BANCA D'ITALIA
EUROSISTEMA

Supplements to the Statistical Bulletin
Methodological Notes

The Measurement of Sampling Error in Bank Interest Rate Statistics

New series
Volume XVII Number 36 - 14 June 2007

SUPPLEMENTS TO THE STATISTICAL BULLETIN

Monetary Financial Institutions: Banks and Money Market Funds (monthly)

The Financial Market (monthly)

The Public Finances (monthly)

Balance of Payments (monthly)

Monetary and Credit Aggregates of the Euro Area: the Italian Components (monthly)

Financial Accounts (quarterly)

Payment System (half yearly)

Public Finance Statistics in the European Union (annual)

Local Government Debt (annual)

Sample Surveys (irregular)

Methodological Notes (irregular)

All the supplements are available on the Bank of Italy's site (www.bancaditalia.it).

Requests for clarifications concerning data contained in this publication can be sent by e-mail to statistiche@bancaditalia.it

CONTENTS

THE MEASUREMENT OF SAMPLING ERROR IN BANK INTEREST RATE STATISTICS

Massimiliano Stacchini[1]

1. Introduction and main conclusions

Regulation ECB/2001/18 allows statistics on euro-area interest rates to be collected according to a sampling approach. In this case, as of the reference month of December 2006 the Regulation requires the minimum national sample size to be in line with the following paragraph:

> The minimum national sample size shall be such that the maximum sampling error for interest rates on new business on average over all instrument categories does not exceed 10 basis points at a confidence level of 90 % (par. 10, Annex I).

If the requirement was not satisfied, the reporting population would have to be enlarged. With that cut-off date in mind, empirical analyses were carried out to see if the sample currently meets the requirements.

This note summarizes the methodology for the measurement of the sampling error of $\hat{\theta}_{mir}$, i.e. the estimator that the Bank of Italy has adopted for the harmonised interest rate statistics. The sampling error is calculated over the period January 2005 – October 2006 (the last reference date available when the exercise was performed).

Results confirm that the sample currently used meets the criteria being introduced as provided for by the ECB. The conclusion holds for each of the three methodologies chosen for the measurement.

2. Sampling error in bank interest rate statistics

For all the instrument categories considered by Regulation /2001/18, the following quantity is calculated:

$$\hat{\theta}_{\text{mir}} = \frac{\sum_{i}^{n} r_i\, w_i}{\sum_{i}^{n} w_i} = \frac{\sum_{h}^{L} \sum_{i}^{n_h} r_{hi} w_{hi}}{\sum_{h}^{L} \sum_{i}^{n_h} w_{hi}}$$

where $\hat{\theta}_{mir}$ is an estimator of θ, the weighted average of the interest rates reported by the potential reporting population. In the observed period it included almost 800 credit institutions; r_i is the

1 Bank of Italy, Research Department. I am grateful to Rosaria Buzzi for her precious help with data on bank interest rates. I wish to thank Paola Battipaglia, Stefano Iezzi and Matteo Piazza for useful comments on a first draft of the paper.

interest rate of the i-*th* credit institution; w_i is the business volume of the i-*th* credit institution; r_{hi} is the interest rate reported by the i-*th* credit institution in stratum h; w_{hi} is the business volume reported by the i-*th* credit institution in stratum h. Finally, *n* is the sample size and n_h the sample size in stratum *h*.

The sampling error (D) is given by:

$$D = Z_{\alpha/2}\sqrt{V(\hat{\theta}_{mir})} \approx Z_{\alpha/2}\sqrt{v(\hat{\theta}_{mir})} \quad (1)$$

where $Z_{\alpha/2}$ is the (1-α/2)*100*th* percentile with the appropriate distribution, $V(\hat{\theta}_{\mathrm{mir}})$ is the variance of $\hat{\theta}_{\mathrm{mir}}$, and $v(\hat{\theta}_{\mathrm{mir}})$ an estimator of that variance. In $\hat{\theta}_{\mathrm{mir}}$ the interest rate reported by the i-*th* bank is weighted by the business volume reported by the bank.

In general, by using weights, the entire potential reporting population is properly represented in the sample so that the weighted mean is an approximately unbiased estimator of the population mean. In unweighted estimators, sampling units have an equal weight of one even if the portions of the population to be represented by reporting agents are of unequal size: some information is overrepresented relative to other information and sample means are biased estimators of population means.

The drawback to using weights in sample means is that it increases the variance of the estimator and the sampling error, owing to the variability of the weights across sampling units.

Two approaches are mainly followed by the literature in dealing with the measurement of the sampling error of weighted estimators. As well as information on the interest rate, the first approach assumes that the weight, the business volume reported by the i-*th* agent, is a characteristic pertaining to the bank. In this perspective, sampling errors depend on how the ratio between the two elements varies across banks: i.e. the flows of interest paid or charged as the numerator, the business volume as the denominator.

A different approach, which does not rely on analytically derived closed forms is pursued by the family of replication-based methods and, among those, by the Jackknife technique (Quenouille, 1956), which we exploited as a second device for the measurement. Finally, we resorted to the procedure used by Battipaglia et al. (2003) at the start of the harmonised interest rate survey. As a natural benchmark for our exercise we compared our results with those obtained by Battipaglia et al. (2003) for the same time span.

In what follows we describe the approaches that are applied for the first time to the best of our knowledge to the measurement of the sampling error in the harmonised interest rate statistics, while the results are illustrated for all the techniques we used in this paper (for Battipaglia et al., see the references).

2.1 The estimator of bank interest rate statistics as a ratio estimator

Suppose we have a population consisting of h strata, where Y_{hi} and X_{hi} are the characteristics Y and X for the unit i in stratum h. We define the *ratio* $R = \frac{Y}{X}$, where $Y = \sum_{h=1}^{L}\sum_{i=1}^{Nh} Y_{hi}$ and $X = \sum_{h=1}^{L}\sum_{i=1}^{Nh} X_{hi}$ are the population *totals* of Y and X. As regard to the sample, we have $\hat{R} = \frac{\hat{Y}}{\hat{X}}$, where $\hat{Y}$ and $\hat{X}$ are *unbiased* estimators of the population *totals* of *Y* and *X*. By setting $\hat{Y} = \sum_{h=1}^{L}\sum_{i=1}^{nh} y_{hi}$, $\hat{X} = \sum_{h=1}^{L}\sum_{i=1}^{nh} x_{hi}$, $y_{hi} \equiv r_{hi} * w_{hi}$ and $x_{hi} \equiv w_{hi}$ we have

$$\hat{R} = \frac{\hat{Y}}{\hat{X}} \equiv \hat{\theta}_{\text{mir}}.$$

In this model $\hat{\theta}_{mir}$ is the ratio estimator of the *total* of *Y* to the *total* of *X*.[2] From the identity between the second and third terms we get the approximated expression for the variance

$$v(\hat{R}) = v(\hat{\theta}_{\text{mir}}).$$

The ratio estimator $\hat{R}$ is a non-linear function in the *totals* of *Y* and *X* (Rao, 2000). Non-linearity makes it difficult to derive an analytic expression for the variance. In this case linearization techniques are helpful and provide approximated expressions for the variance.[3] For stratified sampling design, the Woodruff transformation gives the formula:

$$v(\hat{R}) = \sum_{1}^{L} \frac{n_h (1 - f_h)}{\hat{x}^2} \sum_{1}^{nh} \frac{\left[y_{hi} - \hat{R} x_{hi} - (\bar{y}_h - \hat{R}\bar{x}_h) \right]^2}{n_h - 1}$$

where $\bar{y}_h$ and $\bar{x}_h$ are the averages of y_{hi} and x_{hi} in stratum *h,* respectively, f_h is the sampling fraction, n_h is the sample size in stratum *h*. The expression refers to the *combined* ratio estimator (Cicchitelli, 1993). For large samples the *combined* ratio estimator is unbiased and it is preferred when the sample size of each stratum is not very large[4] (Rao, 2000).

2 This is the view put forward in the Manual on MFI interest rate statistics (BCE 2003): 'It is important to note that in the case of a weighted average the variance of a ratio would have to be calculated'.

3 According to this approach, non-linear estimators are expressed as a function of linear estimators. The 'delta method' is applied to achieve an estimate of the variance.

4 Two different tools – the *separated* estimator and the *combined* estimator – are available for the estimation of ratios in stratified sampling design contexts: the former requires the partial estimates $\hat{R}_h$ for each of the strata to be accurate. Even if the *separated* estimator is more efficient than the *combined* estimator, it is biased when partial estimates are incorrect. When the sample size is small in at least one stratum, the *combined* estimator shows better performances (Rao, 2000). Given the sample size for each of the strata in the Italian sampling design (*n1=34, n2=12, n3=13*) the *combined* estimator has been considered as the appropriate reference model for the measurement of sampling error.

2.2 The Jackknife replication-based method for the measurement of sampling error in bank interest rate statistics

Replication-based methods provide further solutions for the estimation of $v(\hat{\theta}_{mir})$. Information from sampling units are extracted without the 'filter' made up of analytically derived functions. All replication-based methods[5] share the idea of getting the expressions for the variance of $\hat{\theta}_{mir}$ by resorting to a second estimator $\hat{\theta}_{jack}$ of θ, with the following attributes:

$$v(\hat{\theta}_{jack}) \cong v(\hat{\theta}_{mir}) \qquad\qquad v(\hat{\theta}_{jack}) \text{ is well-known.}$$

In other words, the variance of $\hat{\theta}_{\mathrm{mir}}$ is estimated indirectly through knowledge of $v(\hat{\theta}_{jack})$, which is obtained from information provided by the replications of the original sample.[6] The effectiveness of the procedure is conditioned to the reliability of the *Glivenko-Kantelli theorem*:

$$Pr\left\{\lim_{n\to\infty} diff_n = 0\right\} = 1 \qquad \text{where} \qquad diff_n = \sup_{-\infty<x<+\infty} \left|F_{n(x)} - F_{(x)}\right|.$$

When the theorem holds, the sample $F_{n(x)}$ approximates *F(x)* and can 'replace' the potential reporting population. As far as the harmonised interest rate statistics are concerned, the effectiveness of the *Glivenko-Kantelli theorem* has been assumed in view of the high representativeness of the current Italian sample (it accounts for over 80 per cent of the outstanding amounts reported by the population). Wolter (1985) and Cicchitelli (1993) examine the application of the Jackknife replication-based method to stratified sampling designs, under unequal probability sampling. Yung e Rao (1996) further develop applications of the method for *multistage stratified* sampling designs. For the *stratified* sampling case the useful expression is:

$$v(\theta_{jack}) = \sum_{h=1}^{L} (1 - f_h) \frac{n_h - 1}{n_h} \sum_{l=1}^{n_h} \left(\hat{\theta}_{hi} - \hat{\theta}_h\right)^2$$

where $\hat{\theta}_{hi}$ is the estimator of θ which shares the functional form of $\hat{\theta}_{mir}$, obtained by omitting the i-*th* unit in the stratum h from the sample and where $\hat{\theta}_h = \sum_{1}^{n_h} \frac{\hat{\theta}_{hi}}{n_h}$.

5 Bootstrap methods constitute an additional family of replication-based techniques. However applications to stratified sampling designs are not available at the moment (Wolter, 1985; Sarndal et al., 1992).

6 A replication is the result of deleting a certain number of observations from the (original) sample. The maximum degree of accuracy is reached when the size of the replication is equal to *n-1* (in that case the i-*th* replication is obtained by deleting the i-*th* sampling unit from the original sample).

3. Results

In this section the results gained by the methods we have introduced and those obtained using the Battipaglia et al. (2003) technique are shown. As regard the latter, in the preliminary estimates carried out at the start of the harmonised statistics survey, the variance of the estimator was approximated by combining the estimated variability of the average interest rate with the 'intertemporal' variability of the sampling fraction – according to the formula for the product of two stochastic variables. The sampling fraction was defined by the coverage of new business.

Results are displayed in Table 1 and Figures 1-4 (the acronyms *ratio*, *jack*, *bb* refer to the results achieved by the *ratio* estimator, the Jackknife and the Battipaglia et al. procedure, respectively). The sampling error of $\hat{\theta}_{mir}$, at the 10 per cent probability level is provided by (1), where $Z_{\alpha/2}$ is set equal to 1.645.[7] The exercise made use of 52,892 bank-month observations, on both interest rates and business volumes. For each of the instrument categories, data are reported by the 121 credit institutions included in the Italian sample during the period January 2005 – October 2006.

The nature of the sampling error is *point-in-time*, being defined for each of the 22 periods lying in the time span we have considered, and for 28 of the 29 interest rates on new business provided by the Regulation (for one category the sampling error is not defined given that a 'census approach' is used for the sole reporting agent supplying that bank product).

In expression (1), $v\left(\hat{\theta}_{mir}\right)$ was replaced by $v(\hat{R})$ and $v\left(\hat{\theta}_{jack}\right)$.[8] As a whole 1,232 *point-in-time errors D* were measured, (616 by the ratio estimator, 616 by the Jackknife procedure). Battipaglia et al. (2003) estimated an 'intertemporal' type variance: as a consequence they provide 28 sampling errors, each of them representing the error for the entire time span.

Aggregating the results for the 22 periods and for all the phenomena observed, shows that the sampling error for bank interest rates is well below the threshold of 10 basis points set by the Regulation. All the three methods achieve similar results: the sampling errors (average sampling error) are 0.049, 0.042, 0.042 percentage points, respectively.

---

7 In (1) $Z_{\alpha/2}$ was replaced by the 95*th* percentile with the *t*-distribution (1.645). According to Cicchitelli (1993) 'For complex surveys […] results of simulation performed by Frankel (1971), Bean (1975), Campbell e Meyer (1978), must be taken into account. In general, sampling designs are stratified […]. Interval confidence is defined by the percentile with the *t*-distribution.'

8 The sampling fraction f_h was replaced by the ratio of the sample size in stratum *h* to the size of the potential reporting population in stratum *h*, i.e:

$f_h = 34/428 = 0.08$ if h = 1

$f_h = 12/151 = 0.08$ if h = 2

$f_h = 13/146 = 0.08$ if h = 3

$f_h = 64/64 = 1$ if h = 4

Sampling fractions do not vary over time, even if the amounts reported by banks are equal to zero as of a certain reference date. According to this strategy, banks belonging to the potential reporting population which report volumes equal to zero are properly taken into account by the sample.

The average sampling errors decline when the instrument categories are weighted according to their importance. In that case the errors (weighted average sampling error) are 0.024; 0.024; 0.028 percentage points. The lower errors are due to the larger weights in the average of low-error categories, which are usually associated with larger volumes.

The sampling errors are higher for interest rates on loans (0.064; 0.055; 0.053) than for interest rates on deposits (0.020; 0.018; 0.022). The gap could be driven by the differences between the levels of the two groups of statistics.

Table 1. Accuracy of interest rate statistics
(sampling errors in percentage points; time span: Jan. 2005 –Oct. 2006)

Interest rates on new business provided for by Regulation 2001/18			
	ratio	**jack**	**bb**
average sampling error	0.049	0.042	0.042
weighted average sampling error	0.024	0.024	0.028
average sampling error on loans	0.064	0.055	0.053
average sampling error on deposits	0.020	0.018	0.022
Interest rates on new business stricto sensu*			
	ratio	**jack**	**bb**
average sampling error	0.052	0.043	0.043
weighted average sampling error	0.027	0.023	0.024

* *The group excludes bank overdrafts, deposits redeemable at notice and overnight deposits, for which only data on outstanding amounts are available.*

Further aggregations of the results show the sampling errors for interest rates on new business *stricto sensu* (this group excludes bank overdrafts, deposits redeemable at notice and overnight deposits). Results are similar to the previous ones.

The inspection of results at category level (Figures 1-2) shows that for each of the interest rate statistics the sampling error is less than 10 basis points, except for the interest rate on loans for other purposes and the interest rate for consumer credit (both with an i.r.f of less than 1 year). As regards the former, two measures find sampling errors of more than 30 basis points, while as regard the latter, the sampling error is about 12 basis points. These categories account for less than 1 per cent of the overall harmonised new business volumes. They also account for about 3 per cent of the overall harmonised *stricto sensu* new business volumes.

In Figures 3-4 the confidence interval is compared with the threshold provided by Regulation 2001/18. In the same figures we plotted the interest rate on that category. Thanks to the point-in-time nature of the measures, the evolution of the sampling error over time can be monitored.

The comparison is shown only for some categories which are important in the Italian banking system, such as bank overdrafts to non-financial corporations and deposits from households with agreed maturity, but the comparison was carried out for each of the interest rates analyzed in this paper. The figures show that the sampling error is stably below the threshold of 10 basis points for the entire period. The patterns were confirmed after the rise in the minimum rate on main refinancing operations. The stability over time of the sampling errors plotted in Figures 1 and 2 was also found for the interest rates not reported in the figures.

References

Battipaglia P. and F. Bolognesi, (2003), The Harmonization of European Statistics on Bank Interest Rates and the Methodology Adopted by Italy, *Supplements to the Statistical Bulletin,* Bank of Italy.

European Central Bank, (2003), *Manual on MFI Interest Rate Statistics*.

Cicchitelli G., Herzel A. and G. Montanari, (1993), *Il campionamento statistico,* Il Mulino.

Cochran W.G., (1977), *Sampling Techniques,* John Wiley & Sons.

Quenouille M.H., (1956), Notes on bias in estimation, *Biometrika.*

Rao P., (2000), *Sampling Methodologies*, Chapman & Hall.

Sarndal C.E., Swensson B., Wretman J., (1992), *Model Assisted Survey Sampling,* Springer-Verlag.

Wolter K., (1985), *Introduction to Variance Estimation,* Springer-Verlag.

Appendix

Figure 1. Accuracy of interest rate statistics on loans
(sampling errors in percentage points; time span: Jan. 2005 –Oct. 2006)

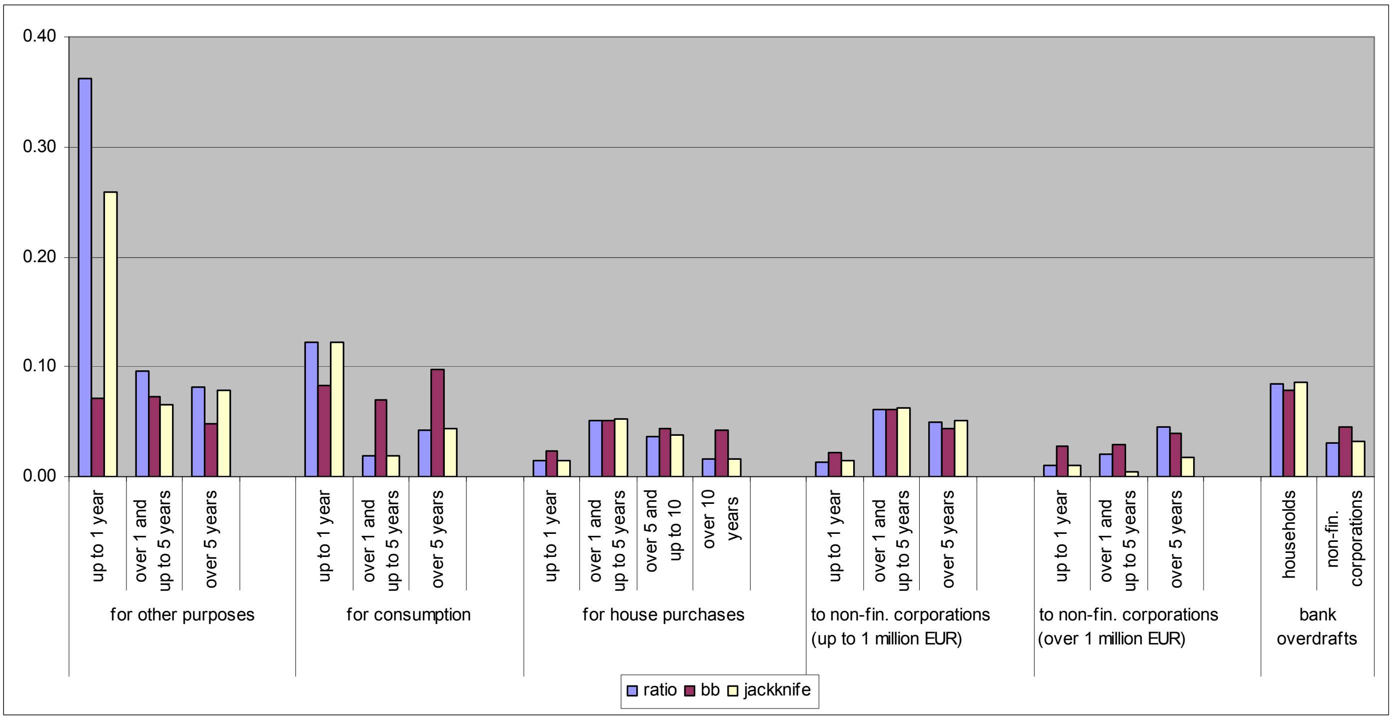

Figure 2. Accuracy of interest rate statistics on deposits
(sampling errors in percentage points; time span: Jan. 2005 –Oct. 2006)

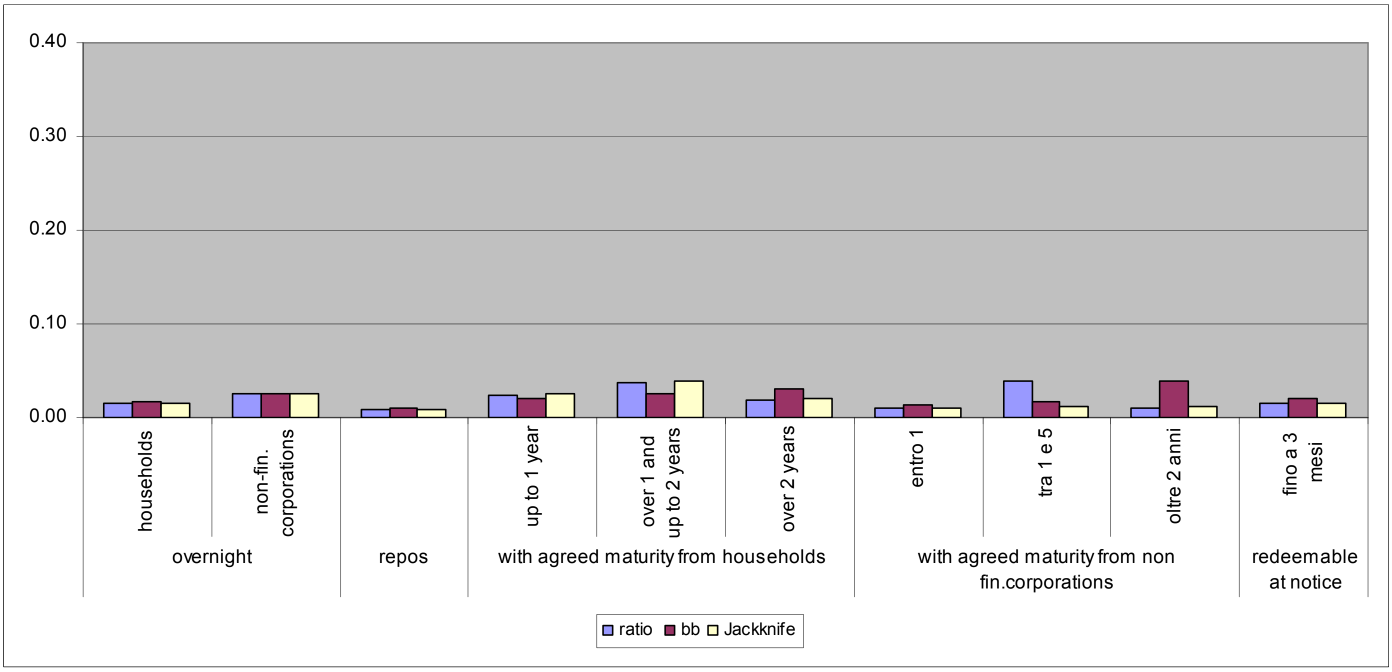

Figure 3. Accuracy of point-in time statistics: confidence interval for Italian interest rate statistics and maximum range allowed by the ECB

(bars delimit the 90% confidence limits allowed by Regulation 2001/18; the shadowed area shows the 90% confidence interval for the interest rate statistics on the labelled category; time span: Jan. 2005 –Oct. 2006)

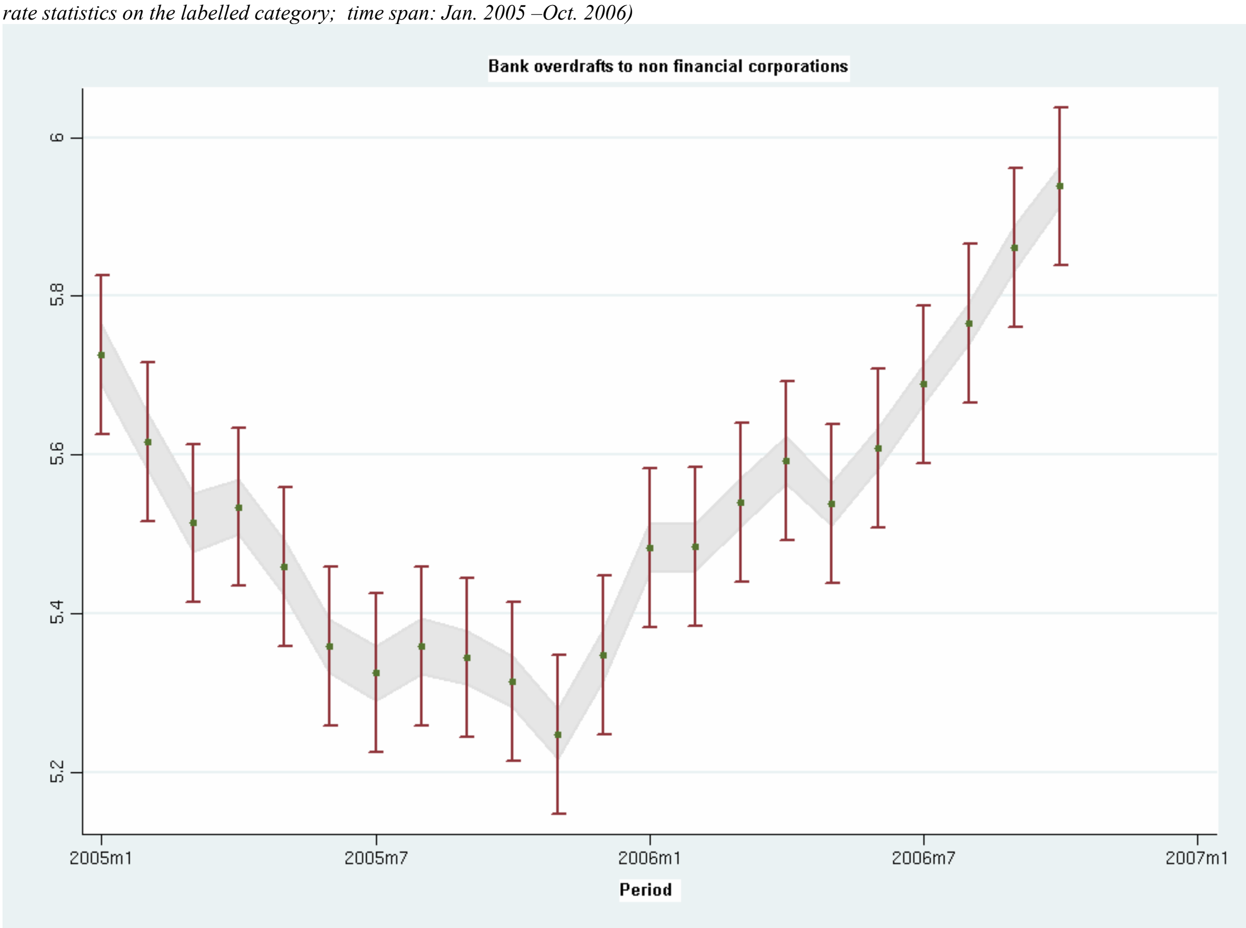

Figure 4. Accuracy of point-in time statistics: confidence interval for Italian interest rate statistics and maximum range allowed by the ECB

(bars delimit the 90% confidence limits allowed by Regulation 2001/18; the shadowed area shows the 90% confidence interval for the interest rate statistics on the labelled category; time span: Jan. 2005 –Oct. 2006)