

# Private Money Creation and Equilibrium Liquidity\*

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## **Abstract**

Can a perfectly competitive issuance of private money fulfill the liquidity needs of the economy? The answer is no. Multiple equilibria are possible: there exist good equilibria with complete satiation of liquidity and absence of default on private money, and bad equilibria characterized by a shortage of liquidity and default. Capital requirements improve welfare provided that leverage is neither too high nor too low. Liquidity regulation can be counterproductive. Government intervention during liquidity crises is beneficial unless fiscal capacity is limited.

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# 1 Introduction

This paper considers an economy that lacks liquidity and studies the conditions under which the creation of private money can provide it. The recent financial crisis has unveiled the existence of a shadow banking sector that for years has been able to provide some form of money-like assets. Suddenly (and this is the very origin of the crisis), transacting parties realized that what was believed to be a safe security – and therefore liquid – did not have appropriate backing in the quality of intermediaries’ assets. What had been acceptable to satisfy liquidity needs became inadequate. The subsequent shortage of liquid assets produced a disruption in the real economy and a deep recession.<sup>1</sup>

Swings in the creation and destruction of private money are not just a recent phenomenon. They have characterized almost every deep financial crisis throughout much of monetary history, with different names given to the intermediaries and their assets and liabilities. Economists have not abstained from the debate, offering opposing opinions on whether private money should be issued and on the restrictions that should be imposed on the financial sector in doing so.<sup>2</sup>

The model of this paper analyzes an economy in which safe and pseudo-safe securities can coexist. In our model, safe assets are forms of private or public debt that are repaid with certainty. Pseudo-safe assets are also debt securities but can be defaulted on in some rare contingencies. The key assumption of our model is the distinction between the liquidity services provided by safe and pseudo-safe assets (i.e., their ability to facilitate transactions). Safe assets always provide liquidity services, whereas pseudo-safe assets do so only in states of nature in which they are not defaulted upon. This key assumption is consistent with the historical evidence discussed by Gorton (2016). Starting in the eighteenth century, certain types of debt have come to serve as liquid assets, including privately produced debt with some risk of default.

The starting point of our analysis is an economy with a shortage of liquidity. There is a supply of safe assets in the form of short-term government

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<sup>1</sup>Brunnermeier (2009) and Stein (2010) provide an interesting account of the 2007-2008 credit and liquidity crunch.

<sup>2</sup>For a comprehensive perspective on the debate, see Aguirre (1985) and Aguirre and Infantino (2013). Moreover, Sargent (2011) offers an interesting historical view on the tension between economic efficiency and financial stability.

debt that is completely backed by real taxes. However, this supply is not enough to fulfill all the liquidity needs of the economy. Private money in the form of deposits issued by financial intermediaries, which invest in risky assets, can be created.<sup>3</sup> The key question we ask is whether this process can succeed in achieving efficiency. According to Hayek (1976), competition in the creation of private money should suffice for the purpose since it eliminates any possible rent in money issuance.<sup>4</sup>

The main result of the paper disconfirms Hayek's view. The creation of private money, left only to the forces of unfettered competition, is not necessarily efficient and is instead consistent with a multiplicity of equilibria. These equilibria can be grouped into two classes. In the first, which we label good equilibria, intermediaries do succeed in issuing safe assets, allowing competition to fulfill all the liquidity needs of the economy and achieve efficiency. In the second class of equilibria, the bad ones, private-money creation takes the form of pseudo-safe assets that satiate liquidity needs only partially; that is, there are states of nature in which intermediaries default, and thus their deposits cannot be used for transactions. Therefore, the first best cannot be achieved.

The source of the multiplicity of equilibria is related to the irrelevance of leverage for intermediaries, similar to the Modigliani-Miller theorem. However, leverage matters for macroeconomic efficiency. If intermediaries are well capitalized, they can supply safe assets to the point where the economy reaches the efficient level of liquidity. Otherwise, they can only issue pseudo-safe assets which completely lose liquidity values under adverse scenarios in which the economy experiences a liquidity crunch.

Motivated by the relevance of intermediaries' leverage for macroeconomic outcomes, we analyze two regulatory tools: capital requirements and liquidity requirements.

Capital requirements that impose bounds on leverage achieve the efficient outcome. However, if intermediaries are forced to hold too much equity, then the first best cannot be achieved. This is because forcing intermediaries to hold too much equity restricts their ability to issue deposits that provide

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<sup>3</sup>This process can also be interpreted as securitization in which risky and illiquid loans are made liquid by creating a senior tranche (which corresponds to the deposits in our model), while the junior tranche (which corresponds to equity) absorbs the illiquid and risky component.

<sup>4</sup>The real bills doctrine also emphasizes perfect competition as a way to achieve efficiency but further requires intermediaries to hold risk-free assets.

liquidity services, therefore reducing welfare. It is key to emphasize that our motivation for capital requirements is related to the dichotomy between the irrelevance of leverage in the financial sector, in the spirit of the Modigliani-Miller theorem, and the relevance of leverage at the macro level in reaching the efficient supply of liquidity. To our knowledge, this justification for capital requirements is novel in the literature and complements the most prominent view that sees capital requirements as a tool to offset the distortion of deposit insurance and other government guarantees.

We then analyze liquidity requirements, such as the new liquidity coverage ratio and net stable funding ratio introduced by Basel III. The main result is that, in some circumstances, these requirements reduce the creation of liquidity, worsening welfare. Without liquidity requirements, intermediaries have the ability to transform risky and illiquid assets into liquid deposits; with liquidity requirements, intermediaries are forced to invest in government-issued public money in order to issue deposits. If the supply of government securities is very low, the issuance of private money is limited and may not be enough to fulfill the liquidity needs of the economy.

We also discuss the policy response to a liquidity crunch that happens when private money is defaulted on and therefore loses its liquidity properties. This type of crisis is different from the classical liquidity crisis due to a debt run, where intermediaries have to liquidate assets early, causing the disruption of capital and real losses. In our model, liquidity can be exchanged for goods, and therefore the liquidity crisis produces an excess supply of goods, which mirrors the shortage of safe assets.<sup>5</sup> The government can restore efficiency by increasing the real value of the remaining safe assets, thereby offsetting the shortage and lowering the nominal price, or by injecting more public money into the economy. Both interventions can succeed only if real taxes are appropriately raised. In the end, the way out of the crisis is to substitute the insufficient backing of private money with more backing of public money. However, if increasing real taxes is not feasible, the ability of the government to stave off a liquidity crisis might be limited.

Our baseline analysis is then extended to address departures from perfect competition by allowing financial intermediaries to have some monopoly power. The interesting conclusion is that the multiplicity of equilibria is eliminated. However, the only equilibrium that arises is of the bad type in

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<sup>5</sup>The role of liquid assets in providing transaction services is also emphasized by Gorton (2016).

which intermediaries create pseudo-safe securities that are defaulted on in some contingencies. Furthermore, monopoly distortions reduce equilibrium liquidity below the level of perfect competition. This extension further reinforces our conclusion that capital requirements are needed to force private intermediaries to create safe securities, although, in this case, the overall supply of liquidity in the economy remains inefficiently low.

Our paper is related to a recent literature spurred by the work of Caballero (2006) that has emphasized the shortage of safe assets as a key determinant of the imbalances of the global economy. Caballero and Farhi (2016) study the macroeconomic effects of a shortage of safe assets, emphasizing that fiscal capacity is the primary source of liquidity creation. They do not model private-money creation.

Recent and concurrent papers, such as Farhi and Maggiori (2016) and Moreira and Savov (2016), instead allow for multiple issuers of liquidity. Farhi and Maggiori (2016) focus on the supply of reserve currency by a monopolist in an international context and on its strategic devaluation decisions. They also consider multiple issuers and the limiting case of perfect competition. In their model, perfect competition always achieves efficiency, contrary to our result. They assume that any issuer is fully backed, eliminating on this ground the possible relationship between leverage and default rates that is key in our framework. Moreira and Savov (2016) show that financial intermediaries can create pseudo-safe securities, but they mainly focus on the macroeconomic consequences of liquidity cycles. In Brunnermeier and Sanikov (2016), private and public money coexist, but money serves as a store of value. More important, the focus of their paper is on the effects of nominal contracts and on the debt-deflation mechanism, whereas a central theme of our paper is the link between private-money creation and the balance sheet of the issuer, including the possibility of default.

Other papers have analyzed the interaction of private money issued by financial intermediaries and public money, mostly using overlapping generations models. Sargent and Wallace (1982) study the real bills doctrine in comparison with the quantity theory in a context in which private and public money are always perfect substitutes. We instead underline the link between the substitutability of private and public money and the balance sheet conditions of intermediaries and government. Bullard and Smith (2003) study the role of outside and inside money (although the latter is always considered default free) in achieving efficiency by using an overlapping generations model in which frictions create spatial separation and limited communication. The

assumed frictions and inefficiencies drive their results in favor of one source rather than another.

The rest of this paper is organized as follows. Section 2 presents a brief overview of the framework and discusses the main mechanism and results. It is followed in Section 3 by a thorough presentation of the model. Section 4 discusses the equilibrium under perfect competition, while Section 5 analyzes capital and liquidity requirements. Section 6 studies government intervention in a liquidity crunch. Section 7 discusses the extension to a market characterized by monopolistically competitive financial intermediaries. Section 8 concludes.

## 2 Equilibrium liquidity

We use this section to expound some of the concepts we are going to develop in the model of this paper and discuss some of the key mechanisms at work. We first introduce the concept of liquidity and then illustrate how to value liquidity before briefly overviewing the equilibrium determination.

First, we assume that only debt can provide liquidity services, whereas other securities such as equity cannot. Second, in each state of nature, a debt security is liquid only if it is not defaulted on in that state. Our assumptions are in the same spirit as Gorton and Pennacchi (1990) and Stein (2012), although ours are more general since they do not allow risky securities to provide liquidity services. In Stein (2012), safe assets are the only liquid securities.<sup>6</sup> We instead directly link the liquidity value of a security with its safety in a particular contingency. The same security can be liquid and therefore accepted in trading goods in a favorable state, but unacceptable in a bad state if the promised payoff is even partially seized. This assumption can be simply justified by the existence of some time requirement to complete the default procedure. These delays are enough to prevent the use of the security in trading goods.

In our model, therefore, both safe and pseudo-safe assets can provide liquidity services.<sup>7</sup> Safe securities are debt paid with certainty, whereas pseudo-

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<sup>6</sup>See also Farhi and Maggiori (2016) and Woodford (2016).

<sup>7</sup>This approach is more in line with the view of Gorton (2016), who argues that privately produced information-insensitive debt, what he defines as safe assets, can carry credit risk and that agencies' ratings can indicate the distance to information sensitivity. To capture this idea, we could clearly put a threshold on the level of riskiness above which

safe securities are debt securities that are defaulted on in some adverse contingencies. Safe assets always provide liquidity services, whereas pseudo-safe assets do so only in the good states of nature.<sup>8</sup> This possible coexistence is appealing since it allows us to model a liquidity crisis as an event in which pseudo-safe assets reveal their risky characteristics, thus experiencing a reduction in their face value and suddenly losing their liquidity properties.<sup>9</sup>

To price securities that have liquidity value, it is necessary to depart from standard asset pricing theory, in which assets are valued only by their pecuniary return.<sup>10</sup> Consider first a security with a promised payoff of one unit. In our model, the security can be issued by the government or by private financial intermediaries. However, for reasons that will be explained later, the publicly issued security is always free of credit risk and, therefore, always liquid. We can think of this security as short-term government bonds or interest-bearing reserves issued by the central bank.<sup>11</sup> The analysis below will show that the household's demand of the publicly issued security is flat at the price  $Q_t^B$

$$Q_t^B = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + \mu_{t+1}) \right\}, \quad (1)$$

where  $\beta$  is the consumer's intertemporal discount factor and  $P_t$  is the general price index. Accordingly,  $\beta P_t/P_{t+1}$  is the stochastic discount factor used to evaluate nominal payoffs between time  $t$  and any contingency at time  $t + 1$ . The nonnegative term  $\mu_{t+1}$  captures the nonpecuniary return of the security. Households are willing to pay more for the publicly issued security given that they can receive its liquidity services in purchasing goods. The liquidity premium can also be described by the nonnegative difference  $Q_t^B - Q_t^f$  where  $Q_t^f = \beta E_t \{P_t/P_{t+1}\}$  is the price of a security with similar credit risk characteristics but no liquidity value.

The key observation is that the equilibrium price  $Q_t^B$  cannot be under-

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debt securities will never be accepted for liquidity purposes. This approach will not have consequences for the generality of our result.

<sup>8</sup>In Section 7, we introduce structured products and assume that they lose liquidity properties even if only one of the primitive securities is defaulted on or partially seized.

<sup>9</sup>In Gennaioli et al. (2012, 2013), crises happen when investors become aware of a neglected tail risk for which what was deemed to be safe debt turns out to be unsafe. However, even in their context, riskiness is an intrinsic characteristic of privately issued securities.

<sup>10</sup>See Lagos (2010).

<sup>11</sup>In our model, the government includes both the treasury and the central bank.

stood simply from equation (1), which reflects only the demand side of a public security. One must also look at the supply side of the security, as well as both the demand and supply of competing private sources of liquidity. Turning first to the central bank's supply of reserves, solvency of the government implies

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( T_{t+j} + (Q_{t+j}^B - Q_{t+j}^f) \frac{B_{t+j}}{P_{t+j}} \right) \right\}, \quad (2)$$

which is indeed critical to understanding why the publicly issued security is free of risk in our model. Given a supply  $B_t$  of securities of unitary promised face value, full backing is always attainable by drawing on three sources. The first is the present discounted value of real taxes, denoted by  $T_t$ , which corresponds to the first term on the right-hand side of the previous solvency condition. The second source results from the liquidity properties of reserves, which produce real rents that can back the value of outstanding obligations.<sup>12</sup> But again, this source might be out of the government's control since it depends on the equilibrium between competing sources of liquidity and therefore on the equilibrium value of liquidity. Third, absent a complete backing derived from the first two sources, the price level can move to adjust the real value of promised obligations to meet available resources, as shown on the left-hand side of the solvency condition. This is why solvency is not an issue for publicly issued securities in our model.

The previous setting is also useful in understanding the demand and supply of private liquidity and equilibrium liquidity overall. In our baseline model, we will show that the household's demand for private liquidity, which takes the form of deposits issued by intermediaries, is flat at the price  $Q_t^D$

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[ (1 - I_{t+1})(1 + \mu_{t+1}) + I_{t+1}(1 - \chi_{t+1}) \right] \right\}, \quad (3)$$

where  $I_{t+1}$  is an indicator function equal to one when the privately issued security is defaulted upon. In this event,  $\chi_{t+1}$  is the seized fraction. In equation (3), the assumption is now evident that the privately issued security provides liquidity benefits, captured by  $\mu_{t+1}$ , only when it is not defaulted on,  $I_{t+1} = 0$ . Therefore,  $Q_t^D = Q_t^B$  whenever financial intermediaries are solvent in all possible states and, therefore, deposits are truly safe assets.

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<sup>12</sup>This source is similar to seigniorage.



It is critical to explain why the privately issued security, unlike the public security, can be seized partially in some or all contingencies. A solvency condition similar to that of the government applies:

$$(1-\chi_t)\frac{D_{t-1}}{P_t} - (1+r_t)K_{t-1}^I = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( T_{t+j}^I + (Q_{t+j}^D - Q_{t+j}^f) \frac{D_{t+j}}{P_{t+j}} \right) \right\} \quad (4)$$

although there are some notable differences.<sup>13</sup> Physical capital,  $K_t^I$ , shows up as an asset with a return  $r_t$ , which is the exogenous force of our model that can trigger default on deposits  $D_t$ . This asset can serve as a backing for deposits but only partially because of its risky nature. Intermediaries have to rely on other sources. The key difference with respect to the government is that these resources may be limited, thereby opening the possibility of (partial) default. First, adjusting the price level is not an option for a single intermediary, unlike the government. What remains are transfers from households,  $T_t^I$ , and the possible rents obtained by issuing liabilities with pecuniary value. However, the first channel can be limited by the fact that positive transfers from the private sector  $T_t^I$  are nothing more than negative dividends or injections of further net worth, and it is reasonable to assume – as we do – that there is a bound on these resources. The nonpecuniary rents are similar to the government's, but with two limitations. On the one hand,  $Q_t^D \leq Q_t^B$ , with strict inequality if the private intermediary has a positive probability of default. On the other hand, and more important, the amount of deposits is determined in equilibrium as a function of households' demand.

Finally, we draw the important distinction between the issuance of public and private liquidity. In the baseline model, we take as given a certain level of public liquidity that is created by the government. A monopoly power is associated with issuing public money. But private liquidity creation arises from the interaction between households and financial intermediaries in money markets. Therefore, while equations (1) and (2) are sufficient to describe public liquidity, equations (3) and (4) are not enough to completely characterize private liquidity creation. Assuming a market characterized by perfect competition, we show that the supply of private liquidity is infinitely

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<sup>13</sup>In the next section, we present a simpler model of intermediaries that live for only two periods in an overlapping way. This framework is designed to maintain tractability without losing generality.

elastic at the price

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} [I_{t+1}(1 - \chi_{t+1}) + (1 - I_{t+1})] \right\}. \quad (5)$$

The demand (3) and supply (5) of private liquidity meet at

$$E_t \left\{ \frac{P_t}{P_{t+1}} [(1 - I_{t+1})(1 + \mu_{t+1})] \right\} = E_t \left\{ \frac{P_t}{P_{t+1}} (1 - I_{t+1}) \right\}.$$

The previous equation delivers on this paper's main result. Private money creation under perfect competition implies a multiplicity of equilibria. There can be equilibria in which deposits are never defaulted on ( $I_{t+1} = 0$  in all contingencies), and correspondingly the equilibrium level of deposits is enough to satiate all needs ( $\mu_{t+1} = 0$  in all contingencies), as shown by the previous equation. There are also equilibria with partial default in some states or in all states ( $I_{t+1} = 1$ ). As consequence, a shortage of liquidity ( $\mu_{t+1} > 0$ ) may arise in these states if public liquidity is scarce, causing an inefficient level of consumption.

Key in our analysis is that the different equilibria stem from different levels of intermediaries' net worth, and therefore by the backing provided to private money. All these results will be derived and discussed in Section 4. We now present the details of the model.

### 3 Model

The model features three sets of actors: households, financial intermediaries and a government. We begin by describing each of these groups and then discuss equilibrium.

#### 3.1 Households

Households are infinitely lived and have the following intertemporal preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + X_t], \quad (6)$$

where  $E_0$  is the expectation operator at time 0 and  $\beta$  is the intertemporal discount factor with  $0 < \beta < 1$ .  $C_t$  and  $X_t$  denote consumption of the same

good but during different subperiods within period  $t$ :  $C_t$  is consumed in the first subperiod,  $X_t$  in the second. Alternatively, this setting can be described as a cash-credit model à la Lucas and Stokey (1987), where  $C_t$  is the cash good and  $X_t$  is the credit good.

A financial friction, however, puts barriers on which securities can be used to purchase consumption goods in the first subperiod. The limitation says that liquidity services can be provided by any publicly and privately created debt security only as long as the security is not defaulted on, even partially, in that particular state of nature in which it is exchanged for goods. Liquidity can then also be supplied by securities that have credit risk, but not in the state of nature in which risk materializes. As discussed in the previous section, the liquidity friction can simply be justified by delays arising from the default procedure.

Two securities can potentially provide liquidity services: a publicly issued security ( $B_{t-1}$ ), which has the interpretation of government debt or interest-bearing central bank reserves; and deposits ( $D_{t-1}$ ), which are privately created by financial intermediaries. At the beginning of a generic period  $t$ , households are subject to the liquidity constraint

$$P_t C_t \leq B_{t-1} + (1 - I_t) D_{t-1}, \quad (7)$$

where  $I_t$  is an indicator function that takes the value of one if the issuer of deposits is in default and zero otherwise;  $P_t$  is the price level of the consumption good.<sup>14</sup> In principle, government debt  $B_{t-1}$  can also lose liquidity value if the government defaults but later we introduce some assumptions such that the government is always solvent in equilibrium.

In the second subperiod, households choose consumption goods  $X_t$  and make portfolio decisions regarding deposits  $D_t$ , government bonds  $B_t$ , capital  $K_t^H$ , and net worth of financial intermediaries  $N_t$ . Their budget constraint is

$$\begin{aligned} P_t X_t + Q_t^B B_t + Q_t^D D_t + Q_t^K K_t^H + N_t &\leq I_t(1 - \chi_t) D_{t-1} \\ + [B_{t-1} + (1 - I_t) D_{t-1} - P_t C_t] &+ Q_{t-1}^K K_{t-1}^H (1 + i_t^K) + N_{t-1} (1 + i_t^N) - P_t T_t, \end{aligned} \quad (8)$$

where  $Q_t^B$ ,  $Q_t^D$ , and  $Q_t^K$  are the nominal prices of government bonds, deposits and capital, respectively. The resources available to households are

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<sup>14</sup>Our result of perfect substitution between the liquidity provided by  $B_{t-1}$  and  $D_{t-1}$  is motivated by the results of Nagel (2014), who estimates a high elasticity of substitution between public and private liquidity.

defaulted deposits  $I_t (1 - \chi_t) D_{t-1}$ , any liquidity  $B_{t-1} + (1 - I_t) D_{t-1}$  not used to buy consumption goods in the first subperiod, capital  $K_{t-1}^H$  bought in  $t - 1$  plus a nominal return  $i_t^K$ , and dividends from holding equity in financial intermediaries  $N_{t-1} (1 + i_t^N)$ ; households pay lump-sum real taxes  $T_t$  to the government. The term  $I_t (1 - \chi_t) D_{t-1}$  appears in the budget constraint ( $\chi_t$  is the rate of default on deposits) because if deposits are partially seized ( $I_t = 1$ ,  $0 < \chi_t < 1$ ), then deposits cannot be used to buy  $C_t$  in the first period, but they become available in the second subperiod.

Capital, which is in fixed supply  $\bar{K}$  in the whole economy, is used to produce output at the beginning of each period with the technology  $Y_t = A_t K_{t-1}$ , where  $A_t$  is a two-state aggregate shock and

$$A_t = \begin{cases} A_h & \text{with probability } 1 - \pi \\ A_l & \text{with probability } \pi \end{cases}.$$

While we do not put any restriction on the probabilities, we interpret  $A_l$  as a rare disaster event with  $A_l < A_h$ . The stochastic process  $A_t$  is i.i.d. over time and is the only stochastic disturbance in the economy. For future reference, denote  $A$  to be the average value of  $A_t$ , that is,  $A \equiv (1 - \pi) A_h + \pi A_l$ . Output can be sold either at price  $P_t$  in the first subperiod or at the same price in the second subperiod. Therefore, the nominal return on capital  $i_t^K$  is defined by

$$1 + i_t^K \equiv \frac{Q_t^K + P_t A_t}{Q_{t-1}^K}, \quad (9)$$

where the payoff is given by the price of capital,  $Q_t^K$ , plus its benefits in terms of good production, equal to  $P_t A_t$ . Capital can be held by both households and intermediaries; therefore,  $\bar{K} = K_t^H + K_t^I$ , where  $K_t^I$  is capital held by intermediaries. See the next section for details.

By investing net worth  $N_{t-1}$  into financial intermediaries, households are entitled to receive a share of dividends  $\Pi_t^D$  from the intermediaries.<sup>15</sup> Accordingly, the return on net worth is defined by

$$1 + i_t^N \equiv \frac{\Pi_t^D}{N_{t-1}}. \quad (10)$$

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<sup>15</sup>The return on net worth invested in intermediaries is given only by dividends and does not include any capital gains since, as will be detailed in the next subsection, intermediaries live for only two periods.

Consumption and portfolio choices are implied by the maximization of (6) under the constraints (7) and (8). Households are risk-averse in the consumption of  $C_t$  but risk-neutral in the consumption of  $X_t$ . This quasi-linear utility simplifies the problem of households, because the marginal utility of wealth is just given by  $\lambda_t = 1/P_t$ , where  $\lambda_t$  is the Lagrange multiplier of the budget constraint (8). Thus, the optimality conditions for the demand of capital and the supply of net worth are

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + i_{t+1}^K) \right\}, \quad (11)$$

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + i_{t+1}^N) \right\}. \quad (12)$$

A further implication of the utility function is that the demand for goods in the first subperiod is

$$C_t = \frac{1}{1 + \mu_t}, \quad (13)$$

where  $\mu_t/P_t$  is the Lagrange multiplier associated with the constraint (7). Since  $\mu_t \geq 0$ , thus  $C_t \leq 1$  and at the first best  $C_t = 1$ . The first-best allocation follows from the fact that the marginal utility of consumption of  $X_t$  in the second subperiod is one, whereas the marginal utility of consumption in the first subperiod is  $1/C_t$ . Therefore, since the price of  $C_t$  and  $X_t$  is the same, the first best is achieved by  $C_t = 1$ .

To conclude the characterization of the household's problem, we derive the demand for government debt and deposits, which also depends on the liquidity value provided by these assets, captured by the Lagrange multiplier  $\mu_{t+1}$  on the constraint (7):

$$Q_t^B = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + \mu_{t+1}) \right\}, \quad (14)$$

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} [I_{t+1}(1 - \chi_{t+1}) + (1 - I_{t+1})(1 + \mu_{t+1})] \right\}. \quad (15)$$

Deposits provide liquidity services, captured by the variable  $\mu_{t+1}$  if positive, only when they are not defaulted on,  $I_{t+1} = 0$ .<sup>16</sup> An implication of (14) and

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<sup>16</sup>In the Appendix, we discuss a case in which deposits are always made liquid through the intervention of brokers who receive a transaction fee for their services.

(15) is that  $Q_t^B \geq Q_t^D$ , with strict inequality when deposits are defaulted on in some contingency. Liquidity services provide benefits to the issuer by lowering borrowing costs.

Finally, a transversality condition applies imposing an appropriate limit on the rate of growth of assets held by households:

$$\lim_{j \rightarrow \infty} \beta^j \left( \frac{Q_{t+j}^B B_{t+j} + Q_{t+j}^D D_{t+j} + Q_{t+j}^K K_{t+j}^H + N_{t+j}}{P_{t+j}} \right) = 0. \quad (16)$$

Equation (16) holds almost surely, looking forward from each time  $t$  and in each contingency at time  $t$ .

### 3.2 Financial Intermediaries

We make the simplifying assumption that financial intermediaries live for only two periods in an overlapping way. Consider intermediaries that start to operate at time  $t$  and end their activity at time  $t+1$ . At time  $t$ , they collect funds by issuing deposits  $D_t$  and raising net worth  $N_t$ . Deposits are issued in the form of one-period zero-coupon bonds with price  $Q_t^D$ . Both sources come from households. Intermediaries invest these resources into capital  $K_t^I$  at price  $Q_t^K$ :

$$Q_t^K K_t^I = Q_t^D D_t + N_t. \quad (17)$$

In the following period  $t+1$ , gross profits of intermediaries  $\Pi_{t+1}$  are given by

$$\Pi_{t+1} = (1 + i_{t+1}^K) Q_t^K K_t^I - (1 - I_{t+1}) D_t - I_{t+1} (1 - \chi_{t+1}) D_t, \quad (18)$$

reflecting the return on capital and the cost of repaying deposits. We are making a distinction here between profits  $\Pi_{t+1}$  and dividends  $\Pi_{t+1}^D$  since the former might include rents retained by intermediaries.

The rate of default,  $\chi_{t+1}$ , is endogenous in our framework and depends on a simple and key assumption that financial intermediaries are subject to limited liability; that is, profits in the last period of their life  $\Pi_{t+1}$  must be nonnegative. Using (18), nonnegative profits imply that the default rate is given by

$$\begin{aligned} \chi_{t+1} &= \max \left( 0, 1 - (1 + i_{t+1}^K) \frac{Q_t^K K_t^I}{D_t} \right) \\ &= \max \left( 0, 1 - (1 + i_{t+1}^K) \frac{N_t + Q_t^D D_t}{D_t} \right), \end{aligned} \quad (19)$$

where the second line follows from the intermediaries' balance sheet (17). Default is more likely when the return on capital, the intermediaries' net worth, and the price of deposits are low enough. Everything else being equal, a higher level of deposits raises the default rate.

We now analyze the market structure in which intermediaries operate. In the baseline model, we assume that the market is perfectly competitive. There is an infinite number of small financial intermediaries that supply a homogeneous product in the form of a deposit security. All these intermediaries are marginal with respect to the size of the overall market in which there is free entry and exit. As a consequence, intermediaries take prices as given and maximize their profits, but competition eliminates any rents from financial intermediation, meaning that profits are equal to dividends in equilibrium,  $\Pi_{t+1} = \Pi_{t+1}^D$ . To understand the source of rents and the role of competition, evaluate profits using the discount factor of households:

$$\begin{aligned} & E_t \left\{ \beta \frac{P_t}{P_{t+1}} \Pi_{t+1} \right\} \\ &= \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[ (1 + i_{t+1}^K) (N_t + Q_t^D D_t) - (1 - I_{t+1}) D_t - I_{t+1} (1 - \chi_{t+1}) D_t \right] \right\} \\ &= N_t + Q_t^D D_t - \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[ (1 - I_{t+1}) + I_{t+1} (1 - \chi_{t+1}) \right] \right\} D_t, \end{aligned}$$

where the first equality uses (17) and (18), and the second equality uses (11) and rearranges. Finally, using (10) and (12), expected profits can be written as the sum of expected dividends and intermediation's rents:

$$\begin{aligned} E_t \left\{ \beta \frac{P_t}{P_{t+1}} \Pi_{t+1} \right\} &= E_t \left\{ \beta \frac{P_t}{P_{t+1}} \Pi_{t+1}^D \right\} + \\ &\quad + Q_t^D D_t - \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[ (1 - I_{t+1}) + I_{t+1} (1 - \chi_{t+1}) \right] \right\} D_t. \end{aligned} \tag{20}$$

Rents are completely abated when nothing is left to the intermediaries once dividends are paid, that is,  $\Pi_{t+1} = \Pi_{t+1}^D$ . The expected discounted value of rents earned by intermediaries is given by the two terms on the second line of (20), which is the difference between what households deposit at time  $t$ , the term  $Q_t^D D_t$ , and the expected value of what intermediaries repay to households in  $t + 1$ .

Zero rents imply a flat supply of deposits at the price

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} [(1 - I_{t+1}) + I_{t+1} (1 - \chi_{t+1})] \right\}. \quad (21)$$

Note from (20) that profit maximization and zero rents have equivalent implications for the price at which deposits are supplied.

### 3.3 Government

The government includes the treasury and the central bank together. For expositional simplicity, the only liabilities are short-term zero-coupon bonds  $B_t$ , which, again, can be interpreted as the treasury's debt or the central bank's reserves. At time  $t - 1$ , the government has to pay back  $B_{t-1}$  using newly issued securities  $B_t$  at the price  $Q_t^B$  and collecting real lump-sum taxes  $T_t$  at the price  $P_t$ . Therefore, its flow budget constraint is

$$B_{t-1} = Q_t^B B_t + P_t T_t.$$

Iterating forward the last expression and combining it with (14), we get

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( T_{t+j} + \beta \frac{\mu_{t+1+j}}{P_{t+1+j}} B_{t+j} \right) \right\} + \lim_{j \rightarrow \infty} \beta^j E_t \left\{ \frac{Q_{t+j}^B B_{t+j}}{P_{t+j}} \right\}. \quad (22)$$

Let us first focus on the second term on the right-hand side. Households' transversality condition (16), together with the balance sheet of intermediaries (17) and the market clearing condition for capital  $\bar{K} = K_t^H + K_t^I$ , implies that

$$\lim_{j \rightarrow \infty} \beta^j E_t \left\{ \frac{Q_{t+j}^B B_{t+j}}{P_{t+j}} \right\} = - \lim_{j \rightarrow \infty} \beta^j E_t \left\{ \frac{Q_{t+j}^K \bar{K}}{P_{t+j}} \right\}.$$

If we focus only on equilibria in which the real price of capital is stationary, then the second term on the right-hand side of (22) is zero and the intertemporal budget constraint of the government simplifies to

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( T_{t+j} + \beta \frac{\mu_{t+1+j}}{P_{t+1+j}} B_{t+j} \right) \right\}.$$



Another way to write it is to use (14):

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( T_{t+j} + (Q_{t+j}^B - Q_{t+j}^f) \frac{B_{t+j}}{P_{t+j}} \right) \right\}, \quad (23)$$

where we have further defined  $Q_t^f = \beta E_t \{P_t/P_{t+1}\}$  to be the price of a risk-free bond that does not provide liquidity services.

The real value of outstanding government debt  $B_{t-1}/P_t$  is backed by two streams of resources. First, the government can levy real taxes on households. Second, as reflected in the second term on the right-hand side of (23), liquidity premia lower the cost of borrowing and enhance the ability to repay debt, captured by a positive difference between the price of bonds and that of similar risk-free but illiquid securities.

Moreover, the nominal value of bonds is always risk-free because the price level can adjust, reducing if necessary the amount of real resources to pay. This is why government debt is always safe in our model and thus always provides liquidity services.<sup>17</sup>

The government chooses two policy instruments: the path of debt and taxes  $\{B_t, T_t\}$ , given an initial condition on  $B_{-1}$ . To simplify our analysis, we find it convenient to assume that the tax rule is of the form

$$T_t = (1 - \beta)T - \left( Q_t^B - Q_t^f \right) \frac{B_t}{P_t}. \quad (24)$$

In each period, real taxes are proportional to a constant,  $T$ , and fall proportionally to the real value of reserves  $B_t/P_t$ . The proportionality factor is captured by the liquidity premium  $Q_t^B - Q_t^f$ . The tax rule greatly simplifies our analysis; once it is substituted into (23), it yields

$$\frac{B_{t-1}}{P_t} = T. \quad (25)$$

A further simplification is to assume that reserves are in constant supply,  $B_t = B$ . It then follows that the specification of the monetary-fiscal policy determines, uniquely, a constant price level  $P = B/T$ . Later, in Section 6, we discuss the implications of alternative monetary-fiscal policy rules.

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<sup>17</sup>This approach is based on the fiscal theory of the price level, which is also convenient since it uniquely determines the price level.

## 4 Equilibrium

We use a standard concept of equilibrium in which households maximize utility, financial intermediaries operate under perfect competition, goods and asset markets clear, and the real value of government debt equals the present discounted value of taxes and seigniorage, as shown in (23), given a monetary and fiscal policy rule. In particular, intermediaries take as given the price of deposits,  $Q_t^D$ . This means that free entry at a price  $Q_t^D$  occurs by intermediaries supplying deposits that are homogeneous to those already in the market. As will be clear later, this implies entering with the same level of net worth as other incumbents.<sup>18</sup>

We have already characterized some equilibrium results, namely that competition in the financial sector implies a flat supply of deposits and that the price level is constant given the monetary-fiscal policy regime. Using the latter result, the demand of capital (11), together with (9), allows us to solve for the real price of capital:

$$\frac{Q^K}{P} = \frac{\beta}{1 - \beta} A,$$

which is also constant (recall that  $A$  is the unconditional expectation of  $A_t$ ). The nominal return on capital (9) simplifies to

$$1 + i_t^K = \frac{\beta A + (1 - \beta) A_t}{\beta A}. \quad (26)$$

Note that real and nominal returns on capital are equal since prices are constant. Denoting  $r_h^K$  and  $r_l^K$  to be the real returns on capital, respectively, in the high and low state, then

$$1 + r_h^K \equiv \frac{\beta A + (1 - \beta) A_h}{\beta A}, \quad 1 + r_l^K \equiv \frac{\beta A + (1 - \beta) A_l}{\beta A}. \quad (27)$$

The following set of equations is what is left to determine the remaining variables. The liquidity constraint (7) now simplifies to

$$B + (1 - I_t) D_{t-1} \geq P C_t, \quad (28)$$

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<sup>18</sup>Section 7 discusses the extension to a market of monopolistic competition in which the fact that deposits are nonhomogeneous allows intermediaries to differentiate their choices.

while first-subperiod consumption and the Lagrange multiplier  $\mu_t$  are related through (13). With constant prices, the demand for government bonds (14) implies the following relationship between their price  $Q_t^B$  and the Lagrange multiplier  $\mu_t$ :

$$Q_t^B = \beta E_t(1 + \mu_{t+1}). \quad (29)$$

Given constant prices, the demand of deposits (15) and the supply (21) simplify to

$$Q_t^D = \beta E_t \{ I_{t+1}(1 - \chi_{t+1}) + (1 - I_{t+1})(1 + \mu_{t+1}) \}, \quad (30)$$

$$Q_t^D = \beta E_t \{ I_{t+1}(1 - \chi_{t+1}) + (1 - I_{t+1}) \}. \quad (31)$$

If  $\chi_t$  were exogenous (and therefore also  $I_t$ ), equations (28)-(31) would suffice to determine the remaining equilibrium variables  $D_t$ ,  $C_t$ ,  $\mu_t$ ,  $Q_t^B$ , and  $Q_t^D$ . However, key in our model is that  $\chi_t$  is endogenous and depends on the limited backing of intermediaries captured by the nonnegative constraint on profits.

With constant prices, equation (19) simplifies to

$$\chi_{t+1} = \max \left( 0, 1 - (1 + r_{t+1}^K) \frac{N_t + Q_t^D D_t}{D_t} \right), \quad (32)$$

where the exogenous force triggering default is the realized return on capital. Equation (32) shows that there are important endogenous feedback effects between the price of deposits  $Q_t^D$ , the default rate  $\chi_{t+1}$ , and the level of deposits  $D_t$ , which we will explore later.

What is not determined is the level of intermediaries' initial net worth,  $N_t$ , opening up the existence of multiple equilibria. More precisely, there exist a continuum of equilibria indexed by  $N_t \geq 0$ . We identify two classes of equilibria: good and bad equilibria. To understand the distinction between the two, equate the demand and supply of deposits, (30) and (31), to get

$$E_t \{ (1 - I_{t+1})(1 + \mu_{t+1}) \} = E_t \{ 1 - I_{t+1} \}. \quad (33)$$

Equation (33) yields one of the key results of this paper. Perfect competition is not enough to avoid instability, in the sense of multiple equilibria. There are indeed equilibria with complete satiation of liquidity in all contingencies; the previous equation is satisfied by  $I_{t+1} = 0$  (no default by intermediaries) and  $\mu_{t+1} = 0$  (demand of liquidity is fully satiated) in all states. But there are other equilibria in which there is no default ( $I_t = 0$ ) and full satiation

( $\mu_{t+1} = 0$ ) in some states, and default ( $I_t = 1$ ) with shortage of liquidity ( $\mu_t > 0$ ) in others. Finally, there are equilibria with default and liquidity shortages in all states. The next two sections characterize all these equilibria.

Before turning to this analysis, we comment on two implications of the model. First, our model is one of coexistence between public and private liquidity. Given the government's backing privileges,  $B$  is always available for liquidity purposes. For private liquidity to play a role, we need to make assumptions that limit the availability of public liquidity. As already discussed, first-best consumption requires  $C_t = 1$ , above which any excess liquidity will be used in the second subperiod. Therefore, using equation (28), if  $B/P \geq 1$ , there is no need to have private liquidity.<sup>19</sup> In what follows, we set an upper bound on taxes,  $T < 1$ , implying  $B/P < 1$ .<sup>20</sup> Under this assumption, we show later that private money can improve upon the equilibrium with public money only. At this point, we justify the bound on  $T$  with political constraints that preclude a very high level of taxation. In Section 6 we will further explore the interaction between the specification of the monetary-fiscal policy rule and equilibrium liquidity.

Second, given the inability of public money to satisfy the demand for liquidity, one of the key questions of our paper is whether competition in private money creation can provide the efficient level of liquidity. The market structure under analysis in this baseline model is of the same form as that advocated by extreme theories of "free banking" discussed by Hayek (1976). Indeed, it is a completely unfettered system characterized by perfect competition that eliminates any operational rents from financial intermediation. Nonetheless, the existence of bad equilibria with default of intermediaries implies that competition does not necessarily deliver the first best.

## 4.1 Good equilibria

In a good equilibrium, intermediaries are always solvent and there is complete satiation of liquidity ( $\mu_t = 0$ ) in all states of nature. It follows that consumption in the first subperiod is at the efficient level  $C_t = 1$ . Prices of government bonds and deposits are equated at  $Q_t^B = Q_t^D = \beta$ , because the liquidity premium is zero (since  $\mu_t = 0$ ) and thus the rate of return required by households

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<sup>19</sup>In our context, this will correspond to the government implementing the Friedman rule by creating enough liquidity.

<sup>20</sup>This bound should be interpreted qualitatively, and it can be significantly higher in a richer model.

to hold liquidity is the same as the one on illiquid assets, equal to  $1/\beta$ . Using (28), the level of deposits is given by  $D/P = 1 - B/P = 1 - T > 0$ , complementing the supply of public money.<sup>21</sup> As a result, the consumption allocation and welfare are the same within the class of good equilibria.

What makes this an equilibrium is the solvency of intermediaries in all states. To check that gross profits are nonnegative in all states, it is sufficient to check them only in the state in which  $A_t = A_l$ :<sup>22</sup>

$$(1 + r_l^K) Q^K K^I - D \geq 0.$$

Using the previous result  $D/P = 1 - T$  and the balance sheet constraint (17), the previous inequality allows us to derive a lower bound  $\bar{N}$  such that a good equilibrium exists if and only if  $N_t \geq \bar{N}$ . The bound  $\bar{N}$  is defined by

$$\bar{N} = P(1 - T) [(1 + r_l)^{-1} - \beta], \quad (34)$$

which is positive since  $(1 + r_l) < 1/\beta$ , as shown in equation (27).

To sum up, net worth should be sufficiently high for the no-default equilibrium to exist. What happens when net worth falls below the threshold  $\bar{N}$ ? This opens the possibility of equilibria with default, explored in the next section.

## 4.2 Bad equilibria

We now analyze default equilibria characterized by insolvency (possibly partial) of financial intermediaries. There are several types of these equilibria depending on (i) the rate of default on deposits and (ii) whether default occurs only in the low state or in all states. The consumption allocation is also state contingent depending on default. We first investigate equilibria in which there is default only in the low state and then consider equilibria with the possibility of default in all states.

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<sup>21</sup>In the model, any level of deposits greater than or equal to  $1 - T$  can arise in equilibrium. That is because households and intermediaries have access to the same technology. Thus, households can either invest directly in capital or hold excess deposits and have banks investing in capital on their behalf. However, if there are intermediation costs, households are better off by holding only the minimum amount of deposits required for liquidity purposes. Our result  $D/P = 1 - T$  can thus be viewed as arising from the limiting case in which the cost of intermediation goes to zero.

<sup>22</sup>If intermediaries are solvent in the low state, they must also be solvent in the high state as well.

The feedback loop in the default equilibrium is related to the lower price of deposits  $Q_t^D$ . Because of the lower price, banks must pay a higher return on deposits. In the low state, though, banks do not have enough resources because of the realized low productivity; therefore, they default on their promises. Anticipating default, households are willing to hold deposits only if their return includes a premium for the possibility of default, or equivalently if the price of deposits is lower than in a good equilibrium. This premium implies a lower price of deposits  $Q_t^D$ , in comparison to the good equilibrium:

$$Q^D = \beta [\pi (1 - \chi_l) + (1 - \pi)] < \beta, \quad (35)$$

where the inequality follows from  $1 - \chi_l < 1$ . The price in (35) satisfies the supply of deposits (31); it also satisfies demand (30) if  $\mu_h = 0$ , that is, if the demand for liquidity is satiated in the  $h$  state. This condition is verified, because households are indifferent about the quantity of deposits to hold (as long as the expected return on deposits is equal to  $1/\beta$ ). Therefore, the amount of deposits held by households is the same as in the good equilibrium,  $D/P = 1 - B/P = 1 - T$ .

However, consumption  $C_t$  is state contingent and given by  $C_h = 1$  and  $C_l = B/P = T < 1$ . A liquidity shortage arises in the low state because transacting parties do not accept defaulted securities for liquidity purposes. These results show that the default equilibrium has some features in common with a liquidity crisis. In the good state, the consumption allocation is the same as in the good equilibrium. However, deposits bear some credit risk. When that risk materializes in the bad state, private money is in default and the economy experiences a liquidity crunch with a sudden fall in consumption  $C_t$ .

To have default in the  $l$  state, the limited liability constraint should bind. Thus, setting intermediaries' profits equal to zero in (18) and evaluating them in the  $l$  state, we obtain

$$(1 + r_l^K) (N + Q^D D) - (1 - \chi_l) D = 0. \quad (36)$$

The easiest approach to characterizing the bad equilibria is to index them by the rate of default on deposits  $\chi_l$  in the  $l$  state. Using (35), (36), and  $D/P = 1 - T$ , we can solve for the level of net worth that is consistent with a given rate of default  $\chi_l$  in the  $l$  state:

$$N = \frac{P(1 - T)}{1 + r_l^K} [(1 - \chi_l) - \beta (1 + r_l^K) (1 - \pi \chi_l)]. \quad (37)$$

Therefore, net worth is equal to  $\bar{N}$  if  $\chi_l = 0$  and is decreasing in  $\chi_l$ . In other words, the rate of default rises as net worth decreases. This weakly negative relationship between net worth and the rate of default is a key result of the baseline model with implications for regulation that we discuss in the next section.

For a given rate of default  $\chi_l > 0$ , the equilibrium values of  $Q^D$  and  $N$  are determined by (35) and (37). However, we next show that a bad equilibrium with default only in state  $l$  and not in state  $h$  can occur if  $\chi_l > \bar{\chi}_l$ , where  $\bar{\chi}_l \equiv 1 - (1 + r_l^K) / (1 + r_h^K)$ . To this end, note from (35) that as  $\chi_l$  increases,  $Q^D$  further falls, which in turn also depresses dividends in the high state; see (18) evaluated at  $t = h$ ,  $I_t = 0$ , and  $\chi_t = 0$ . A bad equilibrium with default only in the low state exists provided that the limited liability constraint is not binding in the high state, that is, using (18), if

$$(1 + r_h^K) (N + Q^D D) - D \geq 0. \quad (38)$$

and the inequality strictly holds. Plugging  $D/P = 1 - T$  and (35) into (36), we can solve for the level of  $\chi_l$  such that the left-hand side of (36) equals zero, which is given by  $\chi_l = \bar{\chi}_l$ . Note also that as  $\chi_l$  tends to  $\bar{\chi}_l$ , equations (26) and (37) imply that net worth  $N$  approaches zero.

Next, we ask what happens when  $\chi_l$  increases above  $\bar{\chi}_l$ . In this case, there is default on deposits not only in the  $l$  state but also in the  $h$  state. The price of deposits is now given by

$$Q^D = \beta [\pi (1 - \chi_l) + (1 - \pi) (1 - \chi_h)] , \quad (39)$$

where  $\chi_h$  is the rate of default on deposits in the high state. To solve for  $\chi_h$  as a function of  $\chi_l$ , we combine (36) with the binding limited liability constraint for intermediaries' profits in the high state, given by

$$(1 + r_h^K) (N + Q^D D) - (1 - \chi_h) D = 0 ,$$

and we obtain

$$\chi_h = 1 - \left( \frac{1 + r_h}{1 + r_l} \right) (1 - \chi_l) . \quad (40)$$

Thus, there are equilibria with default in all states in which  $\chi_l$  is in the range  $[\bar{\chi}_l, 1]$  and  $\chi_h$  is determined by (40). If default also happens in the high state, shortages of liquidity are widespread in the economy, with a drop in first-subperiod consumption in all states in comparison to the first best. That

is,  $C_l = C_h = B/P < 1$  in all contingencies. Nonetheless, households may decide to hold deposits just for their pecuniary return, because the expected return on deposits is  $1/\beta$ .<sup>23</sup>

To conclude the description of the bad equilibria with default in all states, we note that net worth  $N$  is zero in these equilibria since dividends are zero in both states.

### 4.3 Why are there multiple equilibria?

Define a measure of leverage as  $\Gamma \equiv N/D$ . The economy is in a good equilibrium if  $\Gamma \geq \bar{\Gamma}$  with  $\bar{\Gamma} \equiv \bar{N}/D = [(1 + r_l)^{-1} - \beta]$  as is shown in (34) whereas it is in a bad equilibrium when  $0 \leq \Gamma < \bar{\Gamma}$ .<sup>24</sup> The multiplicity of equilibria is thus equivalent to the indeterminacy of leverage of financial intermediaries. In other words, the multiplicity is related to the Modigliani-Miller theorem. This is because the liquidity premium on deposits is always zero in equilibrium; therefore, we have no violation of the assumptions of Modigliani-Miller.<sup>25</sup> In the good equilibrium, the liquidity premium on deposits is zero because the liquidity constraint (7) is never binding. In the bad equilibrium, the liquidity constraint (7) is binding if and only if deposits are defaulted upon. But, in this case, deposits do not provide liquidity services, and thus their liquidity premium is zero as well.

Despite the fact that the Modigliani-Miller theorem holds, welfare depends on leverage. If  $\Gamma$  is below  $\bar{\Gamma}$  and the low state realizes, there is a shortage of safe assets and therefore consumption,  $C_t$ , falls below the efficient level. This relationship between leverage and welfare gives rise to a role for regulation that we now address.

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<sup>23</sup>In this equilibrium, households are indifferent among any level of deposits that is feasible given their budget constraint. However, consistently with footnote 21, if we consider an economy with intermediation costs, deposits held by households are zero even in the limiting case in which intermediation costs are arbitrarily small.

<sup>24</sup>In the Appendix, we further show that if intermediaries are subject to a default cost there is an extra dimension of multiplicity to consider at a given level of leverage.

<sup>25</sup>Limited liability and the possibility of default do not violate the Modigliani-Miller theorem, because the price of deposits  $Q_t^D$  adjusts so that the expected return on deposits is always equal to  $1/\beta$ .



## 5 Regulation

In this section, we analyze two types of regulations: capital requirements and liquidity requirements. These tools are two key pillars of the Basel III regulatory framework. In several countries, including the United States, regulators are currently implementing a transition from the old Basel I and II requirements toward the new rules.

### 5.1 Capital requirements and leverage

According to our results, extreme forms of “free banking” (defined as completely unregulated forces of competition) are not desirable. Perfect competition in our model abates all rents from financial intermediation to zero until net worth is always equal to expected gross profits, which are fully distributed as dividends. But nothing pins down the equilibrium level of leverage. Therefore, to enforce the good equilibrium, regulation is needed.

In particular, to select the good equilibrium, regulators should prescribe that leverage, as identified by  $\Gamma$ , should be in the range  $\bar{\Gamma} \leq \Gamma \leq \tilde{\Gamma}$ , where  $\bar{\Gamma}$  and  $\tilde{\Gamma}$  are respectively defined by

$$\bar{\Gamma} = \left( \min_s \frac{1}{1 + r_s^K} - \beta \right), \quad (41)$$

$$\tilde{\Gamma} = \frac{\beta}{1 - \beta} \frac{A\bar{K}}{(1 - T)} - \beta. \quad (42)$$

The lower threshold (41) is stated for a generalized version of our model in which there are many states of nature (instead of just two), indexed by  $s$ . One needs to compute the worst realization of the distribution of real returns on capital across states  $s$  and compare it with the real risk-free return  $1/\beta$ . Intermediaries’ leverage has to be enough to cover the worst-case scenario. Though useful, this prescription reveals itself to be fragile in practice. Regulators need to understand the full distribution of real returns and evaluate the worst case, which is a difficult task. Mistakes in this computation could lead to lower capital requirements and open the possibility of bad equilibria, characterized by full liquidity in the high state and a liquidity crunch in the low state.

We also want to emphasize the connection between our rule in (41) and the Value at Risk (VaR). In practice, several financial institutions use the

VaR as a simple tool for risk management. The formal implication of our model is that the VaR should be zero. We leave for future research whether a richer model could provide a microfoundation for the optimality of a positive but small VaR.

We can also note that regulation should impose a level of  $\Gamma$  that is not too high and bounded above by  $\tilde{\Gamma}$ . The intermediaries' balance sheet is limited by the total amount of capital  $\bar{K}$  available in the economy. Given the deposit level of the good equilibrium,  $D = P(1 - T)$ , the maximum size of the balance sheet imposes an upper bound on net worth and therefore on our measure of leverage  $\Gamma$ . For instance, when  $\Gamma$  is set at infinity, banks cannot issue any deposits. This is clearly an inefficient outcome in an economy that lacks liquidity, since deposits can indeed provide it. The upper bound on  $\Gamma$  is in contrast with proponents of the idea that banks should be funded with 100% of equity.<sup>26</sup>

Finally, we stress that our motivation for capital requirements is different from, but complementary to, the typical need to offset the distortive effect created by deposit insurance, as in most of the literature.<sup>27</sup> In our model, there is no deposit insurance; yet, there is a clear role for capital requirements in order to preserve the liquidity value of deposits even in the low state and therefore to achieve macroeconomic efficiency. Capital requirements are a means to create privately issued safe assets and prevent a liquidity crisis.

## 5.2 Liquidity requirements

A second key pillar of the Basel III Accords is given by liquidity requirements, such as the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). There are some differences between these two requirements, but they both aim at making sure that a financial institution has enough liquid assets to undergo a stress scenario with intense liquidity outflows.

In this section, we analyze the effect of these two requirements in the context of our model, highlighting one novel insight. We note that, in some circumstances, a liquidity requirement reduces welfare. Consider the LCR applied to our model. The LCR requires financial intermediaries to hold high-quality liquid assets – in our model, government bonds  $B$  – that are greater than or equal to a fraction of deposits. Therefore, the maximum

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<sup>26</sup>See, among others, Cochrane (2014).

<sup>27</sup>See, among others, Begenau and Landvoigt (2016).

amount of deposits that can be issued in the model would be equal to a multiple of  $B$ .<sup>28</sup> As a result, intermediaries in the model will not create enough liquidity to satiate the economy if the supply of government bonds is low enough. The equilibrium will be characterized by an insufficient provision of liquidity, even if intermediaries are well capitalized and never default. This result complements the existing small but growing literature that discusses the benefits and costs of liquidity requirements.<sup>29</sup> In this literature, the costs are mainly in terms of the forgone return on the long-term technology and therefore on the supply of physical capital rather than the supply of safe assets emphasized by our model.

## 6 Policy response to a liquidity crunch

Consider now that intermediaries' net worth is not enough to enforce the good equilibrium but allows an equilibrium with default in the low state. This might be due to an imperfect regulatory environment. In this case, pseudo-safe assets are in circulation that provide liquidity services in the good state but reveal their limited backing if a bad state realizes. How can the government intervene in this situation to limit the costs of the crisis? The only possibility is to relax the upper bound on taxes, at least in the low state.

In the bad equilibrium of Section 4.2 under the assumption of a noncontingent tax level, consumption in the high state is equal to the efficient level,  $C_h = 1$ , whereas consumption in the low state depends on the real value of public liquidity,  $C_l = B/P$ , evaluated at the equilibrium price  $P$ , which is noncontingent as shown in (25). We think of a liquidity crunch as the realization of the low state when the economy is in a bad equilibrium. Thus, in a liquidity crunch, pseudo-safe assets immediately lose their liquidity values

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<sup>28</sup>For instance, if the LCR imposes that financial intermediaries must invest 10% of deposits in government bonds, the total amount of deposits in the economy cannot exceed  $10 \times B$ . The LCR is similar to the reserve requirement that central banks often impose on banks, with the difference that the LCR allows banks to also invest in some interest-bearing liquid assets other than central bank reserves.

<sup>29</sup>See, for instance, Diamond and Kashyap (2015). Because of the simplicity of our model, liquidity requirements give rise only to possible distortions (a reduction of liquidity in some circumstances), but they do not provide any benefits. Thus, our framework can analyze the welfare costs arising from liquidity requirements but cannot analyze the welfare benefits that these requirements may generate.

and consumption drastically falls. The cause is a shortage of safe assets. To counteract the liquidity crisis, we discuss two alternative policies. The first policy is to change the monetary-fiscal policy rule in order to affect the equilibrium price level  $P$ . The second one is a change in the supply of public money  $B$ . We discuss the pros and cons of each policy.

Consider first the adjustment of the price level. To reach efficiency in the liquidity crisis, the price level should move to  $P_l = B$  in the crisis state from a price  $P_h$  in the no-crisis state. This adjustment in prices can be achieved by an appropriate state-contingent specification of the monetary-fiscal policy rule. Consider the simple case in which the path of reserves is kept constant,  $B_t = B$ , and real taxes in the high state are such that  $B/P_h < 1$ , in order to maintain private money creation that is essential in the high state. Given these assumptions, the objective is to find the level of taxes in the low state that implements the desired equilibrium. Since we are seeking an equilibrium in which there is full satiation of liquidity in both states, it follows that  $\mu_h = \mu_l = 0$  and  $Q_t^B = Q_t^f$  at all times. Using these results, we can write the intertemporal budget constraint of the government (23) under the two states as

$$\begin{aligned}\frac{B}{P_h} &= T_h + \frac{\beta}{1 - \beta} [(1 - \pi)T_h + \pi T_l] \\ 1 = \frac{B}{P_l} &= T_l + \frac{\beta}{1 - \beta} [(1 - \pi)T_h + \pi T_l].\end{aligned}$$

It is clear by comparing the two equations that  $T_l > T_h$  since  $B/P_h < 1$ . But why should taxes increase during the liquidity crisis? Corresponding to the shortage of safe assets in the first subperiod, there is a shortage of demand for goods. To increase demand, the purchasing power of the remaining safe assets should increase, lowering the price level. Higher lump-sum taxes reduce the overall wealth of households and decrease the overall demand for consumption goods (first and second subperiod). For a given supply of goods, the price level should fall to equilibrate the goods market. The drop in the price level increases the real value of  $B$ , achieving efficiency. Therefore, the liquidity crisis can be exactly offset by the fall in prices. Note, though, that the only friction in our model is the liquidity constraint. If we posit other frictions such as price rigidity, this will make the adjustment of prices sluggish and unable to completely counteract the liquidity crisis. If instead there are wage rigidities and labor market frictions, the decline in prices and the subsequent

rise in real wages can depress employment. Considering all these arguments, a fall in the price level is not at all desirable during a liquidity crisis.

The other option available to the government is to counteract, at least to some extent, the liquidity crunch through a temporary rise in government debt while keeping constant the price level.<sup>30</sup> Literally, in our model, if a crisis hits unexpectedly in the first subperiod, the central bank has no time to increase reserves to meet demand for that period, and thus liquidity is not sufficient. But, if the crisis is expected to persist, the central bank can raise first-subperiod consumption and reach the efficient level by increasing reserves in such a way that  $B_t/P = 1$ , keeping constant the price level at  $P$ . However, even this intervention requires a backing through higher real taxes.

In either option – temporarily lowering the price level or raising reserves – the government must raise lump-sum real taxes to meet the increase in real public liabilities. The reason lies in the very origin of the liquidity crisis that starts from an insufficient backing of private intermediaries and therefore of private money. To counteract it, the government should supply more of its safe securities. A higher supply of safe government debt requires stronger backing through higher taxes. In the end, the way out of the crisis is to substitute the insufficient backing of private money with more backing of public money. If government intervention is not immediately available or infeasible, there is little hope of averting the liquidity crisis.

## 7 Monopolistic competition

We now discuss how equilibrium changes when the supply side of the deposit market deviates from the assumption of perfect competition. We amend the model to analyze a market in which financial intermediaries are monopolistically competitive. The main result of this section is that only one equilibrium arises, displaying default in the low state. We argue that this outcome reinforces the conclusion we reached under perfect competition, pointing toward adequate capital requirements in order to improve efficiency in the supply of liquidity and thus increase welfare.

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<sup>30</sup>Benigno and Nisticò (2013) reach a similar conclusion, in a different model, when evaluating optimal monetary policy following an exogenous liquidity shock. Reis (2015) also emphasizes the importance of the central bank’s reserves given their safe asset properties during periods of financial disruption. A similar policy is analyzed by Robatto (2016), too, in the context of bank runs and flight to liquidity.

Let us assume that there are  $J$  wholesale financial intermediaries, where each of the  $J$  intermediaries has to pay a fixed real cost  $\Phi$  to operate. A generic type  $j$  supplies  $D_t(j)$  at the price  $Q_t^D(j)$ . The model is also enriched by a retail financial intermediary that invests in a portfolio of all deposit securities by issuing a structured product  $D_t$ , which is the following combination of the simple deposit securities:

$$D_t = \left[ \left( \frac{1}{J} \right)^{\frac{1}{\theta}} \sum_{j=1}^J (D_t(j))^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (43)$$

where  $\theta$ , with  $\theta > 1$ , captures the degree of substitution of the securities  $D_t(j)$  in the structured product. At time  $t$ , the balance sheet of the retail intermediary is

$$Q_t^D D_t = \sum_{j=1}^J Q_t(j) D_t(j),$$

where  $Q_t^D$  is the price of the structured product given by

$$Q_t^D = \left[ \frac{1}{J} \sum_{j=1}^J (Q_t^D(j))^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (44)$$

We assume that households do not have access to the wholesale market and can only invest in the structured security  $D_t$  issued by the retail intermediary at the price  $Q_t^D$ . At time  $t+1$ , the payoff of the retail intermediary's portfolio is fully transferred to the household according to an aggregate default rate  $\chi_{t+1}$  defined by

$$(1 - \chi_{t+1}) D_t = \sum_{j=1}^J (1 - \chi_{t+1}(j)) D_t(j). \quad (45)$$

Given the framework just outlined, nothing changes in the optimization problem of the household, where  $Q_t^D$  and  $D_t$  now have to be understood as the price and quantity of the structured product. As before, if  $D_t$  is partially seized – and this is the case if at least one deposit  $D_t(j)$  is not fully reimbursed – the security  $D_t$  is not accepted in a goods transaction. This is reminiscent of the 2008 financial crisis, where structured securities lost their liquidity value entirely, even if only a few of the embedded primitive securities were defaulted upon.

The supply side changes because wholesale intermediaries face a demand for their deposit securities of the form

$$D_t(j) = \frac{1}{J} \left( \frac{Q_t^D(j)}{Q_t^D} \right)^{-\theta} D_t \quad (46)$$

as a result of how different deposit securities are packaged into the structured product given the cost minimization of the retail intermediary. In a market of monopolistically competitive suppliers, wholesale intermediaries can choose the price of their security  $Q_t^D(j)$  internalizing demand (46) and taking aggregate variables  $Q_t^D$ ,  $D_t$  and  $\chi_{t+1}$  as given.

In what follows, we focus on the case in which the price level is constant at  $P_t = P$ , consistent with the previous analysis. Moreover, we concentrate on symmetric equilibria in which all intermediaries make the same choices.

The budget constraint of wholesale intermediaries (17) is replaced by

$$Q_t^K K_t^I(j) + P\Phi = Q_t^D(j) D_t(j) + N_t(j),$$

taking into account the fixed cost of entry into the market. Equation (18) that describes intermediary  $j$ 's gross profits is given by

$$\begin{aligned} \Pi_{t+1}(j) = & (1 + r_{t+1}^K) Q_t^K K_t^I(j) \\ & - (1 - I_{t+1}(j)) D_t(j) - I_{t+1}(j) (1 - \chi_{t+1}(j)) D_t(j), \end{aligned} \quad (47)$$

where we have also used the fact that the nominal return on capital is equal to the real return,  $i_{t+1}^K = r_{t+1}^K$ , because prices are constant.

The discounted value of profits, (20), is now

$$\begin{aligned} E_t \{ \beta \Pi_{t+1}(j) \} = & E_t \{ \beta \Pi_{t+1}^D(j) \} - P\Phi + D_t(j) Q_t^D(j) + \\ & - \beta E_t \{ 1 - I_{t+1}(j) + I_{t+1}(j) (1 - \chi_{t+1}(j)) \} D_t(j). \end{aligned} \quad (48)$$

A generic monopolist  $j$  internalizes demand (46) but also the fact that its own rate of default  $\chi_{t+1}(j)$  depends on its level of net worth  $N_t(j)$ , its deposits  $D_t(j)$ , and the deposit price  $Q_t^D(j)$ . To understand the link between  $\chi_{t+1}(j)$  and the variables  $N_t(j)$ ,  $D_t(j)$ , and  $Q_t^D(j)$ , there are three possible cases to consider. In the first, the default rate is zero,  $\chi_{t+1}(j) = 0$ , in both the high and low state. In the second, the intermediary  $j$  defaults only in the low state,  $\chi_h(j) = 0$  and  $\chi_l(j) > 0$ . In the third case, there is default in both states,  $\chi_h(j) > 0$  and  $\chi_l(j) > 0$ . We show in the Appendix that only

the second case arises in equilibrium. Here we provide an explanation for why the other two cases are not equilibria.

The case of no default in all states is not an equilibrium because a generic intermediary  $j$  has an incentive to deviate from it by lowering its net worth  $N_t(j)$  and the price charged to depositors  $Q_t^D(j)$ , moving to the region in which it defaults only in the low state. A lower net worth  $N_t(j)$  reduces the backing of the intermediary's deposits, that is, worsens the "quality" of deposits. The intermediary compensates for this lower quality by reducing the price  $Q_t^D(j)$ . More important, lowering the quality and the price allows the intermediary to collect some rents because of its monopolistic power exploiting demand (46). Given that the securities it supplies are substitutes with respect to those of competitors, the intermediary can increase its revenues by lowering the price and raising the demand of its products. Moreover, the lower net worth of  $j$  reduces not only the backing of its own deposits but also the backing of the structured product. The latter negatively affects the demand of all intermediaries. Therefore, intermediary  $j$  benefits from the gains of reducing  $N_t(j)$ , without bearing all the costs.

The prior reasoning, however, does not extend to the case of default in all states. If this happens, the limited liability constraint is always binding and thus profits are always zero. Therefore, intermediaries' rents must be zero as well. Moreover, deposits lose their liquidity value entirely, and households are willing to hold them only for their pecuniary return. But the return that intermediaries offer is lower than the expected return  $1/\beta$  required by households, because intermediaries have to pay the fixed cost  $\Phi$ . As a result, at the supplied price there is no demand, which implies that the case of default in all states is not an equilibrium as well.

In what follows, we focus exclusively on the case in which intermediaries default only in the low state, which is the only equilibrium of the model. A generic monopolist  $j$  chooses the amount and price of deposits to supply,  $D_t(j)$  and  $Q_t(j)$ , respectively, and the level of net worth  $N_t(j)$ . The objective is to maximize expected rents  $\mathcal{R}_t(j)$ , defined as the difference between expected profits and expected dividends:

$$\mathcal{R}_t(j) \equiv \beta E_t \{ \Pi_{t+1}(j) - \Pi_{t+1}^D(j) \} , \quad (49)$$

taking into account the demand schedule (46) and the fact that the limited liability constraint is binding in the low state. The latter constraint implies



that profits (47) evaluated in the low state must be equal to zero:

$$(1 + r_l^K) (Q_t^D(j) D_t(j) + N_t(j) - P\Phi) = (1 - \chi_l(j)) D_t(j). \quad (50)$$

The previous equation implicitly defines  $\chi_l(j)$  as a function of  $N_t(j)$ ,  $D_t(j)$ , and  $Q_t^D(j)$ .

Combining (48) and (50), expected rents (49) are

$$\begin{aligned} \mathcal{R}_t(j) = & \beta(1 - \pi)D_t(j) [Q_t^D(j) (1 + r_h^K) - 1] \\ & - \beta\pi(1 + r_l^K)N_t(j) - \beta(1 - \pi)(1 + r_h^K)P\Phi. \end{aligned}$$

Maximizing the previous expression by choosing  $Q_t^D(j)$  implies the optimal deposit price  $Q_t^D(j) = (1 + \tau)/(1 + r_h^K)$ , where  $1 + \tau \equiv \theta/(\theta - 1) - 1 \geq 1$  is the monopoly markup.<sup>31</sup> Net worth is going to be set at the lowest possible value,  $N_t(j) = 0$ , because  $\mathcal{R}_t(j)$  is decreasing with respect to  $N_t(j)$ . Evaluating expected rents at these optimal choices, we obtain

$$\tilde{\mathcal{R}}_t(j) = \beta(1 - \pi)\tau \left( \frac{1 + \tau}{1 + r_h^K} \right)^{-\theta} \frac{D_t(Q_t^D)^\theta}{J} - \beta(1 - \pi)(1 + r_h^K)P\Phi. \quad (51)$$

Using  $\tilde{\mathcal{R}}_t(j)$ , it is possible to verify that profits (47) evaluated in the high state are nonnegative as long as rents  $\tilde{\mathcal{R}}_t(j)$  are nonnegative as well.

We can now move to the characterization of the equilibrium of the aggregate variables. In a symmetric equilibrium (44) implies that the price of the structured product is the same as that of the deposits of a generic intermediary  $j$ :

$$Q_t^D = \frac{1 + \tau}{1 + r_h^K}. \quad (52)$$

The no-entry condition requires that rents in (51) are equal to zero; therefore, the number of intermediary  $J$  in the market solves the following condition:

$$\tau \frac{D}{J} = (1 + r_h^K)P\Phi.$$

The number  $J$  increases in the monopoly power  $\tau$  and decreases with a higher entry cost  $\Phi$  or a higher supply of public liquidity.<sup>32</sup>

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<sup>31</sup>We have also used the equality  $\beta(1 - \pi)(1 + r_h^K) + \beta\pi(1 + r_l^K) = 1$ , which follows from (27) combined with the definition of  $A$ .

<sup>32</sup>Recall from (25) that  $P$  is increasing in the supply of public liquidity  $B$ .

We can now combine the previous results with those determining households' demand for the structured product  $D_t$ . Using (15), the assumption of constant prices and the fact that default happens only in the low state, households' demand for the structured product is flat at the price

$$Q_t^D = \beta \{(1 - \pi)(1 + \mu_h) + \pi(1 - \chi_l)\}. \quad (53)$$

The equilibrium value of default  $\chi_l(j)$  is the same as the one derived in the case of perfect competition and zero net worth in the low state, that is,  $\chi_l(j) = \bar{\chi}_l$ , where  $\bar{\chi}_l \equiv 1 - (1 + r_l^K) / (1 + r_h^K)$ . This result can be understood intuitively from the fact that intermediaries earn zero rents in equilibrium, because the markup earned on deposits is used to pay the fixed cost  $P\Phi$ . Accordingly, the resources left to depositors, in particular in the low state, are the same as in the case of perfect competition.<sup>33</sup>

Combining the demand of the structured product, (53), with the supply, (52), and using the result  $\chi_l(j) = \bar{\chi}_l$ , we obtain

$$\mu_h = \frac{\tau}{\beta(1 - \pi)(1 + r_h^K)} > 0.$$

Thus, the liquidity constraint is binding in the high state. Combining this result with the fact that  $C_h = 1/(1 + \mu_h)$ , we can get the equilibrium real value of the structured product:

$$\frac{D}{P} = \frac{\beta(1 - \pi)(1 + r_h^K)}{\beta(1 - \pi)(1 + r_h^K) + \tau} - \frac{B}{P} < 1 - \frac{B}{P}. \quad (54)$$

Liquidity is lower compared with that in the case of perfect competition. This difference is driven by two effects. First, the monopolistic power of intermediaries creates a wedge that reduces the supply of deposits; this is a standard result in models with monopolistic competition. The second effect is instead a novel one. The race to the bottom, which leads to  $N_t(j) = 0$ , further reduces the equilibrium value of deposits and thus of liquidity. Indeed, intermediaries pay the fixed cost  $\Phi$  to supply deposits that, in the low state, do not produce liquidity services while just providing a pecuniary return. However, in the low state, households can obtain a pecuniary return by

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<sup>33</sup>Formally, the result can be derived by solving for  $\chi_l(j)$  in (50), after substituting in the optimal choices of intermediaries  $N_t(j) = 0$  and  $Q_t^D(j) = (1 + \tau) / (1 + r_h^K)$ , and using the definition of the structured product in (43) and of its price in (44), the demand for deposits (46), and the fact that intermediaries' rents (51) must be zero in equilibrium.

investing directly in capital without paying any fixed cost. As  $\pi$  increases, the second effect becomes more important and reduces the level of deposits  $D/P$ .

To sum up, we have characterized the existence of only one bad equilibrium with two main features. First, the monopolistic power of intermediaries reduces the supply of liquidity even in the high state. Second, the race to the bottom further reduces liquidity both in the high state (through lower  $D/P$ ) and in the low state (through the default of deposits). Next, we analyze how regulation on the level of intermediaries' capital can fix the second problem but not the first one.

## 7.1 Monopolistic competition and regulation

The existence of only one equilibrium, and a bad one, reinforces the conclusion we reached under perfect competition that regulation is necessary in order for the private sector to create safe assets rather than pseudo-safe assets. In the framework of this section, regulators should require intermediaries to hold a level of capital above a threshold  $\bar{N}^m$  that guarantees solvency in all states. This requirement offsets the distortion that creates the race to the bottom, therefore implying a higher price of deposits (i.e., a lower return on deposits) and, more important, a higher supply of liquidity in both states.

In the Appendix, we show that the equilibrium value of the structured product in the economy subject to regulation is

$$\frac{D}{P} = \frac{1}{(1 + \tau)} - \frac{B}{P}. \quad (55)$$

This level is higher than what the unregulated market can attain; see (54). Whereas the regulator can offset the distortion arising from the race to the bottom, it cannot, however, overcome the monopoly distortion that in the end constrains the supply of liquidity below the level of perfect competition.

The threshold  $\bar{N}^m$  is equal to

$$\bar{N}^m = D^s \left[ (1 + r_l)^{-1} - \beta \right],$$

where  $D^s \equiv D/J$  is the equilibrium value of deposits supplied by each intermediary  $j$ . The capital requirement can be expressed in terms of the leverage measure  $\Gamma = N/D^s$ , and it corresponds to the same lower bound  $\bar{\Gamma}$  that we found in the perfect competition case; see (41). Independently of the

kind of market competition, regulators need to make the same computation regarding the degree of leverage to enforce the good equilibrium.

## 8 Conclusion

We have presented a framework for studying equilibria with private money creation in a model in which both public and private liquidity play a role for transactions.

If the availability of public liquidity is limited because of a restriction on real taxes, there is room for private money creation in normal times. Competition should be supplemented by regulation on intermediaries' capital to enable the economy to reach efficiency in all contingencies. However, under insufficient backing of private money, liquidity crises can occur, featuring a sudden drop in consumption. In this case, we also argue that the same limit on real taxes might give rise to limitations to policy action.

We are aware that we have omitted some important real-world features, but we consider our model as a first step in addressing the important topic of private and public liquidity determination, a debate that has been at the center of economists' thoughts for hundreds of years but which has received little attention in modern economic analysis.

We see at least two possible extensions of our framework. First, we have limited the focus of our analysis only on the consequences that financial disruption has on the liquidity market. There can be, however, important effects on the supply of credit with interesting spillover between credit and money markets that could be explored in more complicated frameworks. Second, we have analyzed stylized models of market interaction such as perfect competition and monopolistic competition. An interesting result under monopolistic competition is that intermediaries might have an incentive to increase their profits relative to other competitors by lowering the price of deposits and therefore increasing the rate of default. But this incentive goes against that of the buyers of deposits, who would prefer instead to have safe assets to satisfy their liquidity benefits. This analysis could then be extended to markets with informational asymmetries between depositors and intermediaries or to other market structures. We leave these extensions for future work.

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## A Appendix

This Appendix has three parts. The first two parts are extensions of the baseline model with perfect competition (one extension adds default costs in the baseline framework; the other analyzes the possibility that brokers can make deposits liquid in all states by collecting a transaction fee for their services). The third part derives some results of the economy with monopolistic competition.

### A.1 Default costs

We discuss how the analysis of the baseline model changes when we consider default costs. This additional feature exacerbates the problems of multiplicity of equilibria along a new dimension.

In the baseline model, intermediaries issue noncontingent deposit liabilities that can be used for transactions in the first subperiod. In the event of default on such securities, the only consequence is that deposits lose their transaction value. In this section, we extend the model by introducing a fixed real cost of default  $c$ , independent of the size of the intermediary's balance sheet, that must be paid using the value of assets before repaying depositors. This cost captures the expenses associated with the bankruptcy process, or a more general unmodeled loss of value associated with default.<sup>34</sup>

A key implication of this extension is that we obtain a multiplicity of equilibria for a given level of net worth  $N$ , for some level of  $N$ .

We now characterize all equilibria with default costs. Consider a financial intermediary born at time  $t$ . Profits at time  $t + 1$  are now

$$\Pi_{t+1} = (1 + i_{t+1}^K)PK_t^H - (1 - I_{t+1})D_t - I_{t+1}(1 - \chi_{t+1})D_t - I_{t+1}Pc,$$

in which the cost  $c$  is incurred only when default happens. Throughout the analysis, we are still making the assumptions that imply a constant price level  $P$ .

The cost of default has important consequences for equilibria. First consider the supply decisions of intermediaries. Expected profits are

$$E_t \{ \beta \Pi_{t+1} \} = N_t + Q_t^D D_t - \beta E_t \{ (1 - I_{t+1}) + I_{t+1}(1 - \chi_{t+1}) \} D_t - \beta E_t \{ I_{t+1} Pc \},$$

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<sup>34</sup>For instance, Veronesi and Zingales (2010) use data from the 2008 financial crisis and estimate that bankruptcy of a financial intermediary would have destroyed about 22% of enterprise value.



which are lowered by default costs. Perfect competition is still assumed, reducing the rents of financial intermediation to zero. The implied supply schedule is of the form

$$Q_t^D = \beta E_t \{ (1 - I_{t+1}) + I_{t+1} (1 - \chi_{t+1}) \} + \beta \frac{Pc}{D_t} E_t \{ I_{t+1} \},$$

showing a negative relationship between the price and the level of deposits. The higher the supply of deposits, the lower the impact of the fixed bankruptcy cost on intermediaries' balance sheets, and the lower the deposit price.

Demand for deposits is unchanged. Supply and demand now meet at

$$E_t \{ (1 + \mu_{t+1})(1 - I_{t+1}) \} = E_t \{ 1 - I_{t+1} \} + \frac{Pc}{D_t} E_t \{ I_{t+1} \}. \quad (\text{A.1})$$

By inspecting the previous equation, it is easy to see that again in the good equilibrium with no default ( $I_{t+1} = 0$  in all states), liquidity is supplied as needed to satiate the consumer,  $\mu_{t+1} = 0$  in all states. It follows that the conditions found before for the lower bound on net worth do not change. The intuition is simple: in the good equilibrium, the cost of default is never suffered, and this is why it does not change the conditions for its existence.

Bad equilibria are now different since intermediaries supply deposits at a higher price to compensate for the default cost. In our simple two-state model, equation (A.1) simplifies to

$$(1 - \pi)(1 + \mu_h) = (1 - \pi) + \frac{Pc}{D} \pi \quad (\text{A.2})$$

when we consider that default can happen only in the low state. Since  $c$  is positive,  $\mu_h$  is also positive. This is the first important difference with respect to the baseline model. There is some shortage of liquidity even in the high state, as opposed to what happens in the baseline model where instead full liquidity was available in that state, even in the bad equilibrium. Therefore, moving from a good equilibrium to a default equilibrium now creates a drop in the level of deposits.

To evaluate the equilibrium level of deposits, note again that  $C_h = d + b = 1/(1 + \mu_h)$ , having defined real variables with lowercase letters,  $d$  and  $b$  for deposit and central bank reserves, respectively. We can substitute  $C_h$  into (A.2) to find that  $d$  is the nonnegative root of a second-order polynomial  $P(d)$  of the form

$$P(d) = d^2 + (b + c\pi_R - 1)d + bc\pi_R$$

in which we have defined  $\pi_R$  as the ratio of the probability of the two states,  $\pi_R \equiv \pi/(1-\pi)$ . Moreover,  $d$  should be in the interval  $[0, 1-b]$ . The study of the roots can be greatly simplified by looking at how they vary with the level of public liquidity available. When  $b$  goes to zero, there are two solutions:  $d = 0$  and  $d = 1 - c\pi_R$ . As  $b$  rises, the smaller root increases while the larger decreases.

This represents another important difference with respect to the baseline model. In the default equilibria, there can be multiple equilibrium levels of deposits all implying some shortage in the high state. However, this multiplicity can easily be reduced. Note first that our economy does not have a store of value, such as currency, which can also provide liquidity services. Assume now that the government can still issue interest-bearing reserves and at the same time can also supply currency. Following previous discussion, both securities always provide liquidity services since they are fully backed. However, demand for currency is zero as long as  $Q$  is less than one since currency is dominated in return by reserves, whereas demand for reserves is zero if  $Q$  exceeds one. Therefore, the value of one is an upper bound for  $Q$ , implying using (29) that  $\mu_h$  is also appropriately bounded.<sup>35</sup> This allows us to disregard the lower root of the polynomial  $P(d)$  because it implies too high a value of  $\mu_h$  in equation (A.2).

In what follows, we restrict attention to the higher root of the polynomial. Still, we find interesting departures from the baseline model when we look at the relationship between net worth and the default rate. Consider the zero-profit condition in the low state of the default equilibria:

$$\begin{aligned} N = & \frac{D}{1+r_l^K} [(1-\chi_l) - \beta(1+r_l^K)(1-\pi\chi_l)] \\ & + \frac{Pc}{1+r_l^K} [(1-\beta(1+r_l^K)\pi)]. \end{aligned}$$

There is now an additional term (on the second line) arising from the positive default cost  $c$ . We must examine whether this level of net worth can be higher than the threshold required to enforce the good equilibria, thus breaking the monotone relationship between net worth and the default rate found in the baseline model. Consider first the case in which  $b$  is close to zero and

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<sup>35</sup>Using (29) and noting that  $C_l = b = 1/(1+\mu_l)$ , the upper bound on  $1+\mu_h$  can be expressed as  $1+\mu_h \leq 1/[\beta(1-\pi)] - \pi_R/b$ . Finally, using (A.2) a lower bound on  $d$  can be found.

equilibrium deposits are equal to  $d \simeq 1 - c\pi_R$ . In the limit  $\chi_l \rightarrow 0$ , the previous equation implies that

$$\tilde{N} = \bar{N} + \frac{Pc}{1 + r_l^K} \left[ \frac{1 - 2\pi}{1 - \pi} + \frac{\pi^2}{1 - \pi} \beta(1 + r_l^K) \right].$$

The threshold  $\tilde{N}$  now exceeds  $\bar{N}$  for several parametrizations (for example, just set  $\pi = 1/2$ ). Critically, default costs can now produce multiple equilibria for the same level of net worth. Indeed, in the previous example, the good equilibrium with no default coexists with equilibria characterized by partial default when net worth is in the range  $[\bar{N}, \tilde{N}]$ . Shifts in confidence that drive expectations to include the possibility of default can be self-fulfilling. The mechanism can be understood as follows. Consider for simplicity a very small probability of realizing the low state. Households expect default in the low state. This expectation feeds into higher borrowing costs to the point at which the current level of net worth is not enough to ensure solvency. Indeed, if the intermediary defaults, it has to pay the cost  $c$ , which is why current net worth is insufficient to cover full reimbursement of deposits. This also explains why partial default is also an equilibrium for the same level of net worth as in the good equilibrium.

Ultimately, capital requirements should require net worth to be greater than the threshold  $\tilde{N}$  to ensure that intermediaries are always solvent. This supports a stricter macro-prudential requirement than that of the baseline model.

To complete the characterization of all the default equilibria, consider that as net worth further decreases, the default rate  $\chi_l$  rises. Zero net worth also triggers default in the high state if combined with a higher level of default in the low state,  $\chi_l \in [\tilde{\chi}_l, 1]$ , where  $\tilde{\chi}_l = 1 - (1 + r_l)/(1 + r_h) + c/d$ . The default rate in the high state is

$$\chi_h = 1 - \left( \frac{1 + r_h^K}{1 + r_l^K} \right) \left[ (1 - \chi_l) + \frac{c}{d} \right].$$

As in the baseline model, when net worth is zero, there is a shortage of private liquidity and deposits can be held only for their pecuniary return.

## A.2 Insurance through brokers

A key assumption of our baseline model is that deposits provide liquidity benefits only in states of nature in which they are not defaulted on (even

partially). We now relax this restriction by assuming the existence of other financial intermediaries – brokers – who can exchange deposits at their fair value before default happens. The security issued by brokers is free of risk and therefore can be used by the bearer to purchase consumption goods in the first subperiod. This additional layer of financial intermediation is inspired by the banking history of the nineteenth century (see Gorton and Mullineaux, 1987). At that time, banks freely issued their notes, which were made liquid in a secondary market by brokers trading them in exchange for specie. Brokers had incentives to monitor the quality of the assets backing bank notes, and the quote in the secondary market revealed that information. The exchange of notes for specie made them indirectly liquid since specie were accepted as a medium of exchange. Brokers were then able to redeem the notes at the issuing bank, making profits or incurring losses.<sup>36</sup>

The timing is as follows. At the beginning of the first subperiod of period  $t$ , after deposits have been issued and before default is realized, brokers propose an insurance contract to households under which they supply a security of value  $1 - E_{t-1}\chi_t$  (the fair value of the deposit at that point) in return for a premium  $f_t$ . The security issued by the broker is free of risk and can be used by households to purchase goods in the first subperiod. Once default happens, brokers are able to recover the value of deposits from the financial intermediary.<sup>37</sup> Brokers' profits are given by

$$\Pi_t^B = ((1 - I_t) + I_t(1 - \chi_t))D_{t-1} - (1 - E_{t-1}\chi_t - f_t)D_{t-1},$$

where both the proportional fee  $f_t$  and the brokers' profits are rebated to households each period. Brokers can make positive expected profits by charging a positive fee  $f_t$ . However, the key assumption is that the security they issue is free of any risk and therefore liquid. This is possible only if they are always solvent. We still assume limited backing of any financial intermediary – and therefore of brokers – which translates into a nonnegative profit

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<sup>36</sup>There is an important distinction between bank notes and demand deposits since the latter, unlike the former, is a claim both on a bank and on an agent's account at that bank. This distinction is not captured in our model, and therefore bank notes correspond to our definition of deposits. Moreover, historically, bank notes are barely liquid without a secondary market. Nevertheless, in the model that follows, we will assume that deposits (or bank notes) have the same liquidity properties as in the baseline model, which, as we will show, can be enhanced by the action of brokers in default states.

<sup>37</sup>Recall that a key assumption made in our baseline model was that depositors could recover the seized value of deposits only in the second subperiod. Here brokers are able to circumvent this restriction and recover the realized value in the first subperiod.

requirement on  $\Pi_t^B$ . Perfect competition in supplying riskless securities ensures that the profits of brokers in the low state are zero.<sup>38</sup> It follows that the fee is determined by  $f = \chi_t - E_{t-1}\chi_t$ .

Depositors should find it convenient to exchange deposits with the security supplied by brokers. For this to happen, the gain in expected utility in the first subperiod should be enough to compensate for the fee. This requirement can be written formally as

$$E_{t-1} \ln \left( \frac{B}{P} + (1 - I_t) \frac{D}{P} \right) \leq \ln \left( \frac{B}{P} + (1 - E_{t-1}\chi_t - f) \frac{D_{t-1}}{P} \right), \quad (\text{A.3})$$

where the left-hand side captures the expected utility from first-subperiod consumption, when no insurance is available, evaluated at the optimal level of deposits,  $D/P = 1 - B/P$ , derived in Section 4.<sup>39</sup> The right-hand side measures the utility under insurance, taking into consideration the possibility of a different optimal level of deposits. Next, we analyze the optimal level of deposits, then come back to evaluate the previous inequality.

Consider first how insurance changes the liquidity constraint of households since all deposits are now exchanged at their fair value through brokers after paying the fee  $f_t$ :

$$B + D_{t-1}(1 - E_{t-1}\chi_t - f_t) \geq PC_t. \quad (\text{A.4})$$

Given the prior liquidity constraint, the household's optimization problem implies a flat demand for deposits at the price

$$Q_t^J = \beta (1 - E_t\chi_{t+1} - f_t) (1 + E_t\mu_{t+1}),$$

whereas supply remains unchanged at (31). They now meet at

$$(1 - E_t\chi_{t+1} - f_t) (1 + E_t\mu_{t+1}) = 1 - E_t\chi_{t+1}, \quad (\text{A.5})$$

implying that the Lagrange multiplier  $\mu_{t+1}$  is no longer state contingent and is now given by<sup>40</sup>

$$\mu = E_t\mu_{t+1} = \frac{f}{(1 - E\chi - f)}.$$

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<sup>38</sup>Perfect competition cannot reduce all profits to zero. Otherwise, in the absence of backing, securities issued by brokers will not be free of risk.

<sup>39</sup>We are still assuming constant  $B$  and  $P$ .

<sup>40</sup>In what follows, we consider stationary equilibria and drop the time index.

The Lagrange multiplier  $\mu$  is zero only if  $f = 0$ , but since  $f = \chi_l - E\chi$ , this is possible only when there is no default, as in the good equilibrium. This result is not surprising. When deposits are free of risk, there is always consumption insurance and therefore no role for brokers unless they operate at a zero fee. The analysis of the good equilibrium would exactly follow the no-insurance case. However, when  $f$  is positive, which happens in default equilibria, the multiplier  $\mu$  is an increasing function of the expected rate of default. With this result in hand, we can derive the optimal level of consumption, which is also not state contingent, given by

$$C = 1 - \frac{f}{(1 - E\chi)} = \frac{1 - \chi_l}{(1 - E\chi)}. \quad (\text{A.6})$$

Insurance clearly works since consumption will be perfectly equalized across states even in the case of default on deposits. However, the somewhat interesting result is that insurance does not reach efficiency unless  $\chi_l = 0$ , which happens only in the no-default equilibria. Otherwise, consumption falls as the default rate rises. This is a new result with respect to the case of no insurance in which consumption dropped only in the low state.

Although the brokers' securities are liquid, the inherent risky characteristics of the original deposit securities are transferred into a positive fee – which is why brokers' securities are liquid – implying an inefficiently low demand of private liquidity given by

$$\frac{D}{P} = \frac{1}{1 - E\chi} - \frac{B/P}{(1 - \chi_l)}. \quad (\text{A.7})$$

Equation (A.7) is obtained using (A.4) and (A.6), noting that  $(1 - E\chi - f) = 1 - \chi_l$ . Equilibrium deposits can even be an increasing function of the default rate, at least for small  $\chi$  and low values of  $B/P$ .<sup>41</sup> As default rises, households are willing to hold more deposits since higher holdings will partially offset the haircut of brokers and provide a buffer of liquidity. However, the increase in deposits does not prevent consumption to fall with the default rate, as discussed before.

We now turn to analyzing the solvency of financial intermediaries. As already noted, the conditions for the existence of the good equilibrium are the same as in the baseline model: net worth should be greater than  $\bar{N}$ , with  $\bar{N}$  given by (34). Consider now equilibria with default only in the low state.

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<sup>41</sup>For deposits to be positive, it is required that  $(1 - \chi_l) - (1 - E\chi)B/P$  is positive.

Again, the critical condition on net worth as a function of other variables is still as in the baseline model:

$$N_t = \frac{D}{1 + r_l^K} [(1 - \chi_l) - \beta(1 + r_l^K)(1 - \pi\chi_l)] ,$$

where now deposits vary with the default rate, as shown in (A.7) setting  $\chi_h = 0$ . Two contrasting channels influence the relationship between net worth and the default rate. On the one hand,  $D$  can rise with the default rate; on the other, the term in the square brackets decreases. This second channel dominates.<sup>42</sup> Therefore, there is a monotone nonincreasing relationship between the level of net worth and the default rate. Further increases in  $\chi_l$  trigger default in the high state as well, once  $\chi_l$  reaches the level  $\bar{\chi}_l = 1 - (1 + r_l)/(1 + r_h)$ . Above  $\bar{\chi}_l$ , the relationship between the two default rates is again given by (40). As in the baseline model, net worth is zero when default also happens in the high state. At this level of net worth, there are multiple default equilibria. However, the important difference with respect to the baseline case is that these equilibria are no longer associated with a drop in consumption in the bad state compared with the good state. It is true, however, that consumption drops with higher default rates, but in a smooth way.<sup>43</sup>

We emphasize that insurance through brokers does not reduce the multiplicity of equilibria. Having characterized all the possible equilibria, we can now study the conditions under which insurance is optimal from the point of view of the consumer. As already discussed, insurance is irrelevant in the good equilibria. In the default equilibria in which  $\chi_l \in (0, \bar{\chi}_l]$ , the inequality (A.3) can be written using (A.7) and the equilibrium premium  $f$  as

$$\left(\frac{B}{P}\right)^\pi \leq \frac{1 - \chi_l}{1 - \pi\chi_l} = 1 - \frac{f}{1 - \pi\chi_l} ,$$

which is in general true for a low enough supply of public liquidity and a relatively high probability of realization of the low state.<sup>44</sup> The key result is that whenever public money is limited, it is then optimal to have private

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<sup>42</sup>This can easily be seen by assuming  $B = 0$ , which is the case in which deposits always increase with the rate of default and at the highest speed.

<sup>43</sup>When  $(1 - \chi_l) - (1 - E\chi)B/P \leq 0$ , equilibrium deposits fall to zero, and only public liquidity will be used for first-subperiod consumption.

<sup>44</sup>Note also that if the previous inequality is true, the condition for a positive level of deposit is also satisfied, since  $B/P \leq (B/P)^\pi$ .

insurance. This is in line with the historical evidence of the early nineteenth century in which public money was not available, and a secondary market for private bank notes developed.

When  $\chi_l$  exceeds  $\bar{\chi}_l$  and there is default also in the high state, the inequality (A.3) can be written using (A.7) and (40) as

$$\left(\frac{B}{P}\right) \leq \frac{1 - \chi_l}{1 - E\chi} = 1 - \frac{f}{1 - E\chi} = \beta(1 + r_{low}),$$

which is exactly the condition required for deposits to be positive. It is satisfied again when public liquidity is low and the variability of the real rate is not very high.<sup>45</sup> In the latter case, the required premium to make brokers' securities free of risk can be small since  $\chi_l$  and  $\chi_h$  are closer.

### A.3 Monopolistic competition: derivations

In this Appendix, we present a detailed derivation of two results of Section 7: the profit maximization of intermediaries and the derivation of the equilibrium with regulation.

#### A.3.1 Profit maximization of intermediaries

Recall, from Section 7, that each intermediary  $j$  internalizes that its own rate of default  $\chi_{t+1}(j)$  depends on  $N_t(j)$ ,  $D_t(j)$ , and  $Q_t^D(j)$ . In order to understand these links, consider three cases. In the first, the default rate is zero,  $\chi_{t+1}(j) = 0$  in both the high and low state. This turns out to be the case when the limited liability constraint at time  $t + 1$  in the low state is not binding, that is, when (47) evaluated in the low state and with no default is strictly positive:

$$\Pi_l(j) = (1 + r_l^K) (Q_t^D(j) D_t(j) + N_t(j) - P\Phi) - D_t(j) > 0, \quad (\text{A.8})$$

in which  $\Pi_l(j)$  denotes profit at time  $t + 1$  in the low state. Since profits in the low state  $\Pi_l(j)$  are strictly positive, profits in the high state  $\Pi_h(j)$  are strictly positive as well.

In the second case, profits in the high state  $\Pi_h(j)$  are still positive and default in that state is zero,  $\chi_h(j) = 0$ . However, default in the low state is positive,  $\chi_l(j) > 0$ . The rate of default  $\chi_l(j) > 0$  is implicitly defined as a

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<sup>45</sup>Recall that  $1/\beta$  is the expected real interest rate.



function of  $N_t(j)$ ,  $Q_t^D(j)$ , and  $D_t(j)$  by the profit condition (47) evaluated in the low state and equated to zero because of the binding limited liability constraint:

$$(1 + r_l^K) (Q_t^D(j) D_t(j) + N_t(j) - P\Phi) = (1 - \chi_l(j)) D_t(j). \quad (\text{A.9})$$

In the third case, default is going to be positive in both states since the limited liability condition always binds, again implying (A.9) in the low state and

$$(1 + r_h^K) (Q_t^D(j) D_t(j) + N_t(j) - P\Phi) = (1 - \chi_h(j)) D_t(j) \quad (\text{A.10})$$

in the high state.

In general, a monopolist  $j$  chooses  $D_t(j)$ ,  $Q_t^D(j)$ , and  $N_t(j)$  in order to maximize (49), subject to the demand schedule (46) and the previous conditions (A.8), (A.9), (A.10), which apply depending on the different cases underlined earlier.

Consider first the region in which there is no default in both states. Expected rents (49) become

$$\mathcal{R}_t(j) = D_t(j) [Q_t^D(j) - \beta] - P\Phi.$$

Given the demand schedule (46), the monopolist sets its optimal deposit price to  $Q_t^D(j) = \beta(1 + \tau)$ , where  $1 + \tau \equiv \theta/(\theta - 1) - 1 \geq 1$  is the monopoly markup. Expected rents evaluated at the optimum, denoted by  $\bar{\mathcal{R}}_t(j)$ , can be written as

$$\bar{\mathcal{R}}_t(j) = \frac{\tau}{1 + \tau} [\beta(1 + \tau)]^{1-\theta} \frac{D_t(Q_t^D)^{\theta}}{J} - P\Phi. \quad (\text{A.11})$$

The case of default only in the low state is analyzed in Section 7. Recall that the optimal choices are  $Q_t^D(j) = (1 + \tau) / (1 + r_h^K)$  and  $N_t(j) = 0$ , and rents evaluated at the optimum are  $\tilde{\mathcal{R}}_t(j)$ , defined by (51).

We now turn to the third case, in which intermediaries default in both the high and low state. Using (A.9) and (A.10), expected rents are given by  $\mathcal{R}_t(j) = -N_t(j)$ . Therefore, rents are minimized by setting net worth at the lowest possible value,  $\hat{N}_t(j) = 0$ , implying zero expected rents,  $\hat{\mathcal{R}}_t(j) = 0$ . The price of deposits  $Q_t^D(j)$  can be computed by noting that the two constraints (A.9) and (A.10), which hold simultaneously, imply

$$Q_t^D(j) D_t(j) - \beta \{1 - E\chi(j)\} D_t(j) = P\Phi - N_t(j) = P\Phi. \quad (\text{A.12})$$

Any  $Q_t^D(j)$  and  $D_t(j)$  that satisfy (46) and (A.12), and that imply default in both states, are a solution. Therefore, the deposit price is  $Q_t^D(j) = (1 + \tau)\beta \{1 - E\chi(j)\}$ .

We have characterized the expected rents in the three regions: (i) the level  $\bar{\mathcal{R}}_t(j)$ , given by (A.11), in the case of no default; (ii)  $\tilde{\mathcal{R}}_t(j)$ , given by (51), when default occurs only in the low state; and (iii)  $\hat{\mathcal{R}}_t(j) = 0$  when there is default in both states. Next, we show that only case (ii) can arise in equilibrium.

First, we note that the region in which there is no default in any state, case (i), cannot arise in equilibrium because intermediaries have an incentive to deviate to case (ii). To see this, assume by contradiction that there is an equilibrium in which all intermediaries are always solvent in all states. In this equilibrium, rents (A.11) must be driven to zero by free entry, implying

$$\frac{D_t(Q_t^D)^\theta}{J} = P\Phi \frac{1 + \tau}{\tau[\beta(1 + \tau)]^{1-\theta}}.$$

We can use this result to check whether a generic intermediary  $j$  can earn strictly positive rents by deviating to choices that imply default only in the low state, that is, case (ii). To do so, we plug the last equation into the expression for rents in case (ii), (51), and rearrange:

$$\tilde{\mathcal{R}}_t(j) = (1 - \pi) \beta (1 + r_h^K) P\Phi \left[ (\beta (1 + r_h^K))^{\theta-1} - 1 \right] > 0,$$

where the inequality follows from  $(1 + r_h^K) > 1/\beta$  and  $\theta > 1$ . Thus, this is a profitable deviation for a generic intermediary  $j$ ; therefore, the case in which all intermediaries are always solvent in all states cannot be an equilibrium.

Second, we show that case (iii), default in both states, cannot arise in equilibrium as well. Using (44) and (45), we note that the price of the structured product and its default rate are the same, respectively, as the price and default rate of a generic intermediary  $j$ . Therefore:

$$Q_t^D = (1 + \tau)\beta \{1 - E\chi\}.$$

The demand for the structured product, (15), becomes

$$Q_t^D = \beta \{1 - E\chi\},$$

where we have used that prices are constant and there is default in both states. Therefore, there is no symmetric equilibrium in this case because  $\tau > 0$ .

Thus, the only remaining possibility is the case in which there is default only in the low state.

### **A.3.2 Equilibrium with regulation**

Under the requirement that net worth must be at least as large as  $\overline{N}^m$ , intermediaries are solvent in all states. As a result, they set the price  $Q_t^D(j) = \beta(1 + \tau)$ , as derived before, and using (44) we obtain  $Q_t^D = \beta(1 + \tau)$  as well. Combining this result with (13) and with the demand from households in (15) evaluated at constant prices, we obtain (55). Finally, the number of intermediaries  $J$  is determined by setting rents in (A.11) to zero, implying the condition  $J = \beta\tau D/(P\Phi)$ .