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cooperation, strategic interactions, and spillovers

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# **CARBON TAXES AROUND THE WORLD: COOPERATION, STRATEGIC INTERACTIONS, AND SPILLOVERS**

by Alessandro Moro\* and Valerio Nispi Landi\*\*

## **Abstract**

We examine the global implications of carbon taxation using a two-country environmental DSGE model, with a specific focus on the strategic interactions among countries, the case for cooperation, and the impact on the balance of payments. From a normative perspective, we show that, assuming a convex pollution disutility, carbon taxes are strategic substitutes across countries: when a country increases carbon taxation, the other country finds it optimal to reduce it. From a positive perspective, a country imposing a unilateral carbon taxation experiences a reduction of its production, a decrease in its interest rate, a depreciation of its currency on impact and an appreciation thereafter, higher debt, and equity outflows to the rest of the world.

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# 1 Introduction<sup>1</sup>

Do environmental policies in some countries affect environmental policies in other countries? What are the international spillover effects of carbon taxes? How do carbon taxes affect open-economy variables such as capital flows and exchange rates? To answer these questions we develop an environmental dynamic stochastic general equilibrium (E-DSGE) model.

The above issues have significant importance in the design of effective environmental policies. Climate change refers to long-term changes in average weather patterns and is characterized by increasing global temperatures, extreme weather events, and widespread ecological disruptions. The scientific consensus is that climate change is driven primarily by human activities, including the emission of greenhouse gases resulting from fossil fuel combustion, deforestation, and industrial processes (IPCC, 2022). Addressing climate change may involve a trade-off, however, as mitigation efforts may require reducing economic activity, i.e. the so-called transition risk.

Climate change exemplifies a global negative externality, given that most countries are too small to effectively reduce global pollution in isolation. Even if national policymakers internalize the domestic costs of climate change through the implementation of carbon taxes, without coordination among countries they do not internalize the global costs, resulting in sub-optimal environmental policies (Hoel, 1991; Van Der Ploeg and De Zeeuw, 1992; Tahvonen, 1994; Nordhaus, 2015). Absent global cooperation, carbon taxes can also lead to unintended spillover effects, influencing capital flows and shifting polluting activities to countries with weaker environmental regulations (Fontagné and Schubert, 2023, Schroeder and Stracca, 2023).<sup>2</sup> While the Paris Agreement has been

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<sup>2</sup>Beyond strategic interactions among countries, a strand of the literature has also focused on the role of intergenerational conflict in preventing the adoption of sound climate policies within economies (Kotlikoff et al., 2021). Moreover, a novel stream of literature is investigating the heterogeneous gains

signed by nearly 200 countries to coordinate environmental policies and limit globally rising temperatures, current progress falls short of the outlined goals (Black et al., 2022).

To examine the effects of carbon taxation from an international economics perspective, we set up a fully-fledged two-country real E-DSGE model. We have two main goals: to analyze the problem of setting the optimal carbon tax; to simulate the domestic and international spillover effects of the optimal tax.

The model incorporates two sectors (green and brown), debt and equity capital flows, and a pollution externality in the utility function. International bond markets are incomplete, as in Gabaix and Maggiori (2015). Atmospheric carbon, accumulating through CO<sub>2</sub> emissions resulting from production in the brown sector, represents pollution in this model. Firms within the brown sector face a carbon tax per unit of emissions and have two options to reduce emissions: i) cut production; ii) spend in abatement activities, i.e. invest in costly technology that enables them to pollute less at any given level of production. In each country, the government sets the carbon tax by following a rule under which the tax increases linearly with GDP. The slope of the rule, i.e. how much the tax is proportional to GDP, is chosen by the government. With this model in hand, we can compute the optimal carbon policy of the two countries, the Nash and the global social planner equilibria, and the spillover effects of different carbon taxes. In order to illustrate the functioning of the model, first we focus on a symmetric calibration with two identical economies. Then, we adopt an asymmetric calibration, considering the partition between advanced and emerging economies.

From a normative point of view, our main novel result is that under a convex pollution disutility carbon taxes are strategic substitutes: higher taxes set in one country reduces the stock of pollution and its marginal cost (as the disutility is convex), in this way inducing the other country to set lower taxes, exacerbating the free-riding problem. If instead the pollution disutility is linear, the marginal cost of polluting is constant and carbon taxes are independent across countries. We also show that, consistently with

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and losses global warming may produce around the world (Krusell and Smith Jr, 2022).



other contributions (e.g. Ferrari Minesso and Pagliari, 2023), the global social planner sets higher taxes compared to the Nash equilibrium, leading to a faster green transition. Moreover, under the asymmetric calibration, we show that a global social planner would increase carbon taxation much more in advanced than in emerging markets, as the latter are poorer and thus have a higher marginal utility of consumption.

From a positive point of view, we find some novel results on the spillover effects of carbon taxation. If only one country – call it Home – sets an optimal policy, capital partially moves to the other country – call it Foreign. Home experiences a gradual reduction in economic activity, driven by the gradual increase in carbon taxation. The Home interest rate on bonds falls, inducing investors to sell Home bonds and buy Foreign bonds. Home faces a real depreciation on impact, which improves the trade balance, and an appreciation thereafter. Home production shifts from the brown to the green sector. Home investors buy equity in the Foreign green and brown sectors, increasing the Foreign Direct Investment (FDI) stock of the country, and giving rise to carbon leakage. In Foreign, the impact on economic activity is slightly negative, as the increase in capital inflows is partially offset by a lower demand from Home. Similarly, Foreign emissions barely change, as the increase in brown FDI from Home is compensated by the reduction in Foreign economic activity. These results hold both in the symmetric and in the asymmetric calibration.

This paper contributes to the literature on E-DSGE models. This class of models are modified versions of the William Nordhaus’ DICE model (see Barrage and Nordhaus, 2023 for the 2023 DICE version). This literature has focused on several aspects of environmental policies, such as: the business cycle effects of environmental policies (Heutel, 2012 and Annicchiarico and Di Dio, 2015; see Annicchiarico et al., 2022 for a literature review); the optimal carbon taxation along a balanced growth path (Golosov et al., 2014; Barrage, 2014); the consequences of carbon taxes on the financial sector (Carattini et al., 2023a; Diluiso et al., 2021; and the macroeconomic effects of green monetary policies (Ferrari and Nispi Landi forthcoming a; Ferrari and Nispi Landi, 2023; Benmir and Roman,

2020; Giovanardi et al., 2023; Bartocci et al., 2022).

Recently, the literature has started to focus on the international aspects of carbon taxation. Ferrari Minesso and Pagliari (2023) estimate a two-country E-DSGE model to study fiscal-monetary interactions. They show that countries do not have an incentive to introduce carbon taxes, as the latter are too costly from a welfare perspective. Under the constraint that they must comply with the Paris agreement, countries should cooperate to minimize welfare losses. Ernst et al. (2023a) calibrate a three-country E-DSGE, analyzing the global macroeconomic consequences of carbon taxes, focusing in particular on quantifying carbon leakage and the role of border adjustment taxes. Compared to these contributions, we include exogenous sector-neutral Total Factor Productivity (TFP) growth, implying that pollution increases over time, without proper regulation. Moreover, we focus our analysis on the impact of the green transition on key open-economy variables, such as the exchange rate, capital flows, and interest rates.

There are also two related works written independently from and simultaneously to our study. Ernst et al. (2023b) include a life-cycle structure to the model of Ernst et al. (2023a), finding that the world interest rate falls when some countries introduce carbon taxation. Carattini et al. (2023b) introduce financial frictions in a two-country E-DSGE to study the financial stability effects of international carbon taxation.

The rest of the paper is organized as follows. Section 2 describes the model in detail. Sections 3 and 4 show the results of the simulation with the symmetric and asymmetric calibration, respectively. Section 5 concludes.

## 2 A two-country model

We set up a DSGE model for the world economy, characterized by two countries (Home and Foreign) and incomplete financial markets. Home (Foreign) consists of households, financiers, final-good firms, green and brown intermediate firms, all of measure  $n$  ( $1 - n$ ). Each country produces a final good which is sold to residents and non-residents for

consumption and investment purposes. The final good is produced with green ( $G$ ) and brown ( $B$ ) intermediate outputs, which in turn are produced using local labor, capital provided by residents, and capital provided by non residents. The production in the two brown sectors generate CO2 emissions, which contribute to global pollution, and as such create disutility to households. The model features debt flows (countries can trade bonds) and equity (FDI) flows (countries can trade physical capital). The model does not feature nominal rigidities, all variables are real.

In what follows, for all variables (except for investment variables) we denote variables that refer to Foreign with the superscript star (\*). For investment variables we denote variables that refer to Foreign investment in Foreign capital with a double star (subscript and superscript); variables that refer to Foreign investment in Home capital with a superscript star; and variables that refer to Home investment in Foreign capital with a subscript star.

## 2.1 Consumption and investment bundles

In each country, there is a consumption bundle  $(c_t, c_t^*)$ , a domestic investment bundle  $(i_t, i_{*t}^*)$ , and an external investment bundle  $(i_{*t}, i_t^*)$ . The consumption and domestic investment bundles read:

$$c_t = \left[ (1 - \gamma)^{\frac{1}{\eta}} c_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} c_{Ft}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad c_t^* = \left[ (\gamma^*)^{\frac{1}{\eta}} (c_{Ht}^*)^{\frac{\eta-1}{\eta}} + (1 - \gamma^*)^{\frac{1}{\eta}} (c_{Ft}^*)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1)$$

$$i_t = \left[ (1 - \gamma)^{\frac{1}{\eta}} i_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} i_{Ft}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad i_{*t}^* = \left[ (\gamma^*)^{\frac{1}{\eta}} (i_{*Ht}^*)^{\frac{\eta-1}{\eta}} + (1 - \gamma^*)^{\frac{1}{\eta}} (i_{*Ft}^*)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (2)$$

where subscripts H and F refer to bundles of Home and Foreign final goods (for instance,  $c_{Ht}^*$  is consumption of Home final good by Foreign households). Parameters  $\gamma, \gamma^* > 0$  measure home bias,  $\eta > 0$  is the elasticity of substitution between Home and Foreign final goods.

Following Ghironi and Kemal Ozhan (2020), we assume that the composition of the external investment bundle reflects the composition of the domestic bundle of the host country. For instance, if Home households invest in Foreign capital, they purchase the bundle  $i_{*t}$ , which has the same composition (and price) of  $c_t^*$  and  $i_t^*$ . If Foreign households invest in Home capital, they purchase the bundle  $i_t^*$ , which has the same composition (and price) of  $c_t$  and  $i_t$ :

$$i_{*t} = \left[ (\gamma^*)^{\frac{1}{\eta}} i_{*Ht}^{\frac{\eta-1}{\eta}} + (1 - \gamma^*)^{\frac{1}{\eta}} i_{*Ft}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (3)$$

$$i_t^* = \left[ (1 - \gamma)^{\frac{1}{\eta}} (i_{Ht}^*)^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} (i_{Ft}^*)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (4)$$

where  $i_{*H}$  and  $i_{*F}$  denote Home purchase of Home and Foreign final goods respectively, for investing in Foreign capital;  $i_H^*$  and  $i_F^*$  denote Foreign purchase of Home and Foreign final goods respectively, for investing in the Home capital. In Home, the following demand functions hold:

$$c_{Ht} = (1 - \gamma) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} c_t, \quad c_{Ft} = \gamma \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} c_t \quad (5)$$

$$i_{Ht} = (1 - \gamma) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} i_t, \quad i_{Ft} = \gamma \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} i_t \quad (6)$$

$$i_{*Ht} = \gamma^* \left( \frac{P_{Ht}^*}{P_t^*} \right)^{-\eta} i_{*t}, \quad i_{*Ft} = (1 - \gamma^*) \left( \frac{P_{Ft}^*}{P_t^*} \right)^{-\eta} i_{*t}, \quad (7)$$

where  $P_{Ht}$  and  $P_{Ft}$  ( $P_{Ht}^*$  and  $P_{Ft}^*$ ) are the prices of the Home and Foreign goods respectively (i.e. the PPI indices), both expressed in their respective currency;  $P_t = [(1 - \gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}}$  is the Home CPI;  $P_t^* = [\gamma^* (P_{Ht}^*)^{1-\eta} + (1 - \gamma^*) (P_{Ft}^*)^{1-\eta}]^{\frac{1}{1-\eta}}$  is the Foreign CPI. Given the symmetric hypothesis, analogous demand functions can be written for the Foreign country. We define the real exchange rate as follows:

$$s_t = \mathcal{E}_t \frac{P_t^*}{P_t},$$

where the nominal exchange rate  $\mathcal{E}_t$  is the price of one unit of Foreign currency in terms of Home currency. By assuming that the law of one price holds ( $P_{Ht} = \mathcal{E}_t P_{Ht}^*$ ,  $P_{Ft} = \mathcal{E}_t P_{Ft}^*$ ), and defining the relative PPI  $p_{Ht} \equiv \frac{P_{Ht}}{P_t}$  and  $p_{Ft} \equiv \frac{P_{Ft}}{P_t}$ , we can rewrite the CPI expressions as follows:

$$1 = (1 - \gamma) p_{Ht}^{1-\eta} + \gamma p_{Ft}^{1-\eta} \quad (8)$$

$$s_t^{1-\eta} = \gamma^* (p_{Ht})^{1-\eta} + (1 - \gamma^*) (p_{Ft})^{1-\eta}. \quad (9)$$

Given that in our model there are no monetary rigidities, there is no role for monetary policy in affecting real variables. Without loss of generality, we can set  $\mathcal{E}_t = 1 \forall t$  and let the Home consumption bundle be the numéraire of the world economy.

## 2.2 Final-good firms

The representative final-good firm in Home uses the following CES aggregator to produce the final good  $y_{Ht}$ :

$$y_{Ht} = \left[ (1 - \zeta)^{\frac{1}{\xi}} (y_{Gt})^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}} (y_{Bt})^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}},$$

where  $y_{Gt}$  and  $y_{Bt}$  denote green and brown intermediate output, respectively;  $\zeta > 0$  is the weight of brown production;  $\xi > 0$  is the elasticity of substitution between green and brown output. Let  $p_{Gt}$  and  $p_{Bt}$  be the price of green and brown production relative to Home CPI  $P_t$ . The optimal demand functions for the intermediate outputs read:

$$y_{Bt} = \zeta \left( \frac{p_{Bt}}{p_{Ht}} \right)^{-\xi} y_{Ht} \quad (10)$$

$$y_{Gt} = (1 - \zeta) \left( \frac{p_{Gt}}{p_{Ht}} \right)^{-\xi} y_{Ht}, \quad (11)$$

where the relative PPI  $p_{Ht}$  is given by:

$$p_{Ht} = \left[ (1 - \zeta) p_{Gt}^{1-\xi} + \zeta p_{Bt}^{1-\xi} \right]^{\frac{1}{1-\xi}}. \quad (12)$$

Analogous conditions hold in Foreign.

## 2.3 Intermediate-good firms

In each sector  $i$ , Home and Foreign firms use domestic capital and labor (provided by residents) and external capital (i.e. FDI, provided by non-residents) to produce the intermediate good  $i$ :

$$y_{it} = A (k_{it-1})^{\alpha_1} (k_{it-1}^*)^{\alpha_2} (h_{it} z_t^W)^{1-\alpha_1-\alpha_2}, \quad i \in \{B, G\} \quad (13)$$

$$y_{it}^* = A^* (k_{*it-1}^*)^{\alpha_1^*} (k_{*it-1})^{\alpha_2^*} (h_{it}^* z_t^W)^{1-\alpha_1^*-\alpha_2^*}, \quad i \in \{B, G\}, \quad (14)$$

where  $y_{it}$  and  $y_{it}^*$  denote Home and Foreign intermediate goods in sector  $i$ ;  $k_{it}$  and  $k_{*it}^*$  denote capital provided by Home residents in Home firms and by Foreign residents in Foreign firms;  $k_{it}^*$  and  $k_{*it}$  denote capital provided by Foreign residents in Home firms and by Home residents in Foreign firms; we can interpret these capital stocks as foreign direct investment (FDI) (Ghironi and Kemal Ozhan, 2020);  $h_{it}$  and  $h_{it}^*$  denote hours worked;  $A$  and  $A^*$  denote constant TFP; all the  $\alpha$ 's are positive parameters measuring the elasticity of production to inputs;  $z_t^W$  is global long-run labor productivity, which evolves following a unit root process:

$$z_t = \iota z_{t-1}, \quad (15)$$

where  $\iota > 1$  is the long-run growth of the economy. Following several two-sector environmental models (for instance Carattini et al. (2023a) and Ferrari and Nispi Landi, 2023),

we assume that brown firms generate CO2 emissions  $e_t$ :

$$e_t = z_e (1 - \mu_t) y_{Bt}, \quad (16)$$

where  $\mu_t$  is the share of emissions abated, and  $z_e > 0$  captures carbon intensity. An analogous equation holds in Foreign. As is standard (Heutel, 2012; Barrage and Nordhaus, 2023), firms pay costs (in terms of the final consumption good) that are convex in the share of emissions abated:

$$AB_t \equiv \frac{\kappa_A}{1 + \nu} (\mu_t^{1+\nu}) y_{Bt}, \quad (17)$$

where  $\nu > 0$  measures the convexity of the cost function, and  $\kappa_A > 0$  governs the relevance of these costs.

The stock of global pollution  $x_t^W$  is fueled by the flow of Home and Foreign emissions. We follow the same carbon cycle model of Hassler et al. (2016):

$$x_t^W - \bar{x}^W = \sum_{j=0}^{t+T} \left[ \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^j \right] [ne_t + (1 - n) e_t^*], \quad (18)$$

where  $\bar{x}^W$  is the pre-industrial level of atmospheric carbon;  $\varphi_L > 0$  is the share of carbon emitted into the atmosphere that remains forever;  $1 - \varphi_0$  of the remaining share exits the atmosphere within one period (i.e. one year); the remaining part  $(1 - \varphi_L) \varphi_0$  depreciates at geometric rate  $\varphi > 0$ . It is as if there are two carbon sinks, one that stays in the atmosphere for ever ( $x_{1t}^W$ ) and one that depreciates at rate  $\varphi$  ( $x_{2t}^W$ ):

$$x_{1t}^W = x_{1t-1}^W + \varphi_L [ne_t + (1 - n) e_t^*] \quad (19)$$

$$x_{2t}^W = (1 - \varphi) x_{2t-1}^W + (1 - \varphi_L) \varphi_0 [ne_t + (1 - n) e_t^*], \quad (20)$$

and  $x_t^W = x_{1t}^W + x_{2t}^W + \bar{x}^W$ .

To save space, we focus on the Home brown firm. The problem for the other three firms (Home green, Foreign green and Foreign brown) is analogous, except that green

firms do not generate emissions. Profits of brown firms read:

$$\Gamma_{Bt} = p_{Bt}y_{Bt} - w_t h_{Bt} - k_{Bt-1}r_{kBt} - k_{Bt-1}^*r_{kBt}^* - \tau_t e_t - AB_t,$$

where  $w_t$  is the hourly wage;  $r_{kBt}$  ( $r_{kBt}^*$ ) is the rental rate of capital installed in Home brown firms and provided by Home (Foreign) residents;  $\tau_t$  is tax per unit of emissions, set by the government. The first order conditions of the profit maximization problem yield the following demand for labor, domestic capital and FDI:

$$h_{Bt} = \frac{(1 - \alpha_1 - \alpha_2) \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{w_t} \quad (21)$$

$$k_{Bt-1} = \frac{\alpha_1 \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{r_{kBt}} \quad (22)$$

$$k_{Bt-1}^* = \frac{\alpha_2 \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{r_{kBt}^*}. \quad (23)$$

Analogous conditions hold for green and Foreign firms. The first order condition with respect to  $\mu_t$  gives the optimal abatement rate:

$$\mu_t = \left( \frac{z_e \tau_t}{\kappa_A} \right)^{\frac{1}{\nu}}. \quad (24)$$

If the tax is zero, firms do not have any incentive to spend in abatement. If the tax is positive, firms trade off tax and abatement costs.

## 2.4 Households

In each country there is a representative household that derives utility from consumption and disutility from global pollution. Including the externality in the utility function rather than in the production function allows us easily to compute the balanced growth path of the economy. Following Pástor et al. (2021), Moro (2021), Avramov et al. (2022), and Ferrari and Nispi Landi (2023), we also assume that households derive utility from holding green assets and derive disutility from holding brown assets.



This assumption captures the taste of investors for specific assets beyond the payoffs (as in Fama and French, 2007) and it also implies that a negative “greenium” emerges in equilibrium: green assets are less remunerative than brown assets, in line with several empirical studies.<sup>3</sup> This assumption is also useful to make the model stationary, without major quantitative implications.<sup>4</sup> The utility function reads:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t - \kappa_X \frac{(x_t^W - \bar{x}^W)^{1+\psi_X}}{1+\psi_X} + \kappa_G \frac{\tilde{K}_{Gt}^{1-\psi_G}}{1-\psi_G} - \kappa_B \frac{\tilde{K}_{Bt}^{1+\psi_B}}{1+\psi_B} \right),$$

where  $\mathbb{E}_t$  is the expectation operator, conditional on information at time  $t$ ;  $0 < \beta < 1$  is the discount factor;  $\kappa_X$ ,  $\kappa_G$ , and  $\kappa_B$  are the weights associated to the utility or disutility of pollution, green assets and brown assets, respectively;  $\psi_X$ ,  $\psi_G$ , and  $\psi_B$  control the curvature of the utility or disutility of pollution, green assets and brown assets, respectively; notice that if  $\psi_X = 0$ , the pollution disutility is linear, and if  $\psi_X = 1$ , it is quadratic;  $\tilde{K}_{Gt}$  ( $\tilde{K}_{Bt}$ ) denotes total holdings of green (brown) capital, installed in Home and Foreign, and detrended by world long-run productivity  $z_t^W$ :

$$\tilde{K}_{it} \equiv \frac{k_{it}}{z_t^W} + s_t \frac{1-n}{n} \frac{k_{*it}}{z_t^W} \quad i \in \{B, G\}. \quad (25)$$

The stock of capital  $k_{*it}$  enters the production function of Foreign firms (equation 14), and it is accumulated purchasing the investment bundle in equation (3), which has the same composition of the Foreign consumption bundle (equation 1): it is then expressed in units of Foreign goods per capita. This is why it is adjusted by  $s_t \frac{1-n}{n}$ , in order to be expressed in terms of Home goods per capita. Dividing by global productivity is instead necessary to ensure a balanced growth path. Capital installed in Home and in Foreign

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<sup>3</sup>See for instance Zerbib (2019), Fatica et al. (2021), Liberati and Marinelli (2021), Pástor et al. (2022), Avramov et al. (2022), Zaghini (2023), and Moro and Zaghini (2023).

<sup>4</sup>See Schmitt-Grohé and Uribe (2003) for a discussion on how to make stationary open-economy models with incomplete markets. Notice that if we assume asset adjustment costs rather than capital in the utility function, the greenium would disappear in steady state (adjustment costs are zero by definition in steady state).

obeys the following laws of motion:

$$k_{Gt} + k_{Bt} = (1 - \delta)(k_{Gt-1} + k_{Bt-1}) + i_t \quad (26)$$

$$k_{*Gt} + k_{*Bt} = (1 - \delta)(k_{*Gt-1} + k_{*Bt-1}) + i_{*t}, \quad (27)$$

where  $\delta > 0$  is the depreciation rate of capital. The representative Home household chooses consumption, investment in the two Home and Foreign sectors, and holdings of a one-period Home bond  $b_t$  (denominated in Home CPI), maximising the utility function in equation 2.4 subject to the laws of motion of capital (equations 26-27) and the following budget constraint:

$$\begin{aligned} c_t + b_t + i_t + \frac{1-n}{n} s_t i_{*t} &= r_{kGt} k_{Gt-1} + r_{kBt} k_{Bt-1} + \\ + s_t \frac{1-n}{n} (r_{*kBt} k_{*Bt-1} + r_{*kGt} k_{*Gt-1}) &+ r_{t-1} b_{t-1} + w_t h_t - t_t + \Pi_t, \end{aligned}$$

where  $r_t$  is the real interest rate on domestic bonds;  $h_t = h$  denote constant hours worked;  $t_t$  are lump-sum taxes;  $\Pi_t$  denotes profits from the financial sector.<sup>5</sup> The first order conditions of the utility maximization problem yield the following Euler equations:

$$1 = \beta \mathbb{E}_t \left( \frac{c_t}{c_{t+1}} r_t \right) \quad (28)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} (r_{kGt+1} + 1 - \delta) \right] + \kappa_G \frac{c_t}{z_t^W} \tilde{K}_{Gt}^{-\psi_G} \quad (29)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} (r_{kBt+1} + 1 - \delta) \right] - \kappa_B \frac{c_t}{z_t^W} \tilde{K}_{Bt}^{\psi_B} \quad (30)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \frac{s_{t+1}}{s_t} (r_{*kGt+1} + 1 - \delta) \right] + \kappa_G \frac{c_t}{z_t^W} \tilde{K}_{Gt}^{-\psi_G} \quad (31)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \frac{s_{t+1}}{s_t} (r_{*kBt+1} + 1 - \delta) \right] - \kappa_B \frac{c_t}{z_t^W} \tilde{K}_{Bt}^{\psi_B}. \quad (32)$$

---

<sup>5</sup>The production sector makes zero profits, as it operates in perfect competition with constant-return-to-scale production functions.

Other things equal, green assets yield lower return than brown assets. Moreover, Home households bear exchange rate risk in investing abroad. In Foreign, analogous conditions hold.

## 2.5 Financiers

There is a representative financier in each country, that intermediates Home and Foreign bonds. The financier adopts a zero-capital carry-trade strategy. In period  $t$ , the Home representative financier buys  $d_{Ht}$  Home bonds and sells  $-s_t d_{Ht}^*$  Foreign bonds such that:  $d_{Ht} - s_t d_{Ht}^* = 0$  (the representative intermediary in Foreign takes position  $d_{Ft}^*$  and  $-\frac{d_{Ft}}{s_t}$  in Foreign and Home bonds, respectively). The representative financier in Home chooses its position in Home and Foreign bonds maximising the expected value of profits in  $t + 1$ :

$$V_t = \max_{d_{Ht}} \beta \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \left( r_t - r_t^* \frac{s_{t+1}}{s_t} \right) d_{Ht} \right]. \quad (33)$$

Following Gabaix and Maggiori (2015), we assume that the intermediary can divert a portion  $\Gamma |\frac{d_{Ht}}{z_t^W}|$  of the position  $d_{Ht}$ , where  $\Gamma > 0$  represents the financiers' risk-bearing capacity and can be interpreted as a measure of the shallowness of FX markets. In order to trust financiers, creditors impose the following incentive compatibility constraint:

$$V_t \geq \frac{\Gamma}{z_t^W} |d_{Ht}| d_{Ht} = \frac{\Gamma}{z_t^W} (d_{Ht})^2. \quad (34)$$

The constraint is always binding, given that the intermediary's value is linear in  $d_{Ht}$  while the constraint is convex. We can write:

$$d_{Ht} = \frac{z_t^W}{\Gamma} \mathbb{E}_t \left[ \beta \frac{c_t}{c_{t+1}} \left( r_t - r_t^* \frac{s_{t+1}}{s_t} \right) \right], \quad (35)$$

which is a modified UIP condition. If  $\Gamma \rightarrow 0$ , the UIP condition holds exactly; in the more general case of  $\Gamma > 0$ , financiers are not able to fully exploit the arbitrage between Home and Foreign bonds, and a spread opens up: in particular, if the Home return is

higher than the Foreign return, financiers are long in Home and short in Foreign bonds, and viceversa. A similar condition holds in Foreign. Profits of Home (Foreign) financiers are transferred to Home (Foreign) households.

We also assume that there are noisy traders that buy  $v_t$  Home bonds and sell  $v_t = -s_t v_t^*$  Foreign bonds. Without loss of generality, we assume that noisy traders transfer profits to Home households. We assume that  $v_t$  is constant along a balanced growth path (i.e.  $v_t = z_t^W v$ ): in our model, the purpose of  $v_t$  is only to pin down a desired steady-state NFA position.

## 2.6 Policy

In each country, the government finances public spending  $g_t$  by raising lump-sum taxes  $t_t$  and emission taxes. In Home, the government's budget constraint reads:

$$p_{Ht}g_t = t_t + \tau_t e_t,$$

and we assume that government purchases are fully biased toward domestic goods. We also set  $g_t$  constant along a balanced growth path (i.e.  $g_t = z_t^W g$ ). When revenues from carbon taxation change, lump-sum taxes are adjusted accordingly. Analogous conditions hold in Foreign.

## 2.7 Market clearing

The market clearing condition for Home final output reads:

$$y_{Ht} = (c_{Ht} + i_{Ht} + i_{Ht}^* + AB_{Ht} + g_t) + \frac{1-n}{n} (c_{Ht}^* + i_{*Ht}^* + i_{*Ht} + AB_{Ht}^*). \quad (36)$$

The right-hand side of the equation consists of two blocks. The first block includes Home output that is used inside the Home economy: for Home consumption, for Home investment in Home firms, for Foreign investment in Home firms, for paying abatement

costs in Home brown firms, and for Home public spending. The second block includes exports: for Foreign consumption, for Foreign investment in Foreign firms, for Home investment in Foreign firms, for paying abatement costs in Foreign brown firms.<sup>6</sup> A similar condition holds for the Foreign final good:

$$y_{Ft}^* = (c_{Ft}^* + i_{*Ft}^* + i_{*Ft} + AB_{Ft}^* + g_t^*) + \frac{n}{1-n} (c_{Ft} + i_{Ft} + i_{Ft}^* + AB_{Ft}). \quad (37)$$

Market clearing in labor market requires:

$$h_t = 1 = h_{Gt} + h_{Bt} \quad (38)$$

$$h_t^* = 1 = h_{Gt}^* + h_{Bt}^*, \quad (39)$$

given that hours of work are supplied inelastically.

Home bonds are purchased by Home financiers, Foreign financiers, Home noisy traders, and Home households, and they are in zero net supply:

$$nd_{Ht} + (1-n)d_{Ft} + nv_t + nb_t = 0. \quad (40)$$

The same holds for Foreign bonds (remind that noisy traders belong to Home households):

$$(1-n)d_{Ft}^* + nd_{Ht}^* + nv_t^* + (1-n)b_t^* = 0. \quad (41)$$

Given that financiers operate without capital, the two previous conditions imply that  $nb_t + (1-n)s_tb_t^* = 0$ .

Finally, we derive the standard equation linking the trade balance to the net financial asset position (NFA) of the Home economy. We define the trade balance  $tb_t \equiv xp_t - mp_t$

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<sup>6</sup>Even if  $i_{Ht}^*$  is purchased by Foreign households, it is not an export for Home, as the resulting capital is used within the Home economy.

where  $xp_t$  and  $mp_t$  are the value of Home exports and imports:

$$xp_t \equiv \frac{1-n}{n} p_{Ht} (c_{Ht}^* + i_{*Ht}^* + i_{*Ht} + AB_{Ht}^*) \quad (42)$$

$$mp_t \equiv p_{Ft} (c_{Ft} + i_{Ft} + i_{Ft}^* + AB_{Ft}). \quad (43)$$

We define the Home total FDI assets ( $K_{*t}$ ) and liabilities ( $K_t^*$ ) as follows:

$$K_{*t} \equiv \frac{1-n}{n} (k_{*Gt} + k_{*Bt}) \quad (44)$$

$$K_t^* \equiv k_{Gt}^* + k_{Bt}^*. \quad (45)$$

Given these definitions, the evolution of the NFA reads:

$$\begin{aligned} b_t + s_t K_{*t} - K_t^* = & tb_t + r_{t-1}^* s_t (d_{Ht-1}^* + v_{t-1}^*) + \frac{(1-n)}{n} r_{t-1} s_{t-1} d_{Ft-1}^* + \\ & + s_t \frac{1-n}{n} [(r_{*kGt} + 1 - \delta) k_{*Gt-1} + (r_{*kBt} + 1 - \delta) k_{*Bt-1}] + \\ & - [(r_{kGt}^* + 1 - \delta) k_{Gt-1}^* + (r_{kBt}^* + 1 - \delta) k_{Bt-1}^*]. \end{aligned} \quad (46)$$

The left hand side features the Home NFA position at time  $t$ : it consists of the debt NFA  $b_t$  plus the equity NFA  $K_{*t} - K_t^*$ . The NFA is financed by the five terms on the right hand side: i) the trade balance; ii) the return on investment in Foreign bonds (made by Home financiers and traders), which yields the Foreign interest rate  $r_t^*$  and bears exchange rate risks; iii) the return  $r_t$  paid by Foreign financiers when they are short in Home bonds; iv) the return gained on FDI green and brown assets, net of depreciation (Home bears exchange rate risk); v) the return net of depreciation on FDI green and brown liabilities, paid to Foreign households, who bear the exchange rate risk.

## 2.8 Equilibrium

In order to make the model stationary, we divide variables with a trend by the labor productivity  $z_t^W$ .<sup>7</sup> The model consists of 50 equations for 50 endogenous variables. There are 2 exogenous variables, the emission taxes  $\{\tau_t, \tau_t^*\}$ . We solve the model assuming perfect foresight. We list the equations in Appendix A.

## 3 Analysis: the symmetric case

In this section, we adopt a symmetric calibration with two identical large economies. This symmetric framework is useful to better understand the mechanics of our model. We then carry out a normative analysis: for different values of  $\psi_X$  – the parameter capturing the curvature of pollution externality – we compute the reaction function of one country to the carbon tax set by the other country; we also find the Nash and the world social planner equilibria. Finally, we simulate the international spillover effects of carbon taxes, when they are set optimally and non-optimally.

### 3.1 Calibration

One period corresponds to one year. The population share  $n$  is equal to 0.5. We normalize Home total factor productivity  $A = 1$ , and by symmetry  $A^* = 1$ . We follow the IMF integrated Policy Framework (Adrian et al., 2021) and set the elasticity of substitution between Home and Foreign goods ( $\eta$ ) equal to 1.5; the shallowness of FX markets ( $\Gamma$ ), equal to 0.02; the steady-state public spending share of GDP equal to 14% (this implies  $g = g^* = 0.1634$ ). As standard, we set the capital depreciation rate ( $\delta$ ) to 10%. Given the symmetry assumption, we set the noisy-traders investment to  $v = 0$ , which implies a debt NFA position equal to 0 in both countries. Trade openness is set to a relatively low value ( $\gamma = \gamma^* = 0.15$ ), as these countries are not small economies.

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<sup>7</sup>Variables with a trend are all types of output, consumption, capital, investment, bonds, emission, pollution, and wages.

We calibrate the values for the interest rate in the two countries ( $r - 1$ ,  $r^* - 1$ ) equal to 2.5%: given our choice of labor productivity  $\iota$  (justified below), these values imply  $\beta = \beta^* = 0.9888$ .

Regarding the parameters of the utility function of green and brown bonds, we follow the empirical evidence in Liberati and Marinelli (2021), and we calibrate ex ante the green premium  $r_{kG} - r_{kB}$  to  $-5$  basis points in both countries, imposing that green (brown) bonds yield a premium of minus (plus) 2.5 basis points compared to public bonds: this implies  $\kappa_G = 5.0271 \cdot 10^{-4}$  and  $\kappa_B = 4.3363 \cdot 10^{-4}$ , in both countries. We assume that the utility function of green bonds is logarithmic and the disutility function of brown bonds is quadratic ( $\psi_G = \psi_B = 1$ ). Using the World Development Indicators Database of the World Bank, we calculate the average investment to GDP ratio (equal to 22% in 2020). Relying on the IMF Coordinated Direct Investment Survey, we set the initial stock of FDI assets equal to around 8.6% of GDP in both countries. Using these quantities, we obtain ex post  $\alpha_1 = \alpha_1^* = 0.2319$  and  $\alpha_2 = \alpha_2^* = 0.0106$ .

Following Carattini et al. (2023a), we assume a convexity parameter in the abatement cost ( $\nu$ ) equal to 1.6, a share of brown goods ( $\zeta$ ) equal to 0.332, and an elasticity of substitution between sectors ( $\xi$ ) equal to 2. We use the same values of Hassler et al. (2016) for the law of motion of pollution, adjusted for an annual calibration ( $\phi_L = 0.2$ ,  $\phi = 0.0023$ ,  $\phi_0 = 0.4010$ ). We set  $z_e = 26.1388$ , such that the initial global emissions are 9.54 GtC, the value in 2020. We assume that the price of one ton of Co2 – the so-called price of carbon – such that all emissions are abated is equal to 514\$, in line with Barrage and Nordhaus (2023): this implies  $\kappa_A = 0.6589$ .

We assume a baseline policy identical to that of Barrage and Nordhaus (2023): starting from 2020, the countries set their initial carbon tax such that the abatement rate is 5% (this implies  $\tau = \tau^* = 2.09 \cdot 10^{-4}$ ). Then, the carbon tax increases by 1% annually until 2200, when the abatement rate is set to 1. We calibrate the long-run gross productivity growth to  $\iota = 1.0135$ : this value ensures that, under the baseline policy, atmospheric carbon evolves similar to that in Barrage and Nordhaus (2023) (Figure 1).

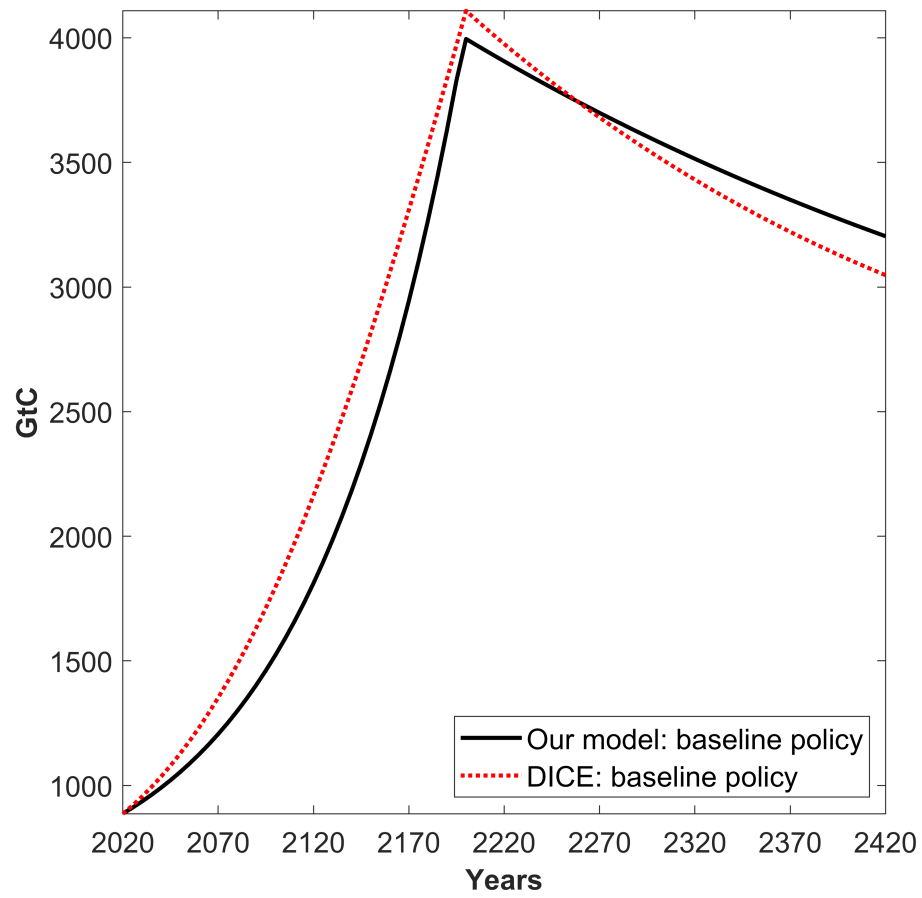


There is huge uncertainty about the disutility arising from high level of pollution. Hassler et al. (2016) consider the linear disutility as the benchmark, but they argue that a convex disutility is more realistic (see the discussion in Section 4.7 in Hassler et al., 2016). We explore different values for the curvature of the pollution disutility:  $\psi_X$ : 0 (linear disutility), 0.5, and 1 (quadratic disutility). The pollution disutility shifter  $\kappa_X$  is set such that in period 0 the disutility from this specification is equivalent to the disutility in Hassler et al. (2016) (see Section 4.6 of that paper):

$$\kappa_X = \gamma^{HKS} (1 + \psi_X) (x_0^W - \bar{x}^W)^{-\psi_X},$$

where  $\gamma^{HKS} = 5.3 \cdot 10^{-5}$  as in Hassler et al. (2016),  $x_0$  is the 2020 level of atmospheric carbon, which we set to 887 GtC as in Barrage and Nordhaus (2023), and  $\bar{x}$  is the pre-industrial level of carbon (equal to 581 GtC). Table 1 reports the chosen parameter values, while Table 2 illustrates the calibrated initial values for the main variable of the model.

## Our model vs Dice: Atmospheric carbon (GtC)



**Figure 1:** The path of atmospheric carbon in the baseline policy in our model (black solid line) and in the DICE model (red dotted line, Barrage and Nordhaus, 2023).

## Calibration: parameters

Parameters	Description	Value
$n$	Population share	0.5
$\beta, \beta^*$	Home and Foreign discount factor	0.9888, 0.9988
$\psi_G, \psi_B$	Curvature of green (brown) asset (dis)utility	1, 1
$\kappa_G, \kappa_G^*$	Weight of green asset utility	$5.0271 \cdot 10^{-4}, 5.0271 \cdot 10^{-4}$
$\kappa_B, \kappa_B^*$	Weight of brown asset disutility	$4.3363e \cdot 10^{-4}, 4.3363e \cdot 10^{-4}$
$\xi$	El. of subst. btw green and brown goods	2
$\zeta$	Share of brown output	33.2%
$\gamma, \gamma^*$	Home and Foreign import quasi-share	0.15, 0.15
$\eta$	El. of subst. domestic vs foreign good	1.5
$\alpha_1, \alpha_1^*$	Share of domestic capital in production	0.2319, 0.2319
$\alpha_2, \alpha_2^*$	Share of external capital in production	0.0106, 0.0106
$\delta$	Depreciation rate of capital	10%
$\iota - 1$	Labor productivity growth	1.35%
$\nu$	Convexity parameter in the abatement cost	1.6
$\kappa_A$	Proportionality constant in the abatement cost	0.6589
$\varphi_L$	Emissions that do not depreciate	20%
$\varphi_0$	Emissions that do not leave the atmosphere in 1 year	40%
$\varphi$	Depreciation rate of remaining emissions	0.23%
$\Gamma$	Shallowness of FX markets	0.02
$g, g^*$	Public spending in SS	0.1634, 0.1634
$z_e$	Emission shifter	26.1388
$v$	Noisy traders	0
$A, A^*$	TFP	1, 1
$\bar{x}^W$	Pre-industrial atmospheric carbon (GtC)	581
$\psi_X$	Pollution disutility convexity	$\{0, 0.5, 1\}$
$\kappa_X$	Pollution disutility shifter	$\{5.30 \cdot 10^{-5}, 4.54 \cdot 10^{-6}, 3.46 \cdot 10^{-7}\}$

**Table 1:** Calibrated parameters

## Calibration: initial values

SS Values	Description	Value
$g/y_H$	Public-spending/GDP	14%
$b/gdp$	Bond NFA/GDP	0%
$\frac{s(1-n)k_*}{n \cdot p_H y_H}, \frac{nk_*}{(1-n)p_F y_F^*}$	FDI/GDP	8.6%, 8.6%
$\frac{i+i^*}{gdp}, \frac{i_*+i_*}{gdp^*}$	Investment/GDP	22%, 22%
$r, r^*$	Interest rate	2.5%, 2.5%
$r_{kB} - r, r_{*kB}^* - r$	Spread brown-public bonds	2.5, 2.5 <i>b.p.</i>
$r_{kG} - r, r_{*kG}^* - r$	Spread green-public bonds	-2.5, -2.5 <i>b.p.</i>
$x^W$	Pollution (GtC)	887
$n \cdot e + (1-n)e^*$	Emissions (GtC)	9.54
$carb.price$	Carbon price under $\mu = 1$ (\$/tCO <sub>2</sub> )	514

**Table 2:** Calibrated initial values.

### 3.2 Normative analysis

Golosov et al. (2014) and Hassler et al. (2016) show that the optimal carbon tax per unit of emission is a linear function of GDP. This result is also robust to different assumptions (Barrage, 2014). In our framework, deriving the optimal environmental policy is computationally unfeasible, but we think that a carbon tax proportional to GDP is a good approximation. Thus, rather than compute a general optimal carbon policy, we focus on the following simple rule:

$$\tau_t = \min \left( \frac{\kappa_A}{z_e}, \tau + F z_t^W \right) \quad (47)$$

$$\tau_t^* = \min \left( \frac{\kappa_A}{z_e}, \tau^* + F^* z_t^W \right) \quad (48)$$

given that  $z_t^W$  is the largest driver of GDP in the model (the other ones are the taxes themselves). The min operator is necessary: if the tax is higher than  $\frac{\kappa_A}{z_e}$ , the abatement rate is higher than one (see equation 24).<sup>8</sup> We assume that the rule is in place until 2200:

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<sup>8</sup>Assuming that the tax responds to an exogenous variable like labor productivity greatly reduces the computational burden.

from 2200 on taxes are such that the abatement rate is 1, as in the baseline policy. The Home and Foreign policy makers choose coefficients  $F$  and  $F^*$ , respectively, to maximise the utility function of their households.

In Figure 2, we show the optimal  $F$  of Home given that of Foreign, for three values of the pollution externality  $\psi_X$ . When the disutility is linear, carbon taxes are independent (blue solid line), as the marginal cost of pollution is constant: tax choices by Foreign do not affect the marginal cost of pollution, hence they do not affect the optimal Home tax.

When the disutility is convex (red dotted and black dashed line), carbon taxes are strategic substitutes: the higher the tax set by Foreign, the lower the Home optimal tax. This occurs as a convex pollution disutility implies a marginal pollution cost that is increasing in pollution itself: when a country increases the carbon tax, it reduces global pollution and its marginal cost, inducing the other country to decrease the carbon tax. When the disutility is convex, a country that is reluctant to set taxes (for irrationality, myopia, or political economy reasons) induces the other country to bear the entire burden of the green transition. The case of a convex disutility rationalizes the current geopolitical situation, in which advanced economies are discussing the implementation of environmental policies, while most emerging countries hesitate, prioritizing development goals instead. In Appendix B, we set up a static model to derive analytically these intuitions.

In Table 3 we show that the tax in the Nash equilibrium is inefficiently low, for the three values of  $\psi_X$  considered: a social planner maximising global welfare would choose a higher value of taxation.

For selected variables, we plot the evolution under the Nash equilibrium (Figure 3, left panel) and the social planner equilibrium (Figure 3, right panel). A linear pollution disutility requires a lower carbon price (blue solid line), relative to the scenario with  $\psi_X = 0.5$  (red dotted line) and with quadratic disutility (black dashed line): under linear utility (and so under lower carbon taxes) the transition is much slower.<sup>9</sup>

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<sup>9</sup>We are assuming that from 2200 on the tax is such that all emissions are abated: this is why we observe a discontinuity around 2200 in some cases (in other cases, full abatement is reached before 2200).

We also quantitatively assess the welfare gain in the social planner allocation. We define with  $\Omega$  ( $\Omega^*$ ) the share of steady-state consumption that Home (Foreign) households should receive under the Nash equilibrium to obtain the same utility as in the social planner equilibrium:

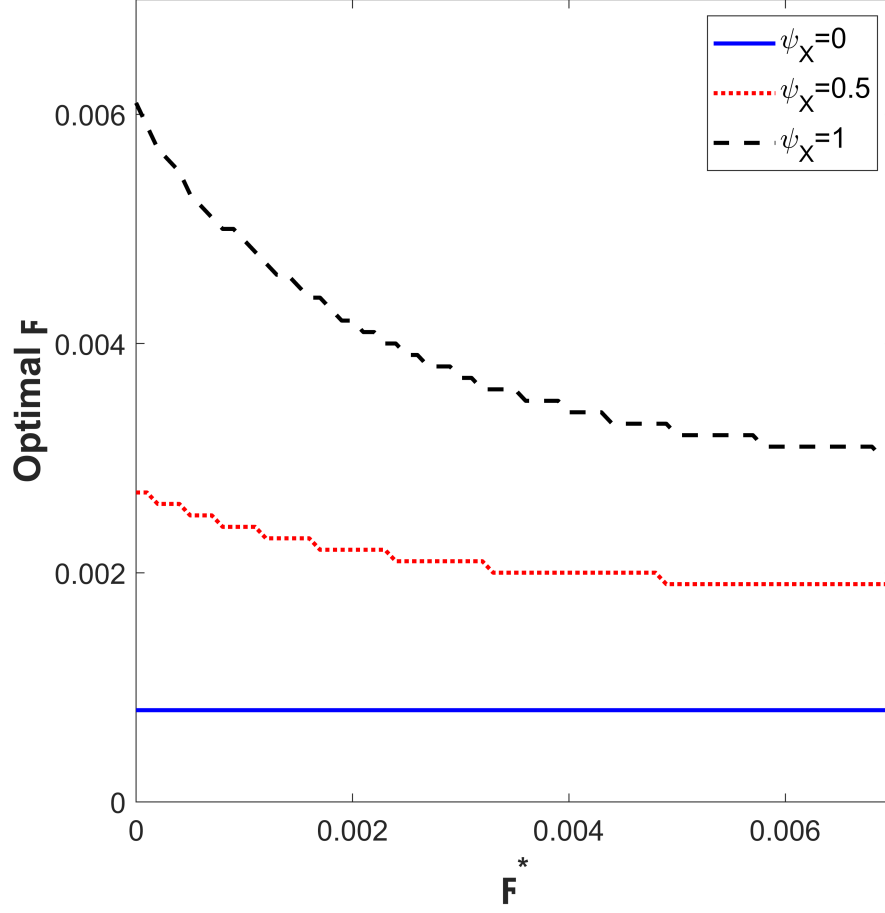
$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ U \left( (1 + \Omega) c_{t+j}^n, \tilde{K}_{Gt}^n, \tilde{K}_{Bt}^n, x_t^{nW} \right) - U \left( c_{t+j}^n, \tilde{K}_{Gt}^n, \tilde{K}_{Bt}^n, x_t^{nW} \right) \right] \right\} = W_t^s - W_t^n,$$

where the index  $n$  refers to the Nash-equilibrium allocation,  $W_t^n$  and  $W_t^s$  are the Home welfare levels in the Nash and in the social planner equilibria.<sup>10</sup> The higher the consumption equivalent  $\Omega$ , the larger the welfare gap between the Nash and the social planner equilibria. When the disutility is more convex, on the one hand the consumption equivalent should be lower, as the pollution reduction obtained by moving from the Nash to the social planner equilibrium is smaller (Figure 3). On the other hand, when the disutility is more convex, reducing pollution yields larger welfare gains: we find that this channel prevails when we move from  $\psi_X = 0$  to  $\psi_X = 0.5$ , while the two channels offset each other when we move from  $\psi_X = 0.5$  to  $\psi_X = 1$  (Table 3, last column). The higher welfare obtained by a global social planner relative to the Nash equilibrium means that there is scope for coordination in setting optimal carbon taxes across countries: in other words, should the two countries coordinate in choosing their carbon taxes, they would be able to achieve higher levels of well-being.

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<sup>10</sup>Given the log-utility assumption, we get  $\Omega = \exp((1 - \beta)(W_t^c - W_t^n)) - 1$ .

## Optimal domestic tax given foreign tax



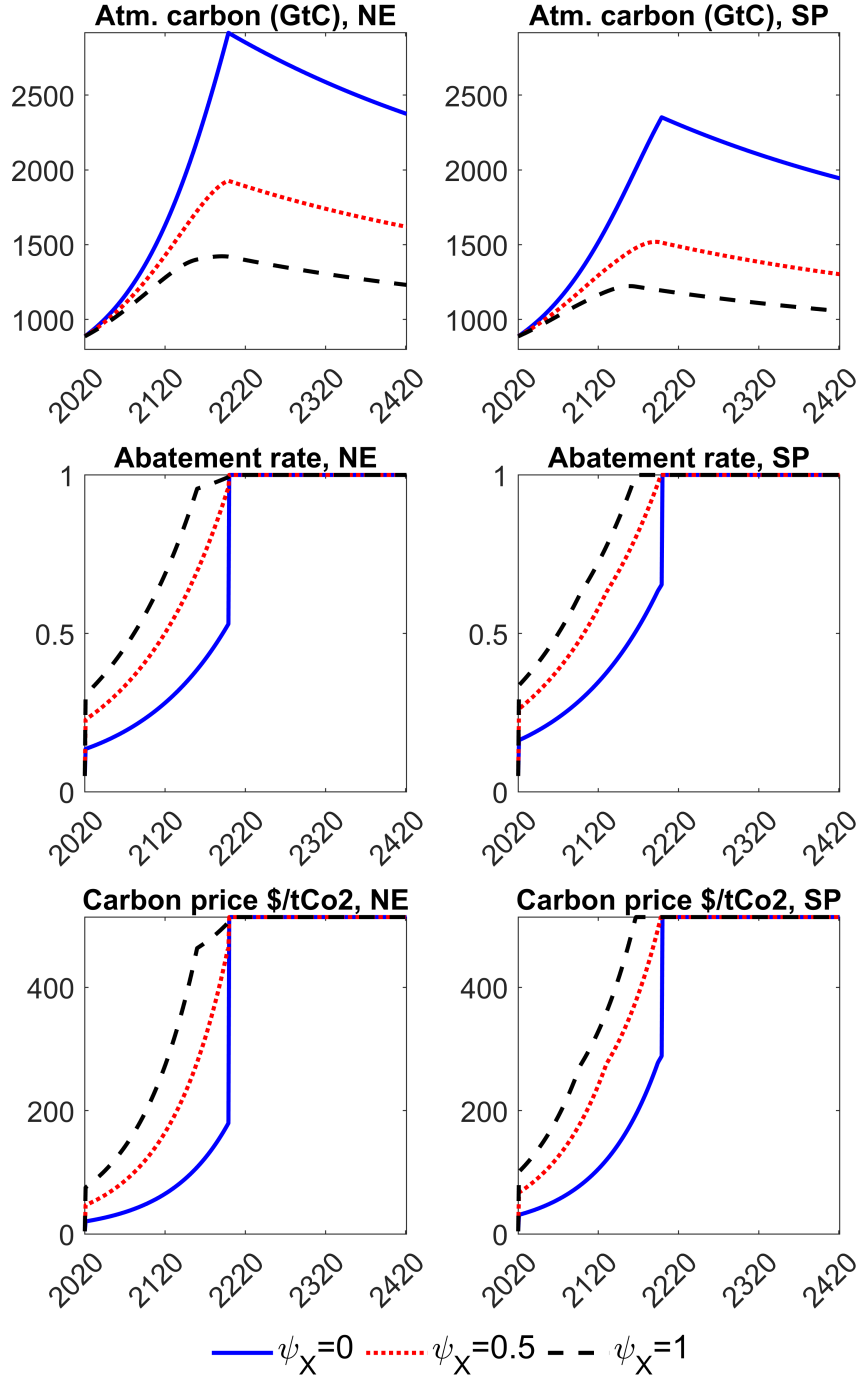
**Figure 2:** Optimal  $F$  given  $F^*$ , under three different values for  $\psi_X$ .

## Optimal carbon taxes

Externality	Nash	Social Planner	CEV
$\psi_X = 0$	$F = F^* = 0.0008$	$F = F^* = 0.0015$ ,	$\Omega = \Omega^* = 0.19\%$
$\psi_X = 0.5$	$F = F^* = 0.0022$	$F = F^* = 0.0035$	$\Omega = \Omega^* = 0.45\%$
$\psi_X = 1$	$F = F^* = 0.0035$	$F = F^* = 0.0051$	$\Omega = \Omega^* = 0.46\%$

**Table 3:** Nash and social planner equilibrium under different externality assumptions. CEV denotes gains or losses, in consumption equivalent terms, under the social planner allocation with respect to the Nash equilibrium.

## Nash vs social planner equilibrium



**Figure 3:** Nash equilibrium (NE, left panel) and social planner equilibrium (SP, right panel), under different  $\psi_X$ .



### 3.3 Positive analysis

What are the macroeconomic consequences for the global economy if only some countries adopt optimal environmental policies? What are the effects on bond and FDI flows? We assume that Foreign follows the sub-optimal baseline policy, whereby the initial carbon tax is such that the abatement rate is 5%; thereafter, the carbon tax increases by 1% annually until 2200, when the abatement rate is set to 1. Home sets a carbon tax as in equation (47), with factor of proportionality  $F = 0.0059$ , which is the optimal response to the baseline policy adopted by Foreign, given the simple rule. We plot variables in percentage deviations from a scenario where Home also follows the baseline policy (Figures 4- 5), comparing Home (blue solid line) and Foreign variables (red dotted line), focusing on the first 60 years of the transition.

The increase in the Home carbon tax depresses output in the brown sector. Production partially shifts to the green sector (Home domestic green capital rises) but not enough to avoid a fall in GDP and, as a result, in consumption, compared to the scenario where also Home follows the baseline policy. Given that the Home carbon tax increases over time as it follows productivity growth, Home GDP, consumption, and capital gradually decrease toward a new lower steady state. The gradual reduction in consumption decreases the Home real interest rate on bonds, as households expect to be poorer in the future: this mechanism resembles the channel shown in the closed economy setting of Ferrari and Nispi Landi (2022), where the green transition reduces the natural interest rate in a textbook New Keynesian model, causing deflation. The lower Home interest rate induces financiers to buy Foreign and to sell Home bonds, improving the Home bond NFA: Home bond outflows increase. The persistent fall of the Home interest rate below the Foreign interest rate drives a gradual expected Home real appreciation over time. Given that international bond markets are frictional, the arbitrage between Home and Foreign bonds is not perfect (equation 35): the resulting reduction in the Foreign rate and the Home expected appreciation (a lower future exchange rate) is not enough to close the interest rate premium  $r_t^* \frac{s_{t+1}}{s_t} - r_t$ , which is higher. As brown firms demand less capital, brown

FDI liabilities fall in Home ( $k_{Bt}^*$  goes down), and are only partially offset by an increase in green FDI assets ( $k_{Gt}^*$  goes up).<sup>11</sup> Home FDI assets slightly increase, both in the green and in the brown sector ( $k_{*Bt}$  and  $k_{*Gt}$  rise).<sup>12</sup> The Home bond and FDI capital outflows are matched with an improvement in the Home trade balance.

The spillover effect on Foreign is overall negative, though the production fall is much smaller compared to Home's. On the one hand, Foreign experiences capital inflows, which sustain production and consumption. On the other hand, Foreign faces a lower demand from Home: this latter channel is stronger after a few periods, contributing to lower production and consumption in the medium term. This also implies that the increase in Home brown FDI assets (i.e higher external capital for Foreign brown firms) is offset by a reduction in domestic investment in Foreign brown firms (i.e lower Foreign capital for Foreign brown firms): Foreign emissions barely move.<sup>13</sup>

We now consider the case of the Nash equilibrium under quadratic disutility, where countries set  $F = F^* = 0.0036$  (Figure 6, red dotted line), comparing it with the previous scenario, where Home follows the optimal and Foreign follows the baseline policy (Figure 6, blue solid line). When both countries optimize, global pollution falls much more, compared to the scenario where Foreign follows the baseline policy. The Home reduction in economic activity is milder compared to the previous scenario, as now Home carbon taxes are lower. Given our symmetry assumption, the macroeconomic response is the same in the two countries and net positions, such as Home bond net assets, do not move. For the same reason, the real exchange rate remains constant.

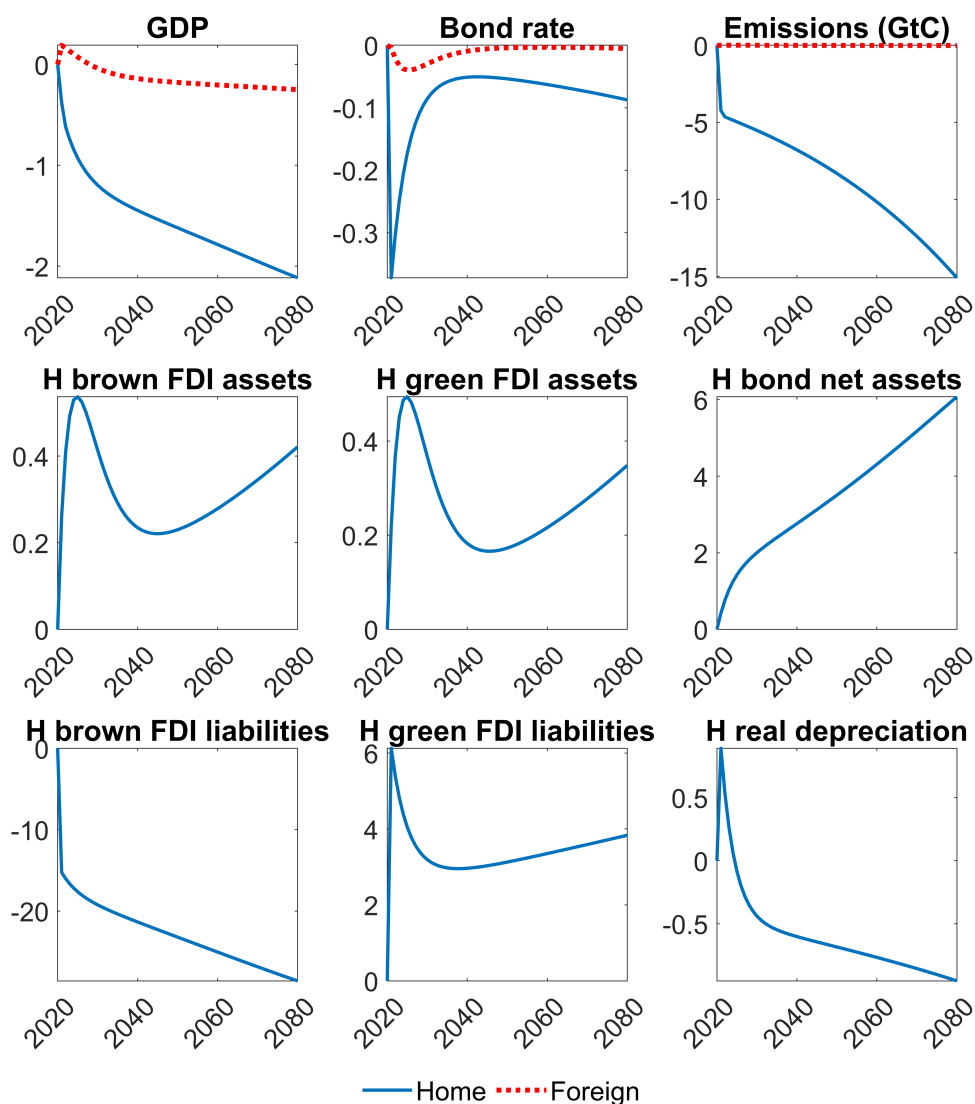
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<sup>11</sup>This implies that Foreign FDI assets decrease in the brown sector and increase in the green sector, as Home FDI assets correspond Foreign FDI liabilities.

<sup>12</sup>This implies that Foreign FDI liabilities decrease.

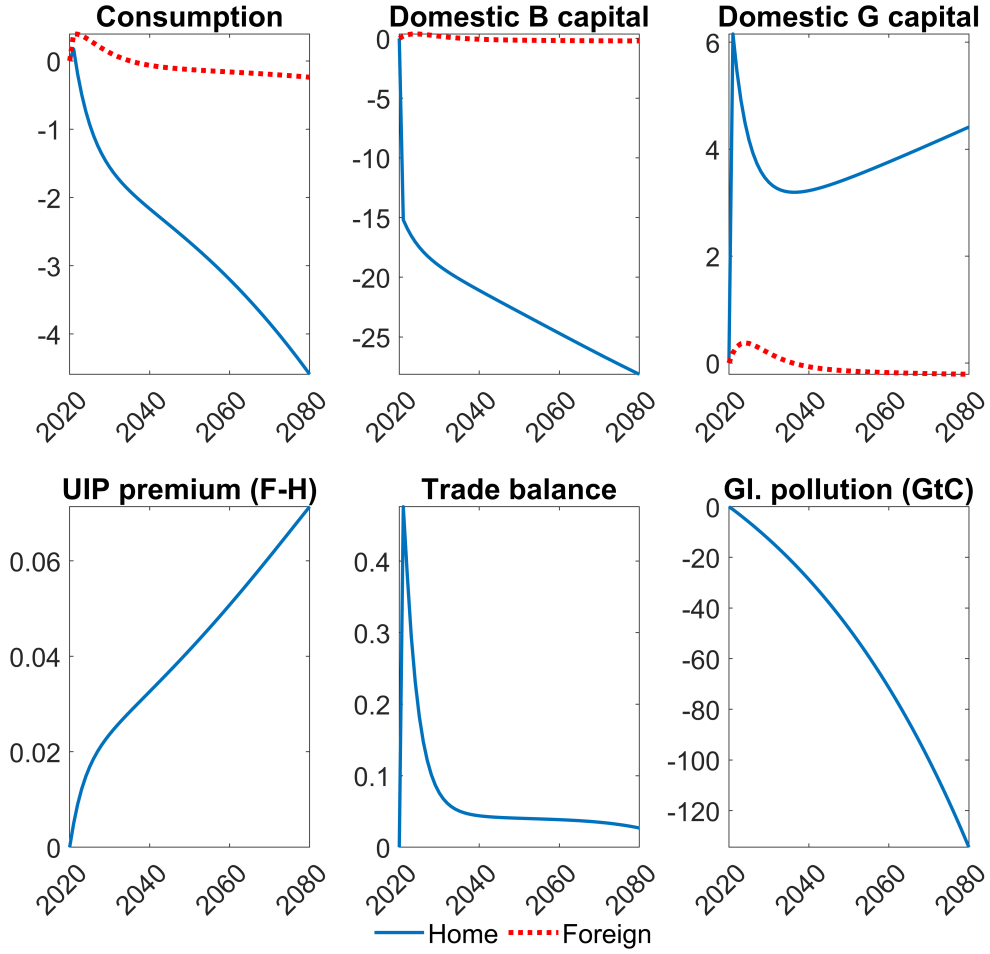
<sup>13</sup>The spillover effect on Foreign would be more benign if the elasticity of substitution  $\eta$  (calibrated to 1.5) would be higher: in this case Home households would find it optimal to greatly increase demand of the Foreign good (that would be a closer substitute to the Home good), boosting Foreign production.

## Macroeconomic effects of the transition: Asymmetric policies (1)



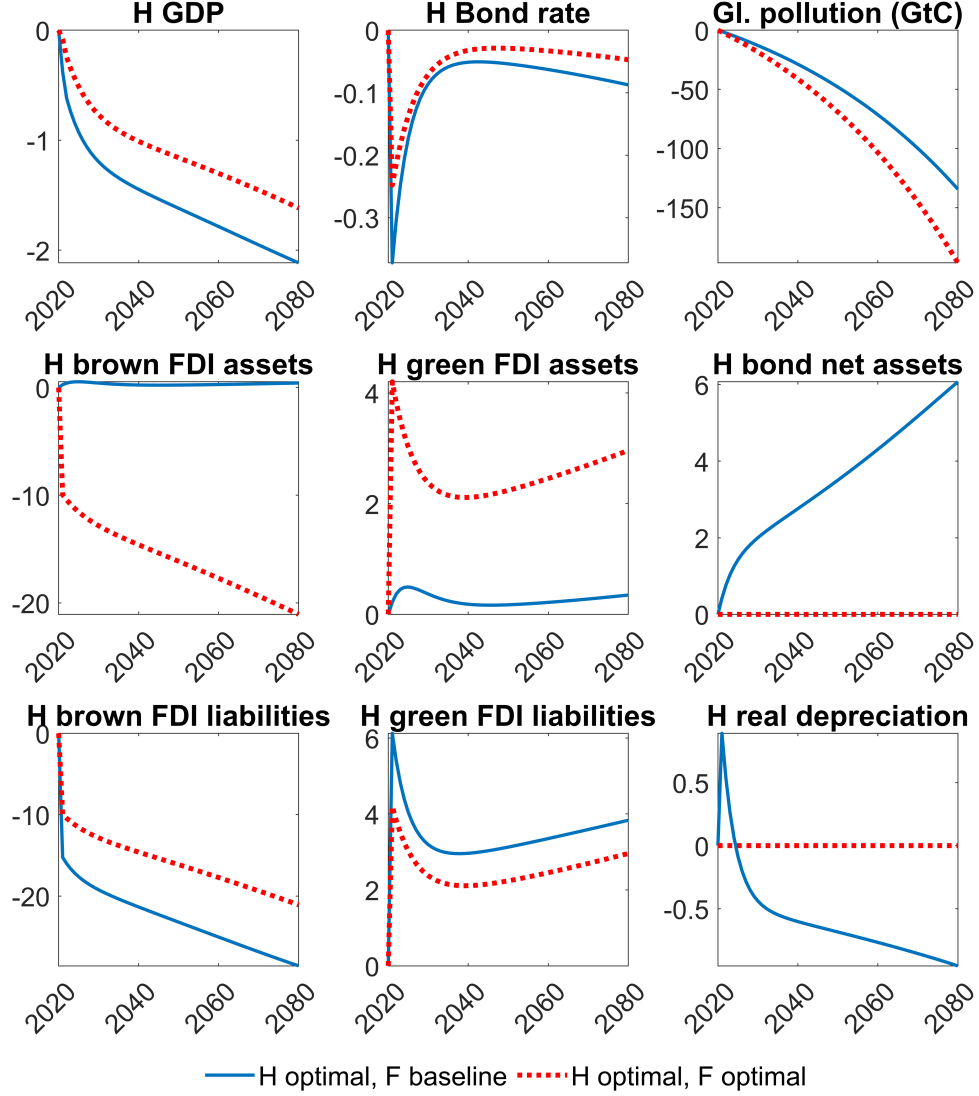
**Figure 4:** Variables are plotted in percentage deviation from the baseline scenario, except for the bond rate and emissions (plotted in level deviations from the baseline scenario), and bond assets (in level deviations from the baseline scenario, as a share of initial GDP). An increase in the Home real exchange rate means a Home real depreciation. Country F follows the baseline policy, country H sets optimally  $F = 0.0059$ .

## Macroeconomic effects of the transition: Asymmetric policies (2)



**Figure 5:** Consumption and capital are plotted in percentage deviation from the baseline scenario, pollution and UIP premium are plotted in level deviations from the baseline scenario, the trade balance is in deviations from the baseline scenario as a share of initial GDP). The UIP premium is defined as  $r_t^* \frac{s_{t+1}}{s_t} - r_t$ . Country F follows the baseline policy, country H sets optimally  $F = 0.0059$ . Red dotted line: Nash equilibrium under  $\psi_X = 1$  ( $F = F^* = 0.0035$ ).

## Macroeconomic effects of the transition: Symmetric vs asymmetric policies



**Figure 6:** Variables are plotted in percentage deviation from the baseline scenario, except for the bond rate and emissions (plotted in level deviations from the baseline scenario), and bond assets (in level deviations from the baseline scenario, as a share of initial GDP). An increase in the Home real exchange rate means a Home real depreciation. Blue line: country F follows the baseline policy, country H sets optimally  $F = 0.0059$ . Nash equilibrium under  $\psi_X = 1$  ( $F = F^* = 0.0035$ ).

## 4 Analysis: the asymmetric case

In this section, we repeat both the normative and positive analysis considering an asymmetric calibration: in this new framework, we interpret Home and Foreign as region of advanced and emerging economies respectively, that choose the same environmental policy within the region.

### 4.1 Calibration

Compared to the symmetric calibration, we change some selected parameters, which we describe as follows. According to the World Economic outlook (WEO), in 2020 the population share in advanced economies is 14%: we set  $n = 0.14$ . In 2020, the GDP per capita (based on PPP) in advanced economies is 4.47 times that of emerging markets (WEO): we set  $A^* = 0.5998$  and  $A = 2.2647$ .<sup>14</sup> We set the real interest rate in Foreign ( $r^*$ ) equal to 0.03 (which implies  $\beta^* = 0.9840$ ). Following the IPF, we set the shallowness of Foreign FX markets ( $\Gamma^*$ ) equal to 0.06 and we calibrate the Home NFA to 22% of Home GDP, which gives the long position of noisy traders  $v = -0.0320$ . Using the World Development Indicators of the World Bank, we set the investment over GDP ratio in Foreign equal to 33%. The stock of FDI outflows over GDP ratio in Foreign is equal to 4.5% using the IMF Coordinated Direct Investment Survey. Given these quantities, we calibrate ex post  $\alpha_1^* = 0.3704$  and  $\alpha_2^* = 0.0077$ . Using the dataset of “Our World in Data”, in 2020 emerging economies account for 70.6% of global CO2 emissions, which gives  $z_e = 18.4249$  and  $z_e^* = 31.7149$ : this implies that advanced economies pollute less, given abatement and brown production. Table 4 reports the parameters and the initial values that are different compared to the symmetric calibration.<sup>15</sup>

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<sup>14</sup>In this way we get the same world GDP arising in the symmetric calibration.

<sup>15</sup>As in the symmetric calibration, some parameters are calibrated ex post, in order to target ex ante some initial values. This implies that these parameters may change their value in the asymmetric calibration (see for instance  $\kappa_A$ ).

## Calibration: parameters and initial values

Parameters	Description	Value
$n$	Population share	0.14
$\beta, \beta^*$	Home and Foreign discount factor	0.9888, 0.9840
$\kappa_G, \kappa_G^*$	Weight of green asset utility	$5.0680 \cdot 10^{-4}$ , $9.0470 \cdot 10^{-4}$
$\kappa_B, \kappa_B^*$	Weight of brown asset disutility	$4.7756e \cdot 10^{-5}$ , $7.6812 \cdot 10^{-4}$
$\gamma, \gamma^*$	Home and Foreign import quasi-share	0.1736, 0.1264
$\alpha_1, \alpha_1^*$	Share of domestic capital in production	0.2349, 0.3704
$\alpha_2, \alpha_2^*$	Share of external capital in production	0.0079, 0.0077
$\kappa_A$	Proportionality constant in the abatement cost	0.7215
$\Gamma, \Gamma^*$	Shallowness of FX markets	0.02, 0.06
$g, g^*$	Public spending in SS	0.4870, 0.1107
$z_e, z_e^*$	Emission shifter	18.4249, 31.7149
$v$	Noisy traders	-0.0320
$A, A^*$	TFP	2.2647, 0.5998
$b/gdp$	Bond NFA/GDP	22%
$\frac{s(1-n)k_*}{n \cdot p_H y_H}, \frac{nk^*}{(1-n)p_F y_F^*}$	FDI/GDP	8.6%, 4.5%
$\frac{i+i^*}{gdp}, \frac{i_*^*+i_*}{gdp^*}$	Investment/GDP	22%, 33%
$r, r^*$	Interest rate	2.5%, 3.0%
$\frac{ne}{ne+(1-n)e^*}$	Home emission share	29.4%

**Table 4:** Calibrated parameters and initial values

## 4.2 Normative analysis

We repeat the normative analysis carried out under the symmetric calibration. As in the symmetric case, when the disutility is linear, carbon taxes are independent, as the marginal cost of pollution is constant (Figure 7, blue solid line). When the disutility is convex (red dotted and black dashed line), carbon taxes are strategic substitutes.

The Home and the Foreign reaction functions now have a different shape when the disutility is convex; in particular, the Home function is steeper. On the one hand, when Foreign sets a low carbon tax, Home sets a relatively higher tax compared to what

Foreign does when Home sets a low tax: assuming  $\psi_X = 1$ ,  $F \approx 0.007$  when  $F^* = 0$ , while  $F^* \approx 0.004$  when  $F = 0$ . This occurs as Home is a richer country in GDP per-capita terms, with a lower marginal utility of consumption compared to Foreign: if Foreign sets a loose environmental policy, Home has a bigger incentive to sacrifice consumption and reduce pollution. If Home sets a loose environmental policy, the incentive of Foreign to sacrifice consumption is much smaller. On the other hand, when Foreign sets a high carbon tax, Home sets a relatively lower tax compared to what Foreign does when Home sets a high tax. This occurs because emerging economies (i.e. Foreign) tend to pollute more than advanced countries (i.e. Home):<sup>16</sup> a relatively high tax set by Foreign is able to greatly reduce pollution costs, decreasing the incentive for Home to set relatively high taxes.

The Nash equilibrium is very close to the symmetric case (Table 5 vs Table 3), with Foreign setting a slightly larger tax compared to Home: this crucially depends on the assumption that Foreign represents all emerging economies as a unique homogeneous block that is responsible for the majority of CO2 emissions.

A global social planner that maximizes world utility internalizes the fact that Home is richer and with a relatively low population share: the planner prefers to set high taxes in Home to not further penalize consumption in Foreign. This implies that Home households are worse off compared to the Nash equilibrium.

We also consider a scenario — labeled as coordination equilibrium — where the social planner maximises global welfare subject to the constraint that economies are not worse off compared to the Nash equilibrium.<sup>17</sup> Under this hypothesis, the Home tax is lower compared to the social planner equilibrium, in order to make Home's welfare as high as under the Nash equilibrium. This also implies that Foreign sets a higher carbon tax.

For selected variables, we plot the evolution under the Nash equilibrium (Figure 8, left panel) and the social planner allocation (Figure 8, right panel). Results are in line

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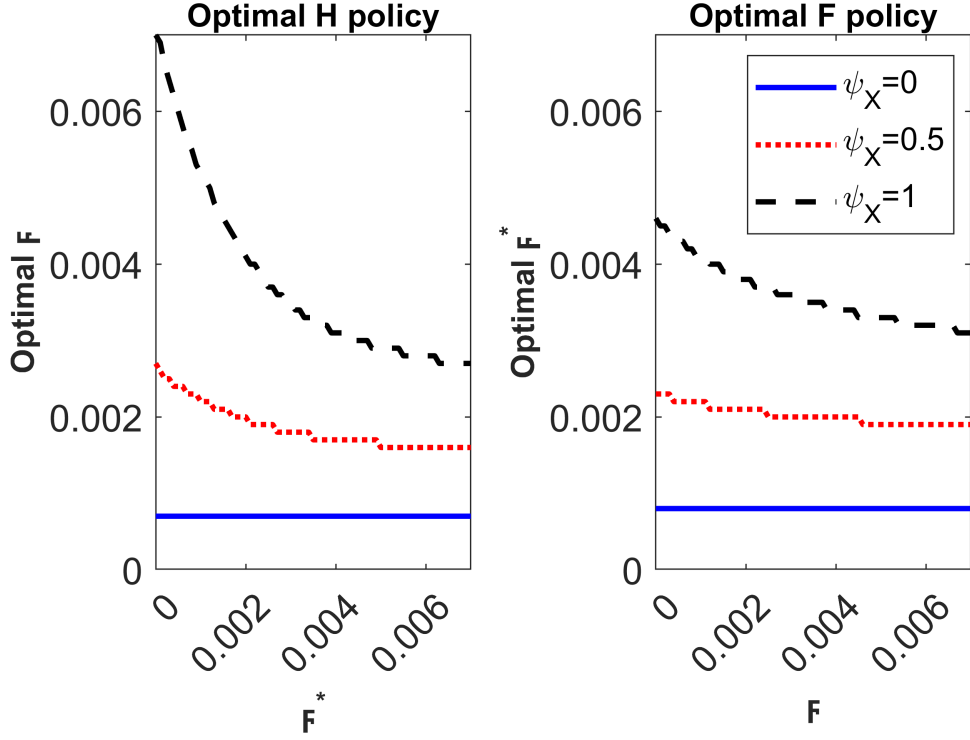
<sup>16</sup>In the calibration, emissions in advanced economies account for around 30% of global emissions.

<sup>17</sup>Under the symmetric calibration, this constraint is never binding, as the countries are equal. This implies that the social planner allocation coincides with the coordination equilibrium.



with the symmetric calibration.

### Optimal tax given other country's tax



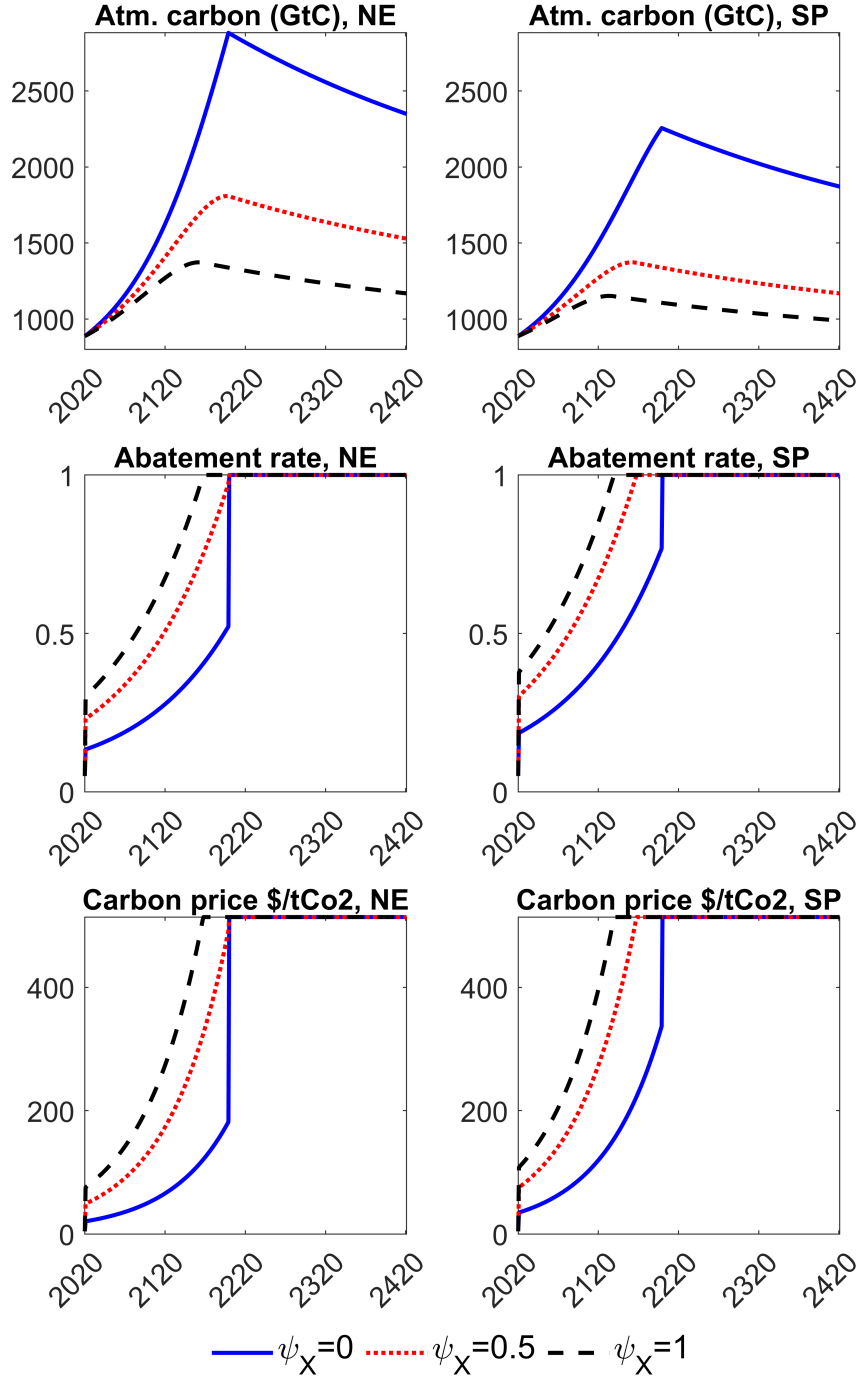
**Figure 7:** Optimal  $F$  given  $F^*$  (left panel) and optimal  $F^*$  given  $F$  (right panel), under three different values for  $\psi_X$ .

### Optimal carbon taxes

Externality	$\psi_X = 0$	$\psi_X = 0.5$	$\psi_X = 1$
Nash	$F = 0.0007, F^* = 0.0008$	$F = 0.0019, F^* = 0.0021$	$F = 0.0033, F^* = 0.0035$
Social planner	$F = 0.0038, F^* = 0.0009$	$F = 0.0090, F^* = 0.0021$	$F = 0.0140, F^* = 0.0032$
CEV	$\Omega = -0.99\%, \Omega^* = 0.62\%$	$\Omega = -2.18\%, \Omega^* = 1.97\%$	$\Omega = -3.10\%, \Omega^* = 2.71\%$
Coordination	$F = 0.0019, F^* = 0.0010$	$F = 0.0046, F^* = 0.0025$	$F = 0.0065, F^* = 0.0041$
CEV	$\Omega = 0.00\%, \Omega^* = 0.27\%$	$\Omega = 0.00\%, \Omega^* = 1.05\%$	$\Omega = 0.00\%, \Omega^* = 1.28\%$

**Table 5:** Nash, social planner and coordination equilibrium under different externality assumptions. CEV denotes gains or losses, in consumption equivalent terms, under the social planner/coordination allocation with respect to the Nash equilibrium.

## Nash vs social planner equilibrium



**Figure 8:** Nash equilibrium (NE, left panel) and social planner equilibrium (SP, right panel), under different  $\psi_X$ . Abatement rates and carbon prices are a world weighted average.

### 4.3 Positive analysis

We repeat the positive analysis carried out under a symmetric calibration. We first assume that Foreign, representing emerging markets, follows the sub-optimal baseline policy (the initial carbon tax is such that the abatement rate is 5%; thereafter, the carbon tax increases by 1% annually until 2200, when the abatement rate is set to 1). Home, which represents advanced economies, sets a carbon tax as in equation (47), with factor of proportionality  $F = 0.0024$ , which is the optimal response to the baseline policy adopted by Foreign, given the simple rule.

The impulse response function are qualitatively similar to those obtained under a symmetric calibration, and we include the figures in the Appendix (Figures E.1-E.2). The increase in the Home carbon tax gradually reduces Home output and consumption. Home households increase savings and this lowers the interest rate on domestic bonds, inducing financiers to buy Foreign bonds and to sell Home bonds. The persistent fall of the Home interest rate below the Foreign interest rate induces an increase in the UIP premium and a real appreciation over time. Brown firms demand less capital and brown FDI liabilities fall in Home, while the green sector receives more FDI. Home FDI assets increase, both in the green and brown sectors. The Home bond and FDI capital outflows are coupled with an improvement in the Home trade balance. The spillover effect on Foreign is overall negative, as the lower demand from Home induces a lower production and consumption in Foreign.

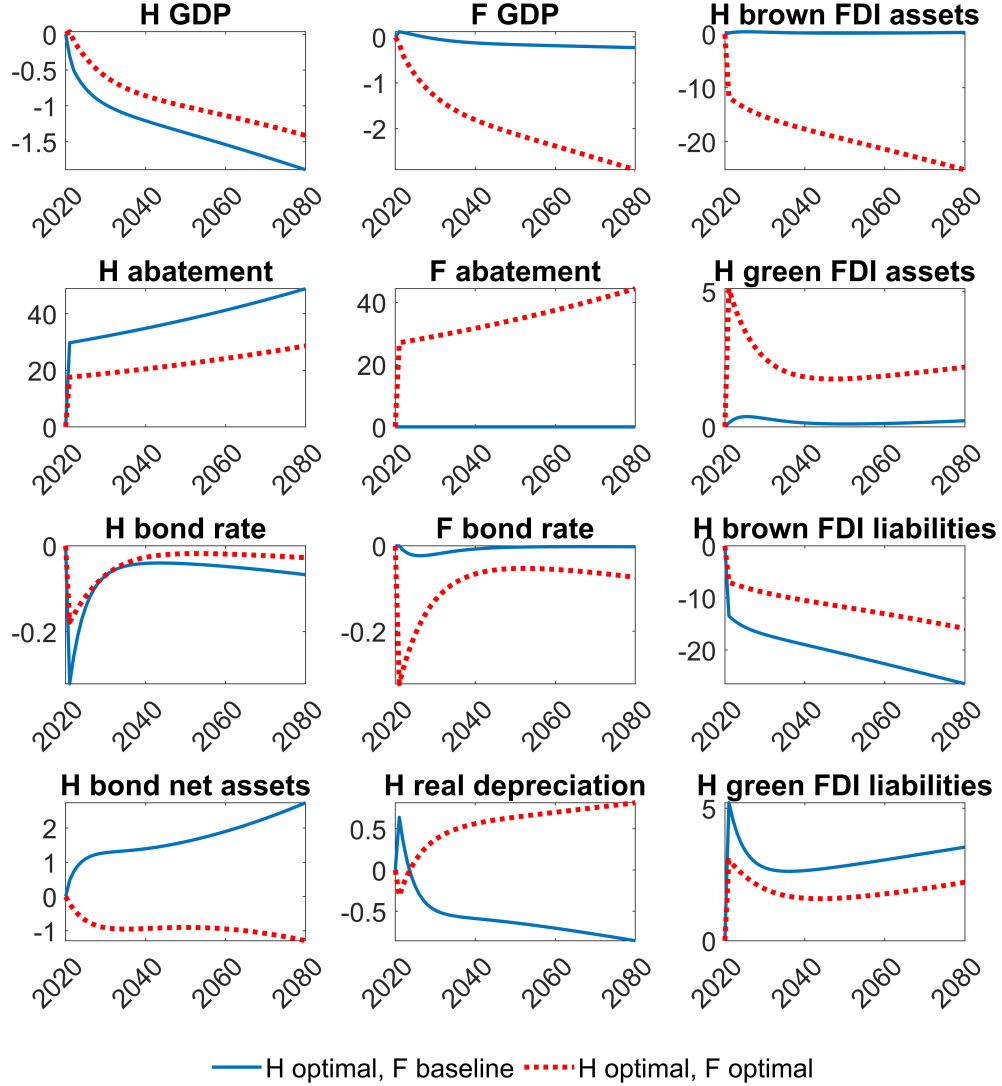
We now turn to the case of the Nash equilibrium under quadratic disutility, where advanced economies (Home) set  $F = 0.0011$  and emerging markets (Foreign) choose  $F^* = 0.0012$  (Figure 9, red dotted line), comparing it with the previous scenario, where Home follows the optimal and Foreign follows the baseline policy (Figure 9, blue solid line). In the Nash equilibrium, Foreign sets a carbon tax slightly higher than that in Home, but this implies a larger abatement given the higher carbon intensity ( $z_e^*$ ) in emerging markets: for any level of abatement and brown production, Foreign pollutes more as it employs a more polluting technology. As a consequence, GDP falls more

in Foreign than in Home, and so does the interest on domestic bonds. Hence, Home experiences bond inflows, currency appreciation, and its bond NFA deteriorates. The higher taxation in Foreign induces lower production in the brown sector, reducing FDI inflows from Home (i.e.  $H$  brown FDI assets); conversely, the relatively higher production in the green sector encourages more FDI inflows (i.e.  $H$  green FDI assets) from Home with respect to the previous scenario.

## Macroeconomic effects of the transition:

### Asymmetric calibration

#### Symmetric vs asymmetric policies



**Figure 9:** Variables are plotted in percentage deviation from the baseline scenario, except for the bond rate and emissions (plotted in level deviations from the baseline scenario), and bond assets (in level deviations from the baseline scenario, as a share of initial GDP). An increase in the Home real exchange rate means a Home real depreciation. Blue line: country F follows the baseline policy, country H sets optimally  $F = 0.0058$ . Nash equilibrium under  $\psi_X = 1$  ( $F = 0.0011$  and  $F^* = 0.0012$ ).

## 5 Conclusions

The introduction and increase of carbon taxes can have significant macroeconomic implications for the global economy. In this study, we explore these implications from both a normative and positive standpoint.

Our results quantify the importance of coordination across countries when it comes to environmental regulation. Compared to the taxes resulting in the Nash equilibrium, a coordinated carbon tax leads to a much faster green transition, reducing the utility costs of climate change. This happens because, in the absence of coordination, a country imposing a carbon tax bears entirely its recessionary costs, while the benefits of a higher taxation, in terms of pollution reduction, are proportional to the size of the country. Moreover, we show that in the Nash equilibrium taxes are inefficiently low especially in high-income countries: in fact, a global social planner would increase carbon taxation more in richer countries, as they are characterized by a lower marginal utility of consumption.

From a positive perspective, our findings indicate that when a large country unilaterally introduces carbon taxation, its consumption gradually falls, the country's interest rate decreases, the exchange rate depreciates, creating expectations of future appreciations. This unilateral action also diverts debt and equity flows towards the other country. In part, these flows include investment in the other country's brown sector, giving rise to carbon leakage. Despite the increase in capital flows, the economic activity falls in the other country too, given the lower external demand.

It is important to note that, for the sake of simplicity, our analysis does not consider certain potentially relevant aspects. These include the endogenous introduction of less polluting technologies and the role of the financial sector in financing the green transition or amplifying the recessionary effects. Moreover, we have ignored other policy instruments – such as the carbon border adjustment mechanism approved in the European Union – that countries could use to limit carbon leakage and other undesired spillovers. We acknowledge that these factors warrant further investigation in future research.

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# Appendix

## A List of equations

We denote with a tilde variables that are divided by the long-run trend  $z_t$ . The model features the following 50 variables:

$$X_t^{end} \equiv \left\{ \tilde{c}_t, \tilde{i}_t, \tilde{i}_t^*, \tilde{y}_{Ht}, \tilde{k}_{Gt}, \tilde{k}_{Bt}, \tilde{k}_{Gt}^*, \tilde{k}_{Bt}^*, h_{Bt}, h_{Gt}, \tilde{e}_t, \mu_t, \tilde{w}_t, p_{Gt}, p_{Bt}, \tilde{y}_{Gt}, \tilde{y}_{Bt}, r_t, r_{kGt}, r_{kGt}^*, r_{kBt}, r_{kBt}^*, \right. \\ \left. \tilde{c}_t^*, \tilde{i}_{*t}^*, \tilde{i}_{*t}, \tilde{y}_{Ft}^*, \tilde{k}_{*Gt}^*, \tilde{k}_{*Bt}^*, \tilde{k}_{*Gt}, \tilde{k}_{*Bt}, h_{Bt}^*, h_{Gt}^*, \tilde{e}_t^*, \mu_t^*, \tilde{w}_t^*, p_{Gt}^*, p_{Bt}^*, \tilde{y}_{Gt}^*, \tilde{y}_{Bt}^*, r_t^*, r_{*kGt}^*, r_{*kBt}^*, r_{*kGt}, \right. \\ \left. r_{*kBt}, p_{Ht}, p_{Ft}, s_t, \tilde{b}_t, \tilde{d}_{Ft}, \tilde{d}_{Ht} \right\}.$$

We list the 50 equations in what follows, dividing the them in three blocks, Home, Foreign, and common equations.

### A.1 Home

#### A.1.1 Households

Euler equation for bonds:

$$1 = \beta \mathbb{E}_t \left( \frac{\tilde{c}_t}{\tilde{c}_{t+1}^\iota} r_t \right). \quad (\text{A.1})$$

Euler equation for capital installed in Home green firms:

$$1 = \beta \mathbb{E}_t \left[ \frac{\tilde{c}_t}{\tilde{c}_{t+1}^\iota} (r_{kGt+1} + 1 - \delta) \right] + \left( \tilde{k}_{Gt} + \frac{1-n}{n} s_t \tilde{k}_{*Gt} \right)^{-\psi_G} \kappa_G \tilde{c}_t. \quad (\text{A.2})$$

Euler equation for capital installed in Home brown firms:

$$1 = \beta \mathbb{E}_t \left[ \frac{\tilde{c}_t}{\tilde{c}_{t+1}^\iota} (r_{kBt+1} + 1 - \delta) \right] - \left( \tilde{k}_{Bt} + \frac{1-n}{n} s_t \tilde{k}_{*Bt} \right)^{\psi_B} \kappa_B \tilde{c}_t. \quad (\text{A.3})$$

Euler equation for capital installed in Foreign green firms:

$$1 = \beta \mathbb{E}_t \left[ \left( \frac{\tilde{c}_t}{\tilde{c}_{t+1} \iota} r_t \right) \frac{s_{t+1}}{s_t} (r_{*kGt+1} + 1 - \delta) \right] + \left( \tilde{k}_{Gt} + \frac{1-n}{n} s_t \tilde{k}_{*Gt} \right)^{-\psi_G} \kappa_G \tilde{c}_t. \quad (\text{A.4})$$

Euler equation for capital installed in Foreign brown firms:

$$1 = \beta \mathbb{E}_t \left[ \left( \frac{\tilde{c}_t}{\tilde{c}_{t+1} \iota} r_t \right) \frac{s_{t+1}}{s_t} (r_{*kBt+1} + 1 - \delta) \right] - \left( \tilde{k}_{Bt} + \frac{1-n}{n} s_t \tilde{k}_{*Bt} \right)^{\psi_B} \kappa_B \tilde{c}_t. \quad (\text{A.5})$$

Law of motion for capital installed in Home firms:

$$\tilde{k}_{Gt} + \tilde{k}_{Bt} = (1 - \delta) \left( \frac{\tilde{k}_{Gt-1} + \tilde{k}_{Bt-1}}{\iota} \right) + \tilde{i}_t. \quad (\text{A.6})$$

Law of motion for capital installed in Foreign firms:

$$\tilde{k}_{*Gt} + \tilde{k}_{*Bt} = (1 - \delta) \left( \frac{\tilde{k}_{*Gt-1} + \tilde{k}_{*Bt-1}}{\iota} \right) + \tilde{i}_{*t}. \quad (\text{A.7})$$

### A.1.2 Final-good firms

Demand for green intermediate output:

$$\tilde{y}_{Gt} = (1 - \zeta) \left( \frac{p_{Gt}}{p_{Ht}} \right)^{-\xi} \tilde{y}_{Ht}. \quad (\text{A.8})$$

Demand for brown intermediate output:

$$\tilde{y}_{Bt} = \zeta \left( \frac{p_{Bt}}{p_{Ht}} \right)^{-\xi} \tilde{y}_{Ht}. \quad (\text{A.9})$$

Total production:

$$p_{Ht} \tilde{y}_{Ht} = p_{Gt} \tilde{y}_{Gt} + p_{Bt} \tilde{y}_{Bt} \quad (\text{A.10})$$

### A.1.3 Brown firms

Production function:

$$\tilde{y}_{Bt} = A \left( \frac{k_{Bt-1}}{\iota} \right)^{\alpha_1} \left( \frac{k_{Bt-1}^*}{\iota} \right)^{\alpha_2} (h_{Bt})^{1-\alpha_1-\alpha_2}. \quad (\text{A.11})$$

Demand for capital supplied by Home households:

$$k_{Bt-1} = \frac{\alpha_1 \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{r_{kBt}}. \quad (\text{A.12})$$

Demand for capital supplied by Foreign households:

$$k_{Bt-1}^* = \frac{\alpha_2 \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{r_{kBt}^*}. \quad (\text{A.13})$$

Labor demand:

$$h_{Bt} = \frac{(1 - \alpha_1 - \alpha_2) \left[ p_{Bt} - \tau_t (1 - \mu_t) z_e - \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \right] y_{Bt}}{w_t}. \quad (\text{A.14})$$

Optimal abatement:

$$\mu_t = \left( \frac{\tau_t z_e}{\kappa_A} \right)^{\frac{1}{\nu}}. \quad (\text{A.15})$$

Emissions:

$$\tilde{e}_t = z_e (1 - \mu_t) \tilde{y}_{Bt}. \quad (\text{A.16})$$

### A.1.4 Green firms

Production function:

$$\tilde{y}_{Gt} = A \left( \frac{\tilde{k}_{Gt-1}}{\iota} \right)^{\alpha_1} \left( \frac{\tilde{k}_{Gt-1}^*}{\iota} \right)^{\alpha_2} (h_{Gt})^{1-\alpha_1-\alpha_2}. \quad (\text{A.17})$$

Demand for capital supplied by Home households:

$$k_{Gt-1} = \frac{\alpha_1 p_{Gt} y_{Gt}}{r_{kGt}}. \quad (\text{A.18})$$

Demand for capital supplied by Foreign households:

$$k_{Gt-1}^* = \frac{\alpha_2 p_{Gt} y_{Gt}}{r_{kGt}^*}. \quad (\text{A.19})$$

Labor demand:

$$h_{Gt} = \frac{(1 - \alpha_1 - \alpha_2) p_{Gt} y_{Gt}}{w_t}. \quad (\text{A.20})$$

#### A.1.5 Financiers

Combine equations (35) and (40) and get the following modified UIP condition:

$$\tilde{b}_t = - \left\{ \frac{1}{\Gamma} \mathbb{E}_t \left[ \beta \frac{\tilde{c}_t}{\tilde{c}_{t+1} \iota} \left( r_t - r_t^* \frac{s_{t+1}}{s_t} \right) \right] + \frac{1-n}{n} \tilde{d}_{Ft} + \tilde{v} \right\}. \quad (\text{A.21})$$

#### A.1.6 Market clearing

Clearing in the labor market:

$$1 = h_{Gt} + h_{Bt}. \quad (\text{A.22})$$

Prices:

$$1 = (1 - \gamma) (p_{Ht})^{1-\eta} + \gamma (p_{Ft})^{1-\eta}. \quad (\text{A.23})$$

Resource constraint:

$$\begin{aligned}
p_{Ht}\tilde{y}_{Ht} = & \tilde{c}_t + \tilde{i}_t + \tilde{i}_t^* + p_{Ht}\tilde{g} + \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \tilde{y}_{Bt} + \\
& + \tilde{b}_t + \frac{1-n}{n} s_t \tilde{i}_{*t} - \tilde{i}_t^* - s_t \frac{1-n}{n} \left( \frac{r_{*kBt} \tilde{k}_{*Bt-1} + r_{*kGt} \tilde{k}_{*Gt-1}}{\iota} \right) + \frac{r_{t-1}^* s_t}{s_{t-1}} \left( \frac{\tilde{d}_{Ht-1} + \tilde{v}}{\iota} \right) + \\
& + \left( \frac{r_{*kBt}^* \tilde{k}_{*Bt-1}^* + r_{*kGt}^* \tilde{k}_{*Gt-1}^*}{\iota} \right) - r_{t-1} \left( \frac{\tilde{b}_{t-1} + \tilde{d}_{Ht-1} + \tilde{v}}{\iota} \right)
\end{aligned} \tag{A.24}$$

The resource constraint states that GDP  $p_{Ht}\tilde{y}_{Ht}$  is equal to the sum of consumption, total investment ( $\tilde{i}_t + \tilde{i}_t^*$ ), public spending, abatement costs, plus the trade balance, which is equal to the last two lines of the previous equation (to see this, combine equations 46, 40, 41, the laws of motion of capital, and divide by  $z_t$ ).

## A.2 Foreign

### A.2.1 Households

Euler equation for bonds:

$$1 = \beta^* \mathbb{E}_t \left( \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} r_t^* \right). \tag{A.25}$$

Euler equation for capital installed in Home green firms:

$$1 = \beta^* \mathbb{E}_t \left[ \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} \frac{s_t}{s_{t+1}} (r_{*kGt}^* + 1 - \delta) \right] + \left( \tilde{k}_{*Gt}^* + \frac{n}{1-n} \frac{\tilde{k}_{Gt}^*}{s_t} \right)^{-\psi_G} \kappa_G^* \tilde{c}_t^*. \tag{A.26}$$

Euler equation for capital installed in Home brown firms:

$$1 = \beta^* \mathbb{E}_t \left[ \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} \frac{s_t}{s_{t+1}} (r_{*kBt+1}^* + 1 - \delta) \right] - \left( \tilde{k}_{*Bt}^* + \frac{n}{1-n} \frac{\tilde{k}_{Bt}^*}{s_t} \right)^{\psi_B} \kappa_B^* \tilde{c}_t^*. \tag{A.27}$$

Euler equation for capital installed in Foreign green firms:

$$1 = \beta^* \mathbb{E}_t \left[ \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} (r_{*kGt}^* + 1 - \delta) \right] + \left( \tilde{k}_{*Gt}^* + \frac{n}{1-n} \frac{\tilde{k}_{Gt}^*}{s_t} \right)^{-\psi_G} \kappa_G^* \tilde{c}_t^*. \tag{A.28}$$



Euler equation for capital installed in Foreign brown firms:

$$1 = \beta^* \mathbb{E}_t \left[ \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} (r_{*k_{Bt+1}}^* + 1 - \delta) \right] - \left( \tilde{k}_{*Bt}^* + \frac{n}{1-n} \frac{\tilde{k}_{Bt}^*}{s_t} \right)^{\psi_B} \kappa_B^* \tilde{c}_t^*. \quad (\text{A.29})$$

Law of motion for capital installed in Foreign firms:

$$\tilde{k}_{*Gt}^* + \tilde{k}_{*Bt}^* = (1 - \delta) \left( \frac{\tilde{k}_{*Gt-1}^* + \tilde{k}_{*Bt-1}^*}{\iota} \right) + \tilde{i}_{*t}^*. \quad (\text{A.30})$$

Law of motion for capital installed in Home firms:

$$\tilde{k}_{Gt}^* + \tilde{k}_{Bt}^* = (1 - \delta) \left( \frac{\tilde{k}_{Gt-1}^* + \tilde{k}_{Bt-1}^*}{\iota} \right) + \tilde{i}_t^*. \quad (\text{A.31})$$

### A.2.2 Final-good firms

Demand for green intermediate output:

$$\tilde{y}_{Gt}^* = (1 - \zeta) \left( \frac{s_t p_{Gt}^*}{p_{Ft}^*} \right)^{-\xi} \tilde{y}_{Ft}^*. \quad (\text{A.32})$$

Demand for brown intermediate output:

$$\tilde{y}_{Bt}^* = \zeta \left( \frac{s_t p_{Bt}^*}{p_{Ft}^*} \right)^{-\xi} \tilde{y}_{Ft}^*. \quad (\text{A.33})$$

Total production:

$$\frac{p_{Ft}}{s_t} \tilde{y}_{Ft}^* = p_{Gt}^* \tilde{y}_{Gt}^* + p_{Bt}^* \tilde{y}_{Bt}^*. \quad (\text{A.34})$$

### A.2.3 Brown firms

Production function:

$$\tilde{y}_{Bt}^* = A^* \left( \frac{\tilde{k}_{*Bt-1}^*}{\iota} \right)^{\alpha_1^*} \left( \frac{\tilde{k}_{Bt-1}}{\iota} \right)^{\alpha_2^*} (h_{Bt}^*)^{1-\alpha_1^*-\alpha_2^*}. \quad (\text{A.35})$$

Demand for capital supplied by Foreign households:

$$k_{*Bt-1}^* = \frac{\alpha_1^* [p_{Bt}^* - \tau_t^* (1 - \mu_t^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu_t^{*1+\nu})] y_{Bt}^*}{r_{*kBt}^*}. \quad (\text{A.36})$$

Demand for capital supplied by Home households:

$$k_{*Bt-1} = \frac{\alpha_2^* [p_{Bt}^* - \tau_t^* (1 - \mu_t^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu_t^{*1+\nu})] y_{Bt}^*}{r_{*kBt}}. \quad (\text{A.37})$$

Labor demand:

$$h_{Bt}^* = \frac{(1 - \alpha_1^* - \alpha_2^*) [p_{Bt}^* - \tau_t^* (1 - \mu_t^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu_t^{*1+\nu})] y_{Bt}^*}{w_t^*}. \quad (\text{A.38})$$

Optimal abatement:

$$\mu_t^* = \left( \frac{\tau_t^* z_e^*}{\kappa_A} \right)^{\frac{1}{\nu}}. \quad (\text{A.39})$$

Emissions:

$$\tilde{e}_t^* = z_e^* (1 - \mu_t^*) \tilde{y}_{Bt}^*. \quad (\text{A.40})$$

#### A.2.4 Green firms

Production function:

$$\tilde{y}_{Gt}^* = A^* \left( \frac{\tilde{k}_{*Gt-1}^*}{\iota} \right)^{\alpha_1^*} \left( \frac{\tilde{k}_{*Gt-1}}{\iota} \right)^{\alpha_2^*} (h_{Gt}^*)^{1-\alpha_1^*-\alpha_2^*}. \quad (\text{A.41})$$

Demand for capital supplied by Foreign households:

$$k_{*Gt-1}^* = \frac{\alpha_1^* p_{Gt}^* y_{Gt}^*}{r_{*kGt}^*}. \quad (\text{A.42})$$

Demand for capital supplied by Home households:

$$k_{*Gt-1} = \frac{\alpha_2^* p_{Gt}^* y_{Gt}^*}{r_{*kGt}}. \quad (\text{A.43})$$

Labor demand:

$$h_{Gt}^* = \frac{(1 - \alpha_1^* - \alpha_2^*) p_{Gt}^* y_{Gt}^*}{w_t^*}. \quad (\text{A.44})$$

### A.2.5 Financiers

Modified UIP condition:

$$\tilde{b}_t = \left\{ \frac{(1-n)s_t}{\Gamma n} \mathbb{E}_t \left[ \beta^* \frac{\tilde{c}_t^*}{\tilde{c}_{t+1}^* \iota} \left( r_t^* - r_t \frac{s_t}{s_{t+1}} \right) \right] - (\tilde{v} + \tilde{d}_H) \right\}. \quad (\text{A.45})$$

### A.2.6 Market clearing

Clearing in the labor market:

$$1 = h_{Gt}^* + h_{Bt}^*. \quad (\text{A.46})$$

Prices:

$$s_t^{1-\eta} = \gamma^* (p_{Ht})^{1-\eta} + (1 - \gamma^*) (p_{Ft})^{1-\eta}. \quad (\text{A.47})$$

By Walras law, the resource constraint of the Foreign economy is redundant and it is not included in the list of equations. We report the equation below:

$$\begin{aligned} \frac{p_{Ft}}{s_t} \tilde{y}_{Ft}^* &= \tilde{c}_t^* + \tilde{i}_{*t}^* + \tilde{i}_{*t} + \frac{p_{Ft}}{s_t} g + \frac{\kappa_A}{1+\nu} (\mu_t^{*1+\nu}) \tilde{y}_{Bt}^* + \\ &\quad - \frac{n}{(1-n)s_t} \tilde{b}_t + \frac{n}{(1-n)s_t} \tilde{i}_t^* - \tilde{i}_{*t} + \left( r_{*kBt} \frac{\tilde{k}_{*Bt-1}}{\iota} + r_{*kGt} \frac{\tilde{k}_{*Gt-1}}{\iota} \right) \\ &\quad - \frac{n}{(1-n)s_t} \left( r_{kBt}^* \frac{\tilde{k}_{Bt-1}^*}{\iota} + r_{kGt}^* \frac{\tilde{k}_{Gt-1}^*}{\iota} \right) + \\ &\quad + \frac{r_{t-1}}{s_t} \left( \frac{\frac{n}{1-n} \tilde{b}_{t-1} + \tilde{d}_{Ht-1} + \tilde{v}_{t-1}}{\iota} \right) - \frac{r_{t-1}^*}{s_{t-1}} \left( \frac{\tilde{d}_{Ht-1} + \tilde{v}_{t-1}}{\iota} \right). \end{aligned}$$

### A.3 Common equations

Market clearing of the Home good:

$$\begin{aligned} n\tilde{y}_{Ht} = n & \left[ (1 - \gamma) (p_{Ht})^{-\eta} \left( \tilde{c}_t + \tilde{i}_t + \frac{\kappa_A}{1 + \nu} (\mu_t^{1+\nu}) \tilde{y}_{Bt} + \tilde{i}_t^* \right) + \tilde{g} \right] + \\ & + (1 - n) \left[ \gamma^* \left( \frac{p_{Ht}}{s_t} \right)^{-\eta} \left( \tilde{c}_t^* + \tilde{i}_{*t}^* + \tilde{i}_{*t} + \frac{\kappa_A}{1 + \nu} (\mu_t^{*1+\nu}) \tilde{y}_{Bt}^* \right) \right]. \end{aligned} \quad (\text{A.48})$$

Market clearing for the Foreign good:

$$\begin{aligned} (1 - n) \tilde{y}_{Ft}^* = (1 - n) & \left[ (1 - \gamma^*) \left( \frac{p_{Ft}}{s_t} \right)^{-\eta} \left( \tilde{c}_t^* + \tilde{i}_{*t}^* + \tilde{i}_{*t} + \frac{\kappa_A}{1 + \nu} (\mu_t^{*1+\nu}) \tilde{y}_{Bt}^* \right) + \tilde{g}^* \right] + \\ & + n \left[ \gamma^* (p_{Ft})^{-\eta} \left( \tilde{c}_t + \tilde{i}_t + \tilde{i}_t^* + \frac{\kappa_A}{1 + \nu} (\mu_t^{1+\nu}) \tilde{y}_{Bt} \right) \right]. \end{aligned} \quad (\text{A.49})$$

Market clearing for Home bonds:

$$\frac{1 - n}{n} \tilde{d}_{Ft} + \tilde{d}_{Ht} + \tilde{v} + \tilde{b}_t = 0. \quad (\text{A.50})$$

Once we find the solution of the model, we can derive global pollution  $x_t^W$ , as follows:

$$x_{1t}^W = x_{1t-1}^W + \varphi_L z_t^W [n\tilde{e}_t + (1 - n)\tilde{e}_t^*] \quad (\text{A.51})$$

$$x_{2t}^W = (1 - \varphi) x_{2t-1}^W + (1 - \varphi_L) \varphi_0 z_t^W [n\tilde{e}_t + (1 - n)\tilde{e}_t^*] \quad (\text{A.52})$$

$$x_t^W = \bar{x}^W + x_{1t}^W + x_{2t}^W, \quad (\text{A.53})$$

where  $z_t^W$  grows at rate  $\iota$  and  $z_0 = 1$ .

## B Static model

We set up a very simple model, in order to explain the main intuitions of the normative results of the analysis. We adopt the Handbook approach of Hassler et al. (2016)

(HKS, henceforth) who set up a basic static model useful to derive sharp and transparent economic intuitions. Compared to their analysis, we assume that the world consists of  $N$  countries that differ only for their contribution to global carbon concentration. First, we consider a climate externality in the production function, second we consider a climate externality in the utility function.

## B.1 Production-damaged externality

In country  $i$ , output  $y_i$  is produced using the following production function:

$$y_i = e^{-\gamma S} k_i^\alpha l_i^{1-\alpha-\nu} E_i^\nu \quad (\text{B.1})$$

where  $k_i$  and  $l_i$  denote capital and labor that are fixed and equal across countries,  $E_i$  denotes fossil energy use (or emissions), and  $S$  is global atmospheric carbon, which negatively affects total factor productivity. The idea is that higher atmospheric carbon increases global temperature, which in turn makes production activities more costly (see HKS for an extensive discussion). All these variables are in per-capita terms. Given that capital and labor are fixed, we normalize  $A \equiv k_i^\alpha l_i^{1-\alpha-\nu} = 1$ . Atmospheric carbon depends linearly on the weighted sum of energy use across countries:

$$S = \phi \sum_{j=1}^N n_j E_j, \quad (\text{B.2})$$

where  $n_j$  is the population share of country  $j$ , and  $\sum_{j=1}^N n_j = 1$ . Parameters  $\nu$ ,  $\gamma$ , and  $\phi$  should be such that the production function is increasing and concave in  $E_i$ . We assume that energy comes from coal, which has a positive extraction cost  $\zeta$ . On top of the extraction costs, the government of country  $i$  imposes a carbon tax  $\tau_i$  for unit of energy. The tax is rebated to households through lump sum transfers  $T_i$ . All countries consume the same good: this assumption and the fact the model is static imply that there is no

trade.<sup>1</sup> The only link between countries is energy, whose use in one country affects the climate externality in other countries.

In each country  $i$ , there is a representative household, owner of the firm, that maximizes a log utility in consumption  $c_i = y_i - (\zeta + \tau_i) E_i + T_i$ , by choosing energy. The representative household does not internalize that energy is damaging its production and that of other countries. The maximization problem is the following:

$$\max_{E_i} \log \left[ e^{-\gamma S} E_i^\nu - (\zeta + \tau_i) E_i + T_i \right].$$

The first order condition yields:

$$\nu \frac{y_i}{E_i} = \zeta + \tau_i, \quad (\text{B.3})$$

where the left-hand side is the marginal benefit of emissions (i.e. the marginal product of energy) and the right-hand side is the marginal cost, given by the extraction cost and the tax. Given the concavity of the production function, a higher tax  $\tau_i$  leads to a lower value of energy employed  $E_i$ .

Compared to the representative household, the social planner takes into account the damage caused by energy use in country  $i$  (without internalizing the damage caused to other countries). By solving the following social planner problem:

$$\max_{E_i} \log \left[ e^{-\gamma \phi \sum_{j=1}^N n_j E_j} E_i^\nu - \zeta E_i \right],$$

we find that the optimal energy use should satisfy:

$$\nu \frac{y_i}{E_i} = \zeta + \gamma \phi n_i y_i, \quad (\text{B.4})$$

where the right-hand side consists of the extraction cost plus the marginal damage, which is given by the damage induced by an additional unit of domestic energy. By comparing

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<sup>1</sup>When country produce the same good, trade is only useful for intertemporal substitution. Given that the model is static, the intertemporal substitution motive is absent.

the social planner allocation in equation (B.4) with the competitive equilibrium allocation in equation (B.3), we derive the optimal tax that makes the decentralized equilibrium optimal in country  $i$ :

$$\tau_i^* = \gamma \phi n_i y_i^*, \quad (\text{B.5})$$

where a star denotes optimality. As in HKS, the optimal tax is proportional to output. However the factor of proportionality is lower, as long as  $n_i < 1$ : given that country  $i$  is relatively small compared to the rest of the world, its influence on atmospheric carbon is relatively weak.<sup>2</sup>

Equation (B.5) is not a closed form solution for  $\tau_i$ , given that the right hand side depends on  $\tau_i$  itself. However, we can see that an increase in the carbon tax set by another country  $\tau_j$  induces a lower value of energy employed in that economy  $E_j$  (by using the country  $j$ 's counterpart of equation B.3), which in turn reduces the global pollution damage (equation B.2), implying a higher value of  $y_i$  (equation B.1), *ceteris paribus*. As the optimal tax is proportional to output, the tax set by government  $i$  must increase. This reasoning proves that under a climate production externality carbon taxes are strategic complements across countries.

## B.2 Disutility externality

Higher atmospheric carbon may also affect households' utility, through several channels, such as health or worse life quality. Suppose that utility function for country  $i$  also depends on atmospheric carbon  $S = \phi \sum_{j=1}^N n_j E_j$ :

$$\log c_i - \gamma_1 \left( \phi \sum_{j=1}^N n_j E_j \right) - \frac{\gamma_2}{2} \left( \phi \sum_{j=1}^N n_j E_j \right)^2,$$

where now atmospheric carbon affects utility directly. Assume also that energy does not damage production, to isolate the implications of pollution disutility. The optimality

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<sup>2</sup>On page 1945, HKS show that the optimal tax is proportional to GDP plus energy costs. Notice that our definition of output is exactly equal to GDP (which is equal to consumption) plus energy costs.

condition for the representative household is identical to equation (B.3). The social planner internalizes the disutility externality and chooses:

$$\nu \frac{y_i}{E_i} = \zeta + (y_i - \zeta E_i) \phi n_i \left( \gamma_1 + \gamma_2 \sum_{j=1}^N n_j E_j \right). \quad (\text{B.6})$$

The optimal tax of country  $i$  reads:

$$\tau_i^* = (y_i - \zeta E_i^*) \phi n_i \left( \gamma_1 + \gamma_2 \sum_{j \neq i}^N n_j E_j + \gamma_2 n_i E_i^* \right). \quad (\text{B.7})$$

The right-hand side consists of the extraction cost plus the marginal disutility of emissions. If the disutility externality is purely linear ( $\gamma_1 > 0, \gamma_2 = 0$ ), the optimal tax is proportional to output minus energy costs (i.e. consumption, or GDP). Notice that the factor of proportionality is identical to that in the production externality case (equation B.5), if  $\gamma_1 = \gamma$ . If the disutility is linear, the optimal tax set in country  $i$  is independent on the environmental policy in the rest of the world, as domestic output does not depend on other countries' emissions anymore.

If the disutility externality is purely quadratic ( $\gamma_1 = 0, \gamma_2 > 0$ ), the tax is still proportional to GDP, but the factor of proportionality is not constant and it depends on global emissions. If atmospheric carbon is higher, the marginal disutility of emissions increases, and the domestic social planner sets higher taxes. In this case, carbon taxes are strategic substitutes: lower taxes in the rest of the world raise global emissions, thus increasing the marginal disutility of pollution in country  $i$  and inducing the domestic social planner to increase the local carbon tax.

### B.3 Simulation

Considering a two-country framework, we simulate the model deriving the optimal tax  $\tau_i$  of country  $i$  as a function of the tax set by the other country ( $\tau_{-i}$ ), in order to compute the Nash and the coordination equilibrium in four scenarios: production externality,



linear disutility, quadratic disutility with low and with high marginal disutility. In the production externality case, we use the same calibration of HKS for  $\nu$ ,  $\phi$ , and  $\gamma$ , where the model period is 100 years: we set  $\nu = 0.04$ ,  $\phi = 0.48$ , and  $\gamma = 5.7 \cdot 10^{-5}$ . As in HKS, we set  $S = 900$  GtC ex ante, in excess of the pre-industrial level: this implies  $\zeta = 2.7 \cdot 10^{-5}$  es post. This calibration leads to a production damage of 5% of output, without carbon taxes. In the other three scenarios we remove the production damage. In the linear disutility scenario, we set  $\gamma_1 = \gamma$  and  $\gamma_2 = 0$  (as in HKS, when they consider a linear externality in the utility function). In the two quadratic disutility scenarios, we set  $\gamma_1 = 0$ . In the first case (low marginal disutility), we set  $\gamma_2 = \frac{\gamma}{S}$ , such that the marginal disutility of atmospheric carbon is the same under linear and quadratic disutility, when there are no carbon taxes. In the second case (high marginal disutility), we set  $\gamma_2 = \frac{2\gamma}{S}$ , such that the disutility under the linear and the quadratic externality are the same, when there are no carbon taxes. Just for illustrative purposes, we consider an asymmetric case, where the share  $n_i$  of economy  $i$  is set to 0.1. In this case, country  $-i$  represents the rest of the world.<sup>3</sup>

We show the tax reaction function of the social planner of country  $i$  as a function of the tax set by the rest of the world (Figure B.1). For convenience, we express all the taxes in terms of the optimal tax set by a global social planner under the production externality. The optimal tax of a global social planner under the production externality can be easily derived setting  $n = 1$  in (B.5), as the global social planner sets the same taxation in both countries, no matter their size, to equate their marginal utilities. We denote this tax as:

$$\tau^{D*} = \gamma \phi y^{D*}, \quad (\text{B.8})$$

where  $y^{D*}$  is output resulting from coordination across countries, under the production externality.

From a qualitative point of view, the four reaction functions have different shapes.

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<sup>3</sup>Here, we are implicitly ignoring the coordination problem within the rest of the world assuming that countries different from  $i$  are behaving like a unique economy.

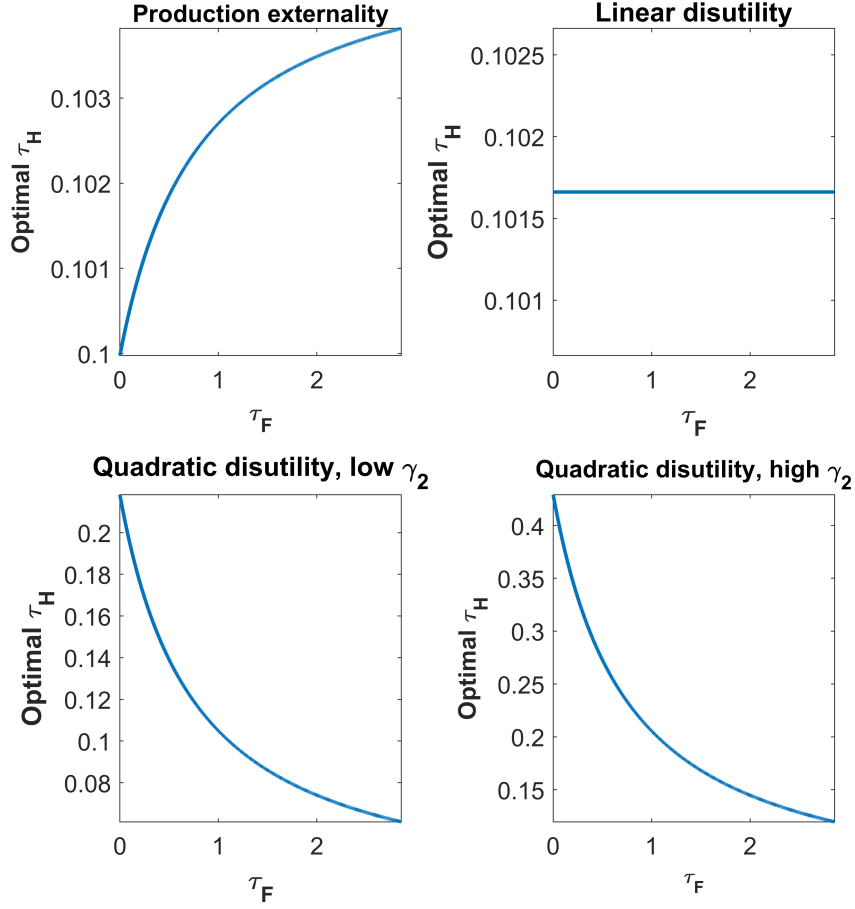
Under the production externality, the reaction function is increasing, meaning that taxes are strategic complements: when taxes in the rest of the world are higher, global damage is lower, domestic GDP is higher, inducing the domestic social planner to raise the tax. Under the linear disutility externality, the reaction function is constant: emissions by other countries do not affect the marginal cost of polluting (which is constant and equal to  $\gamma_1$ ) and the social planner sets the same carbon tax whatever the choice of other countries. Under the two types of quadratic externality, the reaction function is decreasing, meaning that taxes are strategic substitutes around the world: higher carbon taxes in the rest of the world lead to lower emissions, reducing the marginal cost of pollution (equal to  $\gamma_2 S$ ), thus lowering the domestic carbon tax. This implies that countries that set too low carbon taxes, either because they are small or because they adopt sub-optimal policy, may induce other countries to set too costly carbon taxes.

From a quantitative point of view, the reaction function is virtually constant also in the production externality case, being always around 10% of the tax resulting from global coordination. Increasing taxes in the rest of world raises domestic GDP by reducing the damage, but this effect is quantitative small. In practice, under the production externality the domestic social planner sets a tax that is roughly a fraction  $n_i = 0.1$  of the tax set by a global social planner (i.e. the coordination equilibrium). Notice that the domestic tax is also about  $0.1\tau^{*D}$  under the linear disutility case.

Once we have the reaction functions, we can compute the Nash equilibrium between the domestic economy and the rest of the world, and compare it with the coordination equilibrium (Table B.1). We already know that under the production and the linear disutility externality, the tax is roughly a fraction  $n_i$  of the benchmark tax  $\tau^{*D}$  whatever the choices in the rest of the world (which instead always sets  $(1 - n_i)\tau^{*D}$ ): the Nash equilibrium is close to  $\{n_i, 1 - n_i\}$ . By construction, the coordination equilibrium in the production externality case is  $\{1, 1\}$ . In the linear disutility case, the Nash and coordination equilibria barely change compared to the production scenario. If the quadratic disutility is calibrated to have the same marginal disutility of the linear one, taxes are

still close to  $\tau^{*D}$  times the population share in the Nash equilibrium, and the coordination equilibrium is also close to that of the previous two cases. It is worth stressing that in all these three scenarios (production damage, linear disutility, quadratic disutility with low  $\gamma_2$ ), the Nash equilibrium displays lower taxes compared to the coordination equilibrium. This is due to a classic free riding problem: without coordination, when a country imposes a carbon tax, it bears entirely its recessionary costs, while the benefits of a lower pollution are shared with the rest of the world; this leads to a suboptimal level of taxation compared to the coordination equilibrium. Under a quadratic disutility with a high value of  $\gamma_2$ , taxes are much higher both in the Nash and coordination equilibrium. Notice that, in contrast to the previous three cases, the rest-of-the world's social planner sets a tax higher than the global social planner, to take the burden also of the lower tax set by the  $i$ -th economy.

## Optimal Home tax given Foreign tax



**Figure B.1:** Optimal  $\tau_i$  given  $\tau_{-i}$ , under three different pollution externalities. The taxes are expressed in terms of  $\tau^{*D}$ , the optimal tax in the production externality scenario under coordination.

## Optimal carbon taxes

Externality	Nash equilibrium	Coordination
Production	$\tau_i/\tau^{*D} = 0.103$ , $\tau_{-i}/\tau^{*D} = 0.899$	$\tau_i/\tau^{*D} = 1$ , $\tau_{-i}/\tau^{*D} = 1$
Linear disutility	$\tau_i/\tau^{*D} = 0.102$ , $\tau_{-i}/\tau^{*D} = 0.906$	$\tau_i/\tau^{*D} = 1.005$ , $\tau_{-i}/\tau^{*D} = 1.005$
Quadratic disutility, low $\gamma_2$	$\tau_i/\tau^{*D} = 0.107$ , $\tau_{-i}/\tau^{*D} = 0.955$	$\tau_i/\tau^{*D} = 0.961$ , $\tau_{-i}/\tau^{*D} = 0.961$
Quadratic disutility, high $\gamma_2$	$\tau_i/\tau^{*D} = 0.169$ , $\tau_{-i}/\tau^{*D} = 1.493$	$\tau_i/\tau^{*D} = 1.465$ , $\tau_{-i}/\tau^{*D} = 1.465$

**Table B.1:** Nash and coordination equilibrium under different externality assumptions.

## C Initial steady state

First of all, we calibrate parameters  $\gamma$  and  $\gamma^*$  to be proportional to the world share of the other country's GDP:

$$\gamma = \omega \frac{(1-n) p_F \tilde{y}_F^*}{n \cdot p_H \tilde{y}_H + (1-n) p_F \tilde{y}_F^*}$$

$$\gamma = \omega \frac{(1-n) \frac{p_H \tilde{y}_H}{\Lambda}}{n \cdot p_H \tilde{y}_H + (1-n) \frac{p_H \tilde{y}_H}{\Lambda}}$$

$$\gamma = \omega \frac{(1-n)}{\Lambda n \cdot + (1-n)}$$

and

$$\gamma^* = \omega \frac{n \cdot p_H \tilde{y}_H}{n \cdot p_H \tilde{y}_H + (1-n) p_F \tilde{y}_F^*}$$

$$\gamma^* = \omega \frac{n \cdot \Lambda p_F y_F^*}{n \cdot \Lambda p_F \tilde{y}_F^* + (1-n) p_F \tilde{y}_F^*}$$

$$\gamma^* = \omega \frac{n \cdot \Lambda}{n \cdot \Lambda + (1-n)}$$

where:

$$\Lambda \equiv \frac{p_H y_H}{p_F y_F^*}.$$

We set ex ante  $r$  and  $r^*$  and ex post  $\beta$  and  $\beta^*$ . By the Euler equations:

$$\beta = \frac{\iota}{r}$$

$$\beta^* = \frac{\iota}{r^*}.$$

We calibrate ex ante the green and the brown premium (and find ex post  $\kappa_G, \kappa_B, \kappa_G^*, \kappa_B^*$ ), defined as follows:

$$pr_G = r_{kG} + (1 - \delta) - r$$

$$pr_B = r_{kB} + (1 - \delta) - r$$

$$pr_G^* = r_{*kG}^* + (1 - \delta) - r^*$$

$$pr_B^* = r_{*kB}^* + (1 - \delta) - r^*.$$

This implies

$$r_{kB} = \frac{\ell}{\beta} - (1 - \delta) + pr_B$$

$$r_{*kB} = r_{kB}$$

$$r_{kB}^* = \frac{\ell}{\beta^*} - (1 - \delta) + pr_B^*$$

$$r_{*kB}^* = r_{kB}^*$$

$$r_{kG} = \frac{\ell}{\beta} - (1 - \delta) + pr_G$$

$$r_{*kG} = r_{kG}$$

$$r_{kG}^* = \frac{\ell}{\beta^*} - (1 - \delta) + pr_G^*$$

$$r_{*kG}^* = r_{kG}^*.$$

We find the steady state as a function of  $\{\tilde{y}_H, p_B, p_B^*, p_H, z, z^*\}$ . Use the Home CPI equation to find  $p_F$ :

$$1 = (1 - \gamma) (p_H)^{1-\eta} + \gamma (p_F)^{1-\eta}$$

$$p_F = \left[ \frac{1 - (1 - \gamma) (p_H)^{1-\eta}}{\gamma} \right]^{\frac{1}{1-\eta}}.$$

Use Foreign CPI equation to find  $s$ :

$$s = [\gamma^* (p_H)^{1-\eta} + (1 - \gamma^*) (p_F)^{1-\eta}]^{\frac{1}{1-\eta}}.$$

We set  $A^*$  ex post and  $\Lambda$  ex ante. This implies:

$$\tilde{y}_F^* = \frac{p_H \tilde{y}_H}{\Lambda p_F}.$$

We calibrate ex ante:

$$BY = \frac{\tilde{b}}{p_H y_H}.$$

Using the Home bank's condition:

$$\tilde{b}_t = - \left\{ \frac{1}{\Gamma} \mathbb{E}_t \left[ \beta \frac{\tilde{c}_t}{\tilde{c}_{t+1} \iota} \left( r_t - r_t^* \frac{s_{t+1}}{s_t} \right) \right] + \frac{1-n}{n} \tilde{d}_{Ft} + \tilde{v} \right\}$$

$$\begin{aligned}
\tilde{b} &= -\frac{1}{\Gamma} \left[ \frac{\beta}{\iota} \left( \frac{\iota}{\beta} - \frac{\iota}{\beta^*} \right) \right] - \tilde{v} - \frac{1-n}{n} \tilde{d}_F \\
\tilde{b} &= -\frac{1}{\Gamma} \left[ \frac{\beta^* - \beta}{\beta^*} \right] - \tilde{v} - \frac{1-n}{n} \tilde{d}_F \\
\tilde{b} &= -\frac{1}{\Gamma} \left( 1 - \frac{\beta}{\beta^*} \right) - \tilde{v} - \frac{1-n}{n} \tilde{d}_F \\
\tilde{b} + \tilde{v} + \frac{1-n}{n} \tilde{d}_F &= -\frac{1}{\Gamma} \left( 1 - \frac{\beta}{\beta^*} \right) \\
\tilde{d}_H &= \frac{1}{\Gamma} \left( 1 - \frac{\beta}{\beta^*} \right)
\end{aligned}$$

and compute ex post  $v$ . Given the foreign banks conditions:

$$\begin{aligned}
\tilde{b} &= \left\{ \frac{(1-n)s}{\Gamma^* n} \left[ \frac{\beta^*}{\iota} \left( \frac{\iota}{\beta^*} - \frac{\iota}{\beta} \right) \right] - (\tilde{v} + \tilde{d}_H) \right\} \\
\tilde{b} &= \frac{(1-n)s}{\Gamma^* n} \left( 1 - \frac{\beta^*}{\beta} \right) - (\tilde{v} + \tilde{d}_H) \\
\tilde{b} + \tilde{v} + \tilde{d}_H &= \frac{(1-n)s}{\Gamma^* n} \left( 1 - \frac{\beta^*}{\beta} \right) \\
(\tilde{b} + \tilde{v} + \tilde{d}_H) \frac{n}{1-n} &= \frac{s}{\Gamma^*} \left( 1 - \frac{\beta^*}{\beta} \right) \\
\tilde{d}_F &= \frac{s}{\Gamma^*} \left( \frac{\beta^*}{\beta} - 1 \right).
\end{aligned}$$

We find  $v$ :

$$\tilde{v} = - \left( \tilde{b} + \tilde{d}_H + \frac{1-n}{n} \tilde{d}_F \right).$$



We calibrate ex ante the following ratios:

$$\begin{aligned}
GY &= \frac{\tilde{g}}{\tilde{y}_H} \\
GY^* &= \frac{\tilde{g}^*}{\tilde{y}_F^*} \\
FDIY &= \frac{1-n}{n} \frac{s \left( \tilde{k}_{*B} + \tilde{k}_{*G} \right)}{p_H \tilde{y}_H} \\
FDIY^* &= \frac{n}{1-n} \frac{\tilde{k}_B^* + \tilde{k}_G^*}{p_F \tilde{y}_F^*} \\
IY &= \frac{\tilde{i} + \tilde{i}^*}{p_H \tilde{y}_H} \\
IY^* &= s \frac{\tilde{i}_*^* + \tilde{i}_*}{p_F \tilde{y}_F^*}
\end{aligned}$$

and we compute ex post:  $\{\tilde{g}, \tilde{g}^*, \alpha_2, \alpha_2^*, \alpha_1, \alpha_1^*\}$ . From the previous conditions:

$$\begin{aligned}
\tilde{k}_{*B} + \tilde{k}_{*G} &= FDIY \frac{p_H \tilde{y}_H}{s} \frac{n}{1-n} \\
\tilde{k}_B^* + \tilde{k}_G^* &= FDIY^* p_F \tilde{y}_F^* \frac{1-n}{n}.
\end{aligned}$$

By the law of motion of capital:

$$\begin{aligned}
\tilde{i}_* &= \left( \tilde{k}_{*B} + \tilde{k}_{*G} \right) \left( 1 - \frac{1-\delta}{\iota} \right) \\
\tilde{i}^* &= \left( \tilde{k}_B^* + \tilde{k}_G^* \right) \left( 1 - \frac{1-\delta}{\iota} \right).
\end{aligned}$$

Now we can find domestic investment:

$$\begin{aligned}
IY &= \frac{\tilde{i} + \tilde{i}^*}{p_H \tilde{y}_H} \\
\tilde{i} &= IY p_H \tilde{y}_H - \tilde{i}^*
\end{aligned}$$

and:

$$IY^* = s \frac{\tilde{i}_*^* + \tilde{i}_*}{p_F \tilde{y}_F^*}$$

$$\tilde{i}_*^* = IY^* \frac{p_F \tilde{y}_F^*}{s} - \tilde{i}_*.$$

So:

$$\tilde{k}_{*B} + \tilde{k}_{*G} = \frac{\tilde{i}}{1 - \frac{1-\delta}{\iota}}$$

$$\tilde{k}_B^* + \tilde{k}_G^* = \frac{\tilde{i}^*}{1 - \frac{1-\delta}{\iota}}.$$

Find  $g$  and  $g^*$ :

$$\tilde{g} = GY \tilde{y}_H$$

$$\tilde{g}^* = GY^* \tilde{y}_F^*.$$

The price of carbon (in dollar per unit of CO2) is given by:

$$p_{Ct} = \tau_t \frac{con_{GDP}}{3.67}$$

$$p_{Ct}^* = \tau_t^* \frac{con_{GDP}}{3.67},$$

where:

$$con_{GDP} = \frac{gdp_t^{w,\$bil}}{np_H \tilde{y}_H + (1-n) p_F \tilde{y}_F}.$$

The world price of carbon reads:

$$p_{Ct}^W = np_{Ct} + (1-n) p_{Ct}^*.$$

We want to calibrate ex ante  $p_C^W$  when  $\mu = \mu^* = 1$ . This implies:

$$\begin{aligned} p_C^W &= n\tau \frac{con_{GDP}}{3.67} + (1-n)\tau^* \frac{con_{GDP}}{3.67} \\ p_C^W &= \frac{con_{GDP}}{3.67} [n\tau + (1-n)\tau^*] \\ p_C^W &= \frac{con_{GDP}}{3.67} \left[ n \frac{\kappa_A}{z_e} + (1-n) \frac{\kappa_A}{z_e^*} \right] \\ p_C^W &= \frac{con_{GDP}\kappa_A}{3.67} \left[ \frac{n}{z_e} + \frac{(1-n)}{z_e^*} \right]. \end{aligned}$$

We find  $\kappa_A$  ex post:

$$\kappa_A = \frac{3.67 p_C^W}{con_{GDP} \left[ \frac{n}{z_e} + \frac{(1-n)}{z_e^*} \right]}.$$

Set ex ante the abatement rate and find ex post the tax:

$$\begin{aligned} \tau &= \mu^\nu \frac{\kappa_A}{z_e} \\ \tau^* &= \mu^{*\nu} \frac{\kappa_A}{z_e^*}. \end{aligned}$$

Use the brown demand to find  $y_B$ :

$$\tilde{y}_B = \zeta \left( \frac{p_B}{p_H} \right)^{-\xi} \tilde{y}_H.$$

Use the domestic capital demands to find  $\alpha_1$ :

$$\begin{aligned} \tilde{k}_B + \tilde{k}_G &= \alpha_1 \iota \left[ \frac{p_B - \tau(1-\mu)z_e - \frac{\kappa_A}{1+\nu}(\mu^{1+\nu})}{r_{kB}} \tilde{y}_B + \frac{p_G \tilde{y}_G}{r_{kG}} \right] \\ \tilde{k}_B + \tilde{k}_G &= \alpha_1 \iota \left[ \frac{p_B - \tau(1-\mu)z_e^* - \frac{\kappa_A}{1+\nu}(\mu^{1+\nu})}{r_{kB}} \tilde{y}_B + \frac{p_H \tilde{y}_H - p_B \tilde{y}_B}{r_{kG}} \right] \\ \alpha_1 &= \frac{\frac{\tilde{\iota}}{1-\frac{1-\delta}{\iota}}}{\iota \left[ \frac{p_B - \tau(1-\mu)z_e^* - \frac{\kappa_A}{1+\nu}(\mu^{1+\nu})}{r_{kB}} \tilde{y}_B + \frac{p_H \tilde{y}_H - p_B \tilde{y}_B}{r_{kG}} \right]}. \end{aligned}$$

Now we can find  $\tilde{k}_B$  and  $\tilde{k}_G$ :

$$\begin{aligned}\tilde{k}_B &= \frac{\alpha_1 \iota}{r_{kB}} \left[ p_B - \tau (1 - \mu) z_e - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B \\ \tilde{k}_G &= \frac{\alpha_1 \iota}{r_{kG}} p_G \tilde{y}_G.\end{aligned}$$

Similarly:

$$\alpha_2 = \frac{\frac{\tilde{i}^*}{1 - \frac{1-\delta}{\iota}}}{\iota \left[ \frac{p_B - \tau (1 - \mu) z_e - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu})}{r_{kB}^*} \tilde{y}_B + \frac{p_H \tilde{y}_H - p_B \tilde{y}_B}{r_{kG}^*} \right]},$$

and so:

$$\begin{aligned}\tilde{k}_B^* &= \frac{\alpha_2 \iota}{r_{kB}^*} \left[ p_B - z_e \tau (1 - \mu) - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B \\ \tilde{k}_G^* &= \frac{\alpha_2 \iota}{r_{kG}^*} p_G \tilde{y}_G.\end{aligned}$$

Find  $h_B$  using the production function:

$$h_B = \left[ \frac{\tilde{y}_B}{A \left( \frac{\tilde{k}_B}{\iota} \right)^{\alpha_1} \left( \frac{\tilde{k}_B^*}{\iota} \right)^{\alpha_2}} \right]^{\frac{1}{1 - \alpha_1 - \alpha_2}}.$$

Using the labor demand of brown firms we find  $w$ :

$$\tilde{w} = \frac{(1 - \alpha_1 - \alpha_2) \left[ p_B - z_e \tau (1 - \mu) - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B}{h_B}$$

Using the labor demand of green firms we find  $h_G$ :

$$h_G = \frac{(1 - \alpha_1 - \alpha_2) (p_H \tilde{y}_H - p_B \tilde{y}_B)}{\tilde{w}}.$$

We can find  $y_G$  using the production function of green firms:

$$\tilde{y}_G = A \left( \frac{\tilde{k}_G}{\iota} \right)^{\alpha_1} \left( \frac{\tilde{k}_G^*}{\iota} \right)^{\alpha_2} (h_G)^{1 - \alpha_1 - \alpha_2}$$

and  $p_G$ :

$$p_G = \frac{1}{\tilde{y}_G} (p_H \tilde{y}_H - p_B \tilde{y}_B).$$

Use the brown foreign demand to find  $\tilde{y}_B$ :

$$y_B^* = \zeta \left( s \frac{p_B^*}{p_F} \right)^{-\xi} y_F^*.$$

Repeat the previous steps for F:

$$\alpha_1^* = \frac{\frac{\tilde{i}_*^*}{1 - \frac{1-\delta}{\iota}}}{\iota \left[ \frac{p_B^* - \tau^* (1-\mu^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu})}{r_{*kB}^*} y_B^* + \frac{\frac{p_F y_F^*}{s} - p_B^* y_B^*}{r_{*kG}^*} \right]}$$

$$k_{*B}^* = \alpha_1^* \iota \left[ p_B^* - \tau^* (1-\mu^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \frac{y_B^*}{r_{*kB}^*}$$

$$k_{*G}^* = \alpha_1^* \iota \frac{\frac{p_F y_F^*}{s} - p_B^* y_B^*}{r_{*kG}^*}$$

$$\alpha_2^* = \frac{\frac{\tilde{i}_*^*}{1 - \frac{1-\delta}{\iota}}}{\iota \left[ \frac{p_B^* - \tau^* (1-\mu^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu})}{r_{*kB}^*} \tilde{y}_B + \frac{\frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B}{r_{*kG}^*} \right]}$$

$$\tilde{k}_{*B} = \alpha_2^* \iota \left[ p_B^* - \tau^* (1-\mu^*) z_e^* - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \frac{\tilde{y}_B}{r_{*kB}^*}$$

$$\tilde{k}_{*G} = \alpha_2^* \iota \frac{\frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B}{r_{*kG}^*}.$$

Find Home consumption:

$$\begin{aligned} \tilde{c} = & p_H \tilde{y}_H - \left( \tilde{i} + p_H \tilde{g} + \frac{\kappa_A}{1+\nu} (\mu_t^{1+\nu}) \tilde{y}_B + \tilde{b} + \frac{(1-n)}{n} s \tilde{i}_* + r^* \left( \frac{\tilde{d}_H + \tilde{v}}{\iota} \right) + \left( \frac{r_{*kB}^* \tilde{k}_{*B}^* + r_{*kG}^* \tilde{k}_{*G}^*}{\iota} \right) \right) + \\ & + s \frac{(1-n)}{n} \left( \frac{r_{*kB}^* \tilde{k}_{*B}^* + r_{*kG}^* \tilde{k}_{*G}^*}{\iota} \right) + r \left( \frac{\tilde{b} + \tilde{d}_H + \tilde{v}}{\iota} \right). \end{aligned}$$

Use the Home market clearing to find  $c^*$ :

$$\begin{aligned}
& (1-n) \left( \gamma^* \left( \frac{p_H t}{s_t} \right)^{-\eta} \left( \tilde{c}_t^* + \tilde{i}_{*t}^* + \tilde{i}_{*t} + \frac{\kappa_A}{1+\nu} (\mu_t^{*1+\nu}) \tilde{y}_{Bt}^* \right) \right) = \\
& n \left\{ \tilde{y}_H - \left[ (1-\gamma) (p_H)^{-\eta} \left( \tilde{c} + \tilde{i} + \frac{\kappa_A}{1+\nu} (\mu^{1+\nu}) \tilde{y}_B + \tilde{i}^* \right) + \tilde{g} \right] \right\} \\
\tilde{c}^* &= \frac{n}{(1-n) \gamma^*} \left( \frac{p_H}{s} \right)^\eta \left\{ \tilde{y}_H - \left[ (1-\gamma) (p_H)^{-\eta} \left( c + i + \frac{\kappa_A}{1+\nu} (\mu^{1+\nu}) \tilde{y}_B + \tilde{i}^* \right) + \tilde{g} \right] \right\} + \\
& - \left( \tilde{i}_{*}^* + \tilde{i}_{*} + \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \tilde{y}_B^* \right).
\end{aligned}$$

Use the labor demands:

$$\begin{aligned}
(1 - \alpha_1^* - \alpha_2^*) \left[ p_B^* - \tau^* (1 - \mu^*) - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^* &= \tilde{w}^* h_B^* \\
(1 - \alpha_1^* - \alpha_2^*) p_G^* \tilde{y}_G^* &= \tilde{w}^* h_G^* \\
h_B^* + h_G^* &= 1.
\end{aligned}$$

Sum the labor demands:

$$\begin{aligned}
\tilde{w}^* &= (1 - \alpha_1^* - \alpha_2^*) \left[ p_B^* - \tau^* (1 - \mu^*) z_e - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^* + (1 - \alpha_1^* - \alpha_2^*) p_G^* \tilde{y}_G^* \\
\tilde{w}^* &= (1 - \alpha_1^* - \alpha_2^*) \left\{ \left[ p_B^* - \tau^* (1 - \mu^*) z_e - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^* + \left( \frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^* \right) \right\}
\end{aligned}$$

And:

$$\begin{aligned}
h_B^* &= \frac{(1 - \alpha_1^* - \alpha_2^*) \left[ p_B^* - \tau^* (1 - \mu^*) z_e - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^*}{\tilde{w}^*} \\
h_G^* &= 1 - h_B^*.
\end{aligned}$$

We can find  $A^*$  using the production function of Foreign brown firms:

$$A^* = \frac{\tilde{y}_B^*}{\left(\frac{\tilde{k}_{*Bt-1}^*}{\iota}\right)^{\alpha_1^*} \left(\frac{\tilde{k}_{*Bt-1}^*}{\iota}\right)^{\alpha_2^*} (h_{Bt}^*)^{1-\alpha_1^*-\alpha_2^*}}$$

and we can find  $\tilde{y}_G^*$  using the production function of Foreign green firms:

$$\tilde{y}_G^* = A^* \left(\frac{\tilde{k}_{*G}^*}{\iota}\right)^{\alpha_1^*} \left(\frac{\tilde{k}_{*G}^*}{\iota}\right)^{\alpha_2^*} (h_G^*)^{1-\alpha_1^*-\alpha_2^*}.$$

We can find  $p_G^*$ :

$$p_G^* = \frac{1}{\tilde{y}_G^*} \left[ \frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^* \right].$$

Find emissions:

$$\tilde{e} = z_e (1 - \mu) \tilde{y}_B$$

$$\tilde{e}^* = z_e^* (1 - \mu^*) \tilde{y}_B^*.$$

We are left with four equations in six unknowns

$$(1-n) \tilde{y}_F^* = (1-n) \left[ (1-\gamma^*) \left(\frac{p_F}{s}\right)^{-\eta} \left( \tilde{c}^* + \tilde{i}_*^* + \tilde{i}_* + \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \tilde{y}_B^* \right) + \tilde{g}^* \right] + \\ + n \left( \gamma^* (p_F)^{-\eta} \left( \tilde{c} + \tilde{i} + \tilde{i}^* + \frac{\kappa_A}{1+\nu} (\mu^{1+\nu}) \tilde{y}_B \right) \right)$$

$$1 = h_G + h_B$$

$$\tilde{y}_G^* = (1-\zeta) \left(\frac{s p_G^*}{p_F}\right)^{-\xi} \tilde{y}_F^*$$

$$\tilde{y}_G = (1-\zeta) \left(\frac{p_G}{p_H}\right)^{-\xi} \tilde{y}_H.$$

We get the additional two equations by imposing that world emissions are equal to  $e^W$  (which is specified ex ante) and that Home emissions are a share  $e^{share}$  of  $e^W$ :

$$n \cdot e + (1 - n) e^* = e^W$$

$$\frac{n \cdot e}{e^W} = e^{share}.$$

To match these two values, we find ex post  $z_e$  and  $z_e^*$ . Once we have solved the previous problem, we can find emissions:

$$\tilde{e} = z_e (1 - \mu) \tilde{y}_B$$

$$\tilde{e}^* = z_e (1 - \mu^*) \tilde{y}_B^*.$$

and  $\{\kappa_G, \kappa_G^*, \kappa_B, \kappa_B^*\}$ , by using the Euler equations:

$$\kappa_G = \frac{1 - \frac{\beta}{\iota} [(r_{kG} + 1 - \delta)]}{\left( \tilde{k}_G + \frac{1-n}{n} s \tilde{k}_{*G} \right)^{-\psi_G} \tilde{c}^*}$$

$$\kappa_G^* = \frac{1 - \frac{\beta^*}{\iota} [(r_{*kG}^* + 1 - \delta)]}{\left( \tilde{k}_{*G}^* + \frac{n}{1-n} \frac{1}{s} \tilde{k}_G^* \right)^{-\psi_G} \tilde{c}^*}$$

$$\kappa_B = \frac{\frac{\beta}{\iota} [(r_{kB} + 1 - \delta)] - 1}{\left( \tilde{k}_B + \frac{1-n}{n} s \tilde{k}_{*B} \right)^{\psi_B} \tilde{c}^*}$$

$$\kappa_B^* = \frac{\frac{\beta^*}{\iota} [(r_{*kB}^* + 1 - \delta)] - 1}{\left( \tilde{k}_{*B}^* + \frac{n}{1-n} \frac{1}{s} \tilde{k}_B^* \right)^{\psi_B} \tilde{c}^*}.$$



## D Final steady state

We find the final steady state as a function of  $\{\tilde{y}_H, p_B, p_B^*, pr_G, pr_G^*, pr_B, pr_B^*, y_F^*, p_H\}$ .

Use the CPI equations to find the relative prices:

$$p_F = \left[ \frac{1 - (1 - \gamma) (p_H)^{1-\eta}}{\gamma} \right]^{\frac{1}{1-\eta}}$$

$$s = [\gamma^* (p_H)^{1-\eta} + (1 - \gamma^*) (p_F)^{1-\eta}]^{\frac{1}{1-\eta}}.$$

The banks conditions reads:

$$\tilde{d}_H = \frac{1}{\Gamma} \left( 1 - \frac{\beta}{\beta^*} \right)$$

$$\tilde{d}_F = \frac{s}{\Gamma^*} \left( \frac{\beta^*}{\beta} - 1 \right).$$

We find  $\tilde{b}$  using the market clearing of assets:

$$\tilde{b} = - \left( \tilde{v} + \tilde{d}_H + \frac{1-n}{n} \tilde{d}_F \right).$$

Given the premium definitions, we find:

$$r_{kB} = \frac{\iota}{\beta} - (1 - \delta) + pr_B$$

$$r_{*kB} = r_{kB}$$

$$r_{kB}^* = \frac{\iota}{\beta^*} - (1 - \delta) + pr_B^*$$

$$r_{*kB}^* = r_{kB}^*$$

$$r_{kG} = \frac{\iota}{\beta} - (1 - \delta) + pr_G$$

$$r_{*kG} = r_{kG}$$

$$r_{kG}^* = \frac{\iota}{\beta^*} - (1 - \delta) + pr_G^*$$

$$r_{*kG}^* = r_{kG}^*.$$

Use the brown demand to find  $y_B$ :

$$\tilde{y}_B = \zeta \left( \frac{p_B}{p_H} \right)^{-\xi} \tilde{y}_H.$$

Use the domestic capital demands:

$$\tilde{k}_B = \frac{\alpha_1 \iota}{r_{kB}} \left[ p_B - z_e \tau (1 - \mu) - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B$$

$$\tilde{k}_G = \frac{\alpha_1 \iota}{r_{kG}} (p_H \tilde{y}_H - p_B \tilde{y}_B).$$

Domestic investment:

$$\tilde{i} = \left( \tilde{k}_B + \tilde{k}_G \right) \left( 1 - \frac{1 - \delta}{\iota} \right),$$

and

$$\begin{aligned}\tilde{k}_B^* &= \frac{\alpha_2 \ell}{r_{kB}^*} \left[ p_B - z_e \tau (1 - \mu) - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B \\ \tilde{k}_G^* &= \frac{\alpha_2 \ell}{r_{kG}^*} (p_H \tilde{y}_H - p_B \tilde{y}_B) \\ \tilde{i}^* &= \left( \tilde{k}_B^* + \tilde{k}_G^* \right) \left( 1 - \frac{1 - \delta}{\ell} \right).\end{aligned}$$

Find  $h_B$  using the production:

$$h_B = \left[ \frac{\tilde{y}_B}{A \left( \frac{\tilde{k}_B}{\ell} \right)^{\alpha_1} \left( \frac{\tilde{k}_B^*}{\ell} \right)^{\alpha_2}} \right]^{\frac{1}{1 - \alpha_1 - \alpha_2}}.$$

Using the labor demand we find  $w$ :

$$\tilde{w} = \frac{(1 - \alpha_1 - \alpha_2) \left[ p_B - z_e \tau (1 - \mu) - \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \right] \tilde{y}_B}{h_B}.$$

Using the labor demand of green firms we find  $h_G$ :

$$h_G = \frac{(1 - \alpha_1 - \alpha_2) (p_H \tilde{y}_H - p_B \tilde{y}_B)}{\tilde{w}}.$$

We can find  $y_G$  using the production function of green firms:

$$\tilde{y}_G = A \left( \frac{\tilde{k}_G}{\ell} \right)^{\alpha_1} \left( \frac{\tilde{k}_G^*}{\ell} \right)^{\alpha_2} (h_G)^{1 - \alpha_1 - \alpha_2}$$

and:

$$p_G = \frac{1}{\tilde{y}_G} (p_H \tilde{y}_H - p_B \tilde{y}_B).$$

Use the brown foreign demand to find  $\tilde{y}_B$ :

$$\tilde{y}_B^* = \zeta \left( s \frac{p_B^*}{p_F} \right)^{-\xi} \tilde{y}_F^*.$$

As before:

$$\begin{aligned}\tilde{k}_{*B}^* &= \alpha_1^* \iota \left[ p_B^* - z_e^* \tau^* (1 - \mu^*) - \frac{\kappa_A}{1 + \nu} (\mu^{*1+\nu}) \right] \frac{\tilde{y}_B^*}{r_{*kB}^*} \\ \tilde{k}_{*G}^* &= \alpha_1^* \iota \frac{\frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^*}{r_{*kG}^*} \\ \tilde{k}_{*B} &= \bar{\alpha}_2^* \iota \left[ p_B^* - z_e^* \tau^* (1 - \mu^*) - \frac{\kappa_A}{1 + \nu} (\mu^{*1+\nu}) \right] \frac{\tilde{y}_B^*}{r_{*kB}^*} \\ \tilde{k}_{*G} &= \bar{\alpha}_2^* \iota \frac{\frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^*}{r_{*kG}^*}\end{aligned}$$

and:

$$\begin{aligned}\tilde{i}_*^* &= \left( \tilde{k}_{*B}^* + \tilde{k}_{*G}^* \right) \left( 1 - \frac{1 - \delta}{\iota} \right) \\ \tilde{i}_* &= \left( \tilde{k}_{*B} + \tilde{k}_{*G} \right) \left( 1 - \frac{1 - \delta}{\iota} \right).\end{aligned}$$

Find Home consumption:

$$\begin{aligned}\tilde{c}^* &= \frac{n}{(1 - n) \gamma^*} \left( \frac{p_H}{s} \right)^\eta \left\{ \tilde{y}_H - \left[ (1 - \gamma) (p_H)^{-\eta} \left( c + i + \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \tilde{y}_B + \tilde{i}^* \right) + \tilde{g} \right] \right\} + \\ &- \left( \tilde{i}_*^* + \tilde{i}_* + \frac{\kappa_A}{1 + \nu} (\mu^{*1+\nu}) \tilde{y}_B^* \right).\end{aligned}$$

Use the Home market clearing to find  $c^*$ :

$$\begin{aligned}(1 - n) \left( \gamma^* \left( \frac{p_{Ht}}{s_t} \right)^{-\eta} \left( \tilde{c}_t^* + \tilde{i}_{*t}^* + \tilde{i}_{*t} + \frac{\kappa_A}{1 + \nu} (\mu_t^{*1+\nu}) \tilde{y}_{Bt}^* \right) \right) = \\ n \left\{ \tilde{y}_H - \left[ (1 - \gamma) (p_H)^{-\eta} \left( \tilde{c} + \tilde{i} + \frac{\kappa_A}{1 + \nu} (\mu^{1+\nu}) \tilde{y}_B + \tilde{i}^* \right) + \tilde{g} \right] \right\}\end{aligned}$$

$$\begin{aligned}\tilde{c}^* &= \frac{n}{(1-n)\gamma^*} \left(\frac{p_H}{s}\right)^\eta \left\{ \tilde{y}_H - \left[ (1-\gamma) (p_H)^{-\eta} \left( c + i + \frac{\kappa_A}{1+\nu} (\mu^{1+\nu}) \tilde{y}_B + \tilde{i}^* \right) + \tilde{g} \right] \right\} + \\ &- \left( \tilde{i}_*^* + \tilde{i}_* + \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \tilde{y}_B^* \right).\end{aligned}$$

Sum the labor demands:

$$\begin{aligned}\tilde{w}^* &= (1 - \alpha_1^* - \alpha_2^*) \left[ p_B^* - z_e \tau^* (1 - \mu^*) - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^* + (1 - \alpha_1^* - \alpha_2^*) p_G^* \tilde{y}_G^* \\ \tilde{w}^* &= (1 - \alpha_1^* - \alpha_2^*) \left\{ \left[ p_B^* - z_e \tau^* (1 - \mu^*) - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^* + \left( \frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^* \right) \right\}\end{aligned}$$

and:

$$h_B^* = \frac{(1 - \alpha_1^* - \alpha_2^*) \left[ p_B^* - z_e \tau^* (1 - \mu^*) - \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \right] \tilde{y}_B^*}{\tilde{w}^*}$$

$$h_G^* = 1 - h_B^*,$$

and we can find  $\tilde{y}_G^*$ :

$$\tilde{y}_G^* = A^* \left( \frac{\tilde{k}_{*G}^*}{\iota} \right)^{\alpha_1^*} \left( \frac{\tilde{k}_{*G}}{\iota} \right)^{\alpha_2^*} (h_G^*)^{1-\alpha_1^*-\alpha_2^*}.$$

We can find  $p_G^*$ :

$$p_G^* = \frac{1}{\tilde{y}_G^*} \left[ \frac{p_F \tilde{y}_F^*}{s} - p_B^* \tilde{y}_B^* \right].$$

Find emissions:

$$e = z_e (1 - \mu) y_B$$

$$e^* = z_e (1 - \mu) y_B^*.$$

We are left with 9 equations in 9 unknowns

$$(1-n) \tilde{y}_F^* = (1-n) \left[ (1-\gamma^*) \left( \frac{p_F}{s} \right)^{-\eta} \left( \tilde{c}^* + \tilde{i}_*^* + \tilde{i}_* + \frac{\kappa_A}{1+\nu} (\mu^{*1+\nu}) \tilde{y}_B^* \right) + \tilde{g}^* \right] + \\ + n \left( \gamma^* (p_F)^{-\eta} \left( \tilde{c} + \tilde{i} + \tilde{i}^* + \frac{\kappa_A}{1+\nu} (\mu^{1+\nu}) \tilde{y}_B \right) \right)$$

$$\tilde{y}_G^* = (1-\zeta) \left( \frac{sp_G^*}{p_F} \right)^{-\xi} \tilde{y}_F^*$$

$$\tilde{y}_G = (1-\zeta) \left( \frac{p_G}{p_H} \right)^{-\xi} \tilde{y}_H$$

$$\kappa_G = \frac{1 - \frac{\beta}{\iota} [(r_{kG} + 1 - \delta)]}{\left( \tilde{k}_G + \frac{1-n}{n} s \tilde{k}_{*G} \right)^{\psi_G} \tilde{c}}$$

$$\kappa_G^* = \frac{1 - \frac{\beta^*}{\iota} [(r_{*kG}^* + 1 - \delta)]}{\left( \tilde{k}_{*G}^* + \frac{n}{1-n} \frac{1}{s} \tilde{k}_G^* \right)^{\psi_G} \tilde{c}^*}$$

$$\tilde{y}_B^* = A^* \left( \frac{\tilde{k}_{*Bt-1}^*}{\iota} \right)^{\alpha_1^*} \left( \frac{\tilde{k}_{*Bt-1}}{\iota} \right)^{\alpha_2^*} (h_{Bt}^*)^{1-\alpha_1^*-\alpha_2^*}$$

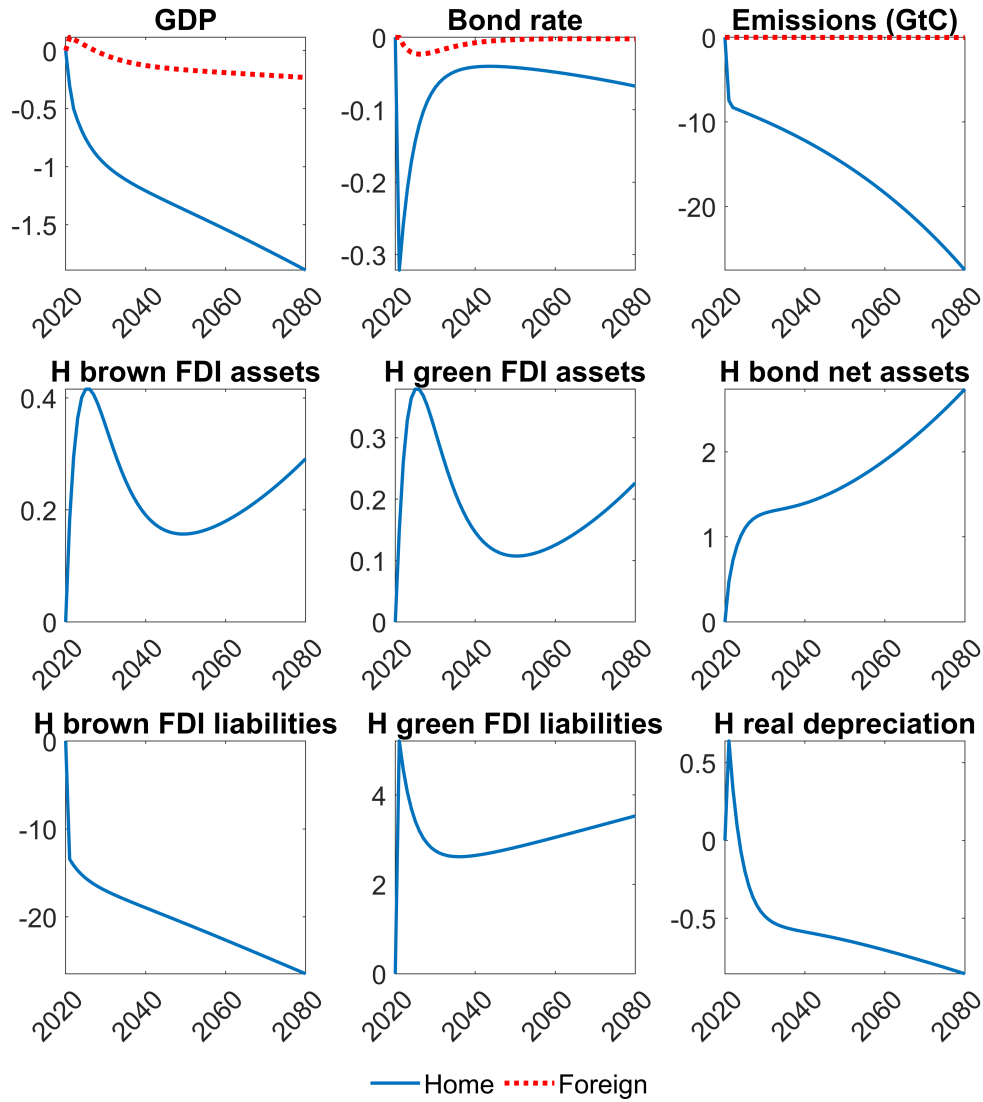
$$\kappa_B = \frac{\frac{\beta}{\iota} [(r_{kB} + 1 - \delta)] - 1}{\left( \tilde{k}_B + \frac{1-n}{n} s \tilde{k}_{*B} \right)^{\psi_B} \tilde{c}}$$

$$\kappa_B^* = \frac{\frac{\beta^*}{\iota} [(r_{*kB}^* + 1 - \delta)] - 1}{\left( \tilde{k}_{*B}^* + \frac{n}{1-n} \frac{1}{s} \tilde{k}_B^* \right)^{\psi_B} \tilde{c}^* x}$$

$$h = h_G + h_B.$$

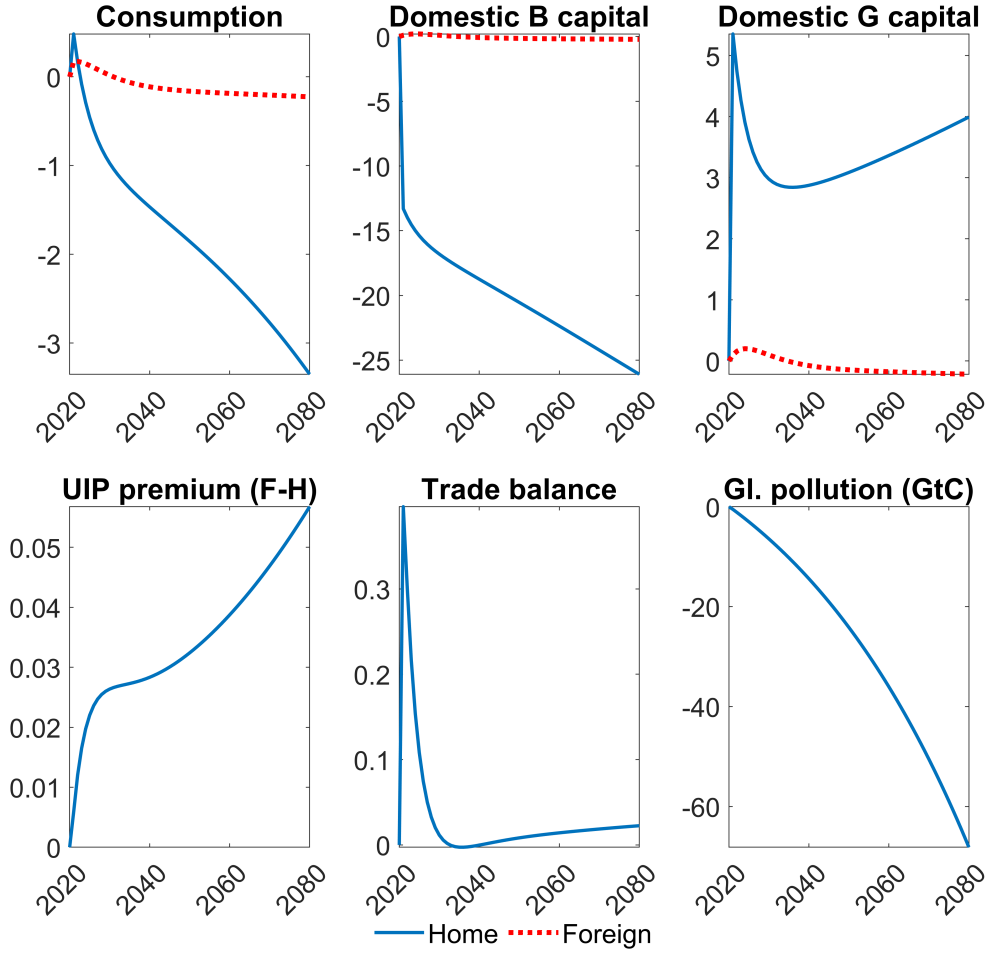
## E Additional figures

### Macroeconomic effects of the transition: Asymmetric calibration, asymmetric policies (1)



**Figure E.1:** Variables are plotted in percentage deviation from the baseline scenario, except for the bond rate and emissions (plotted in level deviations from the baseline scenario), and bond assets (in level deviations from the baseline scenario, as a share of initial GDP). An increase in the Home real exchange rate means a Home real depreciation. Country F follows the baseline policy, country H sets optimally  $F = 0.0024$ .

**Macroeconomic effects of the transition:**  
**Asymmetric calibration, asymmetric policies (2)**



**Figure E.2:** Consumption and capital are plotted in percentage deviation from the baseline scenario, pollution and UIP premium are plotted in level deviations from the baseline scenario, the trade balance is in deviations from the baseline scenario as a share of initial GDP). The UIP premium is defined as  $r_t^* \frac{s_{t+1}}{s_t} - r_t$ . Country F follows the baseline policy, country H sets optimally  $F = 0.0058$ . Red dotted line: Nash equilibrium under  $\psi_X = 1$  ( $F = F^* = 0.0036$ ).



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