FILLING THE GAP
MEASUREMENT OF THE CYCLICAL EFFECT ON BUDGETS

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One of the most vivid discussions in every country is with regard to the assessment of fiscal policy. Since budgets are influenced by business cycles, there is enormous interest in disentangling the underlying fiscal position from the effect of the business cycle. The two main methods used by international institutions to determine this cyclical factor – the aggregated and disaggregated approaches – arguably do not fulfil the necessary requirements for obtaining correct results. In this paper we introduce an alternative disaggregated methodology which is not only able to incorporate theoretical considerations, but is also easily computable, and does not require unavailable data. We also demonstrate that if the deflators of variables are different, then the real cyclical component has to be corrected to obtain the nominal cyclical component. We show that standard cyclical adjustment methods applying constant elasticities are consistent with the broadest definition of discretionary measures, but possibly inconsistent with the underlying deficit. Standard estimations of the cyclical and underlying components can be faulted due to the specific non-linear features of tax systems and unemployment benefit schemes.

1. Introduction

Methods of the cyclical adjustment seek to remove the cyclical effects from budget revenues and expenditures. This can be done in two different ways depending on the objective of such an indicator.

One approach focuses on measuring the degree of government activity. This approach requires removing exogenous effects from the deficit, including effects of the cycle, deflators, exchange rates and interest rates. The problem is that most of these effects can be influenced by government measures, in other words they are not entirely exogenous. Let us consider the case of government consumption, which directly affects both GDP in real terms and the GDP deflator. Other measures, for example changes in indirect tax rates have a direct and an indirect effect on inflation, consumption and profits at the same time.[1] Usually this approach focuses on a one-year definition of government activity instead of taking into account

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[1] For a discussion of such direct and indirect effects see the *Report on Inflation* of the Magyar Nemzeti Bank, August 2003.

discretionary measures with multi-year impacts, for example designing automatic stabilisers in such a manner that budget responses are more than equiproportionate.

Another approach aims at identifying the structural, underlying component of the deficit. In this case temporary effects should be removed from the budget. Usually these efforts are concentrated on filtering out effects of cyclical fluctuations, temporary fiscal measures and interest rates, but fluctuations of deflators are not addressed. It is important to note that cyclical fluctuations and temporary measures can have overlapping parts since temporary budget spending automatically affects tax bases and revenues. Another issue is that budget responses should be assumed to be equiproportionate, since progressivity by definition has only temporary effects.

In practical application there are also two common methods of cyclical adjustment: the aggregate approach and the unconstrained disaggregate approach. The first one is advocated by the IMF, OECD and the European Commission, while the second is applied by the European Central Bank. Using data from the USA, Japan and 25 EU member countries, Kiss and Vadas (2005) demonstrate that both approaches have significant shortcomings, which could be the source of considerable bias. While the aggregate approach cannot cope with different shocks, the unconstrained disaggregate method involves systematic bias and does not contain theoretical consideration. In order to avoid these distortions they established an alternative framework, which is able to incorporate the advantages of both approaches. However, their method assumes an exogenously given output gap and ignores the effect of unemployment on the budget. Finally, none of the recent methods takes into consideration the effect of different deflators. Note that the computation of the cyclical factor involves several GDP elements in real terms, while the budget is evaluated in nominal terms. If the deflators of GDP elements are different then the real and nominal cyclical components are different as well.

In this paper we introduce a methodology that overcomes these limitations.[2] Here we do not address the issues of fluctuations of deflators, but our proposed method also reduces the potential distortions related to them.

2. Overview of recent approaches and their shortcomings

The potential output of an economy is a commonly cited and widely used concept by both policymakers and analysts when seeking to evaluate an economy. Although economists generally agree on the intuitive concept of the cyclical component, there is less consensus on how to measure it, given that it is unobservable, and thus cannot be measured statistically.

Of the various econometric techniques to solve this problem, practically all of which have been tested as possible candidates for measuring trends and cyclical positions, two main methods of estimating the CAB have emerged: the aggregated

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[2] Gauss code of the two official approaches and our method can be downloaded from http://vadasg.extra.hu

approach, as advocated by the International Monetary Fund (IMF), the Organisation for Economic Cooperation and Development (OECD) and the European Commission (EC), and the unconstrained disaggregated approach, as applied by the European Central Bank (ECB). The aggregated approach assumes that a single number, the output gap, is sufficient to evaluate the cyclical effect on the budget balance. The ECB, recognising that this could be misleading in certain cases, advocates the disaggregated approach. Nevertheless, disaggregation raises two essential issues:

(1) the sum of parts should equal the total value, and

(2) different deflators are used in the case of different variables which are not addressed by ECB-type disaggregation.

2.1 Aggregated approach

As mentioned, this method is applied by the European Commission, IMF and OECD. The key idea of their approach is to focus on the aggregate output gap, deriving its effect on the budget. Denis *et al.* (2002) describe the Cobb-Douglas production function using neutral technological progress to estimate potential output:

$$Y_t^* = TFP_t^* \left[L_t (1 - U_t^*) \right]^{\alpha} K_t^{1-\alpha} \tag{1}$$

where Y^*, L, K, U^* and TFP denote output, labour input, capital stock, the trend unemployment rate and total factor productivity respectively. The trend unemployment rate is considered as the NAIRU and estimated by a state-space model (see Denis *et al.*, 2002), while TFP is computed as a Solow residual. Instead of estimating labour (α) and capital ($1-\alpha$) shares, the EC suggests using national accounts to calibrate them. The output gap is computed in the usual way, namely $OG_t = Y_t / Y_t^*$. The aggregated approach applies simple elasticities to compute the cyclical position of the relevant GDP components, such as private wages, consumption, corporate profit, etc. These elasticities are derived by estimating the co-movement between output and corresponding variables.

Unfortunately, this method has four key drawbacks.

- Firstly, it does not take into account and exploit the consequences of choosing the Cobb-Douglas production form, namely its parameters α and $1-\alpha$ determine not only the labour and capital share in level, but also the relative weight of disaggregated gaps. Specifically, the sum of the labour and capital income gap, weighted by labour and capital shares, should be equal to the aggregated output gap. In addition, labour and capital shares cannot be assumed to be constant even in the case of developed economies, not to mention transition ones.
- Secondly, given that the unemployment rates in transition economies have been influenced by several shocks, the standard relations and state-space estimation therefore yield inappropriate results.

- Thirdly, capital stock and/or TFP are not available in several countries. Moroever, even where they are available, their values are already the result of an estimation process, as they are not observable variables.
- Fourthly, and most importantly, in certain periods the disaggregated approach can be identified as a more appropriate method of cyclical adjustment, because the aggregate output gap and its elements, such as consumption, profit, etc., can vary considerably. The significantly different budgetary implications of these "atypical" circumstances have been taken into account in some *ad hoc* analyses (such as EC, 2000), and a few new methods have been introduced (Bouthevillain *et al.*, 2001; Kiss, 2002; Braconier and Forsfält, 2004).

Given these atypical cases, Boije (2004) argues that the aggregate output gap hides underlying developments. While the same output gap can comprise various components, this output gap has a different effect on the economy and the budget. However, the aggregated approach calculates exactly the same effect based on an identical aggregate output gap.[3] This phenomenon may explain Cronin and McCoy's results (1999). They found that the constant elasticities of budgetary revenue and spending on output are not plausible, although these results may be attributable to the above-mentioned composition effect. Even if elasticities on disaggregated gaps are stable, the degree of elasticity in the aggregate differs if the shares of disaggregated gaps are not constant, which is likely to be true for all countries.

Kiss and Vadas (2005) examined the potential bias of aggregation in the United States, Japan and 25 member countries of the EU. Excluding the USA, aggregation bias causes at least 0.1 of a percentage point error in the cyclical component in almost the entire sample. Serious bias, *i.e.* distortion is more that 0.5 per cent of GDP, occurs roughly in half the sample. The distortion becomes more policy-related if we consider the frequency of those cases when two methods, namely aggregated and the disaggregated one, provide different signs, *i.e.* a misleading cyclical indication for fiscal tightening or loosening. In the case of France the aggregate method provides a wrong indication in 33 per cent of cases. Actually this cannot be considered an extreme result, since the average of 27 countries is 15 per cent.

2.2 ECB-type disaggregated approach

Since the aggregated approach can be appropriate under an extremely strict assumption, *i.e.* every GDP component is at the same cyclical position, the ECB proposes using a disaggregated method. In practical terms, Bouthevillain *et al.* (2001) estimate numerous gaps, such as private wages, employment, consumption, corporate profit and the unemployment gaps, by using a univariate Hodrick-Prescott

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[3] For instance, suppose a fictive example in which the first economy is hit by a foreign demand shock, *i.e.* has a negative export gap, while the second economy faces a negative consumption shock. Since exports have a smaller direct effect on the budget position than consumption, the cyclical effect on the budget is smaller in the first economy. Meanwhile, the aggregate approach reports the same cyclical effect.

(HP) filter. However, although this method helps in identifying the various cyclical positions of relevant economic factors and is extremely easy to adapt, there are some problems that weaken its usefulness.

The most important and relevant objection to univariate HP filtering is that there is no theoretical relationship among variables. Bouthevillain *et al.* (2001) and Mohr (2003) argue that the linear nature of the HP filter ensures theoretical consistency among variables, as the weighted sum of disaggregated HP-filtered gaps equals the aggregate gap. Even though the HP filter is linear, this characteristic cannot be exploited in the field of economic time series, since economic time series should be log-transformed in the HP filter[4] and, as a consequence, aggregation constraint is not satisfied.[5]

Another problem is that using only one univariate method may result in an extreme solution that cannot be revealed since there is no control method. Moreover, Darvas and Vadas (2005) prove that better results can be achieved by using several methods. From the point of view of policymaking, the stability of the output gap estimate is crucial. Methods which provide extensive revision in the estimated output gap cannot be used in policy decision-making because they may frequently render previous decisions inadequate. Using a revision-based weighting scheme, Darvas and Vadas (2005) find that a multiple-method approach provides a more stable output gap estimation than any single method.

Kiss and Vadas (2005) estimated the expected bias of not-satisfied aggregation constraint. They argue that, due to the non-linear logarithmic transformation, ECB type disaggregation yields fairly asymmetric bias. According to their calculation the maximum effect of this bias on the cyclical component could be as high as 2 per cent of GDP. Apart from the USA and 10 new EU member countries, where the samples are quite short, the violation of aggregation constraint causes at least 0.1 of a percentage point error in the cyclical component in 16-84 per cent of the sample. Serious bias, *i.e.* distortion is more that 0.5 per cent of GDP, is presented roughly in 2-36 per cent of the sample. In short, unconstrained decomposition could be a considerable source of bias.

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[4] The general form of the univariate HP filter is:

$$\min\left[\sum (x - x^*)^2 + \lambda \sum (\Delta x - \Delta x^*_{-1})^2\right]$$

Note that economic time series generally grow exponentially, which means that Δx_t also increases over time. As a consequence, the second smoothness term in the HP filter would give higher importance to the end of the sample. Log transformation renders the economic time series to I(1) series, implying that Δx_t becomes constant and thus avoids over-weighting.

[5] It is apparent, if:

$$X + Y = Z \qquad \text{and} \qquad HP(X) + HP(Y) = HP(Z)$$

then:

$$x + y > z \qquad \text{when } X, Y > 1$$

thus:

$$HP(x) + HP(y) > HP(z)$$

where small letters denote the logarithm of variables.

2.3 Effect of different deflators

Hitherto we have considered variables in real terms; however, both tax bases and tax revenues are in reality in nominal terms. As a result, real and nominal cyclical positions may have different signs. Therefore, prices need to be introduced; that is, nominal variables are used.

To make the situation more transparent, suppose that the real consumption gap determines the real cyclical position of indirect taxes. Nominal consumption is obtained by multiplying real consumption by the consumer price index (CPI), while indirect taxes are multiplied by the GDP deflator. If the CPI is higher than the GDP deflator, then nominal indirect taxes based on nominal consumption are higher than indirect taxes based on real consumption.

For instance, consider the Hungarian economy in the mid-1990s. Owing to the high inflation rate and tight fiscal policy, the consumption gap was negative in real terms, while the CPI was higher than the GDP deflator. As a result, despite the negative consumption gap, the nominal cyclical position of budget revenues was relatively favourable.

Based on Kiss and Vadas (2005), the price effect caused by the different GDP deflator and consumer price index could have a considerable impact in certain periods. For instance, in Portugal the price effect caused an approximately 6 per cent difference between real and nominal CAB.

3. A new method of measuring the cyclical position

In line with the ECB, we agree that the aggregate output gap could hide relevant underlying processes. We also argue for the importance of the disaggregated approach; however, we additionally insist on the theoretical foundation of the output gap, the existence of a theoretical relationship among cyclical components, and the satisfied aggregation constraint using time-varying labour and capital income shares. In addition, we also suggest taking into account the effect of different deflators in the disaggregated method.

3.1 Deriving the cyclical position [6]

In this part we introduce an easily tractable method, capable of decomposing the output gap. The use of the production function can be favourable, since it is based on various factors that define the aggregate gap. The main drawback of the application of the "full-form" of production function (as in equation (1)) is that it involves several estimated variables, such as capital stock and TFP. Note that since

[6] Gauss code of the two official approaches and our method can be downloaded from http://vadasg.extra.hu

we need only the output gap, these uncertain variables are not necessary. The ratio of actual output to its potential counterpart can be computed by:

$$\frac{Y_t}{Y_t^*} = \frac{(CU_t K_t)^{1-\alpha_t}\left[L_t(1-U_t)TFP\right]^{\alpha_t}(1+\varepsilon_{Y,t})}{(CU_t^* K_t)^{1-\alpha_t}\left[L_t(1-U_t^*)TFP\right]^{\alpha_t}(1+\varepsilon_{Y^*,t})} = \frac{(CU_t)^{1-\alpha_t}(1-U_t)^{\alpha_t}}{(CU_t^*)^{1-\alpha_t}(1-U_t^*)^{\alpha_t}}(1+\varepsilon_{gap,t}) \qquad (2)$$

where *CU* denotes the level of capacity utilisation; here we apply the Cobb-Douglas form with labour-augmenting technical progress.[7]

There are two important deviations from the IMF, OECD and EC approaches. First, we apply a more realistic time-varying capital share, which can be obtained from either estimation[8] or from national accounts. This specification allows us to avoid the assumption of constant labour and capital income shares. It should also be kept in mind that these shares determine how the aggregate output gap should be decomposed into its components. Second, we argue against the HP filtered Solow-residual on both theoretical and practical grounds. If TFP is the explanatory variable of the production function, then estimating equation (1) without it results in an omitted variable problem and misspecification. Using the estimated parameters of this regression and incorporating an HP-filtered residual into the computation of potential GDP cannot, however, be justified. More importantly, in this case the smoothness of potential GDP simply comes from the smoothed disturbance term. Recall that TFP is an unobservable variable. Explaining potential GDP by the deviation of an unobservable variable from its potential level can be cumbersome. Since the CAB plays an important role in policy debate, the usage of the *TFP* variable as the key explanatiory variable could yield albitrary policy argument.[9]

After simplifying and log-transforming equation (2), we obtain:

$$y_t - y_t^* = (1-\alpha_t)\left[cu_t - cu_t^*\right] + \alpha_t\left[\ln(1-U_t) - \ln(1-U_t^*)\right] + \varepsilon_{gap,t} \qquad (3a)$$

where small letters denote the logarithm of corresponding variables. Although equation (3a) can be used to estimate potential GDP, the level of capacity utilisation is not available for every country, as is the case with capital stock. Basu and Fernald (2001) show that working hours contain information about capacity utilisation, thus:

$$y_t - y_t^* = (1-\alpha_t)\left[wh_t - wh_t^*\right] + \alpha_t\left[\ln(1-U_t) - \ln(1-U_t^*)\right] + \varepsilon_{gap,t} \qquad (3b)$$

where *wh* denotes the log of working hours. Obviously, where capacity utilisation time series are available, approximation is not necessary and equation (3a) can be used.

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7 Neutral technological progress is not justified by empirical work.

8 For a Kalman filter estimation of the time-varying capital share, see Kiss and Vadas (2004).

9 Nevertheless, if incorporating *TFP* is desired, then this can be done by simply replacing *TFP* with TFP^* in the denominator and extending equation (3a) with an additional $(1-\alpha_t)(\ln TFP_t - \ln TFP_t^*)$ term.

Similar to the levels of income, the parameters of the production function also identify the relationship among the output gap $(y - y^*)$, labour $(w - w^*)$ and the capital income $(\pi - \pi^*)$ gap. The aggregate output gap equals the weighted sum of labour and capital incomes, where weights are wage (α) and capital shares (1–α). As a consequence, output gap can be decomposed in the following way:

$$y_t - y_t^* = \alpha_t (w_t - w_t^*) + (1 - \alpha_t)(\pi_t - \pi_t^*) \tag{4}$$

where variables with superscript stars denote the potential or trend values of the corresponding variables.[10]

The above-mentioned criteria identify only the share of labour compensation and profit income gaps, not the magnitude of these gaps. Moreover, other real variables and their cyclical components should be determined. In order to achieve this, we have to incorporate a behavioural equation to derive the necessary cyclical component, which is not determined by the parameters of production function.

Obviously, several behavioural equations can be included. However, as (1) the labour-compensation gap determines the direct tax on households, social security contributions and pensions; and (2) the profit gap determines direct tax on corporations, there are only two potential budgetary elements left: unemployment benefit, and indirect taxes on household consumption.

As far as unemployment benefit is concerned, the trend unemployment rate is estimated in line with the output gap (see equations (3a) or (3b)).

Indirect tax on households' expenditure is extremely high, and therefore we incorporate a consumption function, which ensures that the potential value of wages and consumption are connected by theoretical considerations:

$$\Delta ce_t^* = \theta_1 + \theta_2 \left(ce_{t-1}^* + \rho w_{t-1}^* \right) + \theta_3 \Delta ce_{t-1}^* + \theta_4 \Delta w_t^* + \varepsilon_{ce,t} \tag{5}$$

where *ce* denotes the log of private consumption expenditure, and superscript stars again denote the potential of corresponding variables.

In order to incorporate the above equations into our decomposition and to keep our approach tractable and easily reproducible, we develop an alternative framework. Extending the ideas of Laxton and Tetlow (1992), Butler (1996) and

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[10] To understand the derivation of this constraint, divide $Y_t^* = W_t^* + \Pi_t^*$ by Y_t and rearrange the right-hand side to obtain:

$$\frac{Y_t^*}{Y_t} = \frac{W_t}{Y_t}\frac{W_t^*}{W_t} + \frac{\Pi_t}{Y_t}\frac{\Pi_t^*}{\Pi_t}$$

Note that labour and capital income shares then enter into the constraint, namely $W_t / Y_t = \alpha_t$ and $\Pi_t / Y_t = 1 - \alpha_t$. If $x_t - x_t^*$ is small, then $X_t / X_t^* - 1 \approx x_t - x_t^*$, we obtain equation (4).

St. Amant and van Norden (1997) with an aggregation constraint, we apply a multivariate HP filter. The potential value of the wage and profit shares are constrained by equation (4), and the entire system is influenced by theoretical equations (equations (3a) or (3b) and (5)). To achieve this, we embed the above-mentioned equations into the multivariate HP filter:[11]

$$\min_{\substack{y^* \pi^* w^* U^* ce^* \\ \theta_1 \theta_2 \theta_3 \theta_4}} \left[\begin{array}{l} \omega_y \left[\sum_t (y_t - y_t^*)^2 + \lambda \sum_t (\Delta y_t^* - \Delta y_{t-1}^*)^2 \right] + \\ \omega_\pi \left[\sum_t (\pi_t - \pi_t^*)^2 + \lambda \sum_t (\Delta \pi_t^* - \Delta \pi_{t-1}^*)^2 \right] + \\ \omega_w \left[\sum_t (w_t - w_t^*)^2 + \lambda \sum_t (\Delta w_t^* - \Delta w_{t-1}^*)^2 \right] + \\ \omega_U \left[\sum_t (U_t - U_t^*)^2 + \lambda \sum_t (\Delta U_t^* - \Delta U_{t-1}^*)^2 \right] + \\ \omega_{ce} \left[\sum_t (ce_t - ce_t^*)^2 + \lambda \sum_t (\Delta ce_t^* - \Delta ce_{t-1}^*)^2 \right] + \\ \omega_{\varepsilon_y} \sum_t \left[y_t - y_t^* - (1-\alpha_t)\left[cu_t - cu_t^* \right] - \alpha_t \left[\ln(1-U_t) - \ln(1-U_t^*) \right] \right]^2 + \\ \omega_{\varepsilon_{ce}} \sum_t \left[\Delta ce_t^* - \left(\theta_1 + \theta_2 \left(ce_{t-1}^* - w_{t-1}^* \right) + \theta_3 \Delta ce_{t-1}^* + \theta_4 \Delta w_t^* \right) \right]^2 \end{array} \right] \tag{6}$$

subject to

$$y_t - y_t^* = \alpha_t (w_t - w_t^*) + (1 - \alpha_t)(\pi_t - \pi_t^*)$$

Only one question has been left open, namely, how to weight (ω_i) the different parts (lines) in the optimisation. In fact, there are two possible weighting schemes which do not involve an arbitrary assumption. First, we leave every variable its own scale, *i.e.* $\omega_i = \omega_j, \forall i, j$. Second, every variable is normalised, which implies equivalent volatility. Instead of normalising every variable, we set ω_i as $\omega_i = 1/\sigma_i^2$, where σ_i^2 denotes the variance of i^{th} variable.[12]

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[11] Based on the empirical literature, we restrict the cointegration vector to [1 –1] in the consumption equation. Note that other cointegration vectors would imply 100 per cent or minus infinity saving rate, which is unacceptable from both a theoretical and an empirical point of view. However, it is technically possible to assume other cointegration vectors and to estimate ρ in line with the other parameters.

[12] To understand why this weighting scheme provides the same result as the normalisation, consider the normalised x_t

(continues)

The solution to problem (6) provides the potential values of variables and the gaps.

3.2 Correcting the effect of different deflators

Although several methods have been proposed[13] for capturing the trend or potential price level, the actual concept of the potential price level is more difficult to interpret. In this paper we do not address the issue of potential price levels. However, another problem was identified that resembles the composition effect of real variables. We capture this composition effect by recording the difference between the CPI and GDP deflators. In order to understand the basic idea behind our method, it should be noted that nominal variables are first deflated; however, the corresponding deflators differ, variable by variable. For instance, corporate profit is usually deflated by the GDP deflator, while private wages and consumption are deflated by the CPI. As the budget deficit is compared to GDP, the GDP deflator is therefore the relevant one for the budget.

To make the above more explicit, consider $BUD_{R,i} = BASE_{R,i}^{\beta}$ where *BUD*, *BASE*, *R* and β denote i^{th} budgetary revenue or expenditure, its corresponding base (e.g. personal income tax and wages), variables in real terms, and the elasticity of budgetary revenue or expenditure to its base respectively. Note that the cyclical component is expressed relative to the output, so that the cyclical component in real terms (CC_R) can be obtained by:

$$CC_R = \frac{BUD_{R,i}}{Y_R} = \frac{BASE_{R,i}^{\beta}}{Y_R} \tag{7}$$

Since the budget is evaluated in nominal terms, equation (7) has to be reformulated. Presume that the tax base is deflated by the CPI. In this case $BUD_{R,i}P_Y = BUD_{N,i}$, $BASE_{R,i}P_{CPI} = BASE_{N,i}$, where *N* denotes variables in

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$$\tilde{x}_t = \frac{x_t - \bar{x}_t}{\sigma_x}$$

Now the minimisation problem has the following form:

$$\min_{\tilde{x}_t^*}\left\{\sum_{t=1}^{T}(\tilde{x}_t - \tilde{x}_t^*)^2 - \lambda\sum_{t=2}^{T}(\Delta\tilde{x}_t^* - \Delta\tilde{x}_{t-1}^*)^2\right\}$$

Note that x_t^* can be estimated by $x_t^* = \sigma_x\tilde{x}_t^* + \bar{x}_t^*$, which results:

$$\min_{x_t}\left\{\frac{1}{\sigma_x^2}\sum_{t=1}^{T}(x_t - x_t^*)^2 - \lambda\sum_{t=2}^{T}(\Delta x_t^* - \Delta x_{t-1}^*)^2\right\}$$

[13] For instance, Buti and Noord (2003), Kiss (2002) and Denmark in the annex of Bouthevillain *et al.* (2001). Based on their results, the Danish price gap from 1999 to 2000 could lift the cyclical component by 0.3 per cent of GDP.

nominal terms. The cyclical component in nominal terms (CC_N) takes the following form:

$$CC_N = \frac{BUD_{N,i}}{Y_N} = \frac{\left(BASE_{R,i} P_{CPI}\right)^{\beta}}{Y_R P_Y} = CC_R \frac{P_{CPI}^{\beta}}{P_Y} \tag{8}$$

Equation (8) reveals that the real cyclical component has to be corrected to obtain the nominal cyclical component if the deflators, in our case GDP and CPI, are different.

Finally, those budget items which are influenced by this gap should be identified, *i.e.* those which are determined by private wages and consumption (namely direct taxes on households, pension and social security contributions), and indirect taxes on households' consumption. Similar to the cyclical position of the real economy and budget deficit, the whole price gap effect is the weighted average of individual elements deflated by the CPI.

4. Coverage of the adjusted budget items

After obtaining the gaps in tax bases and unemployment the next step is the identification of the coverage of the cyclically influenced budget items. For instance unemployment benefits are obviously connected to the business cycle. In some countries, other expenditures, such as pensions, are also directly influenced by cyclical fluctuations through different kinds of indexation techniques. At the same, time non-tax revenues and the majority of government expenditures are not directly affected by the cycle, or in other words they exhibit zero elasticities to the cycle.

The majority of government expenditures are also included in legal tax bases; therefore, they increase revenue automatically. The actual effects of discretionary spending can be measured by excluding their direct tax content. In principle, both tax bases and revenues can be corrected by government outlays. In this case the indirect taxes and contributions paid by the government and direct taxes and contributions paid by public employees are assumed to have zero elasticities, similarly to the corresponding expenditure items.[14] These data are available at the national level and the adjustments can be done by country experts. (ECB, 2001, Annex) In our disaggregate methodology private and public tax bases cannot be separated. In order to reduce potential distortions caused by different dynamics of these tax bases we propose alternative solutions. If data on taxes paid by the government are available, distortions can be reduced by assuming that these

[14] In like manner, indirect tax revenues should be adjusted with the portion transferred to the EU, because this expenditure item is assumed to have zero elasticity.

expenditure components can have the same elasticities as private taxes (instead of assuming zero elasticity). This solution would produce the same net coverage of the adjusted budget items as the previous method, but distortions can still not be fully removed. For a full correction estimated cyclical components should be adjusted with effects of public wage shocks by calculating a public wage gap between public and private wage indexes. This public wage gap should be subtracted from or added to the cyclical wage gap estimated with our disaggregate methodology.

Despite this issue of consolidation of budget data, indirect, or second-round effects of fiscal policy are still included in the revenue side, for example higher indirect tax revenue due to higher private consumption. In order to obtain a measure of the effect of fiscal policy on various macroeconomic variables, empirical models would be required in which the interrelationships of fiscal policy and economic behaviour are specified.[15] Cyclical fluctuations affect the budget, which in turn has an influence on the cycle through spending programmes or changes in tax rules. Although taxes and tax bases can be easily consolidated with government expenditures, simultaneity cannot be corrected without empirical models.

5. Issues concerning fiscal elasticities

The more difficult task is the estimation of the responsiveness of the chosen budget items to deviations of actual macroeconomic developments from their trends. It can be demonstrated that there is no uniform solution taking into account that some countries have less complex tax systems some others have more complex ones. In the remaining part of this section as a starting point we show three sources of potential problems, then before suggesting solutions, we develop a simple framework of definitions.

In the simplest case macroeconomic tax bases and legally defined tax bases automatically have the same dynamics; furthermore the system of taxes relies exclusively on tax rates instead of employing a set of nominal elements, ceilings, brackets, etc.

In this case unit elasticities can be assumed not only between the trends of taxes and the trends of the corresponding tax bases but also between actual taxes and actual tax bases. It means that cyclical effects can be removed from the actual deficit by applying unit elasticity. Furthermore, the change of this underlying, cyclically adjusted balance can be simply interpreted as the effect of discretionary measures.

In a more realistic scenario, however, tax systems can employ a number of nominal elements and legally defined tax bases can differ significantly from macroeconomic tax bases reflecting tax-payers' decisions. In this case the assumption of unit elasticities between the trends of taxes and the trends of the

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[15] Changes in private saving may partly offset changes in fiscal stance because temporary and permanent or anticipated and unanticipated measures probably affect demand in different ways.

corresponding tax bases remains reasonable. On the contrary, elasticities between actual taxes and actual tax bases should be estimated for each year. In the following three sub-sections we show limitations of this estimation.

5.1 Implications of the nominal elements of the tax system

Nominal elements of tax systems can have two consequences on cyclical adjustment. First, nominal brackets and ceilings may create inconsistency between estimated budgetary effects of nominal and real fluctuations for example in case of surprise inflation. In this case the effect of the drop in real wages would be accelerated by employing higher elasticity of personal income tax (PIT). Since the estimated elasticity of PIT is based on nominal elements, this result would be consistent with a drop in nominal wages, but it would be inconsistent with unchanged nominal wages accompanied with a drop in real wages.[16]

The second consequence is more obvious; the responsiveness of the PIT and social security contributions (SSC) should be reestimated for each year because these elasticities depend on the valorisation of nominal elements. As regards the valorisation of nominal elements, the principle of "no policy changes" does not mean that nominal values should be fixed forever. In this extreme case, the bracket creeping effect[17] would qualify as a neutral policy.

In principle the neutrality of the nominal elements can be achieved by keeping the effective tax rates[18] unchanged, *i.e.* nominal values should be valorised by the expected per capita income each year. In this benchmark case, unit elasticity can be assumed even for PIT. This benchmark case is applicable not only for measuring the structural deficit but also for estimating effects of the multi-year discretionary measures.[19] For example, if the operated tax system is a progressive one, PIT could grow faster than income. The operation of this tax system requires a discretionary decision not only in the first year, but in the subsequent years, too. If we want to catch this multi-year impact, we have to use unit elasticities between taxes and tax bases. By calculating elasticities from tax codes in each year, our results capture only the effects of the discretionary actions of the year in question.

In practice the government may keep the tax burden unchanged over the cycle and therefore nominal values increase in line with medium-term trends in income. This built-in progressivity produces temporarily higher revenue in the case of

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16 Progressive tax systems only enlarge the potential distortion stemmed from the divergence in real and nominal developments. In the case of inflation surprise, all kind of cyclical adjustments would suggest a negative cyclical component, completely ignoring the offsetting effects of the fluctuations in the deflator.

17 In the case of increasing taxable income, nominally fixed (or not fully indexed) tax brackets generate revenues more than equiproportionately because of the higher marginal tax rates.

18 Effective tax rates are equal to actual tax payments as a ratio of the economically defined tax base.

19 Our assumption is that the passive policy would be reflected only in the operation of automatic stabilisers of the budget, but it would be possible to design automatic stabilisers in such a manner that budget responses are more than equiproportionate.

expanding income in a self-reversing way. In contrary to the previous approach, maintaining progressivity may qualify as operating automatic stabilizers. In some case governments actually follow pro-cyclical policies through under- and over-valorisation of nominal elements; tax burden is reduced in good times, while it is increased during slowdown. This practice would be captured by the estimated degressivity of PIT or SSC.

It is an important question, however, how to make yearly estimations of elasticities of PIT and SSC. In the case of PIT and SSC the OECD approach has taken into account tax codes; average and marginal rates adjusted with social allowances are systematically calculated for each level of income. The ratio between the weighted averages of adjusted marginal and average rates provides the elasticity of receipts to gross earnings.[20] It represents the responsiveness of PIT and SSC to additional units of income. However, these calculations based on the tax code of a fixed year, and they are not continuously updated. It can be a problem since valorisation practice can be changed over time, therefore it would be more important for us to capture over- or under-valorisation year by year than calculating effects of additional units of income for specific years. Since the benchmark case for neutrality can be the unchanged effective tax rates, therefore over- or under-valorisation can be approximated by the changes of effective tax rates.

5.2 Implications of private decisions on legally defined tax bases and unemployment

Both tax bases and unemployment have legal and economic definitions. While cyclical gaps are estimated according to the economic definitions, budget items are actually determined by legally defined tax bases and unemployment. There are specific cases when tax-payers' decisions affect only legally defined tax bases without effects on economic tax bases. It has two basic forms:

- the tax avoidance and tax evasion affect only legal tax bases. Its size may change over time both because of cyclical reasons and as a reaction to tax measures (e.g. tax amnesties, tax hikes).
- some optional elements of tax codes affect also only legal tax bases. For example the possibility of receiving investment tax credits depends on the decisions of tax-payers. Both fulfilling the criteria and timing of the claims for tax credits require decisions, which can be affected by cyclical developments or fiscal measures.

If these decisions have important effects on the tax revenue, the trend (underlying) revenue should be estimated in an alternative way. It means that usual indirect approach, *i.e.* calculations of elasticities can be replaced by establishing

[20] Weights of the various income categories are calculated on the basis of an estimated income distribution.

direct links between trends of revenues and the estimated trends of the economic tax bases.[21]

5.3 Implications of asymmetric features of the tax design

In extreme cases not only the indirect approach of elasticities but even direct links between trends of revenues and trends of the economic tax bases can became unreliable. It is due to the fact that deficit imperfectly reflects fluctuations of macroeconomic variables, because the design of tax/benefit systems sometimes renders the operation of automatic stabilisers asymmetric (see Kiss and Vadas, 2004, p. 16). If these asymmetric features are resulted in an identical magnitude of budget responses with an asymmetric time pattern, the direct links between revenue trends and economic trends are not distorted. For example there can be full self-reversing effects with immediate positive but prolonged negative effects on the budget. If these self-reversing effects are incomplete, however, this kind of asymmetry may distort the direct estimation of trend revenue.

In the case of corporate taxation, effective tax rates depends on the severity of recession, *i.e.* it exhibits non-linear features. At a certain point (where there are no taxable profits at all) the effective tax rate temporarily becomes zero for the loss-making companies. While profit-making companies pay their taxes immediately, losses of the others have a negative impact on the budget only on a deferred basis, as the profit has contrasting economic and legal definitions, and the latter allows for carry forward losses.[22] Against this background the composition of the aggregate profits, *i.e.* share of losses does matter. Self-reversing effects may be incomplete due to legal and practical limits of carry forward losses, e.g. loss-making companies can be dissolved.

Another source of asymmetry is the design of unemployment benefit system. The status of "entitled to unemployment benefits" is different from the economic (ILO) definition of unemployment. If the period of entitlement to unemployment benefits is shorter than the business cycle, decrease in ILO unemployment do not necessarily reduce expenditures (elasticity can be close to zero), while increase in ILO unemployment may increase budget expenditures immediately. Here again the composition of aggregate developments (*i.e.* inflows and outflows) does matter. This composition effect can make difficult to establish reliable links between trend of unemployment benefit and trend of ILO unemployment.

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[21] This alternative solution is the direct estimation of the underlying trends of the budget items and receiving the cyclical component as a residual in contrast to the usual indirect approach, namely deducting the estimated cyclical components from actual figures in order to arrive at underlying trends as a residual value.

[22] Similar self-reversing effects may occur in the case of the choice of accelerated depreciation rates, which temporarily reduce the legally defined profits.

5.4 Calculation and interpretation of underlying trends and discretionary measures

After highlighting the complexity of tax/benefit systems we present a simple scheme of a possible decomposition of actual taxes.

Table 1

Decomposition of Taxes into Underlying and Discretionary Determinants

1. Trends of economic tax bases	4. Fluctuations of real economic tax bases and deflators	Non-discretionary components of taxes (1+4)
2. Effects of permanent changes on tax-payers' behaviour	5. Tax-payers' behaviour affected by temporary measures and business cycle	Mixed components of taxes (affected by both measures and private decisions) (2+5)
3. Effects of permanent fiscal measures	6. Effects of temporary fiscal measures (e.g. incomplete valorisation of nominal elements)	Discretionary components of taxes (3+6)
Trend or underlying components of taxes (1+2+3)	Temporary components of taxes (4+5+6)	

This table shows that trends of taxes can be determined by trends of economic tax bases in absence of permanent fiscal measures. While deficits are shaped by the overall impacts of all measures (Table 2/I), definitions tend to focus on direct impacts of those measures, which were implemented in a given year (Table 2/IV). This narrow definition cannot identify any measures with multi-year effects such as permanent changes in the design of the tax/benefit systems or any indirect effects of measures such as tax optimizing reactions of tax-payers.

Table 2

Alternative Definitions of Discretionary Measures

	All measures (including design of tax/benefit systems)	Measures of a given year only
Total impacts of measures (including reactions of tax-payers)	I. Overall impacts of all measures on the deficit	II. Total impacts of new measures
Direct impacts only	III. Direct impacts of all measures	IV. Direct impacts of new measures

The following example (Table 3) shows that standard cyclical adjustment methods applying constant elasticities are not necessarily able to separate impacts of new measures from multi-year impacts of the tax design. If new measures are not implemented in a given year the change of CAB is equal to those effects of the tax design, which partly offset cyclical influences on the deficit.

Table 3

Standard CAB Methods Can Be Consistent with Broad Definition of Measures

	A. Deficit = B + C + D	B. Cyclical loss calculated with constant elasticity	C. Offsetting effects of tax design (carry forward losses)	D. New Measures	E. CAB = A – B
Year $t-1$	0	0	0	0	0
Year t	–0.4	–1	0.6	0	0.6
change	–0.4	–1	**0.6**	**0**	**0.6**

If we are interested in the underlying component of the budget items, we expect to receive cyclically adjusted figures which are close to the trends of the budget items. In fact, without permanent fiscal measures, cyclically adjusted figures should be equal to the trends of the budget items. The following example (Table 4) shows that standard cyclical adjustment methods applying constant elasticities are not necessarily able to completely remove fluctuations from the deficit.

Table 4

Standard CAB Methods Can Be Inconsistent with Underlying Deficit

	A. Deficit = B+C+D	B. Permanent measures	C. Cyclical loss calculated with constant elasticity	D. Offsetting effects of tax design (carry over losses)	E. CAB = A – C
Year $t-1$	0	0	0	0	0
Year t	–0.4	0	–1	0.6	**0.6**

The closer the tax system to the simple scenario we started with, the smaller the distortion caused by employing constant elasticities. An approximation of the distortion is the volatility of cyclically adjusted budget items, therefore it is important to check results whether we can explain major changes with background information about discretionary measures (Kremer *et al.*, 2006). If we cannot explain changes in these residuals we can replace constant elasticities with continuously updated estimations. This approach can solve specific problems related to valorisation of nominal elements.

If unexplained volatility cannot be removed by employing updated estimations this problem can be a consequence of asymmetries in the tax design and/or difference between legal and economic definitions of tax bases and unemployment. In this case capturing the underlying component of the deficit would require employing unit elasticities directly between the trends of taxes and the trends of the corresponding tax bases. In other words the structural tax revenue should change at the same pace as the trend of the tax base. Changes in the effective tax rates would result in a level shift without affecting this co-movement.

A potential problem here is the identification of the permanent levels of the effective tax rates, which can be approximated with the ratio between the trend levels of taxes and trend levels of tax bases. In fact some temporary factors can make this calculation difficult. Since direct estimation of the underlying trends has some uncertainties, these results should be also controlled by background information about discretionary measures. As we have already mentioned the problem here is that estimations are usually available for only direct impacts of those measures, which were implemented in a given year (Table 2/IV). The effects of some measures can be easily estimated (e.g. changes in statutory tax rates or the entitlement period for unemployment benefits). On the contrary, more difficult to estimate the effects of measures related to the "optional" elements of the tax code, which allow for the possibility of receiving investment tax credits, but which depend on the decisions of taxpayers.

6. Summary

In this paper we have surveyed the two main official cyclical adjustment methods, namely the aggregated approach as adopted by the EC, IMF and OECD, and the unconstrained disaggregated approach championed by the ECB.

The main advantage of the aggregated approach is that it uses the production function and hence incorporates a theoretical background into cyclical adjustment. However, it assumes that any other GDP components that are relevant in terms of budget revenue and expenditure are in the same cyclical position as GDP, which is clearly rarely the case. Moreover, aggregated approaches do not exploit the information content of wage and capital shares, which are used to estimate the production function.

The ECB's disaggregated approach is designed to take into consideration the possibility of the different cyclical positions of real variables. It filters each relevant variable, one by one, using the single variable HP filter. However, this procedure can be criticised for its lack of theoretical considerations. In addition, there are serious implications implied by the application of the univariate HP filter. Since economic variables, due to their exponential nature, are log-transformed, the ECB-type disaggregation cannot fulfil the aggregation criterion.

The above-mentioned drawbacks, namely the lack of disaggregation or theory and the violation of the aggregation constraint, produce considerable bias in the estimation of cyclical components. While the first one involves the possibility of wrong policy implications, the latter, due to its non-linear transformation, causes systematic bias.

Since both a theoretical foundation and disaggregation are essential when seeking to obtain appropriate cyclical components, we introduce a method which is able to meet these requirements. First, we insist on the production function-based output gap; however, this implies difficulties owing to the availability of data. Fortunately, since we are only interested in the output gap rather than the full form of the production function, the capital stock and TFP data are not needed in our method. Another important implication of the production function is that the aggregation constraint should not only be satisfied, but that also the constraint is set by the capital and labour income share. In our approach we restrict the estimation procedure by using these shares. Finally, to derive the remaining cyclical component, we apply another behavioural equation, namely a consumption function. The system is estimated by a multivariate HP filter.

We also presented that if the deflators of variables are different then the real and nominal cyclical component can differ significantly. This paper has provided a method that corrects this difference.

We showed that the results of standard cyclical adjustment methods applying constant elasticities are consistent with the broad definition of discretionary measures, but possibly inconsistent with the underlying deficit. Standard estimations of the cyclical and underlying components can be faulted due to the specific non-linear features of tax systems and unemployment benefit schemes.

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