Comments on ”Stochastic Cycles in VAR Processes” by M. Franchi e P. Paruolo.

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The five golden rules of the perfect discussant

1. Translate the paper from math to English: Hard time!
2. Make a summary of the paper: OK.
3. Praise the authors: Easy and pleasant task in this case.
4. Complain about some technical details to show that you've actually read the paper: No way to complain here, the math is extremely accurate.
5. Take the opportunity to promote your own work: I won't miss it!

Let's go MAD!

- Assume that the *n*-vector series X_t is generated by a stable VAR(p)

$$\Pi(L)X_t = \varepsilon_t, \tag{1}$$

where $\Pi(L)$ is a p-order polynomial matrix such that $\Pi_0 = I_n$ and the roots of det$\Pi(L)$ lie outside the unit circle, ε_t is a m.d.s. with $\mathsf{E}(\varepsilon_t) = 0$, and $\mathsf{E}(\varepsilon_t \varepsilon_t') = \Omega$ that is p.d.

- Obtain the Wold representation

$$X_t = C(L)\varepsilon_t$$

where $C(L) = \text{adj}\Pi(L)/\text{det}\Pi(L)$.

- Concentrate on the roots of det$\Pi(L)$. Why? Since the spectral density matrix of X_t is proportional to

$$C(z)\Omega C(z^{-1})', \quad z = \exp(-i\omega), \quad \omega \in [0, \pi]$$

it is clear that these roots are responsible for the dominant cyclical swings of series X_t.

- Cancel out roots that are common to adj$\Pi(L)$ and det$\Pi(L)$ and obtain

$$C(L) = G(L)/g(L)$$

Notice that the existence of "common serial correlation" among elements of X_t implies the presence of such common roots (see *i.a.* C. Hecq & Palm, 2009, JoE).

- Expand $C(L)$ on the q roots of $g(L)$. Since each of these roots can be expressed as $\rho\,\text{exp}(-i\omega)$, where $\rho > 0$, each term of this expansion is associated to a stochastic cycle with frequency ω.

- Substitute the above expansion into the Wold representation, separate "interesting" cycles [i.e. with $\omega \in (0, \pi)$] from "trivial" cycles [$\omega = 0, \pi$] and finally obtain the MAD: Too mad to be written down!

- Importantly, since we're expanding $C(L)$ on (a subset of) the roots of det$\Pi(L)$, $\Pi(\rho z)$ is of reduced-rank $\Rightarrow$ $C(\rho z)$ is of reduced-rank too (similar as in cointegration) $\Rightarrow$ series X_t have "common cycles" at frequency ω.

- Hence, the MAD decomposes series X_t into q common cycles at k ($k \leq q$) different frequencies plus an uninteresting short-memory remainder.

Analogies & differences between MAD and (seasonal) cointegration

- Similar methods and representations as in seasonal cointegration analysis. Suppose X_t is a quarterly time series that is CI(1,1) at frequencies 0, π, and $\pi/2$, then the common trend-seasonals representation is

$$X_t = \underbrace{\gamma_0 \sum_{j=0}^{\infty} \delta_0' \varepsilon_{t-j}}_{\text{common trends}} + \underbrace{C^*(L)\varepsilon_t}_{\text{cycles}} + \underbrace{\gamma_\pi \sum_{j=0}^{\infty} \delta_\pi' \cos(j\pi)\varepsilon_{t-j}}_{\text{common bi-annuals}} +$$
$$\underbrace{(\gamma_{\pi/2} + \bar{\gamma}_{\pi/2}L) \sum_{j=0}^{\infty} [\delta_{\pi/2}' \cos(j\pi/2) + \bar{\delta}_{\pi/2}' \sin(j\pi/2)]\varepsilon_{t-j}}_{\text{common annuals}},$$

see, *i.a.*, Johansen & Schaumburg (1998, JoE) and C. (1999, JAE).

- Common trends & common seasonal cycles are I(1) components whereas the MAD cycles are stationary.

- The presence of (seasonal) cointegration requires that some reduced-rank restrictions apply to the VAR coefficient matrices whereas no additional restrictions are needed to have common MAD cycles.

- Economic theory has something to say about (common) permanent-transitory components in macro variables. Is there any good story to tell about common cycles at different frequencies?

Suggestions for future research and conclusions

- Generalized impulse response analysis could be applied to overcome the issue of non-orthogonality of the shocks at different frequencies.

- Why not extending the MAD to cointegrated VARs?

- It would be extremely interesting and of great practical importance to develop statistical tests for the nullity of the loadings of some cyclical components and to impose the associated restrictions in estimation.

★ and even more importantly...

Have fun, go MAD!