FISCAL PLANNING UNDER UNCERTAINTY – THE IMPLICATIONS OF ECONOMIC AND FISCAL UNCERTAINTY FOR BUDGET FORECASTS

Jenna Robbins, Brian Torgunrud and Chris Matier[*]

This paper presents an analysis of the uncertainty in projections of the federal budget balance arising due to uncertainty in economic, and government revenue and spending projections. Budget projections frequently differ from actual results due to an unavoidable level of uncertainty contained in the underlying economic projections as well as a relationship between economic inputs and fiscal projections that is often volatile. Using a stochastic simulation model that includes a detailed specification for the economy and federal government finances and random variables to proxy uncertainty in these variables, a probability distribution for the budget balance is generated and the probability of achieving a surplus is estimated given various assumptions for fiscal prudence.

1 Introduction

Budget projections frequently differ from actual results due to an unavoidable level of uncertainty contained in the underlying economic forecasts as well as a relationship between economic inputs and fiscal projections that is often volatile. Ensuring that budget forecast errors do not significantly undermine a government's budget planning process requires a careful assessment of these risks and uncertainties.

The Canadian experience of the past fifteen years provides an example of the impact that economic and fiscal forecast uncertainty can have on a country's fiscal position and budget planning practices. Consistently missed economic forecasts in the early part of the last decade contributed to the underestimation of budget deficits, resulting in an increase in the federal debt. To minimize the negative effects of economic and fiscal forecast uncertainty on the budget balance, a more prudent approach to budget forecasting was adopted and refined over time to further guard against deficit outcomes. More recently, federal fiscal forecasts have come under scrutiny for different reasons as the direction of forecast errors has been reversed and has led to persistent and often significant under-estimation of the federal surplus at budget time.

To assess the degree of uncertainty surrounding point estimates of the budget balance some studies have turned to stochastic simulation models. Stochastic

---

[*] Department of Finance/Ministère des Finances, Canada.

The views expressed in this paper are those of the author and do not reflect those of the Department of Finance. The authors would like to acknowledge important contributions made throughout this process by Isabelle Amano, Jian-Guo Cao, Chris Forbes, Phil King, Shaun Peddie and Sally Tewolde.

simulation models provide a means of estimating a range of possible outcomes surrounding point estimate forecasts, given assumptions regarding the degree of uncertainty in forecast inputs. Such information provides policy makers with an increased awareness of the probability of achieving a particular outcome and can serve as an important input into fiscal planning decisions.

This paper presents estimates of the impact of forecast uncertainty on federal budget projections in Canada using a stochastic simulation that encompasses a modified version of Murchinson (2001) for the economy and a modified version of Hermanutz and Matier (2000) for the fiscal structure.

The structure of the paper is as follows. Section 2 provides a review of the relevant literature. Section 3 describes the model and its calibration. Section 4 presents updated estimates of the impact of forecast uncertainty on Canadian federal budget projections. Conclusions are discussed in Section 5.

2 Fiscal forecasting accuracy and stochastic simulation models

2.1 Fiscal forecasting accuracy

The accuracy of budget forecasts in Canada has been the focus of several studies. The most recent major study is a 2005 independent review led by Dr. Tim O'Neill, commissioned by the federal government. In his review, O'Neill assesses the basis for persistent, large federal forecast differences in Canada.

The O'Neill review includes a technical report authored jointly by the Policy and Economic Analysis Program at the University of Toronto and the Centre Interuniversitaire de Recherche en Analyse des Organisations (herein referred to as the report) that provides a quantitative analysis of federal fiscal accuracy from 1994 to 2004. The report provides an analysis of the Canadian government's forecast record over the period for economic and fiscal aggregates as well as the more detailed components. In order to assess forecast errors[1] based on forecasters' knowledge at the time budget projections were made, policy measures introduced post-budget are removed from the actual results and adjustments are made to ensure a consistent basis of accounting is used for forecast-to-actual comparisons throughout the review period. Based on their analysis, the authors conclude that the federal surplus was consistently and significantly under-estimated over the period. The report finds that for some of the fiscal components, such as federal revenues and public debt charges, forecast errors in the broad economic inputs (e.g. nominal GDP, GDP inflation, etc.) were important contributors to forecast errors over the period studied. However, it suggests that other factors, such as volatile tax bases, and lags in the receipt of historical revenue and spending data necessary for an accurate fiscal forecast were also important contributors.

---

[1] Forecast errors are equal to the forecasted amount less the actual amount.

In his final assessment, O'Neill attributes much of the positive federal forecast error to the practice of forecasting under a stringent no-deficit rule while facing considerable economic and fiscal forecast uncertainty. In this environment, he notes that it is understandable that when considering a range of possible outcomes for specific revenue and spending items, forecasters would choose a point estimate at the low-end of the range for revenues and at the high-end for expenditures.

Fiscal projections are subject to two primary sources of uncertainty: uncertainty in the forecasts of the key macroeconomic variables, and uncertainty in the translation of economic forecasts into fiscal forecasts. It is indisputable that economic forecasts are subject to error. Contributing factors include uncertainty regarding: the nature and persistence of economic shocks, the economy's potential growth/level; relationships between economic variables; external developments (foreign economies and commodity prices); and the impacts of data revisions.

Even when economic projections are correct, forecast errors in fiscal components are conceivable. Federal tax revenues are particularly difficult to forecast given volatility in their relationship with underlying economic activity: tax revenues are highly sensitive to the composition and distribution of income, which can be difficult to predict. Furthermore, frequent changes to the tax structure and structural changes in the economy imply a volatile historical relationship between tax revenues and the economy that is of little help in predicting future tax elasticities.

On the spending side, uncertainty surrounding the valuation of liabilities and the appropriation of funds during the forecast year can be significant contributors to forecast errors. Fiscal forecasting is also made difficult as a result of information lags and, in the Canadian context, a full accrual basis of accounting.

2.2 Stochastic simulation models

Several studies in Canada have turned to stochastic simulation models in order to assess the uncertainty in budget projections and evaluate fiscal policy options in an environment of budget uncertainty. Stochastic models proxy the effects of forecast uncertainty by incorporating random variables into the forecasts of the economy and federal government finances. By simulating stochastic models repeatedly, one can generate a probability distribution that describes the range of possible outcomes for the economy and federal finances based on the degree of uncertainty assumed for each component of the model. Furthermore, stochastic simulation models allow one to study fiscal policy options in an environment of forecast uncertainty.

Robson (2006) uses a stochastic simulation model to illustrate an alternative option to point estimates for presenting budget projections and setting fiscal policy. Under Robson's approach to fiscal planning, the government begins its budget planning by deciding upon an acceptable probability of missing its fiscal target (e.g. zero-deficit, target debt-to-GDP ratio), and then chooses a path for spending and

taxes accordingly. Robson uses a simple model of the economy with standard errors calibrated to historical volatility, but also incorporates additional forecast uncertainty on the fiscal side through a stochastic term on the federal tax-to-GDP ratio. Robson then estimates the required growth in spending for a range of fiscal targets and desired probability of meeting each target.

Similar to Robson (2006), Tapp (2006) uses a stochastic simulation model to illustrate an alternative fiscal planning framework. Tapp focuses on a medium-term target for the debt-to-GDP ratio and assesses the probability of hitting that target under various scenarios for spending growth. Tapp's model incorporates two random shocks – a shock on real output growth and a shock on the tax-to-GDP ratio. Tapp assumes the government chooses a trend growth rate for government program spending but that realized program spending deviates from this path based on the impact of changing economic conditions on automatic stabilizers. In the model, a monetary authority is assumed to set the market interest rate. The effective interest rate on federal debt is modeled as a weighted average of the market interest rate (short-term rate) and a constant long-term rate. Tapp uses this model to assess the probability of lowering the federal debt-to-GDP ratio to 25 per cent by 2014-15[2] and 20 per cent in 2019-20. He finds that the targets are reachable in more than 95 per cent of model simulations.

Boothe and Reid (1998) use a stochastic simulation model of the federal budget to assess the probability of recording future deficits given the inclusion of a contingency reserve or buffer against unfavourable budget shocks in budget projections and varying amounts of flexibility in the government's fiscal rule. They employ a simple model of the economy that describes output growth by an AR(1) process and assume that the market interest rate follows a random walk. Both equations include stochastic disturbance terms with standard deviations and correlations determined from regression residuals for the historical period 1953 to 1997. The fiscal variables are assumed to fluctuate based on their relationship with nominal GDP growth and market interest rates. Boothe and Reid simulate their model over a four-year forecast horizon (fiscal years 2000-01 to 2003-04)[3] to assess the probability of realizing a federal deficit under various combinations of contingency reserves and fiscal rules. From their simulations, Boothe and Reid conclude that with a contingency reserve of between $6 and $9 billion, the probability of a federal deficit in Canada would be reduced to close to zero for any given year within the four-year time horizon considered.

Hermanutz and Matier (2000) modify the Boothe and Reid (1998) model to analyse the amount of unused fiscal room that would be required to achieve various probabilities of surplus over the medium term under alternative fiscal rules. Like the

---

[2] Tapp's choice of targets for the federal debt-to-GDP ratio corresponds to federal government medium-term targets announced in the 2005 Economic and Fiscal Update. Since 2005, the medium-term targets have been revised. The current government target is to reduce the federal debt-to-GDP ratio to 25 per cent by 2012-13.

[3] In Canada, the federal government's fiscal year ends March 31.

Boothe and Reid (1998) model, their model includes simple stochastic processes for nominal GDP growth and the market interest rate. However, they calibrate the standard deviation of the disturbance term on nominal GDP to better reflect the low-inflation regime followed by the Bank of Canada since the early 1990s. In addition, Hermanutz and Matier model the effective interest rate such that only a portion of debt rolls over in each year. In contrast, Boothe and Reid assume that all debt rolls over at the current market interest rate, implying a wider range of possible outcomes for the effective interest rate on public debt. As in the Boothe and Reid (1998) model, fiscal variables in the Hermanutz and Matier model are a function of the economic variables and do not contain an idiosyncratic stochastic element, *i.e.* all variation in fiscal forecasts is assumed to be a result of uncertainty in the economic inputs. Using this model, they show that the use of a two-year fiscal planning horizon that allows for the addition of further fiscal measures if future forecasts imply an improved fiscal position, can mitigate some of the impact of uncertainty associated with budget projections relative to a rule that pre-commits the fiscal path several years in advance.

Broadly, these studies use relatively simple stochastic simulation models with varying assumptions and specifications to estimate the uncertainty in budget projections, and then assess various fiscal policy options. This paper builds on these studies by incorporating a more sophisticated model of the economy and by using federal and private sector forecasting records to assess forecast uncertainty and provide a basis for estimates of the dispersion in economic and fiscal shocks used in the model.

3 Specification and calibration of the model

The model used for this paper comprises a reduced-form model of the economy that feeds into the specification of the fiscal structure. The economic and fiscal forecasts are simulated over a five-year forecast horizon. The specifications for the key macroeconomic and fiscal variables are calibrated to reproduce forecasts presented in the November 2006 Economic and Fiscal Update (herein referred to as the November 2006 Update), the federal government's mid-year update of the state of the economy and the federal fiscal position (see Annex 1 for a description of federal fiscal forecasting process in Canada).

3.1 Economy

In the stochastic simulation studies mentioned in the previous section, the models of the economy underlying the simulated fiscal projections are, by design, highly simplified. The specifications of key economic variables are typically univariate, based on estimated stochastic processes. Over the simulation or planning horizon, the standard deviation of the stochastic disturbances are (in most cases) set

equal to the standard deviation of the residuals from the estimated processes and are normally, independently and identically distributed with mean zero.[4] This calibration methodology ensures that future economic outcomes over the planning horizon will deviate from each model's predicted outcome "as they have in the past, with about the same probability distribution of large and small differences".[5] Thus the uncertainty of future economic outcomes depends crucially on each model's (*ex post*) forecast accuracy over history: the smaller a model's historical prediction error, the narrower is the distribution of simulated outcomes in a given year of the planning horizon.[6]

This paper takes an alternative approach to model specification by using a reduced-form forecasting model of the Canadian economy. The approach to calibrating the stochastic disturbances follows previous studies. The model is also calibrated to reproduce economic forecasts from the survey of private sector forecasters used to produce fiscal forecasts presented in the November 2006 Update. Based on our specification and calibration, the distributions of key economic variables generated by our model over the planning horizon are consistent with the historical forecast accuracy of private sector surveys used for budget planning purposes.

The reduced-form model used in this paper is based on Murchison (2001), which was developed to produce quarterly macroeconomic forecasts. The current version of the model consists of six estimated behavioural equations that determine: real GDP growth (IS-curve), GDP inflation, core CPI inflation (Phillips curve), the real exchange rate and short- and long-term nominal interest rates. The model structure allows for correlations among the economic variables, which is an important feature since co-movements in, for example, nominal GDP and interest rates have somewhat offsetting impacts on the budget balance. However, unlike previous studies, for the key economic variables, these correlations are generated by the model itself and are not imposed through the stochastic disturbances. These key equations are as follows:

---

[4] In addition, correlations of the disturbance terms are imposed, calibrated to the correlation of the regression-based residuals. The stochastic processes are also calibrated to produce assumed steady-state values over the planning horizon.

[5] "The Uncertainty of Budget Projections: A Discussion of Data and Methods", Congressional Budget Office, March 2007.

[6] Over the period for which a given equation is estimated, the standard deviation of OLS-based residuals is equivalent to the root mean-squared (static) forecast error, after adjusting for the appropriate degrees of freedom.

Real output growth:

$$\Delta y_t^{CA} = 0.74\Delta yp_t^{CA} + 0.35\Delta YGAP_{t-1}^{CA} + 0.26\Delta y_t^{US} - 0.37\sum_{i=3}^{8}\Delta\left(r_{t-i}^{CA} - \pi_t^{CA}\right) +$$

$$+ 0.05\sum_{i=2}^{6}\Delta e_{t-i}^{CA} + 0.03\sum_{i=3}^{5}\Delta pcom_{t-i}^{XE} + \varepsilon_1 \qquad (1)$$

$R^2_{ADJ} = 0.42$ Std. Deviation of $\varepsilon_1 = 0.54$ Sample: 1976Q1-2006Q2

Output growth[7] (y^{CA}) is a function of potential output growth (yp^{CA}), the change in the output gap ($YGAP^{CA}$), U.S. output growth (y^{US}), the change in real short-run interest rates (defined as the three-month treasury bill rate r^{CA} less quarter-to-quarter logarithmic change in core consumer price inflation π), the real exchange rate e (the nominal Canada-U.S. exchange rate in Canadian dollars multiplied by the ratio of U.S. to Canada GDP inflation) and the production-weighted Department of Finance non-energy commodity price index[8] ($pcom^{XE}$).

Core inflation:

$$\pi_t^{CA} = 0.003D90_t^{CA} + 0.35\pi_t^{*CA} + 0.32\pi_{t-1}^{CA} + 0.13\pi_{t-2}^{CA} + 0.20\pi_{t-3}^{CA} +$$

$$+ 0.03\sum_{i=2}^{3} YGAP_{t-i}^{CA} + 0.03\sum_{i=3}^{4}\Delta e_{t-i}^{CA} + \varepsilon_2 \qquad (2)$$

R2ADJ = 0.48 Std. Deviation of ε2 = 0.27 Sample: 1976Q1-2006Q2

Core inflation is determined by the inflation target π^{*CA} (assumed to be 2 per cent, the midpoint of the 1 to 3 per cent inflation target range), adaptive inflation expectations (using three lags of core inflation), the level of the output gap $YGAP^{CA}$ and the change in the real exchange rate. A dummy variable equalling one from 1990Q1 on is included to capture the impact of the introduction of inflation targeting in the early 1990s.

GDP inflation:

$$\Delta py_t^{CA} = \pi_t^{CA} + 0.21\Delta cpcom_t + 0.16\Delta cpcom_{t-1} + \varepsilon_3 \qquad (3)$$

R2ADJ = 0.25 Std. Deviation of ε3 = 0.48 Sample: 1976Q1-2006Q2

---

[7] Variables given by lower case letters represent natural logs; variables given by upper case letters are levels.

[8] Gaudreault, C. and R. Lamy, *Improvements To Finance's Commodity Price Index and New Commodity Price Indices By Province And Territory*, Department of Finance, 2001.

GDP inflation (py^{CA}) is determined by core inflation and growth in a trade-weighted index of commodity prices (*cpcom*). The latter serves both as a proxy for the food and energy items excluded from core inflation as well as to capture the impact of raw material prices on the terms of trade.

Monetary policy reaction function:

$$r_t^{CA} = 0.98 + 0.75 r_{t-1}^{CA} + 1.97 \sum_{i=0}^{6} \left(\pi_{t-i}^{CA} - \pi_{t-i}^{*CA} \right) + 0.44 \Delta r_t^{US} + \\ + 4.82 \Delta ne_t^{CA} + 1.67 \sum_{i=0}^{6} \left(\Delta YGAP_{t-i}^{CA} \right) + \varepsilon_4 \qquad (4)$$

R2ADJ = 0.92 Std. Deviation of ε4 = 0.40 Sample: 1993Q1-2004Q4

Short-term interest rates are a function of a moving average of the "inflation gap" (actual inflation less target inflation), the change in U.S. short-term interest rates r^{US}, the change in the nominal Canada-U.S. exchange rate *ne*, and a moving average of the change in the output gap. This specification represents a departure from a standard, forward-looking, reaction function: monetary policy reacts gradually to deviations of inflation from its target and to changes in the output gap, as opposed to the level of the output gap. These simplifying assumptions allow for much greater efficiency in simulation (as the model does not have to be solved sequentially one period at a time), and reduces the reliance on an uncertain measure of the output gap. Moreover, the specification describes the data well over the estimation period.

Real exchange rate:

$$\Delta e_t^{CA} = 0.24 \Delta e_{t-1}^{CA} - .005 \left(r_{t-1}^{CA} - r_{t-1}^{US} \right) - .55 \sum_{i=0}^{1} \Delta cpcom_{t-i}^{XE} + 0.12 \Delta e_t^{US} - \\ - 0.11 \left[e_{t-1}^{CA} - 0.40 + 1.93 cpcom_{t-1}^{XE} + 2.94 cpcom_{t-1}^{EN} - 0.29 e_{t-1}^{US} \right] + \varepsilon_5 \qquad (5)$$

R2ADJ = 0.42 Std. Deviation of ε5 = 0.016 Sample: 1976Q1-2006Q2

The real exchange rate equation[9] is modelled in an error correction framework, and is determined by trade-weighted non-energy ($cpcom^{XE}$) and energy ($cpcom^{EN}$) commodity prices, the real U.S. dollar exchange rate excluding Canada (e^{US}) and the U.S.-Canada short-term interest rate differential. The use of trade-weighted commodity price indices allows the equation to account for the variability in the importance of commodities in Canadian terms of trade over time.

---

[9] The real exchange rate equation is based on Helliwell, J., R. Issa, R. Lafrance and Q. Zhang (2005), *NEMO: A Canadian-U.S. Dollar Exchange Rate Equation*, Canada and the Global Economy, proceedings of a conference held at the Bank of Canada, November 2004.

This is particularly important in the case of crude oil, as Canada has gone from being a modest net importer of oil in the early 1980s to being a significant net exporter at present. As a result, the impact of crude oil prices on the exchange rate has become much more significant in recent years.

10-year Government of Canada bond rate:

$$r10_t^{CA} = 0.81r10_{t-1}^{CA} + 0.13r10_{t-2}^{CA} + 0.07\left(rr10_t^{*CA} + \pi_{t-1}^{CA}\right) + 0.09\Delta r_t^{CA} + \\ + 0.07\Delta r_{t-1}^{CA} + 0.08\Delta r_{t-2}^{CA} + 0.78\Delta r10_t^{US} + \varepsilon_6 \quad (6)$$

R2ADJ = 0.95 Std. Deviation of ε6 = 0.26 Sample: 1976Q1-2006Q2

Long-term interest rates (the 10-year Government of Canada bond rate, $r10^{CA}$) are determined by short-term interest rates, U.S. long-term interest rates (the U.S. 10-year government bond rate $r10^{US}$) and a measure of a trend real long-term interest rate (*rr10**) plus core inflation.

Variables exogenous to the Canadian economy, such as U.S. real GDP growth and interest rates, are modelled using the following autoregressive stochastic process, estimated over the period 1990Q1-2006Q2:

$$\Delta z_t = \Delta z^* + \beta\left(\Delta z_{t-1} - \Delta z^*\right) + \varepsilon_z$$

where Δz^* is the average growth rate of variable *Z* over the sample period. To allow for correlations among the exogenous variables, a covariance matrix is constructed using the residuals ε_Z, which is then used to generate multivariate normal random variables for simulation.

The reduced-form model is first calibrated to reproduce the economic forecasts presented in the November 2006 Update.[10] Following previous studies, the standard deviations of the stochastic disturbances over the planning horizon are set equal to the standard deviations of their respective residuals, ensuring that the dispersion of economic outcomes over the planning horizon is consistent with the historical dispersion of the model's predicted outcomes. Moreover, for key economic variables, the model generates distributions over the planning horizon that are also consistent with the historical forecast record of the Department of Finance's survey of private sector forecasters, which has formed the basis for economic assumptions underlying the fiscal projections in budget planning. In other words, for a given Canadian macroeconomic variable, the standard deviation of its simulated distribution over the planning horizon is roughly in line with the (root-mean-square)

[10] Forecasts of the key economic variables based on the non-calibrated version of the model are in line with the private sector forecast presented in the November 2006 Update.

forecast error based on the average forecast from the private sector survey over the period 1994-2006.[11]

3.2 Fiscal structure

The economic results generated by the model feed into a simple fiscal structure derived from Hermanutz and Matier (2000) that produces annual forecasts of government revenues, spending and the budget balance.[12] The model modifies the fiscal structure used by Hermanutz and Matier (2000) by assuming the existence of fiscal forecast uncertainty in addition to that which can be explained by uncertainty in economic inputs.

3.2.1 Overview of Canada's fiscal structure

To facilitate interpretation of the model, it is useful to first review Canada's federal fiscal structure. The composition of federal revenues and spending in fiscal year 2005-06 is presented in Table 1 (see Annex 2 for a detailed decomposition).

Federal revenues are divided into three main components: Tax revenues, revenue from Employment Insurance premiums, and other revenues. Personal income tax, corporate income tax, and the Goods and Services tax (a value-added tax) represent the main components of tax revenues. Employment Insurance premiums are collected from Canadian workers and employers as part of the federal Employment Insurance program, which provides temporary financial assistance for unemployed Canadians while they look for work, as well as other employment benefits (e.g. parental leave). Employment Insurance premiums are determined using a rate-setting mechanism designed to match the level of expected premium revenues with the level of expected program costs. Other revenues include returns on investments, foreign exchange revenues, and revenues from consolidated crown corporations.

Federal spending is divided into four main categories: major transfers to persons, major transfers to other levels of government, direct program spending, and public debt charges. Transfers to persons comprise benefits for seniors under the Old Age Security program, Employment Insurance benefits, and benefits for children. Benefits for seniors and the majority of children's benefits are indexed to inflation, while Employment Insurance benefits (maximum insurable earnings) are indexed to growth in the average industrial wage. Transfers to other levels of government largely comprise major transfers to provinces: equalization payments to provinces, and block transfers in support of provincial health and social programs. Direct

---

[11] Since the distributions of the simulated variables are centred on the forecasts presented in the November Update, their standard deviations are equivalent to the root-mean-square errors.

[12] This is in contrast to the economic forecasts, which are on a quarterly basis. Annual economic projections are derived from the quarterly results.

Table 1

Federal Revenues and Expenses, 2005-06

	Level *($ billions)*	**Share of GDP** *(percent)*
Revenues:		
Tax revenues	186.1	13.6
Employment insurance premium revenues	16.5	1.2
Other revenues	19.6	1.4
Total Revenues	**222.2**	**16.2**
Expenses:		
Major transfers to persons	52.6	3.8
Major transfers to other levels of government	40.8	3.0
Direct program spending	81.8	6.0
Public debt charges	33.8	2.5
Total Expenses	**209.0**	**15.2**
Budget Balance	**13.2**	**1.0**

Source: Annual Financial Report, Fiscal Year 2005-06.

program spending includes expenses for national defence, government enterprises, and departmental operating costs. Public debt charges consist of interest paid on the federal government's interest-bearing debt and other costs associated with servicing the debt.

In 2005-06, federal revenues exceeded federal program spending and public debt charges, resulting in a federal surplus at fiscal year-end of $13.2 billion, or 1.0 per cent of GDP.

3.2.2 Modeling federal revenues and program spending

The relationship between federal revenues (R) and program spending (PE), and the main economic variables[13] – real GDP growth (y_t^{CA}), GDP inflation (py_t^{CA}), and the short-term interest rate (r_t^{CA}) – are estimated using generalized rules of thumb for these relationships presented in the federal government's November 2006

[13] On an annual basis.

Update. Rules of thumb are established using the internal fiscal forecasting model of the Department of Finance and cover three economic shocks: (i) a one-year 1 percentage point decrease in real GDP growth driven equally by lower productivity and employment growth, (ii) a decrease in nominal GDP growth resulting solely from a one-year, 1 percentage point decrease in the rate of GDP inflation, and (iii) a sustained 100-basis point decrease in all interest rates. These rules of thumbs imply relationships for revenues and program spending as depicted below in equations (7) and (8).

$$\Delta \ln R_t = 1.02\Delta y_t^{CA} - 0.34\Delta y_{t-1}^{CA} + 1.05\Delta py_t^{CA} + 0.19\Delta r_t^{CA} + \varepsilon_7, \quad \varepsilon_7 \sim N(0, {\sigma_7}^2) \quad (7)$$

$$\Delta \ln PE_t = c_1 - 0.27\Delta y_t^{CA} + 0.32\Delta py_t^{CA} + \varepsilon_8, \quad \varepsilon_8 \sim N(0, {\sigma_8}^2) \quad (8)$$

Federal revenues grow approximately one-for-one with current real GDP and GDP inflation. The relationship between federal revenue growth and GDP growth is dictated by the relationship between tax revenues and their respective tax bases.

The previous year's real GDP growth influences revenue growth through the rate-setting mechanism for Employment Insurance premium rates. Under this arrangement, a positive shock to real GDP growth leads to higher employment, which leads to stronger growth in premium revenues in year one. However, because Employment Insurance benefits would decline in response to stronger GDP growth, revenue growth is reversed after the first period as the Employment Insurance premium rate is reduced to bring Employment Insurance revenues back to their break-even point with benefits.

Revenues are also a function of the market interest rate, reflecting revenue returns on government financial assets.

In the model, the government targets an annual growth rate for program spending that is in line with the average annual growth rate projected in the November 2006 Update. Deviations from this spending path occur as a result of program spending components whose growth is influenced directly by fluctuations in real GDP growth and GDP inflation. More specifically, growth in program spending is negatively correlated with real GDP growth, reflecting the inverse relationship between the economy and Employment Insurance benefits: stronger real GDP growth and a lower level of unemployment imply lower spending for benefits. Program spending growth increases with GDP inflation given the indexation of statutory programs such as Old Age Security to inflation.[14]

The specifications for both revenues and program spending include stochastic disturbance terms ε_7 and ε_8, assumed to be normally distributed with mean zero and constant standard deviations σ_7 and σ_8. The standard errors, σ_7 and σ_8 are calibrated such that the standard deviation of the simulated distributions for the forecasts of

[14] Indexation may not be exactly in line with GDP inflation due to differences between GDP inflation and CPI inflation. The latter is what is used for the indexation of government programs.

revenue growth and spending growth are consistent with the standard deviation of the federal finance department's revenue and spending forecast errors over the period 1994-2005.[15] Intuitively, ε_7 and ε_8 are intended to capture forecast uncertainty that arises from factors other than errors in economic forecasts.

The period 1994-2005 was chosen because it coincides with the 2004 O'Neill review of federal fiscal forecasting. In that study, series of the forecast errors[16] for the major economic and fiscal components were constructed to assess the federal forecast performance over the period 1994-2003.[17] Forecast errors were adjusted for unforeseen policies announced following the budget forecast, which alter the final result for the year but have no bearing on the forecast performance of the department, and therefore are best excluded from the calculation of forecast errors.

Another benefit of this time period is that fiscal forecasting practices similar to today's practices were in place for most years. For all years considered, the federal government set an annual target for the budget balance as its fiscal rule (deficit targets in the mid-1990s, followed by a target for the surplus once fiscal finances were consolidated) and incorporated some degree of prudence that was explicitly recognized into its budget forecasts. As well, for most years considered, government economic forecasts were based on the average of 15-20 private sector economic forecasts. Considering years with similar forecast practices increases the likelihood that forecast errors over the historical period are representative of forecast errors likely to occur in the future.

As noted in the previous section, the practice of linking future forecast uncertainty to historical forecast errors differs from previous stochastic simulation studies. As well, only a few studies have incorporated the impact of forecast uncertainty beyond that described by economic forecast uncertainty.[18] Those that have, have only considered additional uncertainty in federal revenue forecasts, and have typically relied on the historical series for the revenue-to-GDP ratio to estimate the standard deviation of the disturbance term, which may overstate the forecast uncertainty for revenues due to tax policy changes.[19] By using the federal finance department's historical forecast record rather than the historical series for realized federal revenues, our analysis avoids the issue of additional volatility in revenues

---

[15] The standard deviation of the forecast errors was chosen over the root mean squared error due to the existence of non-zero means for the errors on the fiscal forecasts over the period considered. This methodology, coupled with the model assumption that fiscal forecast errors have mean zero, implies the expectation that going forward forecasts for revenues and program spending will show the same dispersion in errors as that for the historical period, but, unlike the historical period, are unbiased.

[16] The study reports one-year and two-year ahead forecast errors.

[17] For this paper, the series was extended to include 2005 and 2006.

[18] Examples of studies that do incorporate fiscal uncertainty include Hostland (2001), Robson (2006) and Tapp (2006).

[19] Robson attempts to correct for the impact of changing tax policies on the tax yield by considering the standard deviation of the tax yield for the full period (1996-2004) relative to the means of two different fiscal periods within the full sample.

due to tax policy changes. In addition, based on historical forecast accuracy, our model incorporates additional uncertainty in the program spending forecast.

Using the series for one-year ahead fiscal forecast errors from the O'Neill review, the standard deviation on the forecasts of revenue and program spending growth are estimated to be 3.7 and 2.7 percentage points respectively.[20] The implied standard errors for ε_7 and ε_8 are then estimated using equations (7) and (8), and the O'Neill series for the economic inputs.[21] This methodology yields standard errors σ_7 and $\sigma_{8,}$ of 2.7 and 2.6 percentage points respectively for the revenue and program spending growth forecasts.

3.2.3 Estimating non-economic errors for the program spending forecast

One might expect that after accounting for the economic contribution to program spending forecast errors, remaining forecast errors would be minimal given that the government, for the most part, chooses its spending path. However, the estimated standard error on program spending suggests that nearly all of the error on the program spending side is explained by non-economic factors. As such, a few more words on program spending forecast uncertainty and notes to the methodology applied in this paper are necessary to qualify the program spending result.

Uncertainty exists with respect to spending by government departments. Although spending initiatives may be announced, uncertainty still remains regarding whether or not funds will be fully appropriated and spent during the forecasted year. If programs are delayed, funds may lapse or be carried forward to the following year, leading to forecast errors.

Forecast uncertainty on the spending side also exists with respect to the valuation of certain government liabilities, such as liabilities for legal claims, land claims, public service pension plans and more recently, under a full accrual basis of accounting, environmental liabilities and bad debt expenses related to the creditworthiness of debt owed to the Government. When comparing one-year ahead forecasts to actual results, errors may arise due to unforeseen events for which new liabilities are recorded, but also as a result of the revaluation of existing liabilities. Since the value of government liabilities for a given year is determined at year-end after the budget forecast in that year, errors in forecasting program spending are not only conceivable when comparing one-year ahead forecasts to actual results, but

---

[20] The original series were adjusted to accommodate the growth forecasts used for this paper. See Annex 3 for further details regarding computation of growth rates.

[21] O'Neill-calculated forecast errors for the key macroeconomic variables are larger and have greater variance than forecast errors based on the average forecast from the private sector survey for the period 1994 to 1998. This is due to the addition of explicit prudence to the average private-sector forecasts of nominal GDP growth and interest rates for budget planning in those years. When the economic standard errors generated by the model are applied to the fiscal structure, the simulated distribution for the revenue and program spending forecasts exhibit smaller variances than those that existed over the historical period. This methodology makes sense as it removes the impact of deliberate forecast errors from the estimate of forecast uncertainty.

also when comparing in-year forecasts to actual results. Unexpected adjustments to liabilities or the addition of new liabilities can significantly affect forecast accuracy.

Not only do these issues contribute to forecast uncertainty, but for some cases the impact of lapsed or delayed funding on forecast accuracy may not be accurately captured by our model. The use of growth rates in the model implies that errors in the spending level experienced in one year are carried forward to the level forecast for the following years. Unlike the revenue forecast, where this is an appropriate assumption given errors in forecasting the revenue base, on the spending side, forecast errors related to lapsed or delayed funding may not carry forward. For delayed programs, although the spending may not occur in the first year, unless the program is cancelled, some related spending will likely occur in the next year. Consequently, the expression of forecast errors using growth rates may misrepresent some components of program spending errors.

3.2.4 Modelling public debt charges and the budget balance

Public debt charges and the budget balance are modelled as described in Hermanutz and Matier.

As shown in equation (9), the effective interest rate on federal debt (i^D) is a function of the current short-term market rate (r^{CA}), lagged values of the market rate, and a constant rate applied to long-term debt ($\hat{\imath}$). This specification assumes that: a share of debt is short-term in nature and refinanced every year at the current market rate; a share of debt is medium-term debt that is re-financed every three years; and a share of debt is long-term debt that will not be re-financed during the time period covered by the model. Shares are based on the distribution of federal debt as reported for 2005-06. The standard deviation on the forecasted effective interest rate is 0.24 percentage points, and stems solely from the standard deviation on the simulated market interest rate.

$$i_t^D = 0.34 \cdot r_t^{CA} + 0.07 \cdot \left(r_{t-1}^{CA} + r_{t-2}^{CA}\right) + 0.52 \cdot \hat{\imath} \tag{9}$$

Equations 10 through 12 describe public debt charges (*PDC*), federal debt (*D*), and the budget balance (*BB*).

$$PDC_t = i_t^D \cdot D_{t-1} \tag{10}$$

$$D_t = D_{t-1} - 3 \tag{11}$$

$$BB_t = R_t - PE_t - PDC_t \tag{12}$$

Public debt charges are estimated by applying the effective interest rate on federal debt to last period's federal debt. For the purpose of estimating public debt charges, federal debt is assumed to be reduced each year by $3 billion. This practice is consistent with current budget planning practices.

The forecasted budget balance is equal to government revenues less program spending and public debt charges and represents the budget balance for planning

purposes, based on the policy outlook at the time of the forecast and before new measures are introduced.

When interpreting results, the characterization of the projected budget balance is an important consideration. The simulated five-year fiscal paths are based on the policy outlook at the time of the forecast being realized. In some simulations, the fiscal path may show significant deterioration or improvement in the government's fiscal position. Faced with such fiscal outlooks, the government would typically have the opportunity to respond by introducing new policies. For example in simulations where the fiscal position declines significantly over the forecast horizon, the government may chose to lower spending or increase taxes before the forecasted outcome is ultimately realized. As such, results should be interpreted as estimating the impact of uncertainty in budget projections, and not as an indication of the government's fiscal health over the period considered.

4 Model results

Using the November 2006 Update as the starting point for 2006, the model was simulated repeatedly, generating 5000 realizations of the economic and fiscal outcomes for the forecast period 2007 to 2011. The model is calibrated such that the median forecasts match the 2007 to 2011forecasts presented in the November 2006 Update.

4.1 Range of possible outcomes for key economic variables

Table 2 below presents descriptive statistics for key economic variables over the planning horizon 2007 to 2011.

Over the planning horizon (median) real GDP growth is projected to average 2.8 per cent with 90 per cent of the simulated outcomes ranging from 1.1 per cent to 4.3 per cent in 2007. The 90 per cent interval increases with the forecast horizon, ranging from 0.4 per cent to 5.4 per cent in 2011. GDP inflation is projected to average 1.9 per cent with 90 per cent of the outcomes falling within 1.6 percentage points of the median outcome in 2007, increasing to 2.6 percentage points in 2011. The projections of real GDP growth and GDP inflation translate into projected nominal growth averaging 4.6 per cent over the planning horizon with the 90 per cent interval ranging from 2.3 to 7.0 per cent in 2007, rising to 0.9 to 8.3 per cent in 2011. Short-term interest rates are projected to average 4.2 per cent over the planning horizon. The 90 per cent interval increases from 2.8 to 5.1 per cent in 2007 to 0.7 to 8.1 per cent in 2011.

The range and dispersion of outcomes for key economic variables appears "reasonable"; however, without reference to other studies or historical outcomes, it is difficult to gauge the degree to which the simulated distributions may overstate or

Table 2

Economic Outcomes Given in the November 2006 Economic and Fiscal Update

	2006	**2007**	**2008**	**2009**	**2010**	**2011**
Nominal GDP *($ billions)*	1,440	1,507	1,580	1,653	1,731	1,809
95th percentile	-	*1,541*	*1,655*	*1,758*	*1,861*	*1,975*
5th percentile	-	*1,472*	*1,510*	*1,558*	*1,611*	*1,659*
Nominal GDP Growth *(percent)*	5.0	4.6	4.9	4.6	4.7	4.5
95th percentile	-	*7.0*	*8.4*	*8.2*	*8.3*	*8.3*
5th percentile	-	*2.3*	*1.6*	*1.2*	*1.1*	*0.9*
Real GDP Growth *(percent)*	2.8	2.7	2.9	2.9	2.9	2.9
95th percentile	-	*4.3*	*5.2*	*5.3*	*5.5*	*5.4*
5th percentile	-	*1.1*	*0.8*	*0.6*	*0.5*	*0.4*
GDP inflation *(percent)*	2.1	1.9	1.9	1.7	1.6	1.6
95th percentile	-	*3.5*	*4.2*	*4.1*	*4.2*	*4.2*
5th percentile	-	*0.3*	*–0.3*	*–0.7*	*–0.7*	*–0.9*
Short-term Interest Rate *(percent)*	4.1	4.0	4.2	4.2	4.3	4.3
95th percentile	-	*5.1*	*7.0*	*7.9*	*7.9*	*8.1*
5th percentile	-	*2.8*	*1.4*	*0.8*	*0.8*	*0.7*

understate the uncertainty facing fiscal planners. As a check on the reasonableness of our simulated economic outcomes over the planning horizon, for each year of the projection, we compare the root-mean-squared error of the simulated outcomes to the corresponding (root-mean-squared) forecast error based on the average forecasts from the Department of Finance surveys of private sector forecasters over the period 1994-2006 (Table 3). Over most of this period, these surveys have provided the basis for economic assumptions underlying the fiscal projections used for budget planning.

The distribution of possible outcomes for key economic variables in a given year of the planning horizon appears generally consistent with the accuracy of the average private sector forecast over the 1994-2006 period. For real GDP growth, the

Table 3

Dispersion in Simulated Outcomes Compared to Private Sector Forecasts
(percent)

		2007	2008	2009	2010	2011
Real GDP Growth	Simulated RMSE	0.99	1.34	1.42	1.52	1.56
	Survey-based RMSE*	1.44	1.52	1.66	1.31	1.48
GDP Inflation	Simulated RMSE	0.97	1.35	1.5	1.5	1.55
	Survey-based RMSE*	1.05	1.29	1.36	1.37	1.47
Nominal GDP Growth	Simulated RMSE	1.43	2.05	2.16	2.19	2.30
	Survey-based RMSE*	2.08	2.05	2.1	1.96	2.30
Short-term Interest Rate	Simulated RMSE	0.69	1.64	2.12	2.17	2.24
	Survey-based RMSE*	0.71	1.76	1.78	1.81	1.59

* Based on 2006Q4 National Accounts release.

model generates a distribution of outcomes over the planning horizon that is somewhat smaller than the 1-year ahead forecast error would imply.[22] The distribution of GDP inflation outcomes is in line with the dispersion implied by the average private sector forecast. In terms of nominal GDP growth, the distribution of outcomes in 2007 is smaller than the forecast error would imply however the distributions in the remaining years of the planning horizon are in line with the forecast errors at longer horizons. While the simulated root-mean-squared error for

[22] Interestingly, the 4- and 5-year ahead survey-based forecast errors for real GDP growth are smaller than the 2- and 3-year ahead forecast errors. This is also the case for the five-year ahead forecast of the short-term interest rate. We suspect that this result likely reflects changes in the number of survey participants providing longer-term forecasts: almost all participants provide forecasts for outcomes one to three years ahead; however, the sample size falls significantly as the forecast horizon is extended, which would affect the consistency and representativeness of the survey. In addition, while the overall sample size is small – a maximum of 13 observations for the period 1994-2006 – the sample size for 5-year ahead forecasts is limited to only six observations (instead of nine) because 5-year ahead forecasts were not collected in the surveys over the period 2001-03. As a result, the degree of uncertainty facing fiscal planners at longer-term horizons (*i.e.*, 4- and 5-years ahead) may not be adequately represented by the survey-based root-mean-squared forecast errors.

the short-term interest rate is close to the private sector forecast error in 2007 and 2008, its distribution at longer horizons is larger than what the forecast errors would imply. On balance, we suggest that the distributions of economic outcomes generated by our model and calibration appear "reasonable", *i.e.* in line with the historical forecast accuracy of private sector economic forecasts that have been used for budget planning purposes over a two-year time horizon.

4.2 Range of possible outcomes for fiscal variables

Table 4 presents simulated fiscal outcomes over the planning horizon 2007 to 2011.

Over the planning horizon, median budgetary revenues are projected to grow by 3.8 per cent on average with 90 per cent of the simulated outcomes falling between –1.3 per cent and 9.1 per cent in 2007 and –1.6 per cent and 11.1 per cent in 2011. The median forecast for growth in budgetary revenues is similar to that for nominal GDP growth – the underlying tax base. However, the model generates a wider range of possible outcomes for growth in budgetary revenues than that for nominal GDP. This reflects the inclusion in the model of additional variance on the revenue forecast to capture the impact of fiscal forecast uncertainty. The distribution allows for negative growth in revenues in approximately 10 per cent of the simulations. These outcomes appear reasonable given extreme situations where growth in nominal GDP remains low over the planning horizon.

The median forecast for program spending growth follows the path projected in the November 2006 Update. The 90 per cent probability interval ranges from 0.0 per cent to 9.0 per cent in 2007 and from –0.8 per cent to 8.2 per cent in 2011.

The median forecast of the effective interest rate on public debt, which is a function of the short-term interest rate, averages 7.3 per cent over the forecast horizon. It exhibits narrower probability bands than those for the short-term interest rate reflecting the model assumption that less than 100 per cent of government debt is rolled over at the current market interest rate. The median forecast for public debt charges assumes $3 billion in federal debt reduction per year. The range of possible outcomes generated by the model is dictated by the dispersion in outcomes of the forecasted effective interest rate.

The model yields a 90 per cent probability interval for the underlying budget balance of –$6.9 to $21.8 billion (–0.5 to 1.5 per cent as a share of nominal GDP) for 2007. From here, the range of possible outcomes for the underlying surplus grows over time, given growing probability intervals for the levels of federal revenues and program spending.

The end points of the 90 per cent probability interval represent extreme outcomes for the distribution. To provide a more complete picture of the range of possible outcomes, Figure 1 shows the distribution of model outcomes around the median surplus for the forecast horizon considered. The full shaded area represents 90 per cent of model outcomes. Each shaded band represents 10 per cent of model

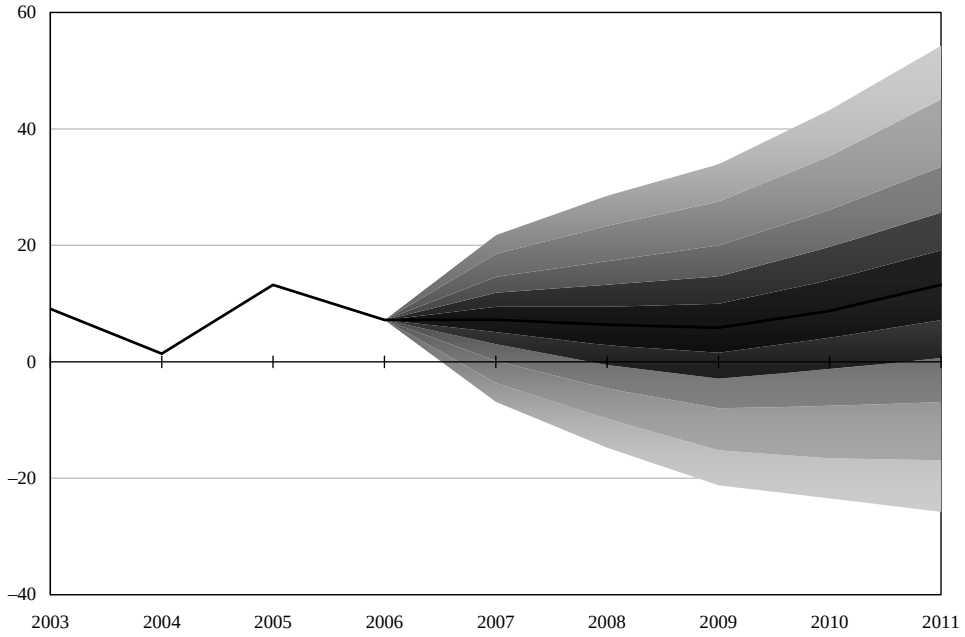
Table 4

Fiscal Outcomes Given in the November 2006 Economic and Fiscal Update

	2006	2007	2008	2009	2010	2011
Budgetary Revenues *($ billions)*	229.4	238.1	245.5	253.7	264.2	276.3
95th percentile	-	*250.3*	*266.1*	*281.0*	*296.5*	*315.8*
5th percentile	-	*226.4*	*227.0*	*230.7*	*236.1*	*241.9*
Budgetary Revenues *(percent of GDP)*	15.9	15.8	15.5	15.4	15.3	15.3
95th percentile	-	*16.5*	*16.6*	*16.7*	*16.7*	*17.0*
5th percentile	-	*15.1*	*14.6*	*14.2*	*13.9*	*13.8*
Growth in Budgetary Revenues *(percent)*	3.2	3.8	3.2	3.4	4.1	4.6
95th percentile	-	*9.1*	*9.3*	*9.6*	*10.4*	*11.1*
5th percentile	-	*–1.3*	*–2.4*	*–2.4*	*–1.9*	*–1.6*
Program Spendine *($ billions)*	187.6	196.0	204.4	213.2	220.6	228.6
95th percentile	-	*204.6*	*217.8*	*230.3*	*241.3*	*252.3*
5th percentile	-	*187.7*	*192.5*	*197.5*	*202.1*	*207.7*
Program Spendine *(percent of GDP)*	13.0	13.0	12.9	12.9	12.7	12.6
95th percentile	-	*13.6*	*14.0*	*14.2*	*14.3*	*14.4*
5th percentile	-	*12.4*	*12.0*	*11.7*	*11.4*	*11.1*
Growth in Program Spendine *(percent)*	7.1	4.5	4.3	4.3	3.5	3.5
95th percentile	-	*9.0*	*9.1*	*9.0*	*8.1*	*8.2*
5th percentile	-	*0.0*	*–0.1*	*–0.2*	*–1.1*	*–0.8*
Effective Interest Rate on Public Debt *(percent)*	7.2	7.3	7.3	7.3	7.4	7.4
95th percentile	-	*7.6*	*8.3*	*8.8*	*8.9*	*9.0*
5th percentile	-	*6.9*	*6.3*	*6.0*	*5.9*	*5.9*
Public Debt Charges *($ billions)*	34.6	34.7	34.6	34.7	34.6	34.5
95th percentile	-	*36.6*	*39.5*	*41.5*	*41.8*	*41.9*
5th percentile	-	*32.9*	*30.0*	*28.2*	*27.7*	*27.6*
Public Debt Charges *(percent of GDP)*	2.4	2.3	2.2	2.1	2.0	1.9
95th percentile	-	*2.4*	*2.4*	*2.4*	*2.3*	*2.2*
5th percentile	-	*2.2*	*2.0*	*1.8*	*1.7*	*1.6*
Underlying Budgetary Bilance *($ billions)*	7.2	7.2	6.4	5.8	8.8	13.2
95th percentile	-	*21.8*	*28.5*	*34.0*	*43.3*	*54.3*
5th percentile	-	*–6.9*	*–14.7*	*–21.2*	*–23.4*	*–25.8*
Underlying Budgetary Bilance *(percent of GDP)*	0.5	0.5	0.4	0.4	0.5	0.7
95th percentile	-	*1.5*	*1.8*	*2.1*	*2.5*	*3.0*
5th percentile	-	*–0.5*	*–1.0*	*–1.3*	*–1.4*	*–1.5*

Figure 1

Underlying Budgetary Balance
Range of Possible Outcomes
Given in the November 2006 Economic and Fiscal Update
(billion dollars)



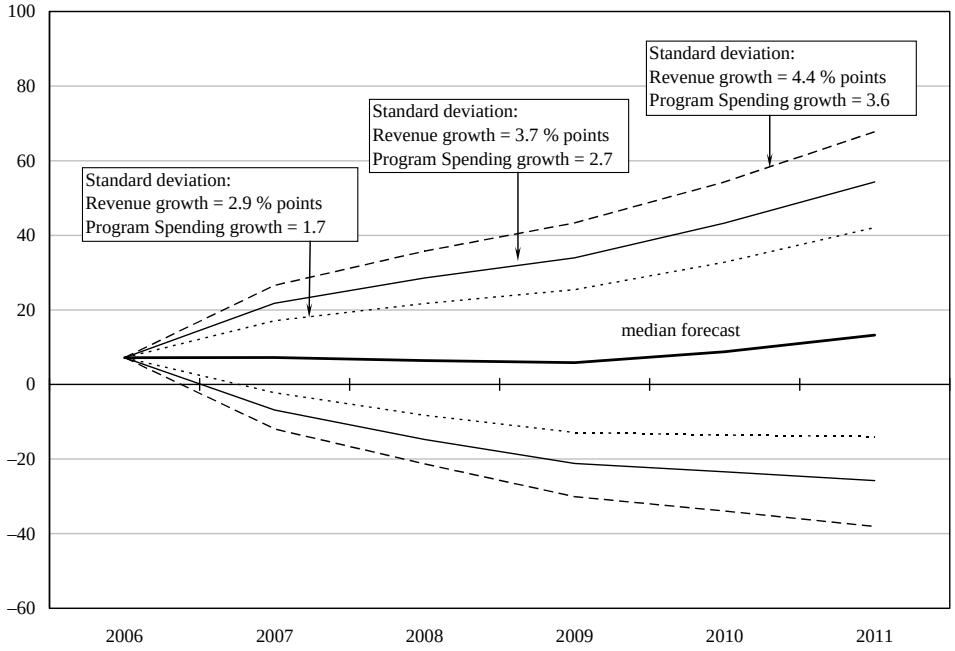
outcomes; with the farthest two bands each representing 5 per cent of outcomes. The narrower bands close to the middle of the distribution illustrate that the probability density is highest near the median forecast. More distant outcomes occur with less frequency, as illustrated by the wider bands farther from the median forecast. The first three bands on either side of the median together represent 60 per cent of possible outcomes. For 2007, this 60 per cent probability interval ranges from $0.2 to $14.6 billion (0 to 1.0 per cent of GDP).

4.3 Sensitivity analysis: revenue and program spending uncertainty

The ranges of possible fiscal outcomes are driven by the assumption that economic and fiscal forecast errors in the future will show similar dispersion as forecast errors over the historical period 1994 to 2005. Although this is a reasonable assumption, depending upon the time period chosen, the dispersion in forecast errors can change significantly. This is particularly true for fiscal forecast errors, which historically have often changed significantly from one year to the next. Furthermore, as discussed in Section 3, quantifying program spending uncertainty is not a

Figure 2

Forecasted Underlying Budgetary Balance
(90 per cent probability intervals given various scenarios for the dispersion of fiscal forecast errors)
(billion dollars)



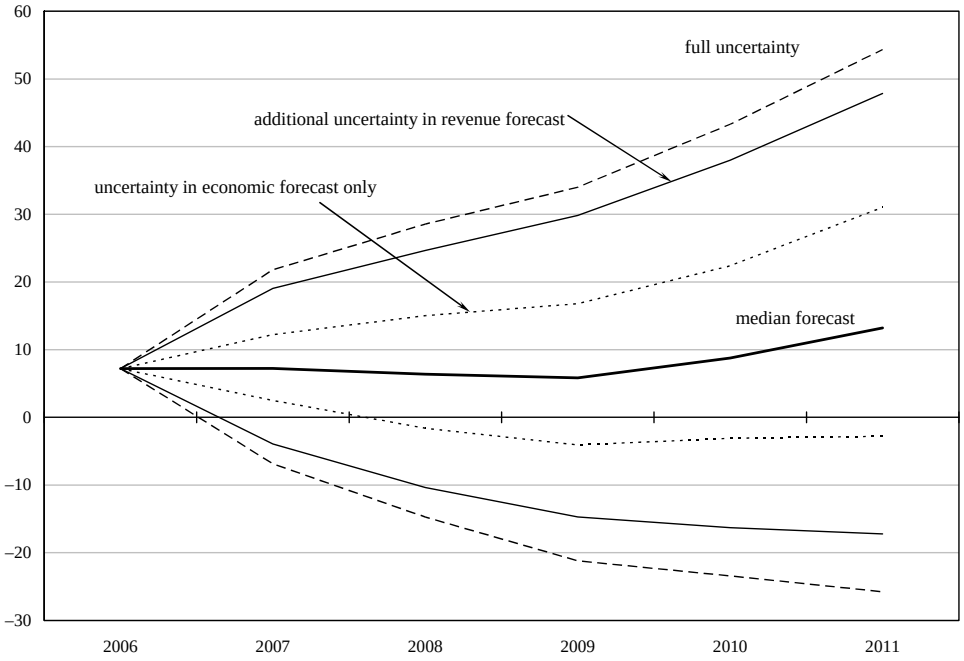
straightforward task. For these reasons, it is useful to provide a sense of the robustness of results under alternative assumptions for the degree of revenue and program spending forecast uncertainty. Figures 2 and 3 show the impact on the range of possible fiscal outcomes of different assumptions for fiscal dispersion.

Figure 2 shows the difference in the 90 per cent probability intervals surrounding the median forecast of the underlying budgetary balance from changing the standard error for the revenue and program spending equations by +/– 1 percentage point. Under these alternative assumptions, the implied standard deviations on the forecast of revenue growth would be approximately 4.4 and 2.9 percentage points respectively. The implied standard deviations on the forecast of program spending growth would be 3.6 and 1.7 percentage points respectively. The implied standard deviations change the total width of the probability interval on the underlying budgetary balance by approximately +/– $10 billion in 2007, increasing to +/– $25 billion in 2011.

Figure 3 shows the impact on the range of possible outcomes for the underlying budgetary balance of economic forecast uncertainty, as well as revenue

Figure 3

Forecasted Underlying Budgetary Balance
(90 per cent probability intervals under the different types of forecast uncertainty)



and program spending uncertainty beyond that which can be explained by economic forecast uncertainty. Based on our model, in an environment of uncertainty in only the economic forecasts, 90 per cent of possible outcomes for the underlying budgetary balance in 2007 would be within +/– $5 billion of the median forecast. This amount increases to +/– $17 billion in 2011.[23]

Uncertainty in the revenue forecast beyond that which is explained solely by economic inputs accounts for the majority of fiscal forecast uncertainty and roughly doubles the width of the probability interval relative to an environment of only economic uncertainty. Once additional uncertainty in the program spending forecast is included, Figure 3 shows that fiscal forecast uncertainty beyond that due to the economic forecast accounts for more than half of the total uncertainty in the fiscal forecast.

---

[23] Since it is assumed that forecast errors are normally distributed around growth forecasts, the distribution around level forecasts is not quite symmetric. For the described scenario, the exact range around the median surplus is +$4.9 billion and –$4.8 billion in 2007-08 and +$17.9 billion and –$15.9 billion in 2011-12.

The model simulations illustrate that economic and fiscal forecast uncertainty influences the accuracy of budget forecasts considerably, and may imply a high probability of deficit outcomes if insufficiently addressed. In this environment the fiscal authority may need to take measures in order to ensure an acceptable fiscal outcome at year-end.

From a technical perspective, the probability of deficit may be reduced by one of two means: by decreasing the dispersion of the distribution of potential outcomes for the planning surplus, or by shifting the mean of the distribution. From a policy perspective, the former may be achieved by making improvements to forecast models and methodologies, or by improving the quality and/or timing of forecast data. The last two government-commissioned reviews of federal fiscal forecasting practices in Canada (the aforementioned O'Neill review in 2004 and the Ernst & Young review in 1994), made several recommendations of this nature, e.g. periodic reviews of federal forecasting models, examination of the causes for revisions to historical GDP data, etc. However, policy reforms of this sort are not always guaranteed to significantly improve forecast performance since a certain degree of uncertainty is unavoidable. An alternative course of action would be to shift the mean of the distribution for the forecasted surplus. This may be done by deciding to allocate less than 100 per cent of the planning surplus to new tax and spending measures, thereby leaving a portion of the forecasted surplus unallocated, *i.e.* targeting a minimum surplus, to guard against negative shocks to the fiscal forecast. Section 5 extrapolates from the model results in order to consider this approach in greater detail.

5 Fiscal planning under uncertainty

In this section, we extrapolate from the distribution for the underlying budgetary balance generated by the model to illustrate how knowledge of forecast uncertainty can be incorporated into governments' fiscal planning assumptions. Specifically we assume that the fiscal authority can target a minimum surplus of various sizes as its fiscal rule. Under this simple fiscal rule, the fiscal authority first sets its minimum surplus target based on an acceptable risk of deficit. It then establishes its unbiased forecast estimates for revenues, program spending, and public debt charges for the five-year period and sets new spending and tax measures, leaving an amount equal to the minimum surplus unallocated to protect against deficit outcomes.

We do not draw conclusions regarding what might constitute an acceptable risk of deficit for each year of the forecast horizon and as such, what might constitute an "optimal" choice for the minimum surplus target. Instead the surplus target is estimated at various probabilities of surplus. Several factors will influence a government's tolerance for deficit outcomes, such as the government's recent fiscal record or its policy goals.

Tables 5 and 6 show the minimum targeted surplus necessary for each of the scenarios presented in Figures 2 and 3 assuming a desired probability of surplus of 50, 60, 70, 80, and 90 per cent. By construction, under all scenarios targeting a balanced budget is required for a 50 per cent chance of achieving a surplus. As the desired probability of surplus increases, so too does the size of the necessary targeted surplus, leaving less room for new tax and spending measures. Although not shown, it is possible to plan for a deficit outcome and still achieve a surplus at year-end, although, based on our model, this will happen less than 50 per cent of the time.

For all scenarios, the necessary targeted surplus increases in the latter years, reflecting growing uncertainty over the forecast horizon. In the Canadian context, the first two years of the horizon are the most relevant, as they coincide with the federal government's budget planning horizon. As well, for later years of the forecast horizon, the fiscal authority has time to react to changes in the fiscal outlook. This, coupled with the two-year planning horizon makes the near-term the more relevant period for assessing the value of possible fiscal rules under forecast uncertainty.

Table 5 shows that incorporating the effects of additional uncertainty in fiscal forecasts beyond that due to uncertainty in economic forecasts significantly increases the targeted surplus necessary to ensure a surplus outcome. Focusing on the first two years, and a desired probability of surplus of 70 per cent, assuming only economic uncertainty, the government would need to set aside $1.6 billion in year one and $2.6 billion in year two. Assuming additional uncertainty in the revenue forecast implies that the targeted surplus increases to $3.7 billion and $5.4 billion for years one and two respectively. Finally, by incorporating further uncertainty in the program spending forecast, one increases the targeted surplus further to $4.2 billion and $6.9 billion for years one and two respectively.

Table 6 shows the impact on the targeted surplus from changing the standard errors (*i.e.* the standard deviation of the random variable) for the specifications of revenue and program spending growth. In other words, the table shows the impact of increasing or decreasing the range of possible outcomes for the underlying surplus. Assuming that a 70 per cent probability of surplus is desired, decreasing the standard errors on the fiscal variables by 1 percentage point reduces the size of the targeted surplus to $2.9 billion in the first year and $4.8 billion in the second year. Alternatively, if the standard errors were to increase, *i.e.* increased forecast uncertainty, then the size of the targeted surplus would increase to $5.7 billion in year one and $9.1 billion in year two.

Relating the results from Table 6 back to possible government policy responses to forecast uncertainty, the effect on the targeted surplus from reducing the standard errors on the fiscal forecasts could also be interpreted as the potential value of improved forecast accuracy, *i.e.* improved quality in forecast data, or improved forecast methodologies, to the fiscal planning exercise. For example, assuming a desired probability of 70 per cent, if the government were able to make

Table 5

Targeted Surplus under Various Scenarios for Forecast Uncertainty

Scenarios: Fiscal forecast uncertainty	Uncertainty in the economic forecast only					Additional uncertainty in the revenue forecast					Full forecast uncertainty				
Probability of surplus in each year	2007/08	2008/09	2009/10	2010/11	2011/12	2007/08	2008/09	2009/10	2010/11	2011/12	2007/08	2008/09	2009/10	2010/11	2011/12
	Targeted surplus *($ billions)*														
50%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60%	0.8	1.3	1.5	1.7	2.4	1.9	2.7	3.4	3.9	4.5	2.1	3.5	4.2	4.6	6.0
70%	1.6	2.6	3.2	3.9	5.2	3.7	5.4	6.8	7.9	9.6	4.2	6.9	8.7	10.0	12.5
80%	2.5	4.1	5.1	6.2	8.3	5.8	8.6	10.6	13.0	15.4	7.0	10.9	13.8	16.3	20.1
90%	3.8	6.3	7.6	9.5	12.5	8.8	13.0	15.9	19.7	23.7	10.8	16.2	21.0	25.3	30.1

Table 6

Targeted Surplus under Various Scenarios for Forecast Uncertainty

Scenarios: Fiscal forecast uncertainty	Full forecast uncertainty					less 1 percentage point on standard errors for revenue and program spending growth					plus 1 percentage point on standard errors for revenue and program spending growth				
Probability of surplus in each year	2007/08	2008/09	2009/10	2010/11	2011/12	2007/08	2008/09	2009/10	2010/11	2011/12	2007/08	2008/09	2009/10	2010/11	2011/12
	Targeted surplus *($ billions)*														
50%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
60%	2.1	3.5	4.2	4.6	6.0	1.5	2.2	2.8	3.3	4.4	2.7	4.5	5.3	6.4	7.8
70%	4.2	6.9	8.7	10.0	12.5	2.9	4.8	6.0	7.2	8.9	5.7	9.1	11.4	13.2	16.4
80%	7.0	10.9	13.8	16.3	20.1	4.8	7.4	9.5	11.5	14.1	9.5	14.3	18.0	21.7	26.0
90%	10.8	16.2	21.0	25.3	30.1	7.5	11.3	14.4	17.9	21.5	14.5	21.3	28.0	33.5	39.5

improvements to the quality of forecast data and methodologies that resulted in a 1 percentage point reduction in the standard errors on the fiscal forecasts (*i.e.* the middle scenario in Table 6), it could free up $1.3 billion ($4.2 billion less $2.9 billion) worth of fiscal room that could be used to introduce new measures without increasing the risk of a deficit outcome. Alternatively, it could leave this extra fiscal room unallocated and increase the probability of surplus to nearly 80 per cent.

6 Conclusions

In this paper we use a stochastic simulation model to assess the effect of forecast uncertainty on federal budget projections. We also investigate the practice of leaving a portion of the planning surplus unallocated as a possible policy response to the existence of forecast uncertainty.

Results show that uncertainty related to both economic and fiscal forecasts imply a wide range of possible outcomes for the actual budgetary balance relative to its value projected at the time of the forecast. Our results also suggest that fiscal forecast uncertainty beyond that expected as a result of uncertainty in economic projections plays an important role, and could contribute more to the overall uncertainty in budget projections than that which can be explained by economic forecasts.

As a policy response, the fiscal authority could choose to mitigate the effects of forecast uncertainty by allocating less than 100 per cent of the planning surplus to new tax and spending measures. This could be done through a simple minimum surplus target as our analysis has illustrated, or other measures such as a ceiling on discretionary spending. Based on our results, an unallocated amount of between $4 billion and $7 billion would imply a probability of surplus of between 70 and 80 per cent in year one. To achieve this assurance of surplus in the second year of the forecast horizon, unused fiscal room would need to be increased to between $7 and $11 billion. Although assurance of a surplus at year-end would be greatly increased under such a policy response, it would come at the cost of reduced funding for new spending and tax measures.

Our results for the 2007-11 forecast horizon are based on the assumption that the federal government faces future forecast uncertainty that is similar to that of the past. It is conceivable that inputs into the forecast process may change in a manner that alters the behaviour of current forecasts compared to the past. In recognition of this possibility, on the fiscal side, we also consider the impact on our results of increasing or decreasing the degree of fiscal forecast uncertainty. Such an analysis also provides some insight into the potential impact of government policies aimed at reducing the degree of uncertainty in budget projections, e.g. model or data improvements.

Regardless of the government's specific choice of policy response, awareness of the degree of uncertainty in budget projections is a critical element of successful

budget planning. Analyses such as the one provided by this paper provide useful perspective for policymakers when developing budget projections and deciding upon new fiscal policies.

Going forward our stochastic model provides a framework for further analysis of fiscal policy in an environment of forecast uncertainty.

ANNEX 1
FEDERAL FISCAL FORECASTING IN CANADA

The federal government updates fiscal projections twice a year – in the federal budget, typically in the spring, and in the Economic and Fiscal Update in the fall. In addition, the fiscal outlook for the year in progress is updated again in June and September.

To ensure objective economic assumptions, fiscal forecasting begins with the use of private sector economic forecasts. The Department of Finance surveys about 20 private sector economic forecasters four times a year, following the release of the quarterly National Income and Expenditure Accounts by Statistics Canada. The average of the private sector economic forecasts from the survey forms the basis for the government's fiscal projections presented in the budget, the Economic and Fiscal Update, and on a quarterly basis.

The Economic and Fiscal Update, prepared in the fall, presents economic and fiscal projections for the year in progress and the next five years in order to show medium-term implications of current policies. These projections are presented alongside projections prepared by four private sector organizations. While the government's projections form the basis for fiscal planning, the fiscal forecasts produced by the private sector organizations allow for comparison of different views of the fiscal outlook.

In the spring, fiscal projections presented in the Economic and Fiscal Update are updated for the budget to reflect the most recent survey of private sector economic forecasters and the most recent financial results. Since a short planning horizon is appropriate for budget decisions, fiscal projections in the budget are presented for the year in progress and the first two years of the forecast horizon. Budget projections include planned annual debt reduction of $3 billion.

Since June 2006, the government has also presented quarterly updates of the government's fiscal forecast for the fiscal year in progress based on financial information regarding the previous fiscal year and information to date for the current fiscal year.

ANNEX 2
COMPOSITION OF FEDERAL FINANCES IN CANADA

Tables 7 and 8 present a decomposition of federal revenues and expenses for the fiscal year 2005-06.

Table 7

Federal Revenues, 2005-06

	Level *($ billions)*	**Share of Total** *(percent)*
Tax revenues		
Income tax		
Personal income tax	103.7	46.7
Corporate income tax	31.7	14.3
Other income tax	4.5	2.0
Total	139.9	63.0
Other taxes and duties		
Goods and services tax	33.0	14.9
Energy taxes	5.1	2.3
Customs import duties	3.3	1.5
Other taxes and duties	4.7	2.1
	46.2	20.8
Total tax revenues	**186.1**	**83.8**
Employment insurance premium revenues	16.5	7.4
Other revenues		
Crown corporation revenues	7.2	3.2
Foreign exchange revenues	2.0	0.9
Other program revenues	10.4	4.7
Total	19.6	8.8
Total revenues	**222.2**	**100.0**

Source: Annual Financial Report, Fiscal Year 2005-06.

Table 8

Federal Expenses, 2005-06

	Level *($ billions)*	**Share of Total** *(percent)*
Transfer payments		
Major transfers to persons		
Elderly benefits	29.0	13.9
Employment insurance benefits	14.4	6.9
Canada Child Tax Benefit	9.2	4.4
Total	52.6	25.2
Major transfers to other levels of government		
Support for health and social programs	27.2	13.0
Fiscal arrangements and other transfers	13.0	6.2
Other transfers	3.3	1.6
Alternative transfers for standing progran	–2.7	–1.3
Total	40.8	19.5
Direct program expenses		
Subsidies and other transfers	24.9	11.9
Other program expenses		
Crown corporations	7.2	3.4
National Defence	15.0	7.2
All other departments and agencies	34.7	16.6
Total	56.9	27.2
Total program expenses	**175.2**	**83.8**
Public debt charges	33.8	16.2
Total expenses	**209.0**	**100.0**

Source: Annual Financial Report, Fiscal Year 2005-06.

ANNEX 3
VARIATION IN REVENUE AND PROGRAM SPENDING FORECASTS

The variation in the forecasts of revenue and program spending growth used in the model was estimated based on O'Neill's 2005 Review of Canadian Federal Fiscal Forecasting. In that review, level forecast errors were estimated over the period 1994-95 to 2003-04 after first making adjustments for policy initiatives announced post-forecast and for accounting changes during the period.

For the purposes of the model used in this paper, it was necessary to translate these series into series of the forecast errors on growth forecasts. Forecasted growth is estimated as the percentage change in the one-year ahead forecast from the in-year forecast for the year before.[24] Actual growth is estimated as the percentage change in the actual level for the year in question adjusted for policy changes unforeseen at the time the forecast was made, from the actual (unadjusted) level for the year before. Actual levels are taken from the Annual Financial Report of the Government of Canada for the year in question. Taking 1994-95 as an example:

- Forecasted growth in revenues is set equal to the percentage change in the forecasted level for 1994-95 from the forecasted level for 1993-94, using forecasts from the February 1994 budget.
- Actual growth in revenues is set equal to the percentage change in the actual level for 1994-95, adjusted for unforeseen policy measures, from the actual (unadjusted) level for 1993-94, using levels as reported at the end of 1993-94 and 1994-95 in the Annual Financial Report.

For some years, accounting adjustments to actual levels were necessary to allow comparisons across years. For example, in 2002-03 and 2005-06 actual outcomes were presented in the Annual Financial Report on a different basis of accounting than the forecast. For these years, actual outcomes were adjusted to the same basis of accounting as that used at the time of the forecast.

The estimated forecast errors for program spending growth assume that the government knows the outcome for transfers to other levels of government with certainty. In the past, provincial/territorial fiscal arrangements (39 per cent of transfers to other levels of government) were formula-based and estimates frequently revised, resulting in payments that were volatile and difficult to forecast, yielding significant forecast errors. In 2004, the program was moved away from its original formula-based approach to ensure more predictable funding. Most importantly for the purposes of forecasting, total funding was legislated to grow by 3.5 per cent annually, implying zero forecast uncertainty. More recently the program has been moved back towards a more formula-based approach. However, unlike the original approach, which resulted in unpredictable and volatile payments, several reforms have been introduced under the new approach to ensure that future funding

---

[24] One-year ahead forecasts refer to the forecast for the next fiscal year. In-year forecasts refer to the forecast for the year in progress. Taking 1994-95 as an example, the one-year ahead forecast was made in the February 1994 budget, while the in-year forecast was provided in the February 1995 budget.

Table 9

Estimated Dispersion in One-year Ahead Fiscal Forecast Errors

	Revenue Growth Error	Program Spending Growth* Error
	(percentage points)	
1994-95	1.7	2.1
1995-96	0.9	0.3
1996-97	–2.3	4.2
1997-98	–6.7	1.7
1998-99	0.5	1.2
1999-00	–6.5	4.3
2000-01**	–7.6	–1.9
2001-02	2.9	–0.7
2002-03**	2.3	2.7
2003-04	–1.4	–0.5
2004-05	–3.2	–5.2
2005-06**	–4.9	2.9
Mean	–2.0	0.9
Standard deviation	**3.7**	**2.7**
Root-mean squared error	4.1	2.8

Source: Review of Canadian Federal Fiscal Forecasting (June 2005), Federal Budgets and Annual Financial Reports, author's calculations.

* Impact of errors in the forecast of transfers to other levels of government excluded.

** Accounting adjustments made to actual levels.

remains stable and predictable. Since historical errors in the forecasts of transfers to other levels of government do not reflect the new regime, they are excluded from our calculations so as not to overestimate the uncertainty that exists in these programs going forward. Although this methodology may underestimate the uncertainty in program spending growth, it is difficult to assess to what extent, or if at all, without historical data under the new regime.

Table 9 presents the modified series from the O'Neill review, with two years added along with the mean, standard deviation, and root-mean-squared error. The model used for this paper is calibrated such that the standard deviations for revenue and program spending growth match the standard deviations for the errors in these forecasts over the period 1994-95 to 2005-06. The standard deviations were chosen over the root-mean squared error to eliminate the effect of forecast bias exhibited in these series (in particular the revenue series) over the historical period, *i.e.* future fiscal forecasts provide the government's "best guess" ex ante and do not contain implicit bias, *i.e.* mean error equals zero.

Going forward, it is possible that forecast bias might persist. For instance, faced with significantly higher tax elasticity than one would expect based on theoretical and empirical evidence, forecasters would likely not respond by adjusting revenue forecasts upwards immediately, but rather would adjust their forecasts gradually over time after gathering more information and knowledge of the source and persistence of the higher tax elasticity. If the higher tax elasticity were to persist, fiscal forecasts would accordingly show some bias for a period of time.

Incorporating the estimated bias going forward would shift the mean of the model-simulated distribution for federal revenue growth higher by 2 percentage points, and increase the estimated probability of surplus for each year of the forecast horizon.

REFERENCES

Boothe, P. and B. Reid (1998), "Fiscal Prudence and Federal Budgeting in the Medium Term", in T.J. Courchene and T.A. Wilson (eds.), *Fiscal Targets and Economic Growth*, Kingston, John Deutsch Institute for the Study of Economic Policy, Queen's University.

Congressional Budget Office (2007), "The Uncertainty of Budget Projections: A Discussion of Data and Methods", http://www.cbo.gov/ftpdocs/78xx/doc7837/03-05-Uncertain.pdf

Department of Finance Canada (1993), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (1994), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (1994), *Annual Financial Report of the Government of Canada*. Ottawa, Department of Finance, Canada.

——————— (2003), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (2004), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (2005), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (2006), *The Budget Plan*, Ottawa, Department of Finance, Canada.

——————— (2006), *Annual Financial Report of the Government of Canada*, Ottawa, Department of Finance Canada.

——————— (2006), *Fiscal Reference Tables*, Ottawa, Department of Finance, Canada.

——————— (2006), *The Economic and Fiscal Update*, Ottawa, Department of Finance, Canada.

Ernst & Young (1994), *Review of the Forecasting Accuracy and Methods of the Department of Finance: Final Report*, Toronto, Ernst & Young.

Gaudreault, C. and R. Lamy (2001), "Improvements To Finance's Commodity Price Index and New Commodity Price Indices By Province And Territory", Ottawa, Department of Finance, Canada.

Helliwell, J., R. Issa, R. Lafrance and Q. Zhang (2005), *NEMO: A Canadian-U.S. Dollar Exchange Rate Equation, Canada and the Global Economy*, proceedings of a conference held at the Bank of Canada, November 2004.

Hermanutz, D. and C. Matier, (2000), "Modelling Federal Finance under Uncertainty", Department of Finance, Working Paper, No. 2000-02, Ottawa, Department of Finance, Canada.

Hostland, D. (2001), "Specification of a Stochastic Simulation Model for the Analysis of Monetary and Fiscal Policy", Department of Finance, Working Paper, No. 2001-14, Ottawa, Department of Finance, Canada.

Murchinson, S. (2001), "NAOMI: A New Quarterly Forecasting Model Part II: A Guide to Canadian NAOMI", Department of Finance, Working Paper, No. 2001-25, Ottawa, Department of Finance, Canada.

O'Neill, T. (2005), *Review of Canadian Federal Fiscal Forecasting: Processes and Systems*, Toronto, O'Neill Strategic Economics.

Policy and Economic Analysis Program, University of Toronto and Centre Interuniversitaire de Recherche en Analyse des Organisations (2005), *Forecasting Processes and Performance of the Department of Finance*, Toronto, Institute for Policy Analysis, University of Toronto.

Robson, W.B.P. (2006), "Beating the Odds: A New Framework for Prudent Federal Budgeting", C.D. Howe Institute, Commentary, No. 230, Toronto, C.D. Howe Institute.

Tapp, S. (2006), "Can a Medium-term Target Improve Canada's Fiscal Policy?", unpublished manuscript presented at the 2006 Annual Meetings of the Canadian Economics Association, Montreal, Concordia University.