The flow of funds in a who-to-whom framework: balance-sheet interlinkages and propagation of quantity shocks

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European Central Bank (**)
Using the w2w framework to study the propagation of shocks across the financial balance-sheets:

e.g. how are bank balance-sheets affected by sales of debt assets (to central banks)?

First (order) impact: balance-sheet reduction…

…but banks finance the sector that acquire the debt instruments sold: second order (positive) effect…

…and finance other sectors that also finance the acquiring sector: third order effect…

… infinite recursive propagation effects that can be decomposed analytically using w2w matrices
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<th>Outline</th>
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<td>3</td>
<td>Shock propagation and network centrality</td>
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</table>
1. Who-to-whom data and debt diffusion matrices

### Who-to-whom data

**Columns** break down a sector’s liabilities by counterparty.

**Rows** break down its assets.

#### Table Representation

<table>
<thead>
<tr>
<th></th>
<th>Debtor (issuer)</th>
<th></th>
<th></th>
<th>Total held</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Banks</td>
<td>Gov’t</td>
<td>Corp.</td>
<td></td>
</tr>
<tr>
<td><strong>Assets:</strong></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>Item 1 ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Item 2 ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt held : A+B+C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Liabilities:</strong></td>
<td>Gov’t</td>
<td>Corp.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Item 1 ...</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>Item 2 ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt issued : B+E+H</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total issued</strong></td>
<td>Banks: A+B+C</td>
<td>Gov’t: D+E+F</td>
<td>Corp.: G+H+I</td>
<td></td>
</tr>
</tbody>
</table>
The **ECB** provides **euro area and country networks** (with data from 13Q4) as data matrices...


A simplified framework to study propagation (I)

<table>
<thead>
<tr>
<th>Assets of,</th>
<th>SN</th>
<th>S12K</th>
<th>S121</th>
<th>S13</th>
<th>Other assets</th>
<th>TOTAL ASSETS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN</td>
<td>$Z_{1,1}$</td>
<td>$Z_{1,2}$</td>
<td>$Z_{1,3}$</td>
<td>$Z_{1,4}$</td>
<td>$n_1$</td>
<td>$t_1$</td>
</tr>
<tr>
<td>S12K</td>
<td>$Z_{2,1}$</td>
<td>$Z_{2,2}$</td>
<td>$Z_{2,3}$</td>
<td>$Z_{2,4}$</td>
<td>$n_2$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>S121</td>
<td>$Z_{3,1}$</td>
<td>$Z_{3,2}$</td>
<td>$Z_{3,3}$</td>
<td>$Z_{3,4}$</td>
<td>$n_3$</td>
<td>$t_3$</td>
</tr>
<tr>
<td>S13</td>
<td>$Z_{4,1}$</td>
<td>$Z_{4,2}$</td>
<td>$Z_{4,3}$</td>
<td>$Z_{4,4}$</td>
<td>$n_4$</td>
<td>$t_4$</td>
</tr>
</tbody>
</table>

- **Non-financial sectors**
- **Central bank**
- **Government**
- **Banks**
### A simplified framework ... (II)

<table>
<thead>
<tr>
<th>Assets of,</th>
<th>SN</th>
<th>S12K</th>
<th>S121</th>
<th>S13</th>
<th>Other assets</th>
<th>TOTAL ASSETS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN</td>
<td>$Z_{1,1}$</td>
<td>$Z_{1,2}$</td>
<td>$Z_{1,3}$</td>
<td>$g_1$</td>
<td>$n_1$</td>
<td>$t_1$</td>
</tr>
<tr>
<td>S12K</td>
<td>$Z_{2,1}$</td>
<td>$Z_{2,2}$</td>
<td>$Z_{2,3}$</td>
<td>$g_2$</td>
<td>$0$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>S121</td>
<td>$0$</td>
<td>$Z_{3,2}$</td>
<td>$0$</td>
<td>$g_3$</td>
<td>$0$</td>
<td>$t_3$</td>
</tr>
<tr>
<td>S13</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

1. Who-to-whom data and debt diffusion matrices
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Diffusion matrix

\[ t = Z \ast 1 + (n + g) \]

\[ a_{i,j} = \frac{Z_{i,j}}{t_j} \]

Financing per unit of investment

\[ t = A \ast t + (n + g) \]

Leontief inverse

\[ t = [I - A]^{-1} \ast (n + g) \]
1. Who-to-whom data and debt diffusion matrices

What is it about? Shares of ...

<table>
<thead>
<tr>
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<th>S121</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN</td>
<td>$a_{1,1}$</td>
<td>$a_{1,2}$</td>
<td>$a_{1,3}$</td>
</tr>
<tr>
<td>S12K</td>
<td>$a_{2,1}$</td>
<td>$a_{2,2}$</td>
<td>$a_{2,3}$</td>
</tr>
<tr>
<td>S121</td>
<td>0</td>
<td>$a_{3,2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

- non-bank financing
- bank financing
- central bank lending
- interbank borrowing
- deposits+
bonds
- currency
- reserves
- bank capital ratio
## 1. Who-to-whom data and debt diffusion matrices

### An example …

<table>
<thead>
<tr>
<th></th>
<th>SN</th>
<th>S12K</th>
<th>S121</th>
</tr>
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<tbody>
<tr>
<td>SN</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>S12K</td>
<td>0.5</td>
<td>0.25</td>
<td>0.7</td>
</tr>
<tr>
<td>S121</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
</tr>
</tbody>
</table>
Back to initial example: how is sector investment (assets) affected by purchases by the central bank of bank assets?

\[ \Delta g = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \]

\[ \Delta t = [I - A]^{-1} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \]

\[ \Delta t = \begin{bmatrix} 2.13 & 1.83 & 1.92 \\ 1.49 & 2.68 & 2.32 \\ 0.07 & 0.13 & 1.12 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0.09 \\ -0.36 \\ 0.98 \end{bmatrix} \]
| 1 | Who-to-whom data and debt diffusion matrices |
| 2 | An analytical decomposition of the propagation of shocks |
| 3 | Shock propagation and network centrality |
2. An analytical decomposition of the propagation of shocks

Continuing with the example, bank investment...

\[-0.36 = -2.68 + 2.32 = \frac{(1 - a_{1,1}) + (a_{2,3}(1 - a_{1,1}) + a_{1,3}a_{2,1})}{\det(I - A)}\]

Diffusion:

\[\Delta t = [I - A]^{-1} \Delta g = \Delta g + A\Delta g + A^2\Delta g + A^3\Delta g + \cdots + A^n\Delta g + \cdots\]

- Purchases, sellings, first order effect
- \((n+1)\) order effects: propagation effects
2. An analytical decomposition of the propagation of shocks

**n-order effects on bank investment** ($\Delta g = [0 \ -1 \ 1]'$)

- **Positive second order**
- **Positive declining n-order propagation**

First order

![Graph showing n-order effects on bank investment](image)

- n-order effect
- Accumulated effect

$\approx -0.36$
2. An analytical decomposition of the propagation of shocks

n-order effects on sectors’ investment ($\Delta g = [0 \ -1 \ 1]'$)

Second and higher order propagation effects are relevant for banks (and non-financial sectors), but not for the central bank… why?
2. An analytical decomposition of the propagation of shocks

Eigenbase representation (*A non-defective*):

n-order effect: “linear operation” *A* applied *n* – 1 times on *g*

\[ A^{n-1} g \]

Decomposing the action of *A*:

\[ Ag = \mathbf{V} \mathbf{E} \mathbf{V}^{-1} g \]

i.e.:

\[ A^n g = \rho_1^n c_1 \mathbf{v}_1 + \rho_2^n c_2 \mathbf{v}_2 + \rho_3^n c_3 \mathbf{v}_3 \]

Second eigenvector, second column of *V*

Second order effect: “linear operation” *A* applied *n* – 1 times on *g*
Eigenvectors in our example

\[ \mathbf{v}_1 = \begin{bmatrix} 0.68 \\ 0.73 \\ 0.05 \end{bmatrix} \]

\[ \rho_1 = 0.76 \]

\[ \mathbf{v}_2 = \begin{bmatrix} 0.75 \\ -0.66 \\ 0.08 \end{bmatrix} \]

\[ \rho_2 = -0.40 \]

\[ \mathbf{v}_3 = \begin{bmatrix} 0.77 \\ 0.16 \\ -0.61 \end{bmatrix} \]

\[ \rho_3 = -0.01 \]
2. An analytical decomposition of the propagation of shocks

Eigenbase decomposition of \((n>1)\)-order effects on bank investment \((\Delta g = [0 \quad -1 \quad 1])\)

Second component (for sector S12K) of second eigenvector

Second component of \(g\) in the eigenbase (second component of \(V^{-1}\Delta g\))
2. An analytical decomposition of the propagation of shocks

Eigenbase decomposition of \((n>1)\)-order effects on sectors’ investment \((\Delta g = [0 ~ -1 ~ 1])\)

Why central bank propagation effects are small?: the mathematical answer

Low components in the large eigenvalues, high component in the small eigenvalue
Accumulated eigenbase decomposition of (n>1)-order effects on bank investment ($\Delta g = [0\ -1\ 1]$)

- Third eigenvector can be ignored: **dimensionality reduction**. For larger matrices, two, three eigenvectors are enough to describe (n>1)-order propagation effects (depending on “eigenvalue jumps”)

- After a few initial propagation effects, **propagation is dominated by the first eigenvector associated to the largest eigenvalue**.
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1. Shock propagation and network centrality

Who-to-whom as a network

ECB website for journalists: www.euro-area-statistics.org
3. Shock propagation and network centrality

Network centrality

- interconnectedness ranking: C-D-A-B
- For more complex networks (in particular weighted networks), the solution is not trivial
- **Eigenvector centrality** provides “interconnectedness” scores/rankings on the basis of the matrix representation of the network: **Perron eigenvector** (principal vector of Perron eigenvalue)
  - here (0.50 0.29 0.61 0.54)
“Eigenvector centrality” is nothing but the eigenvector associated to the largest eigenvalue of the eigenbase decomposition (Perron eigenvalue) of the network matrix.

The lack of propagation effects for the Central bank (S121) in the exercise above is a manifestation of its (relative) lack of centrality. On the contrary banks (S12K) are very central and show large (n>1)-order effects.

Perron-Frobenius theorem guarantees the existence of $\rho$, $\nu$ for irreducible matrices.
Eigenvector centrality in financial data

- indicates sector interconnectedness via direct (first order) investment and financing links, but also indirect (second and higher order) links via financial intermediation.

- Recursive interpretation: “the more a sector is linked to sectors with high score, the higher the score of the sector is”

- Perron’s vector, when calculated on networks …
  - …showing creditor-debtor links, provides rankings of interconnectedness via investment: vulnerability indicator
  - …showing debtor-creditor links (represented by the transposed matrix of a creditor-debtor network), provides rankings of interconnectedness via financing: systemic risk indicator
3. Shock propagation and network centrality

Scores take into account indirect investment-financing links… for debt in the euro area …

Notes:
- Units: components of normalized Perron eigenvectors; network of debt (debt securities, loans and deposits); 16Q4
- S11: non-financial corporations; S12K: MFIs (S121+S122+S123); S124: investment funds; S12O: OFIs (S125+S126+S127); S128: insurance corporations; S129: pension funds; S13: general government; S1M: households and NPISHs (S14+S15); S12: rest of the world

Households are as systemic as government and the rest of the world in spite of having half their liabilities!!!
3. Shock propagation and network centrality

Indirect links matter. Rankings in the euro area …

<table>
<thead>
<tr>
<th>based on volume (degree centrality)</th>
<th>eigenvector centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset volume</td>
<td>Investment centrality</td>
</tr>
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</table>

Households, RoW present substantial differences in investment

Notes:
- S11: non-financial corporations;
- S12K: MFIs (S121+S122+S123);
- S124: investment funds;
- S12O: OFIs (S125+S126+S127);
- S128: insurance corporations;
- S129: pension funds;
- S13: general government;
- S1M: households and NPISHs (S14+S15);
- S12: rest of the world.
3. Shock propagation and network centrality

Eigenvector centrality in the euro area. Debt network. MFIs. Investment

Units: left panel: S12K component in normalized Perron's eigenvector and ranking position; right panel: Perron's eigenvalue; debt network/matrix; 16Q4

- High centrality of MFIs in all euro area countries: high persistence of n-order propagation effects in quantity shocks
- Exceptions are IE, NL, LU, with lower centrality: OFIs to present high n-order effects, particularly persistent in LU (high Perron eigenvalue)
- IT, ES: MFI central and relatively higher n-order propagation effects, but less persistent
Summary and conclusions

- **A Leontief framework and eigenbase representation** applied to who-to-whom matrices allow for analyzing second and higher order indirect, propagation effects of sectors’ asset acquisitions and disposals.

- The eigenbase representation also enables **reduced dimensionality**: propagation n-order effects can be approximated as a linear combination or 2, 3 vectors.

- Perron eigenvector, a standard measure of **network centrality**, is dominating the propagation effects. For large, sparse matrices, the Perron eigenvector is sufficient to characterize propagation.

- A first examination of propagation based on eigenvector centrality for MFIs in the euro area (debt network) show that they are affected by large indirect effects: *e.g. disposals of assets by MFI result in less than proportional decreases in leverage.*


Thank you for your attention!
Reserve slides
Perturbations to the diffusion matrix

**Perturbing diffusion**

\[ \Delta t = \Delta g + A\Delta g + A^2\Delta g + A^3\Delta g + \cdots + A^n\Delta g + \cdots \]

\[ \Delta t = \Delta g + A_0\Delta g + A_1A_0\Delta g + A_2A_1A_0\Delta g + \cdots + A_nA_{n-1} \cdots A_0\Delta g + \cdots \]

\[ A_n = \delta (A_{n-1}) \]

Left eigenvector

\[ \delta (\rho) = \frac{w\delta (A_{n-1})v}{w'v} \]

Angle between right, left eigenvectors

\[ |\delta(\rho)| \leq \frac{1}{\cos \theta} \|\delta(A_{n-1})\| \]
Perron’s eigenvector angle

Note:
- Units: angle between left and right eigenvectors of Perron’s eigenvalue (debt network/matrix); radians; 16Q4