and the loss. In other words, since current endogenous variables in the model depend also on expectations, the effects of inaction on the deviation of inflation and output from targets is amplified. The more persistent are the shocks, the larger will be the effect of inaction on expectations and the cost in terms of welfare. There is a threshold value of persistence, φ^{u*}, at which current gains and future losses offset each other. Beyond that threshold, future losses from inaction would outweigh current gains, so that it pays for monetary policy to be more aggressive. Below that threshold, it pays to be more cautious. In Figure 1 we provide a graphical intuition of these results.

The Figure shows the reaction of the interest rate to a cost push shock in the period when the shock realizes, under various degrees of uncertainty and under various degrees of persistence of the shock itself. The values of the parameters are discussed in the Appendix. On the x-axis we report the persistence of the cost-push shock. On the y-axis instead we plot the response on impact to a cost push shock under the assumption of imperfect information on the parameter of the slope of the Phillips curve in deviation from the perfect information case, for various degrees of uncertainty (ie. for different value of σ_k). For values of φ^u below φ^{u*}(which under our parameterization is equal to 0.7) the response of monetary policy under imperfect information is more cautious than under perfect information. Moreover the higher the uncertainty, the more cautious the response. The opposite happens when φ^u is higher than φ^{u*}: the monetary policy response is stronger than under perfect information and is increasing as uncertainty increases. Finally, notice that the threshold does not depend on the degree of uncertainty. That is, independently from the degree of uncertainty, when the persistence of the cost-push shock is below φ^{u*}, the monetary policy should be more cautious than in the perfect information case; similarly when the persistence is above that threshold,

Figure 1: Difference between the on impact interest rate reaction under imperfect and perfect information to a cost-push shock



Note: in this figure the interest rate response at $t = 0$ to a cost push shock is plotted as a function of the cost-push shock persistence, in deviation from the perfect information response.

the monetary policy should be more cautious.

In the next section we further analyze the welfare implications of optimal discretionary policy when the Central bank is uncertain about the slope of the Phillips curve.

3.1 Welfare analysis

In Proposition 2 we have shown that, when the central bank is uncertain about the slope of the Phillips curve, realized inflation and output gap will deviate from those obtained under perfect information.

In order to characterize the intertemporal welfare losses associated with those deviations we define, for a given level of uncertainty on the parameter k, σ_k^2, and persistence of the cost-push shock, φ^u, the loss under the optimal discretionary policy when the central bank has imperfect information as

$$L_0^{II}\left(\sigma_k^2,\varphi^u\right)=\widehat{E}_0\sum_{t=0}^{\infty}\beta^t L\left[\pi_t\left(i^{II};\sigma_k^2,\varphi^u\right),x_t\left(i^{II};\sigma_k^2,\varphi^u\right)\right]$$

where $L\left[\pi_t\left(i^{II};\sigma_k^2,\varphi^u\right),x_t\left(i^{II};\sigma_k^2,\varphi^u\right)\right]$ is the period t loss function and $\pi_t\left(i^{II};\sigma_k^2,\varphi^u\right)$ and $x_t\left(i^{II};\sigma_k^2,\varphi^u\right)$ are the contingent plans for inflation and output gap under the optimal discretionary policy for a given combination of uncertainty and persistence. Similarly the loss under the optimal discretionary policy and perfect information (i.e., $\sigma_k^2=0$) is defined as

$$L_0^{PI}\left(\varphi^u\right)=\widehat{E}_0\sum_{t=0}^{\infty}\beta^t L\left[\pi_t\left(i^{PI};0,\varphi^u\right),x_t\left(i^{PI};0,\varphi^u\right)\right].$$

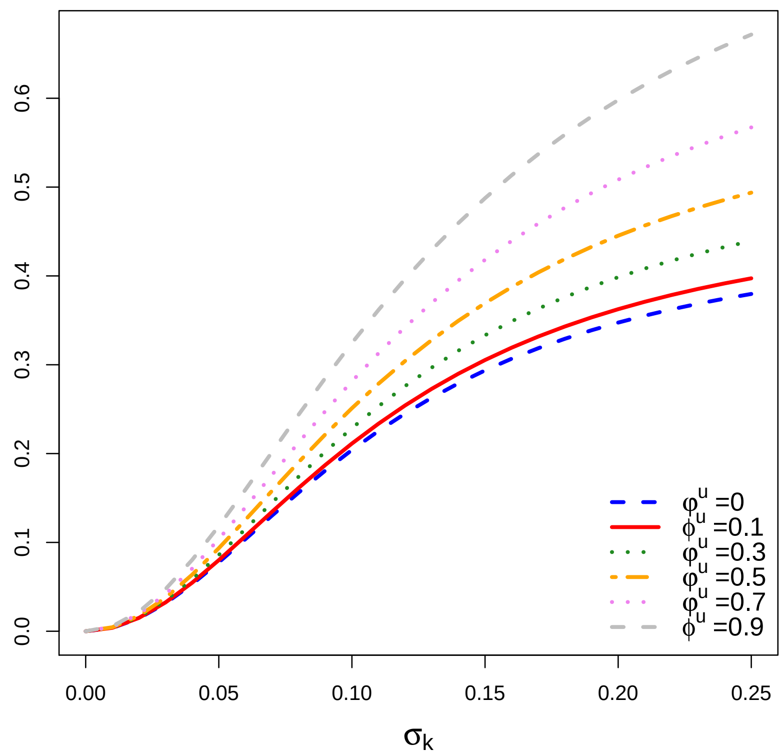
We compare the two welfare losses by computing the fraction of inflation under imperfect information that a central bank is willing to accept above $\pi_t\left(i^{II};\sigma_k^2,\varphi^u\right)$ to be as well off under the optimal discretionary policy with perfect information as under the one with imperfect information. Formally we define the "inflation equivalent", $\gamma^{PI}\left(\sigma_k^2,\varphi^u\right)$, for a given combination of uncertainty and persistence, as

$$L_0^{PI}\left(\varphi^u\right)=\widehat{E}_0\sum_{t=0}^{\infty}\beta^t L\left[\left(1+\gamma^{PI}\left(\sigma_k^2,\varphi^u\right)\right)\pi_t\left(i^{II};\sigma_k^2,\varphi^u\right),x_t\left(i^{II};\sigma_k^2,\varphi^u\right)\right].$$

Figure 2 shows under the parameter reported in Table 1 in the Appendix that the inflation-equivalent measure of welfare loss under imperfect information increases with respect to uncertainty over k.

In the region where σ_k is rather low, the increase in welfare loss is relatively small as

Figure 2: Welfare loss (Inflation equivalent) from uncertainty about the slope of the Phillips curve



persistence of the cost-push shock increases. However, for high levels of uncertainty about k, the increase in the welfare loss drastically increases as persistence increases.

4 Conclusions

In this paper we have considered a very simple and stylized New Keynesian model where the central bank has imperfect information about model parameters. We have developed a procedure to analytically characterize the solution of the planner's problem and the dynamics of the economy. We have shown that such a model is able to provide insightful implications for monetary policy.

The paper provides theoretical support to the view that a pragmatic approach to monetary policy, possibly combining gradualism and conditionality on data, should be pursued in a changing economic environment where the central bank is uncertain about the true shape of macroeconomic relationships and the transmission mechanism.

During periods of high uncertainty but low persistence of shocks, a cautious and gradual attitude of monetary policy allows to contain the volatility of inflation and economic growth, with little risk about the future. On the contrary, an aggressive response is justified when shocks are persistent enough to move agents' expectations away from the central bank's target for a protracted period of time. In such case, it pays for the central bank to accept a higher degree of current macroeconomic volatility in favor of smaller future deviations of macroeconomic variables from the target.

References

ANDĚL, J. (1976): "Autoregressive series with random parameters," *Statistics: A Journal of Theoretical and Applied Statistics*, 7, 735–741.

AOKI, K. (2003): "On the optimal monetary policy response to noisy indicators," *Journal of monetary economics*, 50, 501–523.

BLINDER, A. S. (1999): *Central banking in theory and practice*, Mit press.

BRAINARD, W. C. (1967): "Uncertainty and the Effectiveness of Policy," *The American Economic Review*, 57, 411–425.

CLARIDA, R., J. GALI, AND M. GERTLER (1999): "The science of monetary policy: a new Keynesian perspective," *Journal of economic literature*, 37, 1661–1707.

GALÍ, J. (2008): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press.

GIANNONI, M. P. (2002): "Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model," *Macroeconomic Dynamics*, 6, 111–144.

KIMURA, T. AND T. KUROZUMI (2007): "Optimal monetary policy in a micro-founded model with parameter uncertainty," *Journal of Economic Dynamics and Control*, 31, 399–431.

LORENZONI, G. (2010): "Optimal monetary policy with uncertain fundamentals and dispersed information," *The Review of Economic Studies*, 77, 305–338.

NICHOLLS, D. F. AND B. G. QUINN (1982): *Random Coefficient Autoregressive Models: An Introduction*, vol. 11, Springer Science & Business Media.

ORPHANIDES, A. (2003): "Monetary policy evaluation with noisy information," *Journal of Monetary economics*, 50, 605–631.

SÖDERSTRÖM, U. (2002): "Monetary policy with uncertain parameters," *The Scandinavian Journal of Economics*, 104, 125–145.

SVENSSON, L. E. AND M. WOODFORD (2004): "Indicator variables for optimal policy under asymmetric information," *Journal of Economic Dynamics and Control*, 28, 661–690.

WALSH, C. E. (2004): "Implications of a Changing Economic Structure for the Strategy of Monetary Policy," Santa cruz center for international economics, working paper series, Center for International Economics, UC Santa Cruz.

WOODFORD, M. (2003): *Interest and prices: Foundations of a theory of monetary policy*, Princeton University Press.

Appendix - Parametrization of the model

To perform policy simulations and welfare comparisons, we need to assign values to the parameters of the model. For the sake of comparison, most of the parameters have a mapping in the calibration of the New-Keynesian model of Galí (2008). The discount factor β is set at 0.99 and the coefficient of risk aversion, σ at 1. The parameter ψ in the IS curve is also equal to 1 and it implies that the inverse of the Frisch elasticity of labor supply is equal to 1. Also, we assume that λ is equal to 0.023, a value in line with the one used in Galí (2008).[11]. All other parameters are estimated on US data. More precisely, we collect quarterly data over the sample 1981Q2-2017Q4 on CPI inflation and output gap (source: FRED). To estimate the Phillips curve we also need data on inflation expectations: we take the median of the 5y CPI inflation expectations from the Philadelphia Fed Survey of Professional Forecasters. Lastly, for TFP we rely on the most recently available vintage produced by the San Francisco Fed, filtering the series using an HP filter with smoothing parameter set at 1600. We estimate parameters related to the TFP process and to the Phillips curve separately, by running single equation regressions.

We start considering parameters related to the TFP process. In the data we observe a^T, ie. the time series of the TFP and we need to estimate $\varphi^a, \sigma_a, \sigma_{\phi a}$. We can immediately recover φ^a, ie. the mean of the autoregressive parameter φ_t^a, since

$$\rho_1(a) \equiv \frac{Cov(a_t, a_{t-1})}{Var(a)} = \varphi^a.$$

We take $\sigma_{\phi a}$ to be the standard error of the above estimation of φ^a. Then σ_a is just the standard deviation of the residuals of the regression.

[11] Such value is obtained by dividing k, as estimated in the data (see further), by the hypothetical elasticity of substitution in the goods' market - assumed to be equal to 6.

To estimate uncertainty in the Phillips curve, instead, we rewrite it as follows:

$$\pi_t - \beta E_t \pi_{t+1} = \kappa x_t + \eta_t$$

where η_t is the residual of the regression. The above specification is directly amenable to the data since we observe inflation expectations. We can then construct the following variable: $\Delta\pi_{t,t+1} = \pi_t - 0.99 E_t \pi_{t+1}$ and regress it over the output gap. To recover σ_k we make use of the standard error of the regression. Also, we estimate the autoregressive component of the residuals of the regression and their variance. Interestingly, the estimated value for k is 0.138, a value very close to the one implied by the parameterization of Galí (2008), equal to 0.1275. The values of the parameters are reported in the Table 1.

Parameter	Value	Parameter	Value
Calibrated parameters			
β	0.99	ψ	1
σ	1	λ	0.023
Estimated parameters			
Phillips curve			
k	0.138	σ_k	0.048
ρ_u	0.63	σ_u	0.002
TFP process			
φ^a	0.74	$\sigma_{\phi a}$	0.059
σ_a	0.007		

Table 1: Calibrated and estimated parameter values