Uncertainty about Perceived Inflation Target and Monetary Policy*

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July 2007

Abstract

We analyse the interaction between private agents' uncertainty about inflation target and the central bank's data uncertainty. In our model, private agents update their perceived inflation target and the central bank estimates unobservable economic shocks as well as the perceived inflation target. Under those two uncertainties, the learning process of both private agents and the central bank causes higher order beliefs to become relevant, and this mechanism is capable of generating high persistence and volatility of inflation even though the underlying shocks are purely transitory. We also find that the persistence and volatility become smaller as the inflation target becomes more credible, that is, the private agents' uncertainty about inflation target (and hence the bank's data uncertainty) diminishes.

* The authors are grateful for comments from Nicoletta Baniti, Martin Bodenstein, Anton Braun, Shinichi Fukuda, Bernardo Guimaraes, Athanasios Orphanides, Nobuyuki Oda, Pedro Teles, Hiroshi Ugai, Takashi Ui, seminar participants at Bank of Canada, Bank of England, Bank of Japan, Bank of Portugal, BIS, Cambridge, ESSIM 2005, LSE, University of Tokyo, Hitotsubashi, Kobe, Osaka and Tohoku Universities, as well as participants at the Joint Deutsche Bundesbank/Federal Reserve Bank of Cleveland Conference. The authors also thank Naohisa Hirakata for his excellent research assistance. Aoki thanks hospitality of the Bank of Japan. The paper represents the views and analysis of the authors and should not be thought to represent those of the Bank of Japan.

Contents

1 Introduction

The objective of this paper is to analyse how uncertainty about inflation target affects the degree of uncertainty central bank faces and hence the stochastic properties of inflation. When the private agents are uncertain about inflation target, they generate their belief about the target, which we call 'perceived inflation target'. It is in general quite difficult for the bank to observe perceived inflation target. First of all, there are no direct measures of perceived inflation target that are available for central banks. As proxies, several survey measures of long-term inflation expectations are available, for example, the Michigan Survey and the Survey of Professional Forecasters in the case of the United States. Also, it is possible to extract information on inflation expectations from financial variables such as inflation-indexed bonds. However, private agents' inflation expectations are affected not only by their perceived inflation target but also by their expectations about future conditions of the economy, such as future demand and supply shocks. Therefore, the measures of inflation expectations are only noisy measures of perceived target. Recently, Bekaert et al. (2005) and Kozicki and Tinsley (2005) report their estimates of perceived inflation target in the United States. For the US economy in the late-1970s to early 1980s, Kozicki and Tinsley (2005) report that the perceived inflation target is estimated to be around 8%,[1] while Bekaert et al. (2005)'s estimate is around 14%. From policy point of view, the difference between 8% and 14% is too large — the policy prescription can be very different depending on which to believe. Those examples imply that it is not a trivial task to extract information about perceived inflation target. As a result, the private-sector uncertainty about inflation target creates uncertainty facing the bank about the perceived inflation target. In this paper, we focus our analysis on how this uncertainty distorts bank's stabilisation policy, and draw some implica-

[1] Their estimate of the target is around 3.5%.

tions for inflation dynamics.

It is shown that uncertainty about inflation target contributes to inflation persistence and volatility through two channels. One channel is the private-sector uncertainty about the inflation target. The private-sector learning about the target creates inflation persistence, as is shown in Erceg and Levin (2003). The other, which is our main focus, is the central-bank uncertainty about the perceived inflation target. This uncertainty makes it more difficult for the bank to estimate the state of the economy, leading policy errors. Thus the degree of uncertainty facing the bank is endogenously determined by the credibility problem. In other words, imperfect credibility is bad not only because it fails to fix inflation expectations, but also because it makes the bank's stabilisation policy more difficult.

An intuition behind this can be obtained from the following example. Consider an economy in which the bank wishes to keep track of the natural interest rate, which is the equilibrium real interest rate under flexible prices.[2] By keeping track of the natural rate, the bank can offset the effects of changes in the natural rate on inflation. Suppose now that the bank observes an increase in nominal interest rates. There are two possible reasons. One reason is that the private agents may have revised their expectations about the inflation target. Another reason is that future natural interest rate may have increased. When inflation target is perfectly credible, then the first reason would disappear. If the bank knows that inflation expectation is pinned down by its inflation target, it could infer the natural interest rate from nominal interest rates. However, when the inflation target is not fully credible, the bank cannot tell if the observed increase in the nominal rate is due to a revise in the perceived inflation target or due to a change in the natural rate. This can de-stabilise inflation. With this intuition, we show that under imperfect credibility the learning process of both private agents and the central bank causes higher order beliefs to become relevant, thus increasing the persistence

[2] In section 2 we will consider a model of this kind.

and volatility of inflation. This mechanism is capable of generating high persistence and volatility even though the underlying shocks are purely transitory.[3]

Our model can naturally connect the literature on monetary policy under data uncertainty and that on imperfect credibility. In a series of papers, Orphanides (2001, 2002, 2003) show that the mismeasurement of economic activity, such as the output gap and natural unemployment rate, was responsible for the 'Great Inflation' of the 1970s-1980s in the United States. While his work takes the measurement problem as exogenously given, our paper shows that the measurement problem can arise endogenously from lack of credibility.

Our model also predicts that the measurement problem can decrease as monetary policy gains more credibility. This implies that inflation becomes less volatile and less persistent. In recent years, a large amount of literature has documented changing stochastic properties of inflation and output—the so-called 'Great Moderation'. For example, Benati (2004) reports that output and inflation in the UK has become less persistent and volatile since the UK introduction of inflation target in 1992. Our model can explain this empirical fact. One of the controversial issues in the literature is whether the Great Moderation was caused by good luck (shocks became smaller) or good policy. The existing literature has mainly supported the good-luck hypothesis (See, for example, Ahmed et al. (2004), Stock and Watson (2003)). However, Bernanke (2004) claims that the econometric technique used in the literature may confuse good policy with good luck. His insight is that the existing literature does not take into account the impact of systematic component in monetary policy on inflation expectations. That implies that any fluctuations caused by de-anchored expectations may get confused with genuine non-policy shocks. In our model the change in the stochastic properties of inflation is purely

[3] Woodford (2002) and Amato and Shin (2003) consider the roles of higher order beliefs in monetary models. They consider higher order beliefs among firms under strategic complementarity and how this setting generates persistent effect of monetary policy. On the contrary, we focus on the interaction of beliefs between private agents and the central bank.

driven by learning by both central bank and private agents. By using stochastic simulation based on our model, we confirm Bernanke's conjecture.

The structure of the paper is as follows. The next section layouts the model. Section 3 derives equilibrium, and Section 4 analyses inflation dynamics in our model and draws policy implications. Section 6 concludes.

2 Model

We present a very simple model of inflation determination. It consists of two equations: a Fisher equation that relates the nominal interest rate and the natural interest rate (equilibrium real interest rate), and a monetary policy rule that sets the nominal interest rate as the instrument of monetary policy. The model can be interpreted as a model of inflation determination in an economy under flexible prices.[4]

2.1 Structural equations

The first equation in our model is the linearised Fisher equation of the form:

$$i_t = r_t^n + E_{t|p}\pi_{t+1}, \tag{1}$$

where i_t is nominal interest rate at time t, r_t^n is the natural interest rate at time t, π_t is inflation rate at time t. The expectation operator $E_{t|p}$ represents the expectation conditional on the private-sector information at time t. The information set is defined in Section 2.2.

[4] See, for example, Woodford (2003), Chapter 2.

Equation (1) can be interpreted as a log-linear approximation to the Euler equation of the representative household in an endowment economy with flexible prices. A log-linearised Euler equation in such an economy is given by

$$i_t = \sigma[(E_{t|p}y_{t+1} - y_t) - (E_{t|p}g_{t+1} - g_t)] + E_{t|p}\pi_{t+1}, \tag{2}$$

where y_t and g_t are respectively the log-deviation of exogenous output and demand shock from their steady state values, and σ is the inverse of the elasticity of intertemporal substitution. By denoting

$$r_t^n \equiv \sigma[(E_{t|p}y_{t+1} - y_t) - (E_{t|p}g_{t+1} - g_t)], \tag{3}$$

we obtain (1). Notice that r_t^n represents the equilibrium real interest rate under flexible prices, that is, the natural interest rate.[5]

The second equation in the model is a monetary policy rule. It is assumed that the central bank chooses the nominal interest rate by following the simple monetary policy rule:

$$i_t = \phi(\pi_t - \pi_t^*) + \pi_t^* + E_{t|c}r_t^n, \quad \phi > 1. \tag{4}$$

Here π_t^* is the central bank's inflation target at time t, and $E_{t|c}$ is the expectation operator conditional on the bank's information set at time t. The information set of the bank is specified in Section 2.2. Equation (4) assumes that the bank tries to keep track of the path of the natural interest rate r_t^n conditional on its information set. We interpret this term as representing the stabilisation policy of the bank. As is shown in Section 2.3, the bank can offset the effect of the changes in r_t^n on inflation by keeping track of r_t^n. Finally, by assuming $\phi > 1$, the monetary policy rule (4) satisfies the so-called Taylor principle.

[5]See, for example, Woodford (2003) for this concept.

We assume that the inflation target consists of the long-run component (π^*), and the transitory shock (ε_t):

$$\pi_t^* = \pi^* + \varepsilon_t, \tag{5}$$

where ε_t is i.i.d. with mean zero.This assumption is similar to Erceg and Levin (2003), who also assume that the inflation target varies due to a combination of transitory and highly persistent shocks.[6] Combining (4) and (5), one obtains

$$i_t = \phi(\pi_t - \pi^*) + \pi^* + E_{t|c} r_t^n + u_t, \tag{6}$$

where

$$u_t \equiv (1 - \phi)\varepsilon_t.$$

2.2 Information structure

Now we specify the information structure. On one hand, the private agents are assumed to know the monetary policy rule (4), including the parameter value for ϕ. They observe inflation rate π_t, the nominal interest rate i_t and the natural rate r_t^n. Later we assume that the central bank cannot observe the natural rate. As is shown in Aoki (2006), this assumption can be justified as the equilibrium information structure of a certain island economy.[7]The private agents also observe the bank's estimate of r_t^n at time t that is denoted by $E_{t|c} r_t^n$. The underlying assumption is the situation in which the bank publishes its assessment of the current economic conditions.[8] Following Erceg and Levin (2003), we assume that the pri-

[6]They assume that the auto-regressive root of the persistent shock is 0.999.

[7]Aoki (2006) considers an island economy where information is dispersed such that no one can observe aggregate variables directly. However, the paper shows that the equilibrium of this economy is equivalent to that of economy where there exits the representative household who has perfect information regarding the aggregate states while the central bank does not.

[8]By equation (3), this assumption is equivalent to the assumption that the bank announces its forecasts of the growth rates of demand shocks (such as the government expenditure) and

vate agents cannot directly observe the underlying components of π_t^*, while they can infer π_t^* from knowledge of the bank's reaction function. This may be either because the bank does not explicitly announce its long-run target π^*, or because the target is not fully credible. In this case, for example, when they observe a high nominal interest rate, they cannot tell whether this is because π^* is low or u_t is high. In each period, they update their belief about π^*. Let $E_{t|p}\pi^*$ be the perceived inflation target at time t conditional on the private-sector information at time t.

On the other hand, the bank knows the form of the IS equation (2), including the value of σ. It observes inflation, the nominal interest rate and the inflation target. However it cannot observe the natural rate r_t^n and the private-sector inflation expectations $E_{t|p}\pi_{t+1}$. In our model, the latter assumption is equivalent to the assumption that the bank cannot observe the perceived inflation target $E_{t|p}\pi^*$. This implies that, when the nominal interest rate is high, the bank cannot tell whether this is because the natural rate is high or inflation expectation is high. In each period, the bank estimates the natural rate and the perceived target. Let $E_{t|c}r_t^n$ and $E_{t|c}E_{t|p}\pi^*$ respectively denote the bank's estimate of the natural rate and the perceived target conditional on the bank's information set at time t.

In equilibrium, the endogenous variables $\{i_t, \pi_t\}_{t=0}^{\infty}$ satisfies equations (1) and (6), taking the exogenous variables ($\{r_t^n, u_t\}_{t=0}^{\infty}$) as given. Expectations of each of the bank and the private agents are rational conditional on each of the information set. Our main objective is to analyse the interaction between the uncertainty facing the private sector and the uncertainty facing the central bank, and to draw its implications for inflation dynamics. In order to be able to obtain clear results, let us focus on the simplest case. From now on, we assume the natural interest rate and monetary policy shock are i.i.d. normal and independent

output. Publishing those forecasts are not uncommon in practice. For example, the Bank of England publishes its forecast of GDP in the *Inflation Report*.

from each other. [9] More specifically, we assume

$$r_t^n \sim N(0, \sigma_r^2), \tag{7}$$

$$u_t \sim N(0, \sigma_u^2). \tag{8}$$

It is assumed that the distributions are known and common knowledge.

2.3 Equilibrium when inflation target is observable

As the benchmark case, it is useful to discuss the rational expectations equilibrium when π^* is known and common knowledge. In this case, the agents can also infer u_t from the monetary policy rule (6), implying that the agents have perfect information. It is then shown that information is fully revealed to the bank as well as to the private agents in equilibrium. In order to show this, suppose information is fully revealed in equilibrium. This implies that both the bank and the private agents can infer time-t variables. It also implies that for any future variables x_{t+i}, $E_{t|c}x_{t+i} = E_{t|p}x_{t+i} \equiv E_t x_{t+i}$, where E_t denotes the expectations operator under full information. In particular, the bank can infer r_t^n in equilibrium so that $E_{t|c}r_t^n = r_t^n$. Then, from (1) and (6), it is straightforward to show that the equilibrium inflation is given by

$$\pi_t = \pi^* - \phi^{-1}u_t. \tag{9}$$

Since monetary policy satisfies the Taylor principle, the equilibrium is uniquely determined. Because we assume u_t is white noise (8), equation (9) implies that $E_t\pi_{t+1} = \pi^*$. The bank thus infer that inflation expectations are pinned down

[9] We can extend our analysis to the case in which the natural rate follows an AR(1) process, which does not change our main results. The results in the the case of AR(1) are available upon request.

by π^* even if it cannot directly observe expectations. Then, from (1) it can infer r_t^n because $E_t\pi_{t+1}$ is identified. Therefore we have confirmed that information is fully revealed to the bank even though r_t^n is not directly observable. In other words, when π^* is credible, the bank is not subject to uncertainty regarding the measurement of the natural rate.

Equation (9) implies that inflation fluctuation does not involve any persistence when u_t is white noise. The other disturbance, r_t^n, does not affect inflation fluctuations because the bank fully offsets its effects on inflation. In this sense, the bank's stabilisation policy works perfectly. To summarise, once the bank's inflation target is known to the private sector and becomes common knowledge, all of the uncertainty in the model disappears. Now let us analyse the equilibrium when π^* is not credible.

3 Equilibrium with uncertain inflation target

3.1 Equilibrium given belief

Although beliefs are determined endogenously in equilibrium, it is useful to see how inflation depends on the beliefs of the central bank and the private agents. Complete analysis of the equilibrium is in Section 3.3. From (1) and (6), inflation is given by

$$\pi_t = \phi^{-1}\left[(\phi-1)\pi^* - u_t\right] + \phi^{-1}(r_t^n - E_{t|c}r_t^n) + \phi^{-1}E_{t|p}\pi_{t+1}. \tag{10}$$

Equation (10) shows that π_t depends on the deviation of r_t^n from the bank's estimate $E_{t|c}r_t^n$. If one takes expectation of the both sides of (10) conditional on $E_{t|c}$, one obtains[10]

$$\pi_t = \phi^{-1}\left[(\phi - 1)\pi^* - u_t\right] + \phi^{-1}E_{t|c}E_{t|p}\pi_{t+1}. \tag{11}$$

By solving this equation forward, the bounded solution for π_t in terms of the bank's belief is given by[11]

$$\pi_t = \pi^* - \phi^{-1}u_t + \phi^{-1}(E_{t|c}E_{t|p}\pi^* - \pi^*). \tag{12}$$

Compare (12) with (9). Equation (12) can be reduced to (9) when π^* is common knowledge. In contrast, under our information structure, π_t depends on the second-order belief, namely, the bank's belief about $E_{t|p}\pi^*$. Intuitively, the bank's information problem is the inability to distinguish r_t^n and $E_{t|p}\pi_{t+1}$, and the latter depends on $E_{t|p}\pi^*$. An imprecise estimate of $E_{t|p}\pi^*$ results in an imprecise estimate of r_t^n. This represents the negative feedback effect from the private-sector uncertainty about inflation target to the bank's estimate of the state of the economy (r_t^n).

Alternatively, one can take expectations of the both sides of (10) conditional on $E_{t|p}$ and solve forward to obtain

$$\pi_t = \pi^* - \phi^{-1}u_t + \phi^{-1}(E_{t|p}\pi^* - \pi^*) + E_{t|p}\sum_{j=0}^{\infty}\phi^{-(j+1)}\left[r_{t+j}^n - E_{t+j|c}r_{t+j}^n\right] \tag{13}$$

This expression implies that inflation depends on the perceived inflation target and the private-sector expectations about the bank's current and future estimation errors of the natural rates.

[10] Here we use the fact that $E_{t|c}\pi_t = \pi_t$.

[11] Here we use the fact that $lim_{j\to\infty}[E_{t|c}E_{t|p}E_{t+1|c}E_{t+1|p}...E_{t+j|c}E_{t+j|p}]\phi^{-j-1}\pi_{t+j} = 0$ when π_t is bounded

The interaction between the uncertainty facing private agents and that facing the bank also has implications for inflation expectations. Equation (12) implies that

$$\pi_{t+1} = (1-\phi^{-1})\pi^* - \phi^{-1}u_{t+1} + \phi^{-1}E_{t+1|c}\pi^*_{t+1|p}.$$

Therefore, inflation expectation $E_{t|p}\pi_{t+1}$ is given by

$$E_{t|p}\pi_{t+1} = E_{t|p}\pi^* + \phi^{-1}E_{t|p}\left(E_{t+1|c}E_{t+1|p}\pi^* - E_{t+1|p}\pi^*\right). \tag{14}$$

Equation (14) shows that $E_{t|p}\pi_{t+1} \neq E_{t|p}\pi^*$. Inflation expectation depends on the third-order belief, that is, how the private agents expect that the central bank will mis-estimate the perceived inflation target in the future. Inflation at time t depends on the private-sector expectations about future monetary policy, and therefore it depends on the private-sector expectation about how the bank will mis-estimate the natural rate in the future, as is shown in equation (13). The bank's estimate of the natural rates is related with its estimate of the perceived inflation target. Note that $E_{t|p}E_{t+1|c}r^n_{t+1} \neq 0$ even if $E_{t|p}r^n_{t+1} = 0$. When the private agents expect that the bank's estimate about the perceived inflation target is different from their own perceived target, they will expect that the bank will incorrectly estimate the natural rates. In other words, uncertainty about $E_{t|p}\pi^*$ distorts the bank's estimation of r^n_t, and the private agents understand this information problem the bank faces.

Now we turn to the filtering by private agents and the central bank.

3.2 Evolution of the private-sector belief

In each period, the private agents update their perceived long-run inflation target π^*. From monetary policy rule (6), the observation equation for the private agents is given by

$$i_t - \phi \pi_t - E_{t|c} r_t^n = (1-\phi)\pi^* + u_t. \tag{15}$$

The left hand side of equation (15) is observable, and the right hand side is not. In equation (15), we use the assumption that the bank's estimate of r_t^n is observable. Denote by z_t the linear combination of the observable variables of the private sector:

$$z_t \equiv i_t - \phi \pi_t - E_{t|c} r_t^n.$$

Under assumption (8), the distribution of z_t is given by

$$z_t \sim N\left((1-\phi)\pi^*, \sigma_u^2\right). \tag{16}$$

The filtering problem of the private agents is to distinguish the transitory component u_t from the constant term $(1-\phi)\pi^*$. This is a classic inference problem from a normal distribution with unknown mean and known variance. From now on, it is useful to work with 'precision,' $\gamma_u \equiv 1/\sigma_u^2$, rather than variance σ_u^2. Let the initial prior (at time 0) on $(1-\phi)\pi^*$ be

$$(1-\phi)\pi^* \sim N\left((1-\phi)\pi^*_{0|p}, 1/\tau_{0|p}\right), \tag{17}$$

where $\pi^*_{0|p}$ is the initial prior about π^*, and $\tau_{0|p}$ is the initial precision. Then the posterior mean after t observations is given by (see DeGroot (1970))

$$E_{t|p}\pi^* = \frac{\tau_{0|p}\pi^*_{0|p} + t\gamma_u(1-\phi)^{-1}\bar{z}_t}{\tau_{0|p} + t\gamma_u}, \tag{18}$$

where $\bar{z}_t \equiv \frac{1}{t}\sum_{s=1}^t z_s$. Define

$$a_t \equiv \frac{\tau_{0|p}}{\tau_{0|p} + t\gamma_u}.$$

Then, one can rewrite (18) as

$$E_{t|p}\pi^* = a_t \pi^*_{0|p} + \frac{1-a_t}{1-\phi}\bar{z}_t.$$

This means that $E_{t|p}\pi^*$ is a weighted average of the initial belief $\pi^*_{0|p}$ and the news $\bar{z}$. Furthermore, since $z_t = (1-\phi)\pi^* + u_t$, (18) can be expressed as

$$E_{t|p}\pi^* = a_t \pi^*_{0|p} + (1-a_t)\pi^* + \frac{1-a_t}{1-\phi}\bar{u}_t, \tag{19}$$

where $\bar{u}_t \equiv \frac{1}{t}\sum_{s=1}^{t} u_s$. Notice that $a_t \to 0$ as $t \to \infty$. Also, $\bar{u}_t \to 0$ by the law of large numbers. Therefore, as the private agents observe more information over time, they will eventually learn π^*. Alternatively, we can write (19) in a recursive form:

$$E_{t|p}\pi^* - \pi^* = b_t(E_{t-1|p}\pi^* - \pi^*) + \frac{1-b_t}{1-\phi}u_t, \tag{20}$$

where

$$\begin{aligned} b_t &\equiv \frac{\tau_{t-1|p}}{\tau_{t-1|p} + \gamma_u}, \\ \tau_{t|p} &\equiv \tau_{t-1|p} + \gamma_u. \end{aligned}$$

Equation (20) gives the evolution of the perceived inflation target.

3.3 Evolution of the bank's belief

Next, we compute the optimal filtering problem by the central bank. The bank's observation equation is given by the Fisher equation (1). The bank's observable variable is the nominal interest rate i_t, and the unobservable variables are r_t^n and $E_{t|p}\pi_{t+1}$ that is given by (14). Notice that $E_{t|p}\pi_{t+1}$ is determined endogenously in equilibrium. Here it is necessary to solve the filtering problem and equilibrium simultaneously, since equilibrium depends on the bank's policy, and the bank's policy depends on its estimate of the natural interest rate.[12] This estimate in turn depends on the statistical properties of the bank's observables and the natural rate in equilibrium. In Appendix A, it is shown that the evolution of the bank's estimation error of the natural rate is given by

$$r_t^n - E_{t|c} r_t^n = d_t \frac{B_t}{B_{t-1}} \left(r_{t-1}^n - E_{t-1|c} r_{t-1}^n \right) + (1 - d_t) r_t^n, \tag{21}$$

where B_t and d_t are time-varying deterministic parameters that are given in Appendix A. Equivalently, it is shown in Appendix A the evolution of the bank's estimation error of the perceived inflation target can be written as

$$E_{t|c} E_{t|p} \pi^* - E_{t|p} \pi^* = d_t b_t \left(E_{t-1|c} E_{t-1|p} \pi^* - E_{t-1|p} \pi^* \right) + (1 - d_t) \frac{a_t}{B_t} r_t^n. \tag{22}$$

There is a close relationship between those estimates. Indeed, Appendix A shows that the relationship is given by

$$E_{t|c} E_{t|p} \pi^* - E_{t|p} \pi^* = \frac{a_t}{B_t} \left(r_t^n - E_{t|c} r_t^n \right). \tag{23}$$

Equation (23) implies that imprecise estimate of the perceived inflation target results in imprecise estimate of the natural rate. This is due to the bank's inability to distinguish inflation expectations and the natural rate when it observes the nominal interest rate.

[12] See, Aoki (2003) and Svensson and Woodford (2003) for optimal filtering in forward looking models.

4 Inflation dynamics under uncertain inflation target

4.1 Variance and autocorrelation of inflation

Now we are able to complete our analysis of inflation dynamics. Using (12) and (23), Equilibrium inflation can be written as

$$
\begin{aligned}
\pi_t &= \pi^* - \phi^{-1} u_t + \tilde{\pi}_t, \quad \text{where} \qquad (24) \\
\tilde{\pi}_t &\equiv \phi^{-1} \left[(E_{t|p}\pi^* - \pi^*) + (E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^*) \right] \\
&= \phi^{-1}(E_{t|p}\pi^* - \pi^*) + \phi^{-1}\frac{a_t}{B_t}\left(r_t^n - E_{t|c}r_t^n\right)
\end{aligned}
$$

where $E_{t|p}\pi^* - \pi^*$ and $r_t^n - E_{t|c}r_t^n$ respectively evolve according to (20) and (21). The first two terms ($\pi^* - \phi^{-1}u_t$) of equation (24) are identical to (9), that is, the equilibrium inflation when the inflation target is credible.The term $\tilde{\pi}_t$ represents inflation fluctuations that are caused by the credibility problem. The term $\phi^{-1}(E_{t|p}\pi^* - \pi^*)$ represents fluctuations that are caused by private-sector uncertainty about the inflation target. When the perceived inflation target deviates from the true target, fluctuations in inflation expectations add more fluctuations to the equilibrium inflation. The term $\phi^{-1}a_t/B_t\left(r_t^n - E_{t|c}r_t^n\right)$represents the fluctuations caused by central-bank uncertainty about the perceived target. This term arises from the bank's inability to distinguish the natural rate and inflation expectations when inflation target is not credible.

Equation (20) implies that $E_{t|p}\pi^* - \pi^* \to 0$ as $t \to \infty$, therefore that the private-sector uncertainty diminishes over time and the private agents will eventually learn about π^*. This in turn implies that the uncertainty facing central bank also diminishes over time. Therefore, the contribution of $\tilde{\pi}_t$ to overall inflation persistence and volatility decrease over time and $\pi_t \to \pi^* - \phi^{-1}u_t$ as $t \to \infty$. We can compute the variance and autocorrelation of inflation. In Appendix B, those are given by

$$V[\pi_t] = \phi^{-2}\left[\frac{\sigma_u^2}{(1-\phi)^2}\left\{(b_t-\phi)^2+\sum_{k=0}^{t-1}\left(\prod_{j=k}^{t-1} b_{t+j}^2\right)(b_k-1)^2\right\}\right.$$
$$\left.+\sigma_r^2\left\{h_t^2+\sum_{k=0}^{t-1}\left(\prod_{j=k}^{t-1} g_{t+j}^2\right)h_k^2\right\}\right], \quad (25)$$

$$Cov[\pi_t,\pi_{t-1}] = \phi^{-2}\left[\frac{b_t\sigma_u^2}{(1-\phi)^2}\left\{(b_{t-1}-1)(b_{t-1}-\phi)+\sum_{k=0}^{t-2}\left(\prod_{j=k}^{t-2} b_{j+1}^2\right)(b_k-1)^2\right\}\right.$$
$$\left.+g_t\sigma_r^2\left\{h_{t-1}+\sum_{k=0}^{t-2}\left(\prod_{j=k}^{t-2} g_{t+1}^2\right)h_k^2\right\}\right], \quad (26)$$

where $g_t \equiv b_t d_t$ and $h_t \equiv \frac{a_t}{B_t}(1-d_t)$. Since $\lim_{t\to\infty} g_t = 1$, $\lim_{t\to\infty} b_t = 1$ and $\lim_{t\to\infty} h_t = 0$, it is easily shown that the variance and autocorrelation satisfy[13]

$$\lim_{t\to\infty} V[\pi_t] = (\sigma_u/\phi)^2, \quad \lim_{t\to\infty} Cov[\pi_t,\pi_{t-1}] = 0,$$

which correspond to the variance and autocorrelation of the inflation process when inflation target is observable (equation (9)).

4.2 A reduced-form estimation of model-simulated data

In order to investigate the properties of inflation in our model further, we conduct a simple stochastic simulation of our model. We generated 1000 sets of artificial normally-distributed shocks of 40 periods in each set, and obtained the stochastic process of the inflation under the learning process of the private-sector and the central bank.

Then, we consider the reduced-form dynamics of inflation by estimating an AR

[13] See Appendix B.

model, in line with Ahmed et al. (2004); Stock and Watson (2002).

$$\pi_t = c + \alpha(L)\pi_{t-1} + \eta_t, \quad Var[\eta_t] = \sigma_\eta^2. \tag{27}$$

By model selection criterion (AIC), we set the AR lag order to one. The estimated AR parameter α , the standard deviation of the innovation σ_η^2, and the variance of inflation $Var[\pi_t]$ are calculated for each data set, and the summary statistics are averaged over 1000 sets. In order to examine how these statistics change under the learning process of the private-sector and the central bank, an AR model is estimated over the first 20 periods and the second 20 periods separately. Note that our data frequency should be interpreted as annual rather than quarterly, since our model is a flexible price model.

In the simulation, several parameters must be specified. We choose the central bank's inflation target, π^*, of 2%, and the private-sector prior in (17), $\pi^*_{0|p}$, of 10%. According to the analysis of Kozicki and Tinsley (2001, 2005), these values are roughly in line with the US economy at the beginning of 1980s. With regard to the uncertainty that the central bank initially faces, $E_{0|c}\pi^*_{0|p} - \pi^*_{0|p}$, we assume that it ranges from zero to five.[14] The standard deviation of monetary policy shock, σ_u, is set to 1%, and hence $\gamma_u = 1$. According to Roberts (2004), this is in line with the FED policy between 1960 and 1983. The standard deviation of natural rate of interest, σ_r, is set to 1.5%, and hence $\gamma_r = 0.44$. This value is based on the result of Laubach and Williams (2003), who estimate σ_r as between 1.1 and 2.6 using US data. The policy coefficient, ϕ, is set to 1.5, like Taylor rule. Finally, we set the precision of initial prior, both $\tau_{0|p}$ and $\tau_{0|c}$, to one. In Appendix C we examine some robustness against changing those parameters. Finally, notice that we have assumed the structural shocks r_t^n and u_t are white

[14]This range is in line with differences between the estimate of Bekaert et al. (2005) and Kozicki and Tinsley (2005).

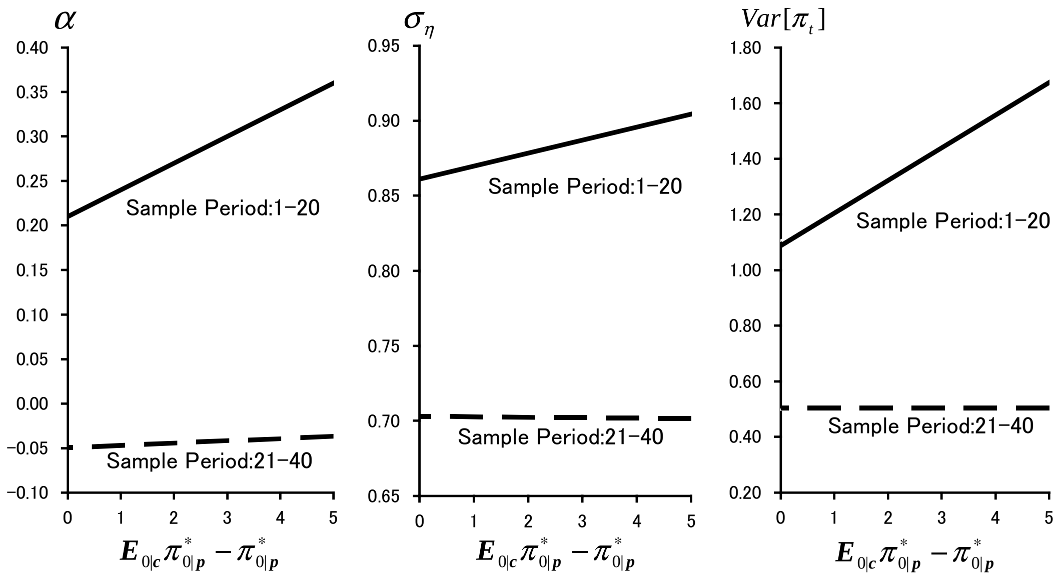

Figure 1: Time-varying persistency and volatility

noise processes. Therefore, the persistence reported below are purely driven by learning by central bank and private agents. In reality, the natural rate can be a persistent process. This implies that the persistence reported below should be interpreted as the lower bound that our theoretical model can generate.

Figure 1 shows the simulation results. The following two features are of particular importance. First, as the difference in the initial beliefs between the private sector and the central bank, i.e. $E_{0|c}\pi^*_{0|p} - \pi^*_{0|p}$, become large, both the estimated AR parameter α and the standard deviation of the innovation σ_η become large in the first-half period. A larger gap in initial beliefs between the private sector and the central bank make the central bank's estimation of the economy more difficult, which results in higher persistence and higher volatility of inflation process. Second, for any value of $E_{0|c}E_{0|p}\pi^* - E_{0|p}\pi^*$, α and σ_η become smaller in the second half period than the first half period. This is because the private agents and central bank eventually learn about the inflation target and the perceived inflation target respectively. Accordingly, both inflation persistence and the volatility of inflation decline in the second half period. Now we draw some implications of this simulation result.

4.3 Nominal anchor and measurement of economic activity

Figure 1 shows that inflation is persistent and volatile in the early phase of learning. (20) shows that u_t has persistent effects on $\tilde{\pi}_t$ through the private-sector filtering. This implies that the private-sector uncertainty about π^* results in persistent inflation through learning. This agrees with Erceg and Levin (2003), who consider a similar economy in which the private-sector learning about inflation target can make inflation process persistent.[15] In our model, there is another channel through which imperfect credibility contributes to inflation persistence and volatility, namely, the bank's uncertainty about perceived inflation target. The fact that the bank cannot directly observe perceived inflation target implies that it cannot identify inflation expectations and the natural interest rates. This fact de-stabilises inflation because it prevents the bank from fully offsetting the effects of the fluctuations in r_t^n on π_t.[16]

Our model shows that the lack of strong nominal anchor and the measurement problem of economic activity are closely related with each other. In a series of papers, Orphanides (2001, 2002, 2003) show that the mismeasurement of economic activity, such as the output gap and natural unemployment rate, was responsible for the 'Great Inflation' of the 1970s-1980s in the United States. He shows that in the 70s the real-time estimate of the natural unemployment rate was too low and the estimate of the potential output was too high. This lead to excess monetary expansion, resulting in high inflation. While his work takes the measurement problem as exogenously given, our paper shows that the imperfect credibility can endogenously amplify the measurement problem. Our model

[15] Erceg and Levin (2003) assume that the central bank has perfect information, which in our model corresponds to the case of $E_{0|c}\pi^*_{0|p} - \pi^*_{0|p} = 0$.

[16] See equation (6). When $E_{t|c}r_t^n = r_t^n$, then inflation does not depend on r_t^n.

implies that the measurement problem becomes serious when monetary policy looses credibility. On the contrary, gaining credibility can make the measurement problem less serious. This has important policy implications. If we take the measurement problem as exogenously given, a policy prescription would be not to respond actively to those economic variables subject to measurement errors. For example, Orphanides and Williams (2005) argue that it is desirable for monetary policy not to respond actively to the unemployment gap because the gap can be subject to large measurement errors. On the other hand, our model implies that gaining credibility helps the bank to reduce the measurement problem.

4.4 Learning and time-varying stochastic process of inflation

Figure 1 shows that the persistence and volatility of inflation decline as both of the agents and central bank learn. It is interesting to compare this observation with some empirical studies. Recently, several articles have documented that macroeconomic volatility in several OECD countries have declined over the past twenty years — the so called 'Great Moderation.' [17]

For example, Benati (2007) found that under inflation targeting inflation persistence declined significantly and exhibits almost no persistence in UK, Canada, Sweden and New Zealand, while it was highly persistent between the breakdown of Bretton Woods and the introduction of inflation targeting. In Benati (2004), he found that the volatility of GDP and inflation in the UK has decreased since the introduction of inflation targeting in 1992. Our model can offer an explanation of his findings. As inflation targeting becomes credible, the private-sector uncertainty about the long-run inflation rate has decreased over time. At the same time, this has a favourable feedback to the stabilisation policy of the Bank of England. A decrease in the private-sector uncertainty has reduced the uncertainty facing the

[17] See, for example, Ahmed et al. (2004); Stock and Watson (2003);Cogley and Sargent (2004)

bank. As a result, the bank's stabilisation policy has improved, making inflation process less volatile and less persistent.

Our simulation, even though it is stylised, has some interesting implication for the econometric analysis of the Great Moderation. In the literature, two competing explanations for the Great Moderation are considered very likely. One is "good policy", i.e. improvements in monetary policy. Another is "good luck", i.e. a fortuitous reduction in exogenous shocks. Several prominent studies have provided support for the good-luck hypothesis. Roughly speaking, those econometric studies interpret the changes in estimated parameters of the model as changes in policy or economic structure, and the changes in the variances of innovations (residuals) as representing the changes in underlying economic shocks. In our simulation, both α and σ_η in equation (27) change, implying that this looks as if both good policy and good luck occur. However, in our simulation, what changed between the first subperiod and the second subperiod is the perceived inflation target and the bank's estimate of the perceived target. We kept the monetary policy rule (6) and variance of shocks (σ_u, σ_r) constant. This agrees with Bernanke (2004)'s view on the literature on the Great Moderation. He argues that the existing studies may incorrectly identify the effect of good policy as good luck. Econometricians typically do not measure exogenous shocks directly but instead infer them form movements in macroeconomic variables that they cannot otherwise explain. When the central bank's inflation target is incredible, the change in de-anchored inflation expectation may result in what appear to be change in exogenous shocks. Shocks in this sense may certainly depend on monetary policy regime. Accordingly, as the inflation expectation becomes to be anchored gradually, the standard deviation of innovation in the reduced form regression may become smaller even when the magnitude of exogenous shock (structural shock) is constant. This makes an econometric analysis based on reduced-form regression incorrectly lead to good-luck bias. This is exactly what happened in our simulation study. Imperfect credibility creates fluctuations in inflation expectations through the private-sector learning. This diminishes over time as agents learn. In addition to this, imperfect credibility creates additional uncertainty facing the bank, which makes policy erratic. This also contributes to innovation variances that diminish over time.

5 Conclusion

In this paper, we analysed how private agents' uncertainty about inflation target and the central bank's uncertainty are related with each other. The model is capable of generating high persistence and volatility even when there is no intrinsic persistence and structural shocks are white noise. We also show that the degree of the bank's data uncertainty is endogenously determined by the degree of private agents' uncertainty about inflation target — the measurement problem can arise endogenously from lack of credibility. Although much of the previous literature has already suggested the great importance of considering data uncertainty in the conduct of monetary policy, it has taken the data uncertainty as exogenously given. Our model implies that decreasing the private agents' uncertainty about inflation target can make the measurement problem less serious for the central bank.

Our model can have some implications for yield curve. As Gurkaynak et al. (2005) shows, long-term nominal interest rates tend to be sensitive to changes in current monetary policy actions when there is uncertainty about nominal anchor. This is because current monetary policy actions bring some news about long-run inflation target. Our model predicts similar results. While their model predicts 'shifts' in the yield curve in response to monetary policy, our model would predict both 'shifts' and 'the change in slopes'. We plan to investigate this question in future research.

Finally, it would be interesting to examine some other monetary policy regimes with stronger nominal anchor. In the present paper, it is assumed that the bank changes the nominal interest rate in response to deviations of inflation from its target value. It would be interesting to analyse price level targeting in the context of interest-rate rules, or monetary-aggregate control instead of interest-rate control. As is well known, price level targeting and monetary aggregate control have stronger nominal anchor than inflation targeting. It would be important to analyse how those different policy regimes perform under imperfect credibility about nominal anchor.

Appendix

A Bank's filtering problem

A.1 Constructing an observation equation of the bank

From (1) and (14), the bank's observation equation is given by

$$i_t = r_t^n + (1-\phi^{-1})E_{t|p}\pi^* + \phi^{-1}E_{t|p}E_{t+1|c}\pi^*_{t+|p}. \tag{A.1}$$

In order to solve the bank's filtering problem, it is convenient to rewrite (A.1) in terms of $\pi^*_{0|p}$. By taking the conditional expectation $E_{t|c}$ of equation (19) and subtracting that conditional expectations from (19), we have

$$E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^* = a_t\left(E_{t|c}\pi^*_{0|p} - \pi^*_{0|p}\right), \tag{A.2}$$

and therefore

$$E_{t|p}E_{t+1|c}\pi^*_{t+1|p} - E_{t|p}\pi^* = a_{t+1}\left(E_{t|p}E_{t+1|c}\pi^*_{0|p} - \pi^*_{0|p}\right). \tag{A.3}$$

Here we use the fact that $E_{t|p}\pi^*_{t+1|p} = E_{t|p}\pi^*$. Equation (A.2) shows that what matters to the difference between the perceived inflation target and the bank's belief the perceived target is the difference of the initial belief ($\pi^*_{0|p}$) between the two of them. As $t \to \infty$, the difference will disappear. This is because (19) implies that the private agents eventually learn about the inflation target, and the bank knows this fact.

Substituting (A.3) into (A.1), one obtains

$$i_t = r_t^n + E_{t|p}\pi^* + \phi^{-1}a_{t+1}\left(E_{t|p}E_{t+1|c}\pi^*_{0|p} - \pi^*_{0|p}\right).$$

Substituting (19) into the above equation, and collecting the variables that are observable to the bank to the left hand side and the unobservables to the right hand side, one obtains

$$i_t-(1-a_t)\pi^*-\frac{1-a_t}{1-\phi}\bar{u}_t = r_t^n+\left(a_t-\phi^{-1}a_{t+1}\right)\pi^*_{0|p}+\phi^{-1}a_{t+1}E_{t|p}E_{t+1|c}\pi^*_{0|p}. \tag{A.4}$$

Equation (A.4) involves the private-sector expectations about the bank's future belief about $\pi^*_{0|p}$. This is an implication of the forward-looking nature of inflation. The term $E_{t|p}E_{t+1|c}\pi^*_{0|p}$ comes from the last term in equation (A.1), which in turn comes from private-sector inflation expectations about future monetary policy. The private sector expectations about future monetary policy depends on their expectations about the bank's filtering in the subsequent periods.

Now the remaining task is to compute the equilibrium and the bank's filtering. The right hand side of equation (A.4) still contains endogenous variables, namely, $E_{t|p}E_{t+1|c}\pi^*_{0|p}$. This should be determined jointly with the bank's filtering. In the next section, we will compute the equilibrium and the filtering by the method of undetermined coefficients. We first guess the equilibrium form and solve the filtering given that guess, and then compute the equilibrium given the filtering.

A.2 Deriving the equilibrium and the bank's filtering

Define X_t as the bank's observable variables (the right hand side of equation (A.4)):

$$X_t \equiv i_t-(1-a_t)\pi^*-\frac{1-a_t}{1-\phi}\bar{u}_t.$$

First of all, notice that, in this economy, the state of the economy can be represented by X_t, r_t^n, $\pi^*_{0|p}$, and $E_{t-1|c}\pi^*_{0|p}$. Therefore, we guess that the equilibrium takes the following form:

$$A_t X_t = r_t^n + B_t \pi^*_{0|p} + C_t E_{t-1|c} \pi^*_{0|p}, \tag{A.5}$$

where A_t, B_t, C_t are time-varying coefficients to be determined. Since $E_{t-1|c}\pi^*_{0|p}$ is observable to the bank at time t, we can define the new observation equation of the bank as

$$A_t X_t - C_t E_{t-1|c} \pi^*_{0|p} = r_t^n + B_t \pi^*_{0|p}. \tag{A.6}$$

Again, the left hand side is observable to the bank, and the right hand side is unobservable. Equation (A.6) shows that the bank's filtering problem reduces to the sequential updating of a constant, $\pi^*_{0|p}$. A slight complication is that it involves a time-varying coefficient B_t. Define the new observable variable as $Y_t \equiv A_t X_t - C_t E_{t-1|c} \pi^*_{0|p}$. From the normality assumption (7), Y_t is again normally distributed:

$$Y_t \sim N\left(B_t \pi^*_{0|p},\ \sigma_r^2\right).$$

Let the prior distribution be:

$$B_{t-1} \pi^*_{0|p} \sim N\left(B_{t-1} E_{t-1|c} \pi^*_{0|p},\ \tau^{-1}_{t-1|c}\right),$$

where $\tau_{t-1|c}$ is the bank's precision at the end of time $t-1$ (i.e., before the bank observes time-t variables). Then the prior for $B_t \pi^*_{0|p}$ is given by (see DeGroot (1970))

$$B_t \pi^*_{0|p} \sim N\left(B_t E_{t-1|c} \pi^*_{0|p},\ \frac{B_t^2}{B_{t-1}^2} \tau^{-1}_{t-1|c}\right). \tag{A.7}$$

Then, the posterior mean of $B_t \pi^*_{0|p}$ is given by

$$B_t E_{t|c} \pi^*_{0|p} = d_t B_t E_{t-1|c} \pi^*_{0|p} + (1 - d_t) Y_t, \tag{A.8}$$

where

$$d_t \equiv \frac{\frac{B_{t-1}^2}{B_t^2}\tau_{t-1|c}}{\frac{B_{t-1}^2}{B_t^2}\tau_{t-1|c} + \gamma_r} \tag{A.9}$$

and $\gamma_r \equiv 1/\sigma_r^2$. The updating equation of $\tau_{t|c}$ can be written as

$$\tau_{t|c} = \frac{B_{t-1}^2}{B_t^2}\tau_{t-1|c} + \gamma_r. \tag{A.10}$$

Using the definition of Y_t, one can rewrite (A.8) as

$$E_{t|c}\pi_{0|p}^* = \left\{d_t - \left(1 - d_t\frac{C_t}{B_t}\right)\right\} E_{t-1|c}\pi_{0|p}^* + (1-d_t)\frac{A_t}{B_t}X_t. \tag{A.11}$$

This implies that, at time $t+1$,

$$E_{t+1|c}\pi_{0|p}^* = \left\{d_{t+1} - \left(1 - d_{t+1}\frac{C_{t+1}}{B_{t+1}}\right)\right\} E_{t|c}\pi_{0|p}^* + (1-d_{t+1})\frac{A_{t+1}}{B_{t+1}}X_{t+1}.$$

Taking expectations conditional on the private-sector information at time t, and arranging terms, one obtains

$$E_{t|p}E_{t+1|c}\pi_{0|p}^* = (1-d_{t+1})\pi_{0|p}^* + d_{t+1}E_{t|c}\pi_{0|p}^*. \tag{A.12}$$

Here we use the assumption that the bank announces $E_{t|c}\pi_{0|p}^*$ at time t. Substituting (A.12) into (A.11), and then substitute the resulting equation into (A.4). Then we can verify that the equilibrium is indeed given by:

$$\begin{aligned} &\left\{1 - \phi^{-1}a_{t+1}(1-d_t)\frac{A_t}{B_t}\right\}X_t \\ &= r_t^n + \left\{a_t - \phi^{-1}a_{t+1}d_{t+1}\right\}\pi_{0|p}^* + \left\{\phi^{-1}a_{t+1}d_{t+1}\left(d_t - \frac{1-d_t}{B_t}C_t\right)\right\}E_{t-1|c}\pi_{0|p}^*. \end{aligned} \tag{A.13}$$

By comparing our guess (A.5) and (A.13), we have the following three identities:

$$A_t = 1 - \phi^{-1} a_{t+1}(1 - d_t)\frac{A_t}{B_t} \tag{A.14}$$

$$B_t = a_t - \phi^{-1} a_{t+1} d_{t+1} \tag{A.15}$$

$$C_t = \phi^{-1} a_{t+1} d_{t+1} \left(d_t - \frac{1 - d_t}{B_t} C_t \right) \tag{A.16}$$

By substituting (A.9), we notice that equations (A.15) and (A.10) represent a system of deterministic (non-linear) difference equation with respect to B_t and $\tau_{t|c}$

$$B_t = a_t - \phi^{-1} a_{t+1} \frac{\frac{B_t^2}{B_{t+1}^2}\tau_{t|c}}{\frac{B_t^2}{B_{t+1}^2}\tau_{t|c} + \gamma_r}, \tag{A.17}$$

$$\tau_{t|c} = \frac{B_{t-1}^2}{B_t^2}\tau_{t-1|c} + \gamma_r. \tag{A.10}$$

Once B_t and $\tau_{t|c}$ are solved, d_t is solved by ($A.9$), equations (A.14) and (A.16) respectively solve A_t and C_t.

Notice that equation (A.15) and (A.10) show that B_t depends both on B_{t-1} and B_{t+1}. Dependence on B_{t-1} is just a direct implication of recursive filtering. Dependence on B_{t+1} is an implication of the interaction between forward-looking nature of inflation and the bank's filtering. In our model, the current equilibrium variables depend on the private-sector expectations about future monetary policy. And future monetary policy depends on how the central bank will estimate the future state of the economy. This is represented in equation (A.12). Therefore, the way the bank will estimate the state of the economy in the next period will affect the current equilibrium. As a result, the bank's filtering in the current period is affected by its filtering in future periods.

The existing literature such as Aoki (2003) and Svensson and Woodford (2003) focus on ‘stationary’ filtering by central bank, so that the Kalman gain is constant over time. In our model, since the bank’s learning is about a constant $\pi^*_{0|p}$, filtering is not stationary. That is the main reason why the bank’s Kalman gain, namely, d_t is not constant over time. In general, even if the bank is estimating time-varying variables, Kalman gains are not necessarily constant. Therefore, the dependence of equilibrium on the future Kalman gain of the central bank is a general property of rational equilibrium models with forward-looking private sector.

Let us finish this section by characterising the stochastic properties of the central-bank uncertainty. By using (A.11), (A.5), and (A.15), the evolution of $E_{t|c}\pi^*_{0|p}$ is given by

$$E_{t|c}\pi^*_{0|p} = d_t E_{t-1|c}\pi^*_{0|p} + (1-d_t)\pi^*_{0|p} + \frac{1-d_t}{a_t - \phi^{-1}a_{t+1}d_{t+1}} r^n_t. \tag{A.18}$$

Substitution of (A.18) into (A.2) yields

$$E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^* = a_t d_t \left(E_{t-1|c}\pi^*_{0|p} - \pi^*_{0|p}\right) + \frac{a_t(1-d_t)}{a_t - \phi^{-1}a_{t+1}d_{t+1}} r^n_t. \tag{A.19}$$

Equation (A.19) clearly shows the dependence on the bank’s current estimate of the economy on its future filtering (d_{t+1}). It also shows that $E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^* \to 0$ as $t \to \infty$.

Finally, as discussed in 3.3, there is a close relationship between the bank’s estimates of the perceived inflation target and the estimates of the natural rate. Taking the conditional expectation $E_{t|c}$ of equation (A.6) and subtracting that conditional expectations from (A.6), we have

$$r^n_t - E_{t|c}r^n_t = B_t \left(E_{t|c}\pi^*_{0|p} - \pi^*_{0|p}\right). \tag{A.20}$$

Using (A.20) and (A.2), we can see that $E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^*$ represents the bank's estimation error about r_t^n

$$E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^* = \frac{a_t}{B_t}\left(r_t^n - E_{t|c}r_t^n\right), \tag{A.21}$$

which is equation (23). Therefore, the central-bank uncertainty about the real rate and about the private-sector belief are related with each other.

B Variance and autocorrelation of inflation

We can rewrite equation (24) as

$$\begin{aligned}\pi_t &= \pi^* + \phi^{-1}\left[(E_{t|p}\pi^* - \pi^* - u_t) + (E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^*)\right] \\ &\equiv \pi^* + \phi^{-1}\left[\xi_t + \zeta_t\right],\end{aligned} \tag{B.1}$$

where

$$\xi_t \equiv E_{t|p}\pi^* - \pi^* - u_t$$

and

$$\zeta_t \equiv E_{t|c}E_{t|p}\pi^* - E_{t|p}\pi^*.$$

By using (20), ξ_t can be written as

$$\xi_t = b_t\xi_{t-1} + b_t u_{t-1} - \frac{b_t - \phi}{1-\phi}u_t. \tag{B.2}$$

By using (22), ζ_t can be written as

$$\zeta_t = g_t\zeta_{t-1} + h_t r_t^n, \tag{B.3}$$

where $g_t \equiv b_t d_t$ and $h_t = (1-d_t)a_t/B_t$. Note that $0 < g_t < 1$, $lim_{t\to\infty}g_t = 1$, and $h_t > 0$, $\lim_{t\to\infty} h_t = 0$.

Since u_t and r_t^n are independent so are ξ_t and ζ_t. Therefore we have

$$V[\pi_t] = \phi^{-2}(V[\xi_t] + V[\xi_t]) \tag{B.4}$$

and

$$Cov[\pi_t, \pi_{t-1}] = \phi^{-2}\left[Cov[\xi_t, \xi_{t-1}] + Cov[\zeta_t, \zeta_{t-1}]\right]. \tag{B.5}$$

Equation (B.2) implies that

$$V[\xi_t] = \frac{\sigma_u^2}{(1-\phi)^2}\left[(b_t - \phi)^2 + \sum_{k=0}^{t-1}\left(\prod_{j=k}^{t-1} b_{j+1}^2\right)(b_k - 1)^2\right] \tag{B.6}$$

and

$$Cov[\xi_t, \xi_{t-1}] = \frac{b_t\sigma_u^2}{(1-\phi)^2}\left[(b_{t-1} - 1)(b_{t-1} - \phi) + \sum_{k=0}^{t-2}\left(\prod_{j=k}^{t-2} b_{j+1}^2\right)(b_{k-1})^2\right]. \tag{B.7}$$

Since $0 < b_t < 1$ and $\lim_{t\to\infty} b_t = 0$ we obtain $\lim_{t\to\infty} V[\xi_t] = \sigma_u^2$ and $\lim_{t\to\infty} Cov[\xi_t, \xi_{t-1}] = 0$.

Equation (B.3) implies that

$$V[\zeta_t] = \sigma_r^2\left[h_t^2 + \sum_{k=0}^{t-1}\left(\prod_{j=k}^{t-1} g_{j+1}^2\right)h_k^2\right], \tag{B.8}$$

and

$$Cov[\zeta_t, \zeta_{t-1}] = g_t\sigma_r^2\left[h_{t-1} + \sum_{k=0}^{t-2}\left(\prod_{j=k}^{t-2} g_{j+1}^2\right)h_k^2\right]. \tag{B.9}$$

Since $0 < g_t < 1$, $\lim_{t\to\infty} g_t = 1$, $0 < h_t$, $\lim_{t\to\infty} h_t = 0$, we obtain $\lim_{t\to\infty} V[\zeta_t] = 0$ and $\lim_{t\to\infty} Cov[\zeta_t, \zeta_{t-1}] = 0$.

C Robustness analysis of Section

In this Appendix, we confirm that the above basic results shown in Section 4 are robust against different parameters. Table 1 shows the parameter for four robustness analysis.

	Benchmark	Case 1	Case 2	Case 3	Case 4
π^*	2	2	2	2	2
$E_{0\|p}\pi^*$	10	**20**	10	10	10
γ_u	1	1	1	**4**	1
γ_r	0.44	0.44	0.44	0.44	0.44
ϕ	1.5	1.5	**1.1**	1.5	1.5
$\tau_{0\|p}$	1	1	1	1	**10**
$\tau_{0\|c}$	1	1	1	1	1

Table 1: Parameter values for sensitivity analysis

Case 1: higher perceived target Initial perceived inflation target is set to 20 %. This may correspond to the case of the introduction of inflation targeting in some emerging countries. When Chile, Israel and Hungary adpted inflation targeting, the inflation rates were about 20 %.

Case 2: less aggressive monetary policy ϕ is set to 1.1, which is lower than the original Taylor rule

Case 3: smaller monetary policy shock σ_u is set to 0.5 % and hence γ_u=0.4. According to Roberts (2004), this is almost in line with the Fed policy after 1984.

Case 4: More stubborn belief The initial value of the private-sector precision parameter, $\tau_{0|p}$, is 10, which means that the private sector is more convinced by their own belief.

Figure 2-5 shows that higher volatility of inflation in the first-half period results from higher perceived target (Case1), less aggressive monetary policy (Case2), larger monetary policy shock (Case3) and more stubborn belief (Case4). In all these cases, as in the benchmark case, the larger $E_{0|c}\pi^*_{0|p} - \pi^*_{0|p}$, the larger the volatility of inflation in the first-half period. Also, the estimated AR parameter α, the standard deviation of the innovation σ_η , and hence the volatility of inflation $Var[\pi_t]$ become smaller in the second half period than the first half period.

Benchmark ($\pi^*_{0|p} = 10$, Black line) vs. Higher perceived target ($\pi^*_{0|p} = 20$, Gray line)
(Solid line: Sample period 1–20, Dashed line: Sample period 21–40)

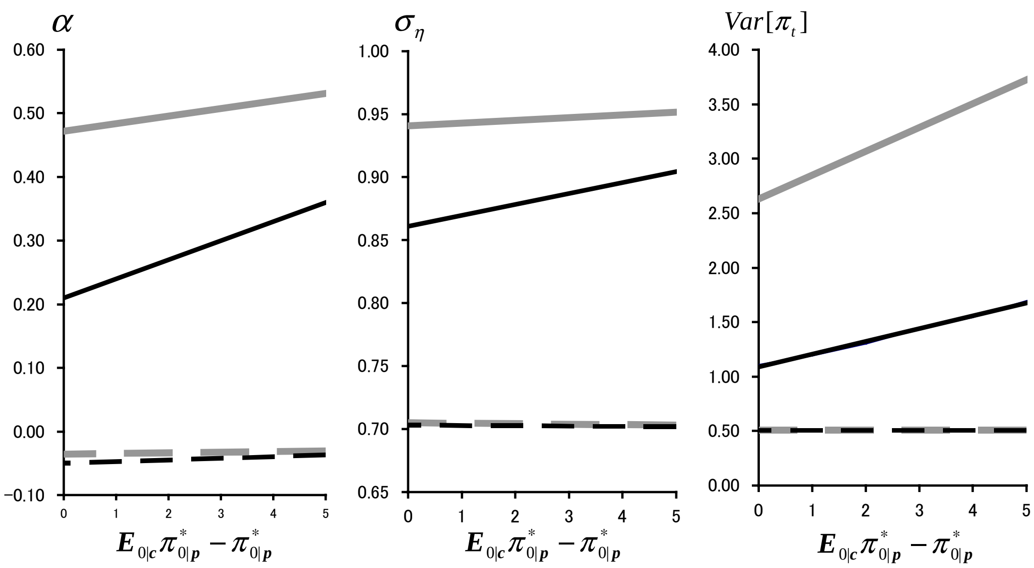

Figure 2: Robustness analysis: case 1

Benchmark ($\phi = 1.5$, Black line) vs. Less aggressive monetary policy ($\phi = 1.1$, Gray line)
(Solid line: Sample period 1–20, Dashed line: Sample period 21–40)

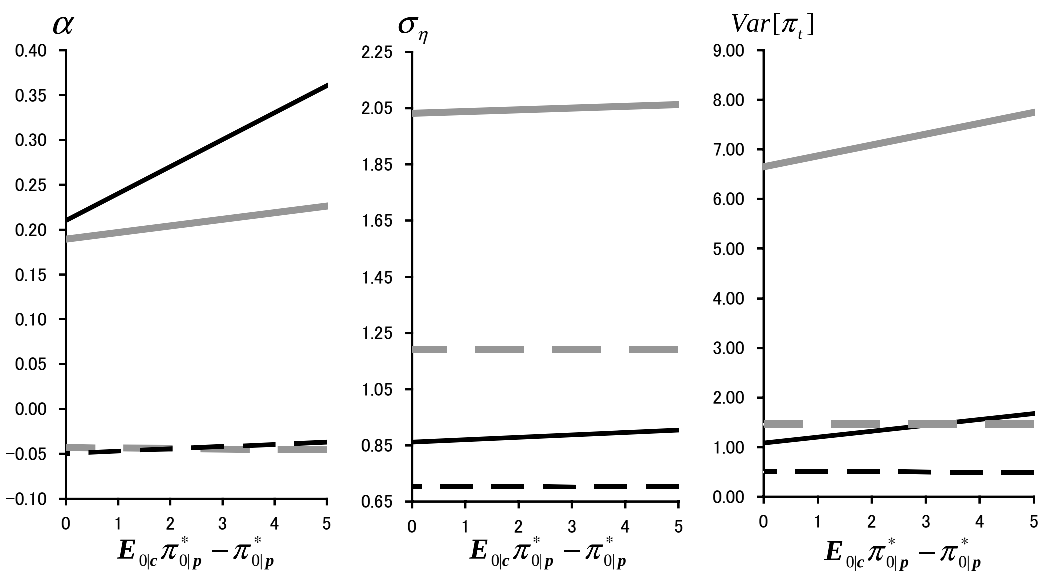

Figure 3: Robustness analysis: case 2

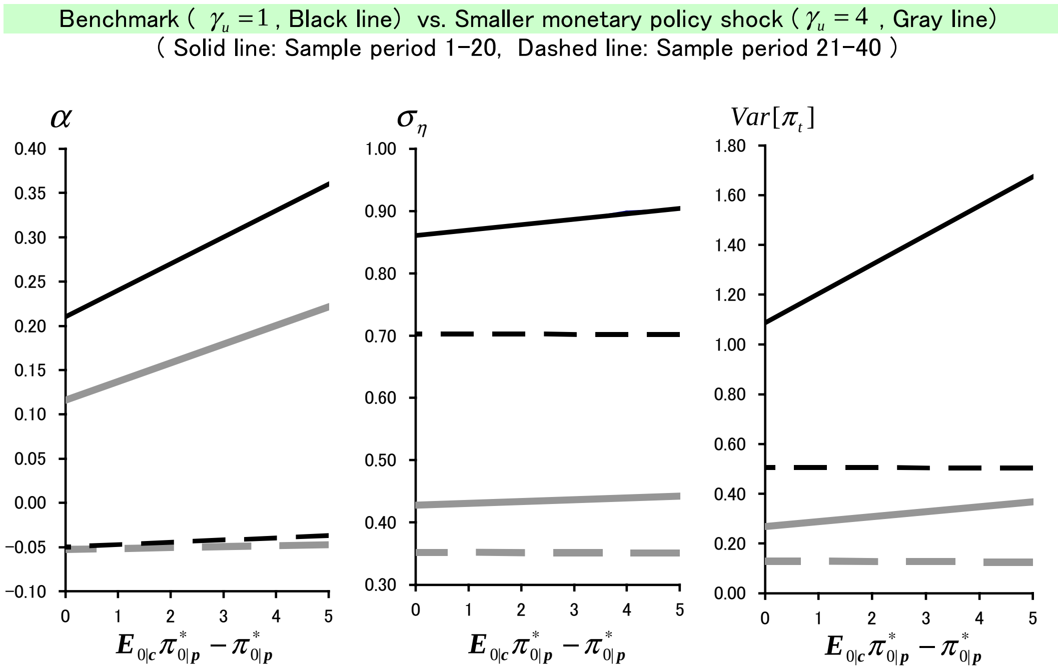

Figure 4: Robustness analysis: case 3

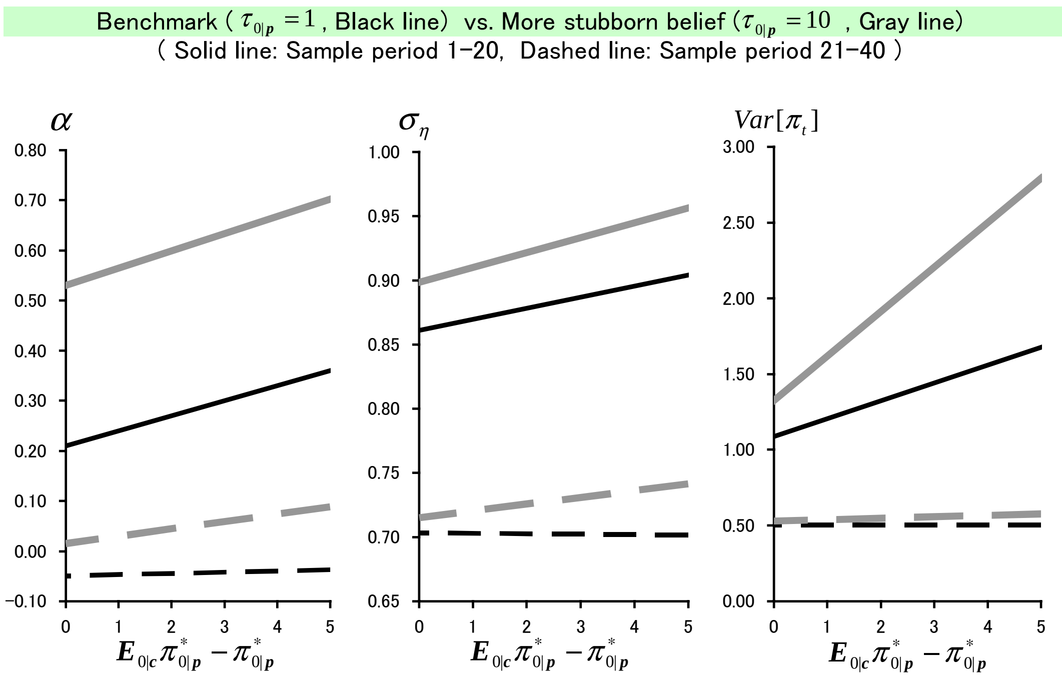

Figure 5: Robustness analysis: case 4

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