Benefits of Gradualism or Costs of Inaction Optimal monetary policy with limited central bank information.

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Central bankers better be cautious!

...so long as persistence of shocks isn't extreme.

Motivation

- Optimal monetary policy crucially depends on model parameters. What if:
    - Parameters are uncertain and (possibly) time-varying;
    - Central bankers have limited information (compared to that of agents).
- Certainty equivalence no longer holds (when randomness does not enter additively).
- Research question:
    - How should optimal policy guard against parameter uncertainty?
    - More cautious / more aggressive / equivalent (to certainty case)?
- Addressing central questions in the current policy debate:
    - Low interest rate environment: has the natural rate decreased permanently?
    - Missing inflation: has Phillips curve flattened or slack been mismeasured?

Literature

- Brainard (1967) - uncertainty about impact of policy $\rightarrow$ **cautious**:
  - Respond less than would be optimal if all parameters were known with certainty;
  - "Brainard conservatism principle" (Blinder);
  - Widely accepted for long, recently challenged.
- Backward-looking models:
  - Craine (1979) - (one-equation) persistence of π $\rightarrow$ **aggressive**;
  - Soderstrom (2002) - (two-equation) persistence of π $\rightarrow$ **aggressive**.
- Forward-looking models (two equation):
  - Kimura-Kurozumi (2007) - (commit) persistence π, x $\rightarrow$ **aggressive**;
  - Moessner (2005) - (discretion) persistence π $\rightarrow$ **aggressive**;
  - Walsh (2003) - (discr) avers to interest rate changes $\rightarrow$ **aggressive**.
- Indicator variables for optimal policy
  - Svensson-Woodford (2003)

Contribution

"*Brainard result was never far from my mind when I occupied the Vice Chairman's office at the Federal Reserve [...] Still, I find these new anti-Brainard results both puzzling and troubling. Though my confidence in the conclusion has been shaken by recent research, my gut still tells me that Brainard was right in practice.*" (Alan Blinder)

- We give theoretical support to Blinder's intuition.
- Optimal monetary policy in a forward-looking (2-eq) New-Keynesian framework, augmented with:
  - Limited information: uncertainty on subset of parameters (stochastic processes);
  - Asymmetric information: planner observes realization of parameter uncertainty with lag (end of period).
  - Agents: observe today's parameters, optimize, prior on random process of parameters from tomorrow.
- Bayesian decision theory problem: minimize "expected" (int. parameter uncertainty) planner's loss function.

Contribution II

- No shortcuts / behavioral assumptions:
  - Fully analitically characterize the underlying rational expectations equilibrium;
  - Essential to characterize the planner's problem;
  - Novelty in the literature.
- General setup to analyze uncertainty of any subset of parameters, so far specialized to:
  - Persistence of technology shock (natural rate): ϕ^a;
  - Slope of the Phillips Curve: κ.
- We find that:
  - Uncertain ϕ^a implies **no change** in optimal policy relative to the full info case (certainty equivalence);
  - Uncertain κ implies **more cautious** response when persistence of cost-push shock ϕ^u is not very high;
  - Uncertain κ implies **more aggressive** response when persistence of cost-push shock ϕ^u is high.

Standard optimal monetary policy problem.

New-keynesian framework without commitment.

Planner's problem:

$$\underset{\{x_t,\pi_t,i_t\}_{t=0}^{\infty}}{Min} E_0 \left\{ \sum_{t=0}^{\infty} [\beta/2]^t \left[\pi_t^2 + \lambda x_t^2\right] \right\}$$

s.t.: chosen policy path $\{i\}_t$ induces new-keynesian equilibrium:

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left[i_t - E_t \pi_{t+1} - r_t^n\right] \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \end{aligned}$$

where r_t^n is the natural interest rate:

$$r_t^n = \rho + \gamma\psi\left[E_t a_{t+1} - a_t\right];$$

x_t dev's from s.s.output; π_t dev's from s.s.inflation; i_t nom.interest rate;

Standard optimal monetary policy problem II

a_t and u_t are exogenous processes for tech. progress and cost-push shocks that evolve according to:

$$\begin{aligned} a_t &= \phi^a a_{t-1} + \varepsilon_t^a \\ u_t &= \phi^u u_{t-1} + \varepsilon_t^u \end{aligned}$$

with $\{\varepsilon_t^a, \varepsilon_t^u\}$ i.i.d. random shocks with mean $\{0, 0\}$ and s.d. $\{\sigma^a, \sigma^u\}$. Substituting, the new-keynesian equilibrium that constrains the planner's problem is:

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left[i_t - E_t \pi_{t+1} - \rho + \gamma\psi\left[1 - \phi^a\right] a_t\right] \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \end{aligned}$$

First Order Conditions

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left[i_t - E_t \pi_{t+1} - \rho - \gamma\psi\left(\phi^a - 1\right) a_t\right] \\ \pi_t &= \beta E_t \pi_{t+1} + k x_t + u_t \\ \lambda x_t &= -k\pi_t \end{aligned}$$

$\{i\}_t$ maintains proportionality of x_t and π_t, offseting the effects of a_t. Solution (dynamic system driven by exogenous dynamics of u_t and a_t):

$$\begin{aligned} x_t &= -\frac{k}{\lambda + k^2 - \lambda\beta\phi^u} u_t \\ \pi_t &= \frac{\lambda}{\lambda + k^2 - \lambda\beta\phi^u} u_t \\ i_t &= \rho + \frac{\left(1 - \phi^u\right) k\sigma + \lambda\phi^u}{\lambda + k^2 - \lambda\beta\phi^u} u_t - \sigma\psi\left(1 - \phi^a\right) a_t \end{aligned}$$

Standard optimal monetary policy problem IV

- Problem is indexed by the set of (constant) parameters

$$\Theta = \{\rho, \gamma, \lambda, \kappa, \psi, \phi^a, \phi^u, \sigma^a, \sigma^u\}$$

  - ρ rate of time preference and $\beta = [1+\rho]^{-1}$ the discount factor;
  - γ coefficient of RRA;
  - λ weight of the output gap relative to that of inflation in the objective function (a convolution of deep paramenters);
  - κ slope of the NKPC (a convolution of deep paramenters);
  - ψ inv. elesticity of labour supply;
  - $\{\sigma^a, \sigma^u\}$ vector of standard deviations of the random shocks;
  - $\{\phi^a, \phi^u\}$ vector of autoregressive parameters laying in the interval $[-1, 1]$.
- For given parameters Θ, dynamics driven by realizations of exogenous stochastic process of shocks $\{\varepsilon_t\} = \{\varepsilon_t^a, \varepsilon_t^u\}$.
- Let $z_t = \{a_t, u_t\}$ be min. set of state variables, then solution is:

$$\{x_t, \pi_t, i_t\} = \{x(z_t), \pi(z_t), i(z_t)\}$$

Optimal monetary policy with asymmetric information.

- Relax the assumptions of the standard problem:
  - Incomplete info: subset $\theta \in \Theta$ is random - stochastic process $\{\theta_t\}$;
  - Asymmetric info: θ_t observed by agents (beginning of period t) before planning, not by planner (end of period t).
- Dynamics driven by realizations of indep. exogenous stochastic processes: shocks $\{\varepsilon_t\}$ and disturbances $\{\theta_t\}$.
- Planner cannot condition expectations on θ_t but only on coarser information set.
  - E_t: expectation operator of agents;
  - $\hat{E}_t$: expectation operator of planner;
  - connected by identity:

$$E_t[\cdot] = \hat{E}_t[\cdot | \theta_t]$$

Optimal monetary policy with asymmetric information.

Planner's problem:

$$\underset{\{x_t,\pi_t,i_t\}_{t=0}^{\infty}}{Min} \hat{E}_0 \left\{ \sum_{t=0}^{\infty} [\beta/2]^t \left[\pi_t^2 + \lambda x_t^2\right] \right\}$$

s.t.: chosen policy path $\{i\}_t$ induces new-keynesian equilibrium:

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left[i_t - E_t \pi_{t+1} - \rho + \gamma\psi\left[1-\phi^a\right] a_t\right] \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \end{aligned}$$

Solution must now have the form:

$$\begin{aligned} \{x_t, \pi_t, i_t\} &= \{x(z_t,\theta_t), \pi(z_t,\theta_t), i(z_t,\theta_t)\} \\ \text{rather than} &: \quad \{x(z_t), \pi(z_t), i(z_t)\} \end{aligned}$$

with additional informational constraints that:

$$\begin{aligned} E_t[\cdot] &= \hat{E}_t[\cdot|\,\theta_t] \\ i\left(z_t,\theta_t^1\right) &= i\left(z_t,\theta_t^2\right) \quad \text{for any arbitrary } \theta_t^1, \theta_t^2 \text{ and any } z_t \end{aligned}$$

- When κ_t is random, the first three FOCs become:
  - using IS and NKPC to substitute for x_t and π_t inside expectations,
  - using the definition of variables $\{\hat{x}_t, \hat{\pi}_t, \hat{\imath}_t\}$,
  - using the LIE (that $\hat{E}_t E_t = \hat{E}_t$ and $\hat{E}_t = \hat{E}_t \hat{E}_{t+1}$):

$$\begin{aligned} \hat{x}_t &= \hat{E}_t \hat{x}_{t+1} - \frac{1}{\gamma}\left[\hat{\imath}_t - \hat{E}_t \hat{\pi}_{t+1} - \rho - \gamma\psi\left[\phi_a - 1\right] a_t\right] \\ \hat{\pi}_t &= \beta \hat{E}_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + \widehat{cov}_t\left(\kappa_t, x_t\right) + \hat{u}_t \end{aligned}$$

$$\left(\lambda + \sigma_\kappa^2\right)\hat{x}_t + \kappa\hat{\pi}_t - \kappa\widehat{cov}_t\left(\kappa_t, x_t\right) + \beta\widehat{cov}_t\left[\kappa_t, E_t\pi_{t+1}\right] + \widehat{cov}_t\left(\kappa_t^2, x_t\right) = 0 \qquad (1)$$

Optimal monetary policy with asymmetric information.

Solution method

- Guess a functional form for all covariances (constant equal zero)
- Then, system (1) is self contained and can be solved for $\{\hat{x}_t, \hat{\pi}_t, \hat{\imath}_t\}$
- Substitute $i_t = \hat{\imath}_t$ everywhere, leaving with two equations (IS and NKPC)
- Solve system IS-NKPC for $\{x_t, \pi_t\}$ (need another layer of guesses)
- Replace solutions back into covariances and verify guess.

$$
\begin{aligned}
\hat{x}_t &= -\frac{k}{\lambda + k^2 - \beta\lambda\phi^u + (1-\beta\phi^u)\,\sigma_k^2} u_t \\
\hat{\pi}_t &= \frac{\sigma_k^2 + \lambda}{\lambda + k^2 - \beta\lambda\phi^u + (1-\beta\phi^u)\,\sigma_k^2} u_t \\
\hat{\imath}_t &= i_t \\
x_t &= \frac{-k}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\phi^u - \beta\phi^u\sigma_k^2} u_t - \psi\left(\varphi_a - \phi_t^a\right) a_t \\
\pi_t &= \frac{k^2 - k_t k + \sigma_k^2 + \lambda}{\lambda + \sigma_k^2 + k^2 - \beta\lambda\phi^u - \beta\phi^u\sigma_k^2} u_t - \psi k_t\left(\varphi_a - \phi_t^a\right) a_t \\
i_t &= \rho + \frac{\lambda\phi^u + k\sigma\left(1-\phi^u\right) + \phi^u\sigma_k^2}{\lambda\left(1-\beta\phi^u\right) + k^2 + \left(1-\beta\phi^u\right)\sigma_k^2} u_t - \sigma\psi\left(1-\varphi_a\right) a_t
\end{aligned}
$$

Standard optimal monetary policy problem III

Complete information comparison.

$$
\begin{aligned}
x_t &= -\frac{k}{\lambda + k^2 - \lambda\beta\phi^u} u_t \\
\pi_t &= \frac{\lambda}{\lambda + k^2 - \lambda\beta\phi^u} u_t \\
i_t &= \rho + \frac{(1-\phi^u)\, k\sigma + \lambda\phi^u}{\lambda + k^2 - \lambda\beta\phi^u} u_t - \sigma\psi\,(1-\phi^a)\, a_t
\end{aligned}
$$

$$
\begin{aligned}
i_t^p &= \rho + \frac{(1-\phi^u)\, k\sigma + \lambda\phi^u}{k^2 + (1-\lambda\beta)\,\phi^u} u_t - \sigma\psi\,(1-\phi^a)\, a_t \\
i_t &= \rho + \frac{k\sigma\,(1-\phi^u) + \lambda\phi^u + \phi^u\sigma_k^2}{k^2 + (1-\lambda\beta)\,\phi^u + (1-\beta\phi^u)\,\sigma_k^2} u_t - \sigma\psi\,(1-\varphi_a)\, a_t \\
\hat{\imath}_t &= i_t
\end{aligned}
$$

As σ_k^2 increases, the reaction of monetary policy to a cost push shock u_t varies.

Whether it becomes more aggressive or cautious, depends on the magnitude of ϕ^u

Let's compare perfect and imperfect information cases

$$
\begin{aligned}
x_t^p &= \frac{-k}{\lambda + k^2 - \beta\lambda\phi^u} u_t \\
x_t &= \frac{-k}{\lambda + k^2 - \beta\lambda\phi^u \left(1 - \beta\phi^u\right) \sigma_k^2} u_t - \psi \left(\varphi_a - \phi_t^a\right) a_t \\
\hat{x}_t &= \frac{-k}{\lambda + k^2 - \beta\lambda\phi^u + \left(1 - \beta\phi^u\right) \sigma_k^2} u_t
\end{aligned}
$$

$$
\begin{aligned}
\pi_t^p &= \frac{\lambda}{\lambda + k^2 - \lambda\beta\phi^u} u_t \\
\hat{\pi}_t &= \frac{\sigma_k^2 + \lambda}{\lambda + k^2 - \beta\lambda\phi^u + (1 - \beta\phi^u)\,\sigma_k^2} u_t \\
\pi_t &= \frac{k^2 - k_t k + \sigma_k^2 + \lambda}{\lambda + k^2 - \beta\lambda\phi^u + (1 - \beta\phi^u)\,\sigma_k^2} u_t - \psi k_t \left(\varphi_a - \phi_t^a\right) a_t
\end{aligned}
$$

Parameterization

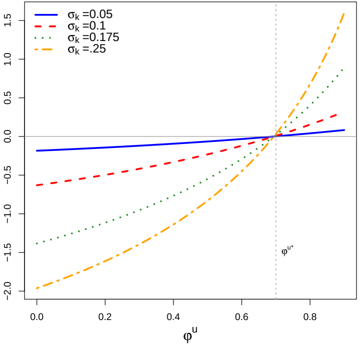
Parameter	Value	Parameter	Value
Calibrated parameters			
β	0.99	ψ	1
σ	1	λ	0.023
Estimated parameters			
Phillips curve			
k	0.138	σ_k	0.048
ρ_u	0.63	σ_u	0.002
TFP process			
φ_a	0.74	$\sigma_{\phi a}$	0.059
σ_a	0.007		

- US data (1981Q2-2017Q4) using SPF inflation expectations

Optimal monetary policy with asymmetric information.

Aggressiveness of monetary policy as a function of ϕ^u

Figure: Difference between the on impact interest rate reaction under imperfect and perfect information to a cost-push shock



- $\varphi^u < \varphi^{u*}$: MP under imperfect information is more cautious
- $\varphi^u > \varphi^{u*}$: MP under imperfect information is more aggressive
- threshold does not depend on the degree of uncertainty

Welfare analysis

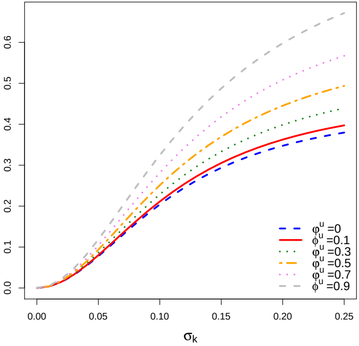
- Compute the inflation-equivalent loss under imperfect information:
- "how much less inflation is needed under imperfect information to equate the welfare loss under perfect information?

$$\mathcal{W}^{pi} = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(1-\gamma)^2 \left(\pi_t^{ii}\right)^2 + \alpha_x (x_t^{ii})^2 \right] \right]$$

- Investigate behavior of inflation-equivalent loss under various degrees of uncertainty wrt to ϕ_a and k...
- ...for different values of ϕ_u

Welfare analysis

- Inflation-equivalent measure of welfare loss under imperfect information with increasing uncertainty over k



- for very low values of σ_k, welfare is comparable with the perfect information case
- the loss drastically increases with high uncertainty over k...
- the more so with a more persistent cost push shock

Conclusions

- very simple and stylized NK model where the central bank has imperfect information about parameters
- analytical characterization of the solution
- theoretical support to a pragmatic approach to monetary policy
- cautious and gradual attitude of monetary policy during periods of high uncertainty but low persistence of shocks...
- ...an aggressive response is justified when shocks are persistent