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A GOODNESS-OF-FIT TEST FOR GENERALIZED ERROR DISTRIBUTION

by Daniele Coin*

Abstract

The Generalized Error Distribution is a widely used flexible family of symmetric probability distribution. Thanks to its properties, it is becoming more and more popular in many fields of science, and therefore it is important to determine whether a sample is drawn from a GED, usually done using a graphical approach. In this paper we present a new goodness-of-fit test for GED that performs well in detecting non-GED distribution when the alternative distribution is either skewed or a mixture. A comparison between well-known tests and this new procedure is performed through a simulation study. We have developed a function that performs the analysis described in this paper in the R environment. The computational time required to compute this procedure is negligible.

JEL Classification: C14, C15, C63.
Keywords: exponential power distribution, kurtosis, normal standardized Q-Q plot.

Contents

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1 Introduction

The exponential power (EP) distribution with mean $\mu \in (-\infty, +\infty)$, variance $\sigma^2 \in (0, +\infty)$ and power parameter $\beta \in (-1; 1]$ is a symmetrical unimodal distributions family. The density function is

$$f_{EPD}\left(x; \mu, \sigma, \beta\right) = \frac{e^{-\frac{1}{2}\left|\frac{x-\mu}{\sigma}\right|^{\frac{2}{1+\beta}}}}{2^{\frac{\beta+3}{2}}\sigma\Gamma\left(\frac{\beta+3}{2}\right)}, \tag{1}$$

and it has domain on $(-\infty, +\infty)$. This family is also known as Generalized Error Distribution (GED) and it is a flexible member of the exponential family (Box and Tiao (1973), Harvey (1990)).

The shape parameter β determines (1) to become the density function of a range of symmetric distributions such as the uniform ($\beta \to -1$) and the double exponential ($\beta = 1$). The shape of the density is more platykurtic than the normal distribution if $\beta < 0$ the converse if $\beta > 0$ (it is normal if $\beta = 0$); to obtain the standard normal distribution we set $\beta = 0$, $\mu = 0$ and $\sigma = 1$ in (1)).

Box and Tiao (1973) have proposed the parametrization reported in (1); another widespread one is given by $v = \frac{2}{1+\beta}$ with $v > 1$ as new shape parameter (see for example Nelson (1991)).

The flexibility properties of the GED family has granted its numerous applications, for example modeling band encoding of audio and video signals (see Sharifi and Leon-Garcia (1995)), the error distribution in time series analysis (see Nelson (1991), Chen *et al.* (2008a)) moreover many application to financial analysis have been suggested (see Theodossiou (2000), Chen *et al.* (2008b), Lee *et al.* (2008) and Marín and Sucarrat (2012)). However before adopting a GED model in an applied problem the most important issue is to determine whether the distribution from which the sample is drawn actually belongs to the GED family. Many procedures for testing the normality of univariate samples have been proposed in the literature, but only graphical methods are commonly used in order to assess GED distribution. This article introduces a new goodness-of-fit statistic test for GED. This procedure is based on a standardized Q-Q plot. By an extensive simulation study we explore the properties of our proposal showing its ability to detect non GED samples.

The paper is organized as follows. In section 2 we summarize the relationship between GED and a normal standardized Q-Q plot. Section 3 presents our contribution, in section 4 an extensive power study is summarized while in section 5 an application to financial data is presented; finally concluding remarks are provided in section 6.

2 Standardized Q-Q Plot and Generalized Error Distribution

Given a set of ordered observations $\mathbf{x}_{(.)} = (x_{(1)}, ..., x_{(n)})$ and $\boldsymbol{\alpha}_n = (\alpha_1, ..., \alpha_n)$, the vector of n expected values of a hypothesized standard ordered n dimensional distribution, a Q-Q plot is constructed by plotting $x_{(i)}$ against the normal scores α_i, given by

$$\alpha_i = \frac{n!}{(i-1)!(n-1)!} \int_{-\infty}^{+\infty} y \left[1 - \Phi(y)\right]^{i-1} \Phi(y)^{n-i} \phi(y) dy, \tag{2}$$

where $\phi(y) = \frac{1}{2\pi} exp\left(-\frac{1}{2}y^2\right)$ and $\Phi(y) = \int_{-\infty}^{y} \phi(z)dz$. In any situation the values assumed by the elements of $\boldsymbol{\alpha}_n$ are only functions of n. Since the normal scores are unknown, we used the numerical approximation algorithm proposed by Royston (1982), that estimates numerically the values of α_i in (2).

The standardized Q-Q plot is instead constructed by plotting

$$z_{(i)} = \frac{x_{(i)} - \widehat{\mu}}{\widehat{\sigma}} \tag{3}$$

against α_i, where $\widehat{\mu}$ and $\widehat{\sigma}$ are the sample mean and the sample standard deviation, respectively. If the estimates of location and scale parameters are selected such that (3) is location and scale invariant, linear transformation of the original data will not alter any point of the plot. This is useful because the intercept and slope of the best fit line have to be 0 and 1 respectively.

GED samples present an S-shape when displayed on a normal standardized Q-Q plot, the slope and the curvature are function of β since it determines kurtosis. In the appendix of Coin (2013) it is proved that symmetrical distributions give rise to symmetrical, with respect to the origin of the axes, inverted S-shaped graphs if they have heavier tails than the normal distribution and symmetrical, with respect the origin of the axes, S-shaped curves if their tails are thinner or shorter than the normal distribution. GED are symmetrical with heavy tails if $\beta > 0$ and thin or short ones for $\beta < 0$ by definition. This property is clearly represented in figure 1.

3 Test for Generalized Error Distribution

The assumption presented in the previous section also entails that the sum of the orthogonal distances between the points $\left(z_{(i)}, \alpha_i\right)$ and the straight line (α_i, α_i) of the Normal Standardized Q-Q Plots are function of β and the sample sizes if $\mathbf{x} \sim GED(\mu, \sigma, \beta, n)$.

Our proposal is based on this intuition: given a set of ordered observations $\mathbf{x}_{(.)} = (x_{(1)}, ..., x_{(n)})$ we test whether $\mathbf{x}$ is GED-distributed as follows: firstly we computed the standardized version of $\mathbf{x}_{(.)}$ obtaining $\mathbf{z}_{(.)} = \frac{x_{(.)} - \widehat{\mu}}{\widehat{\sigma}}$, where $\widehat{\mu}$ and $\widehat{\sigma}$ are the sample mean and the sample standard deviation of $\mathbf{x}_{(.)}$ after we compute the test statistic as follow:

$$T = \frac{\sum_{i=1}^{n} \widehat{z}_{(i)}^2}{\sum_{i=1}^{n} z_{(i)}^2}, \tag{4}$$

where $\widehat{z}_{(i)}$ are the fitted values of z of the following model:

$$z_{(i)} = \alpha_i + \beta_3 \alpha_{(i)}^3 + \epsilon \tag{5}$$

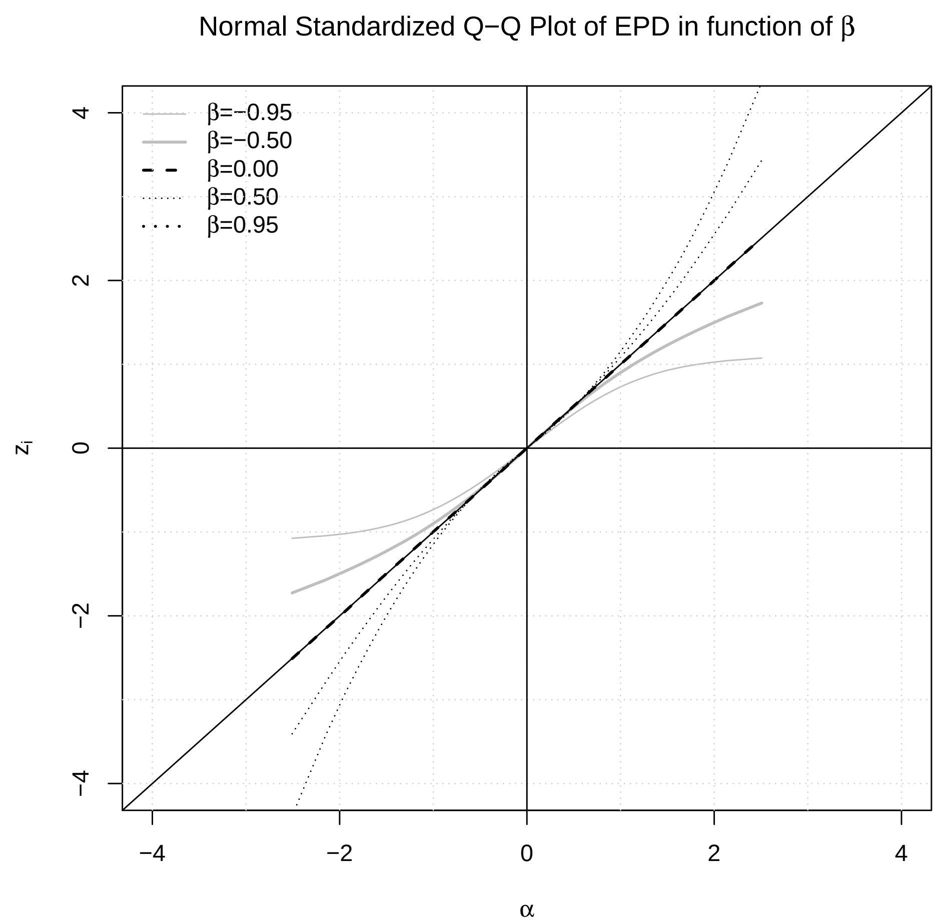

Figure 1: Ordered values of GED plotted on the Normal Standardized Q-Q Plots for different values of β

where ϵ are independent and identically distributed random errors with mean equal to 0. The unknown parameter β_3 in (5) is estimated with Ordinary Least Squares obtaining and $\widehat{\beta}_3$.

The test statistic (4) is a Shapiro and Francia (1972) like test and it can be defined also as the R-squared of the linear model (5). Hence (4) is a statistical measure of how close the $\widehat{z}_{(i)}$s are to the $z_{(i)}$; being also the percentage of the response variable variation that is explained by a linear model (5) then (4) is always between 0 and 1; values close to 1 mean the standardized sample is well approximated by the theoretical values given by the null hypothesis of $\mathbf{x}$ being GED-distributed. A formal proof of this property is provided in appendix A. Finally we are able to formally define our proposed test, the null hypothesis that a sample $\boldsymbol{x}$ is drawn from a GED distribution with unknown parameters, in symbol

$$H_0 : \boldsymbol{x} \sim GED\left(\mu, \sigma, \beta, n\right), \tag{6}$$

is accepted if

$$T \in [q_T(a, \beta, n), 1] \tag{7}$$

for a level of confidence $1-a$; where $q_T(a, \beta, n)$ is the quantile of level a of (4). Since $q_T(a, \beta, n)$ is unknown and function of the sample size n and the true value of β, we estimated $q_T(a, \beta, n)$ simulating 1,000,000 samples for many combination of n and β, where $n = \{20, 21, 22, ..., 1,000\}$ and $\beta = \{-0.995, -0.99, \dots, 0.99, 0.995, 1\}$.

In table 1 we report a short summary of such results for $n = 50$.

	$\widehat{q}_T(a,\beta,n)$					
β	0.5	0.1	0.05	0.01	0.005	0.001
-0.995	0.9840	0.9716	0.9663	0.9521	0.9456	0.9231
-0.8	0.9872	0.9739	0.9680	0.9552	0.9508	0.9386
-0.6	0.9873	0.9740	0.9684	0.9533	0.9469	0.9304
-0.4	0.9870	0.9742	0.9688	0.9574	0.9519	0.9424
-0.2	0.9861	0.9726	0.9667	0.9534	0.9477	0.9335
0	0.9843	0.9680	0.9611	0.9444	0.9396	0.9278
0.2	0.9823	0.9619	0.9529	0.9288	0.9180	0.8991
0.4	0.9801	0.9555	0.9440	0.9185	0.9074	0.8817
0.6	0.9775	0.9485	0.9346	0.9042	0.8900	0.8597
0.8	0.9751	0.9426	0.9270	0.8904	0.8787	0.8222
1	0.9722	0.9331	0.9130	0.8612	0.8453	0.8137

Table 1: Empirical quantiles of T for $n = 50$ and some selected values of β

In figure 2 instead we report the graphical representation $\widehat{q}_T(0.05)$ in function of n and β.

Finally the acceptance region (7) is modified as follows:

$$T \in \left[\widehat{q}_T(a, \widehat{\beta}), 1\right] \tag{8}$$

where $\widehat{\beta}$ is the estimation of β from the sample $\mathbf{x}$ with the estimator presented in Coin (2013).

In table 3 we present the estimated type I errors of the proposed test. The results are obtained by simulating 20,000 samples for different values of β and

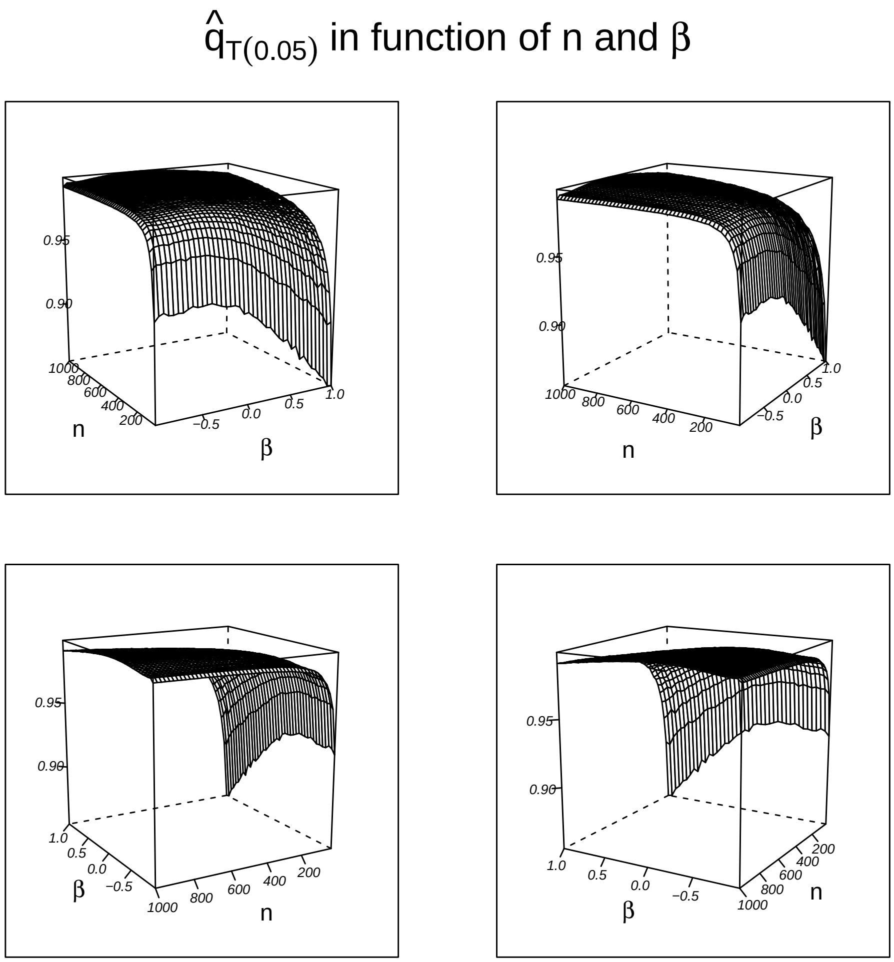

Figure 2: $\widehat{q}_T(0.05)$ in function of n and β

n, the considered sizes of the test are $\alpha = 0.05$ and $\alpha = 0.01$; the estimated type I errors correctly tend to the value of α.

For better understanding we briefly describe the plug-in estimator used for estimating β, given a sample $\mathbf{x} \sim GED\left(\mu, \sigma, \beta, n\right)$ we get standardized ordered sample $\mathbf{z}_{(.)}$ in the same way described above in this section. From the model:

$$z_{(i)} - \alpha_i = \beta_3 \alpha_i^3 + \frac{\epsilon_i}{\sigma}$$

we estimate β_3 with OLS and we substitute in the following plug-in estimator:

$$\widehat{\beta} = f\left(\widehat{\beta}_3\right) = \hat{a}_1 \frac{1}{n} + \hat{a}_2 \frac{1}{n^2} + \hat{a}_3 \widehat{\beta}_3 + \hat{a}_4 \widehat{\beta}_3^2 + \hat{a}_5 \widehat{\beta}_3^3 + \hat{a}_6 \frac{\widehat{\beta}_3}{n}. \tag{9}$$

where n is the sample size and the $\hat{a}_i$ are reported in table 2 (for details how $\hat{a}_i$ are computed see Coin (2013)).

Parameter	Estimates
$\hat{a}_1$	-6.03758
$\hat{a}_2$	-48.41451
$\hat{a}_3$	29.25522
$\hat{a}_4$	219.36466
$\hat{a}_5$	3410.16169
$\hat{a}_6$	-51.11288

Table 2: Estimated parameters of $\hat{a}_i$.

	n=20		n=50		n=100		n=200		n=500		n=1000	
β	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
-0.99	0.0549	0.0104	0.0522	0.0122	0.0519	0.0107	0.0457	0.0086	0.0525	0.0101	0.0498	0.0107
-0.95	0.0535	0.0110	0.0519	0.0095	0.0534	0.0121	0.0500	0.0095	0.0476	0.0098	0.0556	0.0114
-0.9	0.0597	0.0094	0.0486	0.0106	0.0527	0.0104	0.0473	0.0105	0.0513	0.0101	0.0498	0.0080
-0.85	0.0513	0.0078	0.0466	0.0121	0.0486	0.0100	0.0534	0.0104	0.0506	0.0098	0.0550	0.0123
-0.8	0.0475	0.0099	0.0497	0.0140	0.0507	0.0111	0.0503	0.0113	0.0420	0.0095	0.0490	0.0102
-0.75	0.0474	0.0082	0.0490	0.0107	0.0428	0.0084	0.0544	0.0094	0.0531	0.0115	0.0492	0.0126
-0.7	0.0452	0.0087	0.0486	0.0086	0.0462	0.0081	0.0525	0.0126	0.0478	0.0107	0.0546	0.0111
-0.65	0.0483	0.0102	0.0518	0.0117	0.0456	0.0102	0.0503	0.0081	0.0536	0.0089	0.0505	0.0088
-0.6	0.0510	0.0094	0.0438	0.0068	0.0520	0.0123	0.0520	0.0123	0.0574	0.0136	0.0471	0.0084
-0.55	0.0488	0.0107	0.0484	0.0093	0.0509	0.0086	0.0500	0.0120	0.0432	0.0077	0.0529	0.0096
-0.5	0.0476	0.0098	0.0522	0.0106	0.0569	0.0126	0.0476	0.0109	0.0456	0.0104	0.0522	0.0101
-0.45	0.0479	0.0092	0.0501	0.0093	0.0503	0.0094	0.0591	0.0123	0.0476	0.0107	0.0500	0.0073
-0.4	0.0503	0.0084	0.0508	0.0143	0.0508	0.0122	0.0429	0.0152	0.0592	0.0108	0.0500	0.0114
-0.35	0.0476	0.0108	0.0510	0.0126	0.0472	0.0111	0.0525	0.0094	0.0468	0.0092	0.0476	0.0131
-0.3	0.0463	0.0091	0.0495	0.0082	0.0458	0.0081	0.0479	0.0104	0.0513	0.0100	0.0447	0.0118
-0.25	0.0479	0.0114	0.0517	0.0094	0.0523	0.0131	0.0478	0.0088	0.0475	0.0112	0.0530	0.0097
-0.2	0.0481	0.0097	0.0520	0.0115	0.0486	0.0115	0.0583	0.0126	0.0464	0.0099	0.0532	0.0124
-0.15	0.0522	0.0109	0.0598	0.0129	0.0491	0.0110	0.0551	0.0146	0.0481	0.0083	0.0507	0.0097
-0.1	0.0570	0.0106	0.0511	0.0109	0.0527	0.0101	0.0495	0.0115	0.0479	0.0100	0.0549	0.0121
-0.05	0.0481	0.0095	0.0505	0.0108	0.0580	0.0124	0.0459	0.0097	0.0500	0.0117	0.0493	0.0102
0	0.0561	0.0121	0.0482	0.0104	0.0506	0.0102	0.0483	0.0086	0.0500	0.0082	0.0517	0.0097
0.05	0.0491	0.0080	0.0551	0.0097	0.0469	0.0084	0.0476	0.0113	0.0525	0.0114	0.0510	0.0076
0.1	0.0515	0.0106	0.0512	0.0095	0.0413	0.0077	0.0561	0.0134	0.0522	0.0129	0.0462	0.0094
0.15	0.0500	0.0115	0.0494	0.0098	0.0461	0.0086	0.0479	0.0095	0.0452	0.0121	0.0474	0.0089
0.2	0.0519	0.0126	0.0483	0.0080	0.0456	0.0095	0.0539	0.0126	0.0463	0.0106	0.0512	0.0074
0.25	0.0525	0.0109	0.0527	0.0091	0.0552	0.0094	0.0493	0.0089	0.0449	0.0108	0.0461	0.0095
0.3	0.0493	0.0113	0.0505	0.0120	0.0517	0.0105	0.0542	0.0131	0.0481	0.0083	0.0522	0.0115
0.35	0.0472	0.0095	0.0515	0.0123	0.0548	0.0112	0.0479	0.0103	0.0484	0.0093	0.0475	0.0116
0.4	0.0461	0.0091	0.0478	0.0100	0.0460	0.0085	0.0480	0.0091	0.0466	0.0080	0.0508	0.0112
0.45	0.0552	0.0109	0.0498	0.0087	0.0567	0.0132	0.0562	0.0116	0.0463	0.0096	0.0551	0.0121
0.5	0.0471	0.0099	0.0469	0.0089	0.0510	0.0114	0.0510	0.0079	0.0498	0.0098	0.0413	0.0081
0.55	0.0537	0.0104	0.0512	0.0076	0.0505	0.0104	0.0440	0.0088	0.0517	0.0126	0.0458	0.0102
0.6	0.0447	0.0088	0.0481	0.0100	0.0478	0.0083	0.0574	0.0147	0.0520	0.0112	0.0546	0.0103
0.65	0.0518	0.0104	0.0503	0.0129	0.0516	0.0090	0.0471	0.0077	0.0498	0.0086	0.0471	0.0093
0.7	0.0483	0.0110	0.0500	0.0117	0.0440	0.0102	0.0476	0.0071	0.0521	0.0098	0.0552	0.0109
0.75	0.0472	0.0085	0.0473	0.0126	0.0513	0.0131	0.0517	0.0078	0.0525	0.0117	0.0452	0.0112
0.8	0.0529	0.0102	0.0555	0.0128	0.0500	0.0106	0.0465	0.0081	0.0519	0.0118	0.0554	0.0106
0.85	0.0501	0.0093	0.0491	0.0114	0.0462	0.0086	0.0521	0.0118	0.0503	0.0096	0.0501	0.0124
0.9	0.0495	0.0091	0.0509	0.0104	0.0551	0.0106	0.0524	0.0077	0.0470	0.0097	0.0486	0.0112
0.95	0.0467	0.0100	0.0535	0.0100	0.0522	0.0106	0.0478	0.0097	0.0476	0.0110	0.0512	0.0087
1	0.0508	0.0078	0.0554	0.0088	0.0520	0.0103	0.0534	0.0134	0.0493	0.0088	0.0535	0.0134

Table 3: Empirical type I error of T

4 Power Study

In order to evaluate the power of our proposal, a Monte Carlo study was performed. We estimated the power of our testing procedure by simulating 20,000 samples from many alternative distributions, considering the cases of 20, 50, 100, 200, 500 and 1000 units sample size. Power simulations are based on a equal to 0.05 and 0.01 type I error level. Then we compared the power of our test with the ones of two widespread tests based on different distances between theoretical distribution function (Φ_X), given by the null hypothesis, and the empirical distribution function (F_n) of the sample. We will consider the statistic proposed by Kolmogorov (1933) because of its wide diffusion, which is defined as

$$KS = \sup_{\mathbf{z}} |\Phi_Z(\mathbf{z}) - F_n(\mathbf{z})| \,, \tag{10}$$

where Φ_Z is the theoretical standardized distribution function and $\mathbf{z}$ is an n-size ordered standardized sample.

Many authors (e.g. D'Agostino and Stephens (1986) pag. 370-374) suggest the use of the following statistics proposed by Anderson and Darling (1954) for testing normality and adapted to any distribution by Marsaglia *et al.* (2004). It is defined as

$$A = -n - \sum_{i=1}^{n} (2i-1) \frac{\ln P_i + \ln(1 - P_{n+1-i})}{n}, \tag{11}$$

where

$$P_i = \Phi_Z(Z_{(i)}) = \int_{-\infty}^{Z_{(i)}} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt$$

and $z_{(i)}$ is the standardized i ordered value of the sample $\mathbf{x}_n$.

The two summarized above tests can be performed only for simple null hypothesis, in other words the unknown parameters of (1) need to be explicitly defined in H_0, here they are defined by estimation with sample mean and sample variance for μ and σ^2 while β with the method presented in Coin (2013).

The alternative distributions considered in this study were selected to be representative of the various types of distributions considered in many extensive review of power studies for goodness-of-fit tests.

In the first set there are location contaminated normal distributions, $LCN(p, m)$ denotes the case in which an observation is randomly selected with a probability of $1 - p$ from a standard normal distribution and probability p from a normal distribution with mean m and variance 1.

The second set deals with symmetric short-tailed distribution with different shapes ($tnorm(a, b)$ denotes the standard normal distribution truncated at a and b). Set 3 contains distributions that have slightly heavier tails than the standard normal distribution and the standard normal one. Scale contaminated normal distributions are included in sets 4, 5 and 6; $SCN(p, b)$ denotes the case in which an observation is randomly selected with a probability of $1 - p$ from a standard normal distribution and probability p from a normal distribution with variance b and mean 0. Large kurtosis value distribution are in set 7.

The distributions in sets 8 and 9 are skewed unimodal with relative low kurtosis. They are separated by negative and positive skewness values. The location-contaminated normal distributions in sets 10, 11 and 12 are bimodal with positive skewness and relative low kurtosis. Set 13 includes the distributions with the most extreme skewness kurtosis values.

Tables 4, 5, 6, 7, 8 and 9 present the estimated powers of our proposed (T), Kolmogorov-Smirnov (KS) and Anderson-Darling (A) tests versus the alternative distributions summarized above and for different sample sizes equal to 20, 50, 100, 200, 500 and 1000 respectively. The significance levels were: $a = 0.05$ and $a = 0.01$. For better understanding of the simulation results, the highest simulated power for any size among the three tests is highlighted in bold.

For any alternative distribution we also computed the following two indexes of skewness and kurtosis.

We define skewness as the standardized third moment

$$\sqrt{b_1} = \frac{m_3}{m_2^{\frac{3}{2}}}, \tag{12}$$

which is equal to 0 for normal distributions.

The Pearson's measure of kurtosis is the fourth moment and it is equal to 3 for normal distributions:

$$b_2 = \frac{m_4}{m_2^2}, \tag{13}$$

where

$$m_r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^r}{n}, \tag{14}$$

is r-moment of a sample.

Analyzing the simulation results, it emerges that our method is very powerful against skewed alternatives (groups 8 to 13), its power is higher for higher values of (12), it almost always dominates the other two tests specifically for higher sample sizes; (4) presents also good performances when the alternative is a scale contaminated normal distributions (sets 4, 5 and 6), dominating KS and A tests when the sample size is small (20 and 50 units). KS and specifically A perform better against bimodal alternatives (group 1) when sample sizes is small. Finally (4) is ineffective if the alternative distribution is just a normal truncated one (set 2). Group 3 contains distributions that are: or special case of GED family (normal) or very close to it (t with 9 degrees of freedom and logistic) so the percentage of refusal of alternatives tends to the level of confidence of the test.

5 Application to financial data

In order to present an application to real data we performed our test procedure on financial data: from the website $http://finance.yahoo.com$ we got the daily prices of the Standard & Poor's 500 index for 2015. In total we have 252 observations, denoting the daily price with P_i for day i we get the daily returns

Set	Alternative	$\sqrt{b_1}$	b_2	Empirical tests powers $T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	0.9451	0.2756	0.9964	0.7348	**1**	**1**
	LCN(0.5,5)	0	1.51	0.055	0.0004	0.4836	0.0368	**0.9929**	**0.9907**
	LCN(0.5,4)	0	1.72	0.0125	0.0002	0.2546	0.008	**0.9306**	**0.9173**
	LCN(0.5,3)	0	2.04	0.0102	0.0008	0.1028	0.0009	**0.6924**	**0.6584**
	LCN(0.5,2)	0	2.5	0.0286	0.0032	0.032	0.0004	**0.3642**	**0.3292**
	LCN(0.5,1)	0	2.92	0.0568	0.0144	0.0143	0.0002	**0.2095**	**0.1798**
2	beta(0.5,0.5)	0	1.5	0.1414	0.0521	0.4772	0.0277	**0.9893**	**0.9854**
	unif(0,1)	0	1.8	0.0502	0.0106	0.1584	0.0017	**0.8128**	**0.7805**
	tnorm(-1,1)	0	1.94	0.0379	0.0068	0.0927	0.0006	**0.686**	**0.6444**
	beta(2,2)	0	2.14	0.0358	0.0068	0.0485	0.0001	**0.4841**	**0.4402**
	tnorm(-2,2)	0	2.36	0.0351	0.006	0.0242	0.0003	**0.3207**	**0.2816**
	tnorm(-3,3)	0	2.84	0.053	0.012	0.0131	0.0001	**0.2007**	**0.1741**
3	norm(0,1)	0	3	0.0556	0.0138	0.0134	0	**0.1957**	**0.169**
	t(9)	0	4	0.097	0.034	0.0094	0	**0.1323**	**0.1137**
	logis(0,1)	0	4.2	0.1001	0.0382	0.0086	0	**0.12**	**0.1028**
4	SCN(0.05,3)	0	7.65	**0.1824**	0.1024	0.0108	0.0001	0.133	**0.1112**
	SCN(0.05,5)	0	20	**0.3835**	**0.2843**	0.0588	0.0089	0.1736	0.1324
	SCN(0.05,7)	0	31.4	**0.5269**	**0.439**	0.1499	0.0502	0.2731	0.219
5	SCN(0.1,3)	0	8.33	**0.2426**	**0.1389**	0.0124	0.0003	0.1043	0.0845
	SCN(0.1,5)	0	16.5	**0.4617**	**0.3579**	0.0879	0.0124	0.1651	0.1047
	SCN(0.1,7)	0	21.5	**0.5783**	**0.4943**	0.2293	0.0702	0.3061	0.2054
6	SCN(0.2,3)	0	7.54	**0.2514**	**0.1493**	0.0118	0.0005	0.0687	0.051
	SCN(0.2,5)	0	11.2	**0.4053**	**0.311**	0.0984	0.0109	0.121	0.0682
	SCN(0.2,7)	0	12.8	**0.4587**	**0.3787**	0.2695	0.0554	0.24	0.1321
7	laplace(0,1)	0	5.38	**0.1609**	**0.0783**	0.0073	0.0003	0.0574	0.0443
	t(5)	0	6	**0.1445**	0.066	0.0079	0.0004	0.1007	**0.0817**
	t(3)	0	∞	**0.2352**	**0.1454**	0.0345	0.0102	0.0952	0.0729
	t(1)	0	∞	**0.608**	**0.5418**	0.4998	0.3328	0.489	0.3662
	beta(2,1)	0	∞	0.2819	0.1147	0.0924	0.0009	**0.6826**	**0.6431**
8	tnorm(-2,1)	-0.57	2.4	0.0928	0.0234	0.0489	0.0001	**0.4997**	**0.455**
	beta(3,2)	-0.32	2.27	0.0703	0.0174	0.0395	0	**0.4271**	**0.3847**
	tnorm(-3,1)	-0.29	2.36	0.1744	0.0577	0.0337	0.0001	**0.4123**	**0.3747**
	tnorm(-3,2)	-0.55	2.78	0.0537	0.0117	0.0189	0.0001	**0.2566**	**0.2263**
	weibull(4)	-0.18	2.65	0.054	0.0121	0.0164	0.0001	**0.2324**	**0.2017**
	weibull(3.6)	-0.09	2.75	0.0461	0.0094	0.0166	0.0002	**0.2459**	**0.2141**
	weibull(2.2)	0	2.72	0.1372	0.0456	0.0199	0	**0.2943**	**0.2608**
9	weibull(2)	0.51	3.04	0.1824	0.0676	0.0228	0	**0.3207**	**0.2828**
	LCN(0.2,3)	0.63	3.25	**0.3735**	0.1704	0.024	0.0002	0.2855	**0.2346**
10	LCN(0.2,5)	0.68	3.09	**0.9548**	**0.819**	0.2742	0.0094	0.7504	0.6713
	LCN(0.2,7)	1.07	3.16	**0.9998**	**0.9976**	0.7717	0.0892	0.9829	0.9618
	LCN(0.1,3)	1.25	3.2	**0.2823**	**0.1366**	0.0053	0.0001	0.0835	0.064
11	LCN(0.1,5)	0.8	4.02	**0.8153**	**0.6453**	0.0498	0.0027	0.1644	0.0896
	LCN(0.1,7)	1.54	5.45	**0.99**	**0.9624**	0.3035	0.0319	0.5985	0.3865
	LCN(0.05,3)	1.96	6.6	**0.1838**	**0.0769**	0.0036	0	0.0727	0.0615
12	LCN(0.05,5)	0.68	4.35	**0.5675**	**0.3743**	0.01	0.0003	0.0285	0.0175
	LCN(0.05,7)	1.65	7.44	**0.8994**	**0.7784**	0.0716	0.0024	0.1698	0.0935
	chisq(4)	2.42	10.4	**0.5983**	**0.399**	0.0399	0.0005	0.4267	0.3775
13	exp(4)	1.41	6	**0.8537**	**0.7178**	0.1206	0.0074	0.7222	0.6678
	chisq(1)	2	9	**0.9764**	0.933	0.3824	0.1358	0.9653	**0.9514**
	lnorm(0,1)	2.83	15	**0.939**	**0.8735**	0.2653	0.1011	0.8106	0.7571
	weibull(0.5)	6.18	113.9	**0.9982**	0.9924	0.7241	0.5216	0.9972	**0.9956**
	Tukey(10)	6.62	87.7	**0.2927**	**0.194**	0.2383	0.0447	0.1257	0.0556

Table 4: Comparison of Tests power, 20 units samples

				Empirical tests powers					
Set	Alternative	$\sqrt{b_1}$	b_2	$T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,5)	0	1.51	0.7668	0.3154	0.9872	0.9387	**0.9996**	**0.9988**
	LCN(0.5,4)	0	1.72	0.2571	0.0359	0.7522	0.5794	**0.9721**	**0.945**
	LCN(0.5,3)	0	2.04	0.0437	0.0036	0.2325	0.1238	**0.6691**	**0.5614**
	LCN(0.5,2)	0	2.5	0.0358	0.0058	0.0228	0.0083	**0.1794**	**0.116**
	LCN(0.5,1)	0	2.92	**0.0539**	0.0108	0.0034	9e-04	0.0499	**0.0287**
2	beta(0.5,0.5)	0	1.5	0.2432	0.0772	0.9978	0.9674	**1**	**1**
	unif(0,1)	0	1.8	0.0514	0.0118	0.6586	0.4554	**0.9664**	**0.933**
	tnorm(-1,1)	0	1.94	0.0375	0.0076	0.3444	0.1843	**0.836**	**0.7466**
	beta(2,2)	0	2.14	0.0316	0.0054	0.0871	0.0314	**0.4665**	**0.346**
	tnorm(-2,2)	0	2.36	0.0329	0.0068	0.0171	0.0051	**0.1988**	**0.1235**
	tnorm(-3,3)	0	2.84	0.0465	0.0099	0.0023	7e-04	**0.0488**	**0.0246**
3	norm(0,1)	0	3	**0.0588**	0.014	0.0029	5e-04	0.0444	**0.0246**
	t(9)	0	4	**0.1003**	**0.0358**	0.0013	1e-04	0.0153	0.0074
	logis(0,1)	0	4.2	**0.0984**	**0.0331**	0.0012	0	0.0097	0.0049
4	SCN(0.05,3)	0	7.65	**0.2418**	**0.1352**	0.0069	2e-04	0.0273	0.0098
	SCN(0.05,5)	0	20	**0.5077**	**0.3672**	0.1168	0.0355	0.1777	0.0604
	SCN(0.05,7)	0	31.4	**0.6862**	**0.5523**	0.3146	0.1636	0.4043	0.221
5	SCN(0.1,3)	0	8.33	**0.2761**	**0.1483**	0.0218	0.0013	0.0349	0.0031
	SCN(0.1,5)	0	16.5	**0.5141**	**0.3256**	0.3029	0.097	0.4001	0.1169
	SCN(0.1,7)	0	21.5	0.6581	**0.4268**	0.6368	0.3646	**0.7306**	0.425
6	SCN(0.2,3)	0	7.54	**0.2145**	**0.1075**	0.0346	0.0025	0.0413	0.002
	SCN(0.2,5)	0	11.2	0.3203	**0.16**	0.438	0.1322	**0.5221**	0.1121
	SCN(0.2,7)	0	12.8	0.3999	0.192	0.8294	**0.4866**	**0.8811**	0.4755
7	laplace(0,1)	0	5.38	**0.1281**	**0.0494**	0.0176	0.0012	0.0162	6e-04
	t(5)	0	6	**0.1604**	**0.0726**	0.0102	0.0022	0.0181	0.0052
	t(3)	0	∞	**0.269**	**0.1543**	0.0963	0.0383	0.1199	0.0457
	t(1)	0	∞	0.7773	0.6543	0.915	0.8129	**0.9333**	**0.8251**
	beta(2,1)	0	∞	0.6724	0.4315	0.252	0.125	**0.8113**	**0.7101**
8	tnorm(-2,1)	-0.57	2.4	0.2106	0.075	0.0926	0.0336	**0.5269**	**0.3997**
	beta(3,2)	-0.32	2.27	0.1572	0.0513	0.0491	0.0167	**0.3525**	**0.2422**
	tnorm(-3,1)	-0.29	2.36	**0.478**	**0.2446**	0.04	0.0131	0.3195	0.2237
	tnorm(-3,2)	-0.55	2.78	0.0614	0.0141	0.0084	0.0025	**0.0974**	**0.0578**
	weibull(4)	-0.18	2.65	0.0558	0.0145	0.0056	0.0015	**0.071**	**0.0405**
	weibull(3.6)	-0.09	2.75	0.043	0.0091	0.0061	0.0017	**0.0751**	**0.0428**
	weibull(2.2)	0	2.72	**0.3239**	**0.1454**	0.0113	0.0031	0.1315	0.0786
9	weibull(2)	0.51	3.04	**0.4875**	**0.2631**	0.017	0.0056	0.16	0.101
	LCN(0.2,3)	0.63	3.25	**0.719**	**0.5025**	0.0407	0.003	0.1923	0.0928
10	LCN(0.2,5)	0.68	3.09	**0.9999**	**0.9996**	0.7772	0.3021	0.9687	0.8487
	LCN(0.2,7)	1.07	3.16	**1**	**1**	0.9993	0.9564	**1**	**1**
	LCN(0.1,3)	1.25	3.2	**0.5247**	**0.3057**	0.0141	9e-04	0.0191	0.0034
11	LCN(0.1,5)	0.8	4.02	**0.9923**	**0.9661**	0.4938	0.1159	0.5944	0.1495
	LCN(0.1,7)	1.54	5.45	**1**	**1**	0.9799	0.7646	0.9979	0.8982
	LCN(0.05,3)	1.96	6.6	**0.2358**	**0.0953**	0.0017	0	0.0057	0.0028
12	LCN(0.05,5)	0.68	4.35	**0.7893**	**0.5539**	0.0485	0.0025	0.0599	0.001
	LCN(0.05,7)	1.65	7.44	**0.9922**	**0.9502**	0.4146	0.0762	0.6207	0.0951
	chisq(4)	2.42	10.4	**0.959**	**0.8925**	0.0932	0.0114	0.371	0.1845
13	exp(4)	1.41	6	**0.9991**	**0.9953**	0.4045	0.1055	0.8564	0.6042
	chisq(1)	2	9	**1**	**1**	0.9095	0.6054	0.9987	0.9855
	lnorm(0,1)	2.83	15	**1**	**0.9993**	0.7932	0.4759	0.9616	0.8293
	weibull(0.5)	6.18	113.9	**1**	**1**	0.9987	0.9758	**1**	0.9998
	Tukey(10)	6.62	87.7	0.2998	0.1103	**0.8349**	**0.4468**	0.7566	0.2264

Table 5: Comparison of Tests power, 50 units samples

				Empirical tests powers					
Set	Alternative	$\sqrt{b_1}$	b_2	$T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,5)	0	1.51	0.9964	0.9585	**1**	0.9986	**1**	**1**
	LCN(0.5,4)	0	1.72	0.7496	0.363	0.9409	0.7908	**0.9986**	**0.9935**
	LCN(0.5,3)	0	2.04	0.1438	0.0237	0.3027	0.1144	**0.8055**	**0.655**
	LCN(0.5,2)	0	2.5	0.0408	0.0069	0.0113	0.0015	**0.1566**	**0.0796**
	LCN(0.5,1)	0	2.92	**0.0474**	**0.0097**	7e-04	0	0.0187	0.0075
2	beta(0.5,0.5)	0	1.5	0.4665	0.2229	**1**	**1**	**1**	**1**
	unif(0,1)	0	1.8	0.0392	0.0101	0.9374	0.8106	**0.9994**	**0.9974**
	tnorm(-1,1)	0	1.94	0.0238	0.0048	0.6417	0.378	**0.9854**	**0.9504**
	beta(2,2)	0	2.14	0.0249	0.005	0.1339	0.0365	**0.6972**	**0.5046**
	tnorm(-2,2)	0	2.36	0.0247	0.0041	0.0125	0.001	**0.246**	**0.1162**
	tnorm(-3,3)	0	2.84	**0.0371**	**0.0069**	7e-04	0	0.0181	0.006
3	norm(0,1)	0	3	**0.0517**	**0.0132**	8e-04	0	0.0145	0.0049
	t(9)	0	4	**0.1019**	**0.0338**	0.0014	0	0.0052	6e-04
	logis(0,1)	0	4.2	**0.0934**	**0.0273**	0.0018	0	0.0046	2e-04
4	SCN(0.05,3)	0	7.65	**0.3361**	**0.1846**	0.0227	0.0023	0.0811	0.0081
	SCN(0.05,5)	0	20	**0.6827**	**0.4906**	0.3531	0.1376	0.5781	0.2898
	SCN(0.05,7)	0	31.4	**0.8669**	**0.7113**	0.7114	0.4862	0.8515	0.6684
5	SCN(0.1,3)	0	8.33	**0.2918**	**0.1397**	0.07	0.0064	0.2027	0.0257
	SCN(0.1,5)	0	16.5	0.5933	0.3331	0.6884	0.3611	**0.8718**	**0.5861**
	SCN(0.1,7)	0	21.5	0.7824	0.5099	0.9478	0.8126	**0.9832**	**0.9189**
6	SCN(0.2,3)	0	7.54	0.164	**0.0635**	0.1474	0.0151	**0.3566**	0.0398
	SCN(0.2,5)	0	11.2	0.3277	0.1206	0.902	0.5942	**0.9772**	**0.79**
	SCN(0.2,7)	0	12.8	0.4796	0.1992	0.9978	0.9632	**0.9998**	**0.9913**
7	laplace(0,1)	0	5.38	0.0979	**0.0301**	0.0612	0.0062	**0.1469**	0.0091
	t(5)	0	6	**0.1737**	**0.0815**	0.0276	0.0062	0.0742	0.0141
	t(3)	0	∞	0.3321	**0.2063**	0.2482	0.1015	**0.4109**	0.1734
	t(1)	0	∞	0.9125	0.842	0.9976	0.9885	**0.9994**	**0.9956**
	beta(2,1)	0	∞	0.9337	0.8159	0.4866	0.2066	**0.9768**	**0.9186**
8	tnorm(-2,1)	-0.57	2.4	0.3958	0.1896	0.152	0.0415	**0.7772**	**0.5957**
	beta(3,2)	-0.32	2.27	0.2858	0.1204	0.056	0.0124	**0.5037**	**0.3132**
	tnorm(-3,1)	-0.29	2.36	**0.8193**	**0.6275**	0.0408	0.0065	0.4368	0.2413
	tnorm(-3,2)	-0.55	2.78	**0.0892**	0.0212	0.0025	4e-04	0.0777	**0.0281**
	weibull(4)	-0.18	2.65	**0.0573**	0.0141	0.0013	1e-04	0.0367	**0.0142**
	weibull(3.6)	-0.09	2.75	0.0403	0.0074	0.0017	4e-04	**0.0424**	**0.0147**
	weibull(2.2)	0	2.72	**0.6113**	**0.3773**	0.0061	2e-04	0.1218	0.0474
9	weibull(2)	0.51	3.04	**0.8097**	**0.6187**	0.0169	0.0012	0.1689	0.0699
	LCN(0.2,3)	0.63	3.25	**0.9496**	**0.8537**	0.1778	0.0199	0.3793	0.1262
10	LCN(0.2,5)	0.68	3.09	**1**	**1**	0.9985	0.948	**1**	0.9986
	LCN(0.2,7)	1.07	3.16	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,3)	1.25	3.2	**0.8026**	**0.5828**	0.0623	0.0047	0.0692	0.0031
11	LCN(0.1,5)	0.8	4.02	**1**	**0.9999**	0.9541	0.6646	0.9931	0.8245
	LCN(0.1,7)	1.54	5.45	**1**	**1**	**1**	0.9995	**1**	**1**
	LCN(0.05,3)	1.96	6.6	**0.4755**	**0.2424**	0.0083	3e-04	0.0113	2e-04
12	LCN(0.05,5)	0.68	4.35	**0.9915**	**0.9459**	0.4486	0.0934	0.7647	0.1947
	LCN(0.05,7)	1.65	7.44	**1**	**1**	0.9918	0.8537	0.9998	0.9843
	chisq(4)	2.42	10.4	**0.9991**	**0.9972**	0.3393	0.0726	0.7264	0.3172
13	exp(4)	1.41	6	**1**	**1**	0.8742	0.5159	0.9974	0.938
	chisq(1)	2	9	**1**	**1**	**1**	0.9941	**1**	**1**
	lnorm(0,1)	2.83	15	**1**	**1**	0.9962	0.9551	0.9999	0.9975
	weibull(0.5)	6.18	113.9	**1**	**1**	**1**	**1**	**1**	**1**
	Tukey(10)	6.62	87.7	0.7039	0.227	0.9989	**0.9637**	**0.9999**	0.9581

Table 6: Comparison of Tests power, 100 units samples

Set	Alternative	$\sqrt{b_1}$	b_2	Empirical tests powers $T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,5)	0	1.51	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,4)	0	1.72	0.9949	0.9404	**1**	0.992	**1**	**1**
	LCN(0.5,3)	0	2.04	0.3907	0.1336	0.6582	0.2918	**0.9729**	**0.8976**
	LCN(0.5,2)	0	2.5	0.0471	0.0086	0.0181	0.0018	**0.2173**	**0.0856**
	LCN(0.5,1)	0	2.92	**0.0442**	**0.0076**	3e-04	0	0.01	0.0019
2	beta(0.5,0.5)	0	1.5	0.8631	0.6668	**1**	**1**	**1**	**1**
	unif(0,1)	0	1.8	0.0232	0.0055	**1**	0.9978	**1**	**1**
	tnorm(-1,1)	0	1.94	0.0103	0.002	0.9768	0.8571	**1**	**0.9998**
	beta(2,2)	0	2.14	0.0135	0.0022	0.4121	0.1259	**0.9598**	**0.8513**
	tnorm(-2,2)	0	2.36	0.0216	0.0048	0.0365	0.0035	**0.5112**	**0.2479**
	tnorm(-3,3)	0	2.84	**0.0345**	**0.0071**	4e-04	0	0.0115	0.0019
3	norm(0,1)	0	3	**0.0562**	**0.0113**	1e-04	0	0.0057	0.0011
	t(9)	0	4	**0.1072**	**0.0363**	0.0044	4e-04	0.0368	0.0019
	logis(0,1)	0	4.2	**0.0836**	**0.0264**	0.0081	1e-04	0.0575	0.0023
4	SCN(0.05,3)	0	7.65	**0.4292**	**0.2338**	0.1166	0.0135	0.3786	0.1089
	SCN(0.05,5)	0	20	0.8428	0.6584	0.7982	0.5201	**0.9413**	**0.8129**
	SCN(0.05,7)	0	31.4	0.9732	0.9088	0.9765	0.9039	**0.9952**	**0.9798**
5	SCN(0.1,3)	0	8.33	0.3121	0.1295	0.364	0.0714	**0.7393**	**0.3381**
	SCN(0.1,5)	0	16.5	0.743	0.4363	0.9839	0.8912	**0.9993**	**0.9859**
	SCN(0.1,7)	0	21.5	0.9377	0.721	0.9998	0.9973	**1**	**0.9999**
6	SCN(0.2,3)	0	7.54	0.14	0.0403	0.6637	0.2128	**0.934**	**0.6119**
	SCN(0.2,5)	0	11.2	0.4282	0.1462	0.9999	0.9921	**1**	**0.9999**
	SCN(0.2,7)	0	12.8	0.6937	0.3179	**1**	**1**	**1**	**1**
7	laplace(0,1)	0	5.38	0.0891	0.0229	0.3074	0.0448	**0.7141**	**0.2564**
	t(5)	0	6	0.1925	**0.0956**	0.1086	0.0187	**0.3512**	0.0955
	t(3)	0	∞	0.426	0.282	0.6378	0.3253	**0.8881**	**0.6323**
	t(1)	0	∞	0.9864	0.9662	**1**	**1**	**1**	**1**
	beta(2,1)	0	∞	0.999	0.9918	0.9194	0.6402	**1**	**0.9994**
8	tnorm(-2,1)	-0.57	2.4	0.6823	0.4534	0.4822	0.1588	**0.9869**	**0.9308**
	beta(3,2)	-0.32	2.27	0.5213	0.3087	0.1822	0.029	**0.8351**	**0.5979**
	tnorm(-3,1)	-0.29	2.36	**0.9836**	**0.9492**	0.1269	0.0145	0.7859	0.4882
	tnorm(-3,2)	-0.55	2.78	**0.1721**	**0.0562**	0.0029	1e-04	0.0976	0.0269
	weibull(4)	-0.18	2.65	**0.0671**	**0.0185**	0.0021	3e-04	0.0318	0.0079
	weibull(3.6)	-0.09	2.75	**0.0406**	0.0064	0.0013	0	0.0347	**0.0081**
	weibull(2.2)	0	2.72	**0.9048**	**0.7785**	0.0267	0.0016	0.2179	0.0629
9	weibull(2)	0.51	3.04	**0.9838**	**0.9455**	0.0671	0.0045	0.3547	0.1107
	LCN(0.2,3)	0.63	3.25	**0.9995**	**0.9958**	0.6441	0.2396	0.8391	0.4508
10	LCN(0.2,5)	0.68	3.09	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.2,7)	1.07	3.16	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,3)	1.25	3.2	**0.9824**	**0.9243**	0.2996	0.0554	0.4272	0.062
11	LCN(0.1,5)	0.8	4.02	**1**	**1**	**1**	0.9988	**1**	0.9999
	LCN(0.1,7)	1.54	5.45	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.05,3)	1.96	6.6	**0.7712**	**0.5267**	0.0394	0.0022	0.1181	0.0049
12	LCN(0.05,5)	0.68	4.35	**1**	**0.9999**	0.9631	0.6896	0.9992	0.9599
	LCN(0.05,7)	1.65	7.44	**1**	**1**	**1**	**1**	**1**	**1**
	chisq(4)	2.42	10.4	**1**	**1**	0.8646	0.4901	0.9947	0.8894
13	exp(4)	1.41	6	**1**	**1**	0.9998	0.9879	**1**	0.9999
	chisq(1)	2	9	**1**	**1**	**1**	**1**	**1**	**1**
	lnorm(0,1)	2.83	15	**1**	**1**	**1**	**1**	**1**	**1**
	weibull(0.5)	6.18	113.9	**1**	**1**	**1**	**1**	**1**	**1**
	Tukey(10)	6.62	87.7	0.9989	0.9206	**1**	**1**	**1**	**1**

Table 7: Comparison of Tests power, 200 units samples

Set	Alternative	$\sqrt{b_1}$	b_2	Empirical tests powers $T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,5)	0	1.51	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,4)	0	1.72	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,3)	0	2.04	0.8755	0.6338	0.9975	0.949	**0.9999**	**0.9997**
	LCN(0.5,2)	0	2.5	0.0712	0.0147	0.0833	0.0073	**0.517**	**0.2409**
	LCN(0.5,1)	0	2.92	**0.0452**	**0.0086**	0	0	0.0072	4e-04
2	beta(0.5,0.5)	0	1.5	0.9998	0.9973	**1**	**1**	**1**	**1**
	unif(0,1)	0	1.8	0.0052	5e-04	**1**	**1**	**1**	**1**
	tnorm(-1,1)	0	1.94	7e-04	3e-04	**1**	**1**	**1**	**1**
	beta(2,2)	0	2.14	0.0015	4e-04	0.9819	0.8269	**1**	**1**
	tnorm(-2,2)	0	2.36	0.0089	0.0013	0.3745	0.068	**0.9816**	**0.8782**
	tnorm(-3,3)	0	2.84	**0.0246**	**0.0049**	2e-04	0	0.01	8e-04
3	norm(0,1)	0	3	**0.0442**	**0.0092**	2e-04	0	0.0019	2e-04
	t(9)	0	4	0.1237	0.0485	0.0659	0.0056	**0.2958**	**0.0777**
	logis(0,1)	0	4.2	0.0832	0.0237	0.1012	0.0088	**0.4242**	**0.1215**
4	SCN(0.05,3)	0	7.65	0.599	0.334	0.6594	0.3566	**0.9037**	**0.7355**
	SCN(0.05,5)	0	20	0.979	0.9024	0.9996	0.9951	**1**	**1**
	SCN(0.05,7)	0	31.4	0.9998	0.9973	**1**	**1**	**1**	**1**
5	SCN(0.1,3)	0	8.33	0.3436	0.116	0.97	0.8483	**0.9978**	**0.9843**
	SCN(0.1,5)	0	16.5	0.941	0.6867	**1**	**1**	**1**	**1**
	SCN(0.1,7)	0	21.5	0.9987	0.9638	**1**	**1**	**1**	**1**
6	SCN(0.2,3)	0	7.54	0.1153	0.0199	0.9994	0.9884	**1**	**0.9997**
	SCN(0.2,5)	0	11.2	0.7618	0.3413	**1**	**1**	**1**	**1**
	SCN(0.2,7)	0	12.8	0.9651	0.7542	**1**	**1**	**1**	**1**
7	laplace(0,1)	0	5.38	0.0779	0.0175	0.964	0.7057	**0.9988**	**0.9836**
	t(5)	0	6	0.2413	0.1305	0.6591	0.3113	**0.9239**	**0.7351**
	t(3)	0	∞	0.5793	0.4396	0.9969	0.9678	**0.9996**	**0.9987**
	t(1)	0	∞	0.9998	0.9993	**1**	**1**	**1**	**1**
	beta(2,1)	0	∞	**1**	**1**	**1**	0.9998	**1**	**1**
8	tnorm(-2,1)	-0.57	2.4	0.953	0.8648	0.9961	0.921	**1**	**1**
	beta(3,2)	-0.32	2.27	0.8748	0.732	0.8422	0.4223	**0.9998**	**0.9946**
	tnorm(-3,1)	-0.29	2.36	**1**	**1**	0.7363	0.2539	0.9995	0.9915
	tnorm(-3,2)	-0.55	2.78	**0.4463**	**0.2152**	0.0169	8e-04	0.3431	0.0948
	weibull(4)	-0.18	2.65	**0.1065**	**0.0342**	0.0031	1e-04	0.0535	0.0076
	weibull(3.6)	-0.09	2.75	0.0359	0.0067	0.0028	0	**0.0638**	**0.0121**
	weibull(2.2)	0	2.72	**0.9994**	**0.9971**	0.2112	0.0254	0.7395	0.3149
9	weibull(2)	0.51	3.04	**1**	**1**	0.5105	0.118	0.9509	0.6457
	LCN(0.2,3)	0.63	3.25	**1**	**1**	0.9993	0.9744	**1**	0.9982
10	LCN(0.2,5)	0.68	3.09	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.2,7)	1.07	3.16	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,3)	1.25	3.2	**1**	**1**	0.9443	0.6707	0.9934	0.8683
11	LCN(0.1,5)	0.8	4.02	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,7)	1.54	5.45	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.05,3)	1.96	6.6	**0.9926**	**0.9641**	0.3808	0.0637	0.7961	0.3328
12	LCN(0.05,5)	0.68	4.35	**1**	**1**	**1**	0.9999	**1**	**1**
	LCN(0.05,7)	1.65	7.44	**1**	**1**	**1**	**1**	**1**	**1**
	chisq(4)	2.42	10.4	**1**	**1**	**1**	0.9991	**1**	**1**
13	exp(4)	1.41	6	**1**	**1**	**1**	**1**	**1**	**1**
	chisq(1)	2	9	**1**	**1**	**1**	**1**	**1**	**1**
	lnorm(0,1)	2.83	15	**1**	**1**	**1**	**1**	**1**	**1**
	weibull(0.5)	6.18	113.9	**1**	**1**	**1**	**1**	**1**	**1**
	Tukey(10)	6.62	87.7	**1**	**1**	**1**	**1**	**1**	**1**

Table 8: Comparison of Tests power, 500 units samples

				Empirical tests powers					
Set	Alternative	$\sqrt{b_1}$	b_2	$T_{0.05}$	$T_{0.01}$	$KS_{0.05}$	$KS_{0.01}$	$A_{0.05}$	$A_{0.01}$
1	LCN(0.5,10)	0	1.15	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,5)	0	1.51	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,4)	0	1.72	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.5,3)	0	2.04	0.9951	0.9672	**1**	**1**	**1**	**1**
	LCN(0.5,2)	0	2.5	0.1072	0.0258	0.3592	0.0661	**0.8739**	**0.6218**
	LCN(0.5,1)	0	2.92	**0.0415**	**0.0072**	1e-04	0	0.0073	6e-04
2	beta(0.5,0.5)	0	1.5	**1**	**1**	**1**	**1**	**1**	**1**
	unif(0,1)	0	1.8	0.0035	2e-04	**1**	**1**	**1**	**1**
	tnorm(-1,1)	0	1.94	0	0	**1**	**1**	**1**	**1**
	beta(2,2)	0	2.14	1e-04	0	**1**	0.9999	**1**	**1**
	tnorm(-2,2)	0	2.36	0.0026	3e-04	0.9589	0.6402	**1**	**0.9999**
	tnorm(-3,3)	0	2.84	**0.0241**	**0.0042**	3e-04	0	0.0152	7e-04
3	norm(0,1)	0	3	**0.046**	**0.0079**	2e-04	0	0.0018	1e-04
	t(9)	0	4	0.1473	0.0679	0.3137	0.0728	**0.7446**	**0.4213**
	logis(0,1)	0	4.2	0.0724	0.0225	0.4896	0.1355	**0.8831**	**0.6118**
4	SCN(0.05,3)	0	7.65	0.7654	0.5132	0.9741	0.8866	**0.9982**	**0.9882**
	SCN(0.05,5)	0	20	0.9997	0.9969	**1**	**1**	**1**	**1**
	SCN(0.05,7)	0	31.4	**1**	**1**	**1**	**1**	**1**	**1**
5	SCN(0.1,3)	0	8.33	0.4603	0.1906	**1**	0.9996	**1**	**1**
	SCN(0.1,5)	0	16.5	0.9983	0.9779	**1**	**1**	**1**	**1**
	SCN(0.1,7)	0	21.5	**1**	**1**	**1**	**1**	**1**	**1**
6	SCN(0.2,3)	0	7.54	0.1715	0.0331	**1**	**1**	**1**	**1**
	SCN(0.2,5)	0	11.2	0.9791	0.8673	**1**	**1**	**1**	**1**
	SCN(0.2,7)	0	12.8	0.9999	0.9954	**1**	**1**	**1**	**1**
7	laplace(0,1)	0	5.38	0.0836	0.0207	**1**	0.9994	**1**	**1**
	t(5)	0	6	0.2957	0.1848	0.9822	0.8757	**0.9996**	**0.9931**
	t(3)	0	∞	0.737	0.6293	**1**	**1**	**1**	**1**
	t(1)	0	∞	**1**	**1**	**1**	**1**	**1**	**1**
	beta(2,1)	0	∞	**1**	**1**	**1**	**1**	**1**	**1**
8	tnorm(-2,1)	-0.57	2.4	0.9989	0.9929	**1**	**1**	**1**	**1**
	beta(3,2)	-0.32	2.27	0.9878	0.9569	0.9999	0.9855	**1**	**1**
	tnorm(-3,1)	-0.29	2.36	**1**	**1**	0.9984	0.9292	**1**	**1**
	tnorm(-3,2)	-0.55	2.78	0.7793	**0.5457**	0.1093	0.0077	**0.8434**	0.4708
	weibull(4)	-0.18	2.65	**0.1764**	**0.0673**	0.0085	3e-04	0.1544	0.0276
	weibull(3.6)	-0.09	2.75	0.0363	0.0076	0.0096	4e-04	**0.1681**	**0.036**
	weibull(2.2)	0	2.72	**1**	**1**	0.761	0.2829	0.9976	0.9243
9	weibull(2)	0.51	3.04	**1**	**1**	0.9684	0.7057	**1**	0.998
	LCN(0.2,3)	0.63	3.25	**1**	**1**	**1**	**1**	**1**	**1**
10	LCN(0.2,5)	0.68	3.09	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.2,7)	1.07	3.16	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,3)	1.25	3.2	**1**	**1**	0.9999	0.998	**1**	**1**
11	LCN(0.1,5)	0.8	4.02	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.1,7)	1.54	5.45	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.05,3)	1.96	6.6	**1**	**0.9999**	0.9131	0.5465	0.9981	0.9532
12	LCN(0.05,5)	0.68	4.35	**1**	**1**	**1**	**1**	**1**	**1**
	LCN(0.05,7)	1.65	7.44	**1**	**1**	**1**	**1**	**1**	**1**
	chisq(4)	2.42	10.4	**1**	**1**	**1**	**1**	**1**	**1**
13	exp(4)	1.41	6	**1**	**1**	**1**	**1**	**1**	**1**
	chisq(1)	2	9	**1**	**1**	**1**	**1**	**1**	**1**
	lnorm(0,1)	2.83	15	**1**	**1**	**1**	**1**	**1**	**1**
	weibull(0.5)	6.18	113.9	**1**	**1**	**1**	**1**	**1**	**1**
	Tukey(10)	6.62	87.7	**1**	**1**	**1**	**1**	**1**	**1**

Table 9: Comparison of Tests power, 1000 units samples

with:

$$R_i = \frac{P_i}{P_{i-1}}.$$

In figure 3 we present the kernel density estimation of $\boldsymbol{R} = [R_1, ..., R_i, ..., R_{252}]$ and on it we performed the normality test by Shapiro and Wilk (1965) obtaining a value of the statistic test of $W = 0.92727$ and a $p.value < 0.0001$ that leads to reject the null hypothesis of normality. On $\boldsymbol{R}$ we computed (4) obtaining $T = 0.9890673$ with $\widehat{\beta} = 1$. The quantiles of T with $\beta = 1$ and $n = 252$ are $q_T(\boldsymbol{p}) = (0.9889, 0.9773, 0.971, 0.956, 0.9502, 0.9388)$ for $\boldsymbol{p} = (0.5, 0.1, 0.05, 0.01, 0.005, 0.001)$, so we can accept the null hypothesis that $\boldsymbol{R} \sim GED$.

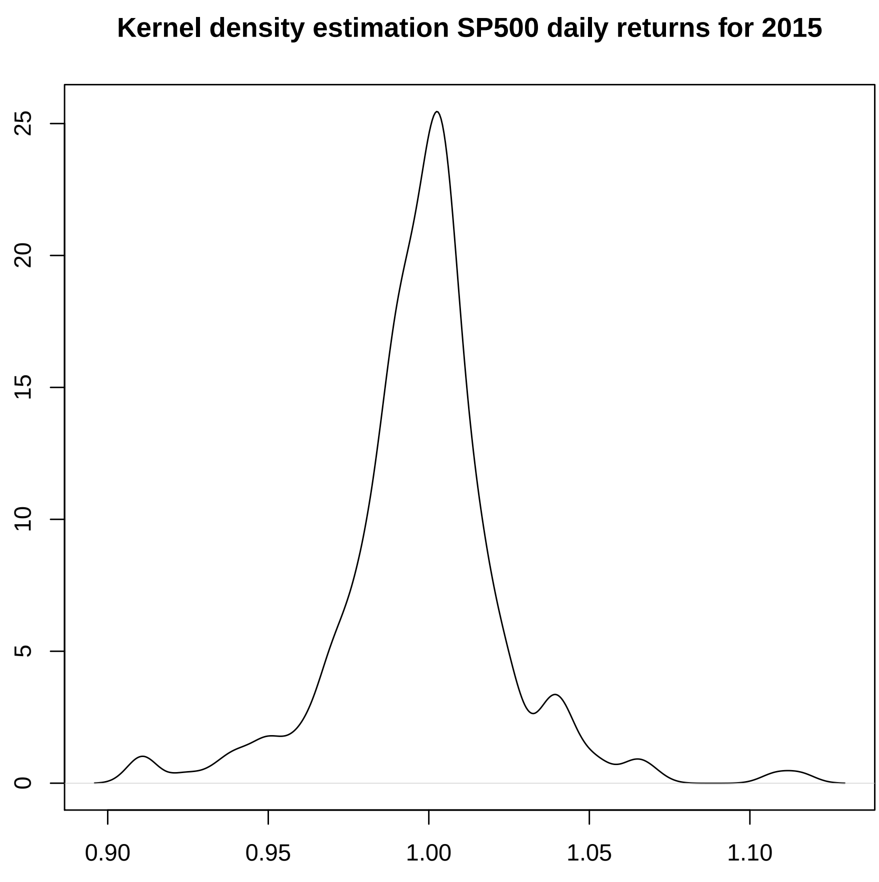

Figure 3: Kernel estimation of the S&P500 daily returns for 2015

6 Concluding Remarks

In this paper we present an original goodness-of-fit test for Generalized Error Distribution. This approach has some appealing features for detecting non GED distribution when the alternative distribution is skewed or a mixture. On the other hand it is quite insensitive to truncated normal alternatives.

The test produces a decision based on the critical values presented in section 3. Unfortunately we are not yet able to supply a method to compute the p-value representing how strongly the hypotheses of GED is rejected.

We have developed a function that performs the analysis described in this paper in the R environment. It can be obtained by e-mail on request to the author. In this R extension it is also possible to find all the critical values computed and cited in this paper. The computational time required to compute this procedure is negligible.

References

Anderson T.W. and Darling D.A. (1954) A test of goodness of fit, *Journal of the American Statistical Association*, 49, 141–160.

Box G. and Tiao G. (1973) *Bayesian inference in statistical analysis.*, Addison-Wesley Ed.; Reading, Massachusetts.

Chen C., Su Y. and Huang Y. (2008a) Hourly index return autocorrelation and conditional volatility in an EAR–GJR-GARCH model with generalized error distribution, *Journal of Empirical Finance*, 15, 4, 789–798.

Chen C.R., Su Y. and Huang Y. (2008b) Hourly index return autocorrelation and conditional volatility in an ear–gjr-garch model with generalized error distribution, *Journal of Empirical Finance*, 15, 4, 789–798.

Coin D. (2013) A method to estimate power parameter in exponential power distribution via polynomial regression, *Journal of Statistical Computation and Simulation*, 83, 11, 1981–2001.

D'Agostino R. and Stephens M. (1986) *Goodness-of-Fit Techniques*, Marcel Dekker, New York.

Davidson R. and MacKinnon J.G. (2004) *Econometric theory and methods*, volume 5, Oxford University Press New York.

Harvey A. (1990) *The econometric analysis of time series*, Mit Press.

Kolmogorov A. (1933) Sulla determinazione empirica di una legge di distribuzione, *Giornale dell'Istituto Italiano Attuari*, 4, 83–91.

Lee M.C., Su J.B. and Liu H.C. (2008) Value-at-risk in us stock indices with skewed generalized error distribution, *Applied Financial Economics Letters*, 4, 6, 425–431.

Marín J.M. and Sucarrat G. (2012) Modelling the skewed exponential power distribution in finance, in: *Mathematical and statistical methods for actuarial sciences and finance*, Springer, 279–286.

Marsaglia G., Marsaglia J. *et al.* (2004) Evaluating the anderson-darling distribution, *Journal of Statistical Software*, 9, 2, 1–5.

Nelson D.B. (1991) Conditional heteroskedasticity in asset returns: A new approach, *Econometrica*, 59, 347–370.

Royston J. (1982) Algorithm AS 177: Expected normal order statistics (exact and approximate), *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 31, 2, 161–165.

Shapiro S.S. and Francia R.S. (1972) An approximate analysis of variance test for normality, *Journal of the American Statistical Association*, 67, 215–216.

Shapiro S.S. and Wilk M.B. (1965) An analysis of variance test for normality (complete samples), *Biometrika*, 52, 591–611.

Sharifi K. and Leon-Garcia A. (1995) Estimation of shape parameter for generalized gaussian distributions in subband decompositions of video, *IEEE Transactions on Circuits and Systems for Video Technology*, 5, 1, 52–56.

Theodossiou P. (2000) Skewed generalized error distribution of financial assets and option pricing, *Available at SSRN 219679*.

A Appendix: a formal proof of the domain of the test statistics

In this section we prove that:

$$0 \leq T \leq 1. \tag{15}$$

The expression (4) can be rewritten as follows:

$$T = \frac{\widehat{\boldsymbol{z}}'\widehat{\boldsymbol{z}}}{\boldsymbol{z}'\boldsymbol{z}} = \frac{Dev(\widehat{\boldsymbol{z}})}{Dev(\boldsymbol{z})}, \tag{16}$$

where $\widehat{\boldsymbol{z}}$ and $\boldsymbol{z}$ are the vectors of $\widehat{z}_{(i)}$ and of $z_{(i)}$ respectively, while $Dev()$ is the deviance operator. The mean of $\boldsymbol{z}$ is 0 since it is a standardized vector as the mean of $\widehat{\boldsymbol{z}}$ is the same of $\boldsymbol{z}$ for the properties of ordinary least squares (see for example Davidson and MacKinnon (2004) chapter 1). Thanks to the geometrical properties of the OLS we can write (see Davidson and MacKinnon (2004) chapter 2):

$$\boldsymbol{z}'\boldsymbol{z} = \widehat{\boldsymbol{z}}'\widehat{\boldsymbol{z}} + (\boldsymbol{z} - \widehat{\boldsymbol{z}})'(\boldsymbol{z} - \widehat{\boldsymbol{z}}), \tag{17}$$

and dividing (17) by $\boldsymbol{z}'\boldsymbol{z}$ we get:

$$1 = T + \frac{(\boldsymbol{z} - \widehat{\boldsymbol{z}})'(\boldsymbol{z} - \widehat{\boldsymbol{z}})}{\boldsymbol{z}'\boldsymbol{z}}, \tag{18}$$

and finally

$$T = 1 - \frac{(\boldsymbol{z} - \widehat{\boldsymbol{z}})'(\boldsymbol{z} - \widehat{\boldsymbol{z}})}{\boldsymbol{z}'\boldsymbol{z}}, \tag{19}$$

since $\frac{(\boldsymbol{z} - \widehat{\boldsymbol{z}})'(\boldsymbol{z} - \widehat{\boldsymbol{z}})}{\boldsymbol{z}'\boldsymbol{z}}$ is a ratio of two squared operations, it has to be ≥ 0, this assumption with the two identities (18) and (19) prove (15).

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2017

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