FORECAST EVALUATION OF SMALL NESTED MODEL SETS

Kirstin Hubrich and Kenneth West

European Central Bank / Federal Reserve Board and University of Wisconsin

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Motivation

- Forecast evaluation often compares a small set of models
  - Criterion: usually mean squared prediction error (MSPE)
  - Usually: Sequence of pairwise model comparisons is carried out, using Diebold-Mariano-West statistic for non-nested models or Clark-McCracken and Clark-West statistics for nested models
  - Our proposal: Comparing models simultaneously; two statistics easy to compute for simultaneously comparing a parsimonious benchmark model to m alternative models that nest the benchmark
  - We take into account potential correlation of model forecasts

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- Non-nested model comparisons: Diebold-Mariano-West tests
  - null hypothesis of equal predictive accuracy (in population)
  - Diebold and Mariano (1995): allow for wide variety of forecast accuracy measures and general assumptions on forecast errors
  - West (1996): allows for estimation uncertainty
  - provide conditions for t-type statistics $\sim A N(0, 1)$
  - nested models: under null of equal predictive accuracy the variance of the forecast error differential is zero
  - derive standard and non-standard limiting distributions
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Example: Our empirical application considers forecasts of U.S. CPI all items inflation, comparing:

- univariate AR forecast ("model 0") vs. m=4 other models
- the m=4 other models are bivariate VARs with CPI inflation and
  - one the component of the CPI (food, energy, commodities or services inflation)
  - output growth, unemployment, commodities or services inflation

⇒ see e.g. Hubrich (2005) and Hendry and Hubrich (2009) for further empirical results and for analytical and simulation results on the use of disaggregate information in forecasting the aggregate...
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### Empirical Example: Forecasting All Items CPI Inflation

<table>
<thead>
<tr>
<th>Method</th>
<th>1984-2004</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSPE (altern)/RMSPE (bench)</td>
</tr>
<tr>
<td>AR$_{(AIC)}$ (bench)</td>
<td>0.187</td>
</tr>
<tr>
<td>Test AR</td>
<td></td>
</tr>
<tr>
<td>vs VAR$_{(AIC)}^{a,f}$</td>
<td>0.999</td>
</tr>
<tr>
<td>vs VAR$_{(AIC)}^{a,e}$</td>
<td>1.097</td>
</tr>
<tr>
<td>vs VAR$_{(AIC)}^{a,e}$</td>
<td>1.048</td>
</tr>
<tr>
<td>vs VAR$_{(AIC)}^{a,s}$</td>
<td>1.027</td>
</tr>
<tr>
<td>vs 4 models</td>
<td></td>
</tr>
<tr>
<td>Critical value</td>
<td></td>
</tr>
</tbody>
</table>
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Illustrative of class of applications relevant for our procedures: MSPE is measure of forecast performance, null model nested in a "small" number of other models

"small" number of models m is greater than 1 but much smaller than sample size

relevant applications include ones that conduct evaluations of this sort simultaneously for several data sets

Relevant asset pricing applications: e.g. Hong and Lee (REStat, 2003), Goyal and Welch (working paper, 2004), Cheung et al. (JIMF, 2005), McCracken and Sapp (JMCCB, 2005), Sarno et al. (JMCCB, 2005), Rapach and Wohar (JEmpFin, 2006)

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Existing procedures

Compute $\chi^2$ statistic (multivariate version of DMW):

- Construct $m \times 1$ vector of MSPE differences; compute long run variance of differences; compute the usual quadratic form $\chi^2$ (unadj.) in our tables (West et al. (1993))
- This statistic has possible problems with size and power:
  - **size**: under our null, the vector of MSPE differences is not centered at zero
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- Reality check (White (2000), proposed for $m \sim T$),
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  - problem: might not account for dependence of predictions on
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- Simulate, including reestimation of forecasting models
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  - possible problem: time intensive

Further alternative: Construct a set of pairwise comparisons,
adjust via Bonferroni or related procedure: low power
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Proposals: MSPE-adjusted t-stats, $\chi^2$

Existing procedures

Proposals

Structure of the Talk

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- construct a vector of **MSPE differences adjusted** as in Clark and West (2006, 2007) to center vector at zero under the null
- compute a variance-covariance matrix for the vector of MSPE differences
- conduct inference via either of the following two options
  1. "max t-stat (adj.)": inference on the largest of the m adjusted t-statistics that compare null model one by one to each of the m larger models via the distribution of the maximum of correlated normals;
  2. "χ² (adj.)": inference via the usual χ² statistic

**Key features:**
- we take estimation uncertainty into account
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Introduction

Proposed procedures: max t-stat (adjusted), adjusted $\chi^2$

Simulation results

Empirical example

Conclusions
MSPE-adjusted t-stats: Intuition

Consider first a comparison of a parsimonious model to a single larger model (m=1, in our terminology); Example:

"model 0": $y_t = \beta_0 + \beta_1 y_{t-1} + e_{0t}$

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$H_0 : \sigma_0^2 - \sigma_1^2 = 0, \ H_A = \sigma_0^2 - \sigma_1^2 > 0$

under the null of equal forecast accuracy attempt to estimate parameters whose population values are zero

⇒ will inflate variance of larger model

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\[ P = \text{number of predictions and prediction errors} \]
\[ R = \text{size of rolling regression sample used to estimate parameter} \]

Smoothed density estimates of

\[
\text{sample MSPE(null)} - \text{sample MSPE(alternative)} \left( \hat{\sigma}_0^2 - \hat{\sigma}_1^2 \right)
\]

across 1000 simulations:
Adjustment Clark and West, Pairwise model comparison

Clark and West (2006, 2007)

- magnitude of the downward shift is estimable; larger the larger number of extraneous regressors in the alternative model
- when comparing a parsimonious model to one other model
  - adjust difference in MSPEs by estimated downward shift
  - after adjustment, conduct inference in standard Diebold-Mariano-West (DMW) fashion

Denote Clark-West adjusted MSPE as
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"Max t-stat (adjusted)" statistic

Our first proposal for comparison of model sets

- Compute adjusted t-statistic for each of the m model comparisons; Suppose \( m = 2 \) alternative models for simplicity. We propose basing inference on

\[
P^{1/2} \begin{pmatrix}
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- Here, \( \rho \) is the correlation between the two t-statistics.
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Inference via the distribution of the maximum of correlated normals

- Let \( \hat{z} \) be the larger of the two t-statistics
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- More generally, for arbitrary $m > 1$, one can obtain a p-value from a set of draws from a normal distribution:
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Proposals: MSPE-adjusted t-stats, $\chi^2$

Simulation results

Empirical example

Conclusions

Further Research

Adjustment

"Max t-stat (adj.)" statistic

$\chi^2$

Theoretical justification

Encompassing Statistic

$\chi^2$ statistic (adjusted)

Second proposal for comparison of model sets

- compute $m$ adjusted differences in MSPEs
- compute variance or long run variance of $mx1$ vector of adjusted differences
- compute usual quadratic form, use critical values from $\chi^2(m)$

"$\chi^2$ (adj.)" = $P\bar{f}'\hat{V}^{-1}\bar{f}$

with $\bar{f} = \hat{\sigma}_0^2 - (\hat{\sigma}_i^2 - (adj.))$ for $i = 1, ..., m$

⇒ possible sacrifice in power given the one-sided nature of the test
**Motivation**

Proposals: MSPE-adjusted t-stats, $\chi^2$

Simulation results
Empirical example
Conclusions
Further Research

**Adjustment**

"Max t-stat (adj.)" statistic

$\chi^2$

Theoretical justification

Encompassing Statistic

$\chi^2$ statistic (adjusted)

Second proposal for comparison of model sets

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Kirstin Hubrich and Kenneth West

FORECAST EVALUATION OF SMALL NESTED MODEL SETS
\( \chi^2 \) statistic (adjusted)

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Theoretical justification for our procedures

- asymptotic validity of use of normal distribution follows from conditions such as in Giacomini and White (2007):
  ⇒ rolling windows (asymptotics: R fixed, P grows); null model: white noise; also for multistep forecasts if direct method used

- approximate asymptotic validity of normal distribution for pairwise model comparison (m=1) follows from Clark and McCracken (2001, 2005) asymptotics:
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• **Note:** Statistic based on adjusted difference in MSPEs is algebraically identical with the encompassing statistic:

\[
\hat{f}_{i,t+1} = \hat{e}_{0,t+1}^2 - (\hat{e}_{i,t+1}^2 - (\hat{y}_{0,t+1} - \hat{y}_{i,t+1})^2)
\]

\[
= \hat{e}_{0,t+1}^2 - \hat{e}_{i,t+1}^2 + (\hat{y}_{0,t+1} - \hat{y}_{i,t+1})^2
\]

\[
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\[
= 2\hat{e}_{0,t+1}^2 - 2\hat{e}_{0,t+1}\hat{e}_{i,t+1}
\]

\[
= 2\hat{e}_{0,t+1}(\hat{e}_{0,t+1} - \hat{e}_{i,t+1})
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Simulation design

Our simulations compare "max t-stat (adj.)" and "χ^2(adj.)" with

- "χ^2(unadj.)" proceeds as does "χ^2(adj.)", but uses raw rather than adjusted differences in MSPEs
- White’s (2000) bootstrap reality check

Simulation set-up (macro and finance DGPs):

- Macro DGP: (y_{1t}, y_{2t}, ..., y_{it})′ follows a VAR(1)
- Null model ⇒ univariate AR(1) in y_t
- m=2 and m=4 alternative models, each of which uses a constant + one lag of y_t + one lag of other variables
- rolling (and recursive) samples; nominal size 10% (5%)
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Simulation results: Size of Tests

<table>
<thead>
<tr>
<th>P</th>
<th>Max t-stat (adj.)</th>
<th>$\chi^2$ (adj.)</th>
<th>$\chi^2$ (unadj.)</th>
<th>Reality check</th>
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<td>40</td>
<td>0.081</td>
<td>0.119</td>
<td>0.157</td>
<td>0.019</td>
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<tr>
<td></td>
<td>$R=40$</td>
<td>$R=100$</td>
<td>$R=200$</td>
<td>$R=400$</td>
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<tr>
<td>100</td>
<td>0.073</td>
<td>0.112</td>
<td>0.241</td>
<td>0.001</td>
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<tr>
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<td>$m=2$</td>
<td>$R=100$</td>
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<tr>
<td>200</td>
<td>0.100</td>
<td>0.134</td>
<td>0.416</td>
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Simulation results: Size of Tests

Empirical Size of Nominal .10 Tests, 1 Step Ahead Predictions

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<th>( m=2 )</th>
<th>( m=4 )</th>
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<td>( R=100 )</td>
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<tr>
<td>40</td>
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<td>0.584</td>
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<td>100</td>
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<table>
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<tr>
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<th>Power</th>
<th>Size</th>
<th>Summary</th>
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<td>R=40</td>
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<td>0.542</td>
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Empirical example: Forecasting Inflation

- predictand = annual U.S. CPI inflation, one month ahead
- two sample periods 1960-1983 and 1960-2004
  - prediction period = 1970-1983; R=120, P=168
  - prediction period = 1984-2004; R=288, P=252
- rolling samples (also recursive)
- null model = univariate AR, lag length selected by AIC
- alternatives are bivariate VARs, lag lengths again selected by AIC. Second predictor (in addition to CPI inflation) is
  - component of inflation (food, energy, commodities, services)
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US CPI Inflation

US CPI year-on-year inflation: aggregate, energy, commodities, food and services

Forecasting inflation

Data
High, volatile inflation period
Low, stable inflation period

Interpretation

Components and real variables

Summary

Motivation

Proposals: MSPE-adjusted t-stats, $\chi^2$

Simulation results

Empirical example

Conclusions

Further Research

Kirstin Hubrich and Kenneth West
Forecasting US Inflation by components

Table 5: Tests of Equal Forecast Accuracy, US year-on-year inflation

<table>
<thead>
<tr>
<th>method</th>
<th>1970-1983</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSPE (altern)/RMSPE (bench)</td>
</tr>
<tr>
<td>AR$_{(AIC)}$ (bench)</td>
<td>0.307</td>
</tr>
<tr>
<td>Test AR</td>
<td></td>
</tr>
<tr>
<td>vs VAR$_{a,f}^{(AIC)}$</td>
<td>1.039</td>
</tr>
<tr>
<td>vs VAR$_{a,e}^{(AIC)}$</td>
<td>1.029</td>
</tr>
<tr>
<td>vs VAR$_{a,c}^{(AIC)}$</td>
<td>1.016</td>
</tr>
<tr>
<td>vs VAR$_{a,s}^{(AIC)}$</td>
<td>0.986</td>
</tr>
<tr>
<td>vs 4 models</td>
<td></td>
</tr>
<tr>
<td>critical value</td>
<td></td>
</tr>
</tbody>
</table>
**Note 2:** It is possible to have the following seemingly paradoxical result:

- sample MSPE from null model $< \text{sample MSPE from alternative model}$
- we reject the null of equal population MSPE in favor of the alternative that the larger model has lower population MSPE
- reason: adjustment larger than MSPE difference
  \[ \bar{f}_1 = \hat{\sigma}_0^2 - \hat{\sigma}_1^2 + P^{-1}\sum_t (\hat{y}_{0,t+1} - \hat{y}_{1,t+1})^2 \]

- Implication for future modeling: there is information in the larger model that is useful for forecasting, but we have not successfully exploited that information
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<table>
<thead>
<tr>
<th>method</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSPE (altern)</td>
</tr>
<tr>
<td>$\text{AR}_{(AIC)}$ (bench)</td>
<td>0.187</td>
</tr>
<tr>
<td>Test AR</td>
<td></td>
</tr>
<tr>
<td>vs $\text{VAR}^{a,f}_{(AIC)}$</td>
<td>0.999</td>
</tr>
<tr>
<td>vs $\text{VAR}^{a,e}_{(AIC)}$</td>
<td>1.097</td>
</tr>
<tr>
<td>vs $\text{VAR}^{a,e}_{(AIC)}$</td>
<td>1.048</td>
</tr>
<tr>
<td>vs $\text{VAR}^{a,s}_{(AIC)}$</td>
<td>1.027</td>
</tr>
<tr>
<td>vs 4 models</td>
<td>1.860</td>
</tr>
<tr>
<td>critical value</td>
<td>1.282</td>
</tr>
</tbody>
</table>
## Forecasting US Inflation: components and real variables

### Table 6: Tests of Equal Forecast Accuracy, US year-on-year inflation

<table>
<thead>
<tr>
<th>method</th>
<th>1970-1983</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSPE (altern)/RMSPE (bench)</td>
<td>t-stat adj.</td>
<td>max t-stat adj.</td>
<td>$\chi^2$ adj</td>
<td>$\chi^2$ unadj</td>
<td>Reality check</td>
</tr>
<tr>
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<td></td>
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</tr>
<tr>
<td>vs VAR$^{a,y}_{(AIC)}$</td>
<td>0.987</td>
<td>2.013*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs VAR$^{a,u}_{(AIC)}$</td>
<td>0.974</td>
<td>3.439*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs VAR$^{a,c}_{(AIC)}$</td>
<td>1.016</td>
<td>1.743*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs VAR$^{a,s}_{(AIC)}$</td>
<td>0.986</td>
<td>2.311*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs 4 models</td>
<td></td>
<td>3.439*</td>
<td>21.762*</td>
<td>2.432</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>critical value</td>
<td></td>
<td>1.282</td>
<td>1.917</td>
<td>7.78</td>
<td>7.78</td>
<td>0.146</td>
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Important to apply appropriate test!

⇒ In low and stable inflation period 1984-2004 disaggregate food inflation and unemployment changes do improve forecast accuracy over simple AR model significantly with pairwise forecast accuracy test, but not with test of model sets.

⇒ Implication of both "max t-stat (adj.)" and "χ²(adj.)".

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Conclusions

We have suggested two tractable methods for:

Comparison of a null (benchmark) model to a small number of alternative models that nest the benchmark

- explicitly account for estimation error in parameters
- easily executed, do not require bootstrap procedures
- Simulation evidence suggests that our procedures have distinctly better size and power than existing procedures
- Empirical examples forecasting US inflation show
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1. Use restrictions in accordance with the nesting properties of the alternative models for test procedure to improve power: Likelihood Ratio test (Granziera, Hubrich and Moon, 2008)
   - all alternative models are nested and nest the benchmark
   - all alternative models are non-nested, but nest the benchmark
   - some alternative models are nested, some are non-nested, all models nest the benchmark

2. Extension to comparison of models that might or might not nest the benchmark (Hubrich, McCracken and West, 2009)
   - implications for asymptotic theory
   - implications for small sample performance
   - wider range of applications

3. "Max t-stat (adj.)" might also be applicable to environments with number of models equal to sample size
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