Non-standard monetary policy, asset prices and macroprudential policy in a monetary union

L. Burlon A. Gerali A. Notarpietro M. Pisani*

6 October 2016

Abstract

This paper evaluates the macroeconomic and financial effects of the Eurosystem’s Asset Purchase Programme and its interaction with a country member macroprudential policy. We assume that some households in a euro area (EA) country are subject to a borrowing constraint, and that the local real estate acts the collateral. In order to highlight the potential synergies and conflicts between the (union-wide) APP and region-specific macroprudential policies, we simulate a situation in which, as the APP is carried out, households in one region of the EA develop overly optimistic expectations about local real estate prices. Our results are as follows. First, a relatively large loan-to-value (LTV) ratio in one region can widely amplify the expansionary effect of the union-wide non-standard monetary policy measures on domestic households borrowing. Second, during the APP implementation, the increase in households’ borrowing in one region can be further magnified by the combination of a high LTV ratio and overly optimistic expectations. Third, region-specific macroprudential measures can stabilize private sector borrowing with limited negative effects on economic activity.

JEL Classification Numbers: E43; E52; E58. Keywords: DSGE models, open-economy macroeconomics, non-standard monetary policy, zero lower bound, macroprudential policy.

*Banca d’Italia. Preliminary. Usual disclaimers apply. We thank for useful comments Stefano Neri, Stefano Siviero, and participants at the Bank of Italy Workshop “Unconventional monetary policy: effectiveness and risks” (March 2016). All errors are ours. Corresponding author: Alessandro Notarpietro. Directorate General for Economics, Statistics and Research, Via Nazionale 91, 00184 Rome, Italy. Phone: +39 06 4792 2623, E-Mail: alessandro.notarpietro@bancaditalia.it.
“Should any threat to financial stability materialise, specific macro-prudential measures should be implemented by national authorities to deal with local risks, without the need to alter the expansionary stance of monetary policy.”

Governor of the Bank of Italy, Ignazio Visco

1 Introduction

The launch of the Eurosystem’s Asset Purchase Programme (APP) has ignited a debate on its direct and indirect effects on both the real economy and the financial sector. The main objective of the APP is to help the Eurosystem achieve its price stability objective in a period of depressed aggregate demand and persistently low inflation. The accommodative stance of monetary policy includes the announcement and commitment to keep the interest rates low for a prolonged period of time (forward guidance).

Some observers have claimed that such monetary policy stance may not be appropriate, once financial markets conditions are taken into account. It has been observed that the current situation characterized by low inflation, subdued growth, and low interest rates stimulates a “search for yield” in financial markets, which can in turn generate financial instability. More specifically, it has been argued that the announced intention to keep short-term interest rates at low levels for a prolonged period of time and the reduction in long-term yields that the APP generates may induce “excessive” (i.e., not driven by fundamental factors) increases in asset prices and private-sector borrowing, at least in some regions of the euro area (EA henceforth).

The related crucial policy question is whether monetary policy should take these side effects into consideration or if local imbalances in financial markets should be addressed by local macroprudential policy authorities. The latter can indeed dampen excessive fluctuations in borrowing, leverage, and asset prices by adjusting country-specific instruments such as the loan-to-value (LTV henceforth) ratios in housing markets. The answer to this question depends on the ability of the regional macroprudential policy to counterbalance the expansionary effects of

1See Visco (2015).
the monetary union-wide APP on regional economic activity and inflation.

This paper contributes to the debate by providing an assessment of the macroeconomic and financial effects of the APP and its interaction with regional macroprudential policy. For this purpose, we simulate a large-scale New Keynesian dynamic general equilibrium model calibrated to the EA and the rest of the world (RW). The EA is modeled as a monetary union of two regions, Home and rest of the euro area (REA), where Home has medium size (its GDP being around 20% of overall EA GDP).

There are three crucial features.

Following Chen, Curdia, and Ferrero (2012), in each EA region some households (labeled as “restricted”) have access only to long-term sovereign bonds. The APP reduces long-term interest rates and induces restricted households to increase consumption and investment via the standard intertemporal substitution effect.

The second feature is the presence, in both EA regions, of “indebted” households. Following Iacoviello (2005), we assume that a fraction of households are subject to a borrowing constraint, where the local real estate is the collateral. The LTV ratio (which affects the borrowing constraint) is region-specific and can be appropriately changed by the local macroprudential authority to favor the financial stability of the region.

The third feature, in line with Dupor (2005), is the presence in the Home region of households that during the implementation of the non-standard monetary policy measures have irrational overly optimistic expectations about the value of real estate. These irrational expectations lead to a non-fundamental increase in domestic real estate prices, and, through the borrowing constraint, in households’ debt. Thus, the increase in Home households’ debt can be considered as “excessive” because it is driven by a shock which is non-fundamental and amplified by a financial friction.

We simulate four scenarios. First, the benchmark scenario corresponds to the simulation of the APP, i.e. euro 180 billion per quarter, that last for seven quarters. Long-term sovereign bonds are held by the EA central bank to maturity

\[ \text{See also In’t Veld et al. (2014).} \]

\[ \text{The scenarios are simulated assuming perfect foresight. Households and firms are surprised by the shock in the first period and fully anticipate shocks perturbing the economy in subsequent periods.} \]
Moreover, the central bank promises to keep the short-term monetary policy rate constant at its baseline level during the first two years (forward guidance); thereafter, it resumes to follow the Taylor rule. The LTV ratio is constant at its baseline (steady-state) value and, in particular, the LTV is larger in the Home region than in the REA.

Second, to highlight the role of the borrowing constraint in making the APP transmission asymmetric across EA regions, we compare the results of the benchmark scenario with those obtained when simulating the APP under the assumption of a lower Home LTV ratio (than in the benchmark scenario).

Third, we add to the benchmark scenario an irrational shock, assumed to hit Home households’ expectations about Home housing prices. We calibrate this shock to generate, during the APP, an overvaluation of housing proportional to those registered during the early 2000s in the EA.

Fourth, we simulate the third scenario under the assumption that Home LTV ratio is endogenous, as it is set according to a macroprudential rule reacting to the domestic (indebted) households’ debt dynamics.

Our results are as follows. First, a higher LTV ratio in one region can widely amplify the domestic propagation of the APP and its expansionary effect on domestic households’ borrowing. Second, during the implementation of the EA-wide non-conventional measures, the increase in households’ borrowing can be further magnified in one EA region by the combination of high LTV ratio and irrational expectations. Third, region-specific macroprudential measures can stabilize private sector borrowing, with limited negative effects on domestic economic activity and no significant effects on inflation.

The paper builds upon several recent contributions. Burlon, Gerali, Notarpietro, and Pisani (2015) evaluate the impact of APP on EA macroeconomic and financial conditions, assuming that some households are subject to a borrowing constraint (the borrowing limit is exogenously set, thus there is no collateral). Chen, Curdia, and Ferrero (2012) introduce financial market segmentation à la Andres, Lopez-Salido, and Nelson (2004) to evaluate the impact of US quantitative easing. We tailor their set-up to a monetary union framework and consider the role of a country-specific borrowing constraint in propagating the shocks. Previous studies on the US such as, e.g., Gertler and Karadi (2011), Gertler and Kiyotaki...
(2010), and Curdia and Woodford (2011) study the effects of security purchase programs in closed-economy settings. Our exercise is calibrated to the EA.

Our contribution is in the same spirit of several recent others that analyze the macroeconomic and financial stability effects of monetary and macroprudential policies interaction with the lens of DSGE models. Among others, Brzoza-Brzezina, Kolasa and Makarski (2015), Beau et al. (2012), Collard et al. (2012), Gelain and Ilbas (2014), and Quint and Rabanal (2014).\footnote{See Neri (2016) for a review of the literature on DSGE models and macroprudential policy.} Rubio and Carrasco-Gallego (2014) develop a DSGE model for the EA with housing and collateral borrowing constraints, where the macroprudential authority sets the LTV ratio according to credit dynamics. Angelini and Gerali (2012) use an estimated DSGE model of the euro area à la Gerali et al. (2010) featuring several financial frictions and a macroprudential authority. We look at the interaction in a monetary union framework between regional macroprudential policy and a union-wide non-standard monetary policy such as the APP, confirming the beneficial interaction in terms of simultaneous macroeconomic and financial stabilization already found in the literature for the standard monetary policy. Finally, as in several previous contributions, our modeling choices do not allow us to consider either risk in general or systemic risk in particular.

The paper is organized as follows. Section 2 describes the main features of the model and the calibration. Section 3 reports the main results. Section 4 concludes.

\section{The model}

We first provide an overview of the model. Second, we illustrate the crucial features for the simulations, that is, the financial fragmentation across heterogeneous agents, the non-fundamental shock to housing prices expectations, and the macroprudential, monetary, and fiscal policies. Third, we define the equilibrium in our economy. Finally, we report the calibration.
2.1 Overview

The model represents a world economy composed of three regions, that is, Home, REA (Home+REA=EA), and RW. The size of the world economy is normalized to one. Home, REA, and RW have sizes equal to $n$, $n^*$, and $(1 - n - n^*)$, with $n$, $n^* > 0$ and $n + n^* < 1$. For each region, the size refers to the mass of households, to the mass of firms operating in each sector, and, in the case of each EA region, the mass of capital producers. Home and REA share the currency and the monetary authority. The latter sets the nominal interest rate according to EA-wide variables (a standard Taylor rule holds) when it does not deliberately enact forward guidance (the policy rate is deliberately kept at its baseline level). The presence of the RW outside the EA allows to assess the role of the nominal exchange rate and extra-EA trade for the transmission of the shocks.

The crucial features of the model are three.

First, we introduce financial segmentation à la Chen, Curdia, and Ferrero (2012), which allows the APP to have real effects. In each EA region there are two types of households, restricted and unrestricted. The restricted households have access only to the domestic long-term sovereign bond market and invest in physical capital accumulation, as they hold a constant (parametric) share of domestic “capital producers.” The unrestricted households (1) have access to markets of domestic short-term (one-period) nominal private and public bonds, and of domestic long-term sovereign bonds, (2) invest in physical capital accumulation (similarly to the restricted households, they hold a constant share of domestic capital producers), and (3) trade a riskless short-term private bond with other regions’ households.\(^5\)

The second feature, following Iacoviello (2005), is that in each EA region there is a third type of households that we label “indebted.”\(^6\) This fraction of households trade in a domestic short-term nominal bond with the (domestic) unrestricted households, and are subject to a borrowing constraint where the local real estate is the collateral. In the borrowing constraint there is an LTV ratio, whose value

\(^5\)The assumed financial market structure allows us to have meaningful EA net foreign asset position and trade balance.

\(^6\)There is no overlap across household types, as the set \{indebted, restricted, unrestricted\} constitutes a partition of the set of households in each region. The labels for these types of households are mainly for exposition purposes, we simply use the terminology of Chen, Curdia, and Ferrero (2012) for unrestricted and restricted households.
is region-specific and can be changed by the local macroprudential authority.

Third, we introduce a non-fundamental shock to housing price expectations in the Home region. Following Dupor (2005), we assume that households have (irrational) overly optimistic expectations about the (future) value of domestic housing. These irrational expectations lead to both non-fundamental movements in real estate prices as well as “excess” transactions in real estate. Because of the non-fundamental increase in the price of Home housing, there is room for enacting macroprudential measures in the Home country.

The remaining features of the model are rather standard and in line with New Keynesian open economy models. Households consume a final good, which is a composite of intermediate non-tradable and tradable goods. The latter are domestically produced or imported. All households supply differentiated labor services to domestic firms and act as wage setters in monopolistically competitive labor markets by charging a mark-up over their marginal rate of substitution between consumption and leisure. The capital producers accumulate physical capital by demanding final investment goods subject to quadratic adjustment costs on investment change (so a “Tobin’s Q” holds). They rent capital to domestic firms producing intermediate goods. They maximize profits with respect to capital and investment taking prices as given, and evaluate returns according to a weighted (by the corresponding shares) average of unrestricted and restricted households’ stochastic discount factors, as in Chen, Curdia and Ferrero (2012). The (net) revenues are rebated in a lump-sum way to domestic unrestricted and unrestricted households, according to their corresponding shares.

On the production side, there are perfectly competitive firms that produce two final goods (consumption and investment goods) and monopolistic firms that produce intermediate goods. Firms are owned by domestic unrestricted households. The two final goods are sold domestically and are produced combining all available intermediate goods using a constant-elasticity-of-substitution (CES) production function. The two resulting bundles can have different composition. Intermediate tradable and non-tradable goods are produced combining domestic capital and labor, that are assumed to be mobile across sectors. Intermediate tradable goods can be sold domestically and abroad. Because intermediate goods are differentiated, firms have market power and restrict output to create excess
profits. We also assume that markets for tradable goods are segmented, so that firms can set a different price for each of the three markets. In line with other dynamic general equilibrium models of the EA, we include adjustment costs on real and nominal variables, ensuring that consumption, production, and prices react in a gradual way to a shock.\(^7\) On the real side, habits and quadratic costs prolong the adjustment of consumption and investment, respectively. On the nominal side, quadratic costs make wages and prices sticky.\(^8\)

In what follows, we report the main new equations for the Home country. Similar equations hold in the REA. Differently from Home and REA, in the RW there exists only one standard representative household. We report other main equations in the Appendix, as they are standard for a New Keynesian model such as ours.

### 2.2 Indebted households

There exists a continuum of indebted households, indexed by \(j'\), with \(j' \epsilon (0, n\lambda_D]\), where \(0 \leq \lambda_D \leq 1\). Their preferences are additively separable in consumption and labor effort. The generic indebted household \(j'\) receives utility from non-durable consumption \(C_D(j')\), housing \(h_D(j')\), which is a durable good, and disutility from labor \(L_D(j')\).\(^9\) The expected lifetime utility is

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{D,t}(j') - \varsigma C_{D,t-1})^{1-\sigma}}{(1-\sigma)} + \chi \log h_{D,t}(j') - \frac{L_{D,t}(j')^{1+\tau}}{1+\tau} \right] \right\},
\]

where \(E_0\) denotes the expectation conditional on information set at date 0, \(\beta_D\) is the discount factor \((0 < \beta_D < 1)\), \(1/\sigma\) is the elasticity of intertemporal substitution \((\sigma > 0)\), \(\chi > 0\) is the weight of housing, and \(1/\tau\) is the labor Frisch elasticity \((\tau > 0)\). The parameter \(\varsigma\) \((0 < \varsigma < 1)\) represents external habit formation in consumption.

Indebted households have access only to the market of domestic short-term

\(^7\)See, among the others, Christoffel, Coenen, and Warne 2008 and Gomes, Jacquinot, and Pisani 2010.

\(^8\)See Rotemberg (1982).

\(^9\)Following common practice in the New Keynesian literature, the assumption of cashless economy holds in the model.
nominal bonds. They supply labor to domestic firms. The implied budget constraint is

\[ B_{D,t}^S (j') - B_{D,t-1}^S (j') R_{t-1}^S = W_{D,t} (j') L_{D,t} (j') - Q^h_t (h_{D,t} (j') - h_{D,t-1} (j')) - P_t C_{D,t} (j'), \]

where \( B_{D,t}^S \) is the (end-of-period) bond that pays the (gross) nominal interest rate \( R_t^S \) (\( B_{D,t}^S < 0 \) is debt). The variable \( W_{D,t} (j') \) represents the nominal wage, the variable \( Q^h_t \) the nominal price of residential real estate \( h_{D,t} \). Finally, \( P_t \) is the price of the consumption bundle.

Home indebted households are subject to the borrowing constraint

\[ -B_{D,t}^S (j') R_t^S \leq m_t E_t \left( Q^h_t+1 h_{D,t} (j') \right), \]

where \( 0 \leq m_t \leq 1 \) is the LTV ratio. The latter is time-varying, because its value is appropriately decided by the domestic macroprudential authority to guarantee the financial stability of the region.

Finally, for simplicity, we assume that indebted households’ wage and labor supply are the same as those optimally chosen by the domestic unrestricted households. As such, indebted households maximize their utility only with respect to consumption, housing and debt while they take labor income as given.\(^{10}\) As the implied first order conditions are rather standard, we do not report them to save on space.\(^{11}\)

### 2.3 Restricted households

There exists a continuum of restricted households, indexed by \( j'' \), with \( j'' \in (n \lambda_D, n \lambda_R] \), where \( \lambda_R \geq 0 \) and \( \lambda_R + \lambda_D < 1 \). Their preferences are the same as those of indebted

\(^{10}\)This assumption may indirectly induce a smaller response of indebted households to demand shocks, as financially indebted households would adjust more their labor supply. Although theoretically appealing a disjoint labor supply choice by indebted households has limited quantitative grip in presence of nominal rigidities. See, e.g., Guerrieri and Lorenzoni (2011) for the importance of the consumption/labor choice of financially constrained agents.

\(^{11}\)They are available upon request.
households (see eq. 1) except that they do not derive any utility from housing. They can differ for the discount factor \((\beta_R \neq \beta_D)\), and have access only to the market of long-term sovereign bonds, modelled as perpetuities, that is, they cost \(P^L_t\) at time \(t\) and pay a coupon \(\kappa\) at time \(t+s+1\), where \(\kappa \in (0,1)\). Each restricted household \(j''\) gets profits from ownership of domestic capital producers, according to the same constant (parametric) share \(0 < \omega < 1\). The budget constraint is

\[
P_t^L B^L_{R,t}(j'') - P_t^L R^L_t B^L_{R,t-1}(j'') = \omega \Pi_t + W^L_{R,t}(j'') L^L_{R,t}(j'') - P_t C^L_{R,t}(j'') - AC^W_{R,t}(j''),
\]

where \(B^L_{R,t}\) is the amount of long-term sovereign bonds and the variable \(\Pi_t\) is the Home capital producers’ aggregate profit. The variable \(R^L_t\) is the gross yield to maturity at time \(t\) on the long-term bond \(R^L_t = 1/P^L_t + \kappa\).

Finally, households act as wage setters in a monopolistic competitive labor market. Each household \(j''\) supplies one particular type of labor services, which is an imperfect substitute to services supplied by other restricted households. It sets its nominal wage taking into account labor demand and quadratic adjustment costs \(AC^W_R\) à la Rotemberg (1982) on the nominal wage \(W^L_R(j'')\):

\[
AC^W_{R,t}(j'') \equiv \frac{\kappa_W}{2} \left( \frac{W^L_{R,t}(j'')/W^L_{R,t-1}(j'')}{\Pi_{W,R,t-1}^{\alpha_W} \Pi_{\bar{\Pi}}^{1-\alpha_W}} - 1 \right)^2 W^L_{R,t} L^L_{R,t},
\]

where \(\kappa_W > 0\) and \(0 \leq \alpha_W \leq 1\) are parameters, the variable \(\Pi_{W,R,t} \equiv W_{R,t}/W_{R,t-1}\) is the wage inflation rate, and \(\Pi_{\bar{\Pi}}\) is the constant long-run inflation target of the EA monetary authority. The adjustment costs are proportional to the per-capita wage bill of restricted households, \(W_{R,t} L_{R,t}\).

Restricted households are crucial for the APP to have real effects in our model.

---

\(^{12}\)See Woodford (2001).

\(^{13}\)As the implied first order conditions are rather standard we do not report them to save on space. They are available upon request.
As they cannot make arbitrage between short-term and long-term bonds, their consumption (and savings) decisions depend only upon the long-term interest rate, which the monetary policy authority can affect by directly intervening in the long-term sovereign bond market.\textsuperscript{14}

### 2.4 Unrestricted households

There exists a continuum of indebted households, indexed by $j$, with $j \epsilon (n\lambda_D + n\lambda_R, n]$. These households have the same preferences as indebted households (see eq. 1), thus they consume non-durable goods and real estate services and supply labor. The only difference is the discount factor, as the unrestricted households’ discount factor $\beta_U$ is strictly larger than that of indebted households ($0 < \beta_D < \beta_U < 1$).\textsuperscript{15}

Home unrestricted households have access to multiple financial assets (all denominated in euro terms): a short-term private bond, $B^S_U$, exchanged with domestic indebted households; a short-term sovereign bond, $B^G$, exchanged with the domestic government; a short-term bond, $B^P$, denominated in euro terms, exchanged with REA unrestricted and RW households; a long-term sovereign bond, $B^L_U$, exchanged with the domestic restricted households, the domestic government and the EA monetary authority, because of the APP. Moreover, they exchange local real estate with the domestic indebted households. Thus, they have several opportunities to smooth consumption when facing a shock. The budget constraint

\textsuperscript{14}We do not allow restricted households to hold real estate to keep the model parsimonious and to get the APP having only an indirect (general equilibrium) effect on the real estate market, associated with the improvement in overall economic conditions. Qualitatively, our results would not greatly change if restricted households would engage in trading real estate.

\textsuperscript{15}The assumption is needed to get a binding borrowing constraint. See Iacoviello (2005).
of the generic unrestricted household $j$ is

$$
P_t^L B_t^L (j) - P_t^L R_t^L B_{t-1}^L (j) 
+ B_t^S (j) - B_{t-1}^S (j) R_{t-1}^S 
+ B_t^G (j) - B_{t-1}^G (j) R_{t-1} 
+ B_t^P (j) - B_{t-1}^P (j) R_{t-1}^P (1 - \phi_t) \tag{6}
= W_{U,t} (j) L_{U,t} (j) + (1 - \omega) \Pi_{t}^{prof} + \Pi_{t}^P (j) - P_t C_{U,t} (j) - Q_t^b (h_{U,t} (j) - h_{U,t-1} (j))
- T A X_t (j) - AC_{U,t}^W (j) - AC_{U,t}^B (j) - AC_{U,t}^h (j),
$$

where the short-term government bond $B_t^G$ pays the EA monetary policy rate $R_t$. The term $\phi_t$ represents an exponential adjustment costs, needed to stabilize the position in that bond.\textsuperscript{16} The variable $\Pi_{t}^{prof}$ represents the Home capital producers’ aggregate profit, and $(1 - \omega)$ the constant (parametric) share, assumed to be the same across unrestricted households. Unrestricted households own all domestic firms. The variable $\Pi_{t}^P (j)$ stands for dividends from ownership of domestic monopolistic firms (claims to firms’ profits are not internationally tradable). The term $T A X_t$ represents lump-sum taxes. The unrestricted households supply labor services under monopolistic competition, and face quadratic adjustment costs $AC_{U,t}^W$ when setting nominal wages (the cost is similar to the one paid by restricted households, see eq. 5). They also pay adjustment costs $AC_{U,t}^B$ on all bond holdings.\textsuperscript{17} The presence of adjustment costs guarantees that the bond holdings follow

\textsuperscript{16}The adjustment cost is defined as

$$
\phi_B \equiv \frac{\phi_{b1}}{\phi_{b2}} \frac{\exp (\phi_{b2} (B_t^P - \bar{B}^P)) - 1}{\exp (\phi_{b2} (B_t^P - \bar{B}^P)) + 1}, \text{ with } \phi_{b1}, \phi_{b2} > 0
$$

where $B_t^P \bar{B}^P$ are the period-by-period and steady-state positions of the representative Home unrestricted household, respectively. Both are taken as given in the maximization problem. A similar cost holds for the RW household.

\textsuperscript{17}We assume a standard quadratic form for the adjustment cost, that is,

$$
AC_{U,t}^B (j) \equiv \frac{\phi_{bL}}{2} (P_t^L B_{U,t}^L (j) - \bar{P}^L \bar{B}^L)^2, \text{ with } \phi_{bL} > 0
$$

where $\bar{P}^L \bar{B}^L$ is the (symmetric) steady-state value of the long-term sovereign bond. The adjustment cost guarantees that the bond holdings follow a stationary process and that the economy converges to the steady state.
a stationary process and that the economy converges to the steady state. Finally, they pay costs $AC_{h,t}^U(j)$ when changing the amount of housing services,

$$AC_{h,t}^U(j) = \frac{\psi_h}{2} \left( \frac{h_{U,t}(j)}{h_{U,t-1}(j)} - 1 \right)^2 Q_t^h h_{U,t}(j),$$

where $\psi_h > 0$ is a parameter.

First order conditions imply no-arbitrage conditions for the unrestricted households. Thus, in equilibrium the interest rates paid by the different bonds are equal to the monetary policy rate $R_t$, except for the spreads induced by the longer maturity and the adjustment costs.

### 2.5 Capital producers

There exists a continuum of mass $0 \leq n \leq 1$ of firms $e$ that produce physical capital. They optimally choose capital $K_t$ and investment $I_t$ to maximize profits subject to the law of capital accumulation, the adjustment costs on investment, and taking prices as given. The law of motion of capital accumulation is

$$K_t(e) = (1 - \delta) K_{t-1}(e) + \left( 1 - AC_t^I(e) \right) I_t(e), \quad (7)$$

where $0 < \delta < 1$ is the depreciation rate. The adjustment cost on investment $AC_t^I(e)$ is

$$AC_t^I(e) = \frac{\phi_I}{2} \left( \frac{I_t(e)}{I_{t-1}(e)} - 1 \right)^2, \text{ with } \phi_I > 0. \quad (8)$$

Capital producers rent existing physical capital stock $K_{t-1}(e)$ at the nominal rate $R_t^K$ to domestic firms producing intermediate tradable and non-tradable goods. Investment is a final non-tradable good, composed of intermediate tradable (domestic and imported) and non-tradable goods. Capital producers buy it in the corresponding market at price $P_I$.  

Capital producers maximize discount future profits with respect to end-of-

---

18 As the implied first order conditions are rather standard we do not report them to save on space. They are available upon request.

19 See Chen, Curdia, and Ferrero (2012) for the details. Our calibration implies that households can modify their financial positions without facing relevant adjustment costs.

20 Because of the adjustment costs on investment, a “Tobin’s Q” holds.
period capital and current period investment, using the stochastic discount rates of restricted and unrestricted households, aggregated according to the corresponding shares $\omega$ and $(1 - \omega)$, respectively.

2.6 The non-fundamental shock to housing prices expectations

Following Dupor (2005), we introduce a non-fundamental shock to expectations about the price of housing in the Home region. Home unrestricted and indebted households have “irrationally optimistic” expectations about the value of Home housing. These irrational expectations lead to both non-fundamental movements in real estate prices as well as excess (relative to fundamental) demand for real estate. Because of the non-fundamental increase in the price of Home housing, there is room for enacting macroprudential measures in the Home country. For the unrestricted households, the housing demand is obtained by maximizing the intertemporal utility function with respect to housing subject to the current and expected future budget constraints. The implied first order condition of the generic Home unrestricted household $j$ is

$$\lambda_{U,t}(j)q^H_t = \frac{1}{R_{U,t}(j)} + \beta E_t \left( \lambda_{U,t+1}(j) \theta_{t+1} q^H_{t+1} \pi_{t+1} \right),$$

(9)

where $\lambda_U$ is the budget-constraint Lagrange multiplier, $q^H$ is the real price of housing (nominal price divided by the deflator of non-durable consumption goods), $\pi$ is the inflation rate of the non-durable consumption bundle, and $\theta > 1$ is the non-fundamental shock to housing price expectations. Because it is larger than 1, households are too optimistic about the future price of the real asset, and increase their demand more than in absence of that shock, to get the capital gain associated with the price increase.

The same shock affects the Home indebted households’ expected future budget constraints and (current and future) borrowing constraints. For the latter, we express the constraint (3) in real terms and add the shock in the expectational term:

$$-b_t(j') R^S_t \leq m_t E_t \left( \theta_{t+1} q^H_{t+1} \pi_{t+1} h_{D,t}(j') \right),$$

(10)
where \( b < 0 \) is the debt in real terms. The housing demand implied by the first-order condition is

\[
\lambda_{D,t} (j') q_t^H = \chi_{1} h_{D,t} (j') + \beta D E_t \left( \lambda_{D,t+1} (j') \theta_{t+1} q_{t+1}^H + \gamma_{D,t} (j') m_t E_t \left( \theta_{t+1} q_{t+1}^H + \pi_{t+1} \right) \right) + \gamma_{D,t} (j') m_t E_t \left( \theta_{t+1} q_{t+1}^H + \pi_{t+1} \right),
\]

where \( \lambda_R \) is the budget-constraint Lagrange multiplier, and \( \gamma_{D,t} (j') \) is the Lagrange multiplier associated with the borrowing constraint. As for unrestricted households, the overly optimistic expectation about future housing prices drives up borrowers’ demand for housing due to the expected capital gain. Moreover, borrowers’ demand for housing increases also because a higher expected value of the real estate allows for a higher borrowing.

The interaction of the non-fundamental shock and the financial friction gives rise to the possibility of enacting macroprudential policy measures in the Home region. Both low interest rates and expectational shock favor the increase in real estate prices and borrowing. In this respect, the value of the LTV ratio is crucial for the amplification of both monetary (fundamental) and expectational (non-fundamental) shocks and, thus, for the increase in Home households’ borrowing.

### 2.7 Macroprudential policy rule

In some simulations it is assumed that the Home macroprudential authority can increase the Home LTV ratio to stabilize the domestic indebted households borrowing, as a ratio to the Home nominal GDP, when it augments. Thus, we do not keep the Home LTV ratio at its baseline level, but we endogenise it according to the feedback-rule

\[
m_t = \max \left( \bar{m}, (1 - \rho_m) \bar{m} + \rho_m m_{t-1} + \rho_B D \left( \int_0^{\lambda D} B_{D,t}^s (j') dj' \right) \right) + \rho_B D \left( \int_0^{\lambda D} B_{D,t-1}^s (j') dj' \right)
\]

where \( 0 \leq \rho_m \leq 1 \) and \( \rho_B > 0 \) are parameters, and \( \bar{m} \) is the LTV ratio in steady state (\( B_{D,t}^s < 0 \) represents a debt). Thus, the larger the increase in borrowing, the larger the reduction in the LTV ratio. By lowering the Home LTV ratio, the macroprudential authority limits the increase in households’ borrowing for a given value of the collateral. The rule is asymmetric, in the sense that the Home
macroprudential authority does not increase the LTV ratio when the borrowing decreases. For the REA, it is assumed that the LTV ratio is constant at its baseline level in all simulations.

### 2.8 Monetary policy

The EA (short-term) monetary policy rate is controlled by the EA monetary authority, which keeps it constant for an announced number of periods (forward guidance on the monetary policy rate) or sets it according to a standard Taylor rule. When the policy rate is not set according to the FG, it reverts to the Taylor rule

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \frac{\Pi_{EA,t}}{\Pi_{EA}} \right)^{(1-\rho_R)\rho_\pi} \left( \frac{GDP_{EA,t}}{GDP_{EA,t-1}} \right)^{(1-\rho_R)\rho_{GDP}},
\]

where \( R_t \) is the gross monetary policy rate. The parameter \( \rho_R (0 < \rho_R < 1) \) captures inertia in interest rate setting, while the parameter \( \bar{R} \) represents the steady-state gross nominal policy rate. The parameters \( \rho_\pi \) and \( \rho_{GDP} \) are respectively the weights of EA consumer price index (CPI) inflation rate (\( \Pi_{EA,t} \)) (taken as a deviation from its long-run constant target \( \bar{\Pi}_{EA} \)) and GDP (\( GDP_{EA,t} \)).

Finally, the EA monetary authority adopts the APP, modelled as exogenous Home and REA long-term sovereign bonds’ purchases. The shock is calibrated so that it corresponds to purchases of euro 60 billion per month, that last seven quarters. In our simulations, Home and REA long-term sovereign bond purchases are proportional to the size of the corresponding region (measured as a share of EA GDP).

---

21The CPI inflation rate is a geometric average of Home and REA CPI inflation rates (respectively \( \Pi_t \) and \( \Pi^*_t \)) with weight equal to the correspondent country GDP (as a share of the EA GDP). The EA GDP, \( GDP_{EA,t} \), is the sum of Home and REA GDPs.
2.9 Fiscal authority

Fiscal policy is set at the regional level. The Home government budget constraint is

\[ B^S_{g,t} - B^S_{g,t-1} R_{t-1} + P^L_t B^L_{g,t} - P^L_t R^L_t B^L_{g,t-1} = P_N t C^g_t - TAX_t, \quad (14) \]

where \( B^S_{g,t} \geq 0 \) and \( B^L_{g,t} \geq 0 \) are respectively the short- and long-term nominal sovereign debt. The variable \( C^g_t \) represents government purchases of goods and services, \( TAX_t > 0 \) are lump-sum taxes to households. Consistent with the empirical evidence, \( C^g_t \) is fully biased towards the intermediate non-tradable good. Hence it is multiplied by the corresponding price index \( P_{N,t} \).\(^{22}\)

The government follows a fiscal rule defined on lump-sum taxes. This rule aims (1) at bringing the short-term public debt, as a percentage \( b^S_g > 0 \) of domestic GDP, in line with its target \( \bar{b}^S_g \) and (2) at limiting its increase \( (b^S_{g,t}/b^S_{g,t-1}) \):

\[ \frac{TAX_t}{TAX_{t-1}} = \left( \frac{b^S_{g,t}}{b^S_g} \right)^{\phi_1} \left( \frac{b^S_{g,t}}{b^S_{g,t-1}} \right)^{\phi_2}, \quad (15) \]

where parameters \( \phi_1, \phi_2 > 0 \) call for an increase in lump-sum taxes whenever the short-term debt level is above target and for a larger increase whenever its dynamics is not converging. A similar rule holds in the REA. We include only the short-term debt in the fiscal rule for two reasons. First, we hold the supply of long-term government bonds \( B^L_{g,t} \) fixed so as to isolate the direct demand effects of the APP, so that changes in the long-term interest rate are entirely due to the non-standard monetary policy measures. Second, we need the fiscal rule to stabilize the short-term debt and, given that the long-term component is exogenous, the overall public debt. In the RW, as there is no distinction between short- and long-term domestic sovereign bonds, the rule holds for the overall public debt.

Finally, lump-sum taxes are paid by unrestricted households only. In this way we are able to isolate the response of restricted and indebted households to the APP from the indirect fiscal adjustment associated with the rule (15) and implied

by the program, as restricted households hold long-term sovereign bonds but are not subject to lump-sum taxes.\textsuperscript{23}

\subsection*{2.10 Key market clearing conditions}

The short-term bond is traded only domestically between indebted and unrestricted households:

\[ \int_{0}^{n\lambda_{D}} B_{D,t}^{S}(j')dj' + \int_{n(\lambda_{D}+\lambda_{R})}^{n} B_{U,t}^{S}(j) dj = 0. \]  \hspace{1cm} (16)

The market clearing condition for the Home long-term government bond is

\[ \int_{n\lambda_{D}}^{n(\lambda_{D}+\lambda_{R})} B_{R,t}^{L}(j'')dj'' + \int_{n(\lambda_{D}+\lambda_{R})}^{n} B_{U,t}^{L}(j) dj + B_{APP,t}^{L} = B_{g,t}^{L}, \]  \hspace{1cm} (17)

where the variable \( B_{EAPP,t}^{L} \) represents the demand for long-term sovereign bonds by the EA monetary authority (see Section 2.8).

The market clearing condition for the short-term internationally traded “private” bond \( B^{P} \) (denominated in euro terms) is

\[ \int_{n(\lambda_{D}+\lambda_{R})}^{n} B_{U,t}^{P}(j) dj + \int_{n(\lambda_{D}+\lambda_{R})}^{n} B_{U,t}^{P}(j^*) dj^* + \int_{n+n^*}^{1} B_{t}^{P}(j^{**}) dj^{**} = 0. \]  \hspace{1cm} (18)

where the variables \( B_{U,t}^{P}(j), B_{U,t}^{P}(j^*), B_{t}^{P}(j^{**}) \) represent the demand by the Home unrestricted households, REA unrestricted households, and RW households, respectively.

The market clearing for the Home short-term sovereign bond is

\[ \int_{n(\lambda_{D}+\lambda_{R})}^{n} B_{t}^{G}(j) dj = B_{g,t}^{S}, \]  \hspace{1cm} (19)

as the short-term sovereign bond is held only by domestic unrestricted households.

\textsuperscript{23}The Ricardian equivalence does not hold in the model. The distribution of lump-sum taxes or, equivalently, the initial distribution of public debt implies that sovereign bond holdings are net wealth.
The market clearing for the Home real estate is

\[
\int_0^{n_{D}} h_{D,t}(j) dj' + \int_{n_{(D+R)}}^{n} h_{U,t}(j) dj = \bar{h},
\]

where \(\bar{h}\) is the constant aggregate supply of housing stock.\(^{24}\)

Similar equations hold in the REA.

### 2.11 Equilibrium

In each country initial asset positions, preferences, and budget constraints are the same for households belonging to the same type and firms belonging to the same sector. Moreover, profits from ownership of domestic firms acting under monopolistic competition are equally shared between unrestricted households. Profits from ownership of domestic capital producers are distributed to restricted and unrestricted households according to the corresponding shares held by each type of households, and are equally shared within each type. Thus, we consider the representative household for each household type (indebted, restricted, and unrestricted). Moreover, we consider the representative firm for each sector (final non-tradable, intermediate tradable, and intermediate non-tradable) and the representative capital producer. The implied symmetric equilibrium is a sequence of allocations and prices such that, given initial conditions and considered shocks, households and firms satisfy their corresponding first order conditions, the Taylor rules, the Home macroprudential rule, the fiscal rules, the government budget constraints hold, and all markets clear.

### 2.12 Calibration

The model is calibrated at quarterly frequency. We set some parameter values so that steady-state ratios are consistent with average euro-area 2014 national account data, which are the most recent and complete available data. For remaining parameters we resort to previous studies and estimates available in the literature.\(^{25}\)

\(^{24}\)As housing is not produced, it does not enter into the computation of GDP.

\(^{25}\)See the New Area Wide Model (NAWM, Christoffel, Coenen and Warne 2008) and Euro Area and Global Economy Model (EAGLE, Gomes, Jacquinot and Pisani 2010)
Table 1 contains parameters for preferences and technology. Parameters with “∗” and “∗∗” are related to the REA and the RW, respectively. We assume perfect symmetry between the REA and the RW unless differently specified. The discount factor of EA unrestricted and RW households is set to 0.9927, so that the steady-state short-term interest rate is equal to 3.0% on an annual basis. The discount factor of EA indebted households is set to 0.9427. The discount factor of restricted households determines the steady-state value of the long-term interest rate and is set to 0.991, so that in steady state the spread between short- and long-term bond is equal to 0.7 percentage points. In each EA region the share of restricted households is set to 0.10 and the share of indebted households to 0.50.

The value for the intertemporal elasticity of substitution, $1/\sigma$, is 1. The Frisch labor elasticity is set to 0.5. Habit is set to 0.75. The weight of housing in utility is set to 0.3, in line with Iacoviello (2005). The shares of indebted, restricted and unrestricted households, and the “equity shares” of capital producers ($\omega$ and $1 - \omega$), are calibrated so that the model yields a response of investment to the (benchmark) APP around four times as large as the response of consumption, in line with standard business cycle facts on response of investment in the EA. The depreciation rate of capital is set to 0.025.

In the production functions of tradables and non-tradables, the elasticity of substitution between labor and capital is set to 0.93. To match investment-to-GDP ratios, the bias towards capital in the production function of tradables is set to 0.56 in Home and, in the REA and in the RW, to 0.46. The corresponding value in the production function of non-tradables is set to 0.53 in Home and 0.43 in the REA and RW. In the final consumption and investment goods the elasticity of substitution between domestic and imported tradables is set to 1.5, while the elasticity of substitution between tradables and non-tradables is set to 0.5, as empirical evidence suggests that it is harder to substitute tradables for non-tradables than to substitute across tradables. The biases towards the domestically produced good and composite tradable good are chosen to match the Home and REA import-to-GDP ratios. In the consumption bundle the bias towards the domestic tradable is 0.68 in Home, 0.59 in the REA, and 0.90 in the RW. The bias towards the composite tradable is set to 0.68 in Home and to 0.50 in the REA and the RW. For the investment basket, the bias towards the domestic tradable is 0.50.
in Home, 0.49 in the REA, and 0.90 in the RW. The bias towards the composite tradable is 0.78 in Home and 0.70 in the REA and in the RW.

Table 2 reports gross mark-up values. In the Home tradable and non-tradable sectors and in the Home labor market the mark-up is set to 1.08, 1.29, and 1.60, respectively (the corresponding elasticities of substitution across varieties are set to 13.32, 4.44, and 2.65). In the REA tradable and non-tradable sectors and in the REA labor market the gross mark-ups are respectively set to 1.11, 1.24, and 1.33 (the corresponding elasticities are set to 10.15, 5.19, and 4.00). Similar values are chosen for the corresponding parameters in the RW.

Table 3 contains parameters that regulate the dynamics. Adjustment costs on investment change are set to 6.00. Adjustment costs on housing change are set to 1.00, to get, in line with other contributions, a response of housing prices of the same magnitude as the response of housing transactions and a response of housing transactions which is twice as large as that of non-durable consumption. Nominal wage quadratic adjustment costs are set to 400. In the tradable sector, we set the nominal adjustment cost parameter to 300 for Home tradable goods sold domestically and in the REA; for Home goods sold in the RW, the corresponding parameter is set to 50. The same parameterization is adopted for the REA, while for the RW we set the adjustment cost on goods exported to Home and the REA to 50. Nominal price adjustment costs are set to 600 in the non-tradable sector.

The parameter regulating the adjustment costs paid by the unrestricted household on deviations of long-term sovereign bond positions from steady-state levels, \( \phi_{bL} \), is set to 0.000039 and to 0.00027 in Home and REA, respectively. The parameters regulating the adjustment cost on private bond position, paid by Home unrestricted households and RW households, are set to 0.055. These parameters have been calibrated following two criteria. First, they should not greatly affect the model dynamics and yet help to stabilize it. Second, the response of the interest rate on long-term sovereign bonds to the benchmark APP should be in line with existing evidence for the EA.

Table 4 reports the parameterization of the systematic feedback rules followed by the fiscal and monetary authorities. In the fiscal policy rule (15) we set \( \phi_1 = 0.05 \) and \( \phi_2 = 10.01 \) for Home, and \( \phi_1 = 0.05 \) and \( \phi_2 = 1.01 \) for the REA and the RW. It is always lump-sum transfers to adjust. The central bank of the EA
targets the contemporaneous EA-wide consumer price inflation (the corresponding parameter is set to 1.7) and the output growth (the parameter is set to 0.1). Interest rate is set in an inertial way and hence its previous-period value enters the rule with a weight equal to 0.87. The values are identical for the corresponding parameters of the Taylor rule in the RW. For the Home LTV ratio, we set it to a relatively large value, 90%, in line with the value of countries that faced a real estate boom in the early 2000s. For the REA ratio, we set it in line with the EA average value, to 50%. The parameter of inertia in the Home macroprudential rule is calibrated to get a rather persistent change in the LTV ratio, in line with common practice of macroprudential authorities not to frequently change the ratio. The parameter measuring the response to the debt-to-GDP ratio is “endogenously” set, to roughly counterbalance the impact of the bubble on households debt.

Table 5 reports the great ratios, which are matched by the model steady state under our baseline calibration. We assume a zero steady-state net foreign asset position of each region. The sizes of Home and REA GDPs as shares of world GDP are set to 5% and to 17%, respectively. So the Home GDP is around 20% of EA GDP.

Indebted households’ debt-to-yearly GDP is set to 85% for the Home country and to 23% for REA. Short-term public debt (ratio to yearly GDP) is set to 13% for Home and 8% for the REA. Long-term public debt is set to 120% and 93% of GDP for Home and the REA, respectively. The parameter $\kappa$ is calibrated to match the duration of this bond, given by $R_{L,t}/(R_{L,t} - \kappa)$, to the average duration of the EA long-term sovereign bond (8 years on average).

We assume that in each country long-term sovereign bond holdings are equally shared between unrestricted and restricted households.

The chosen calibration yields impulse response functions to a standard monetary policy shock (+0.25 basis points) for GDP and inflation in each EA region that are in line with the workhorse estimated models of the EA in the literature.\footnote{\textsuperscript{26}See, for example, the New Area Wide Model (NAWM, Christoffel, Coenen and Warne 2008) and the Euro Area and Global Economy Model (EAGLE, Gomes, Jacquinot and Pisani 2010).}
3 Results

We initially describe the simulated scenarios, and, subsequently, report the results.

In Section 3.1, we report results of the benchmark case, corresponding to the simulation of APP. The shock is calibrated so that it corresponds to overall quarterly purchases of euro 180 billion, that last for seven quarters. It is assumed that long-term sovereign bonds are held to maturity (8 years) and that the central bank starts to gradually sell the bonds afterwards. In the benchmark scenario, the Home LTV ratio is constant at its baseline (steady-state) value equal to 90%. In the alternative scenario, the APP is implemented under the assumption of a lower steady-state value of the Home LTV (50%).

In Section 3.2, we assume that the Home economy is perturbed not only by the non-standard monetary policy measures but also by the expectational (non-fundamental) shock. In Section 3.3 we compare scenarios where the Home LTV ratio is kept constant at 90% or, alternatively, changed by the macroprudential authority according to equation (12).

The scenarios are simulated assuming perfect foresight. Households and firms are surprised by shocks perturbing the economy in the first period of the simulations and fully anticipate subsequent shocks.

3.1 APP and region-specific LTVs

Figure 1 shows results of the benchmark simulation. The inflation rate and the economic activity increase in both EA regions as a consequence of the APP. The effects are rather symmetric across the two regions. Inflation gradually increases and achieves a peak of around 0.6 percentage points (in annualized terms) after four quarters. Thereafter, it gradually decreases. GDP increases by around 1% of its baseline level after four quarters. Thereafter, it gradually returns to its baseline level.

Home and REA consumption and investment benefit from the reduction in long-term real interest rates (not reported). The latter decreases because of the decline in the current and expected long-term nominal interest rates (consistent with the increase in long term sovereign bond prices, associated with the central
bank purchases) and the increase in the expected inflation rates. Consumption is also affected by the intertemporal substitution effect associated with the lower short-term real interest rate that positively affects unrestricted households. The real interest rate decreases because (expected) inflation increases, while the monetary policy rate is kept constant at the baseline level by the monetary authority.

Given the rise in production and in labor demand by firms, labor effort and real wages increase (the latter are not reported to save on space). Both exports and imports increase. The former because the euro depreciates in nominal terms, making goods produced by the EA regions more competitive than goods produced in the RW. Moreover, as each EA region is a relevant trade partner of the other, the increase in each region aggregate demand favors the intra-EA trade.

As shown in Figure 2, in both regions the real estate prices, demand of real estate, (indebted) households’ borrowing, and indebted households’ consumption of non-durable goods increase. More crucially, while the price of housing increases in both regions, households’ debt increases substantially only in the Home region. The reason is that the Home LTV ratio is larger than its REA counterpart, and, thus, amplifies to a larger extent the expansionary effect of the (union-wide) APP on the (region-specific) households’ debt.

To further explore the contribution of the LTV ratio to the transmission mechanism of the APP, in Figures 3 and 4 we report the responses of the Home variables under two alternative calibrations. In one case, the Home LTV ratio is set to 90%, as in the benchmark scenario (reported in the Figures 1 and 2), in the other to 50%. The comparison of the corresponding responses shows the amplifying effect of the LTV ratio (Figure 3): the larger its value, the larger the response of borrowing and, thus, the responses of consumption and real estate prices. The responses of Home aggregate demand and, thus, of Home GDP are affected by the different calibration of the Home LTV ratio, given that relative share of indebted households, set to 50% (a relatively large value), in the Home population (see Figure 4).

Overall, region-specific LTV ratios can amplify the expansionary effect of APP on local households’ borrowing. Moreover, they can also amplify region-specific shocks, for example (fundamental or non-fundamental) shocks that could affect the local housing market during the implementation of the non-standard monetary
3.2 Non-fundamental shock to Home households’ expectations about domestic real estate price

We now assume that it is not only the APP to perturb the Home economy, but also the non-fundamental shock to expectations about the price of Home real estate. The non-fundamental shock is calibrated to get, with respect to the APP-induced increase in Home real estate price, an additional increase (overvaluation of real estate due to the irrational shock) equal to around 5% of the baseline (steady-state) level on average during the first year, when the APP is implemented. The chosen value is line with evidence provided by Hartmann (2015), from which it can be inferred an average over-valuation of EA housing prices equal to around 30% in 2007. This figure implies an average increase in the overvalued component of the price equal to around 5% per year over the 2002-2007 period, characterized by the increase in EA housing prices.

Figures 5 and 6 report the results, comparing the new scenario (“non-fundamental shock” scenario, red dashed line) with the benchmark one (“APP” scenario, black continuous line). In both scenarios the LTV-ratio is at its baseline level (90%).

The combination of APP and expectational shocks induces a wide and persistent increase in the Home real estate price (Figure 5). Both indebted and unrestricted households make overly optimistic predictions about future increases in the value of the real estate. Thus, they immediately increase their demand for housing, driving up the current (first-period) real estate price. The higher value of real estate induces indebted households to borrow more than in the benchmark case. Their debt widely increases, and finances the demand for both non-durable goods and real estate services. The additional increase in indebted households’ consumption of non-durable goods further stimulates Home GDP, which increases more than in the benchmark scenario (see Figure 6).

Overall, the increase in the price of Home real estate is much larger under the assumption that both APP and shock to Home expectations on housing prices perturb the Home economy than under the assumption the APP is the only shock. More crucially, the increase in Home households debt is much larger in the former
case than in the latter, and it can be considered as “excessive.” The reason is that the borrowing is partly driven by a shock which is non-fundamental. Thus, there is room for implementing macroprudential measures in the Home country, as reported in what follows.

3.3 Stabilizing regional financial conditions

We evaluate the role of macroprudential policy by simulating the “non-fundamental shock” scenario, driven by the APP and the overly optimistic expectations about housing prices, under the assumption that the Home LTV ratio is set according to the macroprudential rule (12) instead of being constant at its baseline level (90%). We label it as “endogenous-LTV” scenario.

For the macroprudential rule, we calibrate the parameter $\rho_{BD}$ in eq. (12) to have an increase in the households’ debt in line with the benchmark (fundamental factor-driven) scenario.

Figures 7 and 8 report the results. For comparison, we also report the results of the benchmark scenario (driven by the APP only, and having the Home LTV-ratio constant at its baseline level) and of the non-fundamental-shock scenario (driven by the APP and the non-fundamental shock, where the LTV-ratio is constant at its baseline level).

The increase in Home indebted households’ borrowing is lower in the endogenous-LTV scenario than in the non-fundamental-shock scenario (see Figure 7). Following the increase in the price of housing, households increase their borrowing. At the same time, the macroprudential rule commands a reduction in the Home LTV ratio, from 0.9 to around 0.85, in response to the initial increase in borrowing. The net effect is a lower increase in households’ borrowing. The lower debt increase that makes fewer financial resources available for the households’ expenditure. The indebted households’ demand for housing and non-durable goods increases to a lower extent. Consistently, the increase in the price of Home real estate is reduced.

Relative to the non-fundamental-shock scenario, the lower increase in Home indebted households’ consumption in the endogenous LTV scenario affects the Home aggregate demand and, thus, GDP, whose increase is lower (Figure 8). Households’
labor increases to a lower extent, because of the more contained increase in Home production. Home imports increase to a lower extent as well, consistent with the lower increase in Home aggregate demand.

To the opposite, relative to the non-fundamental-shock scenario, Home investment and exports initially increase more in the endogenous LTV scenario. In the latter case, Home unrestricted households demand long-term sovereign bonds to a larger extent, because they have to finance a lower increase in indebted households’ borrowing. Their relatively large demand of bonds induces a further increase in their price and, thus, a further decrease in the long-term interest rate. The latter constitutes a larger incentive for restricted households to substitute physical capital for sovereign bonds. For Home exports, they increase relatively more because the rather flexible prices of Home extra-EA exports increase to a lower extent, reflecting the lower increase in the marginal production costs associated with the lower increase in Home wages (Home firms increase labor demand to a lower extent, given the lower increase in aggregate demand). Last, but not least, Home inflation is similar across the three scenarios.

Overall, results suggest that macroprudential measures can be implemented at regional level to limit local over-borrowing if the latter is observed during the APP implementation. The macroeconomic implication is the lower increase in regional GDP. The latter should not necessarily be considered a macroeconomic cost, because the macroprudential measure counterbalances a non-fundamental and, thus, inefficient financial shock. A full assessment of costs and benefits associated with the macroprudential measure requires a welfare analysis, which goes beyond the scope of this paper and should include, as a benefit of the macroprudential policy, also the decrease in the likelihood of a sudden bubble burst, which would imply a sudden decrease in consumption and GDP and, thus, an increase in regional financial and macroeconomic instability.

4 Conclusions

This paper argues that there can be synergies between non-standard monetary and macroprudential policies in a monetary union, where the monetary policy appropriately focuses on union-wide economic conditions only. Region-specific
macroprudential policies that stabilize excessive borrowing at regional level can be a helpful complement to the accommodative monetary policy stance, as the combination of the two would favor macroeconomic and financial stability at both regional and union-wide level.

Our results can be further explored. In the framework we use, the regional housing stock is constant. The model can be extended to allow for (endogenous) production of housing, which would then directly enter GDP dynamics. Moreover, from a theoretical perspective, it would be interesting to assess the macroeconomic effects of a systematic and explicit coordination between the union-wide monetary policy authority and the region-specific macroprudential authorities. We leave these issues for future research.
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>H</th>
<th>REA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor $\beta_U, \beta_U^<em>, \beta^</em>$</td>
<td>0.9927</td>
<td>0.9927</td>
<td>0.9927</td>
</tr>
<tr>
<td>Discount factor $\beta_R, \beta_R^*$</td>
<td>0.991</td>
<td>0.991</td>
<td>–</td>
</tr>
<tr>
<td>Discount factor $\beta_D, \beta_D^*$</td>
<td>0.9427</td>
<td>0.9427</td>
<td>–</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution $1/\sigma$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Housing weight $\chi$</td>
<td>0.3</td>
<td>0.3</td>
<td>–</td>
</tr>
<tr>
<td>Share of restricted households $\lambda_R$</td>
<td>0.10</td>
<td>0.10</td>
<td>–</td>
</tr>
<tr>
<td>Share of indebted households $\lambda_D$</td>
<td>0.50</td>
<td>0.50</td>
<td>–</td>
</tr>
<tr>
<td>Share of capital producers held by restricted households $\omega, \omega^*$</td>
<td>0.30</td>
<td>0.30</td>
<td>–</td>
</tr>
<tr>
<td>Inverse of Frisch elasticity of labor supply $\tau$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Habit $\varsigma$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Depreciation rate of capital $\delta$</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>** Tradable Intermediate goods**</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution between factors of production $\xi_T, \xi_T^*, \xi_T^{**}$</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_T, \alpha_T^*, \alpha_T^{**}$</td>
<td>0.56</td>
<td>0.46</td>
<td>0.46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>** Non-tradable Intermediate goods**</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution between factors of production $\xi_N, \xi_N^*, \xi_N^{**}$</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_N, \alpha_N^*, \alpha_N^{**}$</td>
<td>0.53</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>** Final consumption goods**</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution between domestic and imported goods $\phi_A, \phi_A^*, \phi_A^{**}$</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Bias towards domestic tradable goods $a_H, a_H^*, a_H^{**}$</td>
<td>0.68</td>
<td>0.59</td>
<td>0.90</td>
</tr>
<tr>
<td>Substitution between tradables and non-tradables $\rho_A, \rho_A^*, \rho_A^{**}$</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Bias towards tradable goods $a_T, a_T^*, a_T^{**}$</td>
<td>0.68</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>** Final investment goods**</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution between domestic and imported goods $\phi_E, \phi_E^*, \phi_E^{**}$</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Bias towards domestic tradable goods $v_H, v_H^*, v_H^{**}$</td>
<td>0.50</td>
<td>0.49</td>
<td>0.90</td>
</tr>
<tr>
<td>Substitution between tradables and non-tradables $\rho_E, \rho_E^*, \rho_E^{**}$</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Bias towards tradable goods $v_T, v_T^*, v_T^{**}$</td>
<td>0.78</td>
<td>0.70</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*Note: H=Home; REA=rest of the euro area; RW= rest of the world. *∗∗∗* refers to REA, *∗∗* to RW.
### Table 2: Gross Mark-ups

<table>
<thead>
<tr>
<th></th>
<th>Mark-ups and Elasticities of Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tradables</td>
</tr>
<tr>
<td>H</td>
<td>1.08 ($\theta_T = 13.32$)</td>
</tr>
<tr>
<td>REA</td>
<td>1.11 ($\theta_T^* = 10.15$)</td>
</tr>
<tr>
<td>RW</td>
<td>1.11 ($\theta_T^{**} = 10.15$)</td>
</tr>
</tbody>
</table>

Note: H=Home; REA=rest of the euro area; RW=rest of the world.

"*" refers to REA, "**" to RW.
### Table 3: Real and Nominal Adjustment Costs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>H</th>
<th>REA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real Adjustment Costs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment $\phi_I, \phi_I^*, \phi_I^{**}$</td>
<td>6.00</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Housing $\phi_h, \phi_h^*$</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td><strong>Adjustment Costs on bonds</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households’ long-term bond positions $\phi_{bL}, \phi_{bL}^*$</td>
<td>0.00039</td>
<td>0.00027</td>
<td>–</td>
</tr>
<tr>
<td>Households’ private bond positions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{b1}, \phi_{b1}^{**}$</td>
<td>0.055</td>
<td>–</td>
<td>0.055</td>
</tr>
<tr>
<td>$\phi_{b2}, \phi_{b2}^{**}$</td>
<td>0.055</td>
<td>–</td>
<td>0.055</td>
</tr>
<tr>
<td><strong>Nominal Adjustment Costs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wages $\kappa_W, \kappa_W^*, \kappa_W^{**}$</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Home produced tradables $\kappa_H, k_H^*, k_H^{**}$</td>
<td>300</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>REA produced tradables $\kappa_{\text{REA}}^<em>, k_{\text{REA}}^</em>, k_{\text{REA}}^{**}$</td>
<td>300</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>RW produced tradables $\kappa_H, k_H^*, k_H^{**}$</td>
<td>50</td>
<td>50</td>
<td>300</td>
</tr>
<tr>
<td>Non-tradables $\kappa_N, \kappa_N^*, \kappa_N^{**}$</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
</tbody>
</table>

Note: H=Home; REA=rest of the euro area; RW= rest of the world.

$^*$ refers to REA, $^{**}$ to RW

### Table 4: Fiscal, Monetary, and Macroprudential Policy Rules

<table>
<thead>
<tr>
<th>Parameter</th>
<th>H</th>
<th>REA</th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fiscal policy rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_1, \phi_1^*, \phi_1^{**}$</td>
<td>0.05</td>
<td>0.05</td>
<td>–</td>
<td>0.05</td>
</tr>
<tr>
<td>$\phi_2, \phi_2^*, \phi_2^{**}$</td>
<td>10.01</td>
<td>1.01</td>
<td>–</td>
<td>1.01</td>
</tr>
<tr>
<td><strong>Common monetary policy rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged interest rate $\rho_R, \rho_R^*$</td>
<td>–</td>
<td>–</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Inflation $\rho_I, \rho_I^{**}$</td>
<td>–</td>
<td>–</td>
<td>1.70</td>
<td>1.70</td>
</tr>
<tr>
<td>GDP growth $\rho_{\text{GDP}}, \rho_{\text{GDP}}^{**}$</td>
<td>–</td>
<td>–</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Macroprudential rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV ratio $m$</td>
<td>90%</td>
<td>50%</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Lagged LTV ratio $\rho_m$</td>
<td>0.99</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Households’ debt $\rho_{BD}$</td>
<td>1.45</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

Note: H=Home; REA=rest of the euro area; RW= rest of the world.

$^*$ refers to REA, $^{**}$ to RW
Table 5: Main macroeconomic variables (ratio to GDP)

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>REA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private consumption</td>
<td>61.0</td>
<td>57.1</td>
<td>64.0</td>
</tr>
<tr>
<td>Public consumption</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Private investment</td>
<td>18.0</td>
<td>16.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Imports</td>
<td>29.0</td>
<td>24.3</td>
<td>4.25</td>
</tr>
<tr>
<td>Net Foreign Asset Position</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>GDP (share of world GDP)</td>
<td>0.05</td>
<td>0.17</td>
<td>0.78</td>
</tr>
<tr>
<td>Private debt (ratio to annual GDP)</td>
<td>85.0</td>
<td>23.0</td>
<td>–</td>
</tr>
<tr>
<td>Short-term public debt (ratio to annual GDP)</td>
<td>13.0</td>
<td>8.0</td>
<td>–</td>
</tr>
<tr>
<td>Long-term public debt (ratio to annual GDP)</td>
<td>120.0</td>
<td>93.0</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: H=Home; REA=rest of the euro area; RW= rest of the world.
Figure 1: APP. Macroeconomic effects.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized % point deviations.
Figure 2: APP. Effects on real estate and borrowing.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state.
Figure 3: APP and alternative Home LTV ratios. Home real estate and borrowing.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state.
Figure 4: APP and alternative Home LTV ratios. Home macroeconomic variables.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized % point deviations.
Figure 5: APP and expectation shock. Home real estate variables.

Housing prices

Indebted households housing demand

Indebted households debt

Indebted households consumption

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state.
Figure 6: APP and expectation shock. Home macroeconomic variables.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized % point deviations.
Figure 7: APP, expectation shock, and macroprudential policy. Home real estate variables.

Notes: Horizontal axis, quarters; vertical axis, % deviations from the steady state.
Figure 8: APP, expectation shock, and macroprudential policy. Home macroeconomic variables.

- **Inflation**
  - Horizontal axis: quarters; vertical axis: % deviations from the steady state. For inflation, annualized % point deviations.

- **GDP**

- **Consumption**

- **Investment**

- **Exports**

- **Imports**

- **Labor**

- **Nominal exchange rate**

Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized % point deviations.
Appendix

In this Appendix we report a detailed description of the model except for fiscal and monetary policies and households’ optimization problems, which are reported in the main text.\footnote{For a detailed description of the main features of the model see also Pesenti (2008).}

There are three blocs, Home, REA, and RW. In what follows we illustrate the Home economy. The structure of each of the other two regions (REA and the RW) is similar and to save on space we do not report it.

Final consumption and investment goods

There is a continuum of symmetric Home firms producing final non-tradable consumption under perfect competition. Each firm producing the consumption good is indexed by $x \in (0, n]$, where the parameter $0 < n < 1$ measures the size of Home. Firms in the REA and in the RW are indexed by $x^* \in (n, n + n^*)$ and $x^{**} \in (n + n^*, 1]$, respectively (the size of the world economy is normalized to 1).

The CES production technology used by the generic firm $x$ is

$$A_t(x) \equiv \left( \frac{a_T}{\rho_A} \left( a_H^{\frac{a_H-1}{\rho_H}} Q_{HA,t}(x)^{\frac{a_H-1}{\rho_H}} + a_G^{\frac{a_G}{\rho_G}} Q_{GA,t}(x)^{\frac{a_G-1}{\rho_G}} + \frac{1}{\rho_A} Q_{FA,t}(x)^{\frac{a_F-1}{\rho_F}} + (1 - a_H - a_G) \frac{1}{\rho_A} Q_{NA,t}(x)^{\frac{a_N-1}{\rho_N}} \right) \right)^{\frac{1}{\rho_A}}$$

where $Q_{HA}$, $Q_{GA}$, $Q_{FA}$, and $Q_{NA}$ are bundles of respectively intermediate tradables produced in Home, intermediate tradables produced in the REA, intermediate tradables produced in the RW, and intermediate non-tradables produced in the Home country. The parameter $\rho_A > 0$ is the elasticity of substitution between tradables and $\phi_A > 0$ is the elasticity of substitution between tradable and non-tradable goods. The parameter $a_H$ ($0 < a_H < 1$) is the weight of the Home tradable, the parameter $a_G$ ($0 < a_G < 1$) the weight of tradables imported from the REA, and the parameter $a_T$ ($0 < a_T < 1$) the weight of tradable goods.

The production of investment good is similar. There are symmetric Home firms under perfect competition indexed by $y \in (0, n]$. Firms in the REA and in the RW are indexed by $y^* \in (n, n + n^*)$ and $y^{**} \in (n + n^*, 1]$. Output of the generic Home firm is
firm $y$ is

$$E_t(y) \equiv \left( \frac{1}{v_T^{E}} Q^{E}_{H E, t}(y)^{\frac{\phi_E}{\phi_E - 1}} + \frac{1}{v_H^{E}} Q^{E}_{G E, t}(y)^{\frac{\phi_E}{\phi_E - 1}} + \frac{1}{v_G^{E}} Q^{E}_{F E, t}(y)^{\frac{\phi_E}{\phi_E - 1}} \right) \frac{\phi_E}{\phi_E - 1} \frac{\phi_E - 1}{\phi_E} + (1 - v_H - v_G) \frac{1}{v_H^{E}} Q^{E}_{N E, t}(y)^{\frac{\phi_E}{\phi_E - 1}} \frac{\phi_E}{\phi_E - 1} \frac{\phi_E - 1}{\phi_E}$$

Finally, we assume that public expenditure $C^g$ is composed by intermediate non-tradable goods only.

**Intermediate goods**

**Demand**

Bundles used to produce the final consumption goods are CES indexes of differentiated intermediate goods, each produced by a single firm under conditions of monopolistic competition:

$$Q^{H A}(x) \equiv \left[ \left( \frac{1}{s} \right)^{\theta_T} \int_0^n Q(h, x)^{\frac{\theta_T}{\theta_T - 1}} dh \right]^{\frac{\theta_T}{\theta_T - 1}}, \quad (21)$$

$$Q^{G A}(x) \equiv \left[ \left( \frac{1}{S - s} \right)^{\theta_T} \int_n^{n + n^*} Q(g, x)^{\frac{\theta_T}{\theta_T - 1}} dg \right]^{\frac{\theta_T}{\theta_T - 1}}, \quad (22)$$

$$Q^{F A}(x) \equiv \left[ \left( \frac{1}{1 - S} \right)^{\theta_T} \int_{n + n^* + 1}^1 Q(f, x)^{\frac{\theta_T}{\theta_T - 1}} df \right]^{\frac{\theta_T}{\theta_T - 1}}, \quad (23)$$

$$Q^{N A}(x) \equiv \left[ \left( \frac{1}{s} \right)^{\theta_N} \int_0^n Q(i, x)^{\frac{\theta_N}{\theta_N - 1}} di \right]^{\frac{\theta_N}{\theta_N - 1}}, \quad (24)$$

where firms in the Home intermediate tradable and non-tradable sectors are respectively indexed by $h \in (0, n]$ and $n \in (0, n]$, firms in the REA by $g \in (n, n + n^*]$, and firms in the RW by $f \in (n + n^*, 1]$. Parameters $\theta_T, \theta_N > 1$ are respectively the elasticity of substitution across brands in the tradable and non-tradable sector. The prices of the intermediate non-tradable goods are denoted $p(i)$. Each firm $x$ takes these prices as given when minimizing production costs of the final good.
The resulting demand for intermediate non-tradable input \( i \) is

\[
Q_{A,t}(i, x) = \left( \frac{1}{s} \right) \left( \frac{P_t(i)}{P_{N,t}} \right)^{-\theta_N} Q_{NA,t}(x),
\]

(25)

where \( P_{N,t} \) is the cost-minimizing price of one basket of local intermediates:

\[
P_{N,t} = \left[ \int_0^n P_t(i)^{1-\theta_N} di \right]^{\frac{1}{1-\theta_N}}.
\]

(26)

We can derive \( Q_A(h, x), Q_A(f, x), C^g_A(h, x), C^g_A(f, x), P_H, \) and \( P_F \) in a similar way. Firms \( y \) producing the final investment goods have similar demand curves. Aggregating over \( x \) and \( y \), it can be shown that total demand for intermediate non-tradable good \( i \) is

\[
\int_0^n Q_{A,t}(i, x) dx + \int_0^n Q_{E,t}(i, y) dy + \int_0^n C^g_t(i, x) dx
= \left( \frac{P_t(i)}{P_{N,t}} \right)^{-\theta_N} (Q_{NA,t} + Q_{NE,t} + C^g_{N,t}),
\]

where \( C^g_N \) is public sector consumption. Home demands for (intermediate) domestic and imported tradable goods can be derived in a similar way.

**Supply**

The supply of each Home intermediate non-tradable good \( i \) is denoted by \( N^S(i) \):

\[
N^S_t(i) = \left( 1 - \alpha_N \right) \frac{\xi_N}{\xi_N - 1} L_{N,t}(i) \frac{\xi_N - 1}{\xi_N} + \alpha_N \frac{\xi_N - 1}{\xi_N} K_{N,t}(i) \left( \frac{\xi_N}{\xi_N - 1} \right)^{\frac{\xi_N}{\xi_N - 1}}.
\]

(27)

Firm \( i \) uses labor \( L_{N,t}(i) \) and capital \( K_{N,t}(i) \) with constant elasticity of input substitution \( \xi_N > 0 \) and capital weight \( 0 < \alpha_N < 1 \). Firms producing intermediate goods take the prices of labor inputs and capital as given. Denoting \( W_t \) the nominal wage index and \( R^K_t \) the nominal rental price of capital, cost minimization implies that

\[
L_{N,t}(i) = \left( 1 - \alpha_N \right) \left( \frac{W_t}{MC_{N,t}(i)} \right)^{-\xi_N} N^S_t(i)
\]

(28)
and

$$K_{NI,t}(i) = \alpha \left( \frac{R_t^K}{MC_{NI,t}(i)} \right)^{-\xi_N} N_t^{S}(i)$$

where $MC_{NI,t}(n)$ is the nominal marginal cost:

$$MC_{NI,t}(i) = \left( (1 - \alpha) W_t^{1-\xi_N} + \alpha (R_t^K)^{1-\xi_N} \right)^{\frac{1}{1-\xi_N}}. \quad (29)$$

The productions of each Home tradable good, $T^S(h)$, is similarly characterized.

**Price setting in the intermediate sector**

Consider now profit maximization in the Home intermediate non-tradable sector. Each firm $i$ sets the price $p_t(i)$ by maximizing the present discounted value of profits subject to the demand constraint and the quadratic adjustment costs,

$$AC_{NI,t}^p(i) = \frac{\kappa_p^p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Q_{NI,t},$$

which is paid in unit of sectorial product $Q_{NI,t}$ and where $\kappa_p^p \geq 0$ measures the degree of price stickiness. The resulting first-order condition, expressed in terms of domestic consumption, is

$$p_t(i) = \frac{\theta_N}{\theta_N - 1} mc_t(i) - \frac{A_t(i)}{\theta_N - 1}, \quad (30)$$

where $mc_t(i)$ is the real marginal cost and $A_t(i)$ contains terms related to the presence of price adjustment costs:

$$A_t(i) \approx \kappa_p^p \frac{P_t(i)}{P_{t-1}(i)} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)$$

$$- \beta \kappa_p^p \frac{P_{t+1}(i)}{P_t(i)} \left( \frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{Q_{NI,t+1}}{Q_{NI,t}}.$$

The above equations clarify the link between imperfect competition and nominal rigidities. When the elasticity of substitution $\theta_N$ is very large and hence the competition in the sector is high, prices closely follow marginal costs, even though adjustment costs are large. To the contrary, it may be optimal to maintain stable
prices and accommodate changes in demand through supply adjustments when the average markup over marginal costs is relatively high. If prices were flexible, optimal pricing would collapse to the standard pricing rule of constant markup over marginal costs (expressed in units of domestic consumption):

\[ p_t(i) = \frac{\theta_N}{\theta_N - 1} m_{N,t}(i). \]  

(31)

Firms operating in the intermediate tradable sector solve a similar problem. We assume that there is market segmentation. Hence the firm producing the brand \(h\) chooses \(p_t(h)\) in the Home market, a price \(p^*_t(h)\) in the REA, and a price \(p^{**}_t(h)\) in the RW to maximize the expected flow of profits (in terms of domestic consumption units),

\[
E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left[ p_{\tau}(h) y_{\tau}(h) + p^*_t(h) y^*_\tau(h) + p^{**}_t(h) y^{**}_\tau(h) - m_{C,H,\tau}(h) (y_{\tau}(h) + y^*_\tau(h) + y^{**}_\tau(h)) \right],
\]

subject to quadratic price adjustment costs similar to those considered for non-tradables and standard demand constraints. The term \(E_t\) denotes the expectation operator conditional on the information set at time \(t\), \(\Lambda_{t,\tau}\) is the appropriate discount rate, and \(m_{C,H,t}(h)\) is the real marginal cost. The first order conditions with respect to \(p_t(h), p^*_t(h),\) and \(p^{**}_t(h)\) are

\[
p_t(h) = \frac{\theta_T}{\theta_T - 1} m_{C,t}(h) - \frac{A_t(h)}{\theta_T - 1}, \quad \text{(32)}
\]

\[
p^*_t(h) = \frac{\theta_T}{\theta_T - 1} m_{C,t}(h) - \frac{A^*_t(h)}{\theta_T - 1}, \quad \text{(33)}
\]

\[
p^{**}_t(h) = \frac{\theta_T}{\theta_T - 1} m_{C,t}(h) - \frac{A^{**}_t(h)}{\theta_T - 1}, \quad \text{(34)}
\]

where \(\theta_T\) is the elasticity of substitution of intermediate tradable goods, while \(A(h)\) and \(A^*(h)\) involve terms related to the presence of price adjustment costs.
\[ A_t(h) \approx \kappa_H^p \frac{P_t(h)}{P_{t-1}(h)} \left( \frac{P_t(h)}{P_{t-1}(h)} - 1 \right) \]
\[ -\beta \kappa_H^p \frac{P_{t+1}(h)}{P_t(h)} \left( \frac{P_{t+1}(h)}{P_{t-1}(h)} - 1 \right) Q_{H,t+1}, \]
\[ A_t^*(h) \approx \theta_T - 1 + \kappa_H^p \frac{P_t^*(h)}{P_{t-1}^*(h)} \left( \frac{P_t^*(h)}{P_{t-1}^*(h)} - 1 \right) \]
\[ -\beta \kappa_H^p \frac{P_{t+1}^*(h)}{P_t^*(h)} \left( \frac{P_{t+1}^*(h)}{P_{t-1}^*(h)} - 1 \right) Q_{H,t+1}^*, \]
\[ A_t^{**}(h) \approx \theta_T - 1 + \kappa_H^{**} \frac{P_{t+1}^{**}(h)}{P_{t-1}^{**}(h)} \left( \frac{P_{t+1}^{**}(h)}{P_{t-1}^{**}(h)} - 1 \right) \]
\[ -\beta \kappa_H^{**} \frac{P_{t+1}^{**}(h)}{P_t^{**}(h)} \left( \frac{P_{t+1}^{**}(h)}{P_{t-1}^{**}(h)} - 1 \right) Q_{H,t+1}^{**}, \]

where \( \kappa_H^p, \kappa_H^*, \kappa_H^{**} > 0 \) respectively measure the degree of nominal rigidity in the Home country, in the REA, and in the RW.

**Labor Market**

In the case of firms in the intermediate non-tradable sector, the labor input \( L_N(i) \) is a CES combination of differentiated labor inputs supplied by domestic agents and defined over a continuum of mass equal to the country size \( (j \in [0, n]) \):

\[ L_{N,t}(i) \equiv \left( \frac{1}{n} \right)^{\frac{1}{\psi}} \left[ \int_0^n L_t(i, j)^{\frac{1}{\psi} - 1} dj \right]^{\frac{\psi - 1}{\psi}}, \tag{35} \]

where \( L(i, j) \) is the demand of the labor input of type \( j \) by the producer of good \( i \) and \( \psi > 1 \) is the elasticity of substitution among labor inputs. Cost minimization implies that

\[ L_t(i, j) = \left( \frac{1}{n} \right) \left( \frac{W_t(j)}{W_t} \right)^{-\psi} L_{N,t}(j), \tag{36} \]

where \( W(j) \) is the nominal wage of labor input \( j \) and the wage index \( W \) is

\[ W_t = \left[ \left( \frac{1}{n} \right) \int_0^n W_t(h)^{1-\psi} dh \right]^{\frac{1}{1-\psi}}. \tag{37} \]
Similar equations hold for firms producing intermediate tradable goods. Each household is the monopolistic supplier of a labor input $j$ and sets the nominal wage facing a downward-sloping demand obtained by aggregating demand across Home firms. The wage adjustment is sluggish because of quadratic costs paid in terms of the total wage bill,

$$AC_t^W = \frac{\kappa_W}{2} \left( \frac{W_t}{W_{t-1}} - 1 \right)^2 W_t L_t,$$

(38)

where the parameter $\kappa_W > 0$ measures the degree of nominal wage rigidity and $L_t$ is the total amount of labor in the Home economy.