DSGE Models and the Lucas Critique. Discussion

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Structure of the paper

- This paper looks at implications of parameter shifts for econometric policy evaluation, to see whether policy advice derived from DSGE models would have differed fundamentally from that which the policymakers of the 1970s derived from their reduced-form Phillips curves.
  - stability analysis of rolling estimates of parameters in standard DSGE
  - analysis of the implications of observed parameter drift for policymaking, looking at the impulse response functions of the estimated model and analysing how they change over time

The reduction process

Any econometric model can be thought of as the result of a reduction process.
Think of a vector $\mathbf{x}_t$ containing observations on all economic variables at time t. A sample of T time-series observations on all the variables can be represented as follows:

$$\mathbf{X}_T^1 = \begin{bmatrix} \mathbf{x}_1 \\ . \\ . \\ . \\ \mathbf{x}_T \end{bmatrix}.$$

$$D\left(\mathbf{X}_T^1 \mid \mathbf{X}_0, \boldsymbol{\theta}\right) = D\left(\mathbf{x}_1 \mid \mathbf{X}_0, \boldsymbol{\theta}\right) \prod_{t=2}^{T} D\left(\mathbf{x}_t \mid \mathbf{X}_{t-1}, \boldsymbol{\theta}\right).$$

Note that In the case of non-stationarity the unconditional distribution is not defined. On the other hand, in the case of stationarity, the DGP is completely described by the conditional density function $D\left(\mathbf{x}_t \mid \mathbf{X}_{t-1}, \boldsymbol{\theta}\right)$.

The reduction process

The first step of the reduction process can be understood by partitioning $\mathbf{x}$ into three types of variables:

$$\mathbf{x}_t = (\mathbf{w}_t, \mathbf{y}_t, \mathbf{z}_t),$$

where $\mathbf{w}_t$ identifies variables which are unobservable or irrelevant to the problem investigated by the econometrician. In practice these variables are ignored, in theory such a result is obtained by factorizing the joint density and integrating it with respect to $\mathbf{w}_t$:

$$\begin{aligned} D(\mathbf{y}_t, \mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}) &= \iint D(\mathbf{y}_t, \mathbf{z}_t, \mathbf{w}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \mathbf{W}_{t-1,}\boldsymbol{\theta}) \\ & D(\mathbf{W}_{t-1} \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\theta})\, d\mathbf{W}_{t-1} d\mathbf{w}_t. \end{aligned}$$

In this case we have a potential information loss which becomes real when the variables, judged irrelevant for the problem at hand, are relevant. In formal terms, we have no information loss only if

$$D(\mathbf{y}_t, \mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}) = D(\mathbf{y}_t, \mathbf{z}_t, \mathbf{w}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \mathbf{W}_{t-1,}\boldsymbol{\theta}).$$

The reduction process

Suppose that the relevant problem is inference on subset $\boldsymbol{\beta}_1$ of the parameters determining the joint density of $\mathbf{y}_t$, and $\mathbf{z}_t$. In general it is always possible to re-write $D\left(\mathbf{y}_t, \mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}\right)$ as follows:

$$\begin{aligned} & D\left(\mathbf{y}_t, \mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}\right) \\ = \; & D\left(\mathbf{y}_t \mid \mathbf{z}_t, \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2\right) D\left(\mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2\right). \end{aligned}$$

Two conditions need to apply for a model to be robust to the Lucas'critique:

$$\begin{aligned} & D\left(\mathbf{y}_t, \mathbf{z}_t \mid \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}\right) \\ = \; & D\left(\mathbf{y}_t \mid \mathbf{z}_t, \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}_1\right) D\left(\mathbf{z}_t \mid \mathbf{Z}_{t-1}, \boldsymbol{\beta}_2\right). \end{aligned}$$

And that the conditional model $D\left(\mathbf{y}_t \mid \mathbf{z}_t, \mathbf{Y}_{t-1}, \mathbf{Z}_{t-1}, \boldsymbol{\beta}_1\right)$ is structurally invariant, i.e. changes in the distribution of the marginal model for $\mathbf{z}_t$ do not affect the $\boldsymbol{\beta}_1$ parameters

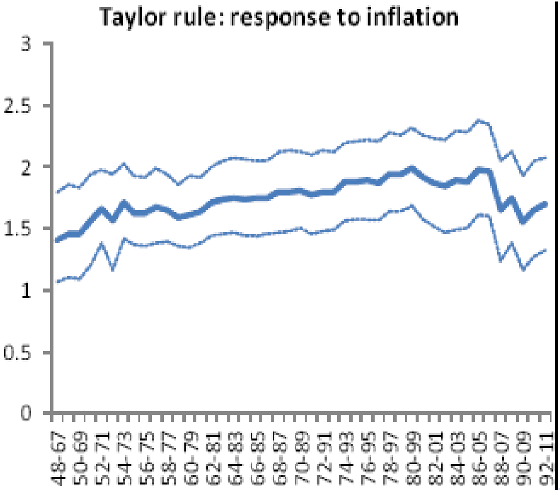
Parameters Instability

There are many different potential sources of parameters' instability

- The Haavelmo distribution does not exist
- something has gone wrong in the first step of the reduction process
- super-exogeneity does not apply

Stability analysis of rolling estimates of parameters in standard DSGE

- Instability is found but some results are unconventional:



Stability analysis of rolling estimates of parameters in standard DSGE

- signs of non-stationarity

AR - stationary TFP shock	0.96	0.91	0.77	0.99

- how are priors calibrated in the rolling analysis ? .

Time variation in impulse response functions

Examine the projection of output and inflation conditional upon a monetary policy shock and then examine how it changes over-time form 1970 onwards

- where is the the policy break ?
- Surico-Benati(2009) note that changes in the coefficients of the monetary policy rule of the DSGE model exert their impact on both the coefficients of the VAR representation of the model, and the elements of the VAR's covariance matrix of reduced-form innovations. As a consequence impulse responses can look stable even across different regimes
- output versus the output gap
- output effect of monetary policy is huge
- no-price puzzle