Liquid Accounts as a Store of Value *

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Abstract

This paper introduces a theory of money as a store of value through a search friction in the goods market. The novel microfoundation helps reconcile the observed amounts of liquidity holdings, with the presence of credit. It also implies a link between the value of money and the demand and supply of goods which explains the joint behaviour of monetary aggregates and real economic activity. In particular, the model provides an explanation of recessions consistent with the surge in liquidity holdings and drop in production capacity utilization, as documented for several recessions including the financial crisis.

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The use of esteemed articles as a store or medium for conveying value may in some cases precede their employment as currency [...] Such a generally esteemed substance as gold seems to have served, firstly, for ornamental purposes; secondly, as stored wealth; thirdly, as a medium of exchange. Jevons (1875)

1 Introduction

As it has long been recognized, money has two primary roles: medium of exchange and store of value. This paper develops a theory of money that exploits the latter motive: the reason why money is demanded even though it is not necessary for transactions and has no intrinsic value, is that goods (consumption or capital) are hard to find because of a search friction. This implies that not all available funds are used to buy goods but the residual is optimally stored into money (or into a deposit account that is like holding money as it gives virtually no interest) not subject to the friction.[1]

To appreciate the role of money as a store of value, it is useful to draw a comparison with the models of money and search (e.g. Kiyotaki and Wright (1989), Shi (1997) and Lagos and Wright (2005)) where, like in this paper, money is microfounded through a search friction in the goods markets. However, these theories microfound a transaction role for money: there, the role for money arises because of the double coincidence of wants due to the fact that anonymity and the lack of enforceability, limits credit (see Kocherlakota (1998) and Aliprantis et al. (2007)).[2] Instead in this model there is no double coincidence problem: I assume enough credit so that the seller is always happy to trade even if the buyer has no money. However, the buyer may not like the good of the seller: this one coincidence problem is enough to generate the need for money as a store of value even in the presence of durable assets that pay a higher return, but which are subject to the friction.[3] Relative to theories based on

[1] The search friction can be broadly interpreted as capturing what hinders the ability to trade quickly, such as information acquisition. Thus that the liquid asset is not subject to the search friction relates to the idea that information insensitive securities should serve as liquidity, Gorton and Pennacchi (1990).

[2] When money has a transaction role, it is also a store of value. For instance, in Lagos and Wright (2005) agents can end up with unspent cash which serves as a store of value. But they would not demand money before entering the decentralized market if it did not facilitate transactions.

[3] One can think of agents having a debit and a credit card. At the end of the period, income is paid in their deposit account (from which they can withdraw with the debit card). Take an agent starting the period with no money and no debt: during the period they buy goods with the credit card. At the end of the period they use their income to clear their credit card debt and —if they

the transaction motive, in some sense this one reverts the issue upside-down: money is not demanded so that people can make transactions more easily, but precisely because people do not make transactions easily. These two theories are not rival and as argued for instance by Wallace (1980), the conceptual distinction between medium of exchange and store of value is not so important per se. But abstracting from the transaction role, this theory offers an explanation for money when trades are not anonymous and the lack of commitment that limits insurance and credit is greatly relaxed.[4] As such, this theory contributes to explaining the large amount of savings held in liquid form and with little return, such as deposit accounts. This is important because deposit accounts amount to a staggering 40% of yearly GDP since the seventies and have in fact increased since the 90's, when electronic payments and credit became more available.[5] These alternative payment means (such as credit cards) curtail the transaction role for money, but do not curtail its role as a store of value as long as they do not lessen the other frictions associated to the purchase of goods, services or commodities.

Money has an important social role and the mechanism also has business cycle implications. The existence of money improves welfare because the possibility to store value in the liquid asset makes agents less preoccupied about not finding goods, but look for better trading opportunities which improves firms' productivity. Technically, the possibility for a buyer to store value in liquid assets leads to a market tightness (sellers over buyers) where the probability of finding goods is lower for buyers, but higher for sellers relative to the non monetary equilibrium.[6] While this channel is desirable because it increases firms' productivity, it is also a natural source for recessions when trading (or matching) conditions deteriorate, leading buyers not to spend their money and sellers not to sell their goods. This link between the "excess sup-

have not found enough consumption or investment opportunities to spend their entire income— they end up with positive current account balance.

[4]It is shown that money is essential with perfect insurance against idiosyncratic matching outcomes and with enough credit to potentially cover all transactions (e.g. available credit equal to 100% of GDP). However, with the maximum feasible credit, money becomes inessential.

[5]The number includes time deposits, arguably a liquid store of value. Checkable deposits alone averaged 8.4% of yearly GDP, or about 100% of monthly GDP. In this model there is only one liquid asset. But in future one could extend the model with several layers of liquidity, from demand and time deposit, to other types of securities with varying degrees of search frictions reflecting their complexity. The model implies that the more severe the friction, the higher the equilibrium return.

[6]The ratio of sellers over buyers can also be rearranged as the value of supply over demand: the presence of money increases the equilibrium value of aggregate demand.

ply" of goods and demand for money formalizes an age old economic intuition which relates to Walras' law and can be traced back to Mill (1844): "It must, undoubtedly, be admitted that there cannot be an excess of all other commodities, and an excess of money at the same time."[7]

This idea, which lies at the core of the neoclassical-keynesian dispute, gained renewed attention in the recent years of increased economic turmoil: according to this view, the financial crisis resulted in a recession because agents stopped spending for consumption and investment but hoarded their wealth in unproductive but safe assets.

Indeed there is evidence that the financial crisis was characterized by a surge in the holdings of liquid assets as is reflected in the large decline in the velocity of money and in the record-high amounts of cash held by firms.[8] That this liquidity surge is related to an excess supply capacity of goods is consistent with the decline in capacity utilization during the Great Recession. Furthermore this pattern is not only true of the financial crisis: Figure 1 shows that in all recessions from the 80s onward, capacity utilization and velocity dropped jointly. The relationship was negative before the 80s; this is also accounted by the model, but through another channel described later.[9]

Despite the appeal of the explanation above, to my knowledge, there is no microfounded model that formally embeds this narrative into a theory of the business cycle. Existing theories of liquidity that study financial crises exploit credit constraints which operate in a rather different way as discussed in the literature review.

[7]Here the excess supply of goods —defined as production capacity minus sales— is an equilibrium outcome given the search friction.

[8]Velocity is an inverse measure of monetary holdings, therefore a drop corresponds to an increase in monetary holdings relative to GDP. Mechanically, declines in velocity are partly explained by the decline in GDP, but it is not clear why the denominator (money) did not decline proportionally. In fact, checkable deposits, M1, M2, and MZM, all *increased* in levels at the onset of the financial crisis and before Quantitative Easing.

[9]Regressing capacity utilization and velocity over the interest rate, inflation, and GDP growth does not explain these facts as the residuals of the 2 regressions exhibit similar patterns: the correlation is 0.43 from the 80s and -0.54 before. Both are strongly significant. Theoretically, to explain fluctuations in velocity has been a long standing challenge since Hodrick et al. (1991). While some progress has been made (see for instance Telyukova and Visschers 2013 and Wang and Shi 2006) Lucas and Nicolini (2015) argue that the interactions between money and financial crises remain poorly understood.

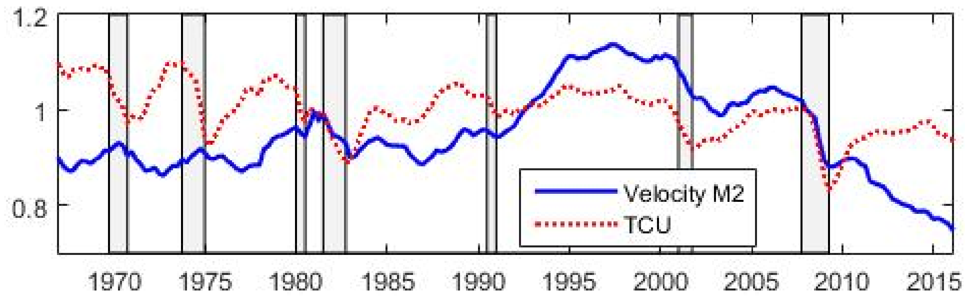

Figure 1: Velocity of M2 and Total Capacity Utilization. Source:FRED
Note: Time series are normalized to be 1 in 2008.

To formally study its business cycle implications, the model is estimated with bayesian methods. While the model is stylised, the quantitative exercise helps illustrate how the theory explains movements in liquidity, capacity utilization, quantities, and prices. As a side, the structural estimation helps showing that the model is sufficiently tractable, despite the microfoundation of money.[10]

While the model is kept simple, it allows for several shocks considered by the literature including technology, labour supply, and demand shocks (similarly to Storesletten et al. 2011). However, a new wedge to the matching process emerges as the primary source of output fluctuations. This wedge is captured by a shock to the "productivity" of the matching function.[11] Intuitively, this is a shock to the intermediation between buyers and sellers; a negative shock captures a more cautious behaviour of buyers and leads to a drop in matches with a resulting increase in excess capacity and in the holding of liquid assets. This shock alone explains 50% of the variance of the growth rate of velocity, 68% of that of capacity utilization, and 37% of that of output between 1967 and 2015. However, this shock is distinguished from a pure demand shock (a shock to search effort similarly to Storesletten et al. (2011)), which is also present and accounts for 20% of the variance of output. In particular, negative demand shocks played a role for the recessions prior to the 80s and explain why velocity did not decline in these recessions as shown in Figure 1: a negative demand shock

[10]The reason why this model is so tractable is that by relaxing the assumption of anonymity, it is possible to have insurance markets and study a representative agent model grounded into a modified neoclassical setting which can be linearized.

[11]While this shock can be seen as a "measure of our ignorance", it helps appreciating the implications of the model. Furthermore, the idea that the matching between demand and supply is the place where much of the business cycle is generated seems worth investigating. In traditional models the uncovered wedge is by and large imputed to TFP shocks.

increases supply over demand, thereby increasing the matching probability of buyers, and reducing excess liquidity / increasing velocity. Instead, the negative matching shock mentioned earlier reduces velocity. Both shocks reduce capacity utilization. Finally, technology shocks explain 28% of the variance of GDP. But they are generally not important during recessions with the exception of the ones of 1973 and 1981.

2 Literature Review

Liquid assets are present in many brunches of the literature. First, a microfoundation of fiat money typically interpreted as a store of value is offered by the overlapping generations (OLG) models pioneered by Samuelson.[12] In the Bewley models (Bewley 1980) consumption uncertainty and the lack of insurance, lead to precautionary liquid savings. This liquidity in excess of expected consumption is usually interpreted as a store of value. However, to make money coexist with other assets that pay dividend, it is necessary to give to money the transaction advantage of being the only asset that can be quickly exchanged for goods to buffer idiosyncratic shocks: see Wen (2015). Other related frameworks where liquidity arises as a combination of timing issues and credit frictions are Holmstrom and Tirole (1998) and Diamond and Dybvig (1983). One contribution relative to these literatures is to offer a motive for liquidity that is robust to the presence of insurance and credit technologies and completely disentangled from transaction needs. These are desirable features given that credit and electronic payments are more and more available. Besides the differences in the microfoundations, the business cycle implications of these theories are rather different.

Also related is the Baumol-Tobin framework (BT) where there is a transaction cost of adjusting the financial portfolio: recent examples include Alvarez and Lippi (2009), Ragot (2014), and Kaplan and Violante (2014). Although often BT models assume that purchases need to be mediated by the liquid asset, this assumption is not necessary so there too money is essentially a store of value. The microfoundation based on search frictions has the following to offer to this exciting research strand: 1. The cost of holding money varies over the business cycle. This way the model predicts when money demand is procyclical and when it is not, and relates this to the endogenous aggregate productivity. 2. With the search friction it is straightforward

[12]However, Wallace (1980) challenges the existence of a clear cut distinction between medium of exchange and store of value in OLG models.

to also assume search effort. Effort costs resemble transaction costs in BT but do not bare the entire responsibility for money holdings as —owing to the search friction— money is demanded even without the BT-like cost. Furthermore, financial innovation (lower effort costs) does not necessarily decrease the level of money demand.

It should be noted that theories where the store of value role of money is more prominent are not mutually exclusive neither between them, nor relative to theories where money is mainly for transactions. For instance Telyukova and Visschers (2013) have both precautionary, and transaction money demand through a cash in advance constraint to account for the variance of velocity.[13] Furthermore, Telyukova (2013) reconciles the coexistence of money holdings and rolled over credit card debt in a model where consumption uncertainty cannot be fully insured through credit cards. See also Wen (2015) for some forms of credit insurance in the Bewley framework.

There is a growing literature that incorporates search frictions in the goods market: examples include Storesletten et al. (2011), Huo and Ríos-Rull (2013), Petrosky-Nadeau and Wasmer (2011) and Den Haan (2014). The main contribution to this literature is to use the framework to construct a theory of liquidity. Furthermore, disciplining the model through monetary quantities elicits the distinction between matching shocks and demand shocks. Also promising is that the model explains 37% of the variance of hours without the labour supply shock and with low labour supply Frisch elasticity. This is an improvement relative to comparable neoclassical business cycle models, see Ríos-Rull et al. (2012). Labour is more volatile here because the income from working cannot be immediately spent. This wedge worsens during recessions consistently with the business cycle accounting of Chari et al. (2007). A growing literature aims at generating recessions with this feature. See for instance Bai et al. (2011), Lopez (2012) and Duras (2014).

The paper is also related to the vast literature that models the financial crisis through credit constraints building on Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In a sense, these theories work in the opposite way to that in this paper. In these models during recessions firms *wish* to produce more but are constrained.[14] Here instead, firms do *not wish* to produce more because of their in-

[13] Wang and Shi (2006) also account for the variance of velocity with search intensity, and with a transaction motive. This paper contributes by especially focusing on velocity during recessions.

[14] See Kiyotaki and Moore (2012), Shi (2012), Jermann and Quadrini (2012), Christiano et al. (2014) and Iacoviello (2015) among others.

ability to sell. These two channels are possibly both real: in the mentioned literature, resources do not reach firms at beginning of production process (to finance input). In the present paper, there is a friction in the sales process. Since this channel is not based on borrowing constraints, but on the incapacity to spend even having access to liquidity, it offers an alternative explanation to the idea that the credit channel has been impaired. In this model too credit declines during recessions, however this does not happen because of tighter credit but because of fewer spending opportunities.[15]

Besides the different explanation for the business cycle, there is also a distinction in the implied notion of liquidity which can be best appreciated by relating to Cui and Radde (2014) and Cui and Radde (2016) who microfound the resaleability constraint in Kiyotaki and Moore (2012) with a search friction. There too, money is valuable because it is not subject to a search friction. However, there is a crucial distinction in the role money plays. There the liquidity advantage of money is that it can be more easily *spent* than other means of payment (this is a transaction role). Here it means that money can be more easily *acquired* than other assets to store value, but money has no advantage in being exchanged for goods relative to other means of payment.[16]

Finally, like the New Keynesian framework, this one may prove useful to study monetary policy. In that framework the main focus is on interest rate policies. An advantage of this framework is to be also able to explain monetary quantities. Thus, it may prove useful to also study unconventional policies such as Quantitative Easing.[17]

The paper proceeds as follows. Section 3 sets up the model. Section 4 offers a theoretical characterization and Section 5 includes the quantitative analysis. Section 6 concludes. The appendixes contain proofs, special cases, figures and tables.

[15]This paper does not study monetary policy (other than showing that money is neutral but not superneutral and that the Friedman rule is optimal) but it is worth pointing out that while the two theories are both consistent with a shrink in loans, they may have different policy implications: open market operations aimed at easing credit conditions can be effective in models with credit constraints (see Kiyotaki and Moore 2012) but may not be as effective in this model which, to a degree, subscribes to the adage: you can lead a horse to water but you cannot make it drink. Evidence on the effects of Quantitative Easing is at best mixed, see Williamson (2015) and references therein.

[16]Relaxing this latter assumption one could capture both roles of liquidity.

[17]In relation to this issue, Woodford (1998) showed in an influential paper that even in the presence of credit that lead to a cash-less limit, the framework remains useful for monetary policy. This raises the issue of whether monetary aggregates are important at all. But to understand the extent to which money matters, it seems important to explain it. This theory accounts for the short run fluctuations in monetary aggregates. Furthermore, monetary aggregates have important consequences for the identification of the matching shock as a new and primary source of the business cycle.

3 The Model

Time is discrete. The economy is populated by a continuum of measure one of households that live forever. In each period static firms produce goods for consumption and investment purposes with a neoclassical production function of labour and capital. Besides consumption and capital, there is a costlessly storable object, called money, which is divisible and intrinsically useless.

Similarly to the standard neoclassical model, in each period firms sell goods to households while labour and capital inputs are supplied by households and demanded by firms. These two latter input markets are competitive. Instead, the market for goods is subject to a search friction.[18]

The market structure for goods is as in Menzio et al. (2013). There is a continuum of submarkets indexed by the terms of trade $(p, q) \in \mathbb{R}_+ \times \mathbb{R}_+$ where p is the price per unit of good paid by the household (the buyer) and q the quantity that goes from the firm (the seller) to the buyer. So pq is the actual payment made by the buyer.[19] A firm chooses how many trading posts to create in each submarket (i.e. how many units of size q to put for sale in each submarket) and a household chooses which submarket to visit. It is convenient to use one of these submarkets as the numeraire. Let p_m be the price of money in terms of the numeraire.

As is typical in search models, the buyer cannot visit multiple submarkets in the same period and can at most find one trading post. So the matching process is such that a household and a trading post meet in pairs; let the matching function μ be concave and homogeneous of degree one in the number of trading posts f and households h, with continuous derivatives. In a sub-market with tightness $\theta = \frac{f}{h}$, let $\psi(\theta) = \mu(f, h)/h = \mu(\theta, 1)$ denote the probability with which a household or buyer finds a trading post, and $\phi(\theta) = \mu(f, h)/f = \mu(1, 1/\theta)$ the probability with which a trading post is matched with a buyer. The function ψ is strictly increasing with $\psi(0) = 0$ and $\psi(\infty) = 1$. ϕ is strictly decreasing with $\phi(0) = 1$ and $\phi(\infty) = 0$.

Search is competitive as in Moen (1997). That is, terms of trade cannot be renegotiated within a submarket (i.e. the price p and the quantity q cannot be renegotiated:

[18]It would be possible to consider search frictions for the inputs markets too. But to isolate the key novelties, the model is kept as close as possible to the neoclassical one.

[19]Terms of trade can be equivalently expressed as the balance pq and the quantity q exchanged. While this latter approach is more conventional in the literature, I prefer to index submarkets by (p, q) because this way it will be more immediate to talk about inflation later.

either there is an exchange of pq for q or there is no deal) and market tightness varies with the terms of trade across the sub-markets according to the equilibrium function $\theta(p, q)$, which is taken as given by firms and households. As a result, the probabilities ϕ and ψ are endogenous functions of (p, q).[20]

Unlike other models of money and search, the payment pq need not take the form of money. To clarify this, it is useful to specify the following timing within the period: the input markets clear at the beginning of the period but payment from firms to households is deferred to the end of the period, after firms revenues are realized. After the input markets clear and before inputs are paid, households and firms make transactions in the frictional market. Households can pay with their money holdings, but also with claims on the end of period income (IOUs).

The ability to pledge the entire income is what is necessary to completely abstract from a cash in advance constraint as agents in equilibrium spend exactly their income.[21] This credit arrangement would make money loose value in models with money for transactions such as the Lagos and Wright (2005) framework: agents would not bring money in the decentralized market if they can promise to pay with their income from the centralized market. While this seems a natural benchmark, even looser credit limits are conceivable: Section 4.5 discusses a credit arrangement that, at the maximum feasible limit, leads to money being inessential.

3.1 Households

During the period, let households available funds in terms of the numeraire be $p_m m + wn + kr$, where m is the amount of money they had from the pervious period, w the wage rate, n hours of work, k the capital stock and r its rental price. So $wn + kr$ is the income from labour and capital that households receive at the end of the period but that they can pledge in exchange for goods.[22]

[20] Competitive search is adopted here because it does not add a bargaining inefficiency, thereby not introducing a further element of departure from the neoclassical framework.

[21] Even though agents are never constrained by their liquidity holdings, the model is consistent with credit shrinkages in recessions as the income they may borrow, shrinks. However, whether agents make transactions with money or with credit is not determined in the model.

[22] Households also own the capital stock, which –like firms production– could in principle be put on sale in the frictional goods market; but they do not wish to sell capital at the market price because for each unit sold, she would then buy $\psi < 1$ goods back and hold the rest in money, which pays a lower return. So in each submarket, the reservation price of the household is higher than the market price charged by firms.

A household enters submarket (p, q) such that

$$pq \leq p_m m + wn + kr. \tag{1}$$

The latter inequality comes from the fact that, since the terms of trade within the submarket are non negotiable, if pq is larger than her available funds $p_m m + wn + kr$, she cannot make the transaction.

With probability $\psi(\theta(p, q))$ there is a match so the household pays pq and buys goods for q which can then be either consumed or used to accumulate capital:

$$c + i = q$$

where c is consumption and i is investment in capital which accumulates according to $k' = i + k(1 - \delta)$, where $\delta \leq 1$ is the depreciation rate. Furthermore, end of period capital $k' \geq 0$. *I.e.* the household can disinvest to the point of consuming up the entire capital stock.[23]

At the end of the period she receives income payments $wn + kr$ (in the form of money or IOUs). Since the household spent pq (in either money or IOUs), her end of period balance is $p_m m + wn + kr - pq$, which is greater or equal to zero given Equation (1). After clearance of IOUs, this balance is stored in money.[24] So the household end of period money holdings m' are

$$p_m m' = p_m m + wn + kr - pq.$$

With probability $1 - \psi(\theta(p, q))$ the household does not make a transaction; in this case she does not buy any goods and at the end of the period she is left with the initial money holdings plus income:

$$\begin{cases} c + k' - k(1 - \delta) = 0, \quad k' \geq 0 \\ p_m m' = p_m m + wn + kr. \end{cases}$$

[23]Since I will introduce insurance markets and Inada conditions in the utility function, this constraint is only avoiding Ponzi schemes, but it does not induce the sort of precautionary savings it would in an incomplete market model à la Aiyagari (1994).

[24]In terms of IOUs, each agent i has a net position of IOUs equal to the IOUs received by firms (and issued by some other household) less IOUs she issued. Call b_i the IOUs issued ba agent i and $b_{-i,i}$ the IOUs agent i receives by firms (the indexes $-i, i$ emphasise that the bond is issued by some household other than i, and passed on to agent i through some firm.) If the net position is positive, she gets money, if it is negative, she pays money (and Equation (1) ensures she has the money). So the money agent i receives is $p_m dm_i = b_{-i,i} - b_i$. The sum of net positions of all households is $\sum_i b_{-i,i} - \sum_i b_i = 0$. So the aggregate money exchange from an household to another clears: $\sum_i dm_i = 0$. See Ljungqvist and Sargent (2012) Section 18.10.3 for a discussion of the exchange and clearance of IOUs in similar settings.

That some agents trade and others do not generates heterogeneity in assets holdings. Ways to maintain tractability in search models are either to assume a big family with many agents as in Shi (1997), or to use the timing and preference structure developed in Lagos and Wright (2005). However, since there is no need to assume anonymity to rule out credit, but actions are monitored, it is possible to have insurance for all households in the same sub-market: assuming the law of large numbers holds so that $\psi(\theta)$ is the exact share of the population that successfully made a transaction, all households that participated in the same sub-market by being ready to pay $x = pq$ for q, receive goods for $\psi(\theta)q$ and pay $\psi(\theta)x$. *I.e.* the share $\psi(\theta)$ of households that made a transaction, transfer $(1 - \psi(\theta))q$ of goods each and are thus left with $\psi(\theta)q$. The transfers sum up to $\psi(1-\psi)q$ which can be divided among the remaining $(1-\psi)$ share of the population so that each receives ψq. In turn, those that receive the goods transfer, make a payment of $\psi(\theta)x$ in liquid assets. It is easy to check that this way each receives goods for $\psi(\theta)q$ and pay liquid assets for $\psi(\theta)x$.[25]

Insurance markets simplify the analysis and clarify that the role of money does not depend on the absence of insurance. However, it is worth discussing the meaning of goods redistribution with search frictions. An interpretation consistent with ex-post redistribution is that goods come in different varieties and each household can only store (and therefore buy) a subset of such varieties but a variety is not known before visiting the trading post. After purchases are made, there can be perfect insurance between households that like the same variety. It should be noticed that this theory of money does not hinge on this insurance assumption: it would be possible to solve the model without the insurance market and allow for heterogeneity to spread.

With insurance markets it is possible to study the problem of a representative household: she starts each period with capital k and money m. For recursive equilibria, the aggregate state Ω is composed of the aggregate capital stock K and money M, and of a vector of shocks with a known Markov process to be defined later.

[25]Monitoring is necessary because without it, a possible strategy is to go to a market that one cannot afford but where one's full balance is equal to the ex-post payment $\psi(\theta)x$. This way the household is not able to pay x in case of a successful match but could then pretend to have not matched and claim a transfer from the other households. Of course, this way overall transactions would not be enough to sustain the insurance scheme. For this reason, anonymity rules out the presence of this insurance market. Notice that the insurance suggested here does not have the incentive issues present in the big family assumption in Shi (1997): there agents do not respond to their individual incentives but act in the interest of the entire family even though they are not monitored.

The household solves the following problem with rational expectations:

$$V(k, m, \Omega) = \max_{\{c,n,d,k',m',q,p,\hat{m}\}\geq 0} u(c, n, d) + E\beta V(k', m', \Omega') \tag{2}$$

$$s.t. \quad p_m\hat{m} + pq \leq p_m m + wn + kr, \tag{3}$$

$$q \leq A_d d \tag{4}$$

$$c + k' - k(1-\delta) \leq \psi(\theta(p, q))q, \tag{5}$$

$$p_m m' \leq pq\,(1 - \psi(\theta(p, q))) + p_m\hat{m}. \tag{6}$$

Where β is the discount factor. The utility function $u(\cdot)$ is increasing in consumption c, and decreasing in market labour n and in shopping effort d. $u(\cdot)$ is concave and has continuous derivatives with $\lim_{c\downarrow 0} u_c = \infty$, $\lim_{n\downarrow 0} u_n = 0$ $\lim_{d\downarrow 0} u_d = 0$.

The household takes input market prices w and r as given. E indicates rational expectations taken over next period aggregate state Ω' given Ω.

Equation (3), states that available funds $p_m m+wn+kr$ are allocated to the purchase of goods pq plus a possible residual the household may wish not to buy good with, which is stored in money: $p_m\hat{m}$. Allowance for $\hat{m}$ is made because searching for goods is costly; this is captured by Equation (4) which states that the more goods one looks for, the more effort d is needed, and $A_d > 0$ is an effort productivity parameter.

Equation (5) shows that only a fraction ψ of demand q is matched with investment and consumption goods. What is left is invested in liquid assets as shown in Equation (6). From this latter equation, it is intuitive why money has value in this economy: $\psi < 1$ implies that not all funds can be spent in goods which gives rise to money demand to store leftover wealth. It should be noticed though that the household effectively chooses ψ (and thereby money holdings) by choosing p and q, which determines market tightness given the equilibrium function $\theta(p, q)$. The first order conditions to the household problem, reported in Appendix A, help clarify this point. In particular, it is useful to discuss the choice of p —Equation (45) in the appendix — which illustrates the key trade-off in the decision of buying goods versus holding money. Rearranging and focusing on an interior solution, the equation can be written as

$$\frac{\partial\psi}{\partial\theta}(\lambda_3 - \lambda_4 p) = \frac{\partial p}{\partial\theta}\left(\lambda_1 - \lambda_4(1-\psi)\right),$$

where λ_1 — λ_4 are the Lagrange multipliers on Constraints (3)—(6).

Why are agents willingly holding money even though it is dominated by capital in return? To substitute money for goods they have to choose a submarket with higher p. This reduces θ (it is shown later that θ is increasing in p) and increase ψ, with the benefit of ending up with more goods and less money as shown in the left-hand-side of the equation above. However, they pay more per unit of good relative to money (right-hand-side). So the choice of the submarket balances the trade-off between the cost of higher prices versus that of being left with money rather than having either consumed or invested in capital.

Later, it is shown that costly effort can induce additional money demand because it saves effort to store wealth in money ($\hat{m} > 0$) rather than searching for q.

3.2 Firms

Firms can choose to open trading posts in any market identified by price and quantity. A trading post in market (p, q) has a match with probability $\phi(\theta(p, q))$, in which case it sells q. To open a trading post, a firm needs production capacity $Ak_d^\alpha n_d^{1-\alpha} \geq q$, where k_d and n_d are the capital and labour inputs. [26]

A trading post in market (p, q) gives profits

$$\pi(p, q) = \max_{k_d, n_d} \phi(\theta(p, q))pq - wn_d - rk_d \tag{7}$$

s.t.

$$q \leq Ak_d^\alpha n_d^{1-\alpha} \tag{8}$$

The first order conditions for capital and labour are

$$\xi(p, q)\alpha A \left(\frac{n_d}{k_d}\right)^{1-\alpha} = r, \tag{9}$$

$$\xi(p, q)(1-\alpha)A \left(\frac{k_d}{n_d}\right)^{\alpha} = w. \tag{10}$$

[26] Otherwise a firm could open many trading posts and exploit the law of large numbers across them to have production capacity only for sales: $Ak_d^\alpha n_d^{1-\alpha} = \phi(\theta(p, q))q$. Ruling this out implies some excess production capacity and an endogenous Solow residual. Following Storesletten et al. (2011), it is assumed that excess production capacity is not storable. This assumption seems reasonable for services and nondurables, which form the majority of GDP. In future it may be interesting to allow for inventories, but to match its rich dynamics (e.g. procyclical inventory investment) the model should be complicated for instance by introducing S-s policies or stockout-avoidance motives; see Wen (2011) for a recent analysis.

Where $\xi(p, q)$ is the lagrange multiplier on the production constraint. The ratio of these two first order conditions implies that $\frac{k_d}{n_d}$ is the same in any trading post. In turn, this implies that $\xi(p, q)$ is equal for all (p, q). Thus it is going to be called ξ from now onward.

Using the 2 first order conditions, maximized profits can be written as

$$\pi(p, q) = \phi(\theta(p, q))pq - \xi q \tag{11}$$

Since firms can choose between any market (p, q), all potentially active markets must give the same profits. Furthermore, free entry implies that such profits must be zero: if profits were positive there would be infinite posts and $\phi(\theta(p, q)) = 0$, which contradicts that profits are positive. Thus Equation (11) implies

$$\phi(\theta(p, q))p = \xi. \tag{12}$$

Since Equation (12) has to hold for a market to be active, it is what defines the function $\theta(p, q)$ that households take into account.

3.3 Equilibrium

Before defining an equilibrium, it is useful to point out a few properties. The next lemma shows that Equation (12) implies that $\theta(p, q)$ trivially depends on q.

Lemma 1 *$\theta(p, q)$ does not depend on q.*

Intuitively, the fact that the technology has constant returns to scale and input prices are taken as given implies that production increases proportionally with costs. Then, if profits per unit of production are zero, for a change in q not to affect profits, ϕ and thereby θ have to remain constant. From now on the dependance on q will be omitted and the function θ will be denoted $\theta(p)$.

It is also immediate from Equation (12) that $\theta(p)$ inherits the differentiability properties of ϕ and that p is a strictly increasing function of θ.

Finally, from Equation (12) it is clear that given θ, p is proportional to ξ. In other words, Equation (12) pins down a functional relationship between θ and p up to a value for revenues per unit of production ξ. This value is free and can be normalized.[27]

[27] For this one has to show that all other prices (r, w and p_m) also change proportionally to ξ,

As a normalization, ξ is chosen to be equal to the equilibrium value of ϕ. This implies $p = 1$ in the equilibrium submarket as is immediate from Equation (12).[28]

To close the model it remains to specify the exogenous stochastic variables. They are z_m, A_d, A, χ_n, and β. z_m and A_d—shocks to the matching function and to effort productivity—are natural wedges implied by this paper, so it is interesting to study their implications. Shocks to technology A, labour supply χ_n, and discount factor β, have been shown to be important drivers of the business cycle and will avoid stochastic singularity in the likelihood estimation, see Fernández-Villaverde et al. (2016).

Definition 1 *An equilibrium is composed of a value function V and policy rules $c, k', n, d, m', \hat{m}, q$ for the household as function of k, m, Ω, where $\Omega = \{K, M, z_m, A_d, A, \chi_n, \beta\}$ with K and M denoting the aggregate capital and money stock. A function $\theta(p; \Omega)$ and prices w, r, p_m, measure of firms f, input demands per firm k_d and n_d, revenues per unit of production ξ, all functions of Ω, such that the following conditions are satisfied:*

1. *Household: The household's decision rules and the value functions V solve the household problem in 3.1*

2. *Firms: k_d, n_d and $\theta(p; \Omega)$ satisfy Equations (9), (10), and (12) with $\xi = \phi(\theta(1; \Omega))$.*[29]

3. *Market Clearing: Households purchases are equal to firms sales:*

$$\psi(\theta(1; \Omega))q = \phi(\theta(1; \Omega))fq; \tag{13}$$

The liquid asset, and inputs markets clear:

$$M = m'.$$

so that no relative price is changed. It is immediate from Equations (9) and (10) that given an allocation, r and w are also proportional to ξ. Expressions for r and w and p from Equations (9), (10) and (12), can then be substituted into the budget constraints —Equations (3) and (6)— to show that p_m is also proportional to ξ. Since no constraint is changed, neither will the optimal choices and thus the equilibrium allocation.

[28]Following Moen (1997), equilibria with multiple active submarkets are ruled out. Those could be possible in principle because both households and firms arbitrage between submarkets call for increasing functions between p and θ, so they may touch more than once. Numerically I find that the case of multiple active markets may only occur in a nife-edge parameterization discussed in Section 5.3.2.

[29]*The latter condition implies that the equilibrium p is normalized to 1.*

$$n = fn_d. \tag{14}$$

$$K = fk_d. \tag{15}$$

4. *Aggregate and individual state variables are consistent:* $k = K$, $m = M$.

5. $z_m, A_d, A, \chi_n, \beta$ *follow stationary stochastic processes of order one.*

Notice that Equation (13) implies $\psi(\theta(1;\Omega))/\phi(\theta(1;\Omega)) = f$. Furthermore, the functions ψ and ϕ are such that $\psi(\theta)/\phi(\theta) = \theta$, so $\theta = f$. This is consistent with the definition of the market tightness $\theta = f/h$ because the measure of households is 1.

4 Characterization

4.1 The role of the search friction and effort for money demand

To highlight the role of the key features of the model (search and effort), it is instructive to study the case when the matching function is $\min(f,h)$ and effort is not costly (*i.e.* the utility function is flat in d). As discussed in Appendix B, in this case the equilibrium is characterized by $\theta = \psi = \phi = 1$, and money looses all value. Furthermore, the equilibrium boils down to the one of the neoclassical model. While perhaps not surprising, it is useful to point this result out because it clarifies what exactly gives rise to valued money: $\psi < 1$ implies that not all funds can be spent in goods which gives rise to money demand. The effort constraint can give a further reason to demand money because it saves effort to store wealth in money ($\hat{m}$) rather than searching for q.

4.2 Changes in money supply

Suppose the government could change the quantity of M. Would it matter? The proposition below shows that money is neutral, but not superneutral. To allow for money to change over time, households receive a lump sum monetary transfer $dm = M' - M$. Thus $p_m dm$ is added to the right hand side of Equation (3).

Proposition 1 *Money is neutral but not superneutral.*

Since money is not superneutral, the following section discusses monetary policy in order to achieve efficiency.

4.2.1 The Friedman rule is optimal

It is first necessary to define efficiency. For that, I construct a planner problem. Since this subsection characterizes deterministic steady state results, for simplicity, the planner problem abstracts from the shocks.

Definition 2 *An allocation $\{c, n, d, q, k', \theta\}$ is said to be efficient if it solves the following planner problem:*

$$\widetilde{V}(k) = \max_{\{q,c,k',\theta,d,n\}\geq 0} u(c, n, d) + \beta E\widetilde{V}(k') \tag{16}$$

s.t.

$$\theta q \leq Ak^{\alpha}n^{(1-\alpha)} \tag{17}$$

$$q \leq A_d d \tag{18}$$

$$c + k' - k(1-\delta) \leq \phi(\theta)\theta q \tag{19}$$

The planner chooses market tightness θ (or equivalently the number of trading posts f as households have measure 1 so $f = \theta$).

Equation (17) ensures that total production is not smaller than the quantity offered by each trading post (q) times the number of trading posts θ.

Constraint (18) states that the planner has to respect the household's effort constraint, this is equivalent to Equation (4) in the household problem and it is repeated for convenience.

The resource constraint, Equation (19), is derived by Equation (5), the equilibrium condition (13), and the fact that θ is equal to the number of trading posts f.

The next proposition shows that in steady state, the first order conditions of the planner and the household coincide at the Friedman rule. If the household problem is concave, this implies that the planner outcome is an equilibrium.[30] It should be noted that concavity does not hold for any parameterization: as discussed in Section 5.3.2,

[30]Without concavity of the household problem in principle the solution to the planner problem may not be an equilibrium, even though it satisfies the household first order conditions.

it is necessary to have a sufficient complementarity in the matching function. While concavity of the household problem is needed to formally conclude that the Friedman rule is efficient, all other theoretical results, including those using the household first order conditions, do not require concavity: to the extent that the first order conditions are necessary, *i.e.* they must hold in an equilibrium, they characterize equilibrium.[31]

Proposition 2 *In the steady state of a monetary equilibrium with positive output, the first order conditions of the planner and the household coincide at the Friedman rule.*

That the Friedman rule is optimal may appear counterintuitive at first glance. Intuitively, inflation could have been beneficial in this framework, because it could have forced households to search more.

To foster intuition, Corollary 1 clarifies how inflation distorts the allocation and Proposition 3 relates the physical allocation, to money holdings. These two results are derived in the special case of no effort costs ($u_d = 0$). This is instructive because inflation implies no wedge between the household first order condition for d—Equation 41—and the one of the planner.

The main intuition behind these results is that with inflation agents do not want to remain stuck with money and so they choose a market with too high θ relative to the planner solution; this way it is easy for the buyer to find goods, but difficult for the seller.

Before moving to the corollary it is useful to discuss the planner solution with no effort costs. In this case it is optimal to put $\theta = 0$. This is because Constraint 18 in the planner problem does not bind and Constraints (17) and (19) imply

$$c + k' - k(1-\delta) \leq \phi(\theta) A k^{\alpha} n^{(1-\alpha)}.$$

From this last equation it is evident that θ approaching zero is optimal because then ϕ tends to one and all production is either consumed or invested, so there is no waste. With θ approaching zero, Equations (17) and (18) imply that q and d approach infinity. Intuitively, the number of trading posts tend to zero, but become

[31] Also, if the constraint set is not convex the Bellman equation may not be differentiable. However, the first order conditions can be derived through a variation method without relying on the Bellman equation. See Stokey et al. (1989) Section 4.5.

large.[32] This extreme result with no effort costs also highlights the role of demand in this model: there is a benefit for the planner to make households search in crowded markets (where the ratio of households per trading posts is high), because the higher the demand for each trading post, the higher ϕ. This also implies high d and low ψ, but it is not a cost if effort is free. Is this implementable? The next corollary shows that θ is chosen optimally at the Friedman rule and not otherwise.[33]

Corollary 1 *Assume that effort is not costly and consider the steady state of a monetary equilibrium with positive output. Then at the Friedman rule, $\theta \to 0$ and $\phi \to 1$. When inflation is above the Friedman rule, $\theta > 0$ and $\phi < 1$.*

The next proposition highlights an implication that at first glance seems counterintuitive: efficiency calls for large amounts of liquid savings. This may seem counterintuitive because in this model money is a measure of savings not matched with goods. However, the possibility to store in money, makes agents choose a lower market tightness, which is efficient because it makes firms sell more goods, but it also implies a low ψ. Put differently, large amounts of liquidity mean large funds available to spend, which improve the allocation.

Proposition 3 *Assume that effort is not costly. In a deterministic steady state with positive output, the value of money tends to infinity at the Friedman rule.*

In summary, an intuition for the Friedman rule and the value of money going to infinity goes as follows: the private benefit from choosing a market with low θ is that p is lower and q is larger. The private cost is that ψ is lower and more savings have to be made in the form of money. When money is a dominated asset in terms of return, this trade-off implies a privately optimal θ greater than zero, so ψ is larger than zero and money is bounded. But at the Friedman rule money gives the same return as capital, so there is no cost in saving in money rather than in capital. So

[32] To understand this it is useful to draw a comparison with models of search in the labour market such as Mortensen and Pissarides (1994); there market tightness is given by the ratio between vacancies and unemployment. If vacancies were free to post, free entry would imply infinite vacancies. Hence the cost of search here takes the role played by vacancy posting costs in Mortensen and Pissarides (1994).

[33] In this case with no effort cost, that θ is optimal at the Friedman rule is proven without assuming concavity: at the Friedman rule the household first order conditions are *only* consistent with the efficient level of θ.

households appreciate the full social benefit of choosing a market with low θ and thus they choose $\theta \to 0$, where q and $p_m m'$ tend to infinity. When effort is costly, there is a further cost of choosing a lower market θ and a larger q because it implies more effort. Thus $\theta > 0$ and q is bounded even at the optimal allocation.

These results also highlight that money has a social role in that it affects people search decisions: when money is a good store of value, agents are not worried about holding it and search optimally, making the matching efficient.[34]

4.3 Lazy money

As mentioned, effort costs ensure that $\theta > 0$ even at the optimal allocation. Effort costs also introduce the possibility that $\hat{m} > 0$. This is an interesting case because it implies further voluntary liquid funds beyond those induced by the matching friction. The following result helps clarifying the role played by effort costs for $\hat{m} > 0$.

Proposition 4 *In a monetary equilibrium when production is positive and $\psi > 0$, costly effort is a necessary condition for $p_m \hat{m} \geq 0$ not to bind.*

Intuitively, with costly effort, to save in money saves the effort from searching for goods. This can be appealing during recessions when the return from capital goods may be very low.

While useful to clarify the mechanisms, in practice the numerical part shows that $\hat{m} > 0$ happens in very rare portions of the state space and never in the simulations run.

It is worth highlighting the assumption $\psi > 0$ in the proposition: with $\psi = 0$, $\hat{m} \geq 0$ is not binding even without costly effort. This is worth mentioning because without effort costs, $\psi = 0$ is an equilibrium at the Friedman rule, as shown in Corollary 1. Intuitively, at the Friedman rule money is as good as capital so one might as well have $\hat{m} > 0$.

4.4 Money is essential

A final way to appreciate the role of money is to think of the non monetary equilibrium: what would happen if people expected future prices $p'_m = p''_m = ...$ equal to

[34] That the matching becomes efficient also hinges on competitive search. Alternative bargaining systems may introduce effort inefficiencies, with possibly different implications for inflation.

zero? With no value for money in the future, from Equation (43), either $p_m = 0$, or $\lambda_4 = 0$, or both.[35]

Firms' first order conditions, the fact that the production technology has constant returns to scale, and market clearing for the capital and labour markets imply that

$$\phi f A k_d^{\alpha} n_d^{1-\alpha} = \phi A k^{\alpha} n^{1-\alpha} = wn + rk. \tag{20}$$

If $p_m = 0$, from Equation (3) $q = wn + rk$, which combined with Equation (20) implies

$$q = \phi A k^{\alpha} n^{1-\alpha}. \tag{21}$$

Equation (13), the binding capacity constraint (8), and the fact that $f A k_d^{\alpha} n_d^{1-\alpha} = A k^{\alpha} n^{1-\alpha}$, imply

$$\psi q = \phi A k^{\alpha} n^{1-\alpha}. \tag{22}$$

Equations (21) and (22) then imply $\psi = 1$ and, through the matching function, $\theta = \infty$ and $\phi = 0$. Since production is bounded, $\phi = 0$ implies that no goods are sold, and from Equation (5) consumption would be equal to negative investment, depleting capital.[36]

This inefficiency helps appreciating the role of money in this model perhaps more than anything else: if buyers cannot store value in the form of the liquid asset, they prefer markets where it is inefficiently too easy to buy goods, but this hinders firms ability to sell goods.

4.5 More credit

In the model set before, agents can issue IOU's for up to their entire income.[37] With this credit limit, agents do not need to use their money stock to make transactions

[35] The case $p_m, \lambda_4 > 0$ and $\lambda_{m'} > 0$ is not a monetary equilibrium as it violates $m' = M > 0$. And if the equilibrium is non monetary, then $p_m = 0$.

[36] It does not seem possible in principle to rule out the case $\lambda_4 = 0$ and $p_m > 0$. In this case Equation (6) is not binding, so it is possible to have $p_m \hat{m} + pq(1-\psi) > p_m m' \geq 0$. With $p_m > 0$, from Equation (3), $q \geq wn + rk$ which combined with Equation (21) implies $q \geq \phi A k^{\alpha} n^{1-\alpha}$. Then from Equation (22) $\psi \leq 1$, *i.e.* ψ can be smaller than 1, so that agents choose $\theta < \infty$, which implies $\phi > 0$ and some production would take place until the period in which p_m becomes equal to zero. Intuitively, households dislike to hold m' and this gives incentives to have $\psi = 1$. But households can deem $p(\theta)$ so steep that it is better to have θ s.t. $\psi < 1$ rather than increasing ψ and thereby also increasing p and having to cut q.

[37] This can be seen from Equation (3): suppose the an agent has $m = 0$, then it can still buy goods for up to $pq = wn + kr$. An agent that spend her entire (end of period) income with no money, is issuing IOU's for her entire income $wn + kr$.

as in equilibrium agents spend exactly their income.[38] While this seems a useful benchmark because it distinguishes the model from models with a cash-in-advance constraint, a natural question is whether there are alternative credit arrangements that could lead to money being inessential.

Money would become inessential if households could commit to the following agreement: each household that is matched with a firm buys goods for $1/\psi$ of her funds $wn + rk + p_m m$. This is more than each households personal funds and thus it requires transfers from other agents, but it is feasible because only a fraction ψ agents are matched. This way the aggregate funds are all invested in goods and money demand is zero.

This arrangement is decentralized below with intratemporal bonds which resemble credit cards through which agents have access to a mutual fund where all households put their joint funds: each household uses the credit card overdraft for $1/\psi$ of her funds if she finds goods. The household can then extinguish the loan at the end of the period, or she can roll it over by means of an intertemporal bond.

It is found that with the maximum feasible credit and without effort costs, money looses all value because all available funds are turned into goods and thus the residual money demand is zero. With effort cost it is less clear because one may want to save in money rather than in goods to save effort. However, it is shown that money looses all value in steady state. Any credit limit below this limiting case makes money valuable.[39]

From this analysis, it should be clear that the fact that credit is not multi-period is not essential for the role of money. For money to loose value, each agent needs to look for more goods than the ones she wants to buy for herself. A necessary condition for this to happen is that the amount of credit available is larger than their income. But whether they pay for these goods with long or short term debt is irrelevant.

4.5.1 Set up with more credit

Since the scheme allows for intratemporal and intertemporal lending, 2 subperiods —one at the beginning and one at the end of the period— have to be made explicit. Furthermore, an agent borrows if she is matched, and lends otherwise, so

[38] This is also the case in Kaplan and Violante (2014) but because there agents are heterogeneous, there are some that spend more than their income.

[39] This means that according to the model, money may loose value if credit was $1/\psi$ times GDP.

the intratemporal bond needs to be contingent on this event: let this contingency be denoted with e if the agent finds a match and u if she does not. It is useful to start setting down the constraints of the household in nominal terms (money is the neumeraire): Constraint (3) becomes

$$\hat{m}(e) + V_1\hat{b}(e) + \widetilde{P}q = Wn + Rk + m + b, \tag{23}$$

and

$$\hat{m}(u) + V_1\hat{b}(u) = Wn + Rk + m + b, \tag{24}$$

depending on whether the agent is matched or not. Agents start with money m and nominal bonds b. W and R are the nominal rental prices of labour and capital inputs. V_1 is the price of bonds at the beginning of the period (sub-period 1) that pays a unit of money at the end of the period after trade (sub-period 2). $\hat{b}(e)$ and $\hat{b}(u)$ denote the positions on these intra-temporal bonds that are exchanged between the first and the second subperiod. $\widetilde{P}$ is the nominal price in the submarket of choice. The bond has some lower bound $\hat{b} \geq \underline{\hat{b}}$. As usual, agents may choose to hold "lazy" money $\hat{m} \geq 0$, which —like bonds— can be sate contingent.

Effort and consumption and investment constraints (4) and (5) are as before.[40]

As before, there is insurance within participants in each submarket $(\widetilde{P}, q)$. It consists of a transfer of $(1-\psi)q$ goods from each matched agent to the rest. Each unmatched agent transfers financial wealth (bonds or money) for $\psi\widetilde{P}q$ so that each matched agent gets $\frac{(1-\psi)}{\psi}\psi\widetilde{P}q$. The end of period financial constraint that corresponds to (6), becomes

$$m'(e) + V_2 b'(e) = \hat{m}(e) + \hat{b}(e) + \widetilde{P}q(1 - \psi(\widetilde{P}, q)) \tag{25}$$

for the matched and

$$m'(u) + V_2 b'(u) = \hat{m}(u) + \hat{b}(u) - \widetilde{P}q\psi(\widetilde{P}, q) \tag{26}$$

for the unmatched. The last terms in the right-hand side in each equation are the insurance transfers. V_2 is the price of an inter-temporal bond b' that pays unity in the next period ($b' < 0$ is equivalent to not clear credit card debt within the period). b' has some lower bound to avoid Ponzi schemes. $m' \geq 0$ is the money carried overt to the next period.

[40]Notice that q in Constraints (23) and in (4) are not state contingent meaning that effort over q is suffered even if the match is not found.

While describing the problem in nominal terms helps fixing ideas, it is best to now turn it in real terms: this way, it is easier to study the case when the value of money goes to zero. Let P be the nominal price in the prevailing equilibrium goods market. Dividing Equations (23)—(26) by P and putting $v_1 = V_1/P$, $p_m = 1/P$, $p = \widetilde{P}/P$, $w = W/P$, and $r = R/P$, one gets

$$p_m\hat{m}(e) + v_1\hat{b}(e) + pq \leq wn + rk + p_m(m+b), \tag{27}$$

$$p_m\hat{m}(u) + v_1\hat{b}(u) \leq wn + rk + p_m(m+b), \tag{28}$$

$$p_m m'(e) + v_2 b'(e) \leq p_m(\hat{m}(e) + \hat{b}(e)) + pq(1-\psi(p,q)), \tag{29}$$

$$p_m m'(u) + v_2 b'(u) \leq p_m(\hat{m}(u) + \hat{b}(u)) - pq\psi(p,q). \tag{30}$$

The Bellman equation is

$$V(k, m+b, \Omega) = \max_{\{n,d,q,p\},\{c(s),k'(s),\hat{m(s)},\hat{b(s)},m'(s),b'(s)\}_1^2} \sum_{s\in e,u} pi(s)[u(c(s),n,d)+E\beta V(k',m'(s)+b'(s),\Omega')] \tag{31}$$

s.t Constraints (27)—(30), (4), (5), as well as the aforementioned corner constraints. $pi(e) \equiv \psi(p,q)$ i.e. the probability of having found a match and $pi(u) \equiv 1-\psi(p,q)$. The timing is captured by the fact that n, d, q, p are independent of s.

To appreciate the implications for the value of money, it is useful to start by focusing on the intratemporal bond. The market clearing condition is

$$\hat{b}(e)\psi + \hat{b}(u)(1-\psi) = 0. \tag{32}$$

Trivially, $v_1 \leq p_m$ is a necessary condition for intratemporal lending, otherwise holding $\hat{m}$ would be more remunerative than holding $\hat{b}$. In particular, if $v_1 = p_m$, agents in state u are indifferent between holding $\hat{m}(u)$ and $\hat{b}(u)$, if $v_1 < p_m$ then agents in state u put $\hat{m}(u) = 0$ and lend all their available funds: $v_1\hat{b}(u) = a \equiv wn + rk + p_m(m+b)$. So the supply of bonds from an agent in state u is infinitely elastic when $v_1 = p_m$, and equal to a when $v_1 < p_m$. $v_1 < p_m$ as an equilibrium outcome cannot be ruled out as for sufficiently low search costs, agents in state e might want to borrow $-v_1\hat{b}(e)\psi > (1-\psi)a$ if $p_m = v$. This drives $v_1 < p_m$ so that the credit market clears at the maximum credit $-v_1\hat{b}(e)\psi = (1-\psi)a$. In this case Proposition 5 shows that money has no value. Intuitively, the agents that get matched manage to spend all existing funds, including those of the unmatched agents, this gives no residual wealth to be stored in money.

If demand for credit is lower so that in equilibrium

$$- v_1 \hat{b}(e)\psi = v_1 \hat{b}(u)(1-\psi) \leq a(1-\psi), \tag{33}$$

then $v_1 = p_m$ because the market clears in the region where the supply of bonds is infinitely elastic. In this case not all funds a may be used so the argument for the case of $v_1 < p_m$ does not hold and money may have value. However, Proposition (6) shows that money looses value in steady state if the borrowing constraint on the intratemporal bond is not binding.

Proposition 5 *If $v_1 < p_m$, money has no value.*

Proposition 6 *In a steady state with no binding borrowing constraints on $\hat{b}$ and with $v_1 = p_m$, money has no value when inflation is above the Friedman rule.*

Notice that also in this case with $v_1 = p_m$ in the end $-v_1 \hat{b}(e)\psi = (1-\psi)a$ i.e. maximum feasible credit.[41]

Obviously, money can have value if the borrowing constraint on $\hat{b}$ binds and $v_1 \hat{b}(u) < a$; this is implicitly the case in the main text where this intratemporal bond was not available (hence implicitly binding). So it is possible to conclude that when credit is at the maximum feasible level $(1-\psi)a$, then money looses value; with less than maximum feasible credit, money has value.

Finally notice that the role of money does not rely on the absence of an intertemporal bond: the presence or absence of b' plays no role in the propositions above. Of course if b' was in positive net supply, it would be money. Similarly, $\hat{b}$ can be seen as inside money, so a way to summarize these results is that if there is enough inside money, outside money looses value. But there is always a need for inside and/or outside liquid assets.

5 Quantitative exercise

It is interesting to study the business cycle properties of this model. To this aim, the model is parameterized.

[41] As shown in the proof of Proposition 5, aggregate money demand is $p_m(\psi m'(e) + (1-\psi)m'(u)) = p_m(\psi \hat{m}(e) + (1-\psi)\hat{m}(u))$. Since $\widehat{m(s)} \geq 0$ for all s, $p_m(\psi m'(e) + (1-\psi)m'(u)) = 0$ implies $\hat{m}(s) = 0$ for all s. From Constraint (28) this implies $v_1 \hat{b}(u) = a$ i.e. maximum feasible credit.

5.1 Preferences

The utility function is: $u = \log(c) - \chi_n \frac{n^{1+1/\nu_n}}{1+1/\nu_n} - \chi_d \frac{d^{1+1/\nu_d}}{1+1/\nu_d}$.
χ_n and χ_d determine average hours and effort. ν_n and ν_d are the Frisch elasticities of labour and effort supply.

5.2 Matching

I assume the following matching function:

$$\mu = z_m^{1/\rho} \left(\alpha_m f^\rho + (1 - \alpha_m) h^\rho \right)^{1/\rho}. (34)$$

where μ is the number of matches and z_m is a parameter. This specification is convenient because as ρ approaches minus infinity, the function converges to $\min(f, h)$, and the model boils down to a perfectly competitive model as discussed in Section B.

5.2.1 Reinterpreting the Matching in terms of aggregate demand and supply

To bring the model to the data, it is useful to point the following out. Multiplying the right and left hand side of the matching function (34) by q one gets

$$y = z_m^{1/\rho} \left(\alpha_m y_s^\rho + (1 - \alpha_m) y_d^\rho \right)^{1/\rho}. (35)$$

Where $y \equiv \mu q$ are total transactions which —since the model abstracts from inventories— are equivalent to GDP. Written this way, the matching function takes as inputs production capacity or supply $y_s \equiv fq$, and households demand $y_d \equiv q$. This is convenient because below I find empirical counterparts to y_s and y_d.

It is also possible to express market tightness in terms of aggregate demand and supply: since $h = 1$, market tightness $\theta = f/h$ can be equivalently expressed as fq/q or y_s/y_d. Finally, probabilities ψ and ϕ can be written as GDP over aggregate demand or supply: $\psi = \frac{y}{y_d}$, $\phi(\theta) = \frac{y}{y_s}$. It is now possible to bring the model to the data.

5.3 Parametrization

Parameter values are summarazide in the table in Appendix E. I focus on quarterly data from 1967.Q1 (when data on total capacity utilization are available) to 2016.Q1.

I assume the following variables to be stochastic: z_m A_d A χ_n and β. Calling any of these variables X, I assume the following stochastic process:

$$X_{t+1} = x_0 \gamma_x^{(1-\rho_x)t} X_t^{\rho}{}_x e^{\varepsilon_{x,t}} \tag{36}$$

with ρ_x smaller than one and innovations ε_x zero mean normal, independent between each other (diagonal covariance matrix), and $i.i.d$ over time.

γ_x is the growth factor of variable X. As is intuitive from Equation (4), to ensure a balance growth path with stationary effort, A_d has to grow at the same rate as q, which in turn grows with technological progress A at a factor $\gamma_a^{1/(1-\alpha)}$, where γ_a is the growth factor of A.

Growth in the other processes would not be consistent with a balanced growth path. Thus it is convenient to have the shock multiply the steady state levels of z_m, χ_n or β: let $\beta = \bar{\beta} x_\beta$ where $\bar{\beta}$ is a fix parameter and x_β is the random variable. And similarly $\chi_n = \bar{\chi}_n x_n$ and $z_m = \bar{z}_m x_m$. This implies that the parameters x_0 and γ_x in Equation (36) are equal to 1 for these trend-less shocks.

I estimate the following parameters: the persistence parameters ρ_x and variances $\sigma(\varepsilon_x)^2$ for each stochastic process, the Frisch elasticities of labour and effort supply ν_n and ν_d, and the complementarity of the matching function ρ.

The remaining parameters are calibrated as follows.

5.3.1 Calibrated parameters and targets

I fix the depreciation rate $\delta = 0.014$ as in Aruoba and Schorfheide (2011) among others, the capital income share $\alpha = 0.34$, and the discount factor $\bar{\beta} = 0.99$; these are conventional values in the DSGE literature. The growth parameter for A is 1.0022 to ensure that the model steady state is consistent with the mean growth rate of per capita real GDP over the sample. The level parameter x_0 in Equation (36) is equal to one for productivity A and that for A_d is set to match a steady state level of effort: similarly to market hours, there is no natural units for their measurement and I put both hours and effort equal to 1/3 in steady state.[42]

The remaining calibrated parameters are pinned down through long run averages, however, these parameters are also function of the parameters to be estimated.

[42]For instance, it is possible to re-scale effort and change A_d with no effect on any other variable as it is clear from Equation (4).

The labour and effort supply parameters $\bar{\chi}_n$ and $\bar{\chi}_d$ depend on the targeted steady state market hours and effort, as well as the Frisch elasticity parameters, to be estimated. To find α_m and $\bar{z}_m$ (the steady state level for z_m), I target steady state values for ϕ, the money output ratio $p_m m/y$, and the consumption output ratio c/y: it is possible to show that given an estimate for the matching function complementarity ρ, there is a unique value of α_m and $\bar{z}_m$, that imply a steady state consistent with the above mentioned targets.

The target for c/y is 0.87, which is the sample average using personal consumption expenditures plus government spending and net exports over GDP. Steady state $p_m m/y$ is 0.55, this it the average of M2, over GDP.[43]

$p_m m/y$ also determines a steady state value for $\psi = 0.65$.[44]

ϕ is constructed through data on total capacity utilization ($\textsc{tcu}$) published by the Federal Reserve Board: ($\textsc{tcu}$) is the percentage of total available capacity being used to produce demanded finished products. This matches closely with what ϕ means in the model. In particular, the literature on capacity utilization measures output as

$$y = (\textsc{tcu}\, k)^{\alpha} n^{1-\alpha}. \tag{37}$$

Since $\textsc{tcu} \in [0, 1]$, it follows that total production capacity y_s is obtained putting $\textsc{tcu} = 1$ in Equation (37). Then $\phi = y/y_s = (\textsc{tcu})^{\alpha}$, whose sample average is 0.93.

The remaining parameters are estimated in the next subsection but with the constructed variables an initial assessment of the matching process is possible. Figure 2 shows a negative relationship between ϕ and θ, and a positive one between ψ and θ. This is consistent with the properties of the matching function but the matching function has not been used to construct these variables.[45]

[43] I experimented with other monetary targets (M1, MZM, Bank assets less loans). They change the size of $p_m m/y$ which determines the level of the search friction, but they have little effects on the propagation of the shocks.

[44] Firms' first order conditions and the fact that $y = \phi f q$ imply $y = wn + kr$. From Equation (3) and with $p_m \hat{m} = 0$ (which has been shown to hold in steady state) one gets $y_d \equiv pq = p_m m + wn + kr$. Then trough $\psi = y/y_d$ it is possible to construct $\psi = (p_m m/y + 1)^{-1}$ and y_d=y/ψ.

[45] To appreciate that this result was not obvious: suppose that y, y_d and y_s were positively correlated (as they indeed are) but changes in y were in general smaller than changes in y_d, which in turn were smaller than changes in y_s. Then θ and ψ would have been negatively correlated.

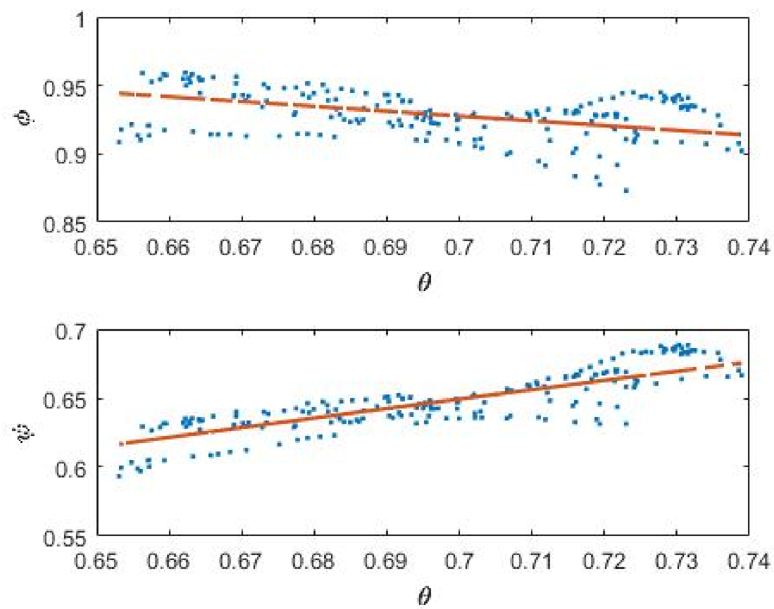

Figure 2: Empirical ϕ and ψ as a function of θ.

Note: the trend is the prediction of the matching function with the estimated parameters and with steady state z_m.

5.3.2 Bayesian Estimation

Observables

The observables are the growth rates of consumption, market hours and real GDP per capita, and the mentioned time series of capacity utilization and money-output ratio. Intuitively, consumption and GDP data should elicit the discount factor shock. Consumption and hours help identify the labour supply shock and the Frisch elasticity. Capacity utilization and the money-output-ratio imply ϕ and θ, which elicit the process for z_m and the matching complementarity parameter ρ. Finally, variations in GDP and θ imply variations in y_d and thus in effort via Equation (4); this disciplines the effort supply shock and the effort Frisch elasticity.

An indication that this methodology works is that an estimation on model generated data (with zero weight on the priors) finds the parameters used to generate the data.

Priors and Posteriors

The table in Appendix E summarizes priors and posteriors.

I assume the Frisch elasticity of labour supply to be Gamma distributed with mean

0.85 and variance 0.1. This ensures that the Frisch elasticity is in line with micro studies, who tend to find smaller elasticities relative to what macro models need to match hours volatility. The posterior mode is 1.02, in line with the studies surveyed by Keane (2011). I use the same prior for the Frisch elasticity of effort given the absence of external evidence. The posterior elasticity is 0.74. Doubling the variance for this prior leaves results essentially unchanged.

I assume a Gamma distribution for $-\rho$. This is because I restrict ρ to be smaller than zero: with too little complementarity (ρ too large), the interior solution of the household's first order condition for p does not maximize the objective function of the household. Increasing complementarity toward the perfect competition case of $\rho = -\infty$ is necessary to make the slope of marginal cost larger than that of the marginal gain. This can be appreciated by rearranging Equation (45) as follows (with $\lambda_p = 0$ in an interior solution):

$$\frac{\partial p}{\partial \theta}(\lambda_1) + \frac{\partial \psi}{\partial \theta}(\lambda_4 p) = \frac{\partial \psi}{\partial \theta}(\lambda_3) + \frac{\partial p}{\partial \theta}(\lambda_4(1-\psi)) \tag{38}$$

The left-hand-side shows the marginal cost of increasing θ: it is due to i. the tightening of Constraint (3) as p increases, ii. the tightening of Constraint (6) as ψ increases. The right hand side shows marginal gain: i. the relaxation of Constraint (5) as ψ increases ii. the relaxation of Constraint (6) as p increases.

Figure D.1 in Appendix D plots for different levels of ρ, the marginal gain and marginal cost of moving θ (which is equivalent to move p given the function $\theta(p)$). In the upper 3 panels, with high complementarity, the marginal cost cuts the marginal gain from below, this is consistent with the objective function being increasing at the left of the crossing point and decreasing thereafter. The opposite is true in the 5th and 6th panels with less complementarity (larger ρ), so that the crossing point is not a max. The 4th panel ($\rho = -0.2$) is close to a nife-edge case in which the marginal cost and marginal gain imply a flat objective function so that many markets give same utility. I find a posterior mode for $\rho = -1.44$.

The other element of departure from the neoclassical model —search effort— does not pose a threat to concavity since the utility function is concave and Effort Constraint 4 is linear.

Following the literature, the persistence parameters of the stochastic processes have a Beta prior and the standard distribution of the measurement error innovations follow an inverse Gamma prior. I set the prior of the measurement error so that the posterior measurement error variance does not exceed 1% of the variance of the observed series. The structural shocks standard deviations have an exponential prior as suggested by Ferroni et al. (2015). As mentioned, the covariance matrix of the innovations is diagonal. The last 5 columns in Appendix E report the mode, median, standard deviation and confidence interval for the estimated parameters.

5.4 Variance Decomposition

To appreciate the importance of each shock the variance decomposition of some variables of interest is reported in Table 1.[46]

The first result is that z_m (in the first row) is the most important shock for velocity and for GDP. It is also the most important one for ϕ, *i.e.* the movements in TFP that are endogenous and not due to the technology shocks A. To understand what z_m does and what does not do, it is useful to study the other shocks.

The effort productivity shock A_d is the most important shock for aggregate demand y_d (5th column) whereas the technology shock A is the most important one for aggregate supply y_s (6th column). This suggests that A_d should really be interpreted as a demand shock (similarly to Storesletten et al. (2011)). Instead z_m has little effect on both aggregate demand and supply, so one should be cautious to interpret it as a demand shock (at least given the notions of demand of this paper); z_m simply affects the matching of demand and supply. A black box that may be interpreted as capturing a more cautious behavior due to frictions such as information, screening, monitoring, agency and retail costs.[47]

Curiously, shocks that have a clear interpretation as supply and demand shocks (A

[46] The model is solved with first order perturbation methods around the deterministic steady state. Brunnermeier and Sannikov (2014) show that this can be an issue with financial models. For instance, in this model the linearized policy function for $\hat{m}$ is always equal to zero. To check that the solution is accurate, the model has also been solved with policy iteration. For reasonable portions of the state space (shocks up to 5 standard deviations and capital 50% below or above the steady state) $\hat{m}$ is zero. Portions outside the mentioned region are never visited during simulations with the identified shocks.

[47] While one can debate on the interpretation of the search friction, there is a further endogenous reason why agents reduce spending: during recessions the return on capital is lower and that of money is higher because of lower inflation. Then, other things equal, it is optimal for households to choose a market with lower probability of finding goods. Furthermore, given the preference for consumption smoothing, the drop in spending is especially absorbed through a substitution of capital investment with money holdings, akin to a Paradox of Thrift. However, with competitive search, the market choice is not distorted and this endogenous reaction is not inefficient as it lessens the drop in firms finding probability ϕ.

and A_d), have effects of similar magnitude on both GDP and velocity. However, the A_d shock is the most important after the labour supply shock for hours volatility.

To appreciate this, it is instructive to rearrange the first order conditions (39)—(46) in Appendix to

$$u_n = -\left(u_c + \frac{u_d}{A_d}\frac{1}{\psi}\right) w,$$

which for simplicity, abstracts from the corner multipliers. The term $\frac{u_d}{A_d}\frac{1}{\psi}$ is a wedge relative to the neoclassical labour supply equation $u_n = -u_c w$. Intuitively, the benefit from working is not just the wage times the marginal utility of consumption, but there is the added issue that goods have to be found which reduces incentives to work. Furthermore, this wedge moves over the business cycle: the correlation between detrended GDP and the wedge constructed simulating the model with the identified shocks is 0.49 and strongly significant. Since a drop in $\frac{u_d}{A_d}\frac{1}{\psi}$ reduces n, the fact that the wedge is procyclical makes hours more volatile.

The model explains 37% of the variance of hours without labour supply shocks and with low labour supply elasticity. While hours movements still require a strong ad hoc labour supply shock, this is an improvement relative to the neoclassical model this paper builds on: with a similar calibration Ríos-Rull et al. (2012) explain about 10% of the variance of hours.

Table 1: Variance decomposition

	GDP	C	N	ϕ	Y_d	Y_s	Velocity
z_m	0.3693	0.1460	0.0538	0.6837	0.0264	0.0188	0.5049
A_d	0.2013	0.0466	0.1622	0.1523	0.7989	0.0592	0.2495
A	0.2797	0.2956	0.0057	0.1098	0.0830	0.6351	0.1799
β	0.0439	0.3194	0.1391	0.0011	0.0642	0.0507	0.0019
n	0.0956	0.1827	0.6315	0.0471	0.0213	0.2314	0.0772

5.5 Impulse Response functions and co-movements

To further appreciate the role of the two shocks z_m and A_d, Figures D.2 and D.3 in Appendix D report the impulse response functions.

Take away points from the figure for z_m are: ϕ increases *i.e.* there is an endogenous surge in the Solow residual $y/\left(k^\alpha n^{1-\alpha}\right)$. This induces the usual real business cycle implications that—consistently with the data—there is comovement of hours, consumption and real input prices with output.[48] Velocity surges (therefore, after a negative shock velocity declines as it has happened in many recessions). p_m drops *i.e.* inflation is procyclical.

Similarly to z_m , A_d shocks are also expansionary, induce an endogenous Solow residual, and comovement of consumption, market hours, and input prices with output. But search effort, ψ, p_m and velocity move in the opposite direction relative to a z_m shock. The shock also causes a large movement in supply over demand θ, which barely moves after a shock to z_m. These are the crucial differences in the propagation of z_m and A_d shocks which identify the two apart: as shown next z_m shocks are of first order importance for recessions characterized by a decline in velocity (or liquidity surge), A_d shocks played an important complementary role in earlier recessions prior to the 80s, when velocity did not decline.

The other impulse responses are not shown because not particularly interesting, but it is worth mentioning that technology shocks A make ϕ countercyclical, which is counterfactual given the path of $\textsc{tcu}$. As a result, technology shocks are estimated to play a minor role in the observed recessions as shown next. This discussion highlights how monetary quantities and $\textsc{tcu}$ are key for the identification of the model.[49]

Storesletten et al. (2011) estimate a model with demand shocks similar to A_d and find it to be more important than technology shocks. They do not use data on money and capacity utilization in their estimation: this would not allow to distinguish between A_d and z_m.

5.6 NBER Recessions

How does the model account for real recessions? Figure 3 shows a peak to trough analysis by depicting a counterfactual path from 2007.IV onward when including only one shock at the time versus the baseline path with all shocks (which generates the

[48]In future one could distinguish the matching function between consumption and investment. But absent further bells and whistles, if the matching shocks were not correlated, the impulse response to each shock would not make consumption and investment co-move. A case for the positive correlation is that consumption and investment products are intertwined. For instance, both consumption and investment often come with credit and insurance contracts, so a shock to the ability to sell financial products would affect both consumption and investment.

[49]Since the mapping between data on capacity utilization and the notion in this model may not be perfect, I experimented with larger measurement error for $\textsc{tcu}$ (40% of its variance) and found negligible differences. What matters is that capacity utilization is procyclical, which would be arguably true of other measures of ϕ.

exact data because shocks for these simulations are identified assuming no measurement error). The figure shows all the observables (ϕ is a monotone transformation of $\textsc{tcu}$ so is essentially an observable) and θ, which is a function of the other observables and —as explained earlier— helps to detect the presence of A_d shocks.[50]

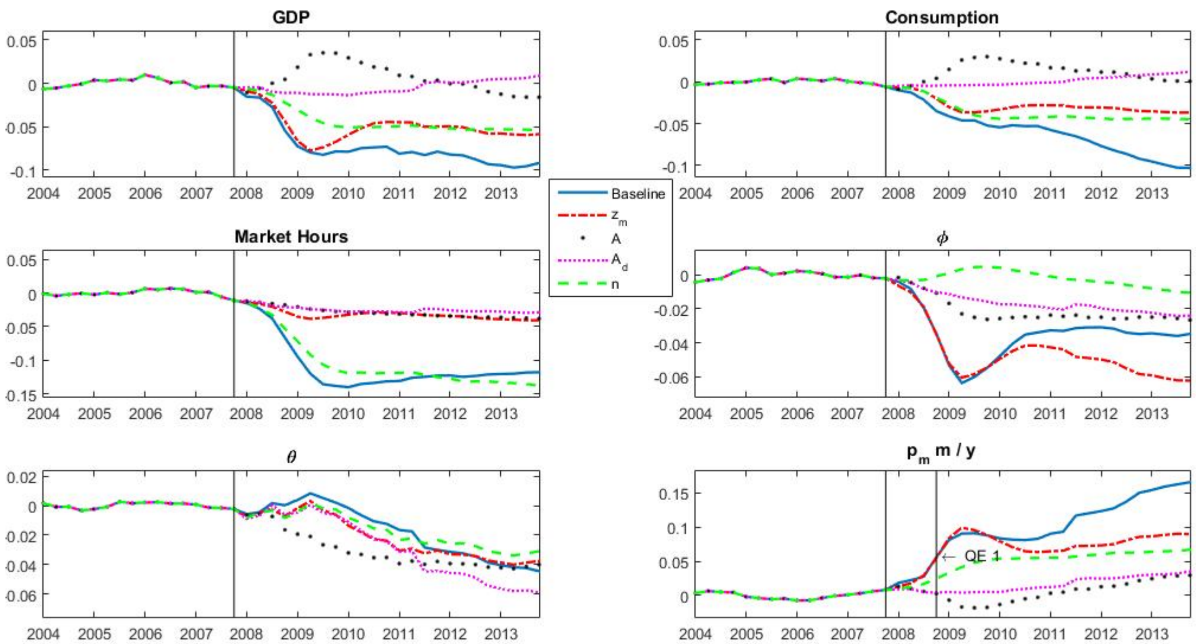

Figure 3: 2007.IV recession due to each shock

First, technology shocks A are not responsible for the crisis. Instead z_m shocks account for virtually the entire drop in output and ϕ (thereby generating an endogenous drop in productivity), as well as the increase in liquidity.[51] z_m shocks alone are also responsible for a sizable fraction of the drop in consumption but for a small fraction of the drop in market hours, which is mainly due to the labour supply shock. The

[50]GDP, consumption and market hours are in logs and linearly detrended, so the figure shows the percentage deviation from a linear trend. Since the other variables are ratios, they are linearly detrended in levels. So if $p_m m/y$ goes from 0.3 to 0.4, the figure shows a 0.1 increase.

[51]Since the model abstracts from monetary policy intervention, which may have contributed to the increase in liquid assets, the figure includes a vertical line at the time of the first round of Quantitative Easing to show that the liquidity surge already toke place. Indeed, it is not obvious if and how a swap of liquid assets between the central bank and private financial institutions affects money supply; see Williamson (2015).

demand shock A_d plaid a negligible role in the Financial crisis. The path with shocks on β is not shown because not important.

A few patterns emerge when also looking at past recessions. Figures D.4—D.8 in Appendix D show a peak to trough analysis carried at previous NBER recession dates.[52] All recessions show a drop in ϕ. Some recessions (all those from the 80s onward) are characterized by a liquidity surge.[53] Earlier recessions do not show a liquidity surge (those started in 1969.IV and 1973.IV).

Which shocks account for these facts? i. They have to be shocks that are recessionary and make ϕ drop; those are negative z_m and A_d shocks; instead negative A shocks make ϕ increase which explains why technology shocks are in general not that unimportant during recessions with the exception of those starting in 1973.IV and 1981.I, where they played a role. ii. As seen in the previous subsection, negative z_m shocks make liquidity surge, negative A_d shocks make it drop. So recessions where liquidity increased are characterized by z_m shocks: this alone generates both the drop in ϕ and the surge in liquidity (as well as most of the drop in GDP). For recessions where liquidity does not increase, a combination of z_m and A_d shocks is necessary: both push ϕ down, but they neutralize each other for liquidity. So early recessions were also characterized by an A_d shock, this is also evident in the figures from the increase in θ. Finally, labour shocks always play a role for market hours (which is to be expected given the low labour supply elasticity).

In summary, the two wedges introduced by this theory —z_m and A_d— are important to account for the business cycle. Furthermore, monetary aggregates and capacity utilization are important to identify these shocks.

6 Conclusions

This paper builds a theory where liquidity resolves the need to carry over value when goods are hard to find. Money offers a way to preserve value for that part of wealth not transformed into goods because of a search friction.

Importantly, this theory does not require trades to be quid pro quo but offers an explanation for money demand when trades are not anonymous and the lack of

[52] The 1981.I and 1981.III recessions are plugged into the same figure to save space.

[53] That a liquidity surge characterized many recessions corroborates the view that such surge is not just an artifact of Quantitative Easing.

commitment that limits insurance and credit is relaxed. Therefore, it can contribute to explaining the large amount of liquidity observed in the data despite the increased availability of electronic payment methods and credit.

In providing a liquid store of value, money plays an important social role: it facilitates transactions because its value increases aggregate demand and thereby, the efficiency of firms. Along with linking demand, supply, and the value of money, the search friction is also a natural source of the business cycle. A shock to the matching function emerges as an important source of recessions, while generating a surge in liquidity: this pattern—recessions characterized by spare production capacity and liquidity hoarding—is documented for the financial crisis and for several earlier recessions.

A virtue of this theory is that it is simple to compute: casted into a neoclassical model with a representative agent, credit and insurance markets, it can be extended in many ways and it may prove useful for monetary policy, especially given that it accounts for monetary quantities, at the center of recent unconventional policies. In particular this framework may provide a rationale for why quantitative easing policies aimed at easing credit conditions may be ineffective at expanding credit: in this model the drop in lending is not due to credit constraints but to the lack of "appetite" from the private sector. This implication stands in contrast to that of models with credit constraints, that are relaxed by quantitative easing policies as shown in Kiyotaki and Moore (2012). While a debate between the two channels may prove healthy, the two are not inconsistent with each other. In fact the model could be extended to include credit constraints. Indeed since the model is tractable, the whole financial sector could be decentralized so that credit and banking frictions could be studied in this framework.

Another related extension is to include other assets differing in their liquidity (captured by the severity of the search friction which stands for differences in information acquisition costs, risk, maturity etc.) reflecting empirical counterparts ranging from government bonds, equity shares and other financial products, to possibly far less liquid assets such as houses.[54]

Finally, it would be interesting to explore other price mechanisms. This paper used

[54] An attractive feature is that the model generates endogenous liquidity premia: agents choose assets trading off their liquidity and their return so that in equilibrium the more liquid the asset the lower its return.

competitive search. This seemed a natural starting choice because it does not add search externalities, thereby keeping it closer to the neoclassical model and isolating the elements of departure: the goods matching function and search effort. With competitive search the Friedman rule is optimal. Perhaps other price mechanisms may lead to different results to induce agents to search more.

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Appendixes

A First order conditions of the household

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$ be the lagrange multipliers on the constraints (3)—(6) and let $\lambda_{k'}, \lambda_{m'}, \lambda_q, \lambda_p, \lambda_{\hat{m}} \geq 0$ be the multipliers respectively on $k' \geq 0$, $p_m m' \geq 0$, $q \geq 0$, $p \geq 0$, $p_m \hat{m} \geq 0$, with complementary slackness between each multiplier and the respective constraint. The households first order condition for c, n, d, k',m',q,p,$\hat{m}$ are

$$u_c = \lambda_3, \tag{39}$$

$$u_n = -\lambda_1 w, \tag{40}$$

$$u_d = -\lambda_2 A_d, \tag{41}$$

$$\lambda_3 - \lambda_{k'} = \beta E \left(\lambda_1' r' + \lambda_3'(1-\delta)\right) \tag{42}$$

$$\lambda_4 p_m - \lambda_{m'} p_m = \beta E \left((\lambda_1' p_m')\right) \tag{43}$$

$$-\lambda_1 p + \lambda_4 p(1-\psi) = \lambda_2 - \lambda_3 \psi - \lambda_q \tag{44}$$

$$-\lambda_1 + \lambda_4(1-\psi) = \frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial p}(\lambda_4 p - \lambda_3) - \lambda_p, \tag{45}$$

$$p_m(\lambda_1 - \lambda_4) = p_m \lambda_{\hat{m}} \tag{46}$$

B Leontief matching function and no effort costs

With matching function $\min(f, h)$, $\phi = 1$ if $\theta \leq 1$ and $\phi = 1/\theta$ if $\theta > 1$. Thus $\phi(\theta)$ is not strictly monotone as assumed in the main text.

Since ϕ is constant for $\theta \leq 1$, firm arbitrage ($\phi(\theta)p = \xi$) implies $p = \xi$ if $\theta \leq 1$, $p = \xi\theta$ if $\theta > 1$. Thus θ is not uniquely identified by p but the relationship is a correspondence. As a result, the household does not just choose p, q, but also θ.

It is easy to argue that with the price correspondence above, the optimal choice of the household is $p = \xi$ and $\theta = 1$. This is because from the matching function, $\psi = \theta$ if $\theta \leq 1$ and $\psi = 1$ if $\theta > 1$. First, ($\theta = 1$, $p = \xi$) is preferred to ($\theta < 1$, $p = \xi$) because the latter implies $\psi < 1$, which induces more savings in money, that pay lower return than capital. On the other hand, $\theta > 1$ would imply $p > \xi$ but would not increase ψ, clearly an inferior choice.

It is trivial that with effort not costing any utils, the households chooses $\hat{m} = 0$ as money is dominated in return by capital. Then Equation (6) implies $p_m m' = 0$, which with positive money supply implies $p_m = 0$.

Since with $\theta = 1$, $\psi = \phi = 1$, it is also trivial to show that the model boils down to the neoclassical one with the familiar neoclassical budget constraint

$$c + k' - k(1 - \delta) \leq wn + kr \tag{47}$$

.

C Proofs

Lemma 1

Pick p, q such that Equation (12) holds. Now suppose $\theta(p, q)$ depended on q. Then it would be possible to change q holding p constant so that $\phi(\theta(p, q))$ increases. But from Equation (11) this makes profits positive, which implies that the assumed $\theta(p, q)$ was not profit maximizing.

Proposition 1

To show neutrality, take an equilibrium allocation with constant money supply $m > 0$. Let p_m and $\hat{m}$ be equilibrium functions. It is possible to change the money supply to zm with $z > 0$ and pick a new price function $p_m^z = p_m/z$ and to change $\hat{m}$ to $\hat{m}^z = z\hat{m}$ so that all equilibrium conditions are satisfied with the same allocation. The equilibrium conditions in which money or p_m appear are Equations (3) and (6), and the Euler equations for m' and $\hat{m}$, (43) and (46) in Appendix A, which must hold in an equilibrium.

Since $p_m^z zm = p_m m$ and $p_m^z \hat{m}^z = p_m \hat{m}$, (3) and (6) are satisfied with the original allocation. In a monetary equilibrium (where $p_m > 0$) Equation (43) can be rearranged so that prices enter as a ratio p_m'/p_m, but this ratio is equal to ${p_m^z}'/p_m^z$. Finally, p_m cancels out in Equation (46). It follows that the original allocation satisfies these conditions.

Superneutrality does not hold because the Euler equation (43) depends on p_m'/p_m which is affected by a change in money growth. Thus the inflation rate affects the dynamics of the Lagrange multipliers λ_1 and λ_4 defined in Appendix A. It follows trivially from the other first order conditions that the allocation is also affected.

Proposition 2

The proof consists of showing that when $\frac{p_m'}{p_m} \to \frac{1}{\beta}$, the first order conditions necessary for a solution to the household problem, are identical to those of the planner in steady state. It is then trivial to show that all other conditions are also identical.

The first order conditions for the household and associated Lagrange multipliers are reported in Appendix A.

Since $p, q > 0$, the Kuhn-Tucker multipliers λ_q and λ_p defined in Appendix A, are both equal to zero. In a monetary equilibrium $p_m m' \geq 0$ is not binding, thus the Kuhn-Tucker multiplier $\lambda_{m'}$ defined in Appendix A is equal to zero. Then the first order condition for m', Equation (43), implies $\lambda_4 = \lambda_1$ in steady state at the Friedman rule. Then Equation (46) implies that $\lambda_{\hat{m}} = 0$.

Since firms first order condition for prices, Equation (12), is true for all p, and using Proposition (1) (that θ is only function of p), one can differentiate Equation (12) with respect to p and get

$$\frac{\partial p}{\partial \theta} = -\frac{\partial \phi}{\partial \theta}\frac{p}{\phi}; \tag{48}$$

Substituting $\frac{\partial p}{\partial \theta}$ from Equation (48) into Equations (44) and (45), normalizing the equilibrium $p = 1$, and noticing that the matching function implies $\psi = \theta\phi$, one gets

$$-\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial \phi}\frac{\psi}{\theta}(\lambda_4 - \lambda_3) = \lambda_2 - \lambda_3\psi. \tag{49}$$

Substituting $\lambda_4 = \lambda_1$ into Equation (44) one gets $\lambda_2 = \psi(\lambda_3 - \lambda_4)$. Substituting this latter condition into Equation (49), one gets

$$\lambda_2\left(\frac{\partial \phi}{\partial \theta} - \frac{\partial \psi}{\partial \theta}\frac{1}{\theta}\right) = \lambda_3\psi\frac{\partial \phi}{\partial \theta}.$$

Substituting ψ and $\frac{\partial \psi}{\partial \theta}$ from $\psi = \theta\phi$ (which implies $\frac{\partial \psi}{\partial \theta} = \phi + \theta\frac{\partial \phi}{\partial \theta}$) one finally gets

$$-\lambda_2 = \lambda_3\frac{\partial \phi}{\partial \theta}\theta^2. \tag{50}$$

This condition characterizes the decentralized choice of θ. Next, I obtain the same condition for the planner.

The Planner first order conditions for θ and q can be arranged as

$$\tilde{\lambda}_3\phi = \tilde{\lambda}_1 - \tilde{\lambda}_3\frac{\partial \phi}{\partial \theta}\theta \tag{51}$$

and

$$\tilde{\lambda}_3\phi\theta = \tilde{\lambda}_1\theta + \tilde{\lambda}_2, \tag{52}$$

where $\tilde{\lambda}_1$, $\tilde{\lambda}_3$ and $\tilde{\lambda}_3$ re the Lagrange multipliers on constraints (17), (18), and (19).[55]

The latter two conditions imply

$$-\tilde{\lambda}_2 = \tilde{\lambda}_3\frac{\partial \phi}{\partial \theta}\theta^2. \tag{53}$$

[55] $d \geq 0$ and $n \geq 0$ and Inada conditions ensure that the non negativity constraints on θ and q do not bind, so Kuhn-Tucker multipliers are not included for θ and q.

This planner condition coincides with the equilibrium condition (50) *iff* $\tilde{\lambda}_i = \lambda_i$ for $i = 2, 3$. It is trivial to verify that this is the case from the first order conditions of the household and of the planner for d and c.

It is equally trivial to verify that the other equilibrium conditions and planner conditions are identical, which completes the proof.[56]

Corollary 1

I first show that $\theta > 0$ and $\phi < 1$ when inflation is above the Friedman rule ($\frac{p'_m}{p_m} < \frac{1}{\beta}$). When inflation is above the Friedman rule, Equation (43) and steady state imply $\lambda_4 - \lambda_1 < 0$. Furthermore, with the marginal utility of effort $u_d = 0$, first order condition for d , Equation (18), implies $\lambda_2 = 0$. Then, Equation (44) implies $\psi > 0$ and hence $\theta > 0$.[57]

I now show that when $\frac{p'_m}{p_m} \to \frac{1}{\beta}$ then $\theta \to 0$. From Equation (43) and steady state, when $\frac{p'_m}{p_m} \to \frac{1}{\beta}$, $\lambda_4 - \lambda_1 \to 0$. Then from Equation (44) $\psi \to 0$ and/or $\lambda_3 - \lambda_4 \to 0$. But from Equation (45), if $\lambda_3 - \lambda_4 \to 0$ then $\psi \to 0$ because $\lambda_4 > 0$ as $\lambda_4 = \lambda_1$ and $\lambda_1 > 0$ from Equation (40). Therefore $\psi \to 0$ and from the matching function $\theta \to 0$.

Proposition 3

It has been shown that $\theta > 0$ tends to zero at the Friedman rule. With total output positive and bounded, Equation (17) implies that θ tends to zero if and only if production per trading post q tends to ∞. Then Equation (6) requires that for θ that tends to 0, $p_m m'$ tends to ∞ because ψ tends to 0.

Proposition 4

Take a monetary equilibrium and any point in the state space where production is positive and $\psi > 0$. One then has to show that if $\lambda_{\hat{m}} = 0$ (which must hold for the constraint to be slack), then effort is costly (the marginal utility $u_d < 0$).

From the household first order condition for $\hat{m}$, Equation (46), $\lambda_{\hat{m}} = 0$ if and only if $\lambda_1 = \lambda_4$. From the first order condition for p, Equation (45), if $\lambda_1 = \lambda_4$ then

$$(\lambda_3 - \lambda_4) q \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial p} = \lambda_4 \psi - \lambda_p$$

$\lambda_4 > 0$ is implied by the first order condition for n, Equation (40). $\lambda_p = 0$ in a monetary equilibrium. The assumptions on the matching function and Equation (12) imply $\frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial p} > 0$. Therefore $\lambda_3 - \lambda_4 > 0$. Then Equation (44) implies that $\lambda_2 > 0$. From the first order condition for d, Equation (41), $\lambda_2 > 0$ implies $u_d < 0$.

[56] As usual, $p'_m/p_m = \beta$ violates the transversality condition for money because the value of money grows too fast but the limit of p'_m/p_m to β is implementable.

[57] In a monetary equilibrium with positive production $p, q, p_m m' > 0$ so $\lambda_p = \lambda_q = \lambda_{m'} = 0$.

Proposition 5
Aggregate money demand is obtained by adding up Equations (29) and (30) for all agents:

$$p_m\left(\psi m'(e)+(1-\psi)m'(u)\right)\leq p_m\left(\psi\hat{m}(e)+(1-\psi)\hat{m}(u)\right)+$$
$$p_m\left(\psi\hat{b}(e)+(1-\psi)\hat{b}(u)\right)+\left(\psi pq(1-\psi)-(1-\psi)pq\psi\right)-v_2\left(\psi b'(e)+(1-\psi)b'(u)\right)\quad(54)$$

The market clearing condition in the intertemporal bonds market is $(1-\psi)b'(u)+(1-\psi)b'(e)=0$. The latter and the market clearing for the intratemporal bond, Equation 32, imply that the last 3 brackets in the right-hand side of Equation 54 are zero. Therefore, money demand depends on the first bracket in the right-hand-side.

It has already been argued that if $v_1<p_m$, then agents in state u put $\hat{m}(u)=0$ and lend all funds a. For this to be an equilibrium, agents in state e must be borrowing, then $\hat{m}(e)=0$ as it would be inefficient to hold cash while borrowing given that to borrow costs $p_m/v_1>1$. Therefore, all terms in the right-hand side of Equation 54 cancel out so that $p_m(m'(e)+m'(u))=0$.

Proposition 6
First order conditions for $\hat{b}_e$ and $\hat{b}_u$ are

$$-\lambda_{1,s}v_1+\lambda_{4,s}p_m+\lambda_{\hat{b}(s)}=0\quad(55)$$

where s is either e or u. $\lambda_{1,s}$ and $\lambda_{4,s}$ are the Lagrange multipliers on either Constraints (27) and (29) if $s=e$, or on (28) and (30), if $s=u$. $\lambda_{\hat{b}(s)}\geq 0$ and $\hat{b}(s)\geq\underline{\hat{b}}$ hold with complementary slackness. With no binding borrowing constraints $\lambda_{\hat{b}(e)}=\lambda_{\hat{b}(u)}=0$. Then from Equation (55), with $v_1=p_m$ $\lambda_{1,s}=\lambda_{4,s}$. The first order condition for $m'(s)$ is

$$\lambda_{4,s}p_m=pi(s)\beta E[(\lambda'_{1,e}+\lambda'_{1,u})p'_m]+\lambda_{m'(s)}.\quad(56)$$

Where $\lambda_{m'(s)}\geq 0$ and $m'(s)\geq 0$ hold with complementary slackness. Notice that the term $E[(\lambda'_{1,e}+\lambda'_{1,u})p'_m]$ in principle may differ depending on the current realization of s. This can be appreciated by writing the latter expression more explicitly as $E[(\lambda_{1,e}(k'(s),m'(s)+b'(s),\Omega')+\lambda_{1,u}(k'(s),m'(s)+b'(s),\Omega'))p_m(\Omega')]$ and noticing that it has not been ruled out that $k'(s)$ or $m'(s)+b'(s)$ may actually differ if s is e or u. I will now assume that with $v1=p_m$ this is not the case and later prove it. Under symmetry, a sum of Equation (56) for u and e gives

$$(\lambda_{4,e}+\lambda_{4,u})p_m=\beta E[\left(\lambda'_{1,e}+\lambda'_{1,u}\right)p'_m]+\lambda_{m'(e)}+\lambda_{m'(u)}.\quad(57)$$

From the first order condition for labour and Inada conditions, it is easy to show that $\lambda'_{1,e}+\lambda'_{1,u}>0$. Now suppose that in a steady state money had value and $\beta p'_m/p_m<1$

(more inflation than Friedman rule). Then Equation (57) and $\lambda_{1,s} = \lambda_{4,s}$ imply $\lambda_{m'(s)} > 0$ for either u or e, or for both. But then symmetry implies $m'(s) = 0$ for both u and e.

This leaves open the possibility of asymmetric equilibria. First, with $v_1 = p_m$, it is easy to check that the right-hand-side of Constraints (29) and (30) attain the same value so that $m'(e) + b'(e) = m'(u) + b'(u)$. But it could be possible to satisfy the latter with agents in one state s having $m' = 0$ and $b' > 0$ and the others holding $m' > 0$ and $b' < 0$.This possibility is ruled out through the Euler equations for $b'(s)$:

$$\lambda_{4,s} v_2 = p1(s)\beta E[(\lambda'_{1,e} + \lambda'_{1,u})p'_m] + \lambda_{b'(s)}.$$

The latter and the first order condition for $m'(s)$ —Equation (56)— imply that for an agent to be willing to hold $m' > 0$ and $b' < 0$, the following arbitrage condition must hold in steady state $v_2 = p_m/p'_m$. But then Equations (56) and (C) imply that agents in the state s that have $\lambda_{m'(s)} > 0$, also have $\lambda_{b'(s)} > 0$. So it is impossible to have $m(s)' = 0$ and $b(s)' > 0$ for the same s. Then it is impossible for the agents in the other state to hold $m' > 0$ and $b' < 0$ as the bond market would not clear.

D Figures

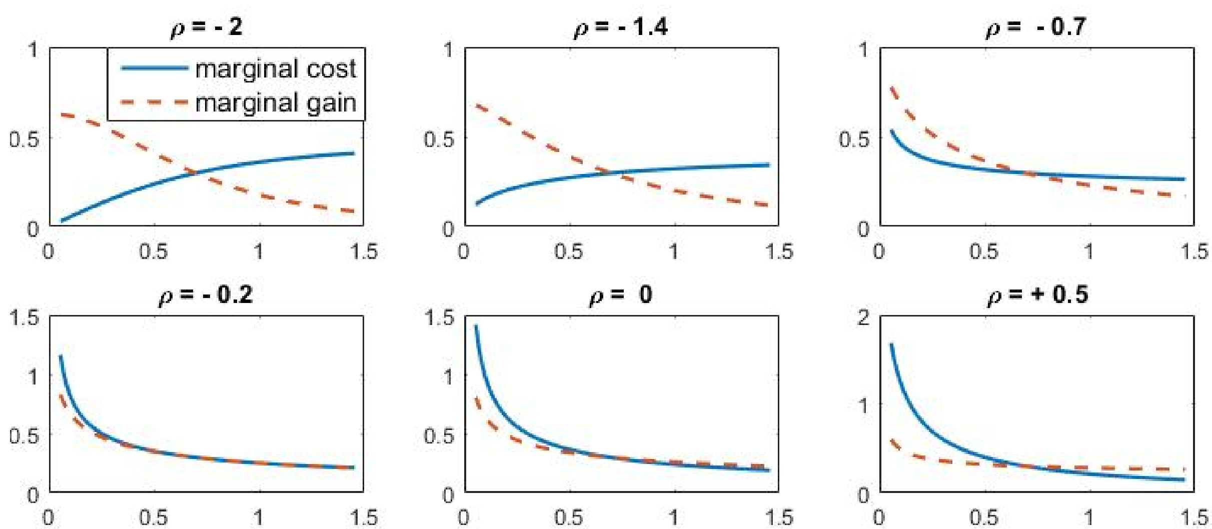

Figure D.1: Marginal gain and marginal cost of submarket choice as a function of θ (condition (38)) for different levels of complementarity in Matching Function 34

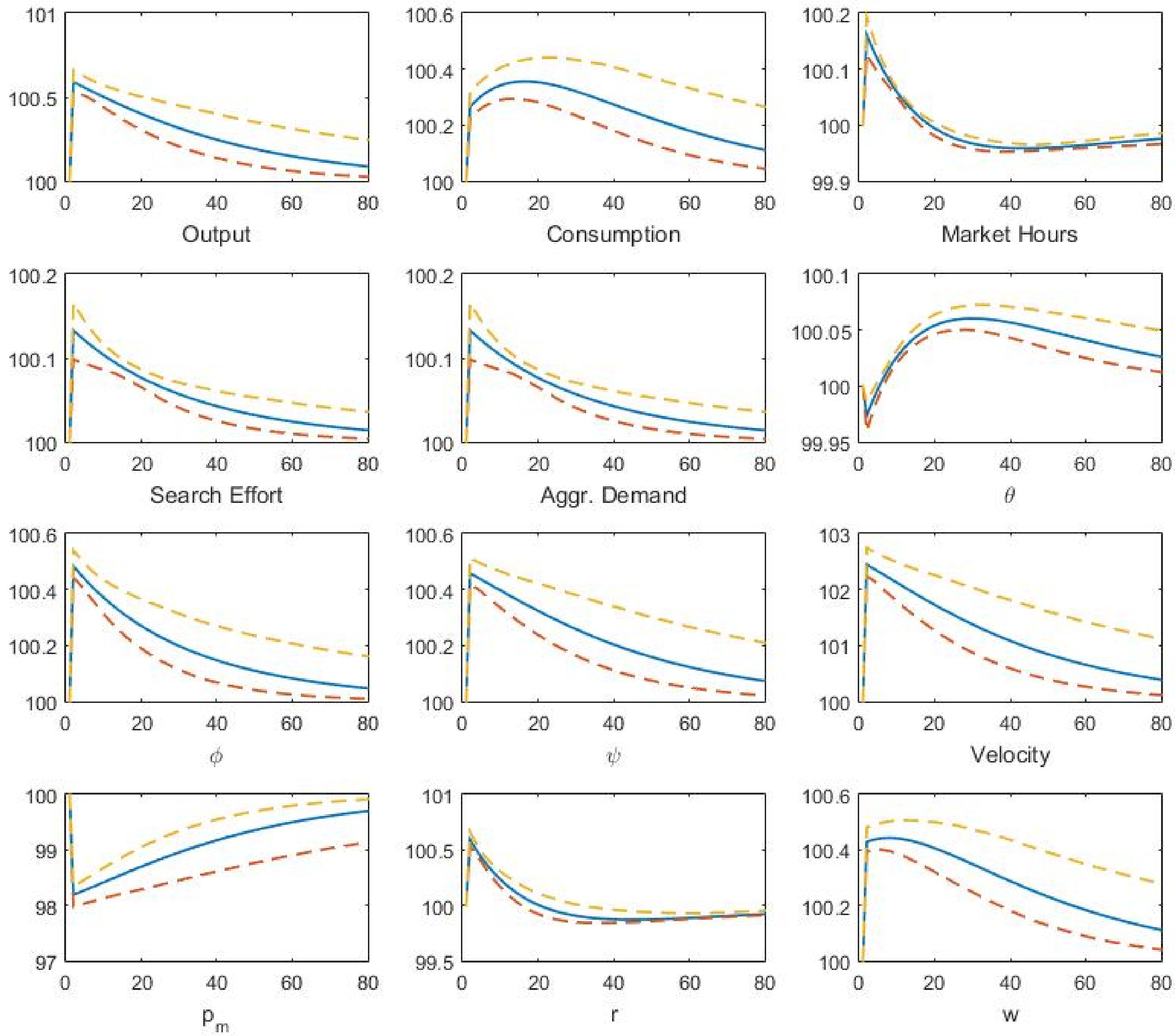

Figure D.2: IRF to Matching Shock z_m

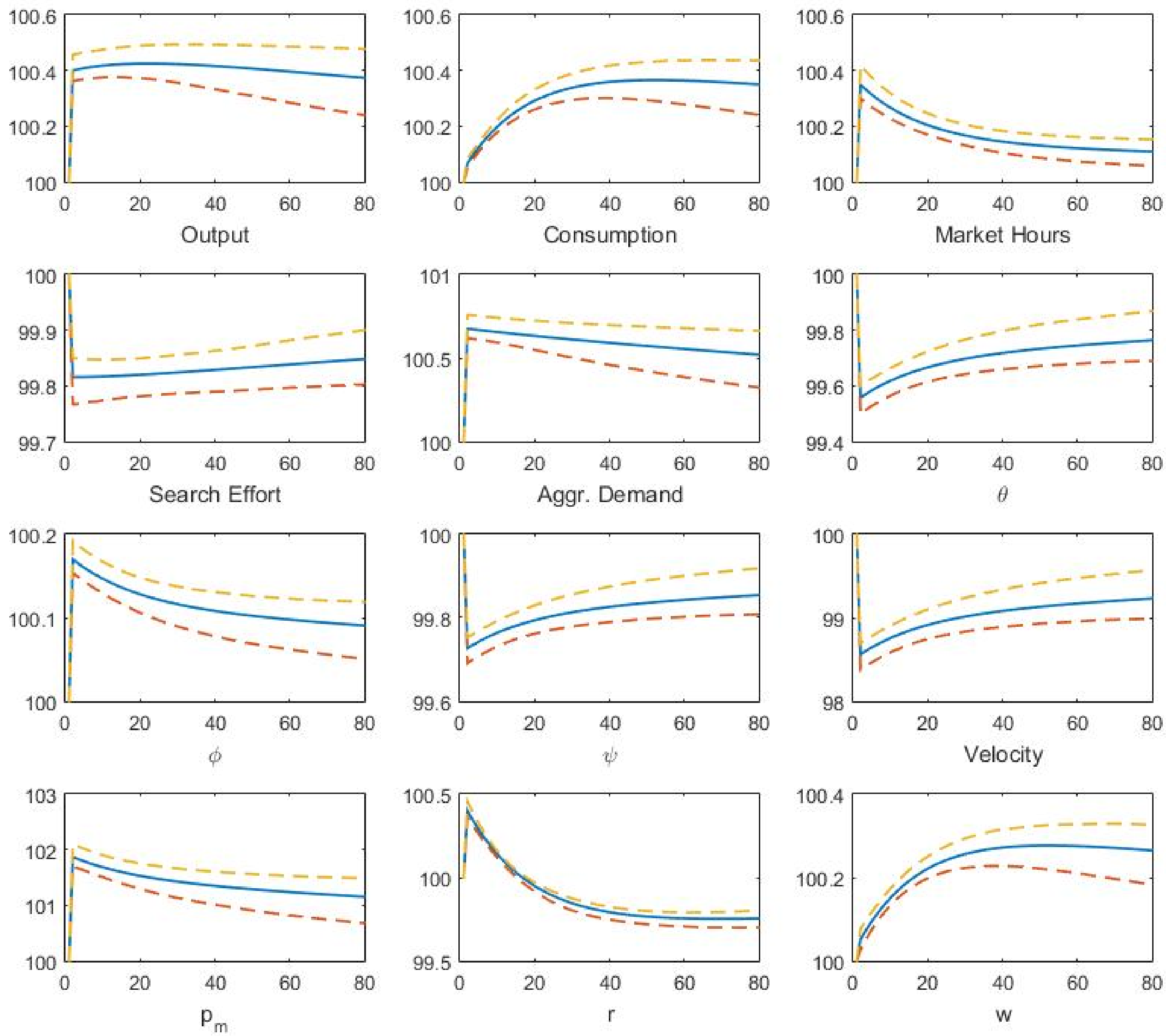

Figure D.3: IRF to Effort Shock A_d

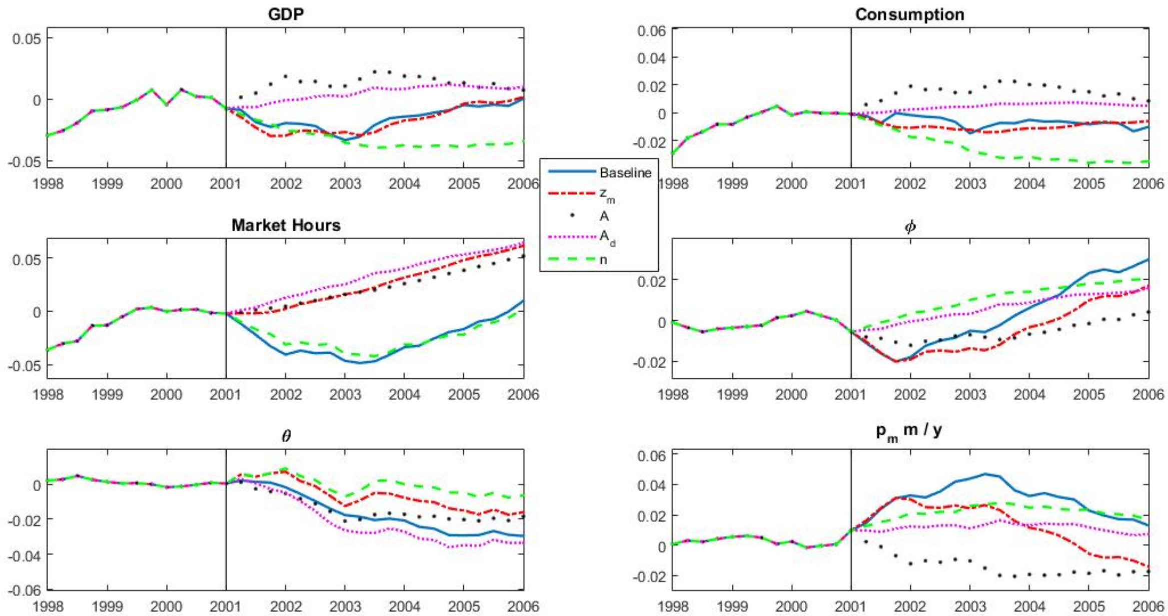

Figure D.4: 2001.I recession due to each shock

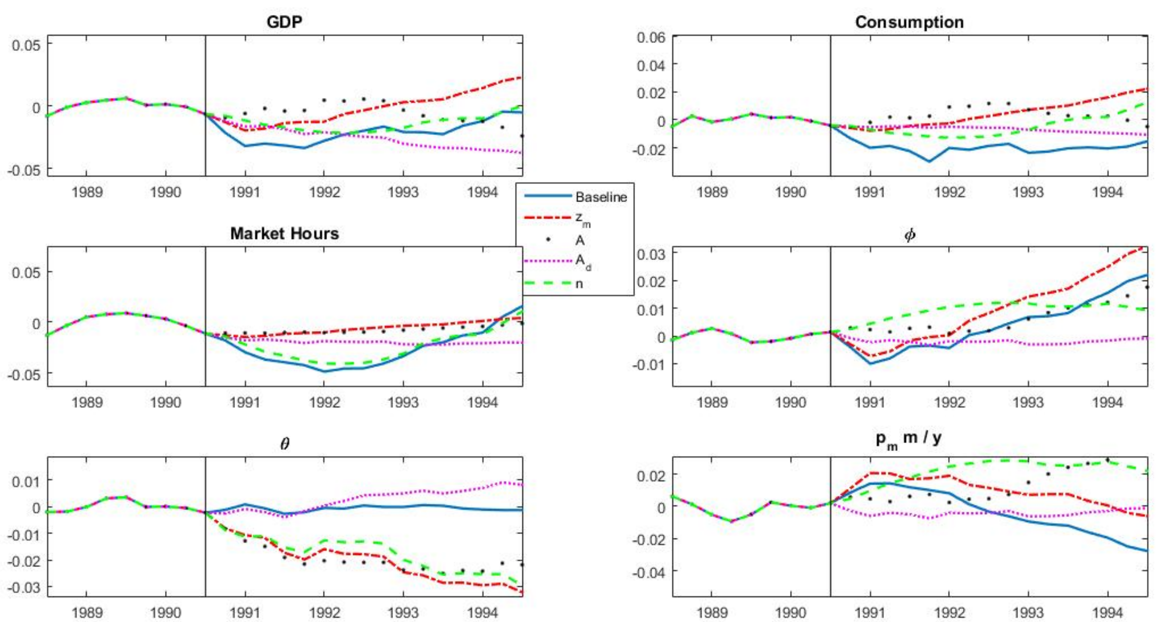

Figure D.5: 1990.III recession due to each shock

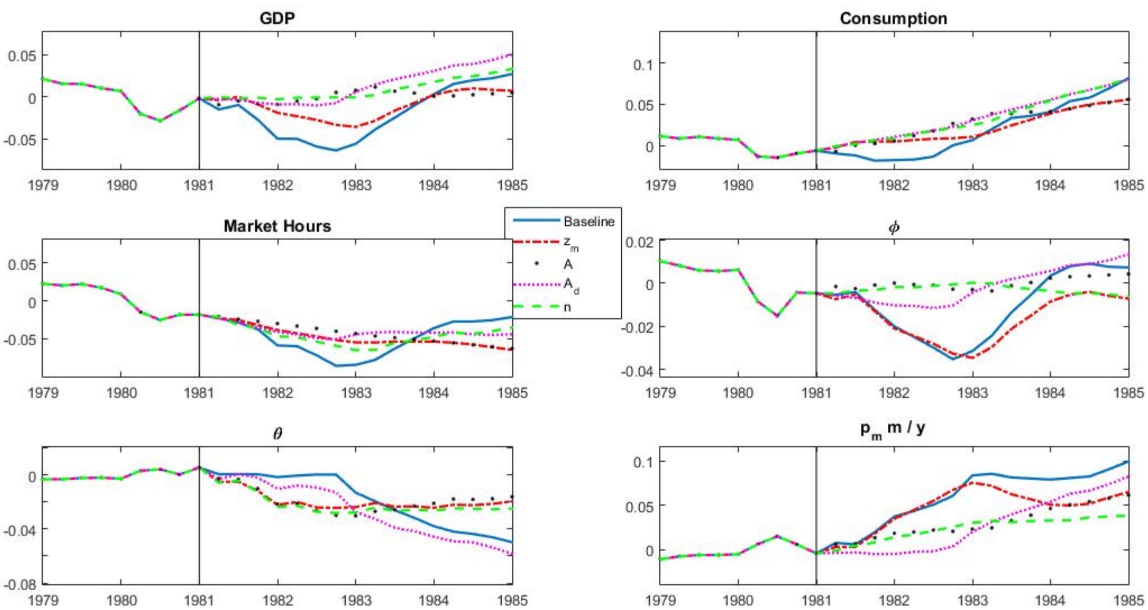

Figure D.6: 1981.I recession due to each shock

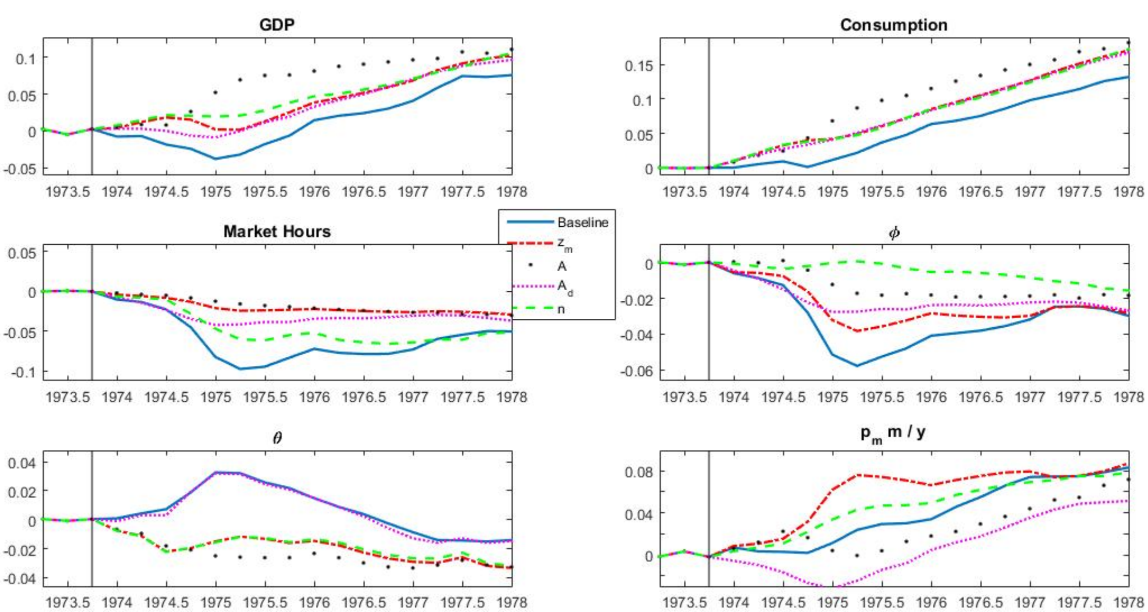

Figure D.7: 1973.IV recession due to each shock

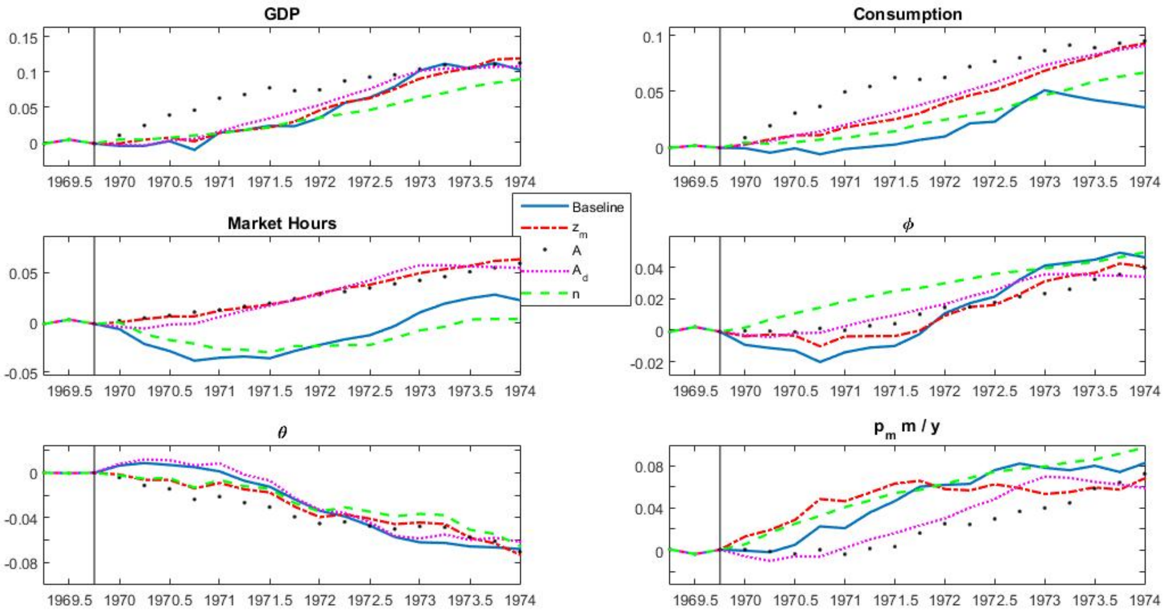

Figure D.8: 1969.IV recession due to each shock

E Data and Summary of Parametrization

Nominal and Real GDP and consumption are taken from the NIPA Tables 1.1.5 and 1.1.6 of BEA. Consumption is defined as personal consumption expenditures on non-durables and services +government spending and net export. Hours per capita are constructed by dividing Hours by population taken from the Bureau of Labor Statistics (BLS). Hours, ID PRS85006033. Civilian Noninstitutional Population, ID LNU00000000. Velocity of M2 Money Stock [M2V], retrieved from FRED, Federal Reserve Bank of St. Louis. Capacity Utilization: Total Industry [TCU], retrieved from FRED, constructed by the Board of Governors.

		Prior			Posterior				
Name	Description	Density	Para(1)	Para(2)	Mode	Median	Std	[5 ,	95]
α	Capital income share	Fixed	0.34						
δ	Capital depreciation	Fixed	0.014						
β	Discount factor	Fixed	0.99						
γ_a	TFP growth	Fixed	1.0022						
ν	Frisch labour supply	Gamma	0.85	0.10	1.02	1.00	0.10	0.85	1.18
ν_d	Frisch effort supply	Gamma	0.85	0.10	0.74	0.75	0.09	0.61	0.91
$-\rho$	Matching Compl.	Gamma	1.00	0.5	1.44	1.41	0.18	1.12	1.73
	Persistence of shocks								
ρ_a	TFP	Beta	0.88	0.10	0.982	0.980	0.008	0.965	0.991
ρ_β	β	Beta	0.88	0.10	0.999	0.998	0.001	0.995	0.999
ρ_n	Labour supply	Beta	0.88	0.10	0.999	0.997	0.002	0.990	0.999
ρ_d	Effort productivity	Beta	0.88	0.10	0.999	0.998	0.001	0.996	0.999
ρ_{z_m}	Matching	Beta	0.88	0.10	0.967	0.967	0.010	0.949	0.981
	Std of shocks								
σ_a	TFP	Exp	1.00		0.67	0.67	0.036	0.62	0.74
σ_β	β	Exp	1.00		0.025	0.026	0.001	0.024	0.029
σ_n	Labour supply	Exp	1.00		1.32	1.33	0.11	1.18	1.52
σ_d	Effort product.	Exp	1.00		0.78	0.78	0.05	0.71	0.87
σ_{z_m}	Matching	Exp	1.00		0.61	0.59	0.08	0.47	0.75

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions. Para (1) indicates the value of the calibrated parameters. For the structural shocks, values in the last 5 columns are multiplied by 100.