When Central Banks Reveal Their Interest Rate Forecasts: Alignment of Expectations Vs. Creative Opacity

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Abstract

We examine the conditions under which a central bank raises welfare by revealing its expected future interest rate in a simple two-period model with heterogeneous information between the central bank and the private sector. The central bank optimally sets the interest rate given its information. The model is designed to rule out common-knowledge and time inconsistency effects. Transparency - when the central bank publishes its interest rate forecast - fully aligns central bank and private sector expectations about the future inflation rate. The private sector fully trusts the central bank to eliminate future inflation and sets the long-term interest rate accordingly, leaving only the unavoidable central bank forecast error as a source of inflation volatility. Under opacity - when the central bank does not publish its interest rate forecast - current period inflation differs from its target not just because of the unavoidable central bank expectation error but also because central bank and private sector expectations about future inflation and interest rates are no longer aligned. Opacity may be creative and raise welfare if the private sector's interpretation of the current interest rate leads it to form a view of expected inflation and to set the long-term rate in a way that systematically offsets the effect of the central bank forecast error on inflation volatility. Conditions that favor the case for transparency are a high degree of precision of central bank relative to private sector information, reasonably good early information and a high elasticity of current to expected inflation.

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1 Introduction

Recently, the Reserve Bank of New Zealand, the Bank of Norway and the Swedish Riksbank have started to publish their expected interest rate paths. The Bank of England has indicated that it might follow suite. One reason for this practice is purely logical. Inflation-targeting central banks publish the expected inflation rate, typically over a two or three-year horizon, but what assumptions underlie their expectation? Obviously, they make a large number of assumptions about the likely evolution of exogenous variables. One of these is the policy interest rate. Most banks - e.g. the ECB and, generally, the Federal Reserve - report that they assume a constant policy interest rate. If, however, the resulting expected rate of inflation exceeds the inflation target, the central bank is bound to raise the policy rate, which implies that the inflation forecast does not really reflect what the central bank expects. Other banks report that their inflation forecasting procedure relies on the interest rate implicit in the yield curve set by the market (this is the case of the Bank of England and of the Riksbank until its recent switch). As long as the central banks agrees with the market forecasts, this might seem to be an acceptable procedure. But what if the market forecasts do not lead, in the central bank's view, to the desirable outcome? Then the inflation forecasts are not what the central banks expect to see and, therefore, the market interest forecasts must differ from those of the central bank. Market interest forecasts do not really improve upon the constant interest assumption.

Why then do most central banks conceal their conditional inflation forecasts by not revealing their expected interest rate paths? Would it not be preferable for central banks to reveal their own expectations of what they anticipates to do? Most central banks reject this idea. Goodhart (2006) offers a number of reasons of why they do so. He first argues that central bank decisions are normally made by committees - the Reserve Bank of New Zealand is an exception among inflation-targeting central banks - which are typically unable to agree on future interest rates. This explanation is not borne out by the case of the Bank of Norway or the Riksbank but their decisions are made by small policy committees that are more likely to agree to an expected interest rate path than larger ones. A simple solution to this problem would be to use, at each horizon, the median interest rate among all committee members, along with the associated range of uncertainty path, as suggested by Svensson (2005a).

Two other arguments, which revolve about the obvious fact that the policy interest rate is not exogenous for a central bank, it is its instrument, are nicely put as follows:

"If, as I suggest, the central bank has very little extra (private, unpublished) information beyond that in the market, [releasing the expected interest rate path forces the bank to chose between] the Scilla of the market attaching excess credibility to the central bank's forecast (the argument advanced by Stephen Morris and Hyun Song Shin), or the Charybdis of losing credibility from erroneous forecasts." (Goodhart, 2006)

The first concern is that the central bank could become unwillingly committed to earlier announcements even though the state of the economy has changed in ways that were then unpredictable. The risk is that either the central bank validates the pre-announced path, and enacts suboptimal policies, or that it chooses a previously unexpected path and loses credibility since it does not do what it earlier said it would be doing. This argument is a reminder of the familiar debate on time inconsistency. The debate has shown that full discretion is not desirable. Blinder *et al.* (2001) and Woodford (2005) argue instead in favor of a strategy that is clearly explained and shown to the public to guide policy decisions.

The second concern is related to the result by Morris and Shin (2002) that the public tends to attribute too much weight to central bank announcements, not because central banks are better informed, but because these announcements are common knowledge. This argument is far from convincing. It is based on the doubtful assumption that the central bank is poorly informed relative to the private sector (Svensson, 2005b). It also ignores that the information here is not general, as in Morris and Shin (2002) but concerns the interest rate to be chosen by the central bank, which must be anyway reveal at least the current interest rate (Gosselin et al., 2006).

A last, related, concern is that revealing future interest rates might create a potential credibility problem. The central bank's announcement is bound to shape the market-set yield curve, but what if the implied short-term rates do not accord with those announced by the central bank? Since it is the long end of the yield curve that affects the economy, and therefore acts as a key transmission channel of monetary policy, it could force the central bank to take more abrupt actions to move the yield curve to match its own interest rate forecasts. Would this note be counter-effective?

We deal with some, not all of these issues. We deliberately ignore the time-consistency issue as well as the Morris-Shin effect, because these aspects have been extensively studied. Instead, we focus on the information role of interest rate forecasts. We focus on how the publication of interest rates affect private sector expectations in a simple model where the central bank and the private sector receive different information about the fundamentals.

We characterize the conditions under which, in our model, revealing future interest rates is desirable. Unsurprisingly, perhaps, we find that publishing the policy interest rate path may lead to welfare losses when central bank information is relatively imprecise. This result, which echoes Goodhart's reservation, reflects a subtle exchange of information between the central bank and the public. In our model, when it announces current and future policy rates, the central bank effectively reveals all its information set. This, in turn, allows the central bank to subsequently recover all of the private sector information set, simply by observing the market-set long-term interest rate. As a consequence, the central bank uses the current short-term rate to achieve the optimal long-term rate, even if that means currently choosing a policy rate that is not optimal for the current period. The strategic use of the current short-term rate may be welfare-reducing when the central bank's own information is imprecise.

Transparency may be welfare-inferior because of the way the private sector interprets the latest central bank interest rate decision. The more accurately the private sector infers the underlying central bank information, the more transparency is desirable. It might seem paradoxical that central bank opacity can be welfare improving when the private sector is poorly informed, but it is not. The more accurate is the private sector, the closer opacity comes to transparency.

This result can be seen as an application of second best theory. Hellwig (2005) has shown that the Morris and Shin (2002) result occurs because the combination of asymmetric information and incomplete markets implies that more information is not necessarily always welfare-increasing. Much the same occurs here. The welfare effect of revealing the interest rate path may increase or reduce welfare depending on the precision of central bank information.

The literature on the revelation of expected future policy interest rates is limited so far. Archer (2005) and Qvigstad (2005) present, respectively, the approach followed by the Reserve Bank of New Zealand and the Bank of Norway. Svensson (2005) presents a detailed discussion of the shortcomings of central bank forecasts based on the constant interest rate assumption or on market rates to build up the case for using and revealing the policy interest rate path. Faust and Leeper (2005) emphasize the distinction between conditional and unconditional forecasts. They assume that the central bank holds an information advantage over the private sector, which in their model implies that sharing that information is welfare-enhancing. They show that conditional forecasts - i.e. not revealing the policy interest path - provide little information on the more valuable unconditional forecasts, for which they find some supporting empirical evidence.

Similarly, Rudebusch and Williams (2006) assume an information asymmetry between the central bank and the private sector regarding both policy preferences and targets.[1] The private sector learns about these factors by running regressions on past information, which may include the expected interest rate path. The paper also allows for a "transmission noise" that distorts its communication. Through simulations, they find that revealing the expected path improves the estimation process and welfare, with a gain that declines as the transmission noise increases. Additionally, they explore the case when the accuracy of the central bank signals is not known by the public. They find that accuracy underestimation limits the gains from releasing the expected interest path while overestimation may be counterproductive. This result is not of the Morris-Shin variety, however, because what is at stake is not the precision of information but the size of the transmission noise, a very different phenomenon.

Walsh (2007) considers a model where the central bank and individual firms receive different signals about aggregate demand and firm-level cost shocks. As a consequence, as in Morris and Shin (2002), the publication by the central bank of its output gap forecasts - which is equivalent in his model to revealing expected inflation - has a large effect on individual firm forecasts, which can be welfare-reducing if the central bank is poorly informed. Walsh examines the possibility that the central bank information is not received by all firms. Partial transparency may allow to offset the common knowledge effect. The optimal degree of transparency - the proportion of firms that receive the central bank's information - depends on the relative accuracy of the central bank's information about demand and supply shocks.

Our contribution differs from Faust and Leeper (2005) and Rudebusch and Williams (2006). They assume the existence of an information *asymmetry*, which makes transparency always desirable as long as the central bank is credible. Instead, we assume that the central bank is credible with known preferences - which fully accord with social preferences - and we focus on information *heterogeneity* between the central bank and the private sector. Like Walsh (2007) we focus on information heterogeneity but, as we consider a single representative private agent, we eliminate the common knowledge effect that is at the center of his analysis.

In our model full central bank transparency is not desirable *per se b*ecause an imperfectly informed central bank policy inevitably makes forecast errors. If the central bank is transparent, central bank and private sector forecasts become aligned. Even though the private sector recognizes that the central bank is subject to forecast errors that result in misguided policy choices and a welfare loss, it fully trusts the central bank to do the best that it can. If, on the contrary, the central bank does not reveal its expected future interest rate, private and central expectations differ. What matters, then, is that the central bank sets the current interest rate while the private sector sets the long-run rate through the yield curve, each one acting on the basis of its own expectations. Opacity can be creative and raise welfare when the private sector mistakenly interprets the choice of the current interest rate as signaling a central bank forecast error in the future.

The next section presents the model, a simple two-period version of the standard New-Keynesian log-linear model. Section 3 looks at the case when the central bank optimally chooses the interest rate and announces its expected future interest rate. In Section 4, the central bank follows the same rule as in Section 3, but does not reveal its expected future

[1] Rudebusch and Williams (2006) also offer an excellent overview of the policy debate about how central banks signal their intentions regarding future policy actions.

interest rate. Section 5 compares the welfare outcomes of the two policy regimes and the last section concludes with a discussion of arguments frequently presented to reject the release of interest rate expectations by central banks.

2 The Model

We adopt the now-standard New-Keynesian log-linear model, as in Woodford (2003). It includes a Phillips curve:

$$\pi_t = \beta E_t^P \pi_{t+1} + \kappa_1 y_t + \varepsilon_t \tag{1}$$

where y_t is the output gap and ε_t is a random disturbance, which is assumed to be uniformly distributed over the real line, therefore with an improper distribution and a zero unconditional mean. In what follows, without loss of generality, we assume a zero rate of time preference so that $\beta = 1$. The output gap is given by the forward-looking IS curve:

$$y_t = E_t^P y_{t+1} - \kappa_2 (r_t - E_t \pi_{t+1} - r^*) \tag{2}$$

where r_t is the nominal interest rate. We do not allow for a demand disturbance because allowing for two sources of uncertainty would greatly complicate the model.[2] We assume that the natural real interest rate $r^* = 0$. Note that all expectations E^P are those of the private sector, which sets prices and decides on output.

We limit our horizon to two periods by assuming that the economy is in steady state at $t = 0$ and $t \geq 3$, i.e. when inflation, output gap and the shocks are nil. This simplifying assumption is meant to describe a situation where past disturbances have been absorbed so that today's central bank action is looked upon as dealing with the current situation ($t = 1$) given expectations about the near future ($t = 2$) - say two to three years ahead - while too little is known about the very long run ($t \geq 3$) to be taken into consideration. Consequently, (1) and (2) imply:

$$\pi_1 = E_1^P \pi_2 - \kappa (r_1 - E_1^P \pi_2 + E_1^P r_2 - E_1^P \pi_3) + \kappa_1 E_1^P y_3 + \varepsilon_1$$

where $\kappa = \kappa_1 \kappa_2$. Note that the channel of monetary policy is the real long-term interest rate, the second term in the above expression. This implies that, when it sets the interest rate r_1, the central bank must take into account market expectations: the longer end of the yield curve $E_1^P r_2$ and expected inflation.

Since the economy is known to return to steady state in period 3, this simplifies to:

$$\pi_1 = (1 + \kappa) E_1^P \pi_2 - \kappa (r_1 + E_1^P r_2) + \varepsilon_1 \tag{3}$$

where $r_1 + E_1^P r_2$ is the long-run (two period) interest rate, and similarly:

$$\pi_2 = -\kappa r_2 + \varepsilon_2 \tag{4}$$

where we also assume that the central bank sets $r_t = r^*$ for $t \geq 3$, which is indeed optimal as will soon be clear.

The loss function usually assumes that society is concerned with stabilizing both inflation and the output gap around some target levels, which allows for a well-known inflation-output trade-off. Much of the literature on central bank transparency additionally focuses on the idea that the public at large may not know how the central bank

[2] A generalization to both demand and supply disturbances, which could preclude obtaining closed-form solutions, is left for future work. Walsh (2007) examines the different roles of these disturbances.

weighs these two objectives. This assumption creates an information asymmetry, which makes transparency generally desirable, as shown in Rudebusch and Williams (2005). Here, instead, we ignore this issue by assuming that the weight on the output gap is zero and that the target inflation rate is also nil. Since the rate of time preference is zero, the loss function is, therefore, evaluated as the unconditional expectation:

$$L = E(\pi_1^2 + \pi_2^2) \tag{5}$$

and this is known to everyone.

The information structure is crucial. Information asymmetry requires that the central bank and the private sector receive different signals about the shock ε_t. We denote $j = CB, P$ the recipient of the signals, the central bank and the private sector, respectively. In addition, in order to meaningfully discuss the publication of interest rate forecasts, we allow for the central bank to discover new information between the release of its forecast and the decision on the corresponding interest rate. To that effect, we assume that two signals are received for each shock ε_t, both of which are centered around the shock: an early signal $\varepsilon_{t-1,t}^j$ is obtained in the previous period, which leads to the forecast $E_{t-1}^j \varepsilon_t$, and a contemporaneous signal $\varepsilon_{t,t}^j$, which is used to update $\varepsilon_{t-1,t}^j$ when forming the forecast $E_t^j \varepsilon_t$.[3]

Figure 1 presents the information structure. At the beginning of period 0, the central bank and the private sector receive an early signal $\varepsilon_{0,1}^j$on the shock ε_1. These signals have known variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$ for the central bank and the private sector, respectively. Equivalently, the signal precisions are $k\alpha$ and $k\beta$. At the beginning of period 1, new contemporaneous signals on $\varepsilon_{1,1}^{CB}$, and $\varepsilon_{1,1}^{P}$, with variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$ respectively, are received by the central bank and the private sector. Using both signals through Bayesian updating, the central bank and the private sector infer expectations $E_1^{CB}\varepsilon_1$ and $E_1^{P}\varepsilon_1$, respectively, with variances α^{-1} and β^{-1} or, equivalently, precisions α and β.[4] We assume that $0 < k \leq 1/2$, which implies that the early signals are less precise than the contemporaneous signals.

[Figure 1 about here]

Much the same occurs concerning the period 2 disturbance ε_2 with a slight but importance difference. At the beginning of period 1, the central bank and the private sector receive, respectively, the early signals $\varepsilon_{1,2}^{CB}$ and $\varepsilon_{1,2}^{P}$ with variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$. The central bank then forms $E_{1,2}^{CB}\varepsilon_2 = \varepsilon_{1,2}^{CB}$ and sets r_1 to minimize $E_1^{CB}L$. The private sector waits until r_1 is set and announced to form $E_{1,2}^{P}\varepsilon_2$ using both its early signal $\varepsilon_{1,2}^{P}$ and whatever information it can extract from r_1. Thus $E_{1,2}^{CB}\varepsilon_2$ and $E_{1,2}^{P}\varepsilon_2$ are formed at different times during period 1: $E_{1,2}^{CB}\varepsilon_2$ before r_1 is known and $E_{1,2}^{P}\varepsilon_2$ afterwards. The reason is that r_1 conveys new information to the private sector, not to the central bank.

At the beginning of period 2, the central bank and the private sector receive contemporaneous signals $\varepsilon_{2,2}^{CB}$ and $\varepsilon_{2,2}^{P}$, with variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$ respectively. We further assume that, at the beginning of period 2, the realized values of π_1 and ε_1 become known to both the central bank and the private sector. The central bank uses all information available - the early and contemporaneous signals $\varepsilon_{1,2}^{CB}$ and $\varepsilon_{2,2}^{CB}$ as well as π_1

[3]The assumption that ε_t is uniformally dsitributed implies that Bayes rule is only applied to the signals - ε_t is uninformative. Note that we have $cov\left(\varepsilon_t, \varepsilon_t^j\right) = 1$.

[4]Note that we assume that the central bank and the private sector form their expectations $E_1^j\varepsilon_1$ before the policy decision on r_1.

and ε_1 - to form its forecast $E_2^{CB}\varepsilon_2$ and sets r_2 to minimize $E_2^{CB}L$. After the central bank decision, the private sector observes r_2, forms its expectations and decides on output and prices.

The focus of the paper is whether, in addition to choosing and announcing r_t, the central bank should also reveal its expectation of the interest rates in the following periods r_{t+i}. This issue is made simpler once we recognized that $r_t = 0$ for all $t \geq 3$, so that we will only need to consider the choice of r_1 and r_2 and whether the central bank reveals $E_1^{CB}r_2$.

Note that we intentionally shut down two channels that figure prominently in the literature and provide arguments against full central bank transparency. The first one is time consistency due to uncertainty about the central bank preferences, presumed to differ from those of the private sector. We know from Cukierman and Meltzer (1986) that this can give rise to "creative ambiguity". Here, instead, we assume that the central bank and the private sector only care about inflation, a special - simple - case of identical preferences. The second channel is the beauty contest effect analyzed by Morris and Shin (2002). This channel arises when the private sector includes a large number of agents who each receive a different signal In that case, they tend to pay excessive attention to central bank signals simply because these signals are seen, and are known to be seen, by all. Here we assume that there is a single representative private sector agent.

3 The Central Bank Reveals its Interest Rate Forecast

We first look at the case where the central banks reveals $E_1^{CB}r_2$, which we refer to as the transparency case. In period 2, the central bank sets the interest rate in order to minimize $E_2^{CB}(\pi_2)^2$ conditional on the information available at the beginning of this period, i.e. after it has received the signal $\varepsilon_{2,2}^{CB}$.[5] The central bank seeks to offset the perceived shock and sets:

$$r_2 = \frac{1}{\kappa}E_2^{CB}\varepsilon_2 \tag{6}$$

The simplicity of this choice is a consequence of our assumption that the economy will return to the steady state in period $t = 3$. It can be viewed either as a rule or as discretionary action given the new information received at the beginning of the period.

Moving backward to period 1, the central bank publishes $E_1^{CB}r_2 = \frac{1}{\kappa}E_1^{CB}\varepsilon_2 = \frac{1}{\kappa}\varepsilon_{1,2}^{CB}$. This shows that publishing the interest rate is equivalent to fully revealing the central bank signal $\varepsilon_{1,2}^{CB}$.[6] As a consequence, in period 1 the private sector receives two signals about ε_2: its own signal $\varepsilon_{1,2}^{P}$ with precision $k\beta$ and, as just noted, the central bank signal $\varepsilon_{1,2}^{CB}$ with precision $k\alpha$. Denoting the relative precision of the central bank and private sector signals as $z = \frac{\alpha}{\beta}$, the private sector uses Bayes rule in period 1 to optimally forecast ε_2:[7]

$$E_1^P\varepsilon_2 = \gamma_1^{tr}\varepsilon_{1,2}^{P} + (1-\gamma_1^{tr})\varepsilon_{1,2}^{CB} = \frac{1}{1+z}\varepsilon_{1,2}^{P} + \frac{z}{1+z}\varepsilon_{1,2}^{CB} \tag{7}$$

[5] Note that we do not allow for the private sector to use newly received information $\varepsilon_{2,2}^{P}$ to update the interest rate. This would considerably complicate matters.

[6] This is so because the model allows for one signal and one policy instrument. If there were more signals than instruments, publishing the expected future value of one instrument would not be fully revealing.

[7] Note a potential notation ambiguity: $E_1^Px_1$, where $x = \varepsilon, \pi$, represents private sector expectations before the central bank sets (and reveals) the interest rate r_1 and reveals $E_1^Pr_2$ while $E_1^Px_2$ is conditionned on this new information.

In order to set the long term interest, the private sector also needs to forecast the future short-term interest rate given by (6) and therefore the central bank's own forecast of the future shock. Conjecture that, similarly to 7, the optimal forecast is:

$$E_1^P E_2^{CB} \varepsilon_2 = \gamma_2^{tr} \varepsilon_{1,2}^P + (1 - \gamma_2^{tr}) \varepsilon_{1,2}^{CB} \tag{8}$$

with unknown coefficient γ_2^{tr} to be determined.

When period 2 starts, π_1 and ε_1 become known. As a consequence, (3) and (6) show that $\pi_1 + \kappa r_1 - \varepsilon_1 = (1+\kappa)(E_1^P \varepsilon_2 - E_1^P E_2^{CB} \varepsilon_2) - E_1^P E_2^{CB} \varepsilon_2$ is known to both the central bank and the private sector. Using (7) and (8) we have:

$$\pi_1 + \kappa r_1 - \varepsilon_1 = [(1+\kappa)(\gamma_1^{tr} - \gamma_2^{tr}) - \gamma_2^{tr}](\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}) + \gamma_2^{tr} \varepsilon_{1,2}^{CB}$$

This implies that, at the beginning of period 2, when π_1 and ε_1 become known, the central bank can recover the private signal $\varepsilon_{1,2}^P$. We have a delayed mirror effect: by revealing the expected future interest rate, the central bank gives out its period 1 information $\varepsilon_{1,2}^{CB}$ and gets in return, in period 2, the private information $\varepsilon_{1,2}^P$. Put differently, by observing how its own information was previously interpreted, the central bank now recovers the signal previously received by the private sector. Importantly, the mirror image is not identical to the original, it adds information to both the central bank and the private sector. This implies that it is to be expected that an optimizing central bank will generally set r_2 at a different level than the initial forecast $E_1^{CB} r_2$. This does not necessarily mean discretionary action: the forecast and the decision both conform to the same rule 6.

Summing up, at the time of setting the interest rate r_2, the central bank uses three signals about ε_2: $\varepsilon_{1,2}^{CB}$ received in period 1 with precision $k\alpha$, $\varepsilon_{2,2}^{CB}$ received in period 2 with precision $(1-k)\alpha$ and now $\varepsilon_{1,2}^P$ with precision $k\beta$. Applying Bayes' rule we have:

$$E_2^{CB} \varepsilon_2 = \frac{z[k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}] + k\varepsilon_{1,2}^P}{z+k}$$

Noting that $E_1^P \varepsilon_{2,2}^{CB} = E_1^P \varepsilon_2$, it follows that $\gamma_1^{tr} = \gamma_2^{tr}$ and therefore:[8]

$$E_1^P E_2^{CB} \varepsilon_2 = \frac{1}{1+z} \varepsilon_{1,2}^P + \frac{z}{1+z} \varepsilon_{1,2}^{CB} = E_1^P \varepsilon_2$$

This key result shows that the private sector's own forecast of the future shock is perfectly aligned with its perception of the future central bank estimate of this shock which, it knows, will lead to the choice of the future interest rate. As they swap signals, both the central bank and the private sector learn from each other. As a consequence, the private sector knows that its own forecast will be taken into account by the central bank when it applies Bayes' rule before deciding on r_2.

[8] Proof:

$$\begin{aligned} E_1^P E_2^{CB} \varepsilon_2 &= \frac{z E_1^P [k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}] + k\varepsilon_{1,2}^P}{z+k} \\ &= \frac{z E_1^P \left[k\varepsilon_{1,2}^{CB} + (1-k)\left(\frac{\varepsilon_{1,2}^P}{1+z} + \frac{z\varepsilon_{1,2}^{CB}}{1+z}\right)\right] + k\varepsilon_{1,2}^P}{z+k} \\ &= \frac{\varepsilon_{1,2}^P}{1+z} + \frac{z\varepsilon_{1,2}^{CB}}{1+z} \end{aligned}$$

Proposition 1 *When the central bank reveals its expected future interest rate, the private sector and the central bank exchange information about their signals received in period 1 about the period 2 shock:*

- in period 1, the central bank fully reveals its signal, which the private sector uses to improve its own forecast

.- in period 2, the central bank can identify the signal previously received by the private sector.

- as a result, central bank and private expectations are fully aligned and, in period 1, both expect future inflation to be zero.

The last statement in the proposition is readily established. In period 2, the interest rate is set by the central bank according to (6) and inflation is set by the private sector based on its own information set, which includes the new signal $\varepsilon_{2,2}^{P}$ received at the beginning of the period and the newly-set interest rate r_2 which reveals $E_2^{CB}\varepsilon_2$, the central bank updated information about the shock ε_2. According to (4):

$$\pi_2 = \varepsilon_2 - E_2^{CB}\varepsilon_2 \tag{9}$$

As a consequence $E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2 = 0 = E_1^{CB}\pi_2$.

In period 1, the central bank sets the interest rates in order to minimize $E_1^{CB}(\pi_1^2 + \pi_2^2)$ conditional on available information. It follows from (9) that r_1 does not affect π_2, so in period 1 the central can simply minimize $E_1^{CB}\pi_1^2$. Since $E_1^P\pi_2 = 0$, from (3) we see that the central bank chooses the short-term interest rate r_1 such that, in expectation, the long-term interest rate - which matters for aggregate demand - fully offsets the current shock:

$$\kappa r_1 + \kappa E_1^{CB} E_1^P r_2 = E_1^{CB}\varepsilon_1$$

Since the central bank has released $E_1^{CB} r_2$, $E_1^P r_2 = E_1^{CB} r_2$, and (6) implies that $\kappa E_1^{CB} r_2 = E_1^{CB}\varepsilon_2$, so the optimal policy decision is:

$$r_1 = \frac{1}{\kappa}\left(E_1^{CB}\varepsilon_1 - E_1^{CB}\varepsilon_2\right) \tag{10}$$

Collecting the previous results, we obtain:

$$\pi_1 = \left(\varepsilon_1 - E_1^{CB}\varepsilon_1\right) + \frac{1}{1+z}\left(\varepsilon_{1,2}^{CB} - \varepsilon_{1,2}^{P}\right)$$

Period 1 inflation depends on two forecasting errors: the period 1 central bank forecasting error and the discrepancy between the central bank and the private sector signals regarding period 2 shock.[9] Note that the impact of this last discrepancy is less than one for one ($\frac{1}{1+z} < 1$) because the revelation of $\varepsilon_{1,2}^{CB}$ by the central bank leads the private sector to discount its own signal $\varepsilon_{1,2}^{P}$ and to bring its forecast $E_1^P\varepsilon_2$ in the direction of $\varepsilon_{1,2}^{CB}$.

It is worth emphasizing that the private sector is well aware that the central bank's interest rate forecast is bound to be inaccurate. Indeed, in general, there is no reason for $E_1^P E_2^{CB}\varepsilon_2$ to be equal to ε_2, but the eventual realization of this difference is irrelevant. The private sector fully understands that the future interest rate will usually differ from what was announced since the central bank will then respond to newly received information $\varepsilon_{2,2}^{CB}$, see (6). This eventual discrepancy is fully anticipated by the private sector

[9] More precisely, inflation is the result of three forecasting errors since $\pi_1 = \left(\varepsilon_1 - E_1^{CB}\varepsilon_1\right) + \frac{1}{1+z}\left[\left(\varepsilon_{1,2}^{CB} - \varepsilon_2\right) - \left(\varepsilon_{1,2}^{CB} - \varepsilon_2\right)\right]$, which includes the central bank and private sector early forecast errors about ε_2.

because the central bank strategy - in other words, its loss function - is public knowledge, so credibility is not an issue here. The difference between the pre-announced rate $E_1^{CB} r_2$ and the actually chosen rate r_2 is well understood to be purely random and therefore uninformative. Importantly, this result holds independently of the degree of precision of the signals received by the central bank and the private sector. What matters is that signal precision be known.[10]

Finally, for future reference, in this case of transparency the unconditional loss function is:

$$L^{tr} = E(\pi_1)^2 + E(\pi_2)^2 = \frac{1}{\beta}\left[\frac{1}{z} + \frac{1}{k}\left(\frac{1}{1+z}\right)^2\left(\frac{1}{z}+1\right) + \frac{1}{z+k}\right]$$

4 The Central Bank Does not Reveal its Interest Rate Forecast

We consider now the case when the central bank does not announce its expectation of the future interest rate. We call this the opacity case. The optimal interest rate in period 2 remains given by (6), formally unchanged from Section 3. The resulting inflation rate is also the same as in (9), although the information available to the central bank is different from that in the previous case, as will be emphasized below.

In period 1, the central bank still reveals the current interest rate, which is set on the basis of the information available to the central bank, i.e. $E_1^{CB}\varepsilon_1$ and $E_1^{CB}\varepsilon_2$. We restrict our attention to the following policy linear rule which optimally uses all available information:[11]

$$r_1 = \mu E_1^{CB}\varepsilon_1 + \nu E_1^{CB}\varepsilon_2 \tag{11}$$

Having observed r_1, the private sector sets the inflation rate according to (3). To that effect, it needs to forecast future inflation, which by (9) depends on $E_2^{CB}\varepsilon_2 = k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}$. In contrast to the previous case, $\varepsilon_{1,2}^{CB}$ is now unknown to the private sector. As a consequence $E_1^P\varepsilon_2$ no longer coincides with $E_1^P E_2^{CB}\varepsilon_2$. In order to form its forecast $E_1^P E_2^{CB}\varepsilon_2$, following Bayes' rule, the private sector uses its three available signals $E_1^P\varepsilon_1$, $\varepsilon_{1,2}^P$ and r_1.[12] It can use $\varepsilon_{1,2}^P$ directly. In addition, the interest rate rule (11) implies that $E_1^{CB}\varepsilon_2 = (r_1 - \mu E_1^{CB}\varepsilon_1)/\nu$ so r_1 can be used to make inference about $E_1^{CB}\varepsilon_2$, but the private sector does not know $E_1^{CB}\varepsilon_1$, it only knows $E_1^P\varepsilon_1$. Still, taking $E_1^P\varepsilon_2$ as a signal for $E_1^{CB}\varepsilon_2$ the private sector can use the linear combination $(r_1 - \mu E_1^P\varepsilon_2)/\nu$ of the two signals r_1 and $E_1^P\varepsilon_2$ to improve its forecast $E_1^P E_2^{CB}\varepsilon_2$.

However, doing so introduces an error since $E_1^{CB}\varepsilon_1 \neq E_1^P\varepsilon_1$. In order to correct for this error, the private sector must forecast $E_1^P(E_1^{CB}\varepsilon_1 - E_1^P\varepsilon_1)$ and adjust Bayes' rule accordingly. Using again the interest rate rule (11), we see that $(r_1 - \nu E_1^{CB}\varepsilon_2)/\mu = E_1^{CB}\varepsilon_1$ so that $E_1^P(E_1^{CB}\varepsilon_1 - E_1^P\varepsilon_1) = (r_1 - \nu\varepsilon_{1,2}^P)/\mu - E_1^P\varepsilon_1$. In the end, the optimal forecast is of the form:

[10] The case when the signal precisions are not known is left for further research. For a study of this case in a different setting, see Gosselin et al. (2006).

[11] There is no reason to presume that a linear rule is optimal. This restrictive assumption, required to carry through the calculations that follow, can be seen as a form of Taylor rule that approximates the optimal policy. This introduces some asymmetry between the transparency and opacity cases: in the former, the rule is optimal, in the latter it may not be. Unfortunately, we are not able to derive the optimal policy choice under opacity.

[12] More precisely, $E_1^P\varepsilon_1$ is not a signal but the expectation formed on the basis of signals $\varepsilon_{0,1}$ and $\varepsilon_{1,1}$.

$$
\begin{aligned}
E_1^P E_2^{CB}\varepsilon_2 &= \gamma_2^{op}\varepsilon_{1,2}^P + (1-\gamma_2^{op})\left(\frac{r_1 - \mu E_1^P\varepsilon_1}{\nu}\right) + \gamma_3^{op}\left(\frac{r_1 - \nu\varepsilon_{1,2}^P}{\mu} - E_1^P\varepsilon_1\right) \\
&= \gamma_2^{op}\varepsilon_{1,2}^P + (1-\gamma_2^{op})\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1)\right] \\
&\quad -\gamma_3^{op}\left[E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB})\right] \qquad (12)
\end{aligned}
$$

with γ_2^{op} and γ_3^{op} to be determined. Note that the two first terms are signals about ε_2, so their weights add up to unity, while the third term corresponds to the adjustment $E_1^P(E_1^{CB}\varepsilon_1 - E_1^P\varepsilon_1)$ and is zero-mean. The same reasoning can be applied to $E_1^P\varepsilon_2$ to obtain:

$$
E_1^P\varepsilon_2 = \gamma_1^{op}\varepsilon_{1,2}^P + (1-\gamma_1^{op})\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right] \qquad (13)
$$

where $\gamma_1^{op} = \frac{k(1+z)+\left(\frac{\nu}{\mu}\right)^2}{(1+z)\left[k+\left(\frac{\nu}{\mu}\right)^2\right]}$.

As in the transparency case, the unknown weighting coefficients γ_2^{op} and γ_3^{op}can be found by identification. In this case, there is no analytical solution. The Appendix shows that:

$$
\gamma_1^{op} - \gamma_2^{op} + \gamma_3^{op}\theta\frac{\nu}{\mu} = 0 \qquad (14)
$$

where $\theta = 1 + \dfrac{1}{(1+\kappa)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_2^{op} + (2+\kappa)\,\dfrac{\nu}{\mu}\gamma_3^{op}}$.

In comparison with the case where the central bank publishes its expected future interest rate, (14) implies that, in general, $\gamma_1^{op} \neq \gamma_2^{op}$ so that $E_1^P\varepsilon_2 \neq E_1^P E_2^{CB}\varepsilon_2$. From (4) and (6), it follows that:

$$
E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2 \qquad (15)
$$

Well aware that its own period 1 forecast of the disturbance ε_2 differs from that of the central bank, $E_1^P\pi_2$ is no longer nil, i.e. the private sector no longer trusts the central bank to achieve its aim and $E_1^P\pi_2 \neq 0$. This is the key difference between transparency and opacity. Private doubt is reflected by the gap between central bank and private sector expectations and is captured by $\gamma_1^{op} - \gamma_2^{op} = (-\frac{\nu}{\mu})\ \gamma_3^{op}\theta$.

The Appendix shows that the optimum interest rate rule in period 1 requires $\mu = -\nu = \kappa^{-1}$. The monetary policy rule is formally identical to (10) in the transparency case. As before, the reason is that, in order to minimize the volatility of π_1, the central bank seeks to set the nominal long-terme interest to offset the first period shock, which it expects to be $E_1^{CB}\varepsilon_1$; to do so, it must take into account its future interest, which it expects to choose so as to offset the future shock expected to be $E_2^{CB}\varepsilon_2$. Thus, even if the central bank is transparent, it must still form a view of its future action. [13]The above results can be summarized as follows:

Proposition 2 *Private and central bank expectations are no longer aligned under opacity. Yet, the interest rate rule is the same irrespective of whether the central bank announces or not its expected future interest rate.*

[13]Note that even though the interest rules are fomally the same under both transparency regimes, this does not imply the same rate interest and inflation rates. Indeed, the information sets of the central bank and of the private sector change with the transparency regime.

The resulting inflation rate in period 1 is:

$$\pi_1 = \frac{1}{\theta - 1}\left[\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right) - \theta\left(E_1^P\varepsilon_1 - \varepsilon_1\right) + \left(E_1^{CB}\varepsilon_1 - \varepsilon_1\right)\right] \tag{16}$$

which combines the forecast errors of both the private sector and the central bank. It follows that:

$$E_1^P\pi_2 = \gamma_3^{op}\left(\theta - 1\right)\left[\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right) - \left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right] \tag{17}$$

which shows the role of the doubt factor $\gamma_1^{op} - \gamma_2^{op} = \gamma_3^{op}\theta$: the private sector will not expect the central bank to eliminate inflation in period 2 unless $\gamma_3^{op} = 0$.

The loss function under central bank opacity is then:

$$\begin{aligned} L^{op} &= E\left(\pi_1\right)^2 + E\left(\pi_2\right)^2 \\ &= \frac{1}{\beta}\left[\left(\frac{1}{\theta - 1}\right)^2\left(\frac{1}{z} + \frac{1}{k} + \frac{1}{kz}\right) + \left(\frac{\theta}{\theta - 1}\right)^2 + \frac{1}{z}\frac{z + k\theta^2\left(1+z\right)}{z + k + k\theta^2\left(1+z\right)}\right] \end{aligned}$$

5 Welfare Analysis

We now compare welfare when the central bank reveals its expected interest rate - labeled transparency - and when it does not - labeled opacity. To do so we study the difference of welfare losses under the two regimes: $\Delta L = L^{op} - L^{tr}$. In spite of the model's extreme simplicity, we cannot derive an explicit condition that determines the sign of ΔL. Consequently, we proceed in three steps. In Section 5.1, we derive a sufficient condition for period 1 loss difference ΔL_1 to be positive; since $\Delta L_2 > 0$ this is also a sufficient condition for transparency to dominate opacity. Then, in Section 5.2, we provide a necessary condition for $\Delta L_1 < 0$. Finally, we present in Section 5.3 the results from the formal analysis of ΔL that is described in the Appendix.

5.1 Preliminary observation

We first compare the welfare losses separately period by period. Starting with period 2, we have:

$$\beta\Delta L_2 = L_2^{op} - L_2^{tr} = \frac{k^2\theta^2(1+z)}{z(z+k)\left[z + k + k\theta^2(1+z)\right]} > 0 \tag{18}$$

Proposition 3 *Transparency is always welfare-increasing in period 2.*

The reason is that the central bank is better informed when it can recover the private sector signal $\varepsilon_{1,2}^P$, see (9).

Thus, a sufficient condition for transparency to be welfare-improving is that the period 1 welfare difference $\Delta L_1 = L_1^{op} - L_1^{tr} \geq 0$. We have:

$$\beta\Delta L_1(\theta) = \left(\frac{1}{\theta - 1}\right)^2\frac{k+1+z}{kz} + \left(\frac{\theta}{\theta - 1}\right)^2 - \frac{1 + k(1+z)}{kz(1+z)} \tag{19}$$

A study of this expression as a function of θ, presented in the Appendix, yields the following sufficient condition for transparency to be welfare improving:

Proposition 4 *A sufficient condition for the release by the central bank of its expected future interest rate to be welfare-improving is that $z > \frac{1+k}{\sqrt{k}}$.*

The more precise is the central bank signal α relative to the private sector signal β - the higher is z - the more likely it is that transparency pays off. Conversely, if central bank information is of poor quality, i.e. when $z < \frac{1+k}{\sqrt{k}}$, the situation becomes ambiguous.[14]

The intuition is as follows. We have seen that transparency raises welfare because it allows for the exchange of early signals between the central bank and the private sector. We have also seen that opacity reduces welfare because it leads to an expectation discrepancy, which is larger the less the central bank signals can be trusted, i.e. the lower is z. Thus k and z act as complementary factors favoring transparency. A higher k means that transparency is achieved with a lower z.

5.2 Why may opacity raise welfare?

We now ask why opacity could ever reduce welfare. It might seem that more information is always better than less. This not necessarily true here since we have two agents, the central bank and the private sector, who strategically interact under heterogeneous information.[15] In order to see why less information may be welfare-increasing, remember first that welfare in period 2 is always higher under transparency because it provides the central bank with more information and therefore a more precise estimate of the period shock. For opacity to welfare-dominate transparency, therefore, it must be that it reduces inflation volatility in period 1 by enough to offset the welfare loss of period 2. From (3) we know that period 1 inflation is driven by expected inflation and the extent to which the central bank fails to stabilize output in period 1. Using (10), (6) and (15), (3) can be rewritten as:

$$\pi_1 = (1+\kappa)E_1^P\pi_2 - \kappa(r_1 + E_1^P r_2) + \varepsilon_1 = (2+\kappa)E_1^P\pi_2 - \psi \tag{20}$$

The "policy miss" term $\psi = \kappa r_1 - (\varepsilon_1 - E_1^P\varepsilon_2)$ measures the private sector's perception of the extent to which the central bank fails to achieve its period 1 objective when it optimally chooses r_1. Without any information about the private signals, the central bank's best forecast of $E_1^P\pi_2$ is $E_1^{CB}E_1^P\pi_2 = 0$, which explains (10). The policy miss term can be rewritten as:

$$\psi = (E_1^{CB}\varepsilon_1 - \varepsilon_1) + \left(E_1^P\varepsilon_2 - E_1^{CB}\varepsilon_2\right) \tag{21}$$

Let us now compare (20) under the two regimes. Under transparency, but not under opacity $E_1^P\pi_2 = 0$, so opacity tends to add volatility to period 1 inflation, the more so the higher is κ. Furthermore, we can show that $Var^{op}(\psi) > Var^{tr}(\psi)$. This is quite intuitive: the first term in (21), the period 1 signal error, is regime-invariant while the second term reflects disagreements between the central bank and the private sector. When it announces $E_1^{CB}r_2$, the central bank fully reveals $E_1^{CB}\varepsilon_2$ and therefore moves $E_1^P\varepsilon_2$ toward $E_1^{CB}\varepsilon_2$.

It follows that opacity always raises period 1 inflation as well, unless $cov(E_1^P\pi_2, \psi)$ under opacity is positive and large enough to offset the other two effects. Thus $cov(E_1^P\pi_2, \psi) > 0$ is a necessary, but not sufficient, condition for opacity to raise welfare. Formally $cov(E_1^P\pi_2, \psi) = \gamma_3^{op}(\theta - 1)\frac{k+1}{\alpha k}$ and the Appendix shows when it is satisfied. Here we provide an intuitive interpretation of this necessary condition.

Imagine that $\varepsilon_1 = \varepsilon_2 = 0$ but $E_1^{CB}\varepsilon_1 > 0$. Mistakenly expecting an inflationary shock, the central bank will raise the interest rate to deflate the economy. How will the private sector react when observing $r_1 > 0$? From (10) we know that the private sector may infer

[14] Note that for k ranging from 0 to 0.5, the threshold value $(1+k)/\sqrt{k}$ ranges from ∞ (when $k = 0$ and the early signals are useless) to 2.1 (when $k = 0.5$ and the early signals are as precise as their later updates).

[15] In Period 1, the central bank acts as a Stackelberg leader in setting r_1 and then the private sector reacts setting π_1 and the long-term interest rate. Then, in Period 2, the central bank reacts and sets r_2.

either that $E_1^{CB}\varepsilon_1 > 0$, or that $E_1^{CB}\varepsilon_2 < 0$, or both. Under transparency, the central bank fully reveals $E_1^{CB}\varepsilon_2 = \varepsilon_{1,2}^{CB}$ so there is no ambiguity. Under opacity, there is room for private sector misinterpretation, which paradoxically may raise welfare.

To see why, note that $E_1^{CB}\varepsilon_1$ combines early and contemporaneous signals while $E_1^{CB}\varepsilon_2 = \varepsilon_{1,2}^{CB}$ is purely an early signal. It follows that if k, the relative precision of early to contemporaneous signals, is low, the private sector will put more trust into the interpretation that $r_1 > 0$ because $E_1^{CB}\varepsilon_1 > 0$. Believing correctly that the high interest rate mostly reflects central bank information about the first period disturbance, the private sector will not change much its forecasts concerning the second period - $E_1^P\pi_2$ and $E_1^P\varepsilon_2$ will be approximately unaffected - and the policy miss will translate into a fall of π_1, with no offsetting effect. When k is low, opacity cannot improve welfare.

If k is large, the private sector will put more weight on the possibility that a high r_1 reflects next period signals, i.e. that $E_1^{CB}\varepsilon_2 < 0$. Believing - wrongly, as it turns out - that the central bank mistakenly expects a future deflationary shock and will attempt to reflate the economy in period 2 ($E_1^P r_2 < 0$), the private sector will expect inflation ($E_1^P\pi_2 > 0$). Since $\pi_1 = (1+\kappa)E_1^P\pi_2 - \kappa(r_1 + E_1^P r_2) + \varepsilon_1$, both forecast changes will offset the impact of the high interest rate r_1 on current inflation. In short, the private sector misinterprets a rise in the current interest rate as signaling an expansionary monetary policy in the future. Creative opacity may raise welfare when unavoidable forecast errors offset each other .[16]

We know from Propostion 4 that transparency always welfare-dominates when z is large. Indeed, in that case, knowing that the central bank information is precise - at least relative to its own - the private sector will not conclude that $E_1^{CB}\varepsilon_2 < 0$ is mistaken; rather it will almost fully trust the central bank to effectively eliminate inflation. The two non-alignment terms $E_1^P\pi_2$ and $\left(E_1^P E_1^{CB}\varepsilon_2 - E_1^{CB}\varepsilon_2\right)$ will be too small to offset the central bank forecast error. In the end, the necessary condition for opacity to raise welfare requires that k be not too small and that z be not too large.

Proposition 5 *(Creative opacity) A necessary condition for opacity to welfare-dominate transparency is that the private sector's own forecasts systematically offset the impact on inflation volatility of the central bank forecast errors. This can be case when the early signals are precise relative to contemporaneous signals and when the relative precision of the central bank information is not too large.*

The covariance condition is necessary, not sufficient, for two reasons. First, even if the correlation is positive, it may be too small to offset the adverse effect of opacity on period 1 inflation volatility. Second, even period 1 inflation volatility is reduced, this may not be enough to offset the fact that inflation in period 2 is always more volatile under opacity than under transparency.

Note the irony of creative opacity: it raises welfare when the central bank's expectation of an inflationary shock in period 1 leads the private sector to expect a contractionary shock in period 2. As a result, current inflation, which combines the mistaken contractionary effect of current monetary policy and the equally mistaken effect of an expected expansionary policy in the future, is less volatile than under transparency when it it only reflects the the mistaken contractionary effect of current monetary policy.

5.3 Welfare ranking

We have derived a sufficient condition for transparency to be desirable and a necessary condition for opacity to dominate. We now study how the condition $\Delta L > 0$ relates to

[16] Formally, even though $E_1^P\varepsilon_2 < 0$ as it partially moves toward what the private sector believes is $E_1^{CB}\varepsilon_2 < 0$, ψ increases because $E_1^{CB}\varepsilon_1 > 0$, and $E_1^P\pi_2 > 0$ so $cov(E_1^P\pi_2, \psi) > 0$.

the three model parameters z, κ and k. Figures 2 and 3 show our results; they are based on a detailed analysis presented in the Appendix. Figure 2 presents the situation when κ is low, Figure 3 corresponds to the case of a high κ. The shaded area corresponds to the case where $\Delta L < 0$, i.e. when opacity is creative and welfare-dominates transparency. The following proposition draws some general implications, which are further detailed and interpreted below.

Proposition 6 *A central bank that follows an optimal linear interest rule (11) raises welfare by revealing the future interest rate in the following cases:*

- when the central bank signal precision is high enough relative to the private signal precision (a high z).

- when the elasticity of current to expected inflation is large (high κ) and the relative signal and early signal precision are not too low (z and k not too low).

[Figures 2 and 3 about here]

The role of relative signal precision. We first look at the role of $z = \alpha/\beta$, the ratio of central bank signal precision α to private sector signal precision β. From Proposition 4 we know that when z is very large transparency dominates, including in period 1. Under opacity, as $z \to \infty$, $\gamma_3^{op}(\theta - 1) \to 0$, so $E_1^P \pi_2 = 0$: the private sector increasingly disregards its own signals and its expectations become aligned with those of the central bank as in the transparency regime. There is little to gain from opacity: from (18) we know that $\lim_{z\to\infty} \Delta L_2 = 0$ and, from (19), $\lim_{z\to\infty} \beta\Delta L_1 = \dfrac{1/k + \theta^2}{(\theta - 1)^2} > 0$ so transparency dominates when z is large enough. As z becomes smaller, the benefit from information disclosure declines as the private sector increasingly doubts any signal from the central bank, for good reason. Opacity increases welfare when, having observed the current interest rate r_1, the private sector sets prices and the long-term interest rate so as to systematically offset the effects of potential large central bank forecast errors.

The role of the elasticity of current to expected inflation. Parameter κ represents the channel through which future expected inflation and the long-term interest rate affect current inflation, see (3). It also determines the central bank decision about the current interest rate. Opacity raises the volatility of these variables because of the non-alignment of central bank and private expectations.

As κ increases, so does the impact of expected inflation and of the long-term interest rate on current inflation, with two consequences. First, it generally makes transparency more desirable. Second, the relative precision z of the central bank signal becomes more important. All in all, the opacity zone shrinks as we move from Figure 2 to Figure 3.

Conversely, when κ is small, the cost of non-alignment of expectations declines, which generally favors the welfare case for opacity. Opacity is desirable when the private sector emphasizes the early signal content of the interest rate to offset the central bank forecast error, i.e. when k is large. As a consequence, the opacity zone spreads to the right beyond $k = 1/2$ as we move from Figure 3 to Figure 2.

The role of early information precision. Remember that transparency leads to an exchange of signals between the central bank and the private sector: in period 1, the private sector discovers the central bank early signal $\varepsilon_{1,2}^{CB}$; in period 2, the central bank recovers the private sector early signal $\varepsilon_{1,2}^{P}$. The welfare value of this exchange of early signals is higher, the more precise they are, therefore the larger is k. This is why transparency is in general more desirable the more precise are the early signals. Yet, the welfare effect of k is not monotonous.

As k increases, more attention is paid by both the central bank and the private sectors to their own early signals. Under opacity, this heightened attention increases the expectation discrepancy, which is the source of a welfare loss. At the same time, because it interprets the current interest rate as conveying information on the central bank early signal when it sets the long-term rate, the private sector may offset the central bank forecast error, which improves welfare. The expectation discrepancy, which rises with k, directly hurts welfare but may be exploited to raise it indirectly. These two opposite effects are a source of non-monoticity.

Let first k tend toward zero - but remain strictly positive because the case $k = 0$ is degenerate. The early signals $\varepsilon_{1,2}^{CB}$ and $\varepsilon_{1,2}^{P}$ are imprecise and largely ignored. As a result, transparency is not particularly helpful but expectations are nearly aligned under opacity. In the end, there is little to choose between the two regimes in period 1. In period 2, under transparency the central bank still receives the signal $\varepsilon_{1,2}^{P}$ which, at the margin, helps it to make a better decision so that $\Delta L_2 > 0$ and transparency dominates.[17]

Consider now higher values of k. The non-monotous effect of k depends on the relative precision z of the central bank signal. When z is high, the expectation discrepancy is low; there is not much that can be exploited under opacity and transparency welfare-dominates. If z is low, the offsetting reaction effect of expectation non-alignment is large, which makes opacity more desirable. As k further rises, the value of exchanging increasingly precise signals comes to dominate and transparency becomes optimal again.

6 Conclusions

The result that the release of interest rate expectations may be desirable is not generally held, especially among practitioners. An articulate presentation of the case against transparency is provided by Goodhart (2005):

"If an MPC's non-constant forecast was to be published, there is a widespread view, in most central banks, that it would be taken by the public as more of a commitment, and less of a rather uncertain forecast than should be the case, (though that could be mitigated by producing a fan chart of possible interest rate paths, rather than a point estimate: no doubt, though, measuring rulers and magnifying glasses would be used to extract the central tendency). Once there was a published central tendency, then this might easily influence the private sector's own forecasts more than its own inherent uncertainty warranted, along lines analysed by Morris and Shin (1998, 2002, 2004). Likewise when new, and unpredicted, events occurred, and made the MPC want to adjust the prior forecast path for interest rates, this might give rise to criticisms, ranging from claims that the MPC had made forecasting errors to accusations that they had reneged on a (partial) commitment."

Part of the argument directly refers to Morris and Shin's common knowledge effect. We do not address this issue here because it has been shown to rest on highly unlikely assumptions. Indeed, it assumes that the central bank is relatively poorly informed (z is low) and that the central bank does not even reveal the current interest rate.[18]

Another part of the argument is that releasing the expected interest rate might lock the central bank into setting its interest rate in the future at forecasted level even though it is no longer desirable given newly available information. The justification is the classic

[17] As $k \to 0$, $\gamma_3^{op} \to 0$ and $\gamma_1^{op} - \gamma_2^{op} \to 0$. At the limit there is no expectation misalignment.

[18] See Svensson (2005b), Hellwig (2005) and Gosselin et al. (2006).

rules versus discretion argument in the presence of time inconsistency, as discussed in Woodford (2005). The private sector will not realize that the central bank's forecast is imprecise and, so goes the argument, any discrepancy between the pre-announced and the realized interest rate decision would reduce the central bank credibility. This does not happen here as we do not allow for time inconsistency: there is no inflation bias and the central bank preferences are known, an assumption that, we believe, is realistic.[19] Under these conditions, when it explicitly recognizes that the central bank forecast is imprecise, the private sector can still improve on both public and private signals by combining them. In our model, this Bayesian signal extraction mechanism is the source of a welfare gain, and the gain is larger the more precise are the early (i.e. one period ahead) signals. By assumption, the private sector knows that the actual interest rate will differ from its forecast; transparency raises welfare because it fully aligns the central bank and the private sector forecasts of future shock.

The effect is further enhanced when the real long-term interest rate has a strong impact on aggregate demand. Through this channel enter private sector expectations of future nominal interest rate and inflation. Under transparency, these expectations reflect only forecasting errors and therefore average out to zero, so the size of the elasticity of current to expected inflation is irrelevant. Under opacity, these expectations depend on the gap between private and central bank expectations, which does not average out to zero. As a consequence, when the elasticity of current to expected inflation rises, so does the variance of inflation under opacity but not under transparency since the private sector does not expect future inflation to deviate from its target. Intuitively, opacity raises the volatility of expected future inflation and therefore the volatility of current inflation.

While these results broadly support the release of central bank interest rate expectations, there can be cases when doing so reduces welfare. Proposition 6 states that this is the case when three conditions are satisfied: aggregate demand is relatively insensitive to the long-term real interest rate (low κ), early signals are imprecise relative to contemporary signals (k not too large), and the central bank signal precision is not too high relative to the private sector signal precision (z not too high). When these three conditions are jointly satisfied, opacity becomes welfare-improving because the expectations alignment discrepancy is negatively correlated with the central bank forecast errors.

Note that the three conditions must be jointly satisfied for opacity to be welfare improving. In contrast transparency is desirable when either the central bank relative signal precision is high or the elasticity of current to expected inflation is large. Relatively precise early signals are not enough to give transparency a hedge irrespective of the two other parameters, but precise early signals lower the thresholds beyond which central bank relative signal precision or the elasticity of current to expected inflation are large enough to favor transparency.

[19] The experience of the Bank of Norway is particularly interesting in this respect. Realizing that credibility is necessary to avoid misinterpretations of the difference between the forecasted and actual interest rate, the Bank of Norway is actively engaged in describing its preferences.

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Appendix

Proof of (13) and (14)

Using (11) note that $E_1^{CB}\varepsilon_1 = (r_1 - \nu\varepsilon_{1,2}^{CB})/\mu$ is a signal about ε_1. In period 1, the private sector observes $\dfrac{r_1 - \nu\varepsilon_2^P}{\mu} = E_1^{CB}\varepsilon_1 + \dfrac{\nu}{\mu}(\varepsilon_{1,2}^{CB} - \varepsilon_{1,2}^P)$, which is therefore also a signal about ε_1 available for the private sector with variance $\dfrac{1}{\alpha} + (\dfrac{\nu}{\mu})^2(\dfrac{1}{k\alpha} + \dfrac{1}{k\beta})$. Similarly, in period 1, the private sector observes $\dfrac{r_1 - \mu\varepsilon_1^P}{\nu} = \varepsilon_2^{CB} + \dfrac{\mu}{\nu}(E_1^{CB}\varepsilon_1 - E_1^P\varepsilon_1)$ which is a signal about ε_2 with variance $\dfrac{1}{\beta}\left[\dfrac{1}{kz} + \left(\dfrac{\mu}{\nu}\right)^2(1+\dfrac{1}{z})\right]$. Using these signals, we can apply Bayes Theorem to obtain:

$$E_1^P\varepsilon_1 = \frac{\left[k + (1+z)\left(\frac{\nu}{\mu}\right)^2\right]E_1^P\varepsilon_1 + kz\left[E_1^{CB}\varepsilon_1 - \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right]}{k(1+z) + (1+z)\left(\frac{\nu}{\mu}\right)^2}$$

$$E_1^P\varepsilon_1^{CB} = \frac{\left(\frac{\nu}{\mu}\right)^2 E_1^P\varepsilon_1 + k\left[E_1^{CB}\varepsilon_1 - \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right]}{k + \left(\frac{\nu}{\mu}\right)^2}$$

$$\begin{aligned} E_1^P\varepsilon_2 &= \frac{\left[k(1+z) + \left(\frac{\nu}{\mu}\right)^2\right]\varepsilon_{1,2}^P + z\left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right]}{(1+z)\left(k + \left(\frac{\nu}{\mu}\right)^2\right)} \\ &= \gamma_1\varepsilon_2^P + (1-\gamma_1)\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right] \end{aligned}$$

which defines $\gamma_1^{op} = \frac{k(1+z)+\left(\frac{\nu}{\mu}\right)^2}{(1+z)\left(k+\left(\frac{\nu}{\mu}\right)^2\right)}$.

$$E_1^P\varepsilon_2^{CB} = \frac{k\varepsilon_{1,2}^P + \left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right]}{k + \left(\frac{\nu}{\mu}\right)^2}$$

It follows that:

$$E_1^P\varepsilon_1 - E_1^P\varepsilon_1^{CB} = \frac{E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB})}{(1+z)\left(k + \left(\frac{\nu}{\mu}\right)^2\right)}$$

Recall (12):

$$\begin{aligned} E_1^P E_2^{CB}\varepsilon_2 &= \gamma_2^{op}\varepsilon_{1,2}^P + (1-\gamma_2^{op})\left[\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right] \\ &\quad -\gamma_3^{op}\left[E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon 1,_2^{CB}\right)\right] \end{aligned}$$

Using (3), (6) and (9), we can now compute π_1, which is necessary to obtain the signal extracted by the central bank at time 2 :

$$\begin{aligned}
\pi_1 &= (1+\kappa)\left(E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2\right) - E_1^P E_2^{CB}\varepsilon_2 - \kappa r_1 + \varepsilon_1 \\
&= (1+\kappa)\left(\gamma_1^{op}-\gamma_2^{op}\right)\left[\varepsilon_{1,2}^P - \left(\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right)\right] \\
&\quad +\gamma_3^{op}(1+\kappa)\left[E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1 + \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right] \\
&\quad -\kappa r_1 - \left[\gamma_2^{op}\varepsilon_{1,2}^P + \left(1-\gamma_2^{op}\right)\left(\varepsilon_{1,2}^{CB} - \frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right)\right] \\
&\quad -\gamma_3^{op}\left[\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right) + \frac{\nu}{\mu}\left(\varepsilon_{1,2}^P - \varepsilon_{1,2}^{CB}\right)\right] + \varepsilon_1
\end{aligned}$$

This expression can be rewritten as:

$$\frac{\pi_1 + \kappa r_1 - \varepsilon_1}{(1+\kappa)\left(\gamma_1^{op}-\gamma_2^{op}\right) - \gamma_2^{op} + (2+\kappa)\gamma_3^{op}\frac{\nu}{\mu}} + \theta\varepsilon_{1,2}^{CB} = \varepsilon_{1,2}^P + \theta\frac{\mu}{\nu}\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)$$

where $\theta = 1 + \frac{1}{(1+\delta)\left(\gamma_1^{op}-\gamma_2^{op}\right)-\gamma_2^{op}+(2+\delta)\gamma_3^{op}\frac{\nu}{\mu}}$. Now note that π_1 and ε_1 become known in period 2 (and r_1 is always known). It follows that the right hand-side in the previous expression is known to the central bank when period 2 starts and it can be used as a signal about ε_2.

We can now use Bayes rule to find $E_1^P E_2^{CB}\varepsilon_2$. Some computations lead to:

$$E_2^{CB}\varepsilon_2 = \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k+\alpha\right)\left(k\varepsilon_{1,2}^{CB} + (1-k)\varepsilon_{2,2}^{CB}\right) + k\left(\varepsilon_{1,2}^P + \frac{\mu}{\nu}\theta\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right)}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k+z+k\right)}$$

so that the composed expectation is given by:

$$E_1^P E_2^{CB}\varepsilon_2 = \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(\alpha+\beta)k+\alpha\right)\left(kE_1^P\varepsilon_{1,2}^{CB} + (1-k)E_1^P\varepsilon_{2,2}^{CB}\right) + k\beta\left(\varepsilon_{1,2}^P + \frac{\mu}{\nu}\theta E_1^P\left(E_1^P\varepsilon_1 - E_1^{CB}\varepsilon_1\right)\right)}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(\alpha+\beta)k+\alpha+k\beta\right)}$$

Using the expressions for the various private expectations, we can deduce by identification:

$$\begin{aligned}
\gamma_2^{op} &= \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k+z\right)\left(\frac{k^2}{k+\left(\frac{\nu}{\mu}\right)^2} + (1-k)\frac{\left(k(z+1)+\left(\frac{\nu}{\mu}\right)^2\right)}{k(z+1)+(z+1)\left(\frac{\nu}{\mu}\right)^2}\right)+k}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k+z+k\right)} \\
\gamma_3^{op} &= -\frac{\frac{\mu}{\nu}\theta\frac{k^2}{k+\left(\frac{\nu}{\mu}\right)^2}}{\left(\left(\frac{\mu}{\nu}\right)^2(z+1)\theta^2k+z+k\right)}
\end{aligned}$$

from which we find:

$$\gamma_1^{op} - \gamma_2^{op} + \gamma_3^{op}\theta\frac{\nu}{\mu} = 0$$

Proof of Proposition 2

The parameters for r_1 are found by minimizing the unconditional loss function $E(\pi_1)^2 + E(\pi_2)^2$.[20] Using (14), the previous expression for π_1 can be rewritten as:

$$\begin{aligned}\pi_1 &= \frac{1}{\theta-1}\left(\varepsilon_{1,2}^P-\varepsilon_2\right)+\frac{\mu}{\nu}\frac{\theta}{\theta-1}\left(E_1^P\varepsilon_1-\varepsilon_1\right)\\ &\quad +(\delta\nu+\frac{\theta}{\theta-1})\left[(\varepsilon_2-\varepsilon_{1,2}^{CB})+\frac{\mu}{\nu}(\varepsilon_1-E_1^{CB}\varepsilon_1)\right]+(1-\kappa\mu)\,\varepsilon_1-(1+\kappa\nu)\,\varepsilon_2\end{aligned}$$

which implies that:

$$E(\pi_1)^2=(1-\kappa\mu)^2\,E(\varepsilon_1)^2+(1+\kappa\nu)^2\,E(\varepsilon_2)^2 \ + \text{other terms}$$

where the other terms depend on k, α, β, μ and ν.

Similarly, note that $\pi_2=\varepsilon_2-E_2^{CB}\varepsilon_2$ and that $E_2^{CB}\varepsilon_2$ is optimally found by the central bank by using the signals $\varepsilon_{1,2}^{CB}$, $\varepsilon_{2,2}^{CB}$, and $\varepsilon_2^P+\theta\frac{\mu}{\nu}\left(E_1^P\varepsilon_1-E_1^{CB}\varepsilon_1\right)$ as indicated above, which gives:

$$E_2^{CB}\varepsilon_2=\frac{\left[\alpha+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2\right]\left[k\varepsilon_{1,2}^{CB}+(1-k)\varepsilon_{2,2}^{CB}\right]+k\beta\left[\varepsilon_{1,2}^P+\theta\frac{\mu}{\nu}\left(E_1^P\varepsilon_1-E_1^{CB}\varepsilon_1\right)\right]}{\alpha+k\beta+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2}$$

so that:

$$\pi_2=\frac{\left[\alpha+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2\right]\left[k(\varepsilon_2-\varepsilon_{1,2}^{CB})+(1-k)(\varepsilon_2-\varepsilon_{2,2}^{CB})\right]+k\beta\left[\varepsilon_{1,2}^P+\frac{\mu}{\nu}\theta\left(E_1^P\varepsilon_1-E_1^{CB}\varepsilon_1\right)\right]}{\alpha+k\beta+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2}$$

and $E(\pi_2)^2$ only includes terms in k, α, β, μ and ν. It follows that the total unconditionally expected loss under opacity can be written as:

$$L^{op}=(1-\kappa\mu)^2\,E(\varepsilon_1)^2+(1+\kappa\nu)^2\,E(\varepsilon_2)^2 \ + \text{other terms}$$

Since both ε_1 and ε_2 are assumed to be uniformly distributed, $E(\varepsilon_1)^2$ and $E(\varepsilon_2)^2$ are arbitrarily large relative to the other terms, in particular the variances α^{-2} and β^{-2}. It follows that the rule that minimizes L^{op} sets these terms equal to zero.

Proof of Proposition 4

The study of (19) shows that $\beta\Delta L_1(\theta)$ reaches a minimum of $\dfrac{kz^2-(1+k)^2}{kz(1+k)(1+z)}$ when $\theta=-\dfrac{1+k+z}{kz}$. This minimum is positive when $z>\dfrac{1+k}{\sqrt{k}}$.

[20] Unconditional because, if it were conditional on central bank information, the coefficients μ and ν would be nonlinear functions of $E_1^{CB}\varepsilon_1$ and $E_1^{CB}\varepsilon_2$ so the rule would not be linear - and impossible to derive in closed form.

Study of the of sign of $\gamma_3^{op}(\theta-1)$

Using the optimality condition $\frac{\mu}{\nu}=-1$, the parameters γ_2^{op}, γ_3^{op} and θ are jointly determined by the following conditions:

$$\gamma_3^{op}=\frac{k^2\theta}{(1+k)\left[k+z+k\theta^2(1+z)\right]}$$

$$\gamma_1^{op}-\gamma_2^{op}+\gamma_3^{op}\theta\frac{\nu}{\mu}=0$$

$$\theta=1+\frac{1}{(1+\delta)\left(\gamma_1^{op}-\gamma_2^{op}\right)-\gamma_2^{op}-(2+\delta)\gamma_3^{op}}$$

Noting that the value of γ_1^{op} is given in the text, we can use these equations to compute γ_2^{op} and γ_3^{op} as a function of θ, but we cannot explicitly solve for θ, which is determined by the following condition (found by computing γ_3^{op} from the above):

$$\frac{k^2\theta}{(1+k)\left[k+z+k\theta^2(1+z)\right]}=\frac{\gamma_1^{op}(\theta-1)+1}{(\theta-1)^2(2+\kappa)}$$

This is a third-degree equation in θ. Examining graphically this equation, we find that the roots are positive if and only if $kz+1-k\kappa-k<0$, and negative in the opposite case.[21] This is the curve is labeled "*Sign of* θ" in Figures 2 and 3. Moreover, when they are positive, the roots are greater than unity when $z>\widetilde{z}(k,\,\kappa)$. Since γ_3^{op} has the same sign as θ, $\gamma_3^{op}(\theta-1)$ has the same sign as $\theta(\theta-1)$. It follows that $\gamma_3^{op}(\theta-1)>0$ when the roots are either negative or positive and larger than unity. We conclude graphically that $\gamma_3^{op}(\theta-1)>0$ when either $kz+1-k\kappa-k>0$ or when $kz+1-k\kappa-k<0$ and $z>\widetilde{z}(k,\,\kappa)$.

Study of ΔL and explanation of Figures 2 and 3

Using the equations that jointly determine the parameters γ_2^{op}, γ_3^{op} and θ, we can write $\Delta L=L^{op}-L^{tr}$ as:

$$\beta\Delta L=\frac{A(\theta,k,z,\kappa)}{zk(\theta-1)^2(z+k)(2+\kappa)(1+z)}$$

where $A(\theta,k,z,\kappa)=a(k,z,\kappa)\theta^2+b(k,z,\kappa)\theta+c(k,z,\kappa)$ is a second-order polynomial in θ with determinant $\Delta(k,z,\kappa)$ and roots $\theta_1(k,z,\kappa)$ and $\theta_2(k,z,\kappa)$, with $\theta_1(k,z,\kappa)<\theta_2(k,z,\kappa)$. Obviously *sign* $\Delta L=$ *sign* $A(\theta,k,z,\kappa)$. Although we cannot compute analytically θ we note that $A(\theta,k,z,\kappa)>0$ when either $\Delta(\theta,k,z,\kappa)<0$ and $a(\theta,k,z,\kappa)>0$ or when $\Delta(\theta,k,z,\kappa)>0$, $a(\theta,k,z,\kappa)>0$ and either $\theta<\theta_1(k,z,\kappa)$ or $\theta>\theta_2(k,z,\kappa)$, or when $\Delta(\theta,k,z,\kappa)>0$, $a(\theta,k,z,\kappa)<0$ and $\theta_1(k,z,\kappa)<\theta<\theta_2(k,z,\kappa)$. These are the conditions that, along with the curve "*Sign of* θ" discussed in the previous section, lie behind the graphical analysis in Figures 2 and 3. More precisely, the figures are based on the following reasoning.

Define $z_1(k,\kappa)$ and $z_2(k,\kappa)$ such that $\Delta(k,z_1,\kappa)=0$ and $a(k,z_2,\kappa)=0$, respectively. It can be shown graphically that $z_1(k,\kappa)>z_2(k,\kappa)\ \forall k,\,\kappa$. It follows that $\Delta L>0$ when:

- $z>z_1(k,\kappa)$, $\Delta(k,z,\kappa)<0$ and $a(k,z,\kappa)>0$.

[21] When κ is very large, there might be negative roots when $kz+1-k\kappa-k<0$, but this does not invalidate the conclusions that follow.

•$z_1(k,\kappa) > z > z_2(k,\kappa)$, $\Delta(k,z,\kappa) > 0$, $a(k,z,\kappa) > 0$ and either $\theta < \theta_1(k,z,\kappa)$ or $\theta > \theta_2(k,z,\kappa)$.

•$z < z_2(k,\kappa)$, $\Delta(k,z,\kappa) > 0$, $a(k,z,\kappa) < 0$ and $\theta_1(k,z,\kappa) < \theta < \theta_2(k,z,\kappa)$.

In the last two cases, we need to check where θ lies with respect to the roots of $A(\theta,k,z,\kappa)$. As already mentioned, this cannot be done analytically. The shape of the opacity zone in Figures 2 and 3 has been determined from a graphical three-dimensional analysis using MathLab and is therefore not precisely known. The figures are also informed by the study of the following limit cases:

•When $k \to 0$, $\Delta L \sim \dfrac{z}{1+z} > 0$. Thus there is exist along the vertical axis a (possibly) thin vertical zone where transparency dominates $\forall$ κ, z.

•When $z \to 0$, there exists a function $k_1(\kappa)$ with $\partial k/\partial\kappa < 0$, such that $\Delta L > 0\ \forall k > k_1(\kappa)$. Thus along the horizontal axis, the opacity zone shrinks to the left as κ increases.

•When $\kappa \to \infty$, then $\theta \to 0$ and $\Delta L \sim \dfrac{2+z}{k(1+z)} > 0$. For a high value of κ transparency always dominates. Graphically in Figure 3, as κ becomes larger the opacity zone shrinks against the vertical transparency zone along the k axis described above in the limit case $k \to 0$.

•When $\kappa \to 0$, for a given value of $z < z_1(k,\kappa)$, $\Delta L > 0$ except for k small. Except for very low, $z_1(k,\kappa) > 1$. Graphically, relatively to Figure 3, in Figure 2 the opacity zone shrinks down and spreads along the horizontal axis beyond $k = 1/2$, except for the narrow band that corresponds to the limit case $k \to 0$. The opacity zone extends beyond the upper limit $k = 1/2$.

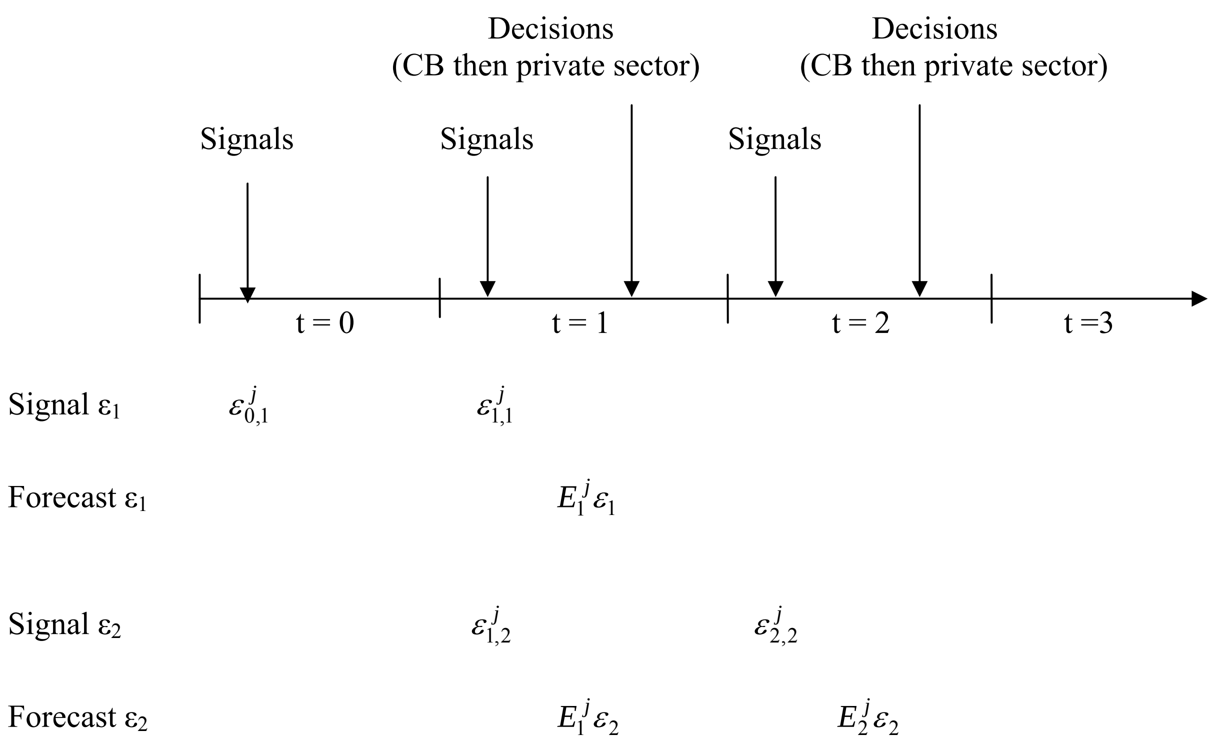

Figure 1. Timing of information and decisions

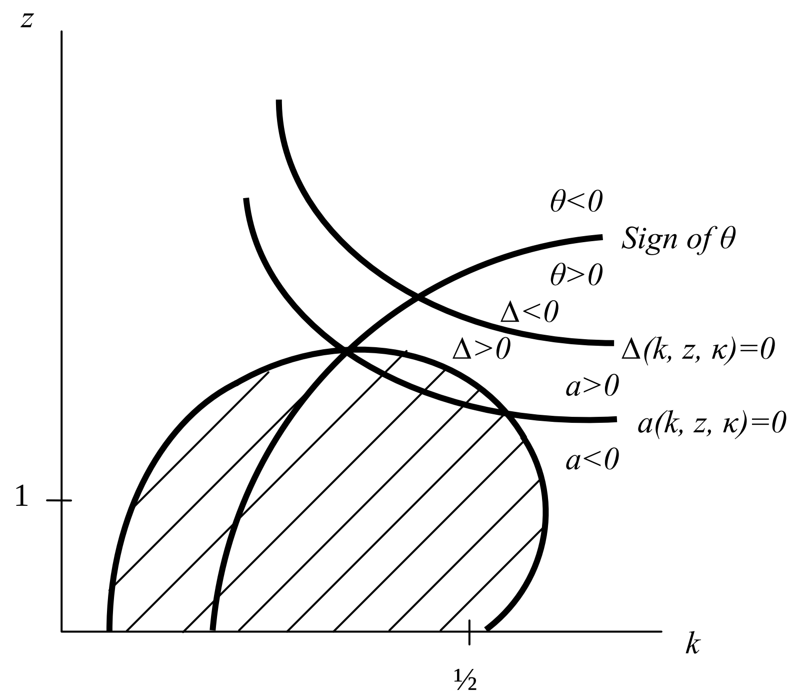

Figure 2. Low κ

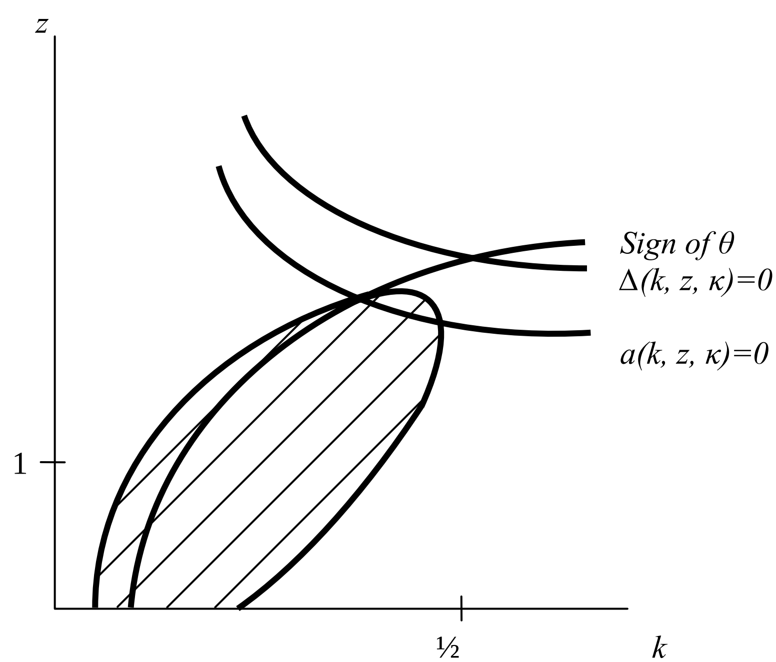

Figure 3. High κ