Confidence Cycles and Liquidity Hoarding

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Abstract

Market confidence has proved to be an important factor during past crises. In this paper, I incorporate a model of the interbank market into a DSGE model, with the interbank market rate and the volume of lending depending on market confidence. I conduct an exercise to mimic some central bank policies: liquidity provision and reduction of the reserve rate. My results indicate that policy actions have a limited effect on the supply of credit if they fail to influence agents' expectations. A policy of a low reserve rate worsens recessions due to its negative impact on banks' revenues.

JEL Codes: E22, E32, G01, G18.

Keywords: Bounded Rationality, DSGE, Financial Sector, Liquidity Provision.

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1. Introduction

The financial agents expectations of economic outlook affect the functioning of financial markets and can propagate shocks to the real economy or become a source of shocks themselves. These expectations are not necessarily perfect. Agents can have limited information or a limited ability to process it. Studies[1] have shown that the expectations of professional forecasters demonstrate inertia, and it takes time for them to learn when changes occur. Therefore, after crisis episodes, agents can have pessimistic forecasts. Imperfect information and/or overly pessimistic expectations influence the efficiency of policy actions aimed at mitigating the recession and can undermine their effect or lead to unintended consequences. During the recent financial turmoil in 2007-2009, European and U.S. banks were reporting tightening of their lending standards largely due to poor expectations of economic activity.[2] While the central banks' were conducting unconventional policy measures, such that liquidity provision and low interest rates among others, to stimulate bank lending, the significant share of liquidity provided by central banks was hoarded in banks' reserves. Banks' pessimism about economic outlook could be one of the factors to affect transmission of unconventional monetary policies and to explain liquidity hoarding.

To address the role of pessimistic expectations, I build a model with heterogeneous banks that learn about economic conditions. In my model, banks lend to the real economy and to each other depending on their heterogeneous return expectations. A decline in return expectations increases their evaluation of counterparty risk on the interbank market. When lenders expect a low return on a risky asset, they assign a high probability to the scenario of their borrowers not being able to honor the debt. These expectations can drive the interbank market rate to a level where no bank is willing to borrow. Without access to the interbank market, the most optimistic banks reduce their lending to the real economy and pessimistic banks just hoard their funds, i.e., keep them in reserves.

The model becomes a tractable extension to the work-horse dynamic stochastic general equilibrium (DSGE) model, as only the moments of the banks' beliefs distribution matter in equilibrium. Within the developed framework, I consider the question of the efficiency of the policy measures applied during the economic downturn. My model allows us to account for the hoarding behavior by banks observed during the crisis, which is often missing from DSGE models analyzing unconventional central bank policy.[3] I consider several types of central bank policy actions that resemble those taken during the crisis and the subsequent recession, including liquidity provision to all banks at a fixed rate and targeted liquidity provision to support lending to the real sector. I also consider the policies of reducing the rate on reserves and relaxing collateral constraints on the interbank market.[4]

My findings suggest that investors' expectations and their uncertainty instigate large swings in the real economy, where manufacturers are dependent on credit. In my model, when banks are concerned about economic prospects the liquidity provision policy dampens the magnitude of the crisis but neither stops nor shortens it. Moreover, a significant share of funds received from the central bank is invested in safe assets instead of flowing into the real economy. This result is in line with the banks' observed behavior. This also suggests that making policy evaluations without accounting

[1] Examples being Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013).
[2] For example in 2009, according to the Bank Lending Survey by the European Central Bank, more than 70% of the banks reported that expectations about economic activity contributed to tightening. A similar figure was reported in the United States, Senior Loan Officer Opinion Survey on Bank Lending Practices by the Federal Reserve Board.
[3] For evidence on hoarding see Gale and Yorulmazer (2013) and Heider et al. (2015) and references therein.
[4] The policy of relaxing collateral constraints by the central banks involved widening the set of assets accepted as collateral by the central bank. In my model, it takes the form of banks being willing to lend up to a larger fraction of borrowers net worth.

for investors' sentiment and market volatility may overstate policy efficiency. Lowering policy rate makes hoarding less attractive, but reduces the banks' revenues, resulting in even worse outcomes than in the case of no central bank action.

This paper is related to several strands of literature. First, it is growing research on adaptive learning in DSGE models. Slobodyan and Wouters (2012), Rychalovska (2016), Milani (2011), and Aguilar and Vázquez (2018) among others show that models with imperfect information explain the data better than the models with rational expectations. Under adaptive learning, agents gradually learn the parameters of the model. Contrary to those studies, I employ, however, a somewhat less sophisticated learning, with the agents only learning about unobservalble component of the shock to returns. At the same time, my model features rich banking sector, making it possible to study how the banks' pessimism transmits into the economy. Second, there are papers on modeling the interbank market. Cui and Kaas (2017) study how expectations about credit conditions become self-fulfilling. Gale and Yorulmazer (2013), Heider et al. (2015), and Allen et al. (2009), consider liquidity hoarding through the interbank market structure. In these models the reason for banks to hoard liquidity is anticipation of a liquidity shock. My interbank market structure can be extended for a liquidity shock, but I focus the role of counterparty risk in breaking the interbank market. Bank heterogeneity in a DSGE model is introduced by Hilberg and Hollmayr (2011), who study liquidity provision and relaxation of collateral constraints. In Hilberg and Hollmayr (2011), bank heterogeneity is caused by exogenous separation into investment and commercial banks; only investment banks are allowed to borrow from the central bank. I consider a different interbank market structure consisting of a number of ex-ante identical banks who differ ex post depending on their subjective interpretation of public information.

There are is a literature on the role of the financial sector and credit in the economy. Studies have incorporated the banking sector into general equilibrium models. Examples include Gertler and Karadi (2011), Curdia and Woodford (2011), Negro et al. (2017), and Gertler and Kiyotaki (2010). Having introduced the financial sector, these papers address central banks' crisis-mitigation policies. While the first two papers consider the effect of policies on the transfer of credit between households and financial intermediaries, the latter two analyze credit supply to entrepreneurs subject to a liquidity constraint of the Kiyotaki and Moore (2008) type. My study also addresses the efficiency of central bank policy, but accounts for the role of investor sentiment.

The question could be if the banks during the crises were overly pessimistic or just rationally predicting the downturn? There is no obvious way to find hard proof of overly pessimistic or overly optimistic expectations. One possible proxy is the survey of professional forecasters. A number of papers studying the survey of professional forecasters, examples being Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013), have found sluggishness in forecasters' expectations. I interpret this finding as meaning that agents form their forecasts based on backward-looking data. It is not surprising, then, that after an episode of low returns or high risk, forecasters underestimate returns or overestimate risks in the next period. Therefore, it is not unrealistic to consider that after the crisis, when central banks started to implement unconventional measures, banks had overly pessimistic expectations.

This paper proceeds as follows. I first analyze a simple model of the interbank market to illustrate the role of market expectations in causing a credit crunch and consider policy actions by a central bank. Next, the general equilibrium model is completed. Within a DSGE model, I show the implications of market mood swings for the propagation of crises and policy efficiency when there is feedback from household decisions and market prices.

2. A Simple Model of Credit and the Interbank Market

In this section I describe the main mechanism of the model in a simplified setting. Later in the paper, the described sector is incorporated into a DSGE model to compare my results with the literature and to consider the general equilibrium effects of policy actions.

There are two time periods and two types of investment opportunities for banks: a storage asset pays R^{res} and a risky asset R^k in the next period. Decisions are made in period 1 and payoffs are realized in period 2. At $t = 1$ banks attract deposits d from the household. I simplify the problem by assuming that deposits are distributed equally among all banks and set $d = 1$. Banks pay R to depositors in the next period. The time subscripts are dropped. There is a continuum of banks normalized to 1 and indexed by i, each with different expectations about the risky asset return, $E^i\hat{R}^k$. In the general equilibrium context the risky asset is credit to the real sector, so in the simple model I sometimes refer to the risky asset position as credit. Banks can participate in the interbank market. If a bank chooses to borrow on the interbank market, I limit its borrowing to its share of liabilities. I call this share λ_b.[5] The interbank market rate, R^{ib}, is determined endogenously by clearing the market. Clearly, portfolio decisions depend on the bank's expectations about the risky asset return, the safe asset return, the state of the interbank market (functioning or not), and the interbank market rate. I show below that, given the safe asset rate, the distribution of banks' beliefs defines their decisions and interbank market conditions. Then I consider possible policy actions and show how they affect liquidity hoarding and credit. In this framework I consider the state of the interbank market and the amount of credit given the moments of the beliefs distribution – the average market belief and its standard deviation.

Lending on the interbank market is risky: there is a probability that due to low portfolio returns borrowers will not repay their loans. I assume that not repaying part of a loan and not paying the full amount are equally costly for borrowers and that the cost is exclusion from the interbank market. That is, the lender only considers the probability that the borrower's return is smaller than his liabilities and disregards the set of possible partial loan repayments. I abstract from the agency problem here for the sake of tractability, assuming that banks will honor their debt unless their returns do not allow it.[6]

Suppose for simplicity that beliefs are distributed uniformly among banks with mean m and variance σ^2.[7] I think of each banker as being a statistician making her best forecast conditional on the available information. Each bank's individual estimate, $E^i\hat{R}^k$, is then assumed to be distributed uniformly with the same variance σ^2. That is, each banker has her own prediction of the risky asset return, $E^i\hat{R}^k$, with variance σ^2, and these predictions are distributed uniformly among banks with mean $\bar{E}\hat{R}^k = m$ and the same variance σ^2. These assumptions are made for the sake of simplicity and more intuitive presentation of the results. Later in the paper I relax these simplifying assumptions and let banks have a model-consistent beliefs distribution.

[5] Risk-neutral borrowers are willing to borrow as much as possible and do not endogenize the effect of such a high demand for loans on interbank market rate. With very high demand for loans, interbank market collapses as interbank market rate spikes. To avoid this situation, I exogenously limit the borrowing. In the general equilibrium model, I calibrate λ_b to match interbank lending series.

[6] I also choose not to model limited liability and default decision in the utility function. In the general equilibrium framework presented in the next section, my banks are owned be housholds, who absorb banks' loses.

[7] The bounds of the uniform distribution a and b are then: $a = m - \sigma\sqrt{3}$ and $b = m + \sigma\sqrt{3}$. In this simplest model, a can be negative.

Every bank is risk neutral and optimizes its next-period return by maximizing the following function:

$$\max_{\alpha^i,h^i,L^i} \alpha^i E^i \hat{R}^k + h^i * R^{res} + \left(1-\alpha^i-h^i\right) p^i R^{ib} + (E^i\hat{R}^k - R^{ib}) * \Lambda^i \tag{1}$$

subject to a collateral constraint on the interbank market: $\Lambda^i = \lambda_b$ if a bank is a borrower or 0 otherwise. Bank i chooses the portfolio shares α^i (the share of the risky asset) and h^i (the share of hoarded or reserve assets), and the rest of the assets $\left(1-\alpha^i-h^i\right)$ are then lent on the interbank market. The return then consists of the expected return on the risky asset $\alpha^i E^i \hat{R}^k$, the return on the safe asset $h^i * R^{res}$, and the expected return on interbank lending $p^i\left(1-\alpha^i-h^i\right) R^{ib}$. The lenders are uncertain whether the borrowers will be able to repay their debt, hence they assign a loan repayment probability p^i. Those banks which are willing to borrow on the interbank market and invest in the risky asset get the expected return on borrowed funds: $(E^i\hat{R}^k - R^{ib}) * \Lambda^i$,[8] where Λ^i is the amount borrowed on the interbank market. Because every bank is risk neutral, the problem results in a corner solution.

Let us now consider the subjective loan repayment probability. The probability of a loan being repaid, p^i, is lender i's subjective probability that the borrower will repay the loan, in other words, that the borrower's return on the risky asset will be higher than her payments on the loan and other liabilities. Because of risk neutrality, all borrowers invest everything in the risky asset. By construction, all banks have an equal amount of deposits and also borrow the same amount on the interbank market: λ_b. That is, from the lender's perspective, each borrower has the same amount of assets and liabilities. And p^i is determined by the lender's belief about the risky asset return, $E^i\hat{R}^k$:

$$p^i = Prob\left((1+\lambda_b)\, E^i\hat{R}^k \geq R + \lambda_b R^{ib}\right) \tag{2}$$

In (2) the borrower's expected return is $(1+\lambda_b)\, E^i\hat{R}^k$, where 1 is the share of borrower's own funds and λ_b is the share borrowed on the interbank market. The liabilities are then $R*1$ to households and $\lambda_b R^{ib}$ to the interbank market lender. If the return is higher than the liabilities, a bank pays interbank market debt. The lender expects that the borrowers return the debt, that is: $(1+\lambda_b)E^l R^k_{t+1} > \lambda_b R^i b + R$. As I have assumed for this section that the banks' estimates of the risky asset return are uniformly distributed with variance σ^2, I can write p^i as a cumulative density function of uniform distribution:

$$p^i = 1 - U_{E^i\hat{R}^k,\sigma_v^2}\left(\frac{R+\lambda_b R^{ib}}{1+\lambda_b}\right) = \frac{1}{2} - \frac{\left(R+\lambda_b R^{ib}\right)}{2\sigma\sqrt{3}(1+\lambda_b)} + \frac{E^i\hat{R}^k}{2\sigma\sqrt{3}} \tag{3}$$

Equation (3) shows the connection between the individual probabilities and the interbank interest rate. Aggregating the interbank market, I show how these individual probabilities translate into the interbank market rate.

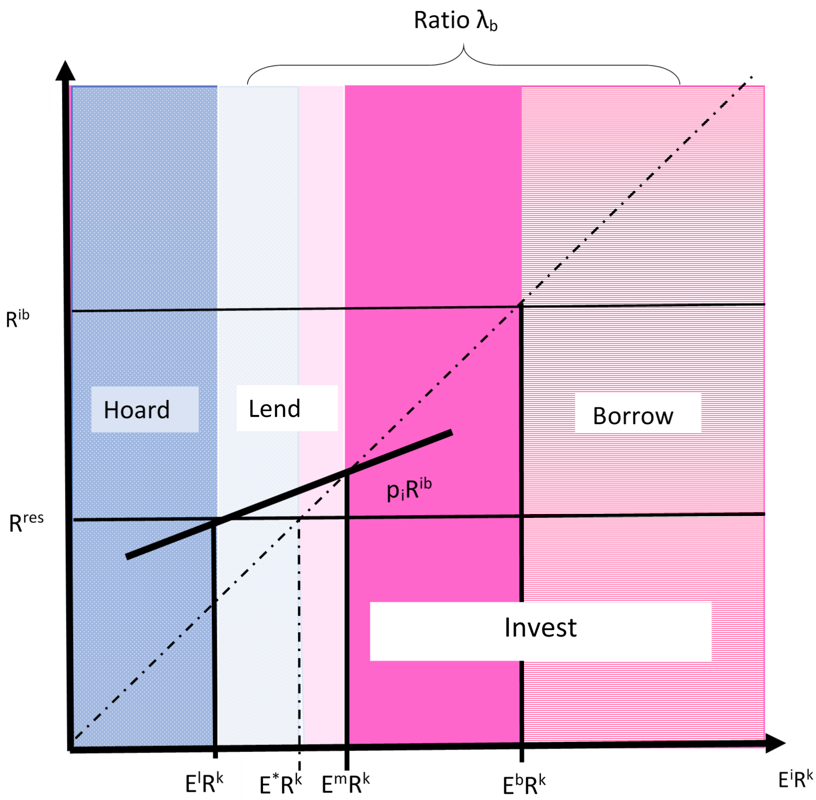
Depending on the beliefs, their mean and dispersion, the interbank market can have three different states: a functioning market, or a market where no one lends, or a market where no one borrows. When the interbank market functions, the marginal investor is indifferent between buying risky asset and lending on the interbank market. I denote her beliefs as $E^m\hat{R}^k = p^m R^{ib}$. A marginal lender is a banker who is indifferent between lending and hoarding with beliefs $E^l\hat{R}^k$ such that $p^l R^{ib} = R^{res}$.

[8] Banks only borrow on the interbank market to invest in the risky asset. Consider the case where a banker borrows and invests in the safe asset. This would mean that $R^{res} > R^{ib}$. In this case, no one would lend on the interbank market. Therefore, the interbank market only functions when $R^{res} < R^{ib}$.

The marginal borrower is indifferent between borrowing on the interbank market or not; and is defined as $E^b\hat{R}^k = R^{ib}$.

Banks' choices are illustrated in Figure 1. On the horizontal axis there are beliefs of individual

Figure 1: Banks' Expectations and Investment Decisions



Dotted blue area – hoarders, white transparent area – lenders, solid pink area – direct investors, red stripes ares – borrowers

banks, on the vertical - rates of return and beliefs. The expected return from the interbank lending, $p^i R^{ib}$, is plotted as a function of $E^i R^k$. When the beliefs are too low for an interbank market to exist, banker are divided into investors (pink and red stripes area) and hoarders (dotted blue area). Crossing of the 45 degree line and the safe asset rate determines the marginal investor when interbank market is not functioning, $E^* R^k$. Bankers to the right of $E^* R^k$ invest, to the left - hoard. When the beliefs rise, increasing probability of the repayment on the interbank market, some of the hoarders and investors become lenders (transparent white area). The marginal investor is then a banker indifferent between investing and lending - the intersection of 45 degree line and $p^i R^{ib}$ curve. Crossing of $p^i R^{ib}$ with safe asset return, gives the beliefs of the marginal lender. The marginal borrower is given by an intersection of the interbank rate and 45 degree line. The area to the right of the marginal borrower beliefs, $E^b R^k$, (red stripes area) shows the share of borrowers among bankers. The interbank market rate is endogenously determined by equalizing share of lenders (white transparent area) with share of borrowers (red stripes area) multiplied by λ_b.

Knowing the distribution of beliefs across banks, in the case of uniform distribution I can write the interbank market clearing condition as follows (a detailed derivation is given in Appendix A):

$$E^m\hat{R}^k - E^l\hat{R}^k = \lambda_b\left(\sigma\sqrt{3} + m - R^{ib}\right) = \lambda_b\left(\bar{R} - R^{ib}\right), \tag{4}$$

where the interbank market rate, R^{ib}, clears the market, and m and σ are the mean and the standard deviation of banks' beliefs distribution. The supply of loans is given simply by the difference between the belief of the marginal investor and that of the marginal lender: $E^m\hat{R}^k - E^l\hat{R}^k$. The demand for loans is the difference between the largest belief in the market, $\bar{R}$, and the belief of the marginal borrower. When the distribution is uniform, the largest belief can be written as the sum of the mean and the standard deviation $\sigma\sqrt{3} + m$. Each individual probability is also a function of the standard deviation.

The solution to the model is then given by the market clearing condition, (4), definition of repayment probability, (3), and the expressions for marginal investors in terms of the interbank market rate:

$$\begin{aligned} E^l\hat{R}^k &= \frac{R}{\lambda_b+1} + \frac{\lambda_b R^{ib}}{\lambda_b+1} - \frac{\sqrt{3}\sigma(R^{ib} - 2R^{res})}{R^{ib}} \\ E^m\hat{R}^k &= \frac{R^{ib}(\sqrt{3}\lambda_b R^{ib} - 3\lambda_b\sigma + \sqrt{3}R - 3\sigma)}{(\lambda_b+1)(\sqrt{3}R^{ib} - 6\sigma)} \end{aligned}$$

Combining these 4 equations gives an equation for the interbank market rate[9]:

$$a*(R^{ib})^3 + b*(R^{ib})^2 + c*(R^{ib}) + d = 0, \tag{5}$$

For the interbank market to function there must be interbank rate, R^{ib}, solving (5), that is real and positive. The necessary and sufficient conditions are summarized in the following proposition.

Proposition 2.0.1. *With $a > 0$, $b < 0$, and $d > 0$, if a positive root exists, it is unique. This positive root exists only if:*

$$R^{res} > A + \frac{R}{\lambda_b+1} < 0, \tag{6}$$

where [10] $A < 0$.

The sufficient condition for the equilibrium with the interbank market is that marginal lender belief satisfies:

$$E^l\hat{R}^k < p^l R^{ib} = R^{res},$$

which implies $\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$.

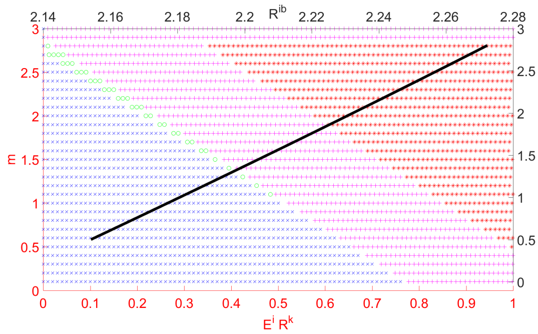
The mathematical proof is given in Appendix A.

Proposition 2.0.2. *Low market beliefs result in a lower interbank rate and lower lending.*

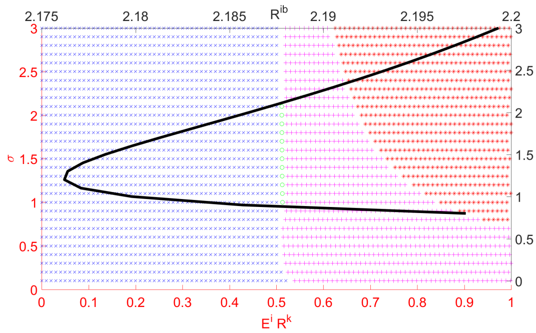
[9] $a = \sqrt{3}\lambda_b(1+\lambda_b)$, $b = -\lambda_b\left(\sqrt{3}(\lambda_b+1)m + 9\lambda_b\sigma + 3\sigma\right)$, $c = 6\sigma\left(\lambda_b\left(\lambda_b m + \sqrt{3}\lambda_b\sigma + m - R^{res}\right) + R - R^{res} - \sqrt{3}\sigma\right)$, and $d = 12\sqrt{3}(\lambda_b+1)R^{res}\sigma^2$

[10] $A = \frac{-3\left(3\lambda_b^3+7\lambda_b+6\right)\sigma^2 - \lambda_b(\lambda_b+1)^2m^2 + 4\sqrt{3}\lambda_b(\lambda_b+1)m\sigma}{6\sqrt{3}(\lambda_b+1)^2\sigma}$

Figure 2: Banks' Beliefs Distribution



(a)



(b)

Numerical simulations. From left to right: x area – hoarders, o area – lenders, + area – investors who do not borrow, * – investors who borrow, solid line – interbank rate (upper horizontal axis). The numerical values for the parameters are $R^{res} = R = 1.0101$, $\lambda_b = 0.1$, for the panel a, $\sigma = 0.98$, for the panel b, $m = 1$.

The mathematical proof is given in Appendix A.

Corollary 2.0.1. *With very low beliefs diversity, there is no lending on the interbank market. With very high beliefs diversity, lending is possible but small.*

Figure 2 summarizes possible scenarios of interbank market functioning, where panel a shows the impact of mean market beliefs on banks' equilibrium allocations and panel b shows the impact of the beliefs dispersion and the standard deviation of banks' forecasts. On the horizontal axis are the shares of bankers: hoarders, lenders, investors that do not borrow, and investors that borrow. The black line is the interbank market rate, measured on the upper horizontal axis. When the interbank market rate is not defined, the interbank market collapses: when the average beliefs and diversity are too small.

If the dispersion is fixed, a decrease in market beliefs about the risky asset return means a shift in the bounds of the beliefs distribution: the most pessimistic banker becomes even more pessimistic and the most optimistic banker becomes less optimistic. Borrowers (the '*' area) use interbank loans to invest in the risky asset. Intuitively, when borrowers expect a lower return on the risky asset they are willing to pay less for the interbank loan. With all bankers being less optimistic about the risky asset return, there is a larger share of those who do not invest themselves and expect a lower loan repayment probability: there is more hoarding (the 'x' area). Those bankers who are considering whether to lend on the interbank market or to invest in the risky asset evaluate these two options at a lower interbank rate. This makes interbank lending less attractive and the share of lenders (the 'o' area) also shrinks.

From proposition 2.0.1 follows that there is a lower bound on beliefs diversity $\sigma\sqrt{3} > (3 + 2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$. The role of the standard deviation is two-fold in the model. First, it measures the dispersion of beliefs among banks. Second, it reflects how each bank is uncertain about its own estimate of the future return. If a lender is almost certain about her low return expectation, she will assign a low loan repayment probability and ask for a high interbank market rate. At the same time, the borrowers are more convinced about their high return expectations and are willing to pay that high rate. Consequently, with a low standard deviation, there is either no lending, or very low lending at a high interbank rate. As the standard deviation increases, so does the uncertainty among the borrowers. They are willing to pay a lower interbank rate. When the uncertainty and dispersion are very large, there is still lending, but its volume is negligible (see Figure 2). If the diversity of beliefs is large, the bounds of the distribution widen and there are some very optimistic borrowers. This pushes the interbank market rate up.

Policy effects

A functioning interbank market

Let us now consider the impact of policy actions in the context of the simple model I have developed. First, suppose that the interbank market is functioning, but some superior agent, which I call the central bank, would like to increase interbank lending and/or stimulate credit to the real economy.

Proposition 2.0.3. *A low policy rate increases lending and lowers the interbank market rate, and increases the supply of credit to the real economy.*

The mathematical proof is given in the Appendix A.

Liquidity provision

In this simple framework without liquidity concerns, the provision of liquidity to banks, whether targeted or untargeted, does not affect the functioning of the interbank market. All the bankers would allocate all their available funds according to the decision rules discussed above. The provision of funds to optimists increases credit to the real economy, given that optimists exist. If the liquidity provision is untargeted and the funds are distributed equally among the banks, the pessimistic banks hoard it, as their main concern is counterparty risk and a low risky asset return. In this regard, targeting only optimistic banks can increase credit. Again, in the general equilibrium context, the feedback from prices and banks' balance sheets reverses the predictions: the untargeted policy results in better general outcomes than the targeted one.

To sum up, in the light of my model, if a central bank wants to increase lending on a market where banks are concerned about counterparty risks, the effect of a policy that does not address those concerns is limited.

Interbank market collapse

Let us now consider the case where the interbank market collapses due to low market expectations about the risky asset return. Obviously, providing banks with additional funds will not revive interbank lending, but, if provided to optimists, such funds could increase credit to the real economy. The size of this effect is conditional on the share of investors. Reduction of the safe interest rate makes safe asset relatively less attractive. But when the banks are pessemistic about risky asset return, the effect of such policy is limited. This is summarized by the next proposition.

Proposition 2.0.4. *The effect of a policy rate reduction is limited by the mean market belief.*

A formal proof is provided in the appendix. The only tool that might have a potential effect is **a reduction in the policy rate**. However, this policy has a very limited or zero effect if market beliefs are very low, which also means a very low interbank rate. In this case, even with a low reserve rate, hoarding is still more attractive than interbank lending.

To sum up, if banks are concerned about a low risky asset return and expect a low loan repayment probability, policy actions have a very limited effect. Liquidity provision policies enhance credit through optimistic bankers only, with the rest of the funds ending up in reserves. A low interest rate policy restores the market only if market beliefs are not very low, and stimulates credit to the real economy among banks expecting the risky asset to pay more than the storage asset.

3. Closing the General Equilibrium Model

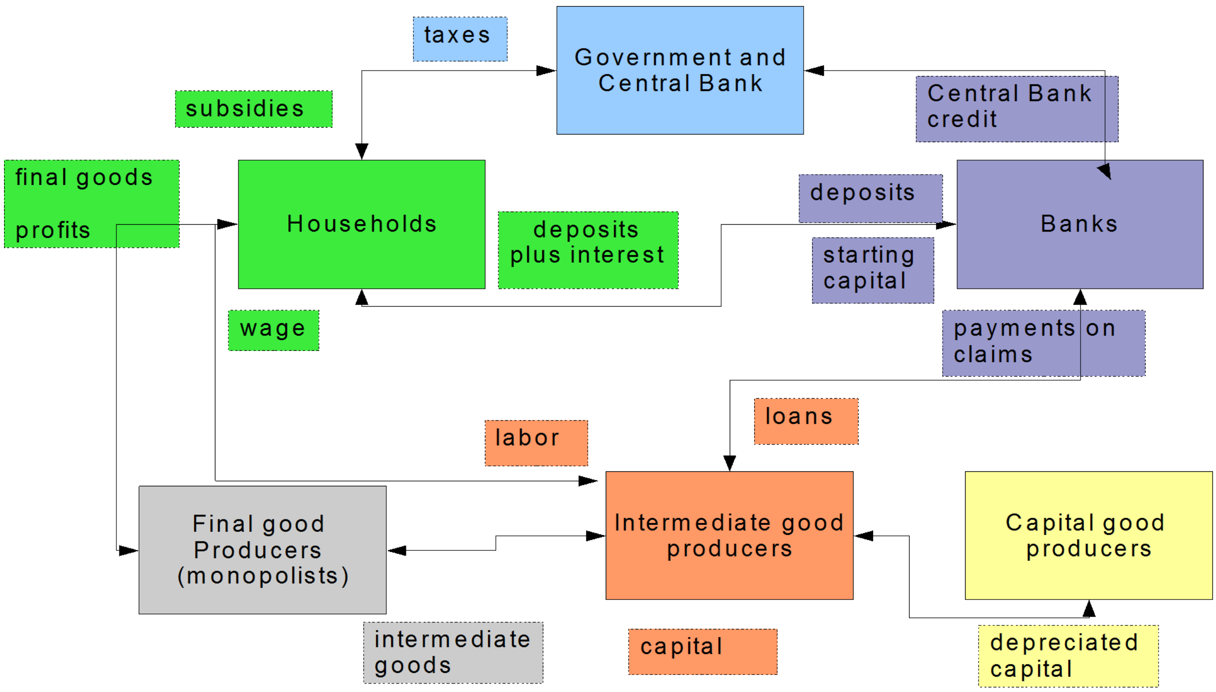
In this section I drop my simplifying assumptions about banks' beliefs. In particular, banks now form expectations based on past data on risky asset returns and private heterogenous signals about future returns. A bank's belief is then the result of the Kalman filter and follows a normal distribution.[11] I then input the banking sector developed above into a linearized DSGE model as in Gertler and Karadi (2011) as shown in figure 3. In their model, agents have perfect expectations about future risky asset returns. I modify it so that risky asset returns are uncertain. Another difference is that, in Gertler and Karadi (2011), banks frictionlessly transfer their liabilities to credit the real sector. In my model, I allow the banks to keep (hoard) liquidity if they choose to. Thus, it is possible to address the question of whether the liquidity provided by the central bank is transmitted to the real economy, or ends up in bank reserves. Last but not least, heterogeneous expectations give rise to an interbank market. In my model, the interbank market serves as a propagation mechanism, increasing or decreasing the credit supply as interbank market conditions change. I start with a description of the main building blocks of the model: the financial sector with heterogeneous beliefs and the interbank market. Then I proceed to complete the general equilibrium model and consider crisis and policy effects when there is feedback from the rest of the economy. The rest of the sectors are standard as in Smets and Wouters (2007) and Gertler and Karadi (2011), so I outline them only briefly. For a more rigorous discussion the reader is referred to these papers.

3.1 The Financial Sector

There is a continuum of banks normalized to one. Every period, a fraction $(1-\theta)$ of banks exit the sector and join the households. At the same time, the same number of household members become bankers and receive starting capital from the households. This starting capital equals a share of total banking sector assets. Banks receive deposits from the households, paying a gross real rate R_t. I model deposits as distributed equally among the banks regardless of their portfolio holdings. Banks allocate their funds between a safe asset paying gross real rate R_t^{res}, a risky asset with an uncertain gross real return, R_{t+1}^k, and interbank market lending with a gross real return R_t^{ib}. Banks are aware of the risk that some borrowers may not repay their debt. The debt repayment probability is reflected in the interbank market rate.[12] In order not to track the distribution of each banker's worth, I treat the bankers as members of one family, where each member maximizes his own return. At the beginning of a new period, before making investment decisions, they all average their net worth.

[11] Switching to normal distribution does not allow us to have analytical solution, but makes shock structure more intuitive and simplifies filtering problem.
[12] Note the timing of the interest. Although it is paid in period $t+1$, the rate on the safe asset and the interbank market rate are set in period t. For the rest of the model description I use the same convention to refer to the timing of the variables when they are decided upon.

Figure 3: Model Overview



The risky asset in the model is credit to the real sector. Banks buy the claims of non-financial firms, S_t, at price Q_t and return R^k_{t+1}. The non-financial firms are intermediate goods manufacturers that need funding to buy capital. They transfer the return on the capital as a payment on S_t. The uncertainty comes from a so-called capital quality shock, ξ_{t+1}, as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). As opposed to physical depreciation, δ_t, it is intended to capture unexplained fluctuations in the value of capital: it influences not only the capital stock $\delta_t \xi_t K_{t-1}$, but also its value $Q_t \xi_t K_{t-1}$. The value of undepreciated capital is then defined as the difference between the value of new capital and the value of depreciated capital:

$$(Q_{t+1} - \delta_{t+1})\, \xi_{t+1} K_t. \tag{7}$$

The return on capital consists of the value of the marginal product of capital, $\alpha \frac{P_{m,t+1} Y_{t+1}}{\xi_{t+1} K_t}$, plus the value of new capital, Q_{t+1}, minus depreciated capital, δ_{t+1}. The quality shock, ξ_{t+1}, then influences expectations of the return on capital:

$$R^k_{t+1} = \frac{\left(\alpha \frac{P_{m,t+1} Y_{t+1}}{\xi_{t+1} K_t} + Q_{t+1} - \delta_{t+1}\right) \xi_{t+1}}{Q_t} \tag{8}$$

Equations (7) and (8) are identical to those in Gertler and Karadi (2011). Now, though, I adjust the process for the quality shock. It is observable by all the sectors, but the composition of the shock is unobservable. With this process I intend to capture developments in capital value which are not predictable by the market. I assume that capital quality is subject to two types of shocks – persistent and transitory. The combination of these two shocks creates uncertainty in predicting future values of capital quality.

$$e^{\xi_t} = e^{\rho_\xi \xi_{t-1}} e^{\mu_t} e^{\varepsilon_{\xi,t}}, \tag{9}$$

μ_t is a persistent shock:

$$e^{\mu_t} = e^{\rho_\mu \mu_{t-1}} e^{v_t}, \tag{10}$$

where ρ_μ and ρ_ξ are persistence parameters, v_t and $\varepsilon_{\xi,t}$ are transitory Gaussian shocks, serially uncorrelated with zero contemporaneous correlation and variances σ_v^2 and σ_ε^2. Neither the intermediate goods producers nor the banks observe either μ_t or $\varepsilon_{\xi,t}$. Next I explain how the banks set their expectations about ξ_{t+1}.

3.1.1 Expectations Formation

Banks do not observe whether the change in ξ in (9) is due to a transitory or a persistent shock. They have access to past data on returns and they use it to form a homogeneous economic forecast. There are, however, private signals - expert adjustments - about the value of μ_t. The inclusion of expert forecast adjustments is motivated by an extensive literature that provides evidence of the widespread use of expert factors in forecasting practice.[13] I model expert adjustment as an additional signal about the value of μ_t:

$$e^{\theta_t^i} = e^{\rho_\theta \theta_{t-1}^i} e^{\eta_t^i}, \tag{11}$$

where η_t^i is the noise in the opinion of bank i's expert, with η_t^i being correlated draws from $N(\mu_t, \sigma_\eta)$.

The noise in expert opinions is correlated with correlation coefficient ρ^c.[14] This correlation can be interpreted in two ways. First, experts tend to react to similar news in a similar fashion – being overly optimistic or overly pessimistic. Second, even though formally they do not share their forecasts with each other, I retain the possibility of convergence of their opinions or coordination on an additional public signal. That is, when the correlation coefficient is one, experts' opinions are fully converged and are the same. Conversely, when ρ^c is zero, they are fully diverged. I assume that the correlation coefficient lies between zero and unity. Appendix B shows that such a correlation among expert errors shifts the average of the draws away from the distribution mean, so that the error in expert opinions is not averaged away. Banks have two sources of information – the past and current realizations of ξ_t and the expert opinion about μ_t. Banks use the Kalman filter to combine the two signals, with the weights of the signals in the final forecasts depending on their relative variance. A description of the Kalman filter setup is given in Appendix C.

3.1.2 The Interbank Market and Banks' Problem

The banks' problem is very similar to the one in the simplified model (1). At time t, banks choose their portfolio allocation: invest a share in the risky asset, α_t^i, leave a share in the reserves (or hoard), h_t, lend on the interbank market, $\left(1 - \alpha_t^i - h_t^i\right)$, or borrow on the interbank market, Λ_t^i:

$$\max_{\alpha_t^i, h_t^i, L_t^i} \alpha_t^i E_t^i \hat{R}_{t+1}^k + h_t^i R_t^{res} + \left(1 - \alpha_t^i - h_t^i\right) p_t^i R_t^{ib} + (E_t^i \hat{R}_{t+1}^k - R_t^{ib}) \Lambda_t^i, \tag{12}$$

subject to

$$\Lambda_t^i = 0 \text{ or } \lambda_b,$$

[13] For an example of this literature and a survey see Franses et al. (2011) or Fildes et al. (2009).

[14] ρ^c is the Pearson correlation coefficient for each pair of experts.

where R_t^{ib} is the gross real interbank market rate to be paid at $t+1$, and p_t^i is the subjective probability that the loan will be repaid in t+1. $E^i\hat{R}_{t+1}^k$ is bank i's subjective expectation about the risky asset return, and R_t^{res} is the gross real safe asset return. If a bank is a borrower on the interbank market, borrowing is restricted to a fraction λ_b of its net worth. Because I assume that net worth is averaged up at the beginning of the period, all borrowers borrow the same amount – λ_b. For a lender or a hoarding bank $\Lambda_t^i = 0$. The interbank market rate, R_t^{ib}, is determined by the market clearing condition. The main modification from the simple case is a different beliefs distribution. The distribution affects the subjective loan repayment probability and the share of bankers hoarding, etc. Recall that each bank's subjective probability of borrowers being able to meet their obligations is:

$$p_t^i = 1 - F_{E_t^i\hat{R}_{t+1}^k, \sigma_R^2}\left(\frac{\left(R_t d_t + \lambda_b R_t^{ib}\right)}{(1+\lambda_b)}\right), \tag{13}$$

where $(1+\lambda_b)\,E_t^i\hat{R}_{t+i}^k$ is the bank's expected risky asset return on its own funds plus those borrowed on the interbank market, λ_b. For a bank to be able to honor its interbank market loan (assuming that debt to the household has priority) the return should be higher than payments to the household, R_t, times the amount of deposits per bank, d_t, and the interbank loan repayment, $\lambda_b R_t^{ib}$, because each bank's belief is distributed normally with variance σ_R^2. Now, p_t^i is the cumulative density function of the normal distribution.

However, to consider what fraction of banks actually invest in credit to the real sector or borrow in the interbank market, I need the distribution across banks. The distribution is Gaussian with mean, m, and variance σ_R^2, coming from the banks filtering problem. The variance of the forecasts is the same for all the banks because they use the same observable and they have the same variance in their expert adjustments. Both the mean and the variance enter the rest of the model as state variables. Thus, I have a continuum of banks with beliefs distributed across banks $E_t^i\hat{R}_{t+i}^k \sim N(m, \sigma_R)$. The share of banks investing in the real economy is simply the share of banks with beliefs equal to or higher than the marginal investor's, $E_t^m\hat{R}_{t+1}^k = p_t^m R_t^{ib}$:

$$s_t^{inv} = \int_{E_t^m\hat{R}_{t+1}^k}^{\infty} f(x)\,dx = 1 - F_{m,\sigma_R^2}\left(E_t^m\hat{R}_{t+1}^k\right), \tag{14}$$

where $F_{m,\sigma_R^2}\left(E_t^m\hat{R}_{t+1}^k\right)$ is normal cdf.

The share of banks borrowing on the interbank market can be defined as the probability that their belief is higher than the interbank interest rate:

$$s_t^b = \int_{R_t^{ib}}^{\infty} f(x)\,dx = 1 - F_{m,\sigma_v^2}\left(R_t^{ib}\right). \tag{15}$$

The share of banks lending is then defined as the probability that the belief is higher than the belief of a marginal lender, $E_t^l\hat{R}_{t+1}^k$ with $p_t^l R_t^{ib} = R_t^{res}$:

$$\begin{aligned} s_t^l &= \int_{E_t^l\hat{R}_{t+1}^k}^{E_t^m\hat{R}_{t+1}^k} f(x)\,dx = \\ &= F_{m,\sigma_R^2}\left(E_t^m\hat{R}_{t+1}^k\right) - F_{m,\sigma_R^2}\left(E_t^l\hat{R}_{t+1}^k\right). \end{aligned}$$

The share of those keeping money in reserves (hoarding) is then defined as those neither investing nor lending $\left(1-s_t^{inv}-s_t^l\right)$. Multiplying these shares by the total funds of the banking family, I get the respective amounts of credit, borrowing, lending, and hoarding.

3.1.3 Interbank Market Clearing

For the interbank market to clear, demand should be equal to supply. Each borrower demands $\Lambda_t^i = \lambda_b$, and each lender supplies 1 if she finds the interbank market rate more attractive than alternative investments (hoarding or risky asset investment). So, market clearing is:

$$s_t^l = \lambda_b s_t^b.$$

Plugging in the expressions for the shares, one can re-write the market clearing condition as:

$$\int_{E_t^l \hat{R}_{t+1}^k}^{E_t^m \hat{R}_{t+1}^k} f(x)\,dx = \lambda_b \int_{R_t^{ib}}^{\infty} f(x)\,dx, \tag{16}$$

where $f(x)$ is a normal density function. Alternatively:

$$F_{m,\sigma_R^2}\left(E_t^m \hat{R}_{t+1}^k\right) - F_{m,\sigma_R^2}\left(E_t^l \hat{R}_{t+1}^k\right) = \lambda_b \left(1 - F_{m,\sigma_R^2}\left(R_t^{ib}\right)\right). \tag{17}$$

The banks' mean beliefs and their variance enter (17) as the moments for the cumulative density function. Then the variance enters the definition of the marginal investor and the marginal lender: $E_t^m \hat{R}_{t+1}^k = p_t^m R_t^{ib}$ and $E_t^l \hat{R}_{t+1}^k$ such that $p_t^l R_t^{ib} = R_t^{res}$. Combining these two definitions and (17) one gets a solution for the interbank market rate and the corresponding amount of lending.

3.1.4 Bank's Net Worth and the Financial Accumulator

Similarly to Gertler and Karadi (2011) I define the bank's net worth as N_t^i, and B_t^i are the deposits from households. In my model, however, banks can invest in three types of assets: risky, safe (hoarding), and interbank loans. So, the bank's balance sheet in my model is given by:

$$Q_t S_t^i + Res_t^i + Lend_t^i = N_t^i + B_t^i, \tag{18}$$

where $\frac{Q_t S_t}{N_t^i + B_t^i} = \alpha_t^i$, $\frac{Res_t^i}{N_t^i + B_t^i} = h_t^i$, and $\frac{Lend_t^i}{N_t^i + B_t^i} = \left(1 - \alpha_t^i - h_t^i\right)$ being the realized return from lending on the interbank market for the lender or $\frac{Lend_t^i}{N_t^i + B_t^i} = -\lambda_b$ for the borrower in (12). Given each asset return, the evolution of a bank's net worth over time can be formulated as

$$N_{t+1}^i = R_{t+1}^k Q_t S_t^i + R_t^{ib} Lend_t^i + R_t^{res} Res_t^i - R_t B_t^i. \tag{19}$$

Note, that for a borrower term $R_t^{ib} Lend_t^i$ is negative and is equal to $R_t^{ib}(-\lambda_b N_t^i)$.

As the agency problem is a slight modification of that in Gertler and Karadi (2011), I put the solution in Appendix D and here present the resulting aggregate constraint:

$$(Q_t S_t + Res_t) = \frac{\eta_t}{\lambda - \nu_t \left(1 - s_t^h\right)} N_t = \varphi_t N_t, \tag{20}$$

where φ_t is the banking sector leverage ratio, λ is a fraction of assets a banker can diverge, and ν_t and η_t are described in Appendix D.

To finalize the law of motion for banks' net worth, recall that in each period a fraction $(1-\theta)$ of the bankers exit and take a $(1-\theta)$ share of the banking family's assets. At the same time, households transfer a fraction ω of the exit value to the new bankers. That is, the law of motion of banks' family net worth is given by:

$$N_{t+1} = \theta \left\{ \left[\left(1 - s_t^h\right) \left(R_{t+1}^k - R_t\right) + s_t^h \left(R_t^{res} - R_t\right) \right] \varphi_t + R_t \right\} N_t + \omega \left(Q_t S_{t-1} + Res_{t-1}\right). \tag{21}$$

The first term on the right hand side in equation (21) (in curly brackets) shows the net wealth accumulation of banks that stay in the banking sector for the next period - there is an exogenous fraction θ of them. Their return equals the return from the risky asset, $\left(R_{t+1}^k - R_t\right)$, and there is share $\left(1 - s_t^h\right)$ of them (those who do not hoard either invest themselves or lend to other banks). Those who hoard, s_t^h share of banks, receive the return on safe asset. The last term in equation (21) is the transfers from households to the new entering bankers.

3.1.5 Credit Support Policies

I consider several credit support policies. Under the first two, the central bank funds asset purchases through intermediaries. The untargeted liquidity provision is modeled as the funding of a share ψ_t of banks' asset purchases:

$$Q_t S_t + Res_t = \varphi_t N_t + \psi_t \left(Q_t S_t + Res_t\right).$$

For targeted credit support, the central bank limits the set of assets to be purchased to risky claims on firms. Let ψ_t^{tar} denote the fraction of risky assets funded by the central bank. Then

$$Q_t S_t + Res_t = \varphi_t N_t + \psi_t^{tar} Q_t S_t.$$

A bank pays R_t for central bank support. There are, however, operational costs of conducting the policy, $\tau\psi_t \left(Q_t S_t + Res_t\right)$ or $\tau\psi_t^{tar} Q_t S_t$. I assume that both policies are equally costly. I model it consistently with Gertler and Karadi (2011), so that the central bank selects ψ_t and ψ_t^{tar} as a proportion of the rise in the risk premium. When there are disturbances in the economy, the risk premium rises above the steady-state level.

$$\psi_t = \kappa \left(R_{t+1}^k - R_t - \left(\overline{Rk - R}\right)\right), \tag{22}$$

where κ is a reaction parameter.

I further consider relaxing the collateral constraint on the interbank market and lowering the real gross return on the safe asset, R_t^{res}, both of which involve no operational costs. Relaxing the collateral constraint takes the form of increasing the fraction of borrowers' liabilities up to which borrowing is restricted – $\lambda_{b.}$ An increase in this fraction, denoted as ∇_t^λ, and a reduction in R_t^{res}, ∇_t^R, follow the same decision rule as the two previous policies considered:

$$\nabla_t^i = \kappa^i \left(R_{t+1}^k - R_t - \left(\overline{Rk - R}\right)\right),$$

where i stands either for λ or for R^{res}. I allow for a different feedback parameter κ^i in the rules.

3.2 The Household

There is a representative risk-averse household in the economy which has utility from consumption and disutility from labor. The household solves the following problem subject to a budget constraint:

$$\max_{C_t,L_t,D_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i-1}^{1+\phi} \right] \tag{23}$$

$$\text{s.t. } C_t + B_t = W_t L_t + R_{t-1} B_{t-1} + \Pi_t + T_t, \tag{24}$$

where C, L, B, and T stand for consumption, labor supply, deposits in banks, and tax, respectively. W and R are the real wage and the real gross return on bank deposits. Π_t is net transfers from financial and non-financial firms to the household. β, ϕ ,$\chi > 0$, and $\beta < 1$.

Bank deposits are guaranteed by the government, which, in the case of bank insolvency, pays the deposits and interest to the household. [15]

The first-order conditions (see Appendix F) state that the marginal disutility of labor is equal to the marginal utility of consumption and that the nominal return on bank deposits should, at the margin, compensate the consumer for postponing consumption to the next period.

3.3 Intermediate Goods Producers

The sector is perfectly competitive. Producers combine labor and capital using the Cobb-Douglas production function:

$$Y_t = A_t \left(U_t \xi_t K_{t-1} \right)^{\alpha} L_t^{1-\alpha}, \tag{25}$$

where K_{t-1} stands for capital, L_t stands for labor, and A_t is total factor productivity. U_t is the utilization rate of capital. That is, shock ξ_t influences effective capital.

Investment in capital should be made one period in advance. To invest in the next period's capital, K_t, intermediate goods producers issue claims S_t at price Q_t^S. The value of the capital they can buy at price Q_t^K is then $Q_t^K K_t = Q_t^S S_t$. In the next period, intermediate goods producers sell the depreciated capital to capital producers at the market price Q_{t+1}^K. Because of the perfect competition among intermediate goods producers, the price of capital equals the price of producers' claims: $Q_t^K = Q_t^S \equiv Q_t$. The amount of depreciated capital is equal to $\delta_t \left(U_t \right) \xi_t K_{t-1}$, where δ_t is the physical depreciation rate and ξ_t reflects the capital quality shock discussed above. At $t+1$, the firm pays a gross return $R_{k,t+1}$ to the bankers per each unit of investment. As firms are identical, investment in capital pays the same return to all banks.

In each period, an intermediate goods producer chooses labor demand and demand for capital to maximize its current and next-period profits. Profit consists of the revenues from production and the resale value of the depreciated capital net of payments on claims S_t and labor costs. The price of a unit of the intermediate good is $P_{m,t}$, and the cost of buying new capital is Q_t. The producer then chooses the utilization rate and labor demand as:

$$(1-\alpha)\frac{P_{m,t}Y_t}{L_t} = W_t,$$

$$(\alpha)\frac{P_{m,t}Y_t}{U_t} = \delta' \left(U_t \right) \xi_t K_{t-1}.$$

[15] If a bank receives negative return, this is reflected in Π_t term.

As firms make zero profit, they distribute the return on capital to holders of their claims as in (8). The wage is then determined by the marginal product of labor. The price of the intermediate good equals the marginal costs.

3.4 Capital-Producing Firms

Capital goods producers are competitive firms. They buy depreciated capital from intermediate goods producers and renovate it at the unit costs and sell it at the unit price. They also produce new capital and sell it at price Q. There are no adjustment costs for renovating worn-out capital, but there are flow adjustment costs when producing new capital. Capital producers are risk neutral and maximize the following utility (first order conditions are in Appendix F):

$$\max_{In_t} E_t \sum_{k=t}^{\infty} \beta^{T-k} \Omega_{t,k} \left((Q_k - 1) I_{nk} - f\left(\frac{In_k + I_{ss}}{In_{k-1} + I_{ss}}\right)(In_k + Iss) \right), \tag{26}$$

where In is net investment, defined as $In_t \equiv I_t - \delta(U_t)\xi_t K_{t-1}$, where $\delta(U_t)\xi_t K_{t-1}$ is the quantity of renovated capital. Iss is steady-state investment and Q_t is the price of capital. Function f is an investment adjustment cost function satisfying the following properties $f(1) = f'(1) = 0$ and $f"(1) > 0$.

3.5 Final Goods Producers (Retailers)

Retailers combine output from intermediate goods producers using the production function:

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{27}$$

where Y_{ft} is composite goods output from retailer f and ε is the elasticity of substitution. I follow the Calvo-pricing convention and each period allow only a fraction γ of firms to optimize their prices. The solution is in the Appendix F.

3.6 The Government and the Central Bank

The government collects lump-sum taxes from households, T_t, and accepts reserves (the safe asset), Res_t. It also bears some costs of conducting policy, Po_t. The government's budget constraint is satisfied when the following holds:

$$G_t + Po_t = T_t + Res_t - R_t^{res} Res_{t-1}. \tag{28}$$

The resources in the economy are then distributed between consumption, investment, and government expenditure on policy:

$$Y_t = C_t + I_t + f\left(\frac{In_t + I_{ss}}{In_{t-1} + I_{ss}}\right)(In_t + Iss) + G_t + Po_t. \tag{29}$$

The central bank conducts monetary policy according to the simple rule:

$$i_t = (1 - \rho_i)\left(i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y_t^*)\right) + \rho_i i_{t-1} + \varepsilon_t, \tag{30}$$

where Y^* is flexible output, ε_t is an exogenous monetary policy shock, and i is the steady-state nominal rate. ρ_i is a smoothing parameter lying between zero and one. The real and nominal interest rates are linked via the Fisher equation: $1 + i_t = R_t E_t(1 + \pi_{t+1})$.

Table 1: Calibrated Parameters Specific to my Model

ω	0.0059	proportional transfer to entering bankers
λ_b	0.24	collateral constraint on interbank market
σ_R^2	0.1	variance of return expectations
σ_v^2	0.001	variance of persistent shock to capital quality
σ_η^2	0.5	variance of expert opinion shock
σ_e^2	0.03	variance of capital quality transitory shock
$\sigma_{\varepsilon\eta}$	0.1	covariance of errors in econometric and expert forecasts
ρ_θ	0.66	persistence of expert opinion shock
ρ_ξ	0.66	persistence of capital quality shock
ρ_μ	0.66	persistence of persistent shock to capital quality
ρ_c	0.62	correlation of experts' opinion
κ^λ	2.5	policy reaction for collateral constraint
κ^R	0.1	policy reaction for reserve rate

4. Calibration and Simulations

To compare my results with the literature, where possible I follow the calibration choices of Gertler and Karadi (2011); I list their parameter choices in Table F1 in Appendix F. There are, however, some parameters specific to my model, described in Table 1. Among them σ_R^2, λ_b, ω, and $\bar{E}\hat{\xi}$. I set average expectations of the capital quality shock, $\bar{E}\hat{\xi}$, to be equal to the steady-state value of $\bar{\xi} = 1$. The variance of banks' forecasts and the dispersion between them, σ_R^2, the collateral constraint, λ_b, and the transfer to new entering bankers, ω, are meant to be suggestive. I set their values to roughly match the following pre-crisis data: the interbank market rate, the share of interbank loans in banks' portfolios, and the share of loans in banks' portfolios. In my model, banks exchange loans for one period on the interbank market, with the period being a quarter. Therefore, the 3-month Euribor is a natural choice for the empirical counterpart for the model interbank rate. The share of interbank loans resembles the share of lenders in my model and is calculated as the ratio of euro area banks' loans to monetary and financial institutions to total assets. Similarly, the share of loans is the ratio of total loans to the total assets of European banks. Total loans include loans to enterprises and to other banks. In my model, what is not lent either to banks or to firms is hoarded. That is, the share of loans is useful for calculating the share of hoarded assets as (1 - share of loans).[16] I choose σ_R^2, λ_b, ω to roughly match an interbank rate of 1.31%, a share of hoarded assets of 40%, and a share of assets lent on the interbank market of 20%, so that σ_R^2, λ_b, and ω to be 0.1, 0.24, and 0.0059, respectively.

The parameters for Kalman filter updating, σ_v^2, σ_η^2, σ_e^2, and $\sigma_{\varepsilon\eta}$, are set to match the steady-state variance of the bank's forecast, σ_R^2. Recall that in my model, ξ is subject to two shocks: a persistent one and transitory one, with the linearized equation: $\xi_t = \rho_\xi \xi_{t-1} + \mu_t + \varepsilon_t$, so the variance of ξ, $\sigma_\xi^2 = \left(\sigma_v^2 + \sigma_\varepsilon^2\right)/(1-\rho_\xi^2)$. At the same time, σ_R^2 is a function of σ_ξ^2, as shown by (8). That is, the steady-state value of σ_R^2 defines the sum of the variances of the persistent and transitory shocks, which are set at 0.001 and 0.03, respectively. I choose the variance of the expert opinion shock, σ_η^2, so that the share of expert adjustments in the final forecast lies within the bounds defined by the

[16] In my model, reserves represent safe assets, but in reality there are a number of assets that can be considered "safe." Consequently, I cannot use the amount of reserves as an empirical counterpart for hoarding.

literature on forecasting.[17] I set σ_η^2 to be equal to 0.5, and the resulting steady state share of expert adjustments is then 0.37. The persistence of the capital quality shock is $\rho_\xi = 0.66$, as in Gertler and Karadi (2011). I set the persistence of all other shocks to be the same, at 0.66 for both the persistent shock and the expert opinions shock.[18]

The policy reaction parameter for the reserve rate is set to match the decline in the real policy rate during 2008–2009 relative to the pre-crisis 10-year average: the resulting deviation from the steady state during the crisis is a fall of 0.23 percentage points. The collateral constraint in my model does not have an intuitive empirical counterpart. The value is set for illustrative purposes and alternative values are discussed. With the parameters described above, I then proceed with an analysis of the linearized model and its performance relative to Gertler and Karadi (2011). The model is simulated using Dynare 4.

4.1 Defining a Crisis

I consider a crisis to be a transitory shock to capital quality, ξ_t. To make the dynamics of my model comparable to the literature, I consider a crisis to be a 5% decline, as in Gertler and Karadi (2011). They set this value to match a 10% decline in the effective capital stock over a two-year period. I consider several types of crises: a drop in ξ_t of 5%, a drop in ξ_t of 5% combined with banks believing that this was a permanent shock,[19] and an unchanged ξ_t with banks believing in a 5% drop in the persistent component of ξ_t. In other words, I consider a crisis without an expectational (pessimistic) shock, a crisis with an expectational (pessimistic) shock, and a pure expectational (pessimistic) shock, respectively. In the simulations without an expectational shock, banks observe a change in ξ_t, but they do not have perfect information on how persistent this change is. They use past and current observations via the Kalman filter to predict ξ_{t+1}. In the simulations with an expectational shock, in addition to a change in ξ_t, banks get a "pessimistic shock": experts start to believe that a persistent shock has occurred. These expert opinions are combined with past observations again via the Kalman filter.

4.2 The Role of Expectations and the Interbank Market

In my economy, expectations determine credit to the real sector. They also affect the functioning of the interbank market: the numbers of borrowers and lenders and the equilibrium interbank market rate. In the general equilibrium model, I consider model responses linearized around the steady state with a functioning interbank market. A decrease in market expectations then results in lower credit supply and lower lending between banks.

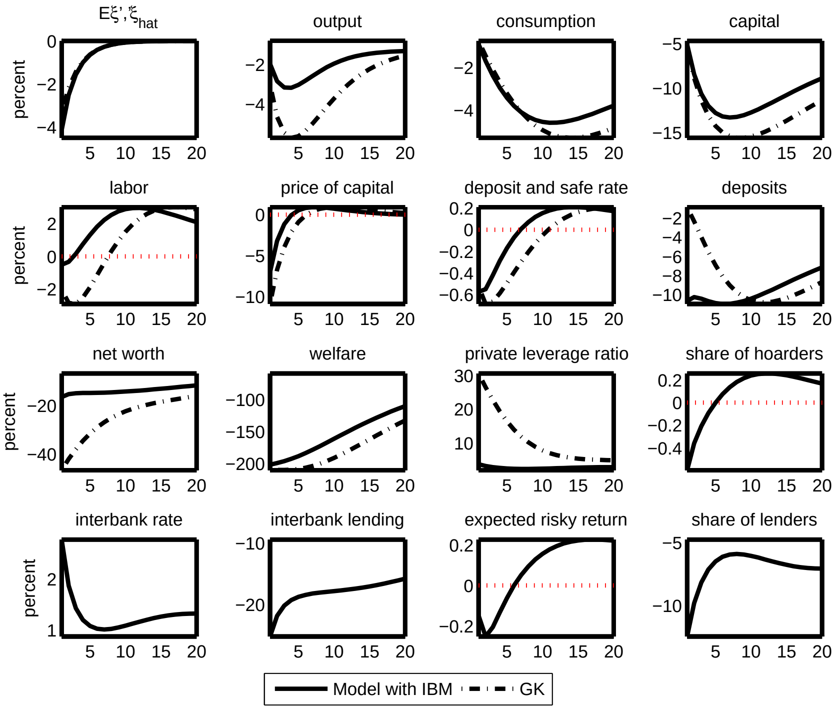
If banks have "rational expectations" as in Gertler and Karadi (2011), there is no interbank market and our models would have identical responses. For this reason, I treat Gertler and Karadi (2011) as a baseline to study the role of expectations and the interbank market. Let us start my analysis with a comparison of the model behavior and the baseline when there is no policy response, and crisis shock is only a shock to ξ_t. In this scenario, the policy rate, R_t^{res}, is set to be equal to the deposit rate so that banks earn nothing on a safe asset. Figure 4 shows the responses of my model and the Gertler

[17] For example, Fildes et al. (2009) analyze a data set containing 70,000 business organizations and their forecasts. They find the mean expert adjustment for monthly forecasts to vary between 18% and 46% depending on the type of business.

[18] In previous research – Audzei (2012) – I calculated the persistence of the expert opinion shock using the SPF GDP forecast. The resulting value was very close to the current calibration – 0.61.

[19] This is modeled as a shock to banks' average belief about the drop in the persistent shock. Recall that in the model this is the average belief that matters for the simulations.

Figure 4: Crisis Simulations: Comparison to the Baseline



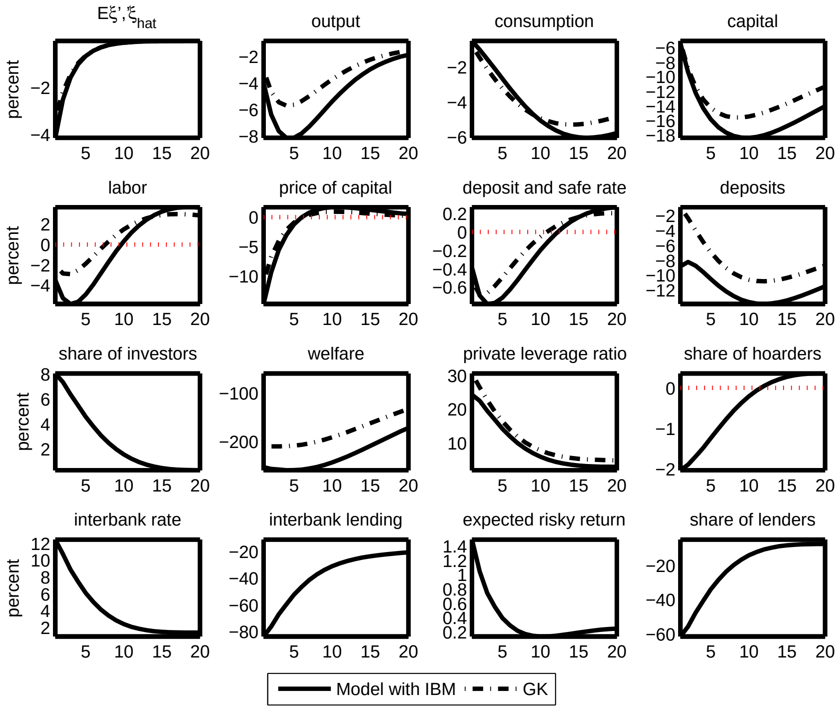
The responses are plotted for 5 % shock to ξ_t.

and Karadi (2011) model [20] where applicable.[21] In period 1 there is a 5% temporary shock to ξ_t. Because ξ_t is itself a persistent process, it remains below the steady-state value for about ten periods. The first subplot shows the expectations about ξ_{t+1}. In a model with perfect expectations, this will be $\overline{E}_t \xi_{t+1} = \rho_\xi * \xi_t$, resulting in a decline of 3.3% in the first period. It coincides with the decline in the ξ_{t+1} in the Gertler and Karadi (2011) model. However, in my model it is not observable whether this was a transitory or a persistent shock. Recall that agents combine two observables to form their forecast: historical values of ξ and expert opinions about the value of the persistent shock to ξ_t. Even when the experts consider the persistent shock to be zero, analysis of historical observations leaves a possibility for it to exist. Consequently, even in a model where expert opinions are not disturbed, the future values of ξ_t are underestimated.

When ξ_t is hit by a shock, there is an immediate decline in the contemporaneous return on banks' investment – R_t^k. This lowers banks' returns and has a negative effect on their net worth. With a lower net worth of banks, current net investment falls and the price of capital decreases. Because capital and, accordingly, net investment fall, there is less demand for capital and capital producers sell it at a lower price. Price of capital also reflects the resale value of capital. Consequently, its fall contributes to a further decline in banks' worth. A fall in the net worth leads to a decline in both safe and risky asset holdings. With smaller net worth, banks are able to attract fewer deposits from households, as their deposits are limited to a fraction of their net worth through the agency problem. Hence, net investment falls even more, as banks simply have less funds for it. With investment falling, the return on starts to rise. Banks would like to invest more with such returns, but their investment is already reduced by the fall in their net worth.

[20] Welfare is calculated as in Gertler and Karadi (2011) as a second order approximation of household utility.
[21] Obviously, such variables as the interbank market rate or safe asset holdings do not exist in their paper.

Figure 5: Crisis Simulations Comparable Net Worth



The responses are plotted for 5 % shock to ξ_t.

Note that in the simple model, a fall in expectations results in lower lending and a lower interbank market rate. The crisis in a general equilibrium results in a fall in lending, but the interbank rate rises. This is because of the feedback from banks' investment to the risky asset and the risky asset return. The share of lenders shrinks because some banks would like to invest themselves. This puts upward pressure on the interbank rate, which therefore rises.

Now compare responses of the model to Gertler and Karadi (2011). In my model, banks do not invest all their funds in the risky asset, but leave some share in the safe one. Consequently, only a proportion of banks' net worth is affected by the fall in the risky asset return. My law of motion of net worth (21) is similar to the dynamics of N_t in Gertler and Karadi (2011), but the term $\left(R_t^k - R\right)$ is also multiplied by the share of banks investing in the risky asset. As a result, my model demonstrates almost half as large a fall in investment and net worth compared to Gertler and Karadi (2011). In table G1 in Appendix G I compare the standard deviations resulting from both models to the standard deviations observed in the data. When it comes to the net worth of the financial intermediaries, the fall in my model is rather close to the data. This is not surprising as banks in reality have a more diversified portfolio than those in the benchmark model holding the risky asset only. A smaller drop in net worth explains smaller a fall in capital and output.

Because the baseline model features only a capital asset and demonstrates a larger fall in the net worth, to isolate the role of the interbank market, I simulate my model controlling for the difference in the net worth.[22] Such a comparison is shown in Figure 5. With the net worth decreased as much as in the baseline, the recession is larger in my model, where the interbank market serves

[22] For this purpose I calculate impulse responses while substituting values for the net worth from the baseline in the state variable.

as a propagation mechanism. The primary effect is borrowers having less net worth, so they can borrow less. This lowers the demand for capital and the price of capital. The increasing (due to a fall in K) expected risky return, which is return on capital, attracts some lenders to become direct investors themselves, pushing the interbank rate further up. This drives even more borrowers out of the market. The higher interbank rate attracts some of the hoarders to become lenders, but it does not offset the fall in lending. An increase in investors' set is responsible for a very similar initial drop in capital in both models. However, the private leverage ratio is smaller in my model as the expected gross rate of banks capital is lower.[23] That is why there are lower deposits, lower safe rate (and lower return on a safe asset), and lower labor and output.

The interbank market serves as a shock amplifier: as lenders are those who would not invest themselves, but are willing to lend their funds to investors, a change in the set of lenders amplifies a fall or rise in credit to the real economy. In a model without the interbank market the capital returns to the steady state much faster. The resulting decline in output is more than 1.5 times more than in the baseline, being the result of both imperfect expectations and the interbank market adjustment to the shock.

Now consider the effect of different crisis shocks in my model: "fundamental" to ξ only, pure expectational shock, and a combination of these two shocks. A comparison of all shocks is shown in Figure 6. Apart from 5% decline in ξ the model with expectational shock features a wave of pessimism among the investors, i.e. a 5% persistent decline in the average banks' prediction about persistent component of ξ, μ_t. The pure expectational shock corresponds to the simulations in which only a pessimistic wave hits the economy, without an actual drop in ξ.

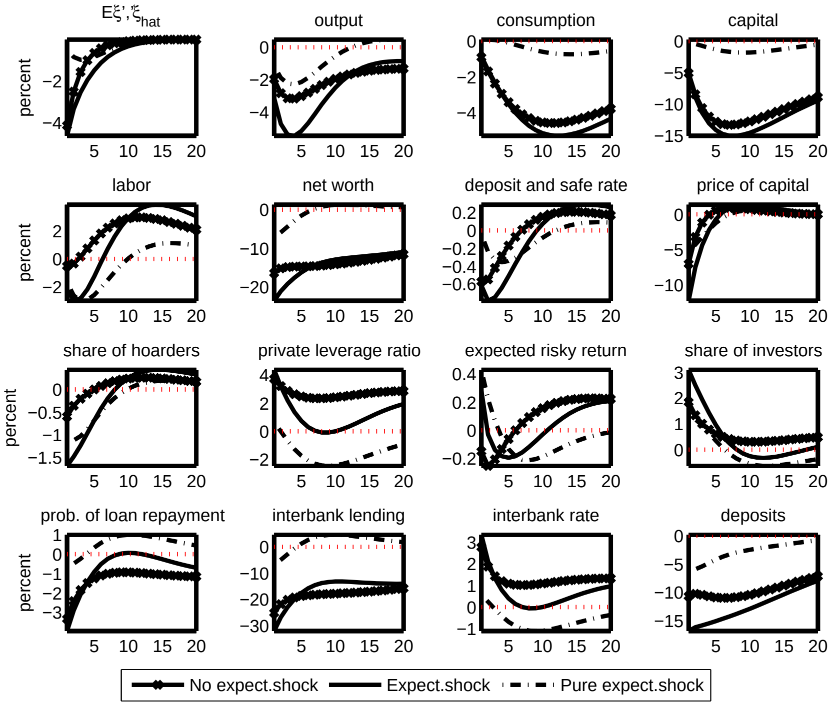
Comparing the responses to a crisis with and without the expectational shock, note that the expected risky asset return falls only without expectational shock because of a smaller decline in capital. Under the pure expectational shock, the expected risky asset return increases as the capital quality is not disturbed by the shock, but the capital investment declines. That increases the marginal return on capital enough to compensate for the pessimistic forecast of ξ. The initial difference in net worth is explained mostly by the price of capital as the shock to the actual ξ_t is the same. That is, net worth falls the most in the model with expectational shocks. Net worth affects banks' ability to attract deposits and influences the deposit rate. Smaller deposits result in a lower interest rate on them, making household savings less attractive. In the model, there are no frictions on the labor side, so it is labor that adjusts, with the fall in consumption being similar under both scenarios. Output falls in response to the fall in capital and labor, with the drop being twice as large as in the scenario with the expectational shock.

When a pure expectational shock hits the economy and there is no actual drop in ξ_t, banks underestimate ξ_t for some period of time. This generates a decline in net investment, a decrease in the price of capital, and a fall in the current return on capital, followed by a decline in net worth. Capital falls initially by 0.1%. The decline in net worth accelerates the fall in capital in the following periods, with the maximum decline being 1.244%. That is, a persistent pessimistic shock can generate a small recession, as investment falls, leading to a decline in output and consumption.

The probability of loan repayment (evaluated for marginal lender) reflects the changes in expected risky return and deposit rates, as borrowers' returns on the interbank market have to be enough

[23] Note that due to the agency problem between households and banks, this ratio depends on banks' continuation value, which is return on banks own capital. Only a share of banks invests in the risky asset (with higher return than the safe asset), and also due to slightly suppressed expectations of ξ, the return is underpredicted.

Figure 6: Crisis Simulations with and without Expectational Shocks



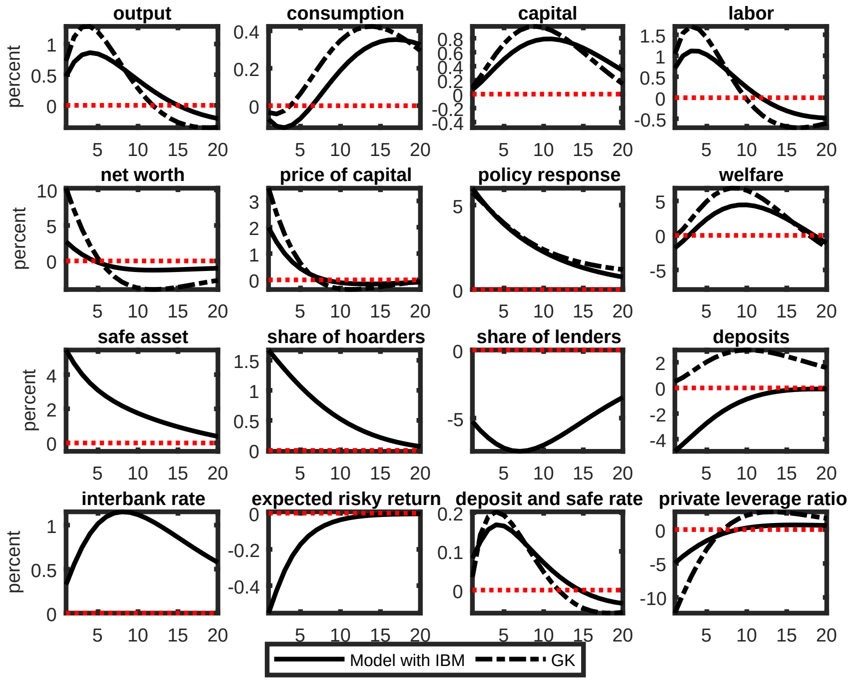
The responses are plotted for 5% shock to ξ. The expectational shock is 5% fall in average expert opinion, $\bar{\theta}_t$.

to compensate for obligations to both household and lenders. Therefore the probability of loan repayment is largest with pure expectational shock and smallest with expectational and ξ shock. The drop and subsequent increases translate then into the interbank rate and market allocations.

Thus, if the crisis is interpreted as a combination of a shock to ξ and a shock to agents' expectations, the resulting responses look like the sum of the pure expectational shock and no expectational shock scenarios. Then the crisis of the observed magnitude could be simulated with a smaller size of the "fundamental" shock, depending on the size and persistence of a wave of pessimism. If one accepts the idea of sluggishness of investors' forecasts as in Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013), meaning overly pessimistic expectations after the crisis episodes, then my model with expectational shock could serve as an illustration of the crisis, generating the same 10% decline in capital over 2 years as in Gertler and Karadi (2011), but with the banks' net worth deviations matching the data due to a more realistic asset structure.

To summarize, the expectational shock alone can generate some need for a policy response by the central bank. Combined with the occurrence of an actual crisis, this leads to a more severe recession. That is, investor sentiment can be an important factor for policy design and evaluation. Without the expectational shock, my model predicts a milder recession than Gertler and Karadi (2011), as banks in my model have an opportunity to diversify their assets and are thus less impacted by the crisis. With the expectational shock, my model has similar predictions to the baseline regarding the dynamics of output, capital, labor, and consumption.

Figure 7: Policy Effects vs Baseline: Untargeted Liquidity Provision



The policy effect is plotted as a percentage difference in model responses with and without policy for 5% shock to ξ_t.

4.3 Policy Results

The main difference from the simple model is the presence of feedback from expectations and investment to asset prices and banks' net worth.

I start my analysis again with a comparison of the baseline model of Gertler and Karadi (2011) and my model with the interbank market without expectational shock. The comparison is complicated due to the different capital structure. If I control for the difference in the net worth as was done in Figure 5, then the crisis is much deeper in my model. Liquidity provision of the same scale leaves my economy in a worse recession than in the baseline model, but the effect relative to the simulation without policy is larger due to the larger initial drop. Therefore, for the comparison, I choose my crisis simulation with expectational shock. In this scenario, the economy is hit by a shock to ξ_t and a wave of pessimism. The resulting simulation gives a very similar drop in output and capital as in the baseline (note that the crisis there is only a shock to ξ_t), but the fall in the net wealth is two times smaller. However, the optimal policy rule as in (22) would be different in my model, requiring a larger policy response in my case. For this reason, I simulated my model using a vector of policy responses similar to the baseline. I present the comparison in Figure 7, where I plot the percentage differences in variables with and without policy ($x = x_{policy} - x_{nopolicy}$). As the figure shows, the policy effects in my model with interbank market are lower lower and delayed. Moreover, the policy even has a negative effect on deposits and interbank market holdings. The policy increases the share of hoarders by decreasing the expected return on capital and increasing the safe rate. As more lenders leave the market to become hoarders, the interbank rate rises, depressing lending. As there are fewer investors (and lower return on banks' aggregate capital), the private leverage ratio falls together with deposits. In a certain sense, the policy "crowds out" interbank lending and borrowing from the households. Also note that the safe asset holding is increased by almost 6 percent, while capital asset less than 1 percent. This hoarding effect undermines the impact of the liquidity provision in my model.

For an analysis of different policies, I consider a "crisis" shock defined as a decline in capital quality, ξ_t, in combination with a wave of investor pessimism. The results are presented in Figure 8. The policy exercises without the expectational shock would have the same qualitative results, but the recession would be smaller and so are the differences between policies.

First, consider the two types of liquidity provision: targeted, solid line; and untargeted, dashed line. The two policies have a very similar effect on output, consumption, and capital, and mitigate the crisis relative to the simulation with no policy response. The policy response is the total amount of funds supplied to banks – $\psi(QK+Res)$ and $\psi^{tar}(QK)$ for untargeted and targeted policies, respectively. The main difference between the two policies is in the share of hoarded assets, $Hoarding$, which is almost twice as large in the case of the untargeted policy. The absolute holdings of a safe asset are also almost twice as large under the untargeted policy. These predictions are in line with the simple model results: the liquidity provision helps restore credit to the real economy, but also increases reserve holdings.

With targeted credit support, banks expect a share of their risky asset purchases to be financed by the central bank. For those with high expectations about the risky asset return, this means less need to borrow from households and on the interbank market. Note that this is only true for banks with high return expectations. In the case of untargeted liquidity provision, where both assets are financed by the central bank, all banks have less need for household funding. Banks' deposits therefore fall more with untargeted support, together with the private leverage ratio and deposit rate. This explains the small differences in output and labor supply.

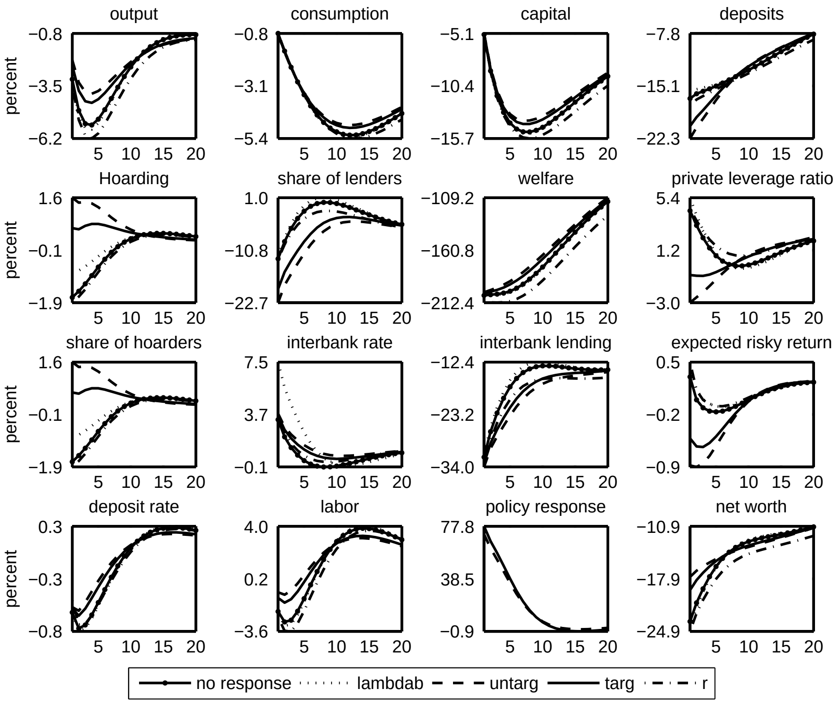
Interest rate policy is the least efficient policy in my simulations. It is modeled as a decline in the reserve rate, R^{res}, below the deposit rate R, meaning that banks are making negative returns on their reserves. In line with the simple model results, such a policy lowers the share of hoarded assets in banks' portfolios. However, it reduces banks' net worth, leading to a large drop in investment. This drop in banks' net worth leads to even worse outcomes than in the case of no policy action.

Relaxing the collateral constraint on the interbank market by raising λ_b allows borrowers to borrow a larger fraction of their net worth. The larger demand for interbank credit drives up the interbank market rate, reducing the number of banks willing to borrow. Thus, there are fewer borrowers on the market, but they borrow more. As a result, interbank market lending increases. The high interbank market rate makes interbank lending more attractive relative to risky asset investment, and thus some potential investors become lenders on the interbank market and the share of those investing in the risky asset falls. As a result, despite the larger volume on the interbank market, credit supply to the real economy is almost unchanged, as are safe asset positions. [24]

To conclude, liquidity provision policies help mitigate the simulated crisis. However, relative to the baseline model of Gertler and Karadi (2011), my model with imperfect information and a storage asset displays low efficiency of liquidity provision, with higher costs, delayed responses and liquidity hoarding. The policy of targeted and untargeted liquidity provisions have a very similar effect in the general equilibrium content because of the feedback from household deposits and labor, with the latter policy resulting in larger hoarding. The policy of low reserve rates makes hoarding less

[24] In alternative simulations I considered different response parameters for relaxing the collateral constraint: from 0.4 to 2.5. The difference in the output and capital responses is negligible.

Figure 8: Policy Effects



The responses are plotted for 5% shock to ξ. The expectational shock is 5% fall in average expert opinion, $\bar{\theta}_t$.

attractive but has a negative impact on banks' net worth, leading to worse outcomes than in the case of no policy.

5. Conclusion

In this paper I address the role of imperfect market expectations in interbank lending and amplifications of economic fluctuations. In particular, I show that assessment of counterparty risk can be one of the factors contributing to a credit crunch.

In a simple finite-horizon model, I consider an expectations-driven credit crunch and show that policy effects in this case are very limited. To study market expectations, I incorporate a heterogeneous banking sector with a continuum of risky asset return expectations. The heterogeneity of expectations gives rise to an interbank market where lenders take into account the possibility of a borrower failing to repay the loan. Imperfect information among the bankers results in higher assessment of counterparty risk after crisis episodes, as bankers are not sure how persistent the negative shock is. The interbank market serves as a shock propagating mechanism as a set of lenders shrinks. I then develop a linearized DSGE model with interbank lending and consider responses around the steady state.

To study how imperfect expectation and/or waves of pessimism amplify crisis shocks I consider several types of crises: with and without pessimism shocks, and purely driven by pessimism. I show that even a pure pessimism shock alone can generate a small recession. Imperfect expectations result in an underestimation of investment opportunities even when the crisis is not accompanied by the waves of pessimism, because agents overestimate the persistence of the shock. The combination of a crisis and a wave of pessimism results in a crisis producing roughly a 10% decline in capital stock over two years (as documented in Gertler and Karadi (2011)).

I consider several types of central bank policy responses, including unlimited liquidity provision, targeted credit support, and varying the interest rate on reserves. Market pessimism dampens the positive effects of policies, making banks hoard central bank funds in reserves instead of transferring them through the bank lending channel. Compared to the model of Gertler and Karadi (2011), the policy effects are smaller and delayed.

A low policy rate (reserve rate) in my model devastates banks' balance sheets and results in a worse recession than in the case of no policy response. Even though it stimulates the interbank market and increases the number of investors, the wealth effect dominates.

References

AGUILAR, P. AND J. VÁZQUEZ (2018): “Term structure and real-time learning.” Working Papers 1803, Banco de España

ALLEN, F., E. CARLETTI, AND D. GALE (2009): “Interbank market liquidity and central bank intervention.” *Journal of Monetary Economics,* 56(5):639–652.

ANDRADE, P. AND H. LE BIHAN (2013): “Inattentive professional forecasters.” *Journal of Monetary Economics,* 60(8):967–982.

AUDZEI, V. (2012): “Efficiency of central bank policy during the crisis: role of expectations in reinforcing hoarding behavior.” Working Paper Series 477, CERGE-EI

COIBION, O. AND Y. GORODNICHENKO (2015): “Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts.” *American Economic Review,* 105(8):2644–78.

CUI, W. AND L. KAAS (2017): “Default Cycles.” Discussion Papers 1716, Centre for Macroeconomics (CFM)

CURDIA, V. AND M. WOODFORD (2011): “The Central-Bank Balance Sheet as an Instrument of Monetary Policy.” *Journal of Monetary Economics,* 58(1):54–79.

FILDES, R., P. GOODWIN, M. LAWRENCE, AND K. NIKOLOPOULOS (2009): “Effective forecasting and judgmental adjustments: an empirical evaluation and strategies for improvement in supply-chain planning.” *International Journal of Forecasting,* 25(1):3–23.

FRANSES, P. H., H. C. KRANENDONK, AND D. LANSER (2011): “One model and various experts: Evaluating Dutch macroeconomic forecasts.” *International Journal of Forecasting,* 27(2):482–495.

GALE, D. AND T. YORULMAZER (2013): “Liquidity hoarding.” *Theoretical Economics,* 8(2): 291–324.

GERTLER, M. AND P. KARADI (2011): “A Model of Unconventional Monetary Policy.” *Journal of Monetary Economics,* 58(1):17–34.

GERTLER, M. AND N. KIYOTAKI (2010): *Financial Intermediation and Credit Policy in Business Cycle Analysis,* volume 3 of *Handbook of Monetary Economics,* chapter 11, pages 547–599., 1 edition.

HEIDER, F., M. HOEROVA, AND C. HOLTHAUSEN (2015): “Liquidity hoarding and interbank market rates: The role of counterparty risk.” *Journal of Financial Economics,* 118(2):336–354.

HILBERG, B. AND J. HOLLMAYR (2011): “Asset prices, collateral and unconventional monetary policy in a DSGE model.” Working Paper Series 1373, European Central Bank

KIYOTAKI, N. AND J. MOORE (2008): “Liquidity, Business Cycles and Monetary Policy.” mimeo, Princeton University and LSE

MILANI, F. (2011): “Expectation Shocks and Learning as Drivers of the Business Cycle.” *Economic Journal,* 121(552):379–401.

NEGRO, M. D., G. EGGERTSSON, A. FERRERO, AND N. KIYOTAKI (2017): “The Great Escape? A Quantitative Evaluation of the Fed’s Liquidity Facilities.” *American Economic Review,* 107(3):824–857.

RYCHALOVSKA, Y. (2016): "The implications of financial frictions and imperfect knowledge in the estimated DSGE model of the U.S. economy." *Journal of Economic Dynamics and Control,* 73:259 – 282.

SLOBODYAN, S. AND R. WOUTERS (2012): "Learning in a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models." *American Economic Journal: Macroeconomics,* 4(2):65–101.

SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review,* 97(3):586–606.

Appendix A: Derivations for the Simple Model

Interbank market clearing. The market clearing condition for the interbank market with a uniform beliefs distribution is:

$$F_{m,\sigma_v^2}\left(E^m\hat{R}^k\right) - F_{m,\sigma_v^2}\left(E^l\hat{R}^k\right) = \lambda_b\left(1 - F_{m,\sigma_v^2}\left(R^{ib}\right)\right)$$

The cumulative distribution function for the continuous uniform distribution is $\frac{x-a}{b-a}$. Then the market clearing condition is rewritten as:

$$\begin{aligned}
\frac{E^m\hat{R}^k - a}{b-a} - \frac{E^l\hat{R}^k - a}{b-a} &= \lambda_b\left(1 - \frac{R^{ib} - a}{b-a}\right) \\
\Rightarrow E^m\hat{R}^k - a - E^l\hat{R}^k + a &= \lambda_b\left(b - a - R^{ib} + a\right) \\
\Rightarrow E^m\hat{R}^k - E^l\hat{R}^k &= \lambda_b\left(b - R^{ib}\right)
\end{aligned}$$

where b is the upper bound on the beliefs distribution, denoted as $\bar{R}$ in the text.

Proposition 2.0.1. The necessary condition for the interbank market to exist is that there is interbank rate, R_t^{ib}, solving

$$a*(R^{ib})^3 + b*(R^{ib})^2 + c*(R^{ib}) + d = 0, \tag{A1}$$

that is real and non-negative.[25] With $a > 0$, $b < 0$, and $d > 0$, if a positive root exists, it is unique. This positive root exists only if:

$$R^{res} > A(m,\sigma) + \frac{R}{\lambda_b + 1} < 0, \tag{A2}$$

where [26] $A < 0$.

The sufficient condition for the equilibrium with the interbank market is that marginal lender belief satisfies:

$$E^l\hat{R}^k < p^l R^{ib} = Res,$$

which implies $\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$.

Proof. Denote $\Delta = b^2 - 3ac$ the disriminant of the cubic equation (A1) and $rib^{(1)}$, $rib^{(2)}$ and $rib^{(3)}$ its 3 roots, where the first one is always real, and the rest can be real. With $a > 0$, $b < 0$ and $d > 0$, the first root is always negative. The last two roots are real and distinct from the first only if $\Delta > 0$. The condition for $\Delta > 0$ is (A2). If the parameters are such that $\Delta > 0$, the second root is always negative and the third one is always positive. Therefore, for the positive real solution to exist the necessary condition is (A2).

[25] $a = \sqrt{3}\lambda_b(1+\lambda_b)$, $b = -\lambda_b\left(\sqrt{3}(\lambda_b+1)m + 9\lambda_b\sigma + 3\sigma\right)$, $c = 6\sigma\left(\lambda_b\left(\lambda_b m + \sqrt{3}\lambda_b\sigma + m - R^{res}\right) + R - R^{res} - \sqrt{3}\sigma\right)$, and $d = 12\sqrt{3}(\lambda_b+1)R^{res}\sigma^2$

[26] $A = \frac{-3\left(3\lambda_b^3+7\lambda_b+6\right)\sigma^2 - \lambda_b(\lambda_b+1)^2m^2 + 4\sqrt{3}\lambda_b(\lambda_b+1)m\sigma}{6\sqrt{3}(\lambda_b+1)^2\sigma}$

However, given the interbank market rate, for the interbank market to exist there must exist marginal lender with the belief, $E^l R^k$ determined form $p^l R^{ib} = R^{res}$, and this belief should be smaller than $p^l R^{ib}$. Otherwise, the marginal lender invests herself and the set of lenders vanishes. Using (3), I analyze the definition of the marginal lender's and then show under which conditions $E^l R^k < R^{res}$: [27]

$$E^l \hat{R}^k = \frac{R^{res} 2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} + \frac{R}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)} R^{ib},$$

$$\frac{R^{res} 2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} + \frac{R}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)} R^{ib} < R^{res},$$

$$\Rightarrow \tag{A3}$$

$$\frac{R^{res} 2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} - \frac{R\lambda_b}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)} R^{ib} < 0,$$

$$\Rightarrow \tag{A4}$$

$$p^l 2\sigma\sqrt{3} - \left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right) + \frac{\lambda_b}{(1+\lambda_b)} R^{ib} < 0, \tag{A5}$$

$$R^{res} 2\sigma\sqrt{3} - R^{ib}\left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right) + \frac{\lambda_b}{(1+\lambda_b)} (R^{ib})^2 < 0, \tag{A6}$$

$$\tag{A7}$$

Where I have used $R^{res} = p^l R^{ib}$. The last inequality could be re-written as:

$$(R^{ib} - x_1')(R^{ib} - x_2') < 0, \tag{A8}$$

With:

$$x_1' = \frac{\left(\sigma\sqrt{3}(1+\lambda_b) + R\lambda_b\right) - (1+\lambda_b)\sqrt{\Delta_1}}{2\lambda_b}, \tag{A9}$$

$$x_2' = \frac{\left(\sigma\sqrt{3}(1+\lambda_b) + R\lambda_b\right) + (1+\lambda_b)\sqrt{\Delta_1}}{2\lambda_b}, \tag{A10}$$

$$\Delta_1 = \left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right)^2 - 8\frac{R^{res}\sigma\sqrt{3}\lambda_b}{(1+\lambda_b)}. \tag{A11}$$

Because LHS in (A6) is parabolas, the inequality is satisfied when:

$$x_1' < R^{ib} < x_2', \tag{A12}$$

given that discriminant, Δ_1, is positive. That is:

$$\left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right)^2 > 8\frac{R^{res}\sigma\sqrt{3}\lambda_b}{(1+\lambda_b)},$$

$$(\sigma\sqrt{3})^2 + 2\sigma\sqrt{3}\frac{R\lambda_b}{(1+\lambda_b)} + {\frac{R\lambda_b}{(1+\lambda_b)}}^2 - 8\frac{R^{res}\sigma\sqrt{3}\lambda_b}{(1+\lambda_b)} > 0,$$

$$(\sigma\sqrt{3})^2 - 6\sigma\sqrt{3}\frac{R\lambda_b}{(1+\lambda_b)} + {\frac{R\lambda_b}{(1+\lambda_b)}}^2 > 0,$$

$$\tag{A13}$$

[27] For the case of exposition here and in the following proofs I substitute steady state assumption on $R^{res} = R$.

which is satisfied for $\sigma\sqrt{3} < (3-2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$ or for $\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$. The first case would contradict $\sigma\sqrt{3} > \frac{R^{ib}}{2} > R^{res}$. Then, the condition on σ is:

$$\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}. \tag{A14}$$

□

Proposition 2.0.2. Low market beliefs result in a lower interbank rate and lower lending.

Proof. In the simple model, lending is given by $E^mR^k - E^lR^k$. Derivating with respect to the average market belief, m:

$$\begin{aligned}\frac{\partial\left(E^mR^k - E^lR^k\right)}{\partial m} &= \frac{\partial E^mR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} - \frac{\partial E^lR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} = \\ &= \frac{\partial R^{ib}}{\partial m}\left(\frac{\partial E^mR^k}{\partial R^{ib}} - \frac{\partial E^lR^k}{\partial R^{ib}}\right) = \frac{\left(\frac{\partial E^mR^k}{\partial R^{ib}} - \frac{\partial E^lR^k}{\partial R^{ib}}\right)}{\left(1+\frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}} - \frac{\partial E^lR^k}{\partial R^{ib}}\right)\right)},\end{aligned}$$

where the last equality is derived from the interbank market clearing condition

$$R^{ib} = m + \sqrt{3}\sigma - \frac{1}{\lambda}\left(E^mR^k - E^lR^k\right),$$

with the derivative with respect to the average market belief being

$$\frac{\partial R^{ib}}{\partial m} = 1 - \frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m} - \frac{\partial E^lR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m},\right)$$

or

$$\frac{\partial R^{ib}}{\partial m} = \frac{1}{\left(1+\frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}} - \frac{\partial E^lR^k}{\partial R^{ib}}\right)\right)}.$$

With the marginal lender and the marginal investor defined, respectively, as $E^mR^k = R^{ib}p^m$ and $R^{res} = R^{ib}p^l$, and $p^i = \frac{1}{2} - \frac{(R+\lambda_b R^{ib})}{2\sigma\sqrt{3}(1+\lambda_b)} + \frac{E^i\hat{R}^k}{2\sigma\sqrt{3}}$, I get

$$\frac{\partial E^lR^k}{\partial R^{ib}} = \frac{\lambda_b}{(1+\lambda_b)} - \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2}. \tag{A15}$$

$$\begin{aligned}\frac{\partial E^mR^k}{\partial R^{ib}} &= \frac{\partial p^m}{\partial R^{ib}}R^{ib} + p^m, \\ \Rightarrow \frac{\partial E^mR^k}{\partial R^{ib}} &= \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}}.\end{aligned} \tag{A16}$$

$\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}$ then can be rewritten as:

$$\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{(1+\lambda_b)} + \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2} =$$
$$\frac{2\sigma\sqrt{3}R^{ib}(p^m - p^l) + 2\sigma\sqrt{3}(2\sigma\sqrt{3}p^l - \frac{\lambda_b}{(1+\lambda_b)}R^{ib})}{(2\sigma\sqrt{3} - R^{ib})R^{ib}} > 0.$$

Because $p^m > p^l$ and $(1+\lambda_b)E^l R^k > \lambda_b R^{ib}$, and the model implies that $R > E^l R^k > \frac{\lambda_b R^{ib}}{1+\lambda_b}$, then $p^l = \frac{R}{R^i b} > \frac{\lambda_b}{1+\lambda_b}$. Moreover, $2\sigma\sqrt{3} > R^{ib}$. Then if follows, that $2\sigma > 0\sqrt{3}p^l - \frac{\lambda_b}{(1+\lambda_b)}R^{ib} > 0$. □

Proposition 2.0.3 *A low policy rate increases lending and lowers the interbank market rate.*

Proof.

$$\frac{\partial R^{ib}}{\partial R^{res}} = -\frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial Rib}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}\right).$$

Note, that the sign of the derivative of the interbank lending $\left(\frac{\partial E^m R^k}{\partial Rib}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}\right)$ is the opposite of the $\frac{\partial R^{ib}}{\partial R^{res}}$. Then:

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}}$$

and

$$E^l R^k = \frac{R}{1+\lambda_b} + \frac{\lambda_b R^{ib}}{1+\lambda_b} - \sqrt{3}\sigma + \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}}$$
$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}}$$

Then

$$\frac{\partial R^{ib}}{\partial R^{res}} = \frac{\frac{2\sqrt{3}\sigma}{R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}} > 0.$$

With $\frac{2\sqrt{3}\sigma}{R^{ib}} > 0$, the size of the derivative is determined by the denominator. $\lambda_b > \frac{\lambda_b}{1+\lambda_b}$ and $2\sigma\sqrt{3}p^m > \frac{\lambda_b R^{ib}}{1+\lambda_b}$ as $2\sigma\sqrt{3}p^m > 2\sigma\sqrt{3}p^l > \frac{\lambda_b R^{ib}}{1+\lambda_b}$, see the proof for proposition 2.0.2. □

Proposition 2.0.4 *The effect of a policy rate reduction is limited by the mean market belief.*

Proof. Suppose that $E^l R^k > R^{res}$. Then, for the policy rate reduction to restore lending, the change should be such that $E^l R^k < R^{res}$

$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{1}{1+\lambda_b} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}} =$$

$$= \frac{\frac{2\sqrt{3}\sigma}{R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\left(\frac{\lambda_b}{1+\lambda_b} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\right) + \frac{2\sqrt{3}\sigma}{R^{ib}} + \frac{1}{1+\lambda_b}$$

$$= \frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\right) + \frac{1}{1+\lambda_b}.$$

The nominator of the first term is larger than the denominator as:

$$\frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}}\right) > 1$$

$$2\sqrt{3}\sigma\left(\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}\right) > R^{ib}\left(\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\right)$$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}p^m - \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}} - \frac{\lambda_b}{(1+\lambda_b)}R^{ib} > -\frac{R^{ib}\lambda_b}{1+\lambda_b}$$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}p^m - \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}} > 0$$

with $\frac{R^{res}}{R^{ib}} = p^l$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}\left(p^m - p^l\right) > 0$$

This is true, as $2\sqrt{3}\sigma - R^{ib} > 0$ (the result from propositions 2.0.1 and 2.0.2) and $p^m - p^l > 0$. That is, the derivative $\frac{\partial E^l R^k}{\partial R^{res}} > 1$.

Now consider how the difference $E^l R^k - R^{res}$ changes with respect to R^{res} :

$$\frac{\partial E^l R^k}{\partial R^{res}} - 1 > 0.$$

That is, the function is increasing in R^{res} and is increasing faster than R^{res}. A downward shift in reserves reduces both the right and left-hand sides of the inequality $E^l R^k > R^{res}$, with $E^l R^k$ declining faster than R^{res}. Thus, if the difference between the marginal lender's belief and the policy rate is small, it is possible to reverse this inequality and restore lending: there will be some banker who would be better off lending on the interbank market at the low policy rate than investing herself or hoarding. However, with a large difference between $E^l R^k$ and R^{res}, which happens with very low market expectations (see proposition 2.0.2), it is not possible to restore lending with a positive policy rate □

Appendix B: Correlation of Experts' Opinions, the Mean Market Belief, and Its Variance

Expert opinions, when linearized, are defined as:

$$\theta_t = \rho_\theta \theta_{t-1} + \eta_t^i,$$

where η_t^i is the noise in the opinion of bank i's expert, with $\eta_t^h \sim N(\mu_t, \sigma_\eta)$. I assume that the noise in experts' opinions is correlated. That is, when one expert overestimates/underestimates the value of a persistent shock, others tend to do the same. Technically, I model correlated draws in the following way. First, there are N[28] independent draws from $N(\mu_t, \sigma_\eta)$. Then, each of the independent draws is rescaled:

$$\bar{\eta}_t^i = \rho^c \eta_t^1 + \sqrt{1-(\rho^c)^2}\eta_t^i,\ h \neq 1, \tag{B1}$$

where η_t^i is one of the independent draws and ρ^c is the correlation coefficient:

$$\rho^c = \frac{Cov(\bar{\eta}_t^i, \bar{\eta}_t^j)}{\sqrt{Var\left(\eta_t^i\right) Var\left(\eta_t^j\right)}}, i \neq j,$$

where $Var\left(\eta_t^i\right) = Var\left(\eta_t^j\right) = Var\left(\bar{\eta}_t\right) = \sigma_\eta^2$. The last equality comes with the observation that $Var\left(\bar{\eta}_t^h\right) = (\rho^c)^2 Var\left(\eta_t^1\right) + \left(1-(\rho^c)^2\right) Var\left(\eta_t^h\right)$. With η_t^h and η_t^1 being drawn from the same distribution, $Var\left(\bar{\eta}_t^h\right) = \left((\rho^c)^2 + 1 - (\rho^c)^2\right) Var\left(\eta_t^h\right) = Var\left(\eta_t^h\right)$.

Using (B1), I thus obtain a sequence of random variables, correlated with each other with correlation coefficient ρ^c. Because in equilibrium only the average shock to market beliefs matters, I now proceed to derive its properties. The expected average belief shock can be defined as:

$$\frac{1}{N} E\left(\eta_t^1 + \sum_{h=2}^{N} \bar{\eta}_t^h\right) = \frac{1}{N} E\left(\eta_t^1 + (N-1)\eta_t^1 \rho^c + \sqrt{1-(\rho^c)^2} \sum_{h=2}^{N} \eta_t^h\right). \tag{B2}$$

Note that η_t^1 and $\eta_t^{h, h\neq 1}$ are independent and drawn from the same distribution. This means that the expectation of their sum equals the sum of their expectations, which are unconditional expectations μ_t. The expected average belief shock is then:

$$\mu_t \frac{1}{N}\left(1 + (N-1)\left(\rho^c + \sqrt{1-(\rho^c)^2}\right)\right).$$

[28] In the text I assume the existence of a continuum of H banks, normalized to 1. Here, for computational purposes, I use N as the number of banks and set it equal to a "large number": $N = 100$.

Note that with $\rho^c = 1$ in the case of perfect correlation and with $\rho^c = 0$ in the case of no correlation, the expected average of the correlated draws corresponds to the unconditional mean. Also, unless μ_t is zero, the average belief shock is not equal to the distributional mean.

The variance of the average belief shock is then:

$$\sigma_\eta^2 \left(1 + (N-1)^2 \left((\rho^c)^2 + 1 - (\rho^c)^2\right)\right) = \sigma_\eta^2 \frac{(2 + N^2 - 2N)}{N^2}. \tag{B3}$$

Appendix C: The Bank's Filtering Problem

The state-space representation of the filtering problem is given by the following equations.

The state equation is:

$$(\mu_t) = \left(\rho_\mu \right) \times (\mu_{t-1}) + (v_t), \tag{C1}$$

where q is the variance of the i.i.d. Gaussian shock v_t.

The measurement vector consists of two types of signals: data on ξ_t and the expert opinion, θ_t. The measurement equation is:

$$\begin{pmatrix} \xi_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \times \mu_t + \begin{pmatrix} \rho_\xi \\ \rho_\theta \end{pmatrix} \begin{matrix} \xi_{t-1} \\ \theta_{t-1} \end{matrix} + \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix},$$

where ε_t and η_t are Gaussian. The measurement equation can be rewritten as:

$$\tilde{\xi}_t = C\mu_t + D\tilde{\xi}_{t-1} + \varpi_t,$$

where

$$C = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, D = \begin{pmatrix} \rho_\zeta \\ \rho_\theta \end{pmatrix}, \tilde{\xi}_t = \begin{pmatrix} \xi_t \\ \theta_t \end{pmatrix}.$$

and $\varpi_t = (v_t,\ \eta_t)'$ is a vector of measurement errors with the variance-covariance matrix:

$$R = \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta}^2 \\ \sigma_{\varepsilon\eta}^2 & \sigma_\eta^2 \end{pmatrix}.$$

where $\sigma_{\varepsilon\eta}^2$ is the covariance of errors in econometric and expert forecasts.

Appendix D: The Agency Problem

Recall that my agency problem differs from that of Gertler and Karadi (2011) in several respects. First, in my model banks have the possibility to put their funds in reserves. Second, banks are heterogeneous, with a share of them investing in a risky asset. Last but not least, some banks participate in the interbank market, transferring some funds from pessimistic to optimistic banks.

The banking family maximizes the terminal worth of each member, discounted by the stochastic discount factor $\beta^j \Omega_{t,t+j}$ arising from the household problem. The value is:

$$V_t = \max E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j} \left(N_{t+1+j}\right) =$$
$$= \max E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j} \left\{ \left(R^k_{t+1+j} - R_{t+j}\right) Q_{t+j} S_{t+j} + \left(R^{res}_{t+j} - R_{t+j}\right) Res_t + R_{t+j} N_{t+j} \right\}. \quad \text{(D1)}$$

Equation (D1) resembles the terminal worth equation in Gertler and Karadi (2011), the only difference being that I are applying it on the average level. Note that the banks' family budget constraint is

$$Q_t S_t + Res_t = N_t + B_t.$$

Also, only those banks with the lowest return expectations hoard funds in reserves (others either invest themselves or lend funds to be invested by others): $Res_t = s^h_t (N_t + B_t) = s^h_t (Q_t S_t + Res_t)$ with s^h_t being the share of hoarders. And $Q_t S_t = \left(1 - s^h_t\right)(Q_t S_t + Res_t)$. The terminal worth is:

$$E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta \Omega_{t,t+1+j} \{ \left(R^k_{t+1+j} - R_{t+j}\right) \left(1 - s^h_{t+j}\right) \left(Q_t S_{t+j} + Res_{t+j}\right) +$$
$$+ \left(R^{res}_{t+j} - R_{t+j}\right) s^h_{t+j} \left(Q_t S_{t+j} + Res_{t+j}\right) + R_{t+j} N_{t+j} \} =$$
$$= E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j} \{ \left(\left(1 - s^h_{t+j}\right) R^k_{t+1+j} + s^h_{t+j} R^{res}_{t+j} - R_{t+j} \right) \left(Q_t S_{t+j} + Res_{t+j}\right) +$$
$$+ R_{t+j} N_{t+j} \}.$$

I then have to restrict banks from borrowing from the household. Otherwise, for a non-negative $\beta^j \Omega_{t,t+j} \left(R^k_{t+1+j} - R_{t+j}\right)$ a bank would like to borrow indefinitely from the household. To avoid this, a moral hazard problem is introduced. At the beginning of the period, a banker can choose to divert a fraction λ of its assets. The depositors can recover the remaining fraction $(1-\lambda)$ of the banks' assets. For a depositor willing to participate, the banks must meet the incentives constraint:

$$V_t \geq \lambda \left(Q_t S_t + Res_t\right),$$

where V_t is the worth the banker would lose by diverting, and $\lambda \left(Q_t S_t + Res_t\right)$ is the gain from diverting. That is, the continuation value should be larger than the gain from deviating. I rewrite (D1) as:

$$V_t = v_t \left(Q_t S_t + Res_t\right) + \eta_t N_t.$$

where

$$\begin{aligned}
v_t &= E_t \left\{ (1-\theta)\,\beta \Omega_{t,t+1} \left(\left(1 - s^h_t\right) R^k_{t+1} + s^h_t R^{res}_t - R_t \right) + \beta \Omega_{t,t+1} \theta \chi_{t,t+1} v_{t+1} \right\}, \\
\eta_t &= E_t \left\{ (1-\theta) + \beta \Omega_{t,t+1} \theta z_{t,t+1} \eta_{t+1} \right\}, \\
\chi_{t,t+1} &= \frac{Q_{t+1} S_{t+1} + Res_{t+1}}{Q_t S_t + Res_t}, \\
z_{t,t+1} &= \frac{N_{t+1}}{N_t},
\end{aligned}$$

and finally I have the expression for the financial accelerator:

$$Q_t S_t + Res_t = \frac{\eta_t}{\lambda - v_t} N_t = \varphi_t N_t.$$

where φ_t is the leverage ratio, limiting the amount of assets an intermediary can acquire as a proportion of net worth.

To determine the leverage ratio, the household needs to form expectations about the future risky asset return. I assume that the household has a belief equal to the mean market belief.

Appendix E: Household, Capital Producers and Retailers

Household The first order conditions for household problem are

$$[C_t]\ \rho_t = (C_t - hC_{t-1})^{-1} - \beta h E_t\,(C_{t+1} - hC_t)^{-1} \tag{E1}$$

$$[L_t]\ \rho_t W_t - \chi L_t^{\phi} = 0 \tag{E2}$$

$$\Omega_{t,t+1} \equiv \frac{\rho_{t+1}}{\rho_t} \tag{E3}$$

$$[B_t]\ E_t \beta \Omega_{t,t+1} R_{t+1} = 1 \tag{E4}$$

where $\Omega_{t,t+1}$ is a stochastic discount factor and ρ_t is the marginal utility of consumption.

Capital Producers The first-order conditions for investment give the price of capital, Q_t:

$$[I_t] : Q_t = 1 + f(\cdot) + \frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} f'(\cdot) - E_t \beta \Omega_{t,t+1} \left(\frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} \right)^2 f'(\cdot) \tag{E5}$$

Retailers Firms are monopolistic competitors and maximize their profit:

$$\max_{P_t^*} \sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right] Y_{ft+i} \tag{E6}$$

subject to demand from households:

$$Y_{ft} = \left(\frac{P_{ft}^*}{P_t} \right)^{-\varepsilon} Y_t \tag{E7}$$

where P_t^* is the optimal price set in period t, γ is the fraction of firms which cannot reset their prices but only index to inflation, and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the one-period inflation rate.

The problem results in the first-order condition:

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p} - \mu P_{m,t+i} \right] Y_{ft+i} = 0 \tag{E8}$$

where $\mu \equiv \frac{1}{1 - \frac{1}{\varepsilon}}$ is a monopolistic mark-up.

The resulting equation for the price dynamics takes the form:

$$P_t = \left[\int_0^1 P_{ft}^{\frac{1}{1-\varepsilon}} df \right]^{1-\varepsilon} \tag{E9}$$

$$P_t = \left[(1-\gamma)\left(P_t^*\right)^{1-\varepsilon} + \gamma \left\{ (1+\pi_{t+k-1})^{\gamma_p} P_{t-1} \right\}^{1-e} \right]^{\frac{1}{1-\varepsilon}} \tag{E10}$$

Appendix F: Calibrated Parameters from Gertler and Karadi (2011)

Table F1: Calibrated Parameters from Gertler and Karadi (2011)

β	0.99	household's discount rate
h	0.815	habit parameter
χ	3.409	relative utility weight of labor
ϕ	0.276	inverse Frisch elasticity of labor supply
λ	0.381	fraction of capital to be diverted
θ	0.972	survival rate of bankers
α	0.33	capital share
U	1	steady-state capital utilization rate
$\delta(U)$	0.025	steady-state depreciation rate
ζ	7.2	elasticity of marginal depreciation with respect to utilization rate
η_i	1.728	inverse elasticity of net investment to price of capital
ε	4.167	elasticity of substitution
γ	0.779	probability of keeping prices fixed
γ_p	0.241	measure of price indexation
κ_π	1.5	inflation coefficient of Taylor rule
κ_y	0.125	output gap coefficient of Taylor rule
ρ_i	0.8	smoothing parameter of Taylor rule
$\frac{G}{Y}$	0.2	steady-state proportion of government expenditure
τ	0.001	cost of government policy
κ	10	reaction parameter for government policy

Appendix G: Comparing Model Generated Moments to the Data

In the table for the output I use GDP per capita, for the consumption - final consumption per capita, for the net worth - net financial assets of financial corporations. All data are from Eurostat for

Table G1: Comparing Model Generated Moments to the Data

	No Crisis Shock		Crisis Shock		Data
	my Model	Baseline	my Model	Baseline	
Output, Y	0.038	0.041	0.109	0.17	0.034
Consumption, C	0.039	0.036	0.222	0.28	0.041
Net Worth, N	0.062	0.108	0.783	1.54	0.817

the Euro area. The standard deviations are calculated for the log differences of the series. The first two columns show the standard deviations from simulations without crisis shocks, but with all other standard shocks in the literature. Namely, monetary policy, shock to government spending, technology shock, shock to banks' net worth, and a shock to the government policy. All shocks with 1 percent standard deviation. The other two columns show the results of simulations with crisis shock - which is 5 percent standard deviation of ξ. The last column corresponds to the Euro area data from 1995Q1 to 2016Q3. When the models are simulated without the crisis shock, the standard deviations of output and consumption are comparable to the moments in the data. The deviation of the net worth, however, is much smaller in the models. When I simulate the models with the crisis shock, conversely, models result in deviations of output and consumption that are several times larger. The deviation in the net worth is matched rather closely by my model, while the baseline model generates twice as large deviation than the observed one. Thus, when the models are simulated with the crisis shock, they overestimate the deviations of output and consumption, as they seem to overestimate the reliance of manufactures on credit.