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# **Evaluating Density Forecasts: Forecast Combinations, Model Mixtures, Calibration and Sharpness**

*by J. Mitchell & K. F. Wallis*

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## The Key Ingredients of the Paper are:

- Density forecasts are becoming popular.

MW says: “*Forecasts for an uncertain future are increasingly presented probabilistically*”.

- This literature rests on the contribution of Dawid (1984) that the probability integral transform values (PITs) of the outcome of the forecasts under correct specification should be uniformly distributed and IID (so called, the complete calibration).
- Diebold, Gunther and Tay (1998) for a time series implementation.

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- The paper is essentially a reply to JRSS paper by Gneiting, Balabdaoui and Raftery (2007, GBR).
  - GBR shows some numerical examples where the PIT histogram “looks” uniform despite the model misspecification.
  - GBR finds the results “disconcerting” and proposes a additional criteria for evaluating density f/c based on density sharpness (ie, the wideness of confidence intervals).

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## The Results:

- MW argument is that the GBR examples are less relevant in a time series framework.
- The Dawid's complete calibration combines the uniformity and IID of the PITs.
- The time series dimension allows a lot of room for testing the IID hypothesis together with the uniformity of the PITs.
- They provide “more encouraging” numerical examples.

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## Comment 1 - GBR

- Are the examples by GBR very compelling? May be no.
- The “climatologist” (i.e., the unconditional density) is the most extreme example.
- All examples are close to fulfil the conditions of the Blackwell and Dubins’ theorem (ie, the distance between alternative distributions is asymptotically negligible, so their forecasts should be almost indistinguishable).
- This seems an instance of the Jeffrey’s law (i.e., observationally indistinguishable statistical approach must be in essential agreement on their assessment about observables).

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## Comment 2 – Uniform and IID

- The fact that the uniformity of the PITs is not a sufficient condition, is not a novelty (see Dawid, 1984). However, this is completely neglected in GBR.
- Tests of IID and uniformity: Hong (2001), Bai (2003), Corradi & Swanson (2005, 2006) and other.
- CS (2006) is an interesting result because CS allow for dynamic misspecification under the null hypothesis (multiple models comparison). CS obtains asymptotically valid critical values even when the conditioning information set does not contain all of the relevant past history.

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## Comment 3

- While sympathetic with MW's points, there would be some benefits to the argument by reassessing the results on term of power of the complete procedure, particularly versus local alternatives.
- The issue of the model estimation error needs to be properly take into account as it gives rise to the tricky issues of nuisance parameters in KS type tests.
- Results of simulation exercises show that power might be an issue for these tests, particularly with macro sample size.

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## Comment 4 – Is density f/c really used in macro?

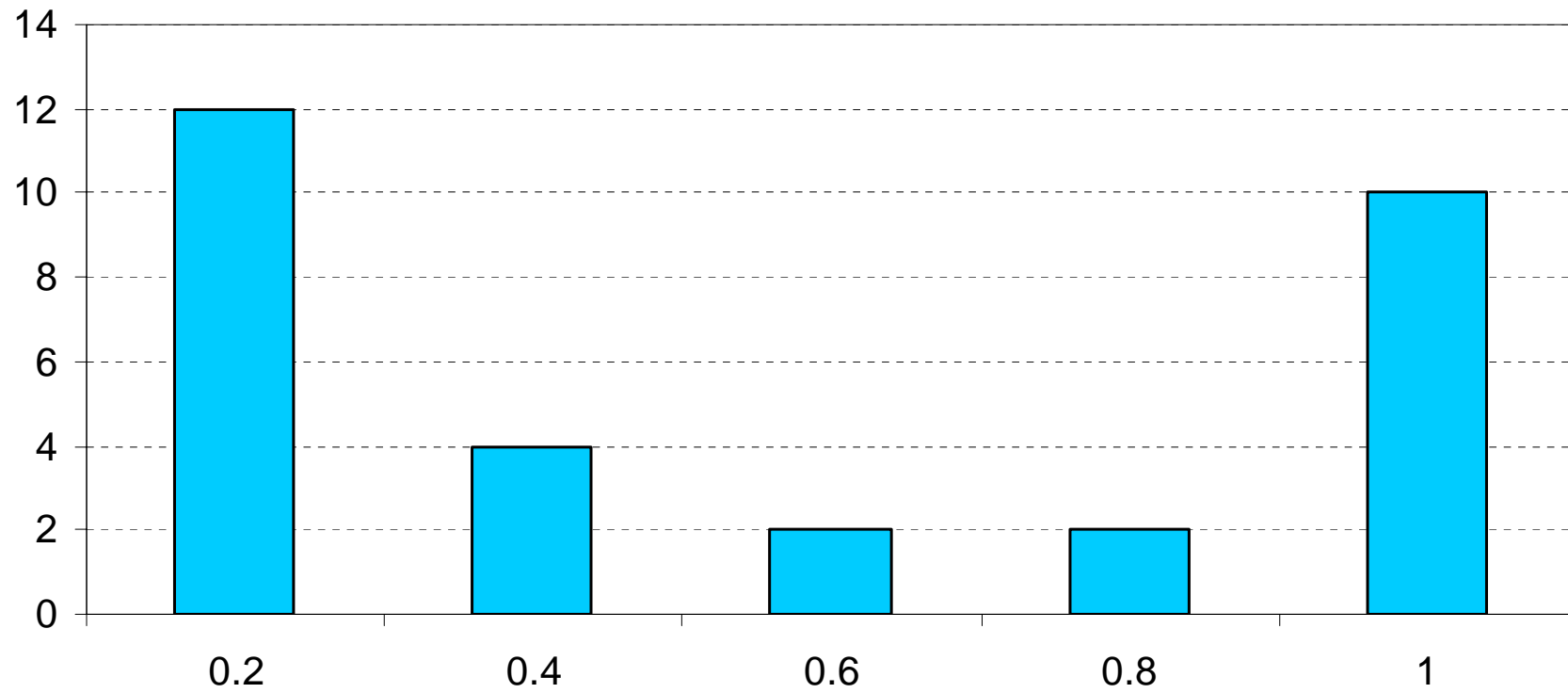
- MW says: “*Forecasts for an uncertain future are increasingly presented probabilistically*”.
- Many central banks are showing nice fan charts which seems to enhance their communication and their presentation of the f/c (BoE, Riksbank, Norges Bank and others).
- However, there is a disconnection between the production of the point f/c and the super-imposition of the density.

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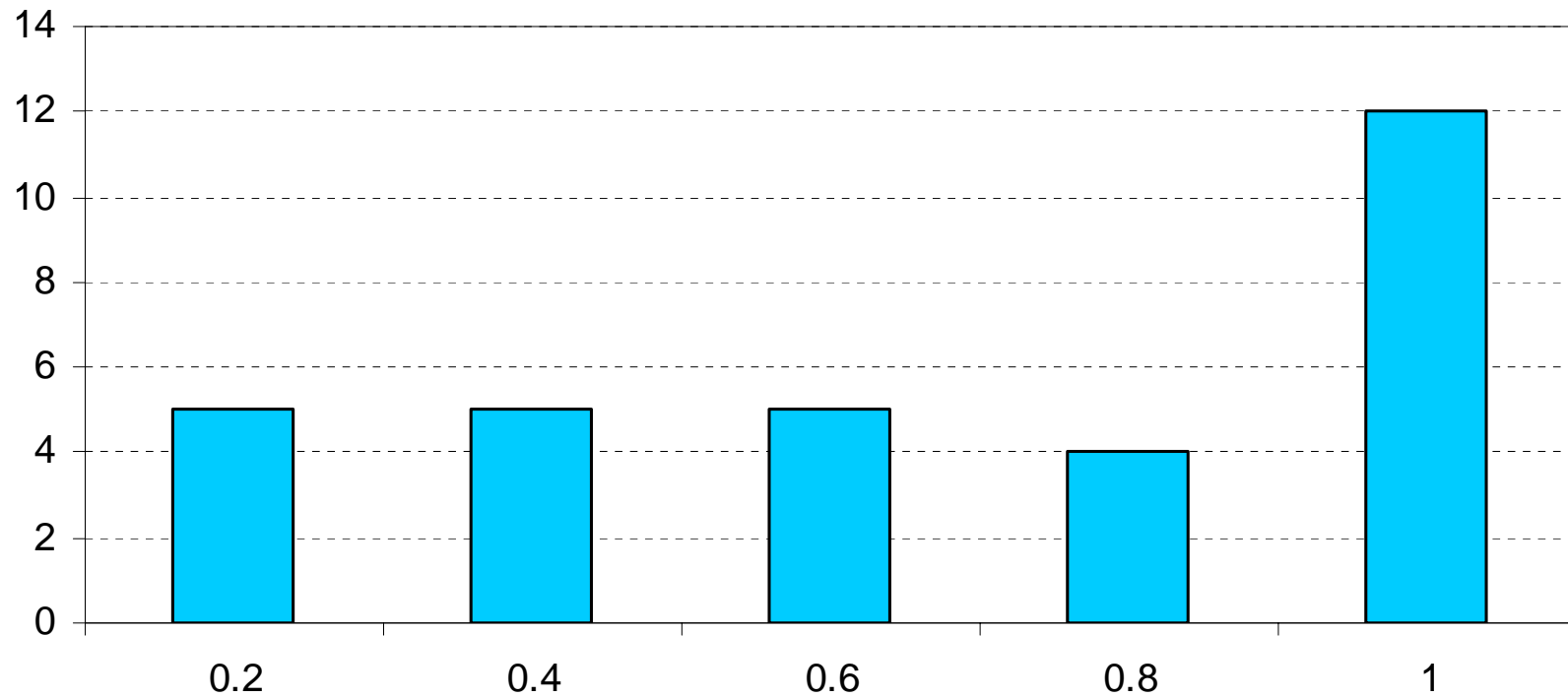
## Comment 4 – Is density f/c really used in macro?

- It is unclear what the super-imposed density represents: model uncertainty, model specification error, parameter uncertainty, or what!!!
- In some cases, the f/c density is only a tool to deliver a policy message.
- Are those densities satisfying complete calibration?  
May be no!!!

## PITs - IFO f/c Market Analysts (2007-2009)



## PITs- EMU HICP f/c Market Analysts (2007-2009)



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# THANKS