DEBT AND GROWTH: NEW EVIDENCE FOR THE EURO AREA

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Against the background of the euro area sovereign debt crisis, our paper investigates the relationship between public debt and economic growth and adds to the existing literature in the following ways. First, we use a dynamic threshold panel methodology in order to analyse the non-linear impact of public debt on GDP growth. Second, we focus on 12 euro area countries for the period 1990-2010, therefore adding to the current discussion on debt sustainability in the euro area. Our empirical results suggest that the short-run impact of debt on GDP growth is positive and highly statistically significant, but decreases to around zero and loses significance beyond public debt-to-GDP ratios of around 67 per cent. This result is robust throughout most of our specifications, in the dynamic and non-dynamic threshold models alike. For high debt-to-GDP ratios (above 95 per cent), additional debt has a negative impact on economic activity. Furthermore, we can show that the long-term interest rate is subject to increased pressure when the public debt-to-GDP ratio is above 70 per cent, broadly supporting the above findings.

Non-technical summary

The fiscal situation remains challenging in much of the developed world, particularly in the euro area. Market concerns with respect to fiscal sustainability in vulnerable euro area countries have grown and spread to other countries. Against this background, empirical research has started to focus on estimates of the impact of public debt on economic activity.

Looking at the debt-growth nexus literature, two characteristics become apparent. First, only few studies focus on euro area countries. This is insofar surprising as the euro area/EMU offers economic dynamics that are rarely found anywhere else in the world. Moreover, this group of countries is in need of special attention given the current sovereign debt crisis. Second, most of the empirical studies still rely on linear estimation frameworks. Only more recently has the focus been shifting to non-linear threshold analyses, inter alia by employing the threshold panel methodology developed by Hansen (1999). However, all of these studies focus exclusively on non-dynamic panel models, which might lead to inconsistent results due to the persistence of GDP growth rates. To our best knowledge our paper is the first to account for this problem through application of a dynamic threshold framework. Comparing the results from dynamic and non-dynamic threshold estimations provides an idea not only about the robustness of the impact of debt on growth, but also about the robustness of the estimated optimal debt ratios.

Our paper adds to the existing literature in the following ways. First, we use a dynamic threshold panel methodology in order to analyse the non-linear impact of public debt on GDP growth. Second, in comparison to the majority of empirical studies we analyse the short-run relationship between public debt and economic growth using yearly data. Third, our focus on EMU

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data provides the opportunity to make specific policy inference, adding to the current discussion on the sustainability of debt dynamics in the euro area.

Our empirical results suggest the following. The short-run impact of debt on GDP growth is positive, but decreases to close to zero beyond public debt-to-GDP ratios of around 67 per cent (*i.e.*, up to this threshold, additional debt has a stimulating impact on growth). This result is robust throughout most of our specifications, in the dynamic and non-dynamic threshold model alike. For really high debt ratios (above 95 per cent), additional debt has a negative impact on economic activity. Confidence intervals for the thresholds are tight, that is (63; 69) for the lower threshold and broader at about (80; 100) for the upper one. Furthermore, we can show that the long-term interest rate is subject to increased pressure when the public debt-to-GDP ratio is above 70 per cent, broadly supporting the above findings.

1 Introduction

The current sovereign debt crisis with its epicenter in the euro area has forcefully revived the academic and policy debate on the economic impact of public debt. Market concerns with respect to fiscal sustainability in vulnerable euro area countries have grown and spread to other countries. Against this background, empirical research has started to focus on estimates of the impact of public debt on economic activity, inter alia by attempting to unveil possible non-linearities.

Nonetheless, the empirical literature on this topic remains scarce (see, for example, Schclarek, 2004; and Reinhart and Rogoff, 2010) and only few studies employ a non-linear impact analysis and are of particular interest for our paper. One of these is a contribution by Kumar and Woo (2010), who use dummy variables for pre-determined ranges of debt to show non-linear effects in a sample of emerging and advanced economies. They find that only very high (above 90 per cent of GDP) levels of debt have a significant and negative impact on growth. Another recent contribution is provided by Checherita and Rother (2010). Expressing growth as a quadratic functional form of debt in a sample of twelve euro area countries over a period starting in 1970, they find significant evidence for a concave (inverted U-shape) relationship. The debt turning point, beyond which debt starts having a negative impact on growth, is found at about 90-100 per cent of GDP.[1]

Papers that relate more closely to the non-linear panel threshold methodology we use in this analysis include the work by Chang and Chiang (2009) and Cecchetti, Mohanty and Zampolli (2011). Both of these papers employ the threshold methodology for non-dynamic panels. Chang and Chiang (2009) analyse a sample of 15 OECD countries and use yearly observations for the period 1990-2004. In a generalisation of the Hansen (1999) multiple regime panel threshold model, they run a regression of GDP per capita growth on the debt-to-GDP ratio and find two debt-to-GDP threshold values, 32.3 per cent and 66.25 per cent. Interestingly, the impact of the debt ratio is positive and significant in all three regimes, higher in the middle regime and lower in the two outer regimes. They thus cannot support the crowding-out view if the debt-to-GDP ratio is more than the threshold value.[2] Cecchetti, Mohanty and Zampolli (2011) use a sample of 18 OECD countries for the period 1980-2010 and obtain a threshold for government debt at 85 per cent of GDP. In contrast to Chang and Chiang (2009), they find a negative impact on growth in the high debt regime.

[1] Confidence intervals for the debt turning points provided in Checherita and Rother (2010) suggest that the negative growth effect of high debt may start already from levels of around 70-80 per cent of GDP.

[2] Chang and Chiang (2009) apply a panel smooth transition regression (PSTR), with a continuous transition function depending on an observable transition variable. In their additive version of this model, the transition function becomes an indicator function, with $I[A] = 1$ when event A occurs, and 0 otherwise. As a consequence, the additive PSTR model is equivalent to the multiple regime threshold model developed by Hansen (1999).

Going through the current empirical debt-growth nexus literature, three characteristics become apparent. First, none of the above mentioned papers uses a dynamic panel threshold approach. Because of the likely persistence in the economic growth rate, the neglect of such a dynamic specification might lead to inconsistent results. Including such dynamics, on the other hand, allows us to capture the effect of debt on growth after controlling for growth persistence, and in this way it is well suited for estimating short-run relationships. To our best knowledge, the current paper is the first to estimate a dynamic threshold model for the debt-growth nexus and then to compare the results of dynamic and static panel estimations. It thus also provides an idea about the robustness of results across different methodologies.

Second, most of the above papers study the long-term impact of debt on growth (Schclarek, 2004; Reinhart and Rogoff, 2010; Kumar and Woo, 2010; Checherita and Rother, 2010).[3] So far, the only exception has been Chang and Chiang (2009), who use exclusively yearly data and thus capture a short-term impact comparable to our focus. On the same note, most of the literature on short-term growth effects analyses fiscal multipliers of shocks to government expenditure or taxes (see Hemming *et al.*, 2002; and van Riet, 2010, for relevant surveys), and if the role of debt is accounted for, its influence is indirect. IMF (2008), for instance, finds that the impact of discretionary fiscal impulses on real GDP growth is contingent on the level of debt, *i.e.*, it is positive and larger at low government debt levels (relative to the sample average). Differently from these studies, the objective of the present paper is to investigate the direct (short-term) impact of debt on growth.

Third, Checherita and Rother (2010) has been so far the only paper focussing exclusively on euro area countries. This is surprising as the EMU offers economic dynamics that are rarely found elsewhere in the world. Moreover, with the current sovereign debt crisis, the euro area would be in need of particular attention, while averaging across OECD countries makes policy inferences difficult.

To summarise, our paper adds to the existing literature in the following ways. First, we use a dynamic threshold panel methodology, inter alia by adapting the methodology proposed in Hansen and Caner (2004), and use it to analyse the non-linear impact of public debt on GDP growth. To our best knowledge, a comparable approach has been applied only once before, in a contribution by Kremer, Bick and Nautz (2009), who analyse the non-linear impact of inflation on growth. Second, we study the short-run relationship between public debt and economic growth using yearly data. Third, our focus on EMU data provides the opportunity to make specific policy inference, adding to the current discussion on the sustainability of debt dynamics in the euro area.

The paper is organised as follows. Section 2 describes the employed methodology and Section 3 presents the data. The estimation results are shown in Section 4. Section 5 employs several robustness exercises, including a broad extension of the explanatory variable set and an analysis of the impact of debt on long-term interest rates. Section 6 concludes.

2 Methodology

In order to account for the persistence of the growth rate, we need a threshold model that allows for endogeneity. Caner and Hansen (2004) develop a threshold methodology for dynamic models, which has to be extended to a panel framework. With several differences as explained

[3] Checherita and Rother (2010) use both yearly data for the dependent variable (and one year-lagged debt data), as well as 5-year overlapping and non-overlapping averages (with debt measured at the beginning of the 5-year period and estimates corrected in all cases for time autocorrelation), but do not find radically different results across the various specifications. Cecchetti, Mohanty and Zampolli (2011) use the (less conventional) long-term approach by employing only the 5-year overlapping average growth rates.

below, the extension we apply here has been first suggested by Kremer *et al.* (2009), who analyse the non-linear impact of inflation on growth within an Arellano and Bover (1995) estimation.[4]

The starting point for the threshold analysis is the specification of a linear model, which in the present case is a balanced panel of the form:

$$y_{it} = \mu_i + \chi y_{i,t-1} + \alpha X_{it} + u_{it} \quad (1)$$

$y_{i,t}$ is the dependent variable of country i at time t, $y_{i,t-1}$ is the endogenous regressor, in our case the lagged dependent variable, μ_i are the country specific fixed effects and X is a set of explanatory regressors. The error term μ_{it} is independent and identically distributed with mean zero and finite variance. The linear model can be estimated following the Arellano and Bond (1991) dynamic panel approach.[5]

We estimate the dynamic threshold model following the approach by Caner and Hansen (2004), who develop an estimator and an inference theory for models with endogenous variables and an exogenous threshold variable. Since Caner and Hansen (2004) do not apply their procedure to panel data we first have to make their framework suitable to deal with the country-specific fixed effects. While in a non-dynamic panel model the individual effects μ_i can be removed by mean differencing, in the dynamic panel mean differencing leads to inconsistent estimates due to the fact that the lagged dependent variable will always be correlated with the mean of the individual errors and thus with all of the transformed individual errors (see Arellano, 2003, p. 17). As an alternative we apply a strategy as first suggested in Kremer *et al.* (2009) and use forward orthogonal deviations[6] (1995). The method subtracts the average of all future available observations of a variable and makes it possible to maintain the uncorrelatedness of the error terms.[7]

The dynamic panel threshold model can be represented with:

$$y_{it} = \mu_i + \chi y_{i,t-1} + \alpha' x_{it} + \beta_1 d_{it} I(z_{it} \le z^*) + \beta_2 d_{it} I(z_{it} > z^*) + u_{it} \quad (2)$$

where x is a set of regime independent control variables, d is the set of variables allowed to switch between regimes, and l is an indicator function taking on the value 1 if the value of the threshold series z is below a specific threshold value z^*.

In the estimation of the dynamic panel model, we first run a reduced form regression of the endogenous variable on a set of instruments. For the lagged GDP growth rate we use higher lags of GDP growth as instruments and we can then replace $y_{i,t-1}$ in equation (2) with its predicted values $\hat{y}_{i,t-1}$.

After the reduced form regression the threshold model can be estimated, with the specific threshold value being determined following the strategy by Hansen (1999). The procedure includes three essential steps:

1) first, we conduct a series of least squares (LS) minimisations. That is, we estimate model (2) with 2SLS for each value of the threshold series z. The corresponding LS estimates of the

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4 An alternative approach for a dynamic threshold model can be found in Cimadomo (2007). He extends the Hansen (1999) approach by a two stage procedure. In the first step, the autoregressive coefficient is estimated from a linear regression. In the second stage this coefficient is treated asknown and fixed in the non-linear panel regression model.

5 In contrast to our paper, Kremer *et al.* (2009) employ the Arellano and Bover (1995) estimator, as they focus on the central role of initial income for growth convergence. Due to the endogeneity of the lagged level of GDP, the application of Arellano and Bover (1995) is necessary. Since we focus on growth persistence and a short-run impact analysis, the Arellano and Bond (1991) estimation is more appropriate.

6 Programming codes for forward orthogonal deviations can be obtained from http://www.cemfi.es/ arellano.

7 An empirical Monte Carlo proof for the advantage of orthogonal deviations over mean deviations is found in Hayakawa (2009).

parameters and the sum of squared residuals are kept;[8]

2) in a second step the threshold value z^* is selected as the one which minimises the sum of squared residuals;
3) in a third step we test for the significance of the chosen z^*. Since the threshold value is not identified under the null of linearity, the distribution of a standard *F*-statistic is not chi-square. Hansen (2000) therefore proposes a bootstrap procedure with which the asymptotic null distribution of the heteroscedasticity adjusted test statistic can be approximated.[9]

Hence, we test for the threshold significance using the test statistic:

$$F_T = \sup_{z \varepsilon S} F_T(z) \tag{3}$$

where:

$$F_T(z) = T(\frac{\tilde{\sigma}_T^2 - \hat{\sigma}_T^2(z)}{\hat{\sigma}_T^2(z)}) \tag{4}$$

where $\hat{\sigma}_T^2 = \frac{1}{t}\sum_{t=1}^{T} \hat{u}_t^{\ 2}$ is the estimated residual variance of the threshold and $\tilde{\sigma}_T^2$ is the residual variance of the corresponding linear model. Details of the testing procedure are described in the Appendix.

If we find a significant threshold value z^*, the slope coefficients of equation (2) are estimated with GMM.[10] For a more efficient weighting matrix in the coefficient estimation, we prefer the general GMM to the 2SLS estimator, and repeatedly predict the residuals to construct new covariance matrices of the moments after the initial 2SLS estimate.

We also allow for the possibility of more than one threshold and therefore more than two regimes (see Hansen 1999), but since a second threshold value turns out to be insignificant in most of the specifications it will be ignored in the following analysis, unless specified otherwise.

3 Data

3.1 Structural considerations

The model is estimated for 12 euro area countries Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain for yearly data starting with 1990. Using this relatively short time span offers a couple of advantages. First, the shorter period covers more accurately the process of EMU preparation and implementation and is thus less prone to structural changes and more comparable with today's economic conditions. More importantly, the debt-to-GDP ratio is found to be non-stationary upon inclusion of the previous decade (the 1980s). Using the longer time span we would not be able to fully rely on the results of

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8 This step is repeated for each value of the threshold series on a specified subset of the series, which should be trimmed in order to assure a minimum number of observations in the resulting subsamples. In the non-dynamic model, the 2SLS estimator reduces to the simple LS estimator.

9 We test the null hypothesis of linearity against threshold non-linearity also allowing heteroscedasticity in the error terms. Caner and Hansen (2004) provide evidence that the distribution theory in Hansen (2000) is applicable to the case of 2SLS estimation. However, a full distribution theory for dynamic panels has not yet been provided (we thank Bruce Hansen for his comments). The specific coefficients on the explanatory variables of the dynamic model should thus be considered carefully. Since on the other hand the non-dynamic panel estimation might give inconsistent results due to omitted lagged variables, the direct comparison of both approaches will give an idea about the range in which the coefficients lie.

10 The slope coefficients of non-dynamic model are estimated by OLS.

the threshold testing procedure and, consequently, on the obtained threshold values.[11] Given the above, we base our main estimation models on the period 1990-2007/10 (we do, however, include a discussion on results from the year 1980 onwards in the robustness section).

We are analysing the impact of one-year lagged debt-to-GDP ratios on annual real GDP growth rates. We thus obtain a near contemporaneous effect, which gives us an idea of the short-term debt impact. Hence, a positive impact of debt on growth could be interpreted as a stimulating effect of additional debt. However, the possibility that long-term effects of high debt might be negative cannot be ruled out based on the yearly analysis.

3.2 *Endogenous, regime-dependent variable and other control variables*

The data used originates primarily from the European Commission AMECO database. The endogenous variable is the real GDP growth rate. As the single regime-dependent and threshold variable we use the debt-to-GDP ratio. Since this can be correlated with a range of other factors impacting on growth, we also control for a broad set of other explanatory variables. In the benchmark specification, we include the gross fixed capital formation as a share of GDP, trade openness (defined as imports plus exports as a share of GDP), and a dummy signalling the period of effective EMU membership. Moreover, under the robustness tests we control for other potentially relevant variables as identified in the theoretical and empirical growth literature, such as the initial level of GDP per capita, population growth, secondary education, a measure for the old dependency ratio, the unemployment rate, the budget balance and long- and short-run interest rates.

4 Estimation

4.1 *Benchmark model*

The benchmark model for the 12 EMU countries over the period 1990-2007 (first, excluding the current crisis years) is estimated in the following specification:

$$y_{it} = \mu_i + \chi y_{i,t-1} + \alpha_1 OPEN_{i,t-1} + \alpha_2 GCF_{i,t-1} + \alpha_3 EMU_{it}$$
$$+ \beta_1 d_{i,t-1} I(d_{i,t-1} \le d^*) + \beta_2 d_{i,t-1} I(d_{i,t-1} > d^*) + u_{it} \qquad (5)$$

where y is the GDP growth rate, $OPEN$ is the trade openness measure, GCF is the ratio of gross capital formation to GDP, EMU is the dummy variable which signals the EMU membership, and d is the debt-to-GDP series, with d^* being the debt-to-GDP threshold value. For the dynamic model, $y_{(t-1)}$ is replaced by the predicted values $\hat{y}_{(t-1)}$ obtained from the structural first stage regression of $y_{(t-1)}$ on the lags of $y_{(t-2)}$ to $y_{(t-8)}$. Of course, GDP growth in the structural equation could be dependent on more than one lag. However, we find a second and higher lags to be insignificant in all of our specifications, and therefore they will be ignored in the following analysis.

Table 1 shows the benchmark results for the non-dynamic and the dynamic panel threshold estimation. We can see some differences between the two models, but for both the direction and the significance of the coefficients are comparable. As such, trade openness has a significantly positive effect on GDP growth, the coefficient on investment is positive but insignificant and the EMU dummy is significantly negative. In the dynamic model the strongest impact on current growth comes from the past growth rate itself.

[11] Details on the distribution theory can be found in Hansen (2000).

Table 1

Benchmark Results, 1990-2007

Variable	Non-dynamic Panel		Dynamic Panel	
$y_{(t-1)}$			0.4583***	*(0.1055)*
Openness	0.0148**	*(0.0064)*	0.0172**	*(0.0078)*
GCF	0.0539	*(0.0401)*	0.0184	*(0.0396)*
EMU	–0.0070**	*(0.0034)*	–0.0099***	*(0.0031)*
$d_{(t-1)}$ if $d \le d^*$	0.0697***	*(0.0209)*	0.0668***	*(0.0148)*
$d_{(t-1)}$ if $d > d^*$	0.0082	*(0.0095)*	0.0124	*(0.0104)*
Threshold Estimate	$d^* = 0.6640$		$d^* = 0.6644$	
Bootstrap *p*-value	0.0630		0.0780	
Confidence Intervals	$0.6287 < d^* < 0.6831$		$0.6287 < d^* < 0.6908$	

Standard errors in brackets.
*/**/*** indicate significance levels at the 10/5/1 per cent level.

Independent of the specifications, both models find a debt threshold value of around 0.664, which is significant at the 10 per cent level with *p*-values of 0.063 and 0.078 for the non-dynamic and the dynamic model, respectively. This threshold value splits the observations of the non-dynamic (dynamic) panel into 128 (125) observations in the lower, and 88 (91) observations in the upper regime. When the debt ratio is below 66.4 per cent of GDP, the impact of additional debt is significantly positive in both specifications, with coefficients corresponding to around 0.07 percentage point increase in the annual growth rate after a 1 percentage point increase in the debt-to-GDP ratio. If the debt ratio is above the threshold value, the impact reduces to values around zero, which are therefore insignificant.

This is a very strong result. Additional debt might have a positive impact on GDP growth due to stimulus effects of fiscal policy. However, once a debt threshold is reached this positive effect disappears or becomes insignificant.

4.2 Including the years 2008-10

We re-estimate the model including the crisis years 2008 to 2010. The results for the two threshold models are presented in Table 2. The threshold value of the non-dynamic model increases slightly to 71.7 per cent. At the same time, the regime-independent coefficients change notably compared to the benchmark results, with the GCF being the only positive and significant variable. The impact of debt on GDP growth also changes substantially. For the extended period, it is significantly positive in the lower regime, and significantly negative in the upper regime, while now diverging more in absolute size between the two specifications.

Including the years 2008 to 2010 in the dynamic specification gives the high threshold value of 95.6 per cent, which is significant at the 10 per cent level with a *p*-value of 0.098, resulting in

Table 2

Benchmark Results, 1990-2010

Variable	Non-dynamic Panel		Dynamic Panel	
$y_{(t-1)}$			0.3218^{***}	*(0.1245)*
Openness	–0.0082	*(0.0072*	0.0014	*(0.0058)*
GCF	0.1126^{**}	*(0.0529)*	0.0147	*(0.0568)*
EMU	–0.0071	*(0.0045)*	-0.0091^{**}	*(0.0036)*
$d_{(t-1)}$ if $d \leq d^*$	0.0470^{***}	*(0.0182)*	0.0351^{***}	*(0.0107)*
$d_{(t-1)}$ if $d > d^*$	-0.0411^{***}	*(0.0144)*	-0.0588^{***}	*(0.0200)*
Threshold Estimate	$d^* = 0.717$		$d^* = 0.956$	
Bootstrap *p*-value	0.0960		0.0980	
Confidence Intervals	$0.6287 < d^* < 0.7809$		$0.8140 < d^* < 1.0344$	

Standard errors in brackets.
$^{*}/^{**}/^{***}$ indicate significance levels at the 10/5/1 per cent level.
Threshold of 0.717 splits the sample into 168 observations in the lower and 85 in the upper regime.
Threshold of 0.956 splits the sample into 198 observations in the lower and 55 in the upper regime.

198 observations in the lower, and 55 observations in the upper regime.[12] Except for trade openness the regime-independent coefficients are more robust to changes in the time span than in the non-dynamic model (hence, the lagged GDP is significantly positive, GCF insignificant and the EMU dummy significantly negative). However, the changes for the regime dependent debt variable are comparable to the non-dynamic panel. In the lower regime, the impact of debt is positive at 0.035 per cent, while in the upper regime we obtain a larger negative impact of –0.059 per cent (both values being significant).

With a coefficient of 0.035 the impact in the lower regime decreases strongly compared to the value of 0.067 in the specification without the years 2008-10. However, since the introduction of the higher debt threshold leads to an average estimate over almost the entire original sample (plus a few new observations), we re-estimate the dynamic model with a second threshold, combining the multiple threshold estimation strategy by Hansen (1999) with our framework. We fix the first threshold at 95.6 per cent, and test for a second threshold in the lower sample. We indeed find a second threshold corresponding to the smallest sum of squares again to be 0.664, but it is insignificant with a *p*-value of 0.147. For illustration purposes the estimation results including the second threshold are shown in Table 3. Compared to the results of the dynamic model presented in Table 2, the debt impact in the lowest sample is now higher (0.0496), while the value of the second regime is insignificant and close to zero up to the threshold of 95.6 per cent of GDP. Afterwards, the debt impact remains negative, highly statistically significant and similar in size.

Hence, our results suggest that debt can have a stimulus effect on growth in the EMU up to a value of between 60 and 70 per cent of GDP. Above that, the growth impact becomes first insignificant, before turning negative for very high debt-to-GDP ratios.

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12 The reason for a higher threshold when the years 2008-10 are included is that the point of highest significance of the one break we are looking at shifts upwards. Using a data set up to 2007, we had only few observations with debt higher than 95 per cent of GDP.

Table 3

Second Threshold Value – Dynamic Panel

Variable	Dynamic	
$y_{(t-1)}$	0.3221***	(0.1245)
Openness	–0.0001	(0.0058)
GCF	0.0200	(0.0567)
EMU	–0.0092**	(0.0037)
$d_{(t-1)}$ if $d \leq 0.664$	0.0496***	(0.0137)
$d_{(t-1)}$ if $0.664 \leq d \leq 0.956$	0.0146	(0.0114)
$d_{(t-1)}$ if $d > 0.956$	–0.0591***	(0.0200)

Standard errors in brackets.
*/**/*** indicate significance levels at the 10/5/1 per cent level.
The two thresholds split the sample into 154 observations in the lower regime, 44 in the middle regime, and 55 in the upper regime.

5 Robustness

To make sure that our results are robust throughout a broader range of specifications, we conduct a variety of additional tests. Those include further explanatory variables, an extension of the time frame, further endogeneity tests, an analysis of influential euro area countries, and an analysis employing the real sovereign long term interest rate as the dependent variable. For most of the robustness tests, the results of the benchmark specification can be supported and remain consistent.

5.1 Including further explanatory variables

Next to lagged GDP growth, trade openness, gross capital formation and the dummy for EMU membership, we consecutively include further explanatory variables to test for robustness of the results. These are population growth, the old dependency ratio, the unemployment rate, secondary education, GDP per capita, the general government budget balance and primary budget balance (in ratios to GDP), private gross capital formation (replacing the aggregate variable) and the long and short term interest rates. All variables included are lagged one year compared to the dependent variable in order to avoid further endogeneity. Table 4 shows the results for the threshold dynamic model. Altogether, there are comparatively few changes in the coefficients and their significance, no matter which other variable is included.[13] Furthermore, for all the specifications the estimated threshold associated with the smallest sum of squares is 66.4 per cent, and the threshold value remains significant at the 10 per cent level. The debt coefficients of the two regimes are mostly comparable to the benchmark specification. Only for the last two columns the debt impact is smaller, but it is still significant and positive in the lower, and very close to zero in the upper regime.

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[13] This is also true if the explanatory variables are used without or with two lags instead.

Table 4

Robustness, Dynamic Model – Non-linear, 1990-2007

Variable	(1)	(2)	(3)	(4)	(5)[(a)]	(6)	(7)	(8)	(9)	(10)[(b)]
$y_{(t-1)}$	0.4679*** (0.1080)	0.4448*** (0.1046)	0.4473*** (0.1414)	0.4592*** (0.1073)		0.4278*** (0.1224)	0.4551*** (0.126)	0.4183*** (0.1115)	0.3496*** (0.1008)	0.4135*** (0.1081)
Openness	0.0176** (0.0079)	0.0176** (0.0077)	0.0188** (0.0088)	0.0176** (0.0077)	0.0326*** (0.0139)	0.0153* (0.009)	0.0141 (0.0090)	0.0158* (0.0088)	0.0214*** (0.0074)	0.0285*** (0.0098)
GCF	–0.0024 (0.0599)	0.0004 (0.0476)	–0.0218 (0.0492)	0.0193 (0.0402)	–0.0182 (0.0472)	–0.0408 (0.0468)	–0.0411 (0.0475)		–0.0144 (0.0382)	–0.0318 (0.0362)
EMU	–0.0096*** (0.0031)	–0.0095*** (0.0032)	–0.0112*** (0.0041)	–0.0101** (0.0031)	–0.00001 (0.0042)	–0.0052 (0.0041)	–0.0046 (0.0037)	–0.0053 (0.0035)	–0.0117*** (0.0030)	–0.0120 (0.0114)
Population growth	0.2643 (0.5179)									
Old ratio		–0.1693 (0.2067)								
Unemployment			–0.1024 (0.1288)							
Secondary education				0.0000 (0.0001)						
GDP per Capita					–1.8948 (1.278)					
Budget balance						–0.0153 (0.0668)				
Primary budget balance							–0.0409 (0.0623)			
GCF Private								–0.0543 (0.0544)		
Long run interest rates									–0.4230*** (0.1086)	
Short run interest rates										–0.3390*** (0.0781)
$d_{(t-1)}$ if $d \leq 0.664$	0.0697*** (0.0159)	0.0670*** (0.0144)	0.075**** (0.0191)	0.0669*** (0.0152)	0.0626*** (0.0142)	0.0718*** (0.0151)	0.0707*** (0.0145)	0.0730*** (0.0145)	0.0491*** (0.0137)	0.0396*** (0.0114)
$d_{(t-1)}$ if $d \leq 0.664$	0.0157 (0.0122)	0.0120 (0.0105)	0.0211 (0.0139)	0.0137 (0.0115)	0.0054 (0.015)	0.0125 (0.0117)	0.0112 (0.0119)	0.0133 (0.0115)	0.0048 (0.0127)	0.0055 (0.0106)
Bootstrap *p*-value	0.085	0.069	0.10	0.075	0.10	0.084	0.092	0.10	0.070	0.080
Confidence Region	$0.6287<d^*<0.6908$	$0.6287<d^*<0.6908$	$0.6287<d^*<0.6831$	$0.6287<d^*<0.6908$	$0.6287<d^*<0.6698$	$0.6287<d^*<0.6908$	$0.6287<d^*<0.6908$	$0.6287<d^*<0.6908$	$0.6127<d^*<0.6831$	$0.6287<d^*<0.7210$

Standard errors in brackets.
a) Non-dynamic estimation since lagged GDP per capita and lagged GDP growth rate are highly correlated.
b) Estimation excludes Luxembourg due to data limitations.
*/**/*** indicate significance levels at the 10/5/1 per cent level.

Table 5

Alternative Endogenous Variables, Threshold Panel, 1990-2007

Dependent Variable	**Potential GDP Growth**		**GDP Growth**		**GDP Growth**	
Estimation Method	**Dynamic Panel (a)**		**Dynamic Panel (b)**		**IV 2SLS (c)**	
$y_{(t-1)}$	0.8562***	(0.0344)	0.2209**	(0.1008)	0.4234***	(0.1102)
Openness	0.0038**	(0.0018)	0.0310**	(0.0089)	0.0132**	(0.0058)–
GCF	–0.0356***	(0.0096)	–0.0569	(0.0572)	0.0246	(0.0413)
EMU	–0.0020***	(0.0008)	–0.0117***	(0.0043)	–0.0077**	(0.0037)
$d_{(t-1)}$ if $d \leq d^*$	0.0163***	(0.0028)	0.0867***	(0.0177)	0.0583***	(0.0119)
$d_{(t-1)}$ if $d > d^*$	0.0041	(0.0030)	0.0185	(0.0149)	–0.0016	(0.0161)
Threshold Estimate	$d^* = 0.6644$		$d^* = 0.6640$		$d^* = 0.6640$	
Bootstrap p-value	0.026		0.085		0.058	
Confidence Intervals	$0.6287 < d^* < 0.7170$		$0.6287 < d^* < 0.6908$		$0.6287 < d^* < 0.6831$	

(a) $y_{(t-1)}$: potential GDP growth; (b) $y_{(t-1)}$: output gap;
(c) $y_{(t-1)}$: GDP growth, debt/GDP as second endogenous variable.
Standard errors in brackets.
*/**/*** indicate significance levels at the 10/5/1 per cent level.

5.2 *Including the period 1980-1989*

As discussed above, the non-stationarity of the debt-to-GDP variable if the years 1980-89 are included causes the resulting threshold estimates to be potentially unreliable. We do, however, re-estimate the model including the foregoing decade to examine whether our implications are generally stable. The estimation suggests that while the obtained linear (regime-independent) coefficients do not change significantly, including the previous decade leads to insignificant threshold estimates.[14] Although insignificant, the two debt-to-GDP ratios associated with the lowest sum of squares lie on average around 0.20 and 0.67, depending on the specification. The lower values can be explained by the lower average debt ratios prevailing in the 80s.

[14] This result does not change if dummy variables for the 90s or the years 2008-2010 are included. The results in this subsection are available upon request.

5.3 An alternative endogenous variable / Dealing with endogeneity

In addition to using the GMM estimation[15] to further control for the possibility of endogeneity problems we estimate the dynamic panel with the growth rate of potential GDP instead, where the first lag of the dependent variable, $y_{(t-1)}$, is instrumented with longer lags of the GDP growth rate. The results are shown in the first column of Table 5. The employed endogenous GDP variable has little impact on the significance and size of the threshold value and the debt coefficients, as well as on the direction of the regime-independent variables (the only change is observed in the significance of GCF, which is now significantly negative). The threshold estimate is again 66.4 per cent, being statistically significant at the 5 per cent level. The impact of debt below the threshold decreases, but is still positive and significant, while the impact above 66.4 per cent remains insignificant.

As an alternative, we replace the lagged GDP growth rate in the benchmark specification with the lagged output gap, which is again instrumented with further lags of GDP growth. The results are shown in the second column of Table 5. The coefficient on the output gap series, $y_{(t-1)}$ is positive and significant, while the threshold value and all of the remaining coefficients are comparable to the benchmark specification.

Another endogeneity issue might arise from the debt variable itself. That is, we can expect reverse causation between GDP growth rates and debt levels (low growth rates are likely to result in higher debt-to-GDP ratios). Even though the positive values of the debt coefficients in the benchmark estimation rule out the possibility of reverse causation almost entirely, we still control for endogeneity to check if the results are altered significantly. If this was the case, we could suspect further endogeneity problems. We would like to continue estimating the dynamic panel when debt endogeneity is taken into account. Unfortunately it is impossible to split the instrumented debt-to-GDP series within the construction of the GMM estimator. Therefore we have to limit our estimation to a less efficient (albeit still consistent) 2SLS estimation of the following form:

1) in a first step, the lagged GDP growth rate and the lagged debt-to-GDP series are regressed on higher lags of both variables plus all the exogenous regressors. We then predict the values for both lagged GDP growth and lagged debt-to-GDP;
2) the threshold testing procedure is similar to the benchmark estimation, only with the regime dependent series being the predicted values for debt/GDP;
3) based on the threshold value, the coefficients are estimated using OLS. The resulting coefficients are the 2SLS estimators.

The third column of Table 5 shows the results from the described regression approach. As can be seen, the coefficients differ only negligibly from the benchmark results.

5.4 Influential countries

Based on the benchmark specification, we first exclude two sets of countries, those with the highest and those with the lowest debt-to-GDP ratios over time. Excluding Luxembourg – the country with the lowest debt-to-GDP ratios – has no significant impact on the results. The same is true if we exclude Belgium or Italy, the two countries with the highest average debt ratios. Even if the two countries are excluded together (resulting in a sample with only 10 countries) the

[15] See Caselli *et al.* (1996) who proposed to use GMM as a way to deal with endogeneity problems in the context of panel growth regressions and Durlauf *et al.* (2005) for a related discussion.

coefficients change only marginally and the significant debt-to-GDP threshold value is again 66.4 per cent.

Next to the outliers of high and low debt ratios, we conduct the exclusion exercise for all the remaining countries (excluding one country at a time). Only two countries seem to have an impact on the debt threshold: Greece and Ireland. Excluding Greece or/and Ireland results in a debt threshold of 45 per cent.[16] The coefficients of debt on GDP growth in the two regimes are comparable to the benchmark results, positive and significant for debt ratios below, insignificant and close to zero above the threshold value.

However, we would like to mention that the exclusion of countries is conducted only as an econometric exercise and is of limited value to our analysis. Not only could we lose significant spillover effects, but we are also specifically interested in the most significant values for the (old) euro area as a whole over the period of our analysis and not only for a subset of countries.

5.5 *Influence on the interest rate*

Finding a significant debt threshold gives rise to the question why its impact on growth becomes smaller once a certain threshold value is reached. Among other channels, higher public debt is likely to be associated by investors with higher sovereign risk premia, which could be translated into higher long-term interest rates. In turn, this may lead to an increase in private interest rates and a decrease in private spending growth, both by households and firms (see Elmendorf and Mankiw 1999), which is likely to dampen output growth. While the empirical findings on the relationship between public debt and long-term interest rates are diverse, a significant number of recent studies suggest that high debt may contribute to rising sovereign yield spreads (see Codogno *et al.* 2003; Schuknecht *et al.* 2010 and Attinasi *et al.* 2009, among others) and ultimately sovereign long-term interest rates (Ardagna *et al.* 2007, Laubach 2009).

In order to examine this hypothesis, we run a non-dynamic threshold estimation of the form:

$$INT_{it} = \mu_i + \alpha_1 INT^s_{i,t-1} + \alpha_2 GDP_{i,t-1} + \alpha_3 OPEN_{i,t-1} + \alpha_4 EMU_{it}$$
$$+ \beta_1 d_{i,t-1} I(d_{i,t-1} \leq d^*) + \beta_2 d_{i,t-1} I(d_{i,t-1} > d^*) + u_{it} \,, \qquad (6)$$

INT is the sovereign long-term real interest rate, INT^S is the short-term real interest rate, which is included to capture monetary policy effects, GDP is the growth rate of GDP, and as before $OPEN$ is the trade openness measure, EMU is the dummy variable which signals the EMU membership, and d is the debt-to-GDP series, with d^* being the threshold value. The explanatory variables are broadly in line with Ardagna *et al.* (2007).[17]

Both interest rate series are de-trended, applying linear trend filtering from 1990. The resulting coefficients for the two periods 1990-2007 and 1990-2010 are presented in Table 6. For both time periods we find a threshold value of 73.8 per cent, significant at 10 per cent, and respectively, at 1 per cent level. Below this threshold, the impact of additional debt decreases the long-run interest rates.[18] Once the threshold is reached, we observe an increasing pressure on the

[16] The results of estimations with Greece and Ireland excluded one at a time are comparable with those resulting from a combined exclusion.

[17] Ardagna *et al.* (2007) estimate the response of long-term interest rates in a panel of 16 OECD countries, over the years 1975-2002. Comparable to our specification, they use the nominal interest rate on 10-year government bonds as the dependent variable, and GDP growth, interest rates on 3-month Treasury bills, inflation and deficit as explanatory variables, a baseline specification which is close to the one employed in our paper.

[18] For a detailed discussion on reasons for the negative impact of debt on interest rates below a threshold value, we refer to Section 3, specifically 3.2 in Ardagna *et al.* (2007).

Table 6

Interest Rates, Non-dynamic Threshold Model

Years	1990-2007		1990-2010	
INT^S	0.2860***	(0.0551)	0.3881***	(0.0442)
GDP	–0.0801	(0.0509)	–0.0491	(0.0452)
Openness	–0.0172**	(0.0073)	–0.0087	(0.0059)
EMU	0.0077**	(0.0030)	0.0062**	(0.0028)
$d_{(t-1)}$ if $d \leq d^*$	–0.0406***	(0.0089)	–0.0288***	(0.0077)
$d_{(t-1)}$ if $d > d^*$	0.0079	(0.0122)	0.0283***	(0.0086)
Threshold Estimate	$d^* = 0.7380$		$d^* = 0.7380$	
Bootstrap *p*-value	0.078		0.009	
Confidence Intervals	0.6287< d^* < 0.7709		07220< d^* < 0.8180	

Dependent variable: long-term real sovereign interest rates.
Standard errors in brackets.
*/**/*** indicate significance levels at the 10/5/1 per cent level.

interest rate. This is true especially for the longer period, for which the coefficient on the upper regime debt ratio is highly statistically significant and positive. These results are broadly in line with Ardagna *et al.* (2007): using debt in a quadratic functional form, they find a non-linear effect of public debt on long-term interest rates, with a negative impact when the debt-to-GDP ratio is below 65 per cent and a positive impact when the ratio is above this threshold.[19] The resulting crowding-out of economic activity helps explaining why the impact of additional debt on the economy decreases with the size of debt, and might even become negative above certain threshold values.

6 Conclusion

Our paper analyses the short-run impact of debt-to-GDP ratios on GDP growth, using one year lagged debt ratios in a non-linear threshold panel model. The empirical results suggest the following. The short-run impact of debt on GDP growth is positive, but decreases to close to zero and loses significance beyond public debt-to-GDP ratios of around 67 per cent. This result is robust throughout most of our specifications, in the dynamic and non-dynamic threshold models alike. For high debt ratios (above 95 per cent) the impact of additional debt has a negative impact on economic activity. The confidence intervals for the thresholds are generally tight, at about (63; 69) for the lower threshold and broader at about (80; 100) for the upper threshold.

[19] Ardagna *et al.* (2007) further include a panel VAR estimation, which does not account for any form of non-linearity. Clearly, applying the threshold methodology to a VAR specification would be an interesting extension. It is, however, beyond the scope of this paper.

Various robustness tests show that the lower threshold value reacts only marginally to changes in the number of control variables and countries included. The only departure from 67 per cent as the most significant debt threshold value occurs when we include the years before 1990 and the crisis years 2008-10. However, in both cases tests for further thresholds reveal that 67 per cent is associated with the value resulting in the (second) smallest SSR. We further show that the long-term interest rate is subject to increased pressure when the public debt-to-GDP ratio is above 70 per cent, broadly supporting the above findings.

Our results suggest that the positive short term economic stimulus from additional debt decreases drastically when the initial debt level is high, and might even become negative. The reverse would imply that when the debt ratio is very high, reducing it would have beneficial effects for annual growth. On the other hand, in case of low debt levels, reducing the debt further would tend to reduce growth in the short run, in line with conventional Keynesian multipliers (while the long-term effect may differ). Hence, in light of the attempt to defend increasing debt with economic stimulus reasons, our results are supportive only if the initial debt level is below a certain threshold.

APPENDIX
THRESHOLD TESTING

The pointwise *F*-statistic is:

$$F_T = \sup_{z \varepsilon S} F_T(z) \tag{7}$$

where:

$$F_T(z) = T\left(\frac{\tilde{\sigma}_T^2 - \hat{\sigma}_T^2(z)}{\hat{\sigma}_T^2(z)}\right) \tag{8}$$

with $\tilde{\sigma}_T^2$ being the estimated residual variance of the corresponding linear model. The threshold value is not identified under the null of linearity and consequently the distribution of the standard *F*-statistic is not chi-square (Hansen 2000). We can approximate the asymptotic distribution with the following bootstrap procedure:

Compute y_t^* iid $N(0,1)$ random draws and regress y_t^* on X_t and on $X_t(z)$ to obtain the residual variances $\tilde{\sigma}_T^{*2}$ and $\tilde{\sigma}_T^{*2}(z)$, respectively. Repeated bootstrap draws from the test statistic:

$$F_T^* = \sup_{z \varepsilon S} F_T^*(z) \tag{9}$$

with:

$$F_T^*(z) = T\left(\frac{\tilde{\sigma}_T^{*2} - \hat{\sigma}_T^{*2}(z)}{\hat{\sigma}_T^{*2}(z)}\right) \tag{10}$$

can then be used to approximate the asymptotic null distribution of F_T. The distribution of F_T^* converges weakly in probability to the null distribution of F_T under the alternatives for Γ_2 and the asymptotic bootstrap *p*-value is obtained by counting the percentage of bootstrap samples for which the bootstrap statistic F_T^* exceeds the statistic F_T.

Accounting for possible heteroscedasticity in the error terms, the standard *F*-statistic is replaced by a heteroscedasticity-consistent Wald or Lagrange Multiplier test:

$$L_T = \sup_{z \varepsilon S} L_T(z) \tag{11}$$

with:

$$L_T(z) = (R\hat{\delta}(z))'[R(M_T(z)^{-1}V_T(z)M_T(z)^{-1})R']^{-1} R\,\hat{\delta}(z) \tag{12}$$

where $R = (1\ I)$ is the selector matrix, $M_T(z) = \sum X_t(z)X_t(z)'$ and $V_T(z) = \sum X_t(z)X_t(z)'\hat{u}_t^2$.

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