DESIGNING MODEL-BASED FISCAL POLICY RULES

Javier J. Pérez and Paul Hiebert

1. Introduction

In order to generate solvency for the fiscal sector, leading macroeconomic forecasting models employ a fiscal rule. The rule is designed to guarantee that the intertemporal budget constraint of the government is satisfied – thereby generating model closure. In addition to effectively ruling out the possibility of an explosive path for fiscal variables such as the government debt ratio, the adopted rule can strongly influence the adjustments of fiscal variables against shocks or policy changes. The choice of fiscal rule thus entails potentially significant consequences for the intertemporal behaviour of fiscal variables, as well as effects on macroeconomic and financial variables in the model. More importantly perhaps, policy choices can be influenced by the specification of the fiscal rule, as the impact of policy changes and reforms is often assessed on the basis of a macroeconomic model that can be influenced by the specification of the fiscal rule.

Existing fiscal rules employed in leading macroeconomic forecasting models are generally imposed *exogenously*, and involve backward-looking behaviour on the part of the fiscal sector, despite the widespread use of a forward-looking framework in modelling households, firms and the monetary authority. This modelling strategy can, in principle, lead to inconsistencies in the fiscal sector with other sectors of the model as the functional form and calibration of the rule is largely determined outside the auspices of the model.

In this paper, we describe a recent methodological proposal put forth by Perez & Hiebert (2001) to identify the appropriate fiscal rule *endogenously* in a stochastic model, based solely on the properties of the model itself. Specifically, we discuss how a state-contingent policy rule can be obtained, relating fiscal instruments to different shocks affecting the

---

* European Central Bank. The views expressed in this paper are those of the authors and do not necessarily reflect those of the European Central Bank (ECB). The authors would like to thank F. Orlandi, R. Johnson, J.-P. Vidal, M. Catenaro, M. Tujula, P. Cour-Thiman, R. Strauch, along with seminar participants at the 3rd Banca d’Italia Workshop on Public Finance and the Fiscal Policies Division of the ECB for helpful discussions and comments. Any remaining errors are of course the sole responsibility of the authors.
economy and expectations of future developments of the economy – all on the basis of the existing set-up of the model at hand. This endogenous fiscal policy rule is inherently consistent with the fundamental structure of the model on which it is based, wholly integrated with all agents and sectors in the economy, and with the structural parameters of the model. In addition, the proposed fiscal rule, with a fiscal authority systematically reacting to disturbances to the economy is consistent with the standard role of government of fiscal stabilisation in the face of economic fluctuations.

The paper is organised as follows. In Section 2, we discuss the rationale for fiscal policy rules, in the context of uncertainty in public finance – and how to capture these concepts in a macroeconomic model. In Section 3, we explore the need for fiscal rules in macroeconomic models, and on the diversity of existing rules. In Section 4 we offer the rationale for an alternative specification in the form of an endogenous fiscal rule. Finally, in Section 5 we present some concluding remarks.

2. Main features of fiscal policy rules

2.1 Sources of variability in public finances

Government budgets are subject to considerable uncertainty, given various shocks which have an impact on public finances, both of a permanent and temporary nature. The effects of on public finances of uncertainty may first come from non-budgetary sources. Shocks may be of the standard macroeconomic type – involving, for instance, shocks to technology, demand, energy, and labour supply. They may also be due to changes in key financial variables, such as interest rates and exchange rates. The effects of on public finances of uncertainty may also come from budgetary sources. This could include uncertainty which often is inherent in developing public spending plans, particularly relating to unforeseen developments in military spending (e.g. wars) and agricultural spending (e.g. compensation for droughts, epidemics). In addition to these sources of temporary unpredictability, shocks of a more permanent nature to public finances can include demographic shifts (e.g. pressure on public expenditure expected to accompany population ageing) and underlying changes in tax collection (e.g. the development of “underground” economies).

Given the numerous sources of unpredictability, there is considerable scope for forecasting errors in developing and executing fiscal
plans. Kilpatrick (2001) postulates that in a stochastic world, in order to maintain budgetary stability in any given period, the fiscal authority should react to any shocks that affect public spending or the tax base. This implies that either implicitly or explicitly, governments should have a type of contingency fund to cater for these sources of uncertainty. Although in reality, the identification of shocks is not straightforward – see, for example, Blanchard and Perotti (1999), in a macroeconomic model, the source of all shocks hitting public finances or the economy is clearly identifiable.

2.2 Modelling uncertainty in public finances

In theory, the government reaction to shocks affecting the budget in a model should be shock-specific. As such, in a model characterised by optimising forward-looking agents, the intertemporal fiscal rule should include policy reactions to the different shocks affecting the economy. In the absence of an active monetary authority monetising shocks to debt, as is in the case of most industrialised countries, the fiscal authority should react to any innovation affecting the fiscal sector through the adjustment of budgetary items.

For instance, we may expect that different unforeseen shocks, such as those to output (e.g. an oil shock) or those to government spending (e.g. a war or drought) would have differing impacts on the government budget and therefore elicit a nuanced reaction based on the source of the shock. Moreover, the rule should, in principle, be introduced in a forward-looking manner consistent with the other sectors of the model. As such, the rule should be based on expectations of future values of relevant variables. Thus, the rule would amount to some combination (either linear or nonlinear) of endogenous and exogenous variables in the model. We import and discuss these a rule which satisfies the above criteria in Section 4.
3. **On the need for fiscal rules in macroeconomic models, and on their diversity**

In establishing the basis for including a fiscal rule in a macroeconomic model, one must look first to the government budget constraint, which takes the following standard form in discrete time:

\[ \frac{B_t}{P_t} + \tau_t = g_t + I_{t-1} \frac{B_{t-1}}{P_t} \]

where \( B_t \) stands for time-\( t \) nominal debt, \( g_t \) is real primary spending, \( \tau_t \) tax collection in real terms, and \( I_t = 1 + i_t \) the nominal interest rate on bonds\(^1\). Simply put, this condition entails that the government has to issue debt to pay for spending in excess of tax collection. The aggregate variables defined above may of course be broken down into their subcategories in macroeconomic models used in practice. Solving this equation forward we have that:

\[ \frac{B_t}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{j=0}^{j} \frac{\pi_{t+j+1}}{I_{t+j+1}} \right) \tau_{t+j+1} - g_{t+j+1} \left( \lim_{j \to \infty} \left( \prod_{j=0}^{j} \frac{\pi_{t+j+1}}{I_{t+j+1}} \right) \frac{B_{t+j+1}}{P_{t+j+1}} \right) \]

where \( \pi_t \) is the time-\( t \) inflation rate. For the government to be solvent, the second term of the right-hand side of the previous expression has to be equal to zero. In other words, any shock affecting spending or real debt should be covered by tax changes\(^2\). In any standard model with optimising debt-holders, this is the condition close to the exact form of one of the optimality conditions (the transversality condition attached to bond-holdings) that has to be verified.

3.1 **General formulation of a fiscal rule**

The fiscal rules used in existing macroeconomic models are based on maintaining budgetary solvency required by (1). As the rules

---

1 Seigniorage revenue is neglected for the sake of simplicity.

2 Effectively, this can be considered as a no-Ponzi game condition, whereby in order to guarantee solvency, the government must be able to back all debt through its tax and spending system.
traditionally involve adjustment on the revenue side of the government budget, we can write any generic tax system as:

\[ \tau_t = \tau(c_t, y_t, \ldots) + \tau_{rule} \]

where the first part of the equation embodies the normal tax system of the economy (income taxes, consumption taxes, etc.), while the second component represents the revenue adjustment by the government to guarantee solvency, and react to shocks. Permanent shocks to the economy would show up in the first part of the equation, while transitory shocks would be catered for by the second. In theory, fiscal closure rules – captured here by the \( \tau_{rule} \) – can take various functional forms, including several types of variables, not only lagged values of certain state variables. These rules for model economies approximate the actual reaction to shocks by the fiscal authority.

3.2 Traditional specification of fiscal closure rules in macro models

Budgetary adjustment is generally either in the form of either a tax-difference rule – as in MULTIMOD (IMF) and NIGEM (National Research Institute) – whereby the change in taxation is a function of the objective variable; or tax level rule – as in MSG2 (a model developed by McKibbin and Sachs) – whereby the tax rate itself is adjusted in reference to the objective variable. To illustrate, a tax-difference rule would take some variant of the following generic form:

\[ \tau_{rule} - \tau_{rule-1} = a(x_{t-1} - x^*_{t-1}) + b\Delta(x_{t-1} - x^*_{t-1}) \]

where \( x \) is the objective variable (i.e. government debt or deficit), with an asterisk denoting the steady state value, and \( a \) and \( b \) the speed of adjustment parameters, which are calibrated. \( \Delta = I - L \) stands for the first difference operator.

The calibration of the exogenous rules currently used in practice requires the calibration of the so-called speed of adjustment parameters, controlling the behaviour of the adjustment variable to deviations of target values in the rule (e.g. deficit, debt) from their steady-state values. Mitchell et al. (2000) find that this calibration may be somewhat informal or ad-hoc, although some modellers have pursued more formal exercises in the derivation of their fiscal rule – see, for example, the derivation based on a
quadratic loss function in Barrell et al. (1994). In any case, when designing the rules and calibrating the key parameters, these existing rules do not consider explicitly who the debt holders are in the models they are analysing, and tend to focus the stability analysis on the system formed by the budget constraint, (1), while the calibration of the speed of adjustment parameters is then done on the basis of some advocated properties of the model solution and responses to shocks. Although this is a practical and partially valid approximation, it does not guarantee that the resulting fiscal rule is fully consistent with the properties of the model it is trying to close.

The rules used in practice are quite diverse in specification, and some recent studies have found through standardised simulations that the various specifications of these rules can lead to widely divergent results. For instance, Mitchell et al. (2000) compare the response of standardised version of the fiscal rules of leading macroeconomic forecasting models, including NIGEM, MULTIMOD and MSG. They find that the impulse response function to a shock in government expenditure differs widely, ranging from a relatively monotonic adjustment to a nonlinear adjustment path. Bryant and Zhang (1996) also find that the response of variables can differ quite substantially on the basis of alternative standardised fiscal rules. They conclude on the basis of this evidence that generally, there is a particularly imprecise understanding of how economies respond to fiscal policy actions. Lastly, Barrell et al. (1994) also find that the implementation of the fiscal policy rule has a significant effect on model properties in comparing the tax rules of NIGEM, MULTIMOD and MSG.

Despite the variation in results, little consensus exists on the proper formulation in terms of the dynamic adjustment component of the fiscal rules in the literature. As noted in Mitchell et al. (2000) and Johnson (2000), this wide variety of fiscal rules found in the literature highlights the lack of agreement amongst modellers regarding the appropriate functional form for these rules. This at least can partially be attributed to their formulation which, to a certain extent, may lack rigorous theoretical underpinnings fully consistent with the model in which they are used. Their formulation is imposed outside the confines of the model, and can involve the considerable use of judgement in some cases. In this sense, their derivation cannot be entirely consistent with all of the other economic variables in models by design. This type of lack of internal consistency in modelling has been criticised by many for its lack of microfoundations starting with Lucas (1976). A more fundamental criticism of exogenously imposed fiscal rules is their inherent vulnerability to the points raised in the
Lucas paper, as changes in the baseline parameters of the model may not directly lead to a change in the form or calibration of the fiscal rule.

4. A proposal to identify model-based fiscal closure rules

Based on the principles outlined in Section 2, it is sensible to postulate that a fiscal closure rule code for a government could take the form of a given reaction to transitory shocks affecting their budgets, $\tau_t = f(\text{shocks}_t)$. This rule would imply that, on average over the simulation horizon, any increase in tax collection due to shocks of one sign would be offset by decreasing tax collection (transfers) coming from shocks of the opposite sign. A policy reacting to innovations would be countercyclical by nature, and non-distortionary, as it should be that $E_{t+1} [\tau_t] = E_{t+1} [f(\text{shocks}_t)] = 0$. If the fiscal authority were to react to time-$t$ shocks, this would be enough to ensure stability of the model economy.

4.1 Derivation of Ramsey-type optimal policies

In order to endogenously calibrate the form of $\tau_t$, and as an alternative to the exogenous imposition of a fiscal rule, one solution would be to derive a fiscal rule based entirely on the design of the markets in the model, in a fully-fledged optimising framework entirely consistent with the microeconomic foundations of the model, so that one could obtain the coefficients in $\tau_t$ optimally and also the optimal form of $\tau_1, \tau_2, \ldots$ in (3). For the development and implementation of such a rule in simple models see, for example, Chari et al. (1994) or Manzano and Ruiz (2000).

Although the strategy pursued in this literature would entail many desirable characteristics, it is generally limited to the analysis of fairly simplified economies – and would be impractical for large-scale macro models, given the level of complexity of the economy in these models and their level of disaggregation. In fact, solving a dynamic optimisation model in which the government maximises agents’ utility subject to all Euler conditions in agents’ problems would be cumbersome, if not impossible, with the level of disaggregation in large-scale macroeconomic forecasting models.
4.2 Fiscal theory of the price level

Another alternative that goes beyond the standard practice outlined in the previous Section uses the stability properties of the model under analysis to constrain the coefficients of rules of the form (4). In other words, the coefficients a and b are chosen on the basis of agents’ decisions ensuring stability of real debt. Nonetheless, these coefficients cannot be uniquely calibrated using this strategy, and the formulation of the functional form of the rule itself remains ad-hoc. This shortcoming is natural, as this approach has been used for alternative purposes of macroeconomic modelling, mainly to stress the close link that the government budget constraint imposes between monetary and fiscal policies – see, for example, Leeper (1991), Sims (1994), Woodford (1995), Mc Callum (2001) or Andrés et al. (2001).

4.3 An identification methodology based on the analysis of forecast errors

This methodology to retrieve endogenous fiscal policy rules in models with imperfect foresight on the part of agents generates rules which are formulated entirely on the structural parameters and framework of the model. In this way, the rule adjusts automatically in response to any changes to structural parameters of the model, thereby reducing the susceptibility of this sector to the Lucas Critique.

The rule is derived using standard stability analysis theory for rational expectations models, based on Blanchard and Khan (1981), Sims (2000) and Novales et al. (1999). It is constructed based on the expectations errors of agents within the model. It can be expressed in implicit form or explicit form, whereby the fiscal authority systematically reacts to individual shocks to the economy via a state-contingent lump sum tax on households3.

---

3 See Perez and Hiebert (2001) for an illustration of this on the basis of a simplistic standard model. Nevertheless, it should be stressed that the proposed identification methodology is, in principle, general enough to be applied to any given large-scale macroeconomic model with optimising agents.
Box
The four steps required to construct a model-based fiscal rule

1. Perform the stability analysis of the model at hand to determine the stability needs of the system, assuming \( \tau^\text{rule} = 0 \);

2. Identify the expectation error relevant for debt stability, from the relevant row(s) of the stability conditions, and then use them to guess a first tentative relation between \( \varepsilon^\text{rule} \) and the relevant expectation error(s).

3. Determine the value of the coefficient(s) of that relation on the basis of the stability analysis of the system, and it has to compute the stability conditions of the system including the guess (initialised, for example, to one); then the parameter(s) are calibrated in such a way that the stability conditions are exactly the initial set (computed in the first step).

4. Compute the fiscal closure rule by using the calibrated coefficient and the relevant condition relating expectation errors and shocks.

4.3.1 Government solvency and stability analysis in rational expectations models

Any given dynamic stochastic rational expectations model can be written, without lack of generality, in the following implicit form:

\[
F(u_{t+1}, u_t, \varepsilon_t, \eta_{t+1}) = 0
\]

where the vector \( u_t \) contains the endogenous and exogenous variables in the model, as well as the conditional expectations in the model; they may be decision variables of the economic agents, such as consumption or real debt holdings, or variables obtained as functions of decisions, such as real interest rates, or exogenous variables like random shocks or policy variables decided by the government. The vector \( \varepsilon_t \) contains the innovations in the laws of motion of the exogenous states, and \( \eta_t \) is the vector of expectational errors, satisfying \( E_t(\eta_{t+1}) = 0 \), where the operator
$E_t(\bullet)$ denotes the expected value of the argument given the information set available up to time-$t$.

Proceeding to conduct a stability analysis of the above system, we express the linearised/ log-linearised version of the system around the deterministic steady-state can as:

$$\Gamma_0 u_{t+1} = \Gamma_1 u_t + \psi \varepsilon_{t+1} + \Pi \eta_{t+1}$$

plus a set of transversality conditions:

$$\lim_{j \to \infty} \varphi' u_{t+j} = 0$$

where $\varphi$ is the appropriate discount rate for this model. For the transversality conditions to hold, we need to add a set of stability conditions to the system described in (6). For this model, these stability conditions are defined by the eigenvectors associated with the unstable eigenvalues of the system (6). Assuming invertibility, the stability analysis is based on $\Gamma_0^{-1} \Gamma_1$.$^4$ The key to obtaining the stability conditions is obtaining the transversality conditions attached to the unstable eigenvalues, given that those attached to the stable eigenvalues are always satisfied. Expressing the stability conditions as a linear (or log-linear) relationship between the expectational errors and the structural shocks affecting the economy, a unique stationary equilibrium satisfies the condition:

$$P' \Gamma_0^{-1} (\psi \varepsilon_{t+1} + \Pi \eta_{t+1}) = 0 \text{ for all } t$$

where $P'$ denotes the rows of the decomposed matrix of $\Gamma_0^{-1} \Gamma_1$ which amounts to a particular linear (or log-linear) combination of the endogenous and exogenous variables in the model. The above closing condition would be needed to solve for all expectational errors in the model. As discussed in Sims (2000), for the equilibrium to be uniquely determined, one such condition should be present for each expectational error in the model. The stable paths of the approximated model economy can be simulated given (6) just by appending (8).

In order to construct the fiscal rule, we would need to detect the relevant stability condition for debt given by (8), and then identify the necessary fiscal policy reaction, linking the tax instruments to the structural

\footnote{Note that invertibility is not strictly required – a more generalised solution algorithm is available in the form of $QZ$-decompositions (see, for instance, Sims (2000) or Novales \textit{et al.} (1999)).}
innovations in the economy in order to endogenously determine policy responses by identifying the coefficients $\gamma_s$ in:

$$\tau_i^\text{rule} = \gamma_s \eta_i^s$$

and then identify an implied relation:

$$\tau_i^\text{rule} = \gamma_s \epsilon_i^s$$

4.3.2 Obtaining an endogenous fiscal rule in implicit form

Consider a standard neoclassical growth model with agents maximising their discounted utility derived from consumption, and with debt, in which the transversality condition associated with debt takes the form:

$$\lim_{j \to \infty} E_t \left[ \varphi^{**} \frac{B_{i+j}}{P_{i+j}} \right] = 0$$

Using the standard analysis outlined in Pérez and Hiebert (2001) would imply that for this transversality condition to hold, one equation summarising the relevant stability conditions should be added to the system of first order conditions and constraints. This condition would take the form of a linear/non-linear combination of real debt with other variables in the economy, such as consumption, ($c_i$) or the capital stock ($k_i$):

$$\text{function}_{\text{debt}} (B_t/P_t, c_t, E_t [f_{\text{debt}}(c_{t+1}, k_{t+1}, ...)], ..., \text{shocks}_t) = 0$$

and should hold in each single period of time. This condition would be unique, and would replace the transversality condition in the set of optimality conditions used to solve for all the variables in the model. One way to give some economic interpretation to this type of condition would be as follows. Once agents internalise that the government commits itself to be solvent, they behave in such a way that indeed the resulting equilibrium is stationary and the government debt is valued and held by the agents.

From an economic point of view, and for the purposes of policy analysis, the intuition behind a pure analysis of the stability conditions might be considered a bit obscure. Although imposing such conditions to solve for the variables in the model is technically correct, it is somewhat
more difficult to give some economic meaning in the framework of the model being analysed. Specifically, when imposing the transversality conditions for bonds, one may wonder which instrument the government would be moving on the event of, for instance, a recession. The implicit formulation above reflects how agents internalise the commitment from the part of the government to be solvent. When this commitment is internalised, solvency is automatic. Indeed, a way to rewrite the stability condition for bonds would be:

$$E_t [f_{debt}(c_{t+1}, k_{t+1}, ...)] = \text{function}_{debt}^{-1} (B_t/P_t, c_t, ..., \text{shocks}_t)$$

so that imposing solvency implies a certain form to solve for agents’ beliefs. This is why for the solution to be unique there should be one such condition per expectation error or expectation in the model.

4.3.3 Expressing the endogenous fiscal rule in explicit form

From the fiscal policy point of view we would be interested in knowing what amount of revenue given by $\tau^\text{rule}$ would stabilise debt and make the transversality condition hold. To do so, first note that the condition including debt of the type outlined above has a counterpart involving either a linear or non-linear relationship between the expectational error associated with agents’ interest rate forecasts, $\xi^l$, a subset of $\eta$ – the structural shocks. Using this as an example, by construction,

$$\xi^l \equiv f_{debt} (c_t, k_t, ...) - E_{t+1} [f_{debt} (c_t, k_t, ...)]$$

we can then postulate that the government should raise or decrease revenue in line with agents’ relevant expectational error:

$$\tau^\text{rule} = -\gamma^h \xi^l$$

and we can identify the first coefficient on the right hand side of the above expression out of the stability analysis of the system at hand. Combining the coefficient identified by the means outlined above with the implied relation between $\xi^l$ and the structural innovations would give us a relationship of the form:

$$\xi^\text{rule} = \gamma^h \xi$$
that can be identified as the fiscal policy closure rule. Taking the set of first order conditions and constraint corresponding to the model under analysis, and appending this rule would produce stable outcomes fully consistent with the model solution, where all of the behavioural elements that the literature on fiscal rules normally imposes on the fiscal rule would be transferred back to the properties of the model itself.

5. Conclusions

In principle, the methodology outlined in this paper is applicable to a wide range of macroeconomic forecasting models, given its requirement only of rational expectations frameworks in stochastic imperfect foresight models – and, accordingly, the presence of expectations errors. Specifically, we explain how the presence of forward-looking agents, combined with some other mild conditions, is sufficient to generate model closure and intertemporal behaviour consistent with the foundations of the model at hand.

The model-based rules which would result from an application of the methodology proposed in this paper would share many of the desirable features of exogenously-imposed rules. Most importantly, they guarantee solvency on the part of the government and rule out instrument instability. In addition, the proposal presented here possesses some additional appealing properties not shared by exogenously imposed fiscal rules. Firstly, the rules are forward-looking and in a manner consistent with the specification of other sectors in the economy. More generally, the rules are consistent with the setup of the model in which they are implemented, by design, meaning that a change in structural parameters will automatically be reflected in the fiscal rule. Secondly, they are state-contingent. Exogenously imposed fiscal rules may involve acyclical features, where, for example, adjustment of taxes is dependent solely on the observed deviation of the deficit or debt from its target value. The endogenous fiscal rules derived here, on the other hand, are exhibit shock-specific fiscal policy responses, which is a desirable property from an economic point of view. Thirdly, the rules, in principle, ought to produce relatively smooth adjustment processes for taxes consistent with the behaviour of households with concave utility functions - which gives the result that households smooth consumption. Unlike many exogenously imposed fiscal rules, the impulse response of variables is consistent with the optimal time path of adjustment of agents within the model, and adjustment is not dependent on
calibrated parameters. Lastly, the rules would embody only counter-cyclical automatic adjustments on the part of the fiscal authority unless a discretionary component is assumed – and in this way can be considered as mimicking automatic stabilisation properties of government budgets.
REFERENCES


Kilpatrick, A. (2001), ”Transparent frameworks, fiscal rules and policy-making under uncertainty”, paper included in this volume.


Congress of the European Economic Association held in Bolzano, Italy, September.


