

# Wealth Inequality in the US: the Role of Heterogeneous Returns

Inês Xavier

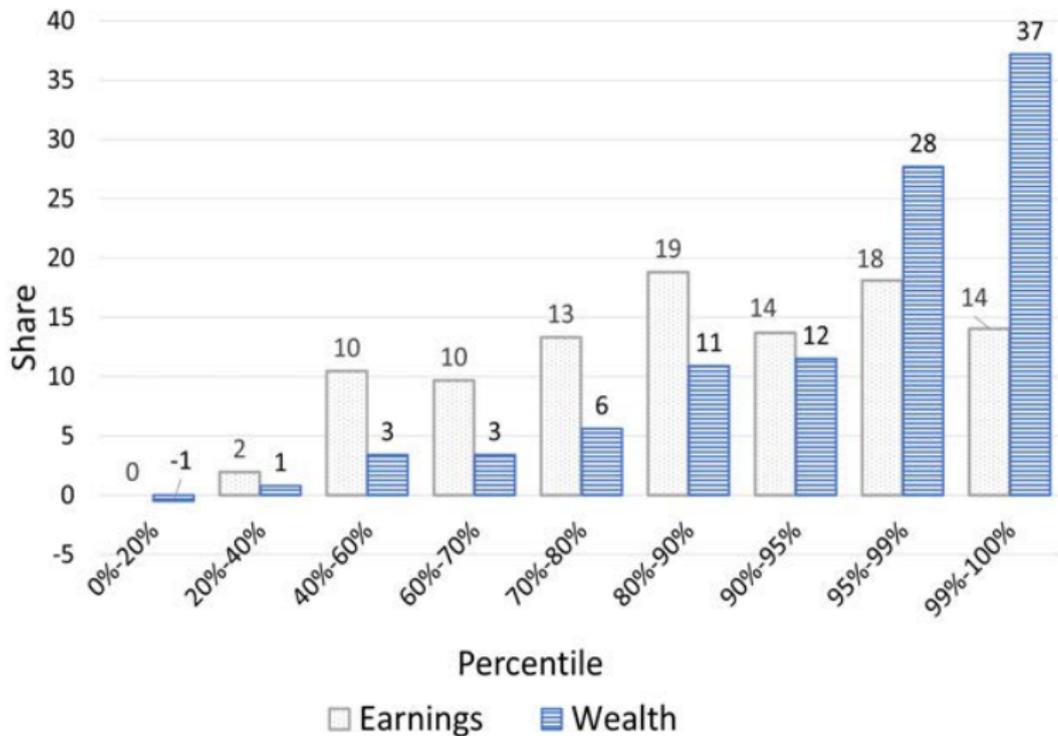
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## Motivation: U.S. Wealth is highly concentrated...more so than Earnings



Source: U.S. Survey of Consumer Finances (2019)

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- ▶ Provide evidence for U.S. (expand on evidence for Scandinavian economies)
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## 2. Implications for wealth inequality through PE model of earnings + return heterogeneity

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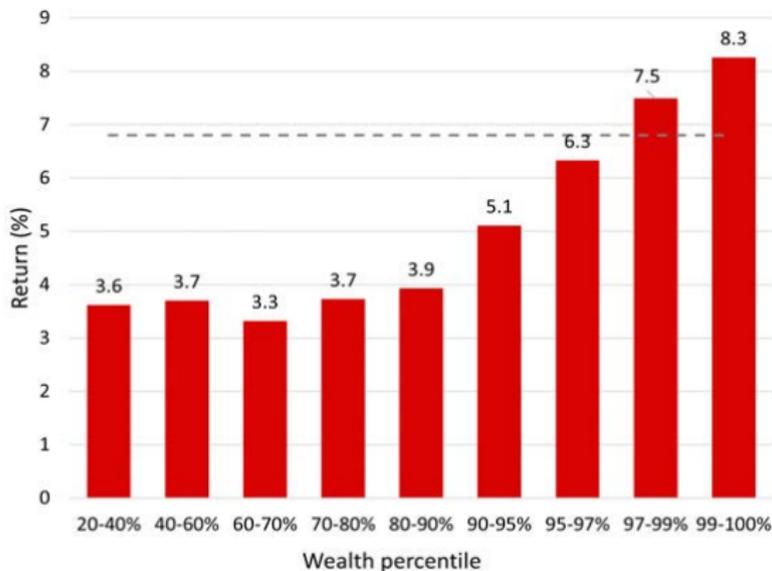
- ▶ Model with earnings & return heterogeneity + calibrate returns to match empirical evidence for U.S.
- ▶ Benhabib, Bisin and Luo (2019), Hubmer, Krussel, Smith (2020), Gabaix, Lasry, Lions, Moll (2016), Achdou, Han, Lasry, Lions, Moll (2020)

## **Main findings (I):** Returns to wealth in the data

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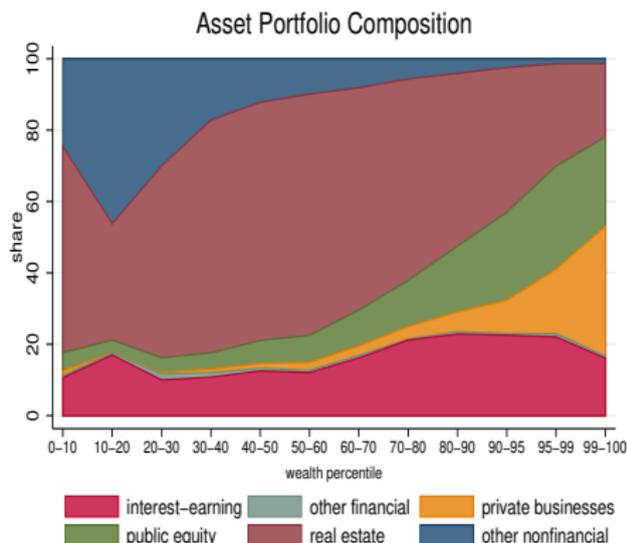
- ▶ Average return gap of **4.7 percentage points** between 20th and 99th percentiles

# Main findings (I): Two important sources of return differentials

## 1. Heterogeneous portfolios

Aggregate yearly return, 1990-2019

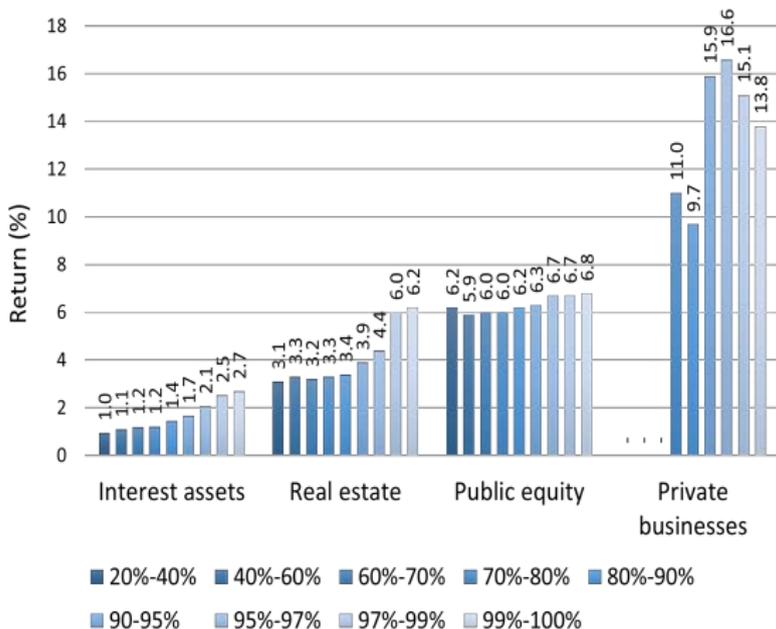
Wealth component	Return
Interest-earning assets	2.1%
Public equity	<b>6.7%</b>
Private businesses	<b>13.4%</b>
Real estate	<b>5.3%</b>
Debt	2.7%
Other financial assets	0.4%
Other nonfinancial assets	1.9%
<b>Aggregate portfolio</b>	<b>6.8%</b>



Rich own + equity → higher returns than real estate

# Main findings (I): Two important sources of return differentials

## 2. Heterogeneous returns within asset classes



Private businesses and Real estate

**Main findings (II):** A **model** to study importance of return heterogeneity for wealth inequality

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**Individuals.** Continuum of individuals indexed by  $i$  choose the path of consumption that maximizes

$$\mathbb{E}_0 \int_0^{\infty} e^{-\rho t} u(c_{it}) dt \quad (1)$$

- ▶ Preferences display constant relative risk aversion (CRRA), i.e.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \text{ with } \gamma > 0.$$

- ▶ Individuals accumulate wealth  $a_{it}$  over time according to

$$\dot{a}_{it} = y_{it} + r_{it} a_{it} - c_{it} \quad (2)$$

- ▶ individuals face a borrowing limit

$$a_{it} \geq \underline{a} \quad (3)$$

with  $-\infty < \underline{a} < 0$ .

**Labor income**  $y_t$  evolves stochastically over time according to the stationary diffusion process

- ▶ Log-earnings,  $z_t \equiv \log(y_t)$ , follow Ornstein-Uhlenbeck (O-U) process:

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**Returns**  $r_t$  evolve stochastically over time according to the stationary diffusion process (O-U)

$$dr_t = \theta_r(\bar{r}_j - r_t)dt + \sigma_{r,j} dZ_t \quad (5)$$

- ▶ Two sources of return differences: (1) risk,  $Z_t$ ; and (2) return **types**
- ▶ Baseline: three return *types*  $j$

Stationary Equilibrium is given by

- ▶ **Policy functions**  $\{c_i(a, y, r), s_i(a, y, r)\}$ : solve individual optimization problem given exogenous processes for  $y$  and  $r$
- ▶ **Stationary distribution** over wealth, labor income and returns  $g_i(a, y, r)$ : consistent with individual choices and the exogenous processes for  $y$  and  $r$

## 1. **Earnings:** from literature

$$dz_t = \theta_z(\bar{z} - z_t)dt + \sigma_z dW_t \quad (6)$$

- Autocorrelation of log-earnings equal to 0.9:  $\theta_z = 0.11$
- Standard deviation of log-earnings:  $\sigma_z = 0.2$

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## 2. Returns: target return moments from SCF

$$dr_t = \theta_r(\bar{r}_j - r_t)dt + \sigma_{r,j} dZ_t \quad (7)$$

▶ Parameters:  $\theta_r, \bar{r}_j, \sigma_{r,j}, \delta_j, j=1,2,3$

▶ Target **average returns by wealth**:

20%-40%, 40%-60%, 60%-70%, 70%-80%, 80%-90%, 90%-95%,  
95%-97%, 97%-99%, top 1%

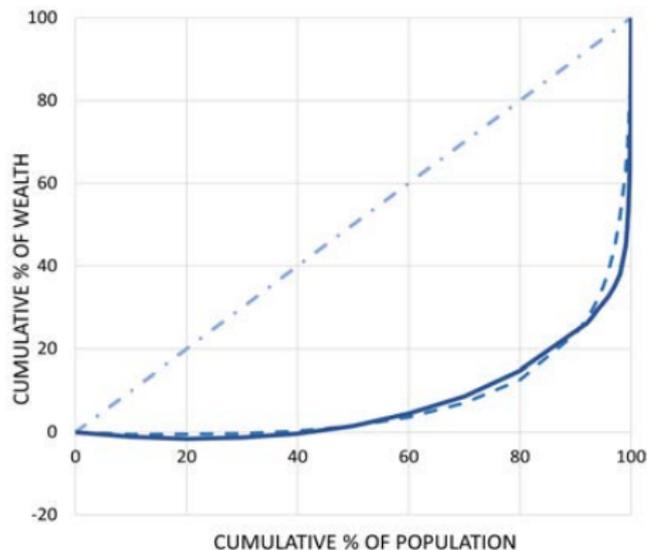
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### Lorenz curve



— Equality

- - Data

— Heterogeneous Earnings and Heterogeneous Returns

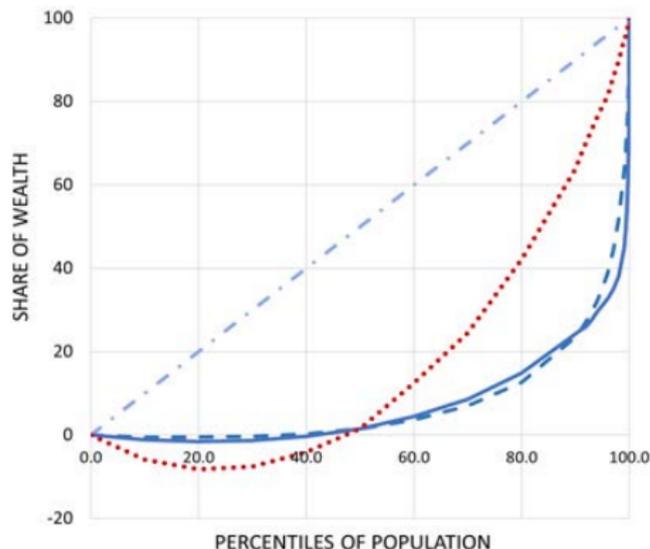
	Model	Data
Bottom 50%	1.5%	1.5%
Middle 40%	22.8%	22.1%
Top 10%	75.7%	76.4%
Top 5%	68.9%	64.9%
Top 1%	55.5%	37.2%

Table: Wealth shares: model and data (2019)

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### Lorenz curves



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	Homogeneous returns	Baseline
Bottom 50%	1.5%	1.5%
Middle 40%	62.3%	22.8%
Top 10%	36.2%	75.7%
Top 5%	21.1%	68.9%
Top 1%	5.2%	55.5%

Table: Wealth shares: Homogeneous Returns, Baseline and Data

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- ▶ Going forward: deep drivers of return differences (skills, portfolios, technology, frictions,...)

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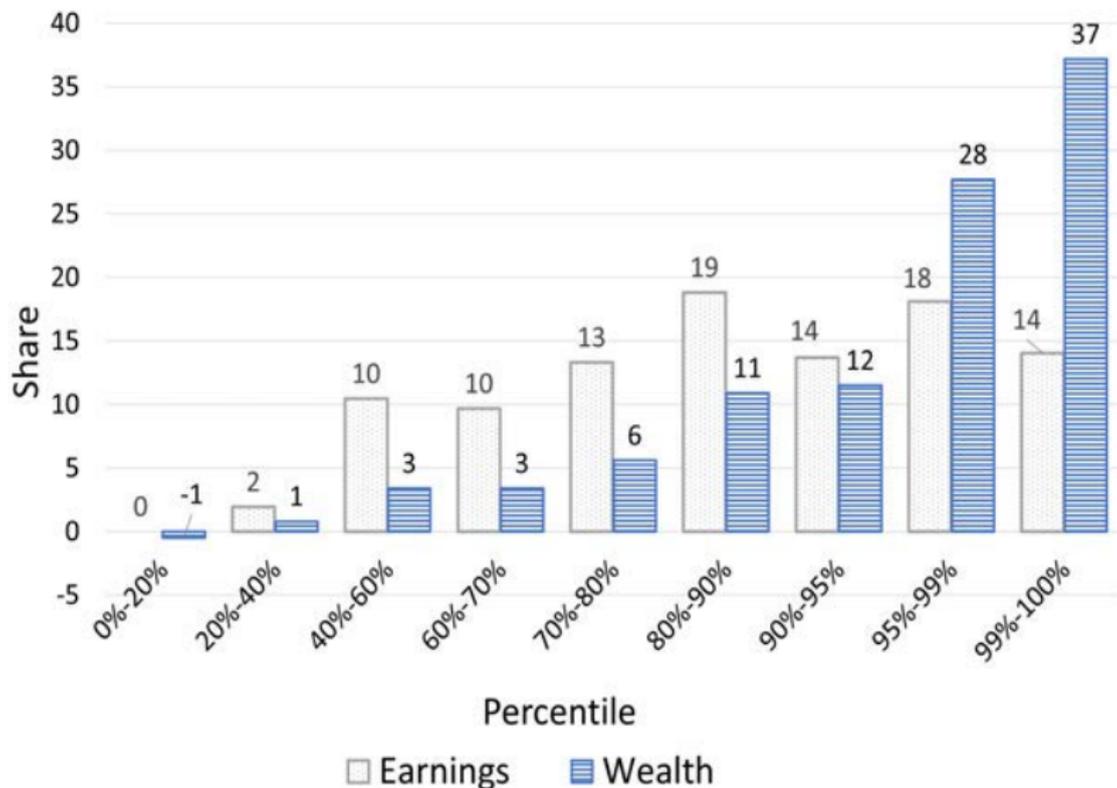
### 2. Important implications for distribution of wealth

- ▶ Return differences as in the data can rationalize observed **large top wealth shares**

- ▶ Returns to wealth in SCF data: [Appendix 1: data](#)
- ▶ Model: [Appendix 1: model](#)
- ▶ Parameterization: [Appendix 1: parameterization](#)

# Appendix

## Motivation: U.S. Wealth is highly concentrated...more so than Earnings



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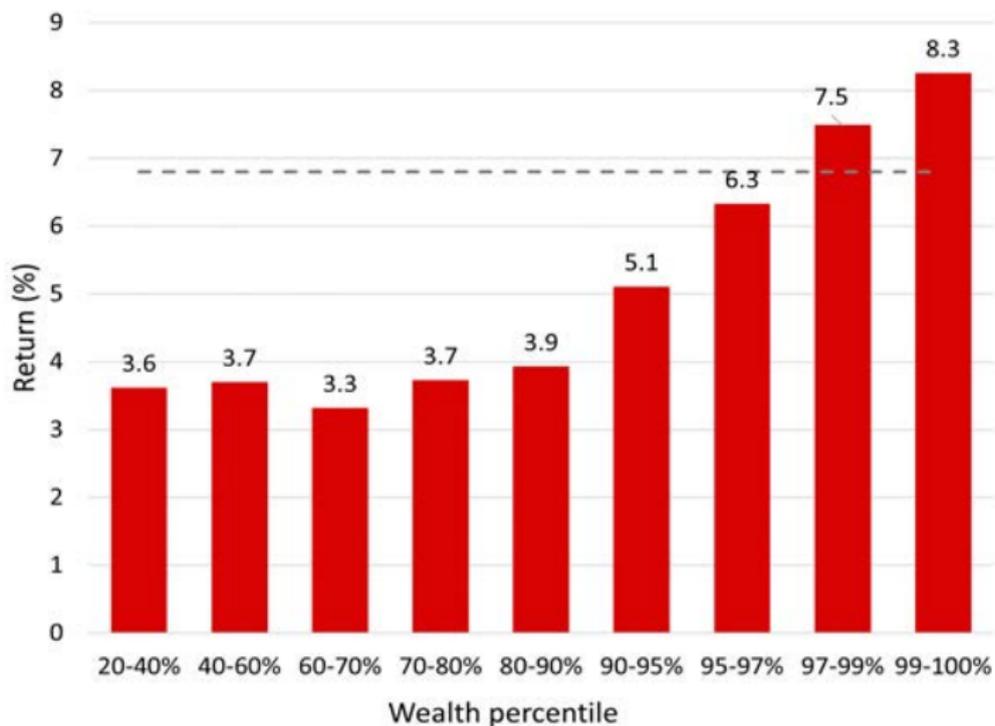
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- ▶ Build on theoretical mechanisms proposed in literature + calibrate returns guided by empirical evidence for U.S.
- ▶ Benhabib, Bisin and Luo (2019), Hubmer, Krussel, Smith (2020), Gabaix, Lasry, Lions, Moll (2016), Achdou, Han, Lasry, Lions, Moll (2020)

**Finding 1.** Returns to wealth are heterogeneous and increase with net worth (US)

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Return on wealth by percentile of wealth



## Finding 2.

- ▶ **Return heterogeneity** + **Earnings inequality**, calibrated to U.S. data, can rationalize degree of wealth concentration in data

	Data (2019)	Homogeneous returns	Heterogeneous returns
Bottom 50%	1.5	1.5	1.5
Middle 40%	22.1	62.3	22.8
<b>Top 10%</b>	<b>76.4</b>	<b>36.2</b>	<b>75.7</b>

*Wealth shares: model and data*

- ▶ Simple model with 2 sources of heterogeneity can replicate high degree of wealth concentration
- ▶ Return differences are strong force for wealth concentration

- ▶ **Survey of Consumer Finances (SCF)**, 1989-2019: Every 3 years, cross-section of US households' assets, liabilities and income
- ▶ Random sample of US households + oversampling of wealthy ( $\approx 4000 - 6000$  households)
- ▶ At each survey-period, data on households' **income** and **wealth**
  - ▶ **Income**: Wages, dividends, profits, interest, ...
  - ▶ **Wealth**: bank deposits, stocks, bonds, ...

## Wealth components

Wealth component	Detail
Interest-earning assets	transaction accounts, certificates of deposit, government, corporate and foreign bonds, other financial securities, cash value of life insurance
Public equity	directly or indirectly held (e.g. mutual funds)
Private businesses	corporate and non-corporate
Real estate	primary homes and other real estate
Other financial assets	residual
Other nonfinancial assets	e.g. vehicles, artwork, precious metals
Debt	mortgage debt, consumer debt, other debt

(i) What is the return on wealth?

$$R_W = \sum_c \omega_c R_c \quad (8)$$

(ii) What is the return on each wealth component,  $R_c$ ?

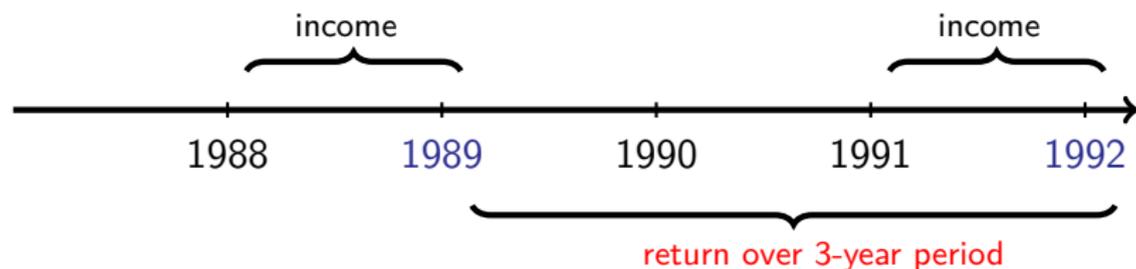
$$R_c = R_c^{\text{Yield}} + R_c^{\text{KG}} \quad (9)$$

Yield: SCF

Capital gains/losses: Aggregate price data (by asset class)

(i) **The Yield component:** average annualized returns over three-year intervals

For eg., over the period 1990-1992, the average annualized return  $R$  is computed as the geometric average of returns  $R_1$  and  $R_2$  as follows



$$R_1 = \left(1 + \frac{3NI_{1988}}{P_{1989}}\right)^{\frac{1}{3}}$$

$$R_2 = \left(1 + \frac{3NI_{1991}}{P_{1989}}\right)^{\frac{1}{3}}$$

$$R = (\sqrt{R_1 \cdot R_2} - 1) \cdot 100$$

$NI$  = total income flow generated by the asset

$P$  = market value of the asset stock.

Table: Yield component of returns, 1990–2019

<b>Wealth component</b>	<b>Net income</b>	<b>Yield</b>
Interest-earning assets	Interest income	2.1%
Public equity	Dividends	1.8%
Private businesses	Net profits	9.0%
Real estate	Rental income	4.2%
Debt	Loan interest payments	2.7%

Private businesses

## (ii) Capital gains and losses

- ▶ Use external data to impute capital gains/losses on different assets

**Table:** Capital gains and losses, 1990–2019

<b>Wealth component</b>	<b>Source</b>	<b>KG</b>
Public equity	Shiller (2015)	4.9%
Private businesses	US Financial Accounts	4.4%
Real estate	Shiller (2015)	1.1%
Other financial	SCF	0.4%
Other nonfinancial	SCF	1.9%

## The aggregate return on wealth and its components

Aggregate yearly return, 1990-2019

Wealth component	Yield	Capital gain	Return
Interest-earning assets	2.1%	—	2.1%
Public equity	1.8%	4.9%	<b>6.7%</b>
Private businesses	9.0%	4.4%	<b>13.4%</b>
Real estate	4.2%	1.1%	<b>5.3%</b>
Debt	2.7%	—	2.7%
Other financial assets	—	0.4%	0.4%
Other nonfinancial assets	—	1.9%	1.9%
<b>Aggregate portfolio</b>	<b>4.1%</b>	<b>2.7%</b>	<b>6.8%</b>

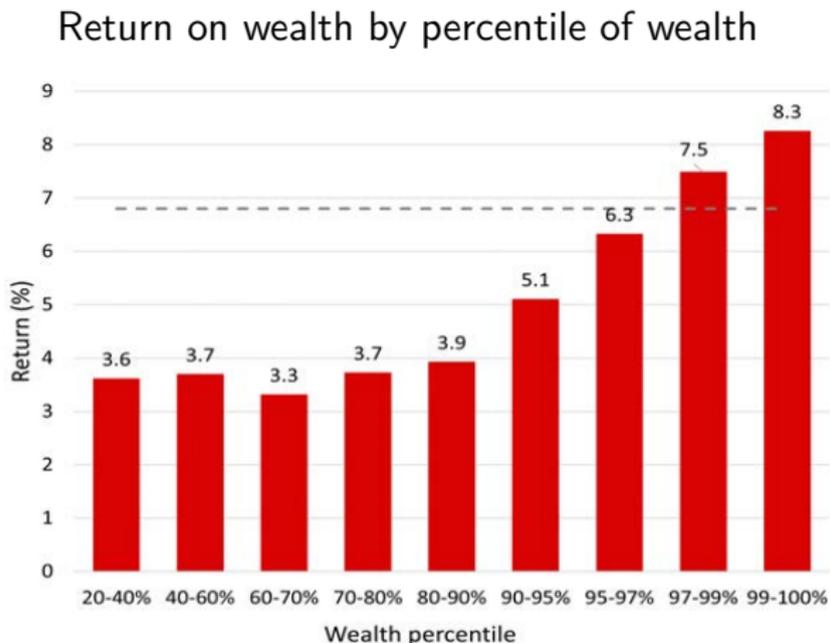
Next:

1. Return heterogeneity?

## Heterogeneous returns?

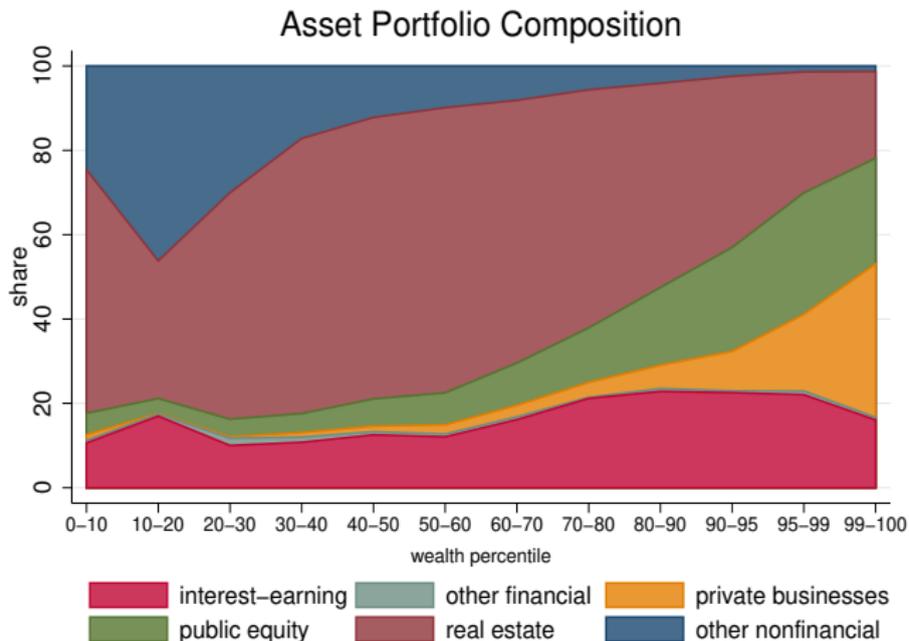
## Heterogeneous returns?

- ▶ Repeat calculations at different points of wealth distribution

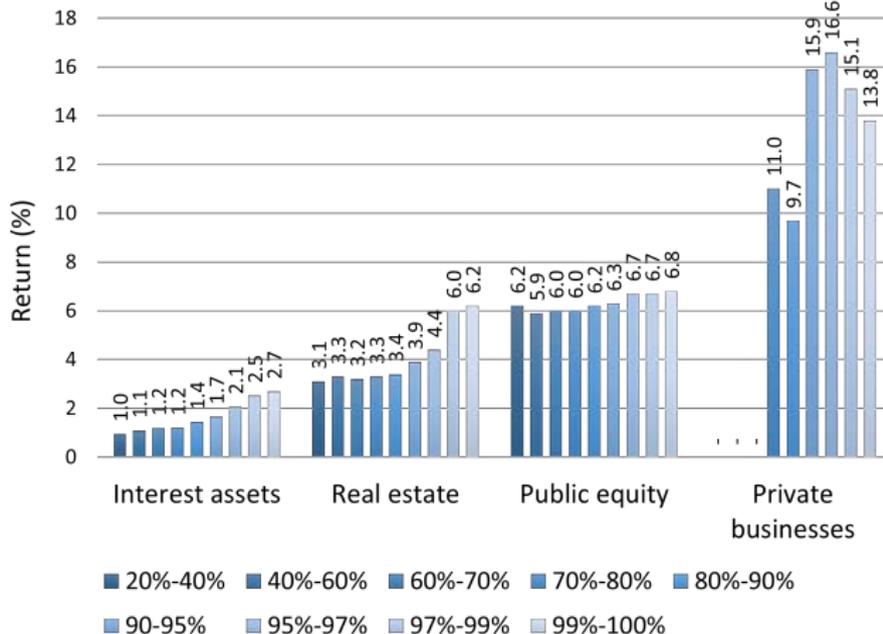


- ▶ Average return gap of **4.7 percentage points** of top relative to bottom group

## 1. Heterogeneous composition of wealth portfolio

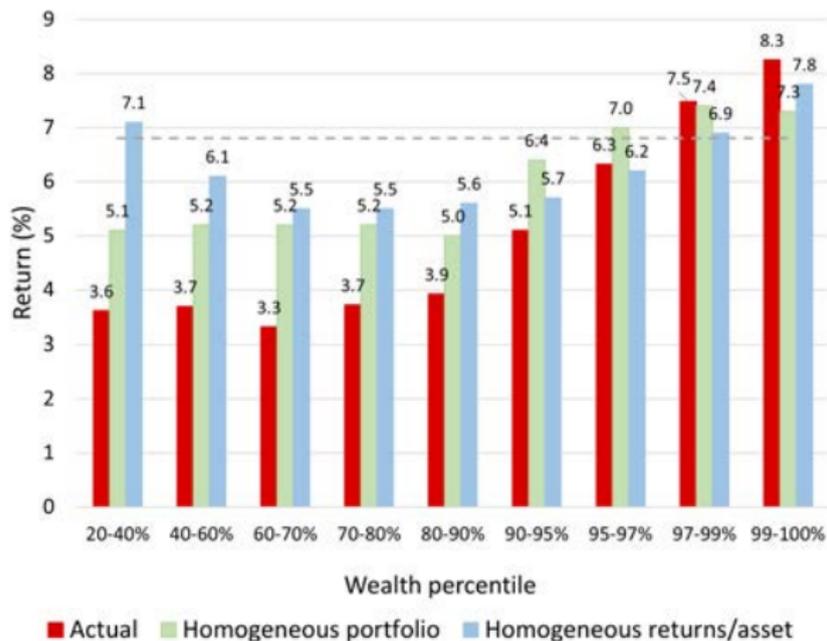


## 2. Heterogeneous returns within asset classes



# Heterogeneous portfolios vs. heterogeneous returns/asset

## ▶ Counterfactuals:



Back to [Chapter 1: further details](#)

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  - ▶ **Return differences within asset classes**
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  - ▶ Add **return heterogeneity** motivated by empirical evidence
    - ▶ Positive correlation between returns and wealth + estimated differences

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  - ▶ Add **return heterogeneity** motivated by empirical evidence
    - ▶ Positive correlation between returns and wealth + estimated differences
- ▶ My model: **return “types” + return risk**

**Individuals.** Continuum of individuals indexed by  $i$  choose the path of consumption that maximizes

$$\mathbb{E}_0 \int_0^{\infty} e^{-\rho t} u(c_{it}) dt \quad (10)$$

- ▶ Preferences display constant relative risk aversion (CRRA), i.e.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \text{ with } \gamma > 0.$$

- ▶ Individuals accumulate wealth  $a_{it}$  over time according to

$$\dot{a}_{it} = y_{it} + r_{it} a_{it} - c_{it} \quad (11)$$

- ▶ individuals face a borrowing limit

$$a_{it} \geq \underline{a} \quad (12)$$

with  $-\infty < \underline{a} < 0$ .

**Labor income**  $y_{it}$  evolves stochastically over time according to the stationary diffusion process

$$dy_{it} = \mu_y(y_{it})dt + \sigma_y(y_{it})dW_{it} \quad (13)$$

- ▶ Functions  $\mu_y$  and  $\sigma_y$  determine the mean and standard deviation of the growth rate of earnings
- ▶  $W_{it}$  is a standard Brownian motion

**Returns**  $r_{it}$  evolve stochastically over time according to the stationary diffusion process

$$dr_{it} = \mu_{r,i}(r_{it})dt + \sigma_{r,i}(r_{it})dZ_{it} \quad (14)$$

- ▶ Flexible formulation that allows drift and diffusion of return process to potentially differ across individuals (“type dependence”)
- ▶  $Z_{it}$  is a standard Brownian motion

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- ▶ **Policy functions**  $\{c_i(a, y, r), s_i(a, y, r)\}$  that solve individual optimization problem given exogenous processes for  $y$  and  $r$

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- ▶ **Stationary distribution** over wealth, labor income and returns  $g_i(a, y, r)$  that is consistent with individual choices and the exogenous processes for  $y$  and  $r$

Back to [Chapter 1: further details](#)

## 1. Externally calibrated parameters

- ▶ CRRA risk aversion parameter:  $\gamma = 2$
- ▶ Log-earnings,  $z_t \equiv \log(y_t)$ , follow Ornstein-Uhlenbeck (O-U) process

$$dz_t = \theta_z(\bar{z} - z_t)dt + \sigma_z dW_t \quad (15)$$

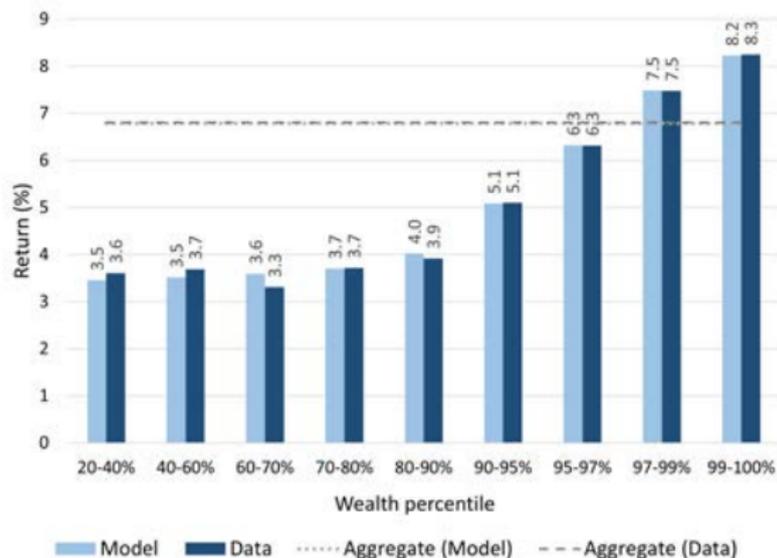
- Autocorrelation of log-earnings equal to 0.9:  $\theta_z = 0.11$
- Standard deviation of log-earnings:  $\sigma_z = 0.2$
- Normalize aggregate earnings to 1:  $\bar{z} = 0.78$

## 2. Fitted parameters

- ▶ Discount rate:  $\rho$
- ▶ Borrowing limit:  $\underline{a}$
- ▶ Return process:
  - ▶ Returns follow O-U process:  $dr_t = \theta_r(\bar{r}_j - r_t)dt + \sigma_{r,j}dZ_t$
  - ▶ Baseline: three return types
  - ▶  $\theta_r, \bar{r}_j, \sigma_{r,j}, \delta_j, j = 1, 2, 3$
- Targets:
  - **Aggregate rate of return:** 6.80%
  - **Wealth share bottom 50%:** 1.5%
  - **Average returns by wealth:** 20%-40%, 40%-60%, 60%-70%, 70%-80%, 80%-90%, 90%-95%, 95%-97%, 97%-99%, top 1%

Table: Targeted Moments

	Model	Data
<i>Aggregate return</i>	6.79%	6.80%
<i>Wealth bottom 50%</i>	1.5%	1.5%

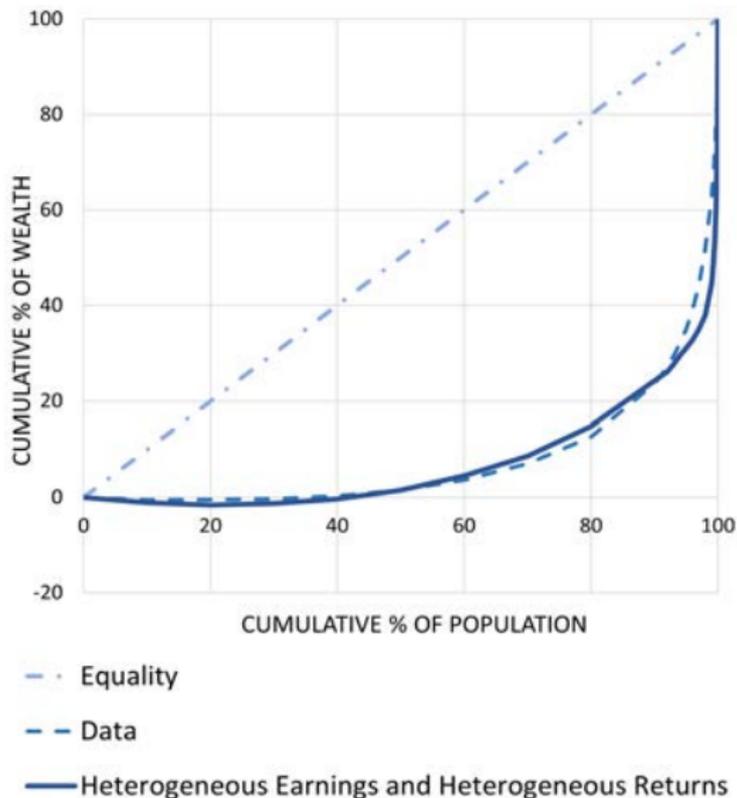


	<b>Type 1</b>	<b>Type 2</b>	<b>Type 3</b>
Mean, $\bar{r}_j$	0.033	0.058	0.082
SD, $\sigma_{r,j}$	0.056	0.202	0.057
$\theta_r$	3.08	3.08	3.08
Share, $\delta_j$	0.80	0.18	0.02

- ▶ Majority (80%) of households are “low” return type
- ▶ 18% of households are “mid” return type
- ▶ 2% of households are “high” return type

## Results: **Steady-State Wealth Inequality**

- ▶ Model-implied distribution close to empirical distribution of wealth



## Results: **Steady-State Wealth Inequality**

	Model	Data
Bottom 50%	1.5%	1.5%
Middle 40%	22.8%	22.1%
Top 10%	75.7%	76.4%
Top 5%	68.9%	64.9%
Top 1%	55.5%	37.2%

Table: Wealth shares: Model and Data (2019)

- ▶ Model replicates overall distribution of wealth.

## How important are heterogeneous returns for wealth inequality?

- ▶ Counterfactual: Homogeneous Returns

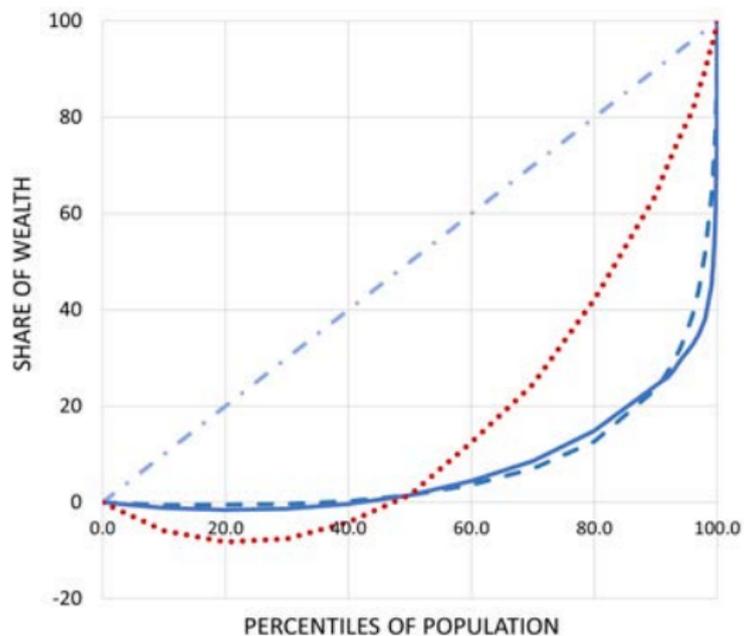
## How important are heterogeneous returns for wealth inequality?

► Counterfactual: Homogeneous Returns

	Homogeneous returns	Baseline	Data
Bottom 50%	1.5%	1.5%	1.5%
Middle 40%	62.3%	22.8%	22.1%
Top 10%	36.2%	75.7%	76.4%
Top 5%	21.1%	68.9%	64.9%
Top 1%	5.2%	55.5%	37.2%

Table: Wealth shares: Homogeneous Returns, Baseline and Data

## Wealth Distribution graphically: Lorenz Curves



- · Equality
- - Data
- Heterogeneous Earnings and Heterogeneous Returns
- ··· Heterogeneous Earnings and Homogeneous Returns

Return heterogeneity is key to understand wealth inequality in the United States.

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1. **Empirically, return differences are large**

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- ▶ Average return increases with wealth (up to 4.7 p.p. difference)
- ▶ Portfolio composition + return differences within asset classes
- ▶ Further things to learn: deep drivers of return differences (skills, portfolios, technology, frictions,...)

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### 2. Large implications for distribution of wealth

- ▶ Return differences as in the data can rationalize observed **large top wealth shares**

# Appendix 1.1.

1. **Accounting for labor income:** some entrepreneurs do not report own salary
  - ▶ Impute salary to active entrepreneurs
  - ▶ **adjustment:** multiply annual hours worked by estimated wage rate for **similar individuals** who worked in paid employment
    - “Similar” individuals: Age, Education (HS, College), Gender
2. **Corporate tax adjustment:** convert pre-tax profits into after-tax.

$$\text{tax rate}^1 = \begin{cases} 0.3 & , \text{C corporations} \\ 0 & , \text{S corporations \& partnerships} \end{cases}$$

---

<sup>1</sup>measure of average effective corporate tax rate in United States.

3. **Retained earnings:** subtract fraction of earnings retained in the firm

$$\text{retention rate}^2 = \begin{cases} 0.4 & , \text{C corporations} \\ 0.2 & , \text{S corporations \& partnerships} \end{cases}$$

Back to [Returns](#).

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<sup>2</sup>estimate of ratio of retained earnings to after tax profits in NIPA data. Use values from *VJ (2002)* and *Kartashova (2014)*.

	<b>P20</b>	<b>P50</b>	<b>P99</b>	<b>Diff. P99-P20</b>
SCF (1989-2019)	3.6%	3.7%	8.3%	4.7%
Sweden (2000-2007) <sup>3</sup>	3.8%	4.7%	8.1%-9.8%	4.3%-6%
Norway (2005-2015) <sup>4</sup>	-1.5%	3.8%	5.7%	7.2%

Back to [Returns](#).

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<sup>3</sup>Bach et al. (2020)

<sup>4</sup>Fagereng et al. (2020)

## Comparison to Bach et al. (2020) and Fagereng et al. (2020)

- ▶ No immediate counterpart of different types
- ▶ Idiosyncratic volatility Bach et al. (2020):

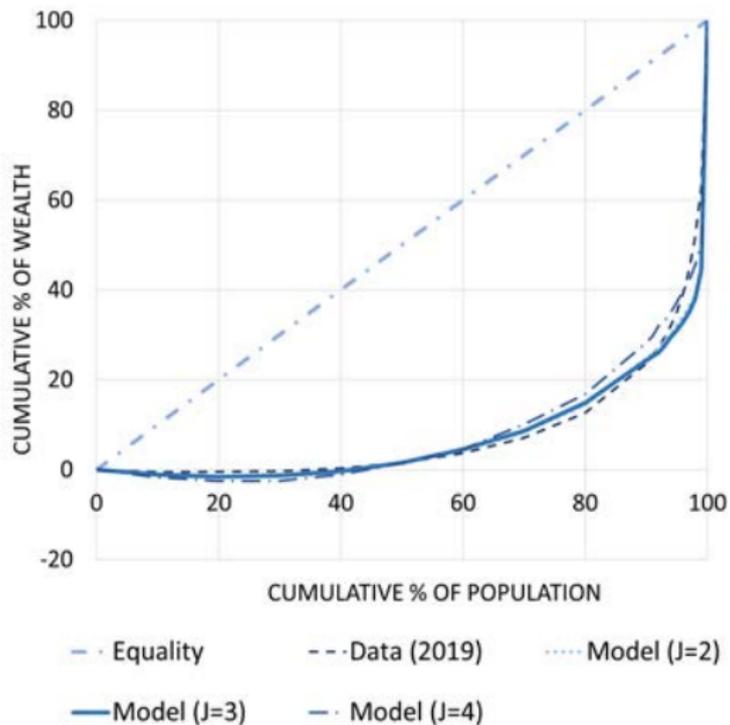
	<b>P20</b>	<b>P90</b>	<b>P99</b>
Model	6.5%	14.5%	5.8%
Bach et al. (2020)	8%	6%	8.7%-27.5%

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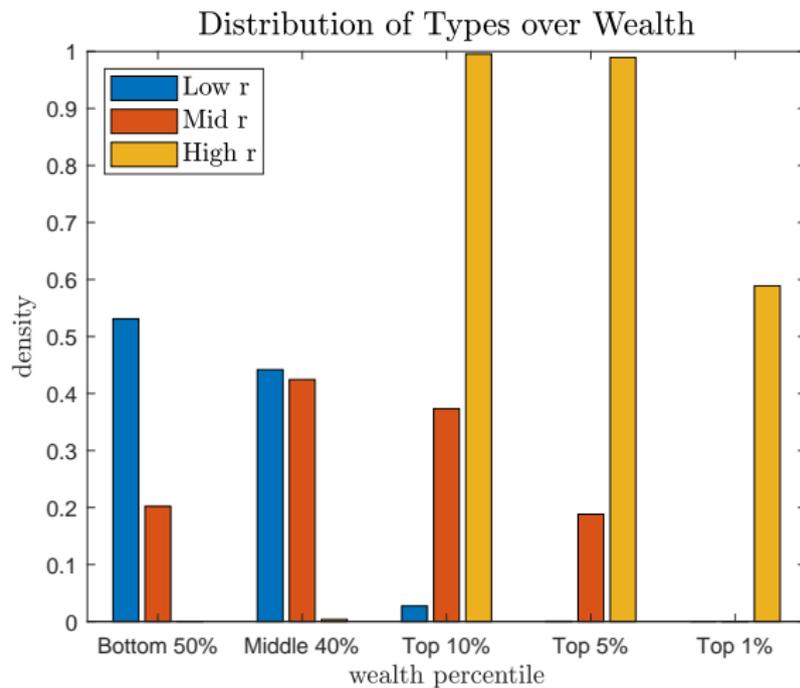
Wealth percentile	Two types	Three types	Four types
20%	20.4%	6.9%	7.4%
90%	21.0%	14.5%	8.3%
99%	23.9%	5.8%	9.7%

Back to [Parameters](#).

## Alternative specifications: Two, Three and Four return types

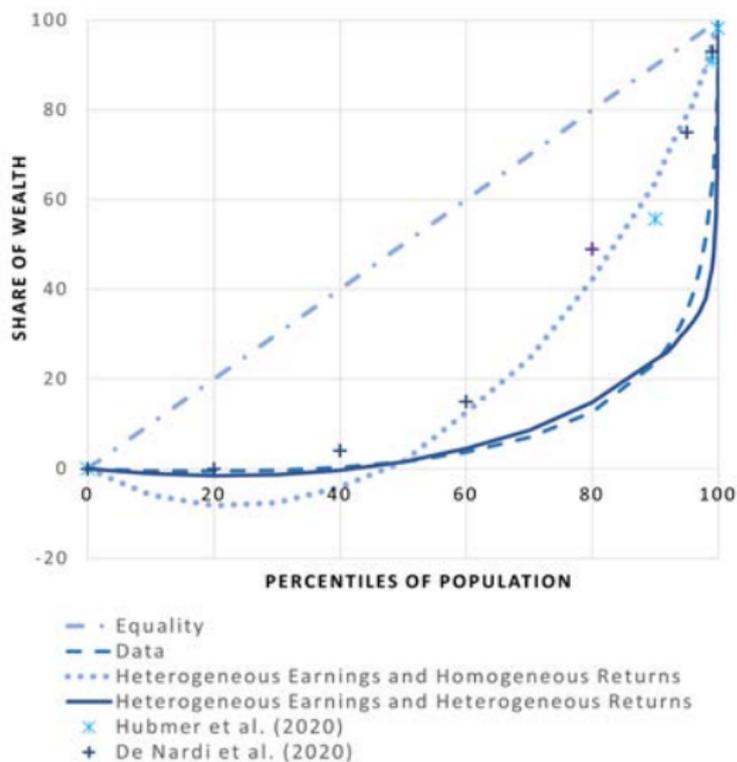


## “High return types” more likely to become rich



Back to [Lorenz Curve](#) .

## Richer earnings processes



Back to [Results](#).