ON THE SUSTAINABILITY OF THE SPANISH PUBLIC BUDGET PERFORMANCE

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1 Introduction[1]

The most common definition of fiscal sustainability, and that employed in this paper, is based on the need for a fiscal deficit to be financed, i.e. on the concept of inter-temporal budget constraint, which requires that the current market value of debt be equal to the discounted sum of expected future primary surpluses. In this context, fiscal policy is sustainable if the discounted value of debt reaches zero at the limit.

The issue of whether the current fiscal policy can be maintained indefinitely, i.e. whether it is sustainable or not, is an important one, since, on the one hand, it will determine the need for future discretionary policy actions. In this sense, since the concept of sustainability relies on the fact that governments need enough resources to ensure their ability to carry out the functions attributed to them, sustainability analysis helps to determine whether a current policy can be maintained in the long run

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with the ongoing ability to generate financial resources. On the other hand, fiscal sustainability has clear implications for other macroeconomic variables. In particular, a non-sustainable fiscal policy involves a risk of future interest rate rises leading to a slowdown in economic growth. Moreover, in the context of EMU, there are risks that non-sound fiscal policies of individual members may have adverse effects on the rest of the member economies. Furthermore, testing sustainability in the Spanish case is of particular relevance due to the fact that several policy actions have been adopted in Spain since 1975, aimed at implementing a European Welfare State model and a modern tax system, that has led to a sharp increase in public expenditure and revenue.

Different tests of sustainability are proposed in the literature. These pay special attention to integration orders of deficit and debt processes, and to the underlying stochastic structures and the existence of cointegration relationships between revenues and expenditures. Earlier tests in the literature indicated that the condition for fiscal sustainability is the stationarity of the debt (Hamilton and Flavin, 1986) or that the discounted debt process follows an I(0) process without drift (Wilcox, 1989)[2]. Later work developed alternative conditions for fiscal sustainability: provided that total public revenue and expenditure are first-order integrated, sustainability requires both series to be cointegrated (Hakkio and Rush (1991), Haug (1991), Smith and Zin (1991), Trehan and Walsh (1988,1991)). More recently, Quintos (1995) extended this literature by introducing “strong” and “weak” conditions for fiscal sustainability. On one hand, a “strong” condition corresponds to those previously mentioned: stationarity of the debt process or, alternatively, cointegration between revenue and expenditure. However, these only refer to sufficient conditions for sustainability. On the other, a “weak” condition requires the growth rate of debt to be lower than the growth rate of the economy.

From an economic point of view, there are important differences between the concepts of strong and weak sustainability. Strong sustainability is understood as a situation in which no future

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2 Wickens and Uctum (1993) develop a test for sustainability when a feedback rule between the deficit and debt is introduced.

problems in the deficit's behaviour are expected to arise in the absence of significant changes in the processes followed by both public expenditures and revenues. However, weak sustainability implies that governments may have future problems in marketing their debt, involving a substantial risk of a rise in interest rates that may have perverse effects on economic growth and the public budget, necessitating fiscal reforms. Accordingly, the difference between both concepts of sustainability is quite relevant, both from a positive and normative analysis of fiscal policy developments and the associated consequences for the macroeconomic variables of interest.

Camarero et al. (1998) apply the aforementioned tests to the Spanish case, showing that public revenues and expenditures are cointegrated only when the possibility of structural shifts in this relationship is taken into account. According to their analysis, the deficit process is found to be sustainable in the weak sense. However, since over the sample period many fiscal reforms have taken place in Spain, a deeper univariate analysis of the series involved might be of great interest and may provide useful information for deriving sounder conclusions about the sustainability of Spanish fiscal policy in recent years. In this context, the existence of changes in the order of integration, which can be associated with fiscal reforms or with gradual fiscal adjustments, might modify previous results and thus our conclusions. For this reason, in this paper we apply the traditional tests of sustainability, following Quintos´ approach. In addition, we introduce a univariate analysis of the series, consisting of testing whether breaks in the stochastic trend, which may bias both unit-root tests and cointegration relationships, have taken place, and whether the series have undergone changes in the order of integration that may, in some way, invalidate the cointegration analysis.

The rest of the paper is structured as follows. Section 2 presents a summary of fiscal policy performance during the sample period (1964-1998), which may be of particular interest for better understanding the results. Section 3 outlines the theoretical framework. Section 4 contains the empirical results. Finally, Section 5 draws the conclusions.

2 Deficit and debt in Spain[3]

Since 1964, five distinct periods can be identified in relation to public finances in Spain:

1964-75: a period of strong economic expansion, with average real GDP growth of 6.4%, characterised by small budget surpluses, owing to the steady growth of government revenue and expenditure.

1975-85: against a background of economic crisis and political change, the previous situation changed in 1976, with the appearance of a budget deficit. Although it was small in the first two years, it grew continuously, except in 1979 and 1983, to reach 5.8% of GDP in 1985. On the one hand, public expenditure as a percentage of GDP almost doubled in this period (from 23.5% of GDP in 1974 to 41.6% en 1985, which represents an average annual increase of 1.6% of GDP) due to low economic growth (average real GDP growth of 1.6%) and the building of the Welfare State. On the other hand, public revenues also increased significantly as a consequence of the 1977 and 1978 fiscal reforms, but at lower rates than in the case of expenditure (total revenues moved from 23.6% of GDP in 1974 to 35.8% en 1985, which represents an average annual growth of 1% of GDP).

As a consequence of this budgetary imbalance, public debt also spiralled, from 12.1% of GDP in 1979 to 43.7% in 1985. However, this increase in debt did not lead to a similar rise in the interest burden because, until 1982, around two-thirds of the budget deficit was funded by the Banco de España and financial institutions, primarily through compulsory reserve requirements. In fact, public debt assumed by the private and external sectors under orthodox financing arrangements played a very limited role, covering less than 25% of the state-borrowing requirement. Nonetheless, as from 1983, the deficit was funded in a more orthodox fashion, and the government came to rely more heavily on Treasury bill issuance. This, together with the high interest rates

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[3] See Argimón, Gómez, Hernández de Cos and Martí (1999) for a deeper analysis of fiscal policy in Spain.

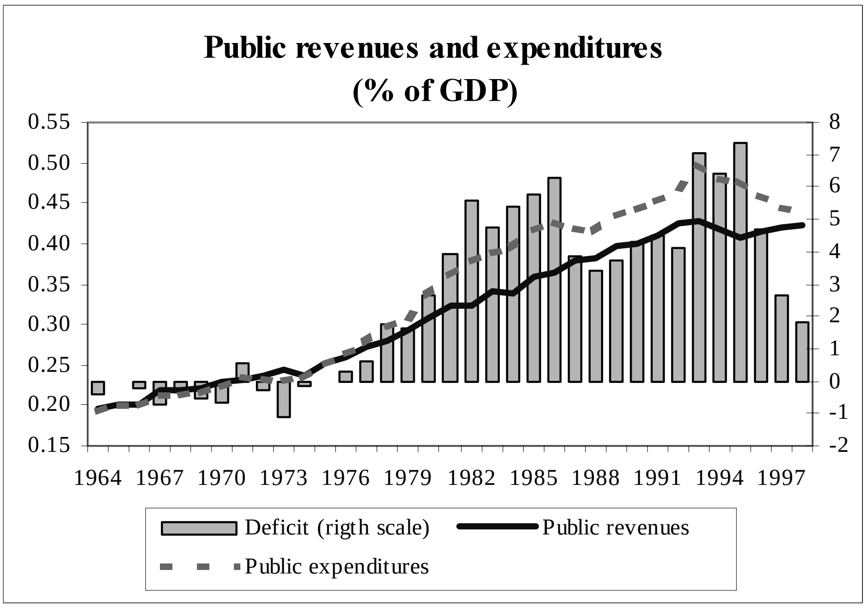
prevailing in the period, caused the interest burden to double as a proportion of GDP between 1982 and 1985 (to 2% in 1985).

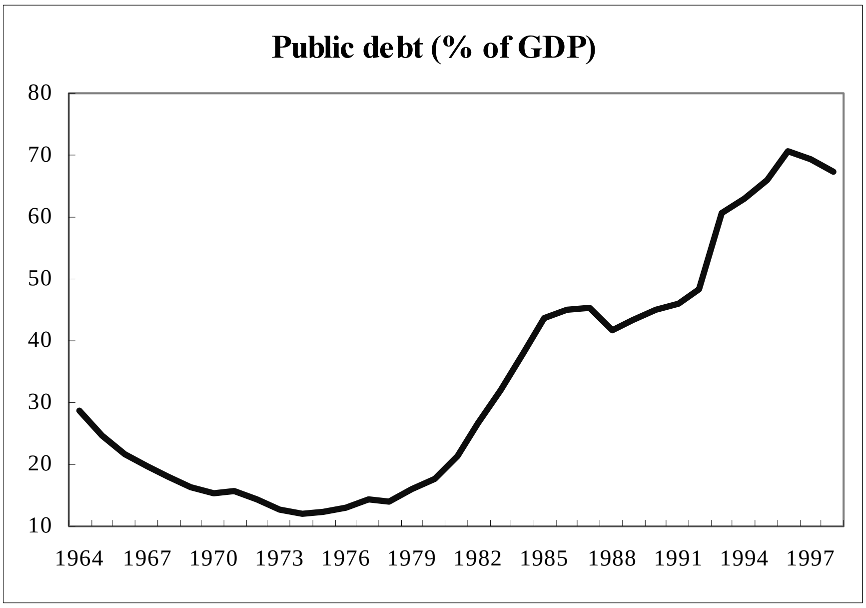
1986 to 1988: following Spain's accession to the European Community and the commencement of a new cyclical expansion, there was a change in direction in Spanish fiscal policy. The budget deficit was reduced from 5.8% in 1985 to 3.4% in 1988, essentially due to the growth of government revenue. In fact, public revenue as a percentage of GDP increased 2.2 percentage points while public expenditure fell by only 0.2 percentage points. Moreover, there was a significant improvement in the primary balance, which swung from -3.8% in 1985 to a small surplus in 1988, enabling public debt to be whittled down to 41.7% in 1988.

1989 to 1993: the aforementioned period of fiscal restraint came to an end in 1989, when the budget deficit began to grow again, to reach 7% at the height of the economic crisis in 1993. The primary balance followed a similar path to the deficit. After small surpluses between 1987 and 1989, it moved into deficit in 1990, rising to 1.8% of GDP in 1993. As regards public revenues and expenditures, similarly to the period 1975-1985, both increased significantly, reaching 42.8% and 49.8% of GDP, respectively, in 1993. Finally, there was only a slight increase in public debt, to 45.9% of GDP, primarily as a consequence of the strong growth in GDP between 1989 and 1991 (11% in nominal terms), and despite the increase in the cost of debt during this period. Thereafter, however, it rose to exceed 60% of GDP in 1993, as a consequence of the increase in the budget deficits, the fall in nominal GDP growth and the prohibition on monetary financing of the deficit as from 1994, under the Treaty on European Union. At the same time, the interest burden rose, reaching 5.2% of GDP in 1993.

1994 to 1998: fiscal policy was constrained in this period by the commitment to meet the convergence criteria set out in the Treaty on European Union to regulate access to the Third Stage of EMU. In accordance with this commitment, the tendency to imbalance in public finances came to an end in 1994, with a moderate reduction in the deficit. However, this was reversed again in 1995, when the budget deficit reached 7.3%. Thereafter, there was a gradual decline in the deficit, which reached 1.8% in 1998, in a context of economic recovery. The reduction in the public deficit was the result of a drop in spending, which fell by 5.7 points relative to GDP. Meanwhile, the share in GDP of total general government revenue declined slightly. Finally, public debt

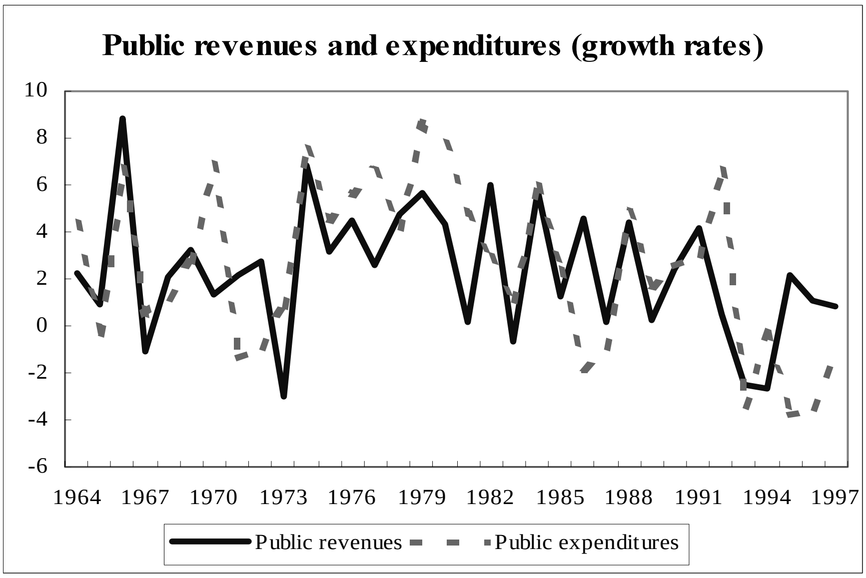
Figure 1. Fiscal variables as % of GDP

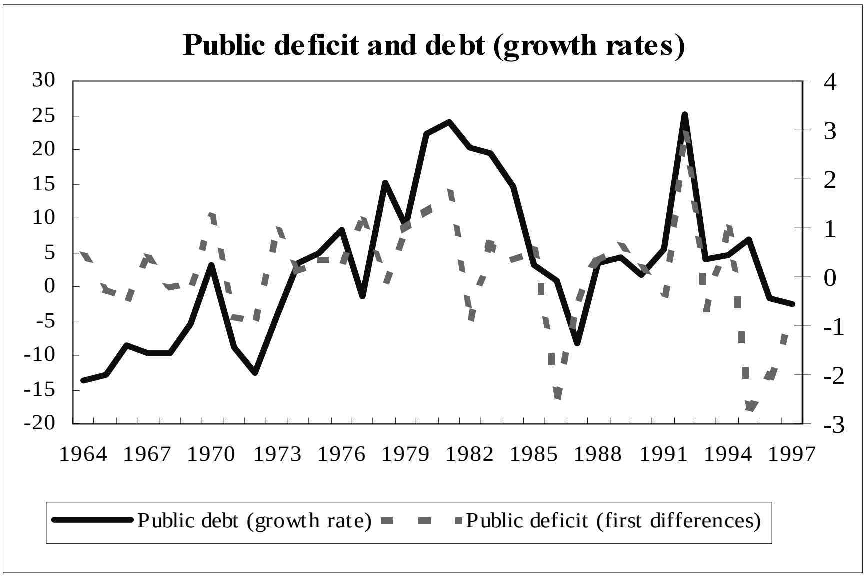




Source: Banco de España.

Figure 2. Growth rates of fiscal variables





Source: Banco de España.

peaked at over 70% of GDP in 1996 and then declined slightly to 67.4% of GDP in 1998. The factors responsible for this decline include the existence of primary surpluses in 1997 and 1998, the fall in interest rates and the revenue obtained from the privatisation of state-owned firms.

Finally, interest payments, which peaked in 1995 (at 5.6% of GDP), fell to 4.1% of GDP in 1998. This can be explained by both the reduction in the level of public debt in 1997 and 1998 and the decline in interest rates from 1995 (the average interest rates on Treasury bills fell from 9.1% in 1995 to 3.7% in 1998, while those on government bonds fell from 11.1% in 1994 to 6.4% in 1998).

3 Theoretical framework

As stated in the introduction, the definition of sustainability employed in this paper is based on the concept of the fulfilment of the intertemporal borrowing constraint of the government. In period the budget constraint can be expressed as follows[4]:

$$\Delta B_{t+1} = r_t B_t + G_t - R_t$$

with B_t being the stock of debt at the end of period t-1 in real terms, G_t real public expenditure excluding interest payments, R_t real public revenues and r_t the average real interest rate on the debt in period t-1. Thus, the term G_t - R_t is defined as the primary deficit. Accordingly, total public expenditures are

$$G_t^R = r_t B_t + G_t$$

Therefore, public deficit is defined as $D_t = G_t^R - R_t$. However, the latter variables are not the most accurate ones in a sustainability analysis. In fact, few or no conclusive results can be drawn from

[4] In this paper, seigniorage is not considered as a source of public revenues since the current institutional framework in EMU avoids the possibility of deficit financing through monetisation. Nevertheless, we are aware that, in the sample period of analysis, the fiscal policy has been often conducted by extensive use of seigniorage.

variables that show an upward trend if the economy shows a similar pattern. In other words, the relevant variables must be considered by taking into account the size of the economy, and any sustainability analysis should thus be performed using the latter variables as percentages of GDP and focusing on the burden that public debt imposes on the economy. Therefore, the budget constraint in t and the definition of total public expenditures, both in GDP terms, are now

$$\Delta B_{t+1} = \lambda_t B_t + G_t - R_t \qquad [1]$$

$$G_t^R = \lambda_t B_t + G_t$$

where $\lambda_t = \dfrac{r_t - g_t}{1 + g_t}$, which can be understood as the addition to net debt due to the excess of the real interest rate over g_t , the real GDP growth rate.

Taking the excess of the real interest rate over the growth rate of the economy as stationary around a mean λ, [1] can be expressed as

$$\Delta B_{t+1} = \lambda B_t + X_t - R_t \qquad [2]$$

where $X_t = G_t + (\lambda_t - \lambda) B_t$. Solving forward [2], the intertemporal borrowing constraint is obtained as

$$B_t = E_t \sum_{j=0}^{\infty} \gamma^{j+1} (R_{t+j} - X_{t+j}) \quad ; \quad \gamma^{\mathrm{j}+1} = (1+\lambda)^{-(j+1)} \qquad [3]$$

where the transversality condition $E_t \lim_{j \to \infty} \gamma^{j+1} B_{t+j+1} = 0$ is assumed to hold. Such transversality condition has a very well defined economic sense. It implies that, for a process to be sustainable, the current debt must equal the expected present value of future primary surpluses. Otherwise, stabilisation measures will be required in order to coax the public deficit back to a sustainable path.

If our focus is on the deficit variable, taking first differences in [3] yields:

$$G_t^R - R_t = E_t \sum_{j=0}^{\infty} \gamma^{j-1} (\Delta R_{t+j} - \Delta X_{t+j}) \qquad [4]$$

The left-hand side of [4] represents the public deficit, and the transversality condition that verifies the equation is then

$$E_t \lim_{j \to \infty} \gamma^{j+1} \Delta B_{t+j+1} = 0 \qquad [5]$$

Sustainability tests in literature aim to verify whether this transversality condition in the government budget constraint holds. These tests pay special attention to integration orders of deficit and debt processes, and to the underlying stochastic structures and the existence of cointegration relationships between revenues and expenditures. A usual procedure consists of testing the stationarity of ΔB_t in various forms, as Hamilton and Flavin (1986) propose, or alternatively the stationarity of $G_t^R - R_t$ if both are I(1), according to the method employed by Trehan and Walsh (1988). In both cases, the transversality condition holds because $\Delta B_t = Op(1)$ and, accordingly, the limit term in [5] behaves as

$$E_t \lim_{T \to \infty} \exp(-T\, k) = 0 \qquad [6]$$

where k is a constant and $Op(\cdot)$ the rate at which a stochastic sequence converges in probability to a non-stochastic sequence.

The procedure employed by Trehan and Walsh implies testing cointegration between revenues and expenditures when the cointegrating vector (1,-1) is imposed. An alternative procedure would be to test cointegration in

$$R_t = \alpha + \beta G_t^R + \varepsilon_t \qquad [7]$$

and afterwards, test the null H_0: β=1. According to these methods, the deficit would be non-sustainable if ΔB_t is non-stationary, or if cointegration in [7] does not hold with cointegrating vector (1,-1).

However, as Quintos (1995) shows, these methods only refer to sufficient conditions for sustainability. In general, it is not necessary for

ΔB_t to be I(0) for [5] to hold. If ΔB_t is I(d), being d a finite order of integration, it verifies[5] $\Delta B_t = Op(T^{d/2})$. In this case, the limit term in [5] behaves as

$$E_t \lim_{T\to\infty} \exp(-T\ k)\, T^{d/2} = 0 \qquad [8]$$

This result determines that if ΔB_t is an integrated process of any finite order, the discount factor decreases at a higher rate than ΔB_t, making the transversality condition, and thus the intertemporal borrowing constraint, hold, although the limit term in [5] approaches zero at a lower speed than in the case when ΔB_t is I(0). Consequently, using Quintos' terminology we will say that a deficit process is sustainable in its strong form if the limit term in the transversality condition behaves as [6], whereas if this limit behaves as [8] the process will be said to be sustainable in its weak form. Therefore, only when ΔB_t contains explosive roots of high enough magnitude to offset the discount factor will the deficit be non-sustainable.

As stated before, strong sustainability means that no future problems, according to the current state of affairs, are likely to arise, whereas a weakly sustainable budget performance could lead in the future to problems in debt-marketing, that would involve a risk of interest rates increases. Should this occur, macroeconomic stability would be endangered and severe fiscal reforms should be adopted.

In this context, Quintos shows that β=1 in [7] is only a sufficient condition for sustainability, in that it implies that the transversality condition behaves as [6]. However, it is not a necessary condition. Therefore, the necessary and sufficient condition is 0<β≤1, whereas cointegration is only a sufficient condition. Substituting [7] in [1] we obtain

$$B_{t+1} = (1+\lambda_t(1-\beta))B_t + (1-\beta)G_t - \alpha - \varepsilon_t \qquad [9]$$

5 Quintos proves this result for d=1. The general result is proved in De Castro and Peruga (1999).

or equivalently

$$\Delta B_{t+1} = \lambda_t (1-\beta) B_t + (1-\beta) G_t - \alpha - \varepsilon_t = (1-\beta) G_t^R - \alpha - \varepsilon_t \qquad [10]$$

If G_t^R is I(1), 0<β<1 implies, given [10], that ΔB_t is I(1), no matter whether ε_t is I(0) or I(1). In other words, cointegration in [7] plays no role, and consequently the transversality condition will behave as [8], being the deficit process sustainable only in its weak form. On the contrary, ΔB_t will be I(0) and thus the deficit strongly sustainable, when simultaneously β=1 and ε_t are I(0), i.e. cointegration between public revenues and expenditures holds. If we reject cointegration in [7] and β equals 1, the deficit will be sustainable in its weak form, because according to [10], ΔB_t will be I(1) as well. Finally, if β=0 the deficit is not sustainable. A summary of all the possibilities is found in Table 1.

Table 1. Quintos' test

Cases for $G_t^R \sim$ I(1)

Values for β	**and Cointegration in [7]**	**Yields ΔB_t**	**⇒ Conclusion for sustainability**
β=1	Yes	I(0)	Strong sustainability
β=1	No	I(1)	Weak sustainability
0<β<1	Plays no role	I(1)	Weak sustainability
β=0	Plays no role	I(1)	No sustainability

According to the process described earlier, Quintos suggests first to analyse the orders of integration of the variables G_t^R and R_t , and provided that they are I(1), to estimate [7] and test the null H_0: β=0 against the alternative Ha: β>0. If H_0 is accepted the deficit is not sustainable, whereas if it is rejected the null H_0: β=1 against Ha: β<1 should be tested. Should H_0 be rejected, the result 0<β<1 is obtained and the transversality condition would behave as [8], and accordingly the deficit would be weakly sustainable. In this case, as [9] shows, the undiscounted debt process contains an explosive root. On the other hand, if one cannot reject H_0: β=1, one should test for a cointegration relationship in [7]. In case cointegration is accepted, the transversality condition will behave as [6], and therefore, the strong sustainability result will hold. If, on the contrary, cointegration is rejected in [7], the transversality condition will behave again as [8], and thus the deficit will be weakly sustainable.

4 Empirical results

The following empirical results are based on annual data for Spain of public debt, public deficit and public revenues and expenditure for the period 1964-1998. We are aware that any long-run analysis based on such a small number of observations may be somewhat troublesome. Moreover, the well-known lack of power of unit-root tests added to this problem obliges us to treat the results with the greatest care. Without forgetting these difficulties, we will present our main findings in what follows.

4.1 Full sample analysis without structural breaks

In Table 2 we summarise the unit root tests for the variables used in the analysis. In none of the cases do the tests reject the null hypothesis of the existence of one unit root. Since no constant or deterministic trend turned out to be significant for R_t , G_t or D_t, the tests reject the null of the existence of two unit roots.

The tests performed on the debt process do not clearly reject the existence of a constant and a deterministic trend. Thus, the ADF test does not offer conclusive results about the existence of one or two unit roots in the process followed by this variable, although it seems to favour the I(2) hypothesis. The Phillips-Perron method offers a different view in that it rejects the null of two unit roots against the alternative of only one unit root. However, given that D_t can be considered as I(1), according to [1], we might expect B_t to be I(2), or accordingly ΔB_t to be I(1).

Table 2. Unit root tests

	I(1) vs. I(0)					
	ADF statistics			Phillips-Perron statistics		
	t_{α}	$t_{\alpha*}$	$t_{\alpha**}$	Z(t_{α})	Z($t_{\alpha*}$)	Z($t_{\alpha**}$)
B_t	0.92	-0.26	-2.46	2.07	0.77	-2.54
R_t	3.53	-0.76	-0.99	4.35	-0.78	-1.19
G_t^R	1.38	-1.12	-0.93	2.30	-1.14	-0.39
D_t	-0.84	-1.45	-1.40	-0.82	-1.45	-1.31
	I(2) vs. I(1)					
B_t	-2.26**	-2.59	-2.44	-2.83***	-3.19**	-3.25*
R_t	-1.86*	-3.34**	-3.38*	-3.94***	-6.47***	-6.48***
G_t^R	-1.98**	-2.46	-2.63	-2.97***	-3.73***	-3.92**
D_t	-3.16***	-3.10**	-3.19	-5.25***	-5.18***	-5.27***

*The symbols *, ** and *** denote rejection of the null at the 10%, 5%, and 1% significance levels, respectively. The number of lags used has been set to 1.*

Critical values

	10%	5%	1%
Z(t_{α})	-1.60	-1.95	-2.64
Z($t_{\alpha*}$)	-2.62	-2.97	-3.68
Z($t_{\alpha**}$)	-3.21	-3.55	-4.27

The lack of power of unit-root tests, together with the difficulty of distinguishing between a I(2) process and a I(1) process with drift and time trend, prevents us, for the moment, from drawing further conclusions on this issue.

Given that G_t and R_t are I(1) processes, we perform Quintos' test and estimate [7] for the whole sample. We estimated [7] by OLS, the maximum likelihood procedure suggested by Johansen, and by the non-parametrical procedure proposed by Phillips and Hansen (1990)[6]. Once [7] was estimated, we performed several cointegration tests based on the ADF and Phillips (1987) statistics and the Trace Statistic suggested by Johansen. The estimation and cointegration tests results are summarised in Table 3 and none of the tests reject the null of absence of cointegration between both variables. Moreover, the estimated coefficient is between zero and one, which, according to Quintos, would lead us to conclude that the deficit is sustainable in the weaker form[7]. However, such a result is not at all informative because the absence of cointegration yields a spurious estimation of β. In order to avoid this problem and to complete Quintos' test, we estimated [7] in first differences (Hamilton, 1994), yielding an estimated coefficient of β=0.31. Given that the condition $0<\beta<1$ holds, the transversality condition behaves as [8], and accordingly the deficit process is sustainable in its weaker form. Furthermore, by [9] we know that the debt process should have an explosive root, which is consistent with the positive t-ratios obtained in the unit-root tests on B_t without considering either a constant or a deterministic trend. The estimated explosive root in this case has been 1.01, and its small magnitude can explain why the process followed by this variable can be better approximated by a I(1) rather than by a I(2) one.

The result so far obtained are not conclusive in that the power of the ADF and other cointegration tests diminishes in the presence of structural breaks[8] Over the course of sample period, on the other hand, many fiscal

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6 The latter may be advisable when the regressors may be endogenous, which leads to a second order asymptotic bias in the OLS estimators. The second-order asymptotic bias arises because the estimators are still consistent when cointegration holds. In order to correct this bias, they suggest estimating by instrumental variables, but the instruments do not fully eliminate the asymptotic bias when the regressors are endogenous. Therefore, they suggest semi-parametric corrections in the long run covariance matrix, which lead to asymptotically unbiased-in-median estimators. These fully-modified estimators form the basis of the so-called fully-modified Wald tests, which can be used for testing general linear hypotheses of the coefficients in cointegrating regressions, and their asymptotic distributions are χ^2. The correction in the long-run covariance matrix is based on the procedure suggested by Andrews and Monahan (1992).

7 The results derived from our estimations are not very different from those of Camarero, Esteve and Tamarit (1998).

8 See Hansen (1992), Hansen and Johansen (1992), Gregory, Nason and Watt (1996) and Gregory and Hansen (1996).

reforms have taken place in Spain. In particular, since the late seventies, fiscal policy in Spain changed from a system in which the general government budgets were formally balanced, or even showed a small surplus, to another –as from 1976- with public deficits, linked to the expansion of spending as a consequence of moving towards European welfare state models. Also, the tax system was thoroughly overhauled, with the introduction of personal income tax in 1978 and VAT in 1986. Further, the progressive move as from 1983 from monetisation to a more orthodox financing of the deficit, which coincides in time with high budget imbalances, resulted in the emergence of a significant pubic debt balance and a subsequent increase in the interest burden. Lastly, the so-called "State of Regional (Autonomous) Governments" has been established further to the 1978 Constitution, hand in hand with the decentralisation of spending, the result of ongoing negotiations regarding regional government financing for arrangements. Therefore, in the following section we will consider the possibility of presence of structural breaks and explore whether the conclusions are substantially affected.

4.2 Structural-break tests on the cointegration relationship

The Gregory and Hansen test

Gregory and Hansen (1996) are concerned with the possibility that the cointegrating vector may change during the sample period at a single unknown point in time. If this is true, the standard ADF and Phillips Z_α and Z_t tests lose power. As a consequence, if the true model is cointegrated with a regime shift, standard analysis consisting of estimating [7] and performing ADF or Phillips Z_α and Z_t tests on the cointegrating residuals does not reject the null of no cointegration. Gregory and Hansen propose a statistic that attempts to test the null hypothesis of no cointegration against the alternative of cointegration with a structural break at an unknown point in time, and accordingly consider three possible models. Thus, they allow only for changes in the intercept, without and with time trend, and shifts in both the intercept and slope coefficients. These are referred to as Level shift (C), Level shift with trend (C/T) and Regime shift (C/S) models, respectively. In this context, the stable cointegration relationship without structural breaks is only a particular case. Their procedure consists of estimating by OLS and

Table 3. Long run relationship between R_t and G_t

$$R_t = \alpha + \beta G_t^R + u_t$$

		Cointegration tests			
	β	ADF[a]	Z_α[b]	Z_t[a]	Trace Statistic[c]
OLS (Phillips-Ouliaris)	0.77	-2.01	-16.96	-1.05	
Phillips-Hansen	0.79	-1.92	-17.01	-1.79	
Johansen	0.74				19.54/4.41
Fully modified Wald test for the null (P-H)[d]	β=0 β=1	704.09 51.97			

a) *Critical values for the ADF and Z_t statistics are –3.51, -3.80 and –4.36 at the 10%, 5% and 1% significance levels, respectively. They have been taken from Phillips and Ouliaris, 1990.*

b) *Critical values for the Z_α statistic are –23.19, -27.08 and –32.19 at the 10%, 5% and 1% significance levels, respectively. Also taken from Phillips and Ouliaris, 1990.*

c) *Johansen LR critical values at the 5% and 1% significance levels for the null hypothesis of no cointegration relationships are 19.96 and 24.60, respectively, while the critical values for the null hypothesis of at most one cointegration relationship are 9.24 and 12.97.*

d) *Critical values for the χ^2_1 are 2.71, 3.84 and 6.63 at the 10%, 5% and 1% significance levels, respectively.*

computing the cointegration tests for every possible break point and selecting as the most probable break point that associated with the highest absolute value for these tests[9].

Table 4 shows the results from the implementation of these tests to the Spanish case. As can be seen, none of the statistics turned out to be significant. These results, together with those in Table 3, support the null hypothesis of no cointegration in [7]. However, the statistic InfADF

9 See Appendix A for details on the implementation of this test.

applied to the (C/S) model shows that, albeit non-significant, between 1987 and 1988 a change in the fiscal policy regime may have taken place.

The Hansen test

Hansen (1992) also considers the possibility of a structural break at an unknown point in time, although the null hypothesis is the existence of cointegration, in contrast with the Gregory and Hansen test. Hansen provides three tests for parameter instability based on the information derived from the fully-modified residuals in the cointegrating equation[10]. The statistics of these tests are complementary to those proposed by Gregory and Hansen, in that Hansen tests the null of cointegration with no regime shift against the alternative that a regime shift has occurred. Following Hansen we will call these statistics SupF, MeanF and L_c. Their values are reported in Table 5. Accordingly, the L_c and the SupF statistics show evidence of parameter stability, although the MeanF statistic could suggest that a gradual change in the behaviour of the fiscal variables may have taken place.

Note that the null can be rejected not only because there is a regime shift, but also because cointegration does not hold in [7]. Therefore, the L_c statistic can also be understood as a LM cointegration test. According to its value, it suggests a stable long- term relationship between public revenues and expenditures, in contrast with the results derived from the standard cointegration tests and those proposed by Gregory and Hansen.

The Hansen and Johansen Test

Hansen and Johansen (1993) do not examine directly the stability of the parameters in the cointegration equation, but the stability of the eigenvalues associated with the Error Correction Model (ECM henceforth) that yield to the estimation of the cointegrating vector. They propose a recursive Likelihood Ratio test with null of cointegration for every subsample[11]. The statistics to be considered will be referred to as

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10 See Appendix B for a detailed explanation of these tests.

11 More detailed information on this test can be found in Appendix C. Programs have been provided by J.L. Fernández.

Table 4. Gregory-Hansen tests for structural breaks

Model	InfADF[a]	Ninf	Mean ADF[b]	Z_t[a]	Z_α	Nzinf
Level shift (C)	-3.79	1990	-2.62	-3.36	-20.46	1987
Regime Shift (C/S)	-4.56	1988	-2.76	-4.21	-26.31	1987

a) *Critical values for the ADF and Z_t statistics are –4.34, -4.61 and –5.13 at the 10%, 5% and 1% significance levels for the (C) model, whereas the critical values for the same significance levels for the (C/S) model are –4.68, -4.95 and –5.47, respectively. Taken from Gregory and Hansen, 1996.*

b) *Critical values for the MeanADF statistic are –3.53, -3.80 and –4.26 at the 10%, 5% and 1% significance levels for the (C) model, whereas the critical values for the same significance levels for the (C/S) model are –3.52, -3.78 and –4.27, respectively.*

c) *Critical values for the Z_α statistics are –36.19, -40.48 and –50.07 at the 10%, 5% and 1% significance levels for the (C) model, whereas the critical values for the same significance levels for the (C/S) model are –41.85, -47.04 and –57.17, respectively. Taken from Gregory and Hansen, 1996.*

Table 5. Hansen test

Statistic	Value	P-value[a]
L_c	0.10	0.20
MeanF	3.05	0.17
SupF	6.88	0.20

a) *This column shows the probability of parameter instability. Probability equal to 0.20 means ≥0.20. According to Hansen (1992), a P-value over 0.20 can be taken as evidence of parameter stability.*

Table 6. Hansen and Johansen test

Statisic	Critical values		
	10%	5%	1%
SupHJ	3.69	4.81	7.39
Break point			
MeanHJ	0.69	0.98	1.65

SupHJ and MeanHJ, which are the maximum and mean of the sequence of all the HJ(t) statistics for every possible break point.

The results drawn from this test are presented in Table 6. Only the MeanHJ statistic turned out to be significant at the 10% level which could indicate a gradual regime shift, although the evidence on parameter instability is not conclusive at all.

The results from the tests above do not support the hypothesis of the existence of a structural break in the behaviour of the fiscal variables. Rather, they could be taken as evidence of a gradual shift, although the evidence on this latter issue is far from conclusive. Moreover, we do not have clear evidence of cointegration. Neither the Gregory and Hansen test nor the standard cointegration tests allow us to reject the null of no cointegration, although the L_c test would suggest the contrary. Nevertheless, according to Gregory and Hansen (1996), it would be advisable to test first the null of no cointegration and, if it is rejected in favour of the (C) or the (C/S) models, then perform the Hansen test in order to obtain deeper evidence of a regime shift.

So far, our results do not qualitatively differ from those obtained by Camarero et al. (1998). However, given the above results we consider that a deeper univariate analysis of the series involved is of great interest and may provide us with useful information in order to derive our conclusions. If we consider that in a given period the order of integration of the series involved in the analysis has changed, then the latter analysis is misleading. In this context, changes in the order of integration can be associated with fiscal reforms or even with the achievement of a given level, after a gradual adjustment, in the variables considered after the implementation of the above-mentioned reforms. This is our justification for considering an alternative way of addressing the problem of fiscal sustainability.

4.3 Univariate analysis and structural breaks

We will address our univariate analysis in two different ways. First, we check whether the order of integration of the variables that was obtained in 4.1. may be misleading due to the presence of instability in the stochastic trend. Second, we consider the possibility of changes in the order of integration of the variables.

Structural-break tests on the stochastic trend

The presence of instability in the stochastic trend makes the standard ADF test to be biased to accept the null hypothesis of stationarity in many cases (Hendry and Neale (1990), Perron (1989, 1990), Perron and Vogelsang (1992), among others). In order to avoid these problems, sequential procedures have been suggested in the literature due to the power they show against alternatives such as rolling regressions and recursive methods. Examples in this field can be found in Banerjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992) and Perron and Vogelsang (1992). These procedures are based on the ADF statistic and select the minimum of the sequence for all the possible break points in the sample. Another statistic based on the mean of the sequence of ADF statistics is suggested by Hansen (1992). While the first shows power in the presence of well located breaks, the second shows a comparative advantage in detecting gradual changes in the stochastic trend. From each of these sequential statistics the maximum (minimum for the ADF) and mean ones will be considered, and they will be referred to as Inf t_ρ, Mean t_ρ, Sup $|t_{\mu'}|$, Mean $|t_{\mu'}|$, Sup $|Rt_{\mu'}|$ and Mean $|Rt_{\mu'}|$[12].

The results derived from these tests[13] are reported in Table 7.

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[12] See Appendix D for a detailed explanation of these tests.

[13] These tests have been kindly provided by J.L. Fernández.

Table 7. Stochastic trend stability analysis

Statistic	ΔB_t	R_t	G_t^R	D_t
Inf t_ρ	-3.43	-2.65	-2.73	-3.35
Year	1976	1973	1973	1978
Mean t_ρ	-2.53	-0.43	-0.97	-1.63
Sup $\|t_{\mu'}\|$	2.23	3.72	3.48	2.87
Year	1976	1991	1992	1978
Mean $\|t_{\mu'}\|$	1.07	1.69	1.28	1.16
Sup $\|Rt_{\mu'}\|$	2.58	3.09*	3.69**	2.31
Year	1991	1992	1992	1993
Mean $\|Rt_{\mu'}\|$	0.76	1.08	1.10	0.87

Note: (*) and (**) mean rejection of the null at the 10% and 5% significance levels, respectively.

Critical values (Taken from Fernández , 1999)

	10%	5%	1%
Inf t_ρ	-4.06	-4.38	-5.03
Mean t_ρ	-2.62	-2.92	-3.54
Sup $\|t_{\mu'}\|$	3.88	4.22	4.87
Mean $\|t_{\mu'}\|$	1.77	1.95	2.34
Sup $\|Rt_{\mu'}\|$	3.04	3.37	4.04
Mean $\|Rt_{\mu'}\|$	1.48	1.70	2.11

We can see that little evidence of jumps of noticeable magnitude can be found.

However, a deeper analysis shows that a level shift in revenues and expenditures seems to have taken place around 1992, which becomes more evident in the case of public expenditure. These conclusions are supported not only by the significance of the Sup $|Rt_{\mu'}|$ statistic, but also by the high values that the Sup $|t_{\mu'}|$ take, which are very close to significance. Moreover, the high Mean $|t_{\mu'}|$ for the revenues also seems to favour the hypothesis that, at least, a gradual change has taken place.

Tests of changes in the order of integration

If the order of integration varies over time the implications for a sustainability analysis may be very important, because it can make us change our initial view derived from the estimation of [7] with or without structural breaks. Concretely, if ΔB_t is not stationary in the first part of the sample but becomes stationary in the last part, although a global analysis would lead us to conclude that, according to [9], the deficit process is sustainable in a weak sense, the relevant issue for analysing the future behaviour will be the current process followed by this variable. As a result, we should say that the sustainability seems to be turning to its strong form and no future fiscal problems seem to arise in the horizon.

Earlier work on this issue has been done by Leybourne, McCabe and Tremayne (1996) and Maeso (1997). Here, we follow Fernández (1999) and use a sequential procedure in order to get the following statistics: $\mathrm{Supt}_{\delta 1}$, $\mathrm{Meant}_{\delta 1}$, $\mathrm{Supt}_{\delta 2}$, $\mathrm{Meant}_{\delta 2}$, $\mathrm{Supt}_{\alpha 1}$, $\mathrm{Meant}_{\alpha 1}$, $\mathrm{Supt}_{\alpha 2}$ and $\mathrm{Meant}_{\alpha 2}$. As before, the Sup statistics have power for a unique break point, whereas the mean ones have power for gradual changes. According to Zivot and Andrews (1992), the break point is associated with the observation that corresponds to the Sup[14].

The results derived from these tests are shown in Table 8. The general conclusion that can be extracted is that the Mean statistics for the last part of the sample tend to reject the null of I(1) at the 10%

[14] See Appendix E for further explanation of these tests.

Table 8. Partial integration tests[15]

Statistic	ΔB_t	R_t	G_t^R	D_t
Supt$_{\delta 1}$	-2.35	-0.59	-1.26	-1.89
Year	1993	1974	1988	1976
Meant$_{\delta 1}$	-0.97	0.98	0.25	-1.21
Supt$_{\delta 2}$	-3.89*	-3.38	-3.59	-3.33
Year	1987	1991	1988	1980
Meant$_{\delta 2}$	-2.96**	-2.42*	-2.44*	-2.40*
Supt$_{\alpha 1}$	-2.35	-0.68	-1.23	-1.67
Year	1968	1970	1968	1990
Meant$_{\alpha 1}$	-0.73	0.62	0.07	-0.68
Supt$_{\alpha 2}$	-3.71*	-3.55	-3.37	-3.22
Year	1987	1991	1988	1980
Meant$_{\alpha 2}$	-2.80***	-2.09*	-2.23*	-2.11*

Note: (*), (**) and (***) mean rejection of the null at the 10%, 5% and 1% significance levels, respectively.

Critical values (Taken from Fernández , 1999)

	10%	5%	1%
Supt$_{\delta 1}$	-3.44	-3.76	-4.44
Meant$_{\delta 1}$	-2.37	-2.46	-3.23
Supt$_{\delta 2}$	-3.80	-4.12	-4.76
Meant$_{\delta 2}$	-2.36	-2.59	-3.07
Supt$_{\alpha 1}$	-3.18	-3.48	-4.12
Meant$_{\alpha 1}$	-2.09	-2.35	-2.88
Supt$_{\alpha 2}$	-3.60	-3.91	-4.52
Meant$_{\alpha 2}$	-2.09	-2.28	-2.65

15 The programs for performing these tests have also been provided by J.L. Fernández.

significance level, whereas the null is not rejected for the first part. This indicates that the processes followed by the relevant variables are becoming stationary, and accordingly the debt in GDP percentage points is becoming I(1). The change is taking place in a gradual form and begins between the late 80s and the early 90s. This result has a direct economic interpretation since the first sample period, which covers from 1964 to the early 90s, was characterised by the implementation of a modern fiscal policy in Spain, which implied the building of the Welfare State and a new tax system, moving towards European models, that required a rapid expansion of public revenues and expenditures. Once this target was achieved and with the additional constraint derived from the commitment to meet the convergence criteria set out in the Treaty on European Union, consolidation issues become more relevant in conducting fiscal policy. (section 4.1) and the structural-break tests on the cointegrating relationship (section 4.2) must be questioned because the variables in [7] are not always I(1), and, consequently, the cointegrating analysis may lose sense at least for the whole sample period. Furthermore, our results show that in recent years the "sustainability in the weak sense" seems to be changing into "sustainability in a strong sense", according to Quintos' terminology, and no future problems in marketing public debt are expected to arise as far as this trend is confirmed.

5 Conclusions

This paper attempts to ascertain whether the current fiscal policy in Spain is sustainable or not, in the sense of the current market value of debt being equal to the discounted sum of expected future surpluses. For this purpose, we apply the traditional tests of sustainability, following Quintos´ approach. In addition, we introduce a deeper univariate analysis of the series. Our findings can be summarised as follows.

As regards the unit root tests applied to the variables used in the analysis, the null hypothesis of the existence of one unit root is accepted, while all the tests reject the null of the existence of two unit roots. Only in the case of the public debt, do the tests not offer conclusive results about the existence of one or two unit roots in the process followed by this variable. Furthermore, the tests tend to reject cointegration between public revenues and expenditures, and given that the condition $0<\beta<1$ holds, the interpretation has to be that the deficit process is sustainable in its weak form. Thus, the debt process has an explosive root but one of small magnitude (1.01). Moreover, the results from the tests do not support the hypothesis of the existence of a structural break in the long-term relationship between public revenues and expenditures.

The weak sustainability result would imply that, although the transversality condition holds, the government could eventually have problems in marketing its debt, and this involves a risk of rising interest rates in the future. On the hand, that would increase the primary deficit via interest payments. And, on the other, the prospects of future increases in interest rates would cause a slowdown in economic growth and in the government's capacity to generate resources. This second channel also tends to increase future deficits and could ensue in a non-sustainable path, necessitating a fiscal adjustment.

Finally, the tests applied to find changes in the order of integration of the series indicate that the processes followed by revenues, expenditures and deficit are becoming stationary, and the debt as percentage of GDP is thus turning from I(2) to I(1), making the transversality condition hold in its strong sense, in contrast with the results obtained by Camerero et al. (1998). This change is taking place gradually and starts between the late 80s and early 90s. Therefore, the general conclusion drawn from the cointegration estimation and the structural-break tests on the cointegrating relationship must be questioned because the variables are not first-order integrated for the whole sample period, and the cointegrating analysis may, therefore, be somewhat lacking in meaning for this whole period.

Consequently, our results show that in recent years the "sustainability in the weak sense" seems to be changing into "sustainability in a strong sense", according to Quintos' terminology. However, any conclusion to be derived from these results should take into account the limitations of the analysis, in particular the fact that this is based on past data. Therefore, under this approach, a sustainable fiscal policy would indicate that if the variables involved follow the pattern of the past in the future, no problems in marketing public debt are expected to arise.

Appendix A

The Gregory and Hansen Test (1996)

Gregory and Hansen (1996) propose a statistic that attempts to test the null hypothesis of no cointegration against the alternative of cointegration with a structural break at an unknown point in time, and accordingly consider three possible models. The first one is referred to as "level shift" (C), which is expressed as:

$$y_{1t} = \mu_1 + \mu_2 D_{t\tau} + \alpha y_{2t} + e_t \qquad \text{[A.1]}$$

where

$$D_{t\tau} = \begin{cases} 0 & t \leq [\tau T] \\ 1 & t > [\tau T] \end{cases} \quad , \quad \tau \in (0,1)$$

with [ ] meaning "integer part" of the argument inside. Thus, [A.1] aims to test whether there is a change in the intercept term at the time of the shift. The second possibility is called "level shift with trend" (C/T) with the form:

$$y_{1t} = \mu_1 + \mu_2 D_{t\tau} + \beta t + \alpha y_{2t} + e_t \qquad \text{[A.2]}$$

The last model considered is known as "regime shift" (C/S) and takes the form:

$$y_{1t} = \mu_1 + \mu_2 D_{t\tau} + \alpha_1 y_{2t} + \alpha_2 y_{2t} D_{t\tau} + e_t \qquad \text{[A.3]}$$

These models are estimated recursively by OLS for all possible break points in the interval $\tau \in$ [0.15,0.85]. A sequence of ADF and Phillips Z_α and Z_t residual-based tests is computed, and they calculate the highest absolute value of the sequence. The observation associated with this statistic is taken as the most probable break point.

More recently Fernández (1999) has tabulated the distribution for the Mean of the ADF test (MeanADF) which was not originally tabulated by Gregory and Hansen. This latter statistic could be used for testing a gradual change in the policy regime and shows that this statistic has an acceptable power in finite samples. He also shows that the test proposed by Gregory and Hansen has more power than the tests proposed by Hansen (1992) and Hansen and Johansen (1992) (which we shall refer to later) in finite samples for detecting parameter instability in cointegrated relationships, although all of them have lower power the lower the sample period.

Appendix B

The Hansen Test (1992)

This test also considers the possibility of a structural break at an unknown point in time, although the null hypothesis is the existence of cointegration, in contrast with the Gregory and Hansen test. Thus, the alternative is the existence of a structural break. He considers the following relationship between the variables:

$$y_t = A_t x_t + u_{1t} \qquad [B.1]$$

with

$$x_t = (x'_{1t}, x'_{2t})'$$

$$x_{1t} = 1$$

$$x_{2t} = x_{2t-1} + u_{2t}$$

He proposes four tests for instability. The first two are called F_t and SupF for the alternative of a single structural break in A_t, which yields:

$$A_i = \begin{cases} A_1 & \text{i} \le \text{t} \\ A_2 & \text{i} > \text{t} \end{cases}$$

where $1<t<n$. These tests for parameter instability are based on the *scores* obtained from the fully-modified residuals in the cointegrating equation and a long-run estimation of the covariance matrix as suggested by

Andrews and Monahan (1992), which uses a prewhitened kernel estimator with a plug-in bandwidth.

The test F_t assumes that the break point is known and takes the expression:

$$F_t = trace\{S'_{nt} V_{nt}^{-1} S_{nt} \hat{\Omega}_{1\cdot 2}^{-1}\}$$

and S_{nt} and V_{nt} are

$$S_{nt} = \sum_{i=1}^{t} \left(x_t \hat{u}_{1t}^{+}{}' - \begin{pmatrix} 0 \\ \hat{\Lambda}_{21}^{+} \end{pmatrix} \right)$$

$$V_{nt} = M_{nt} - M_{nt} M_{nn}^{-1} M_{nt}$$

and

$$M_{nt} = \sum_{i=1}^{t} x_i x_i'$$

where $\hat{u}_{1t}^{+}{}'$ are the fully-modified residuals from the estimation of [B.1], corrected by the endogeneity bias of the regressors and $\hat{\Omega}_{1\cdot 2}$ is semi-parametric estimation of the long term variance of u_{1t} conditioned to u_{2t} as suggested by Andrews and Monahan.

$$\Omega = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} E(u_j u_t')$$

Under the null of stationarity of A_t this contrast follows a χ^2 with degrees of freedom equal to the number of cointegrating vectors. This test is similar to the Chow test, but can only be used when t can be chosen independently of the sample size, and thus has a low power. Therefore, when the break point is unknown Hansen proposes the statistic

$$SupF = \sup_{t/n \in \xi} F_{nt}$$

where ξ is a compact subset of the interval (0,1). Hansen suggests considering the F_{nt} statistics in the interval ξ = [0.15 , 0.85] in order to avoid distortions induced by break points close to the first and final observations. The observation associated with *SupF*, *NsupF*, can be interpreted as an indicator of the possible break point. The *SupF* statistic has power against a swift shift in regime. On the other hand, when the parameter Hansen suggests the *MeanF* statistic, which takes the form:

$$MeanF = \frac{1}{n^*} \sum_{t/n \in \xi} F_{nt} \qquad \text{where} \quad n^* = \sum_{t/n \in \xi} 1$$

The last test posed by Hansen is an LM one called L_c, which is appropriate when the likelihood of parameter variation is relatively constant throughout shifts gradually over time, when A_t follows a martingale process, the sample, and takes the form:

$$L_c = trace\left\{ M_{nn}^{-1} \sum_{t=1}^{n} S_t \hat{\Omega}_{1\cdot 2}^{-1} S_t' \right\}$$

This test does not require specification of an interval for t a and can be taken as a cointegration test with the null of existence of cointegration.

Appendix C

The Hansen and Johansen Test (1993)

The Hansen and Johansen test is a recursive test that can be applied to the maximum-likelihood method proposed by Johansen (1988,1991) for estimating cointegrating vectors. It examines the stability of the eigenvalues associated with the Error Correction Model, which measures the correlation between the vector of variables in levels and in first differences.

A vector with p I(1) variables, whose dynamics are defined by a VAR, has the form:

$$\Delta X_t = \alpha\beta' X_{t-1} + \Lambda Z_t + \varepsilon_t, \qquad \text{with } t=1,\dots T \qquad [C.1]$$

where

$$Z_t = (\Delta X_{t-1},...,\Delta X_{t-k+1}, D_t, 1)'$$

$$\Lambda = (\Lambda_1,...,\Lambda_{k-1}, \Psi)'$$

D_t is a set of seasonal dummies, β is the cointegrating vector and α a vector of adjustment coefficients for transitory deviations from the long-term relationship. Regressing ΔX_t and X_{t-1} over Z_t the residuals R_{0t} and R_{1t} are obtained. Using these residuals the matrixes of moments and the eigenvalues are obtained:

$$S_{ij} = \sum R_{it} R_{jt}'$$

$1 > \hat{\lambda}_1 > ... > \hat{\lambda}_p > 0$ and the corresponding eigenvectors $\hat{V} = (\hat{v}_1,...,\hat{v}_p)$

by solving the equation

$$|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0 \qquad [C.2]$$

These eigenvalues and eigenvectors yield the estimation of β and the range of the matrix of cointegrating vectors, r. They propose the following statistic:

$$HJ(t) = t \sum_{i=1}^{r} \ln \left| \frac{1-\hat{\rho}_i(t)}{1-\hat{\lambda}_i(t)} \right|$$

where $\hat{\lambda}_i(t)$ are the eigenvalues without restrictions obtained from [C.2] for the subsample 1,…,t, while $\hat{\rho}_i(t)$ are the eigenvalues obtained for the same subsample according to:

$$|\rho\beta' S_{11}(t)\beta - \beta' S_{10(t)} S_{00}^{-1}(t) S_{01}(t)\beta| = 0$$

or in other words, imposing the restriction that the cointegrating-vectors matrix in the subsample 1,…t equals β, the cointegrating-vectors matrix for the whole sample. For every possible break point, the *HJ* statistic is a

LR test that compares the eigenvalues obtained with and without restrictions, and follows a χ^2 distribution with $(p\text{-}r)r$ degrees of freedom. As t approaches the end of the sample, the statistic converges to 0, so it is expected that its asymptotic power is greater for structural breaks in the beginning of the sample.

In their paper Hansen and Johansen do not tabulate the empirical distribution associated with these statistics, but they have recently been obtained by Fernández (1999).

Appendix D

Structural-Break Tests On The Stochastic Trend

These tests are based on the following set of equations. The first one is

$$\Delta y_t = \mu + \mu' D_{\tau t} + \rho y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t \qquad \text{[D.1]}$$

where

$$D_{\tau t} = \begin{cases} 1 & \text{t} < [\tau T] \\ 0 & \text{t} \geq [\tau T] \end{cases} \quad , \quad \tau \in (0,1)$$

with [ ] meaning “integer part” of the argument inside. Following Fernández (1999) two statistics for every break point will be calculated from [D.1]: t_ρ and $|t_{\mu'}|$, which are the pseudo-standard ADF statistic and the absolute value of the t-statistic for the null of μ'=0, as a test for the stability of the stochastic trend suggested by Banerjee, Lumsdaine and Stock (1992). These authors also consider a second statistic, $|Rt_{\mu'}|$, which is the absolute value of the t-statistic for the null μ'=0 in the restricted version

$$\Delta y_t = \mu + \mu' D_{\tau t} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t \qquad \text{[D.2]}$$

From each of these sequential statistics the maximum (minimum for the ADF) and mean ones will be considered, and they will be referred to as Inf t_ρ, Mean t_ρ, Sup $|t_{\mu'}|$, Mean $|t_{\mu'}|$, Sup $|Rt_{\mu'}|$ and Mean $|Rt_{\mu'}|$.

Depending on the form of the underlying process, under some alternatives of I(1) and I(2) processes a spurious correlation between the first difference and the level of the variable may arise, leading to a misleading rejection of the null of I(0) in favour of false stationarity. In other cases, the ADF statistic does not have power to distinguish between unstable I(1) processes and stable I(2) processes. In accordance with these considerations, Fernández (1999) provides a good description of the behaviour of these statistics in several cases, and it can be summarised as follows:

a) If the process is I(0), the latter statistics behave well in levels and in first differences.

b) I(0) processes with an unstable mean are well detected by the Sup $|t_{\mu'}|$.

c) The statistics of change in the stochastic trend do not reject the null in the case of pure I(1) processes.

d) In the case of I(1) processes with a change in μ, sequential statistics in levels may or may not reject the null depending on the case at hand. However, the Inf t_ρ statistic will, in general, reject the null in first differences. Moreover, the Sup $|Rt_{\mu'}|$ rejects the null of stability in levels, whereas Sup $|t_{\mu'}|$ will do so in first differences.

e) The statistics under consideration do not reject the null in first differences for I(2) processes. In levels the unrestricted ones are preferable, whilst both restricted and unrestricted do not indicate instability.

Accordingly, the following method is proposed:

- If the first difference is stationary the underlying process may be either I(0) or I(1), but never I(2). Therefore, if the null is not rejected in first differences the process is I(2).

- Conditioned to not being I(2), if the null is not rejected in levels, then the process cannot be I(0). Thus, if it is a stable I(1) process the statistics of change in stochastic trend do not reject the null, whereas the contrary occurs if it is unstable. In this latter case, restricted statistics are recommended for convex I(1) processes. Moreover, unstable I(1) series will be considered a I(0) with unstable mean.

- In fact, rejection of I(1) in levels does not definitely indicate stationarity because we could be in the presence of an unstable I(1) series. In this case the restricted statistics of change in the stochastic trend are expected to reject the null in levels.

- Finally, I(0) stable processes will be accurately detected by the standard ADF and Mean t_ρ statistics, whereas unstable I(0) will be better evidenced by the Inf t_ρ.

Appendix E

Tests on Changes in the Order of Cointegration

Earlier work on this issue has been done by Leybourne, McCabe and Tremayne (1996) and Maeso (1997). The former paper tests the null of I(1) with invariant coefficient against the alternative of random coefficient. The latter tests the same null against the alternative of a constant coefficient with a different level since a given date, using rolling regressions. Here we follow Fernández (1999) and use a sequential procedure consisting of estimating the following set of equations:

$$\Delta y_t = \mu + \delta_1 D_\pi y_{t-1} + \delta_2 (1 - D_\pi) y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t \qquad [E.1]$$

$$\Delta y_t = \mu + \alpha_1 D_\pi y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t \qquad [E.2]$$

$$\Delta y_t = \mu + \alpha_2 (1 - D_\pi) y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t \qquad [E.3]$$

where D_π has been defined above. [E.1] aims to test the existence of one unit root in both subsamples simultaneously. On the contrary, [E.2] and

[E.3] impose I(1) in one part of the sample. Accordingly, a sequence for the pseudo-t-ratios $t_{\delta 1}$, $t_{\delta 2}$, $t_{\alpha 1}$ and $t_{\alpha 2}$ associated with the coefficients for every possible break point is obtained. From every sequence two summary statistics are calculated: the mean and the lowest one. Following Fernandez these will be referred to as $Supt_{\delta 1}$, $Meant_{\delta 1}$, $Supt_{\delta 2}$, $Meant_{\delta 2}$, $Supt_{\alpha 1}$, $Meant_{\alpha 1}$, $Supt_{\alpha 2}$ and $Meant_{\alpha 2}$. As before, the Sup statistics have power for a unique break point, whereas the mean ones have power for gradual changes. According to Zivot and Andrews (1992), the break point is associated with the observation that corresponds to the Sup.

Fernández (1999) shows that the standard ADF behaves badly with changes in the order of integration, and his results are thus summarised:

- When the non-stationarity appears in the second part of the sample and there is no stochastic trend, the statistics that show most power are $Supt_{\delta 1}$ and $Meant_{\delta 1}$, followed by $Supt_{\alpha 1}$ and $Meant_{\alpha 1}$.
- When there is a stochastic trend and the first part of the sample is I(0), the $Supt_{\delta 1}$ and $Supt_{\alpha 1}$ become significant.
- If the second part is I(0) and we cannot reject a stochastic trend then the $Supt_{\delta 2}$, $Meant_{\delta 2}$, $Supt_{\alpha 2}$ and $Meant_{\alpha 2}$ are expected to be significant.

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