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by Sara Cecchetti, Marco Rocco and Laura Sigalotti

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CONTAGION AND FIRE SALES IN BANKING NETWORKS

by Sara Cecchetti*, Marco Rocco** and Laura Sigalotti*

Abstract

The paper develops a theoretical framework to analyze the connection between the structure of banking networks and their resilience to systemic shocks. We base our analysis on the model of interbank contagion proposed by Cifuentes, Ferrucci and Shin (2005), which accounts for the impact of illiquid assets' fire sales. We develop this model along three main lines: (i) analytically proving, in a general setting, the existence of an equilibrium and the convergence of the algorithm that can be used to compute it; (ii) extending the scope of the simulations (e.g., including an assessment of the resilience of different stylized network topologies and a sensitivity analysis); (iii) generalizing the model to deal with the case where more than one illiquid asset is available on the market.

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1 Introduction¹

The global financial crisis brought to the attention of both academics and policy-makers the importance of taking into account the intertwined nature of financial systems. Interconnections between financial institutions are varied and arise both from the asset side and the liability side of their balance sheets. For instance, connections between banks include direct exposures in interbank loans and indirect links stemming from holding similar portfolios or sharing the same group of depositors.

Network analysis provides a convenient way to model linkages among financial institutions. Since the seminal paper by Allen and Gale (2000), a network perspective has been increasingly used in Economics and Finance and network analytical tools have become part of the policymaker’s toolkit, especially for financial stability purposes. A careful assessment of the externalities that risks associated with single institutions can create for the entire system is necessary for the purposes of financial supervision. Network analysis can contribute to the assessment of such externalities and to the implementation of adequate regulatory initiatives; for instance, measures of interconnectedness were introduced by the Financial Stability Board in its regulatory framework on systemically important financial institutions (see e.g. Gai, 2013, for a discussion of financial stability and systemic risk based on network methods).

In general a network is simply a collection of nodes, linked with one another by arcs, or edges. A banking system can be represented as a network in which nodes correspond to banks and arcs stand for financial interdependencies among those banks, such as bilateral loans on the interbank market or more complex interactions (see Newman, 2010; Jackson, 2010, for an introduction to network analysis and its applications to social and economic networks). Network topology, i.e. the arrangement of links among nodes, accounts for the system’s “emerging properties”, i.e. those features of the network that cannot be inferred from the characteristics of its individual components, but only from the way in which these units are related to each other. Network theory provides a conceptual framework to analyze the different patterns of interconnections among financial institutions

¹This publication should not be reported as representing the views of the European Central Bank (ECB) or the Bank of Italy. The views expressed are those of the authors and do not necessarily reflect those of the ECB or the Bank of Italy. We are grateful to Piergiorgio Alessandri, Lorenzo Burlon, Antonio Di Cesare, Giovanni di Iasio, Giorgio Gobbi, Giuseppe Grande, Giancarlo Mazzoni, Marcello Pericoli and Stefano Battison for useful comments and suggestions.

and to assess their implications in terms of systemic risk.

The academic literature that studies financial systems using network analytical tools has grown at a fast pace over the last few years. Following Allen and Babus (2009), we distinguish among two main approaches, depending on the type of questions addressed by network theory: (i) papers studying network effects, focusing on phenomena taking place in fixed networks; (ii) papers addressing the mechanism of network formation.² Our paper relates to the first strand of literature, which includes extensive research on financial stability and contagion: by investigating the responses of different financial network structures to a shock hitting one or more banks and propagating through the system, the most resilient structures are identified.³ Among the main contributions to the literature on contagion in financial networks and its implications for financial stability, we recall Allen and Gale (2000), Freixas et al. (2000), Cifuentes et al. (2005a,b), Gai and Kapadia (2010), Upper (2011), Gai et al. (2011), Allen et al. (2012), Amini and Minca (2012), Gauthier et al. (2012), Cabrales et al. (2014), Elliot et al. (2014), Acemoglu et al. (2015a). These papers explore the mechanisms through which solvency and liquidity shocks may generate a domino effect in a financial network, where distress can flow through a variety of channels.⁴ In particular, Cifuentes, Ferrucci, and Shin (2005a,b, CFS hereafter) is a pioneering paper on the interaction between two important channels of contagion: direct balance-sheet exposures between banks and price deterioration of bank assets due to fire sales. The authors develop a stylized model in which, on one hand, banks can be affected by direct exposure on the interbank market, where the default of some of their counterparties can occur; on the other hand, banks hold similar portfolios, whose market value can be depressed by a fire sale mechanism.

Cifuentes et al. (2005a,b) relate to the literature on asset fire sales, which began with the paper by Shleifer and Vishny (1992), where asset market liquidity is linked to the funding of financial intermediaries. This field of research gained

²The literature on strategic network formation in financial markets developed in very recent years; important contributions include Zawadowski (2013); Farboodi (2014); Acemoglu et al. (2015b); Babus (2015). Our paper adopts a “network effects” perspective, thus diverging from this growing literature.

³In this paper we adopt a rather broad and informal definition of contagion as the transmission of (severe) negative shocks affecting one or more banks to other banks in the system. This is in line with “what has evolved into the most common usage of the term contagion – the transmission of an extreme negative shock in one country to another country (or group of countries)” (Forbes, 2012).

⁴In addition to the theoretical investigations, a number of empirical works studied the stability of national banking systems by simulating the effects of financial contagion on those systems. For instance, Mistrulli (2011); Battison et al. (2015) deal with the Italian interbank market.

greater importance after the recent financial crisis, when fire sales contributed to aggravate the fragility of the financial system by deteriorating the balance sheets of financial institutions.⁵ An incomplete list of the recent theory on asset fire sales in financial markets include Brunnermeier and Pedersen (2009); Adrian and Shin (2010); Shleifer and Vishny (2011); Jotikasthira et al. (2012); Caballero and Simsek (2013); Geertsema (2014).

The paper by CFS is the first one in the literature in which price effects play an important role in the modelization of contagion in interbank networks; we will describe their model in details in Section 2 since the present paper builds on that framework. We elaborate on the layout by CFS mainly along three directions: (i) we provide complete analytical foundations to CFS framework, presenting a constructive proof of the existence of an equilibrium, along with the convergence of the algorithm to such equilibrium (Section 3); (ii) we extend the scope of the simulations and perform a thorough investigation of the resilience of different stylized network topologies to exogenous systemic shocks; we also conduct a sensitivity analysis of the model to its main parameters, along with related policy implications (Section 4); (iii) we generalize the model to deal with the case where banks are endowed with different classes of illiquid assets, having different prices, and each kind of illiquid assets can undergo price deterioration phenomena at the same time; this assumption adds realism to the model but increases significantly both theoretical and computational complexity (Section 5). Within our analysis, in line with a wide share of the related literature, we do not model possible strategical anticipation of the default of a bank by other institutions (banks can only take action once they are hit by a shock or by contagion) and we do not model all relevant channels of contagion (in particular, we do not consider liquidity hoarding). These extensions are left for future research.

In the empirical section of this paper we investigate the fragility of different stylized network topologies hit by an exogenous shock; the importance of the structure of the lending relationships among banks for the stability of the financial system was studied in many papers, including Allen and Gale (2000); Allen et al. (2012); Roukny et al. (2013); Cabrales et al. (2014). In particular, Cabrales et al. (2014) adopt a theoretical perspective to assess the resilience of banking network characterized by different stylized configurations. In our empirical analysis we consider four archetypal topologies: totally interconnected, circle, star and core-

⁵See, e.g. United States Treasury (2009), which states that the outburst of the housing bubble triggered a wide-scale deleveraging in financial markets and subsequent fire sales.

periphery topologies; despite being overly simplistic, this stylized configurations can provide useful insights on the differences in the fragility of banking networks with different arrangements.

Consistently with the literature, in this paper the main results of the simulation exercises highlight the crucial role of the network topology in determining how the banking system may be affected by systemic shocks. In particular, simulation results confirm that networks with a star or core-periphery structure may be more resilient, but are more fragile with respect to targeted shocks that affect core banks (Section 4.2). Concerning the distribution of the initial shock, we also show that, even in the case of homogeneous banks (i.e., banks with identical balance-sheets), the impact of contagion following an exogenous shock of fixed aggregate magnitude depends on the number of banks among which the shock is distributed. The dependence is non-linear and varies according to the network topology (Section 4.3). Furthermore, we show that the network structure also affects the relative importance of the two main parameters of the model (i.e., the degree of liquidity of non-interbank assets and the requirement on the leverage ratio) in the transmission of the shock. For instance, in the star topology the two parameters are equally important, while in the circle and totally interconnected configurations either the former or the latter plays a preeminent role (Section 4.5). As a consequence, the relative effectiveness of policy instruments (e.g. capital or liquidity requirements) employed to tackle different risks in the banking system may depend on the topology of the interbank network.

Finally, when several illiquid assets are available, we model the liquidation strategy of each bank as a solution to a convenient optimization problem. In this way we are able to: (i) analytically justify the behavioral rule assumed in the CFS model for the single-asset case; (ii) highlight possible discrepancies between microfounded optimal strategies and liquidation strategies that are often assumed as sensible behavioral rules. In particular, since bank assets are marked to market, the liquidity of an asset is not an intrinsic quality of the asset (fully described by the elasticity of its price) from the point of view of a bank that has to deleverage, but depends on both the composition of the bank's portfolio and the liquidation strategies adopted by the other banks in the system.

The remainder of the paper is organized as follows. Section 2 presents CFS model, on which our work builds. Section 3 presents our formalization of the model in the case of a single illiquid asset, with the proof of the existence of an equilibrium and the convergence of the algorithm. Section 4 discusses the results

of simulation exercises in the single asset case based on the theoretical framework built in the previous section, while Section 5 generalizes that framework to the case of multiple illiquid assets. Finally, Section 6 concludes.

2 CFS model

The starting point of this work is the paper by Cifuentes et al. (2005a,b, CFS), which deals with liquidity risk in a network of financial institutions and explore the effects of asset fire sales. They consider a system of interconnected institutions (which can be thought of as stylized banks) subject to a regulatory constraint on their leverage ratio, calculated marking-to-market the value of assets. The institutions in the network can hold interbank claims and two kinds of assets (one liquid and one illiquid);⁶ moreover, they can borrow funds from the interbank market and collect deposits. When the system is hit by an exogenous shock, some of the institutions may go in distress and become unable to meet the capital requirement; in this case they can ease their balance sheet positions by selling part of their non-interbank assets. If the market demand for such assets is not perfectly inelastic, then sales by distressed institutions can lower the corresponding price. Since the regulatory ratio is computed using marked-to-market assets, a further round of endogenously generated sales can occur, possibly leading to contagious failures. The paper by CFS is a pioneering contribution capturing the importance of price effects in the modelization of contagion in interbank networks. This section describes in detail CFS model, since our work builds on that framework.

The model considers a system of N interconnected banks. Each institution can hold interbank assets and liabilities, described by a $N \times N$ matrix

$$\mathbf{L} = (L_{ij})_{1 \leq i, j \leq N},$$

where the entry L_{ij} is the face value of the liability of bank i to bank j . By definition $L_{ij} \geq 0$ and $L_{ii} = 0$ for all i, j . Let L_i be the face value of the total interbank liabilities of bank i , i.e.

$$L_i = \sum_j L_{ij},$$

⁶In the model, the degree of liquidity is represented by a parameter accounting for price elasticity, as shown in equation (4).

and π_{ij} be the proportion of bank i 's liabilities associated with bank j :⁷

$$\pi_{ij} = \begin{cases} L_{ij}/L_i & \text{if } L_i > 0 \\ 1/N & \text{if } L_i = 0. \end{cases}$$

Let $\mathbf{\Pi} = (\pi_{ij})_{1 \leq i, j \leq N}$. We denote by x_i the market value of bank i 's interbank liabilities, which can be lower than L_i because the bank can be unable to repay all of its debts. The model assumes that all the interbank claims have equal seniority, so the payments made by a bank in distress are proportional to its notional liabilities: the borrower i pays the amount $x_i \pi_{ij}$ to the lender j .

In addition, bank i is endowed with a certain amount c of liquid assets and a fixed holding e of illiquid assets, which can be sold at the market price p (which is endogenously determined by the model). Moreover, bank i collects d_i units of deposits. Then the equity value of bank i is given by

$$pe_i + c_i + \sum_j x_j \pi_{ji} - x_i - d_i. \quad (1)$$

Besides equal priority of interbank claims, the model assumes two additional conditions: limited liability of the shareholders, which implies that bank i 's equity value is nonnegative; and priority of debt over equity, which implies that the equity value is strictly positive only if $x_i = L_i$.

For a fixed value of p , based on the previous assumptions the vector of interbank payments is given by

$$x_i = \min \left\{ L_i, pe_i + c_i + \sum_j x_j \pi_{ji} - d_i \right\}; \quad (2)$$

either a bank is able to pay its liabilities in full (L_i), or it pays the marked-to-market asset value.⁸ Equation (2) shows that bank i 's equity value depends on the payments x_j of the other institutions, as well as on the market price p of the illiquid assets. The price p is endogenously determined by the model and depends on the amount of assets liquidated in the market by the banks in distress.

In this model each institution is subject to a capital adequacy constraint: the ratio between the equity value, where the interbank assets and liabilities are

⁷The definition of π_{ij} when $L_i = 0$ ensures that row-sums of $\mathbf{\Pi}$ equal one in any case.

⁸Actually, when deposits are added to the balance-sheet, the quantity $pe_i + c_i + \sum_j x_j \pi_{ji} - d_i$ may take negative values if the counterparties of bank i do not repay their liabilities in full. Then, (2) should be replaced by $x_i = \max \{0, \min \{L_i, pe_i + c_i + \sum_j x_j \pi_{ji} - d_i\}\}$, as in (10).

calculated in terms of the expected payments, and the marked-to-market value of the assets held by the bank must be above a pre-specified threshold \bar{r} . If a bank violates the capital requirement, then it starts selling its assets to ease its balance-sheet position (in this kind of models this is the only feasible option, since raising fresh capital is not allowed). Let t_i be the units of the liquid asset sold by the distressed institution (at full price 1) and s_i be the units of illiquid asset sold by bank i (at price $p \leq 1$). Then, after the sale of t_i units of liquid assets and s_i units of illiquid assets, the regulatory leverage ratio reads

$$r_i = \frac{pe_i + c_i + \sum_j x_j \pi_{ji} - x_i - d_i}{p(e_i - s_i) + (c_i - t_i) + \sum_j x_j \pi_{ji}} \geq \bar{r}. \quad (3)$$

In equation (3), the denominator is the marked-to-market value of the assets after the sale of t_i and s_i : it is assumed that assets are sold for cash (giving $t_i + ps_i$) and that cash does not attract capital requirement, so the ratio takes into account only the residual assets held by the bank.

The goal of the model is to determine an equilibrium solution $(\mathbf{x}, \mathbf{s}, p)$, which is defined as follows.

Definition 2.1. *The triple $(\mathbf{x}, \mathbf{s}, p)$, with \mathbf{x} = vector of payments, \mathbf{s} = vector of sales of illiquid assets, p = price of the illiquid asset, is an equilibrium if the following conditions are satisfied:*

1. *for all i , $x_i = \min \{L_i, pe_i + c_i + \sum_j x_j \pi_{ji} - d_i\}$;*
2. *for all i , s_i is the smallest amount of illiquid assets sold by bank i which guarantees that the regulatory constraint is satisfied, upon selling all the liquid asset. If $r_i < \bar{r}$ for all values of s_i , then s_i is set equal to e_i .*
3. *the equilibrium price satisfies $p = \xi^{-1}(\sum_i s_i)$, where ξ^{-1} denotes the inverse demand function.*

In CFS the inverse demand curve for the illiquid asset is assumed to be exponential,

$$p = \exp\left(-\alpha \sum_i s_i\right), \quad (4)$$

where α is a fixed exogenous parameter mirroring the semi-elasticity of the price. An equilibrium price is defined as a value p such that the aggregate sale $s(p) = \sum_i s_i(p)$ of illiquid assets equals the demand $\xi(p)$, $s(p) = \xi(p)$.

The existence of an equilibrium triple (x, s, p) is a nontrivial issue, even in a stylized interbank network. In fact, the problem is highly nonlinear in the three unknowns, which are defined by implicit conditions. The paper by CFS deals with two subproblems separately:

- on one hand, it states that equation (2) can be solved, for fixed p and s , using a fixed point theorem, as in Eisenberg and Noe (2001);
- on the other hand, it explores the existence of the equilibrium price p (and simultaneously of the optimal quantity s), keeping the payment vector x fixed. The paper provides some conditions under which, for fixed x , the problem admits an equilibrium price lower than 1 (i.e. the trivial solution in which no fire sale occurs) and gives a graphical intuition of the price dynamics.

However, the work by CFS does not address explicitly the existence of the whole equilibrium solution (x, s, p) . In addition, when non-bank deposits are added to the balance-sheet, the arguments used in Eisenberg and Noe (2001) do not apply any longer and the proofs need to be deeply modified. The paper by CFS states that the equilibrium can be determined computationally through an iterative procedure, whose results are shown in a simulation exercise. In the next section we will present a complete analytical foundation which is not present in Cifuentes et al. (2005a,b), including a constructive proof of the existence of the equilibrium, which will be the basis for our simulations.

3 Analytical foundations for the single asset case

In this section we provide a complete analytical foundation for a CFS-like model, presenting a constructive proof of the existence of an equilibrium, along with the convergence of the algorithm used to find it. None of the following proofs is shown or sketched in CFS. As we will show in this section, the existence of an equilibrium is not trivial at all, despite the highly stylized model.

The main result is stated in Theorem 3.8: we construct a non-increasing sequence of prices p^k and a non-increasing sequence of payment vectors x^k which converge to an equilibrium of the problem. Moreover, we describe in detail the algorithm we will implement in the simulations, which generalizes CFS to the case of a general demand function and explicit exogenous shocks. The iterative

procedure is composed of two intertwining algorithms: a modification of Eisenberg and Noe's algorithm to compute the interbank payment vector and a fixed point argument to determine the price of the illiquid asset through a fire sale mechanism.

3.1 Notation and preliminaries

1. Sales of illiquid assets. Let $\xi = \xi(p)$ be a demand function for the illiquid asset, with $p \in (0, 1]$. We assume that ξ is continuous, invertible and such that ξ^{-1} is non-increasing.⁹ Let $s = s^x(p)$ be the supply function, defined as the sum of the units of illiquid asset sold by the banks in the systems.

Bank i is subject to the regulatory requirement defined in Equation (3). If the bank is in distress and violates the regulatory condition, then it liquidates (part of) its assets to ease its balance sheet position. In some cases it is sufficient to sell a share of the liquid assets endowment to comply with the regulatory condition. If bank i 's leverage ratio r_i is still below the regulatory ratio \bar{r} even after selling all the available liquid assets ($t_i = c_i$), then we can compute the minimum amount of illiquid assets the bank needs to liquidate to match exactly the regulatory ratio \bar{r} . Let $g_i^x(p)$ be the amount of illiquid assets which makes the condition $r_i = \bar{r}$ hold for given x and p , after all liquid assets are liquidated: by Equation (3),

$$g_i^x(p) = -\frac{(1 - \bar{r})e_i}{\bar{r}} + \frac{x_i + d_i - (1 - \bar{r})\sum_j x_j \pi_{ji} - c_i}{\bar{r}p}. \quad (5)$$

Note that $g_i^x(p)$ is negative whenever $r_i > \bar{r}$; in this case the bank does not need to sell any illiquid assets. The amount of illiquid assets bank i can sell is bounded from above by its endowment e_i , while the variable $g_i^x(p)$ can take values greater than e_i . Then the share of illiquid assets sold by bank i is given by

$$s_i^x(p) = \begin{cases} e_i & \text{if } x_i < L_i \\ \max\{\min\{e_i, g_i^x(p)\}, 0\} & \text{otherwise.} \end{cases} \quad (6)$$

Note that for all i and x , $s_i^x(p)$ is a non-increasing continuous function of p and that there exist two values $0 \leq q_i^1 \leq q_i^2 \leq 1$ such that $s_i^x(p) = e_i$ for $p \leq q_i^1$, $s_i^x(p) = 0$ for $p \geq q_i^2$ and $s_i^x(p) = g_i^x(p)$ for $p \in (q_i^1, q_i^2)$.

⁹This assumption are sufficient to prove the main results and generalize the exponential demand function used in CFS.

Although the price p can vary in $(0, 1]$ *a priori*, actually we can restrict the range of feasible prices to a smaller interval, which depends on the network structure and on the initial shock which hits the system.

- The upper bound p_0 of the price set depends on the extent of the initial shock which hits the system, possibly triggering a fire sale mechanism. In this setting an exogenous shock is modeled by canceling part of the illiquid assets held by the banks of the network; it can be concentrated on one financial institution or widespread over a number of banks.

The after-shock price p_0 is a function of the magnitude of the shock. If it were equal to 1 regardless of the extent of the shock, we might obtain some incongruous results: a large shock on the illiquid assets of one bank could lead to a small overall effect, since the after-shock quantity of illiquid asset in the bank's portfolio could not be sizable enough to trigger a fire sales mechanism; on the other hand, a smaller shock to the same bank could generate more contagious effects, since in this case the bank's after-shock endowment of illiquid asset could suffice to trigger widespread fire sales, if liquidated.

Let σ be the overall amount of illiquid assets canceled by the exogenous shock. We then assume that, when the shock hits the system, the price of the illiquid asset drops to the after-shock price

$$p_0 = \xi^{-1}(\sigma) = \xi_{\sigma}^{-1}(0), \quad (7)$$

where we introduce the additional notation

$$\xi_{\sigma}^{-1}(s) := \xi^{-1}(\sigma + s).$$

Note that the parameter σ affects the function ξ_{σ}^{-1} only by horizontal shifts (it determines the starting point of the inverse demand function, corresponding to $s = 0$); the shape of the function is the same for all values of the initial shock.

Note that p_0 is the price of the illiquid assets prior to the spread of the contagion and the asset fire sales. Since the fire sale mechanism can only depress the market value of the assets, any feasible price p must be lower than or equal to p_0 .

- On the other hand, the lowest feasible price corresponds to the case in which all the banks in the system liquidate their endowment of illiquid assets in full:

$$p_{\min} = \xi_{\sigma}^{-1} \left(\sum_i e_i \right). \quad (8)$$

We define

$$\Psi^x(p) = \xi_{\sigma}^{-1}(s^x(p)), \quad (9)$$

with $s^x(p)$ as in (6). From the previous observations we deduce that

$$\Psi^x : [p_{\min}, p_0] \rightarrow [p_{\min}, p_0].$$

Since it is the composition of two non-increasing functions, Ψ^x is non-decreasing in p . By Tarski's fixed point theorem (see Theorem A.1) we can deduce the existence and uniqueness of the maximal fixed point of Ψ^x , for a given x :

Lemma 3.1. *For fixed x , there exists a unique $p^* \in [p_{\min}, p_0]$ such that*

$$\Psi^x(p^*) = p^*$$

and $p^* \geq q$ for all $q \in [p_{\min}, p_0]$ satisfying $\Psi^x(q) = q$.

Proof. The interval $[p_{\min}, p_0]$ is a complete lattice; the function $\Psi^x : [p_{\min}, p_0] \rightarrow [p_{\min}, p_0]$ is non-decreasing by construction. By Tarski's fixed point theorem, Ψ^x admits a unique maximal fixed point. ■

2. Clearing payments. Having fixed a price p for the illiquid asset, the vector x is a clearing payment vector if it is a fixed point of the map

$$\Phi_p : [0, L_1] \times \dots \times [0, L_N] \rightarrow [0, L_1] \times \dots \times [0, L_N]$$

defined as

$$\Phi_p(x) = \max\{\min\{L, \Pi'x + pe + c - d\}, 0\}. \quad (10)$$

The function Φ_p is non-decreasing in $[0, L_1] \times \dots \times [0, L_N]$ by construction. The existence of a maximal fixed point of Φ_p can be obtained by Tarski's fixed point theorem (the proof runs as in Lemma 3.1).

Lemma 3.2. *For fixed p , there exists a unique vector $x \in [0, L_1] \times \dots \times [0, L_N]$ such that*

$$x = \Phi_p(x)$$

and that $x \geq y$ for all $y \in [0, L_1] \times \dots \times [0, L_N]$ satisfying $y = \Phi_p(y)$.

The function Φ_p can be written in an equivalent form, which will be convenient to describe the algorithm; to this end, we introduce some additional notation. Let

$$D_p(x) = \{i \in 1, \dots, N : (\Phi_p(x))_i < L_i\} \quad (11)$$

and let

$$(\Lambda_p(x))_{ij} = \begin{cases} 1 & \text{if } i = j \text{ and } i \in D_p(x) \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

For fixed p , given $x, y \in [0, L_1] \times \dots \times [0, L_N]$ we define

$$\begin{aligned} F_{p,y}(x) &= \Lambda_p(y) \left(\min\{L_i, \max\{0, \Pi'(\Lambda_p(y)x + (I - \Lambda_p(y))L) + pe + c - d\}\} \right) \\ &\quad + (I - \Lambda_p(y))L. \end{aligned} \quad (13)$$

By construction we have

$$F_{p,x}(x) = \Phi_p(x), \text{ for all } x. \quad (14)$$

Note that, for fixed p, y , the map $F_{p,y}(\cdot)$ is non-decreasing.

We introduce a definition which will be useful in the proofs.

Definition 3.3. *The vector x is a supersolution for Φ_p if*

$$x \geq \Phi_p(x).$$

We now prove a lemma on the monotonicity of $F_{p,y}$ which will be useful in what follows.

Lemma 3.4. *Let $F_{p,y}(x)$ be defined as in (11)-(13), for $p \in [p_{min}, p_0]$, $x \in [0, L_1] \times \dots \times [0, L_N]$ and $y \in [0, L_1] \times \dots \times [0, L_N]$. Then the function $F_{p,y}(x)$ is non-decreasing:*

- (a) in p , for fixed x, y ;
- (b) in y , for fixed p, x ;

Proof.

- (a) Notice first that, if $p \leq q$, $D_p(y) \supseteq D_q(y)$, as $\Phi_p(y)$ is non-decreasing in p for fixed y , where the two sets are defined in (11). As a consequence,

$\Lambda_p(y) \geq \Lambda_q(y)$ component-wise, the function Λ being defined in (12). We can distinguish three cases:

- if $(\Lambda_p(y))_{ii} = (\Lambda_q(y))_{ii} = 0$, then $(F_{p,y}(x))_i = L_i = (F_{q,y}(x))_i$;
- if $(\Lambda_p(y))_{ii} = (\Lambda_q(y))_{ii} = 1$,

$$\begin{aligned} (F_{p,y}(x))_i &= \min \left\{ L_i, \max \left\{ 0, \sum_j \pi_{ji} ((\Lambda_p(y))_{jj} x_j + (1 - (\Lambda_p(y))_{jj}) L_j) \right. \right. \\ &\quad \left. \left. + p e_i + c_i - d_i \right\} \right\} \\ &\leq \min \left\{ L_i, \max \left\{ 0, \sum_j \pi_{ji} ((\Lambda_q(y))_{jj} x_j + (1 - (\Lambda_q(y))_{jj}) L_j) \right. \right. \\ &\quad \left. \left. + q e_i + c_i - d_i \right\} \right\} = (F_{q,y}(x))_i, \end{aligned}$$

as $(\Lambda_p(y))_{jj} x_j + (1 - (\Lambda_p(y))_{jj}) L_j \leq (\Lambda_q(y))_{jj} x_j + (1 - (\Lambda_q(y))_{jj}) L_j$
and $p e_i \leq q e_i$;

- if $(\Lambda_p(y))_{ii} = 1$ and $(\Lambda_q(y))_{ii} = 0$,

$$\begin{aligned} (F_{p,y}(x))_i &= \min \left\{ L_i, \max \left\{ 0, \sum_j \pi_{ji} ((\Lambda_p(y))_{jj} x_j + (1 - (\Lambda_p(y))_{jj}) L_j) \right. \right. \\ &\quad \left. \left. + p e_i + c_i - d_i \right\} \right\} \leq L_i = (F_{q,y}(x))_i. \end{aligned}$$

- (b) If $y \leq z$, then $D_p(y) \supseteq D_p(z)$, since $\Phi_p(y)$ is non-decreasing in y for fixed p . Therefore, $\Lambda_p(y) \geq \Lambda_p(z)$ component-wise. The proof is then on the same line of (a). ■

We finally present a definition of *equilibrium* coherent with our framework.

Definition 3.5. A pair $(x^*, p^*) \in [0, L_1] \times \dots \times [0, L_N] \times [p_{\min}, p_0]$ is said to be an *equilibrium* if

$$x^* = \Phi_{p^*}(x^*) \quad \text{and} \quad p^* = \Psi^{x^*}(p^*).$$

3.2 The algorithm

We assume that an interbank system as in the previous sections is hit by an exogenous shock σ on the illiquid assets. Let e_i^0 be the initial endowment of illiquid assets for bank i and e_i be the after-shock one. Then $\sigma = \sum_i e_i^0 - \sum_i e_i$. The algorithm runs as follows:

- Step 0 (Initialization): we set

$$x^0 = L, p^0 = p_0, s^0 = (0, \dots, 0), t^0 = (0, \dots, 0).$$

- Step $k, k \geq 1$: in the previous step we determined the payment vector x^{k-1} , the amount of illiquid assets sold by each bank $s^{k-1} = (s_1^{k-1}, \dots, s_N^{k-1})$ and the corresponding price p^{k-1} . Moreover we calculated the amount of liquid assets sold by the banks in the network $t^{k-1} = (t_1^{k-1}, \dots, t_N^{k-1})$.

Now, we first compute the value of the leverage ratio for each bank:

$$r_i^k = \frac{p^{k-1}e_i + c_i + \sum_j x_j^{k-1}\pi_{ji} - x_i^{k-1} - d_i}{p^{k-1}(e_i - s_i^{k-1}) + (c_i - t_i^{k-1}) + \sum_j x_j^{k-1}\pi_{ji}}.$$

We define the vector of indicator functions I^k as $I_i^k = \chi_{r_i^k < \bar{r}}$, i.e. $I_i^k = 0$ if bank i complies with the capital requirement, $I_i^k = 1$ otherwise.

- ◊ If $t_i^{k-1} = c_i$ and $s_i^{k-1} = e_i$ for all i such that $I_i^k = 1$, then we terminate the algorithm: either a bank is satisfying its regulatory ratio (i.e. $I_i^k = 0$), or it has sold all the available assets (i.e. $I_i^k = 1, t_i^{k-1} = c_i$ and $s_i^{k-1} = e_i$). In this case there is no further sale of illiquid assets and the corresponding price remains constant. We set:

$$x^* = x^{k-1} = L, p^* = p^{k-1}, t^* = t^{k-1}, s^* = s^{k-1}. \quad (15)$$

- ◊ If, for some $i, I_i^k = 1$ and $s_i^{k-1} < e_i$, then we compute the amount of assets sold by the banks in distress, the resulting price of the illiquid assets and the payment vector.
 - **Computation of p^k, s^k and t^k .** Namely, we first calculate the units of liquid assets sold by each bank, which is given by

$$t_i^k = \max \left\{ \min \left\{ \frac{d_i + x_i^{k-1} - (1 - \bar{r})(p^{k-1}e_i + c_i + \pi'x^{k-1})}{\bar{r}}, c_i \right\}, 0 \right\}.$$

The formula above states that bank i sells the smallest amount of available liquid assets which guarantees the fulfillment of the capital adequacy condition. If the regulatory ratio is still violated after selling the entire endowment c_i , then bank i needs to start selling its illiquid assets.

Now, s^k and p^k are defined as follows:

$$s^k = \sum_i s_i^k,$$

$$s_i^k = s_i^{x^{k-1}}(p^k) = \begin{cases} e_i & \text{if } x_i^{k-1} < L_i \\ \max\{\min\{e_i, g_i^k(p^k)\}, 0\} & \text{otherwise,} \end{cases} \quad (16)$$

with

$$g_i^k(p) = -\frac{(1-\bar{r})e_i}{\bar{r}} + \frac{x_i^{k-1} - (1-\bar{r})\sum_j x_j^{k-1}\pi_{ji} - c_i}{\bar{r}p}, \quad (17)$$

and p^k is the greatest fixed point of

$$\Psi^k(p) = \xi_\sigma^{-1}(s^k(p)) = \xi_\sigma^{-1}(s^{x^{k-1}}(p)) \text{ in } [p_{\min}, p_0].$$

In order to find p^k numerically, we implement a sub-algorithm starting from the initial guess $P_0 = p_0$. Then for $n \geq 1$ we set the recursive condition $P_n = \Psi^k(P_{n-1})$. By Lemma A.2, the sequence $\{P_n\}$ converges to p^k , so we can stop the iterations when $|\Psi^k(P_n) - P_n|$ is lower than a pre-assigned tolerance level ε .

- o **Computation of x^k .** We define x^k as the greatest fixed point of the map $F_{p^k, x^{k-1}}$, i.e.

$$F_{p^k, x^{k-1}}(x^k) = x^k$$

and $x^k \geq z$ for all z such that $F_{p^k, x^{k-1}}(z) = z$. In order to find a numerical approximation of x^k , we start a sub-algorithm from $X_0 = L$ and we set $X_n = F_{p^k, x^{k-1}}(X_{n-1})$ for $n \geq 1$. By Lemma A.2, the sequence $\{X_n\}$ converges to x^k , so we can stop the iterations when $|F_{p^k, x^{k-1}}(X_n) - X_n|$ is below a pre-assigned tolerance level ε .

Lemma 3.6. *If $p^{k+1} \leq p^k$ and $x^k \geq \Phi_{p^k}(x^k)$, then*

$$x^{k+1} \leq x^k \quad \text{and} \quad x^{k+1} \geq \Phi_{p^{k+1}}(x^{k+1}).$$

Proof. Let \hat{x}^{k+1} and x^{k+1} be the greatest fixed points of the maps $F_{p^k, x^k}(\cdot)$ and $F_{p^{k+1}, x^k}(\cdot)$, respectively, i.e.

$$F_{p^k, x^k}(\hat{x}^{k+1}) = \hat{x}^{k+1}, \quad \hat{x}^{k+1} \geq z \text{ for all } z \text{ such that } z = F_{p^k, x^k}(z),$$

and

$$F_{p^{k+1}, x^k}(x^{k+1}) = x^{k+1}, \quad x^{k+1} \geq z \text{ for all } z \text{ such that } z = F_{p^{k+1}, x^k}(z).$$

Note that \hat{x}^{k+1} and x^{k+1} are well-defined by Theorem A.1.

We firstly show that

$$x^{k+1} \leq \hat{x}^{k+1} \leq x^k, \tag{18}$$

thus getting the first part of the proof. Recall that $F_{p, y}(\cdot)$ is non-decreasing and, by Lemma 3.4, $F_{p^{k+1}, x^k}(y) \leq F_{p^k, x^k}(y)$ for all y . Then, by Lemma A.3,

$$F_{p^{k+1}, x^k}^{(n)}(L) \leq F_{p^k, x^k}^{(n)}(L).$$

By passing to the limit as $n \rightarrow +\infty$ on both sides of the previous inequality and using Lemma A.2, we obtain $x^{k+1} \leq \hat{x}^{k+1}$.

The second inequality can be proved analogously. By Lemma 3.4, $F_{p^k, x^k}(y) \leq F_{p^k, x^{k-1}}(y)$ for all y . As a consequence,

$$F_{p^k, x^k}^{(n)}(L) \leq F_{p^k, x^{k-1}}^{(n)}(L),$$

from which, letting n tend to infinity, $\hat{x}^{k+1} \leq x^k$; this concludes the proof of (18).

It remains to show that x^{k+1} is a supersolution, i.e. $x^{k+1} \geq \Phi_{p^{k+1}}(x^{k+1})$. We have just shown that $x^{k+1} \leq x^k$; by Lemma 3.4 we get

$$F_{p^{k+1}, x^k}(x^{k+1}) \geq F_{p^{k+1}, x^{k+1}}(x^{k+1}).$$

Since $F_{p^{k+1}, x^k}(x^{k+1}) = x^{k+1}$ and $F_{p^{k+1}, x^{k+1}}(x^{k+1}) = \Phi_{p^{k+1}}(x^{k+1})$, the proof is complete. ■

Lemma 3.7. *If $x^k \leq x^{k-1}$, then $p^{k+1} \leq p^k$.*

Proof. We recall that, for fixed p and i , $s_i^k(p)$ and $g_i^k(p)$ are defined as in (16) and (17). If $x_i^k = L_i$, the assumption $x^k \leq x^{k-1}$ implies that $x_i^{k-1} = L_i$. There follows that $g_i^{k+1}(p) \geq g_i^k(p)$, hence $s_i^{k+1}(p) \geq s_i^k(p)$. On the other hand, if $x_i^k < L_i$ then $s_i^{k+1}(p) = e_i$ by definition, so $s_i^{k+1}(p) \geq s_i^k(p)$ for all p . Since ξ_σ^{-1} is non-increasing,

we get that

$$\xi_\sigma^{-1}(s^{k+1}(p)) \leq \xi_\sigma^{-1}(s^k(p)), \quad \text{i.e. } \Psi^{k+1}(p) \leq \Psi^k(p).$$

Since Ψ^{k+1} is non-decreasing, by applying Lemma A.3 we get

$$(\Psi^{k+1})^{(n)}(p_0) \leq (\Psi^k)^{(n)}(p_0) \quad \text{for all } n \geq 1.$$

By Lemma A.2, we can pass to the limit as n tends to infinity and get

$$p^{k+1} \leq p^k,$$

as desired. ■

Theorem 3.8. *There exist a non-increasing sequence of prices $\{p_k\}$ converging to a price p^* and a non-increasing sequence of payment vectors $\{x_k\}$ converging to a vector x^* , such that (p^*, x^*) satisfy*

$$x^* = \Phi_{p^*}(x^*) \quad \text{and} \quad p^* = \Psi^{x^*}(p^*) = \xi_\sigma^{-1}(s^{x^*}(p^*)). \quad (19)$$

The pair (x^, p^*) is then an equilibrium according to Definition 3.5.*

Proof. Let $p_0 = \xi^{-1}(\sigma)$ and $x^0 = L$. For $k \geq 1$, we define p^k as the greatest fixed point of the function $\Psi^k = \Psi^{x^{k-1}}$ and then x^k as the greatest fixed point of the function $F_{p^k, x^{k-1}}$.

We can show by induction that

$$p^{k+1} \leq p^k, \quad x^{k+1} \leq x^k, \quad \text{and} \quad x^{k+1} \geq \Phi_{p^{k+1}}(x^{k+1}). \quad (20)$$

- For $k = 0$ we have $x^0 = L$, $p^0 = p_0$ and $x^0 = L \geq \Phi_{p_0}(x^0)$. Then by definition $p^1 \leq p^0 = p_0$ and $x^1 \leq x^0 = L$. Moreover, $x^1 \geq \Phi_{p^1}(x^1)$ by Lemma 3.6.
- Now, we assume that $p^k \leq p^{k-1}$, $x^k \leq x^{k-1}$ and $x^k \geq \Phi_{p^k}(x^k)$ and we show that this implies (20).

By Lemma 3.7, we have that $x^k \leq x^{k-1}$ implies $p^{k+1} \leq p^k$. Now, by Lemma 3.6, we deduce that $x^{k+1} \leq x^k$ and $x^{k+1} \geq \Phi_{p^{k+1}}(x^{k+1})$, as desired.

The sequence $\{x^k\}$ is non-increasing and takes values in the set $[0, L_1] \times \dots \times [0, L_N]$, hence it admits a limit $x^* \in [0, L_1] \times \dots \times [0, L_N]$. The sequence $\{p^k\}$ is non-

increasing and $p^k \in [p_{\min}, p_0]$ for all k ; hence it admits a limit $p^* \in [p_{\min}, p_0]$. It remains to show that (x^*, p^*) satisfies (19).

By the definition of x^{x+1} , the monotonicity of $F_{p^{k+1}, x^k}(\cdot)$ and the fact that x^k is a supersolution, we get

$$x^{k+1} = F_{p^{k+1}, x^k}(x^{k+1}) \leq F_{p^{k+1}, x^k}(x^k) \leq F_{p^k, x^k}(x^k) = \Phi_{p^k}(x^k) \leq x^k.$$

Since $x^k \rightarrow x^*$ and $x^{k+1} \rightarrow x^*$, by comparison $\{\Phi_{p^k}(x^k)\}$ admits a limit and

$$\lim_k \Phi_{p^k}(x^k) = x^*.$$

The function $\Phi_p(x)$ is continuous in $(x, p) \in \mathcal{D} = [0, L_1] \times \dots \times [0, L_N] \times [p_{\min}, p_0]$ by construction, hence

$$x^* = \lim_k \Phi_{p^k}(x^k) = \Phi_{p^*}(x^*).$$

By definition of p^k we have

$$\lim_k \Psi^k(p^k) = \lim_k p^k = p^*. \quad (21)$$

It remains to show that $p^* = \Psi^{x^*}(p^*)$. Recall that $\Psi^x(p) = \xi^{-1} \circ s^x$. In this case we cannot claim that Ψ is continuous in (x, p) in the whole domain \mathcal{D} , because s is not continuous as a function of x (see equation (6)). However, we can show that the continuity of Ψ on a subset of \mathcal{D} is sufficient to prove the desired result.

Let

$$\mathcal{A} = \left\{ \left(\prod_{i=1}^N A_i \right) \times [p_{\min}, p_0] : A_i = \{L_i\} \vee A_i = [0, L_i) \text{ for all } i \right\}.$$

We can show that:

1. $\Psi = \Psi^x(p)$ is continuous in B for all $B \in \mathcal{A}$;
2. there exist $\bar{k} \in \mathbb{N}$ and $\bar{B} \in \mathcal{A}$ such that $(x^k, p^k) \in \bar{B}$ for all $k \geq \bar{k}$. In other words, the sequence (x^k, p^k) is eventually contained in one of the sets of continuity of the function $\Psi^x(p)$.

By (6), for fixed i the function $s_i^x(p)$ is continuous with respect to the variables (x, p) in the set

$$A_1 \times \dots \times [0, L_i) \times \dots \times A_N \times [p_{\min}, p_0],$$

with $A_j = [0, L_j]$ for all $j \neq i$, since the only set of discontinuity is $\{x_i = L_i\}$. Moreover, the restriction of $s^x(p)$ to the hyperplane $\{x_i = L_i\}$ is continuous by definition, hence $s_i^x(p)$ is continuous with respect to (x, p) in

$$A_1 \times \dots \times \{L_i\} \times \dots \times A_N \times [p_{\min}, p_0],$$

with $A_j = [0, L_j]$ for all $j \neq i$. By repeating this argument for each bank and summing up $s(x, p) = \sum_i s_i(s, p)$, we get claim 1.

We can now prove claim 2. Let $x^* \in [0, L_1] \times \dots \times [0, L_N]$ be $x^* = \lim_k x^k$. For fixed i , either $x_i^* = L_i$ or $x_i^* \in [0, L_i)$. In the first case, since the sequence x^k is non-increasing, then we have $x_i^k = L_i$ for all k , we set $k_i^0 = 0$. In the latter case, by definition of limit there exists $k_i^0 \in \mathbb{N}$ such that $x_i^k < L_i$ for $k \geq k_i^0$. Up to permutations of the indexes, we can assume that $x_i^* = L_i$ for $i = 1, \dots, I$ and that $x_i^* < L_i$ for $i = I + 1, \dots, N$. Then for all $k \geq \bar{k} := \max_i k_i^0$ we have

$$x^k \in \{L_1\} \times \dots \times \{L_I\} \times [0, L_{I+1}) \times \dots \times [0, L_N).$$

There follows that there exist $\bar{k} \in \mathbb{N}$ and $\bar{B} \in \mathcal{A}$ such that $(x^k, p^k) \in \bar{B}$ for all $k \geq \bar{k}$, and $\Psi^x(p)$ is continuous in \bar{B} . Then we have $\lim_k \Psi^k(p^k) = \Psi^{x^*}(p^*)$; on the other hand, $\Psi^k(p^k) = p^k \rightarrow p^*$ by (21), hence

$$\Psi^{x^*}(p^*) = p^*,$$

as desired. ■

4 Simulations for the single asset case

In this section we present the results of a few simulation exercises that are meant to illustrate how the model works and how asset fire sales and direct balance-sheet exposures may interact, spreading contagion through the system. In this connection, a caveat is in order: the model has a theoretical purpose and is more suitable for a qualitative analysis than for exact calibration to real data. As a

consequence, the numerical values assigned to the parameters in the simulations and the results presented have an illustrative aim only and are not meant to provide quantitative estimates of potential losses in real banking systems.

4.1 Simulations set-up and stylized network topologies

The CFS model was tested by the authors on a uniform, totally interconnected network of ten banks, i.e. a banking system in which all banks have equal balance-sheets and lend to or borrow from one another the same amount of money. This framework is useful to discriminate patterns in the results that are due to intrinsic characteristics of the model from patterns that might emerge as a consequence of specific features of the banking system at hand. Therefore, we will consider this benchmark case as well in our assessment of the model.

On the other hand, the purpose of introducing networks in economic and financial models relates to the need to explicitly account for the structure of interconnections that characterizes the financial system. Indeed, this structure (the topology of the network) may deeply influence the behavior of the system. In particular, it may contribute to the resilience of the financial system to exogenous shocks, or, on the contrary, increase havoc (see e.g. the seminal paper by Allen and Gale, 2000). It is then important to consider how the model behaves when performing contagion analysis on different network topologies. To this end, in the following paragraphs we consider four main archetypal topologies (Figure 1):

- (i) *totally interconnected topology* – each bank is linked to all other banks (as described above);
- (ii) *circle topology* – each bank borrows from only one bank and only lends to another bank, i.e. bank i borrows from $i - 1$ and lends to $i + 1$, for $i = 1, \dots, N$ (bank 1 borrows from bank N);
- (iii) *star topology* – there is only one big bank (the core of the star) that borrows from and/or lends to all other banks in the system, while non-core banks are not interconnected with one another;
- (iv) *core-periphery topology* – similar to the star topology, there is a core of big banks that interact on the interbank market with smaller banks (peripheral banks).

While the first three topologies are highly stylized, the last one represents a situation that is somewhat closer to real banking systems (see Craig and von Peter, 2014; in ‘t Veld and van Lelyveld, 2014; Fricke and Lux, 2015, for a discussion on core-periphery topologies in different financial markets). Though differentiating the way in which banks are interconnected, we assume a uniform structure in terms of balance-sheet dimensions and composition, except for the banks belonging to the core of the star or the core-periphery networks. We consider networks of 100 banks, with the initial (i.e., pre-shock) balance-sheet of a representative bank in the system as in Table 1. In the case of the star topology, we add to the 100 uniform banks a core bank, the total assets of which are five times bigger in value than those of the representative bank. Peripheral banks are assumed to be split in two equinumerous groups, according to whether they lend to or borrow from the core bank. Finally, in the core-periphery network, the core consists of 10 uniform banks, with total assets ten times those of peripheral banks. The latter are divided in 10 groups (of 10 banks each). The banks in each group exchange money in the interbank market between one another (assuming total interconnectedness inside each group) and with one and only one bank belonging to the core. Core banks, similarly, lend to and borrow from any other bank belonging to the core, in addition to peripheral banks belonging to the associated group.

Table 1: Representative bank’s balance-sheet

Assets		Liabilities	
Liquid asset:	$c_i = 40$	Non-interbank liabilities:	$d_i = 160$
Illiquid asset:	$e_i = 130$	Interbank liabilities:	$L_i = 30$
Interbank assets:	$\sum_{j \neq i} x_j \pi_{ji} = 30$	Equity:	$K_i = 10$

In the following sections we present four main simulation exercises implemented with this basic set-up. Unless otherwise specified, we set the regulatory leverage ratio equal to 4% (a figure broadly in line with leverage ratios of many commercial banks and with the 3% requirement envisaged by the Basel 3 framework) and the minimum price that can be attained by the illiquid asset equal to 0.9 (the initial price, when no liquidation has occurred yet, is normalized to 1, as the price of the liquid asset).¹⁰ The inverse demand function is assumed to be

¹⁰Obviously, these parameters play a key role in determining the outcomes of any simulation exercise (a sensitivity analysis is presented in Section 4.5). In particular, the choice of the

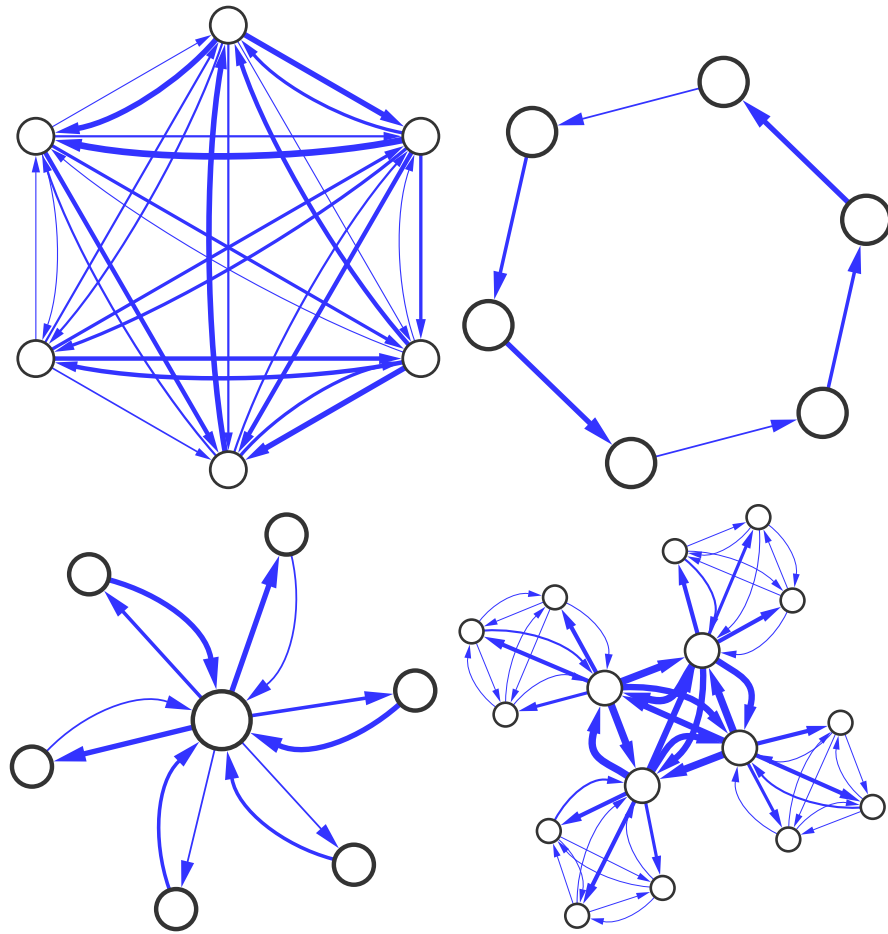


Figure 1: Stylized network topologies

quadratic and is accordingly fitted to the minimum price as:¹¹

$$p(s) = 1 - \alpha s^2, \quad \text{where} \quad \alpha = \frac{1 - p_{min}}{\left(\sum_{j=1}^N e_j\right)^2}.$$

This functional form represents the easiest way to instance the assumption of a concave inverse demand function, which better fits the empirical behavior of prices in case of fire sales than a convex function (like the exponential one) does. Indeed, the price of an asset is usually little affected by small sales, but it may significantly deteriorate, and at a faster pace, in case there are substantial sales.

Furthermore, in each exercise contagion analysis is performed in a standard way: an exogenous shock hits one or more banks in the system and the ensuing propagation of distress is measured. A bank defaults if, after liquidating all of its non-interbank assets, it does not comply with the regulatory leverage ratio. In this connection, note that:

- (i) As is common in the literature, we adopt a “timeless” perspective, according to which contagion spreads instantaneously (there is no time dimension in the model) and banks do not have the possibility to readjust the composition of their assets in response to a shock or to defaults of other banks (they are only allowed to sell as many assets as needed to comply with the regulatory leverage ratio).
- (ii) The rate of recovery for investors exposed to a defaulting bank is determined endogenously in the model. As a consequence, the magnitude of the initial shock matters and we do not need to restrict ourselves to consider the complete wipe out of a bank’s total assets as an initial trigger of the contagion process.

(iii) We only consider here shocks to the asset side of the balance-sheet.¹² More

minimum price is not straightforward. For instance, in Cifuentes et al. (2005a), the extended working paper version of the CFS model, in most simulations the authors set the minimum price equal to 0.5. On the other end of the scale, Gauthier et al. (2012) set it equal to 0.98. While modeling real fire sales would require to pick a low value of the minimum price, the stylized structure of the model, in which only one type of illiquid asset is available on the market, imposes a trade-off: on one hand, low values of the minimum price would produce the unrealistic result that even a modest shock could trigger an endemic contagion yielding to the wipe-out of the whole system; high values, on the other hand, are unrealistic but allow to study a wider range of outcomes assuming reasonable shock values.

¹¹Section 4.5 provides an example of how a different assumption on the functional form may affect the results of the simulations. In particular, comparative statics for other values of the minimum price attainable by the illiquid asset will be shown.

¹²The financial crisis has shown that considerable problems to bank balance-sheets may stem

precisely, the initial shock is modeled as a loss of a given share of the illiquid asset, affecting one or more banks in the system.¹³

- (iv) In the CFS model, distress may be transmitted even if no default occurs: as soon as a bank is forced to sell part of the illiquid asset in its portfolio in order to comply with the regulatory ratio, the price of that asset deteriorates and, as a consequence, the value of other banks' illiquid assets diminishes as well.

Finally, when evaluating the impact of fire sales and balance-sheet contagion on the system, we will compute the following metrics:

- the share of liquid assets sold by bank i , $\%Liq = t_i/c_i$;
- the share of illiquid assets sold by bank i , $\%Illiq = s_i/e_i$;
- the share of interbank liabilities that bank i is unable to repay, $\%IB = 1 - x_i/L_i$;
- the negative of the change in total asset value, with respect to the value of assets recorded after the shock hits and before the contagion process starts, namely

$$\Delta_{TA} = 1 - \frac{\sum_{j=1}^N (c_j - t_j + p \cdot (e_j - s_j) + \Pi'x)}{\sum_{j=1}^N (c_j + p_0 \cdot e_j + \Pi'L)}$$

where p_0 and e_j denote respectively the price of the illiquid asset and bank j 's endowment of illiquid asset immediately after the initial shock, before any liquidation process starts;

- the (positive) loss of senior creditors (e.g., depositors or bondholders)

$$Loss_D = \frac{\sum_{j=1}^N \max\{0, d_j - (c_j + p \cdot e_j + \Pi'x)\}}{\sum_{j=1}^N d_j};$$

from the liabilities side and part of the recent literature has shifted towards the consideration of funding liquidity issues. However, we do not include this extension here in order to avoid complicating the model excessively, before having clarified the structure of its original version (see also the comments in Amini and Minca, 2012, on this topic).

¹³Once the shock hits the system, the price of the illiquid asset is updated, as if the amount of illiquid asset eliminated by the shock were sold on the market. Running simulations without this adjustment, one could draw the paradoxical conclusion that a more severe shock would be better than a lighter one. In fact, a severe shock would leave the hit bank with less illiquid asset available for sale, i.e. with a smaller potential to negatively impact other banks via the fire sales channel (see Section 3.1).

- the number of defaults, i.e. the number of banks $j = 1, \dots, N$ such that $t_j = c_j$, $s_j = e_j$ and

$$r = \frac{c_j + p \cdot e_j + \Pi'x - d_j - L_j}{\Pi'x} < \bar{r}.$$

4.2 Contagion analysis

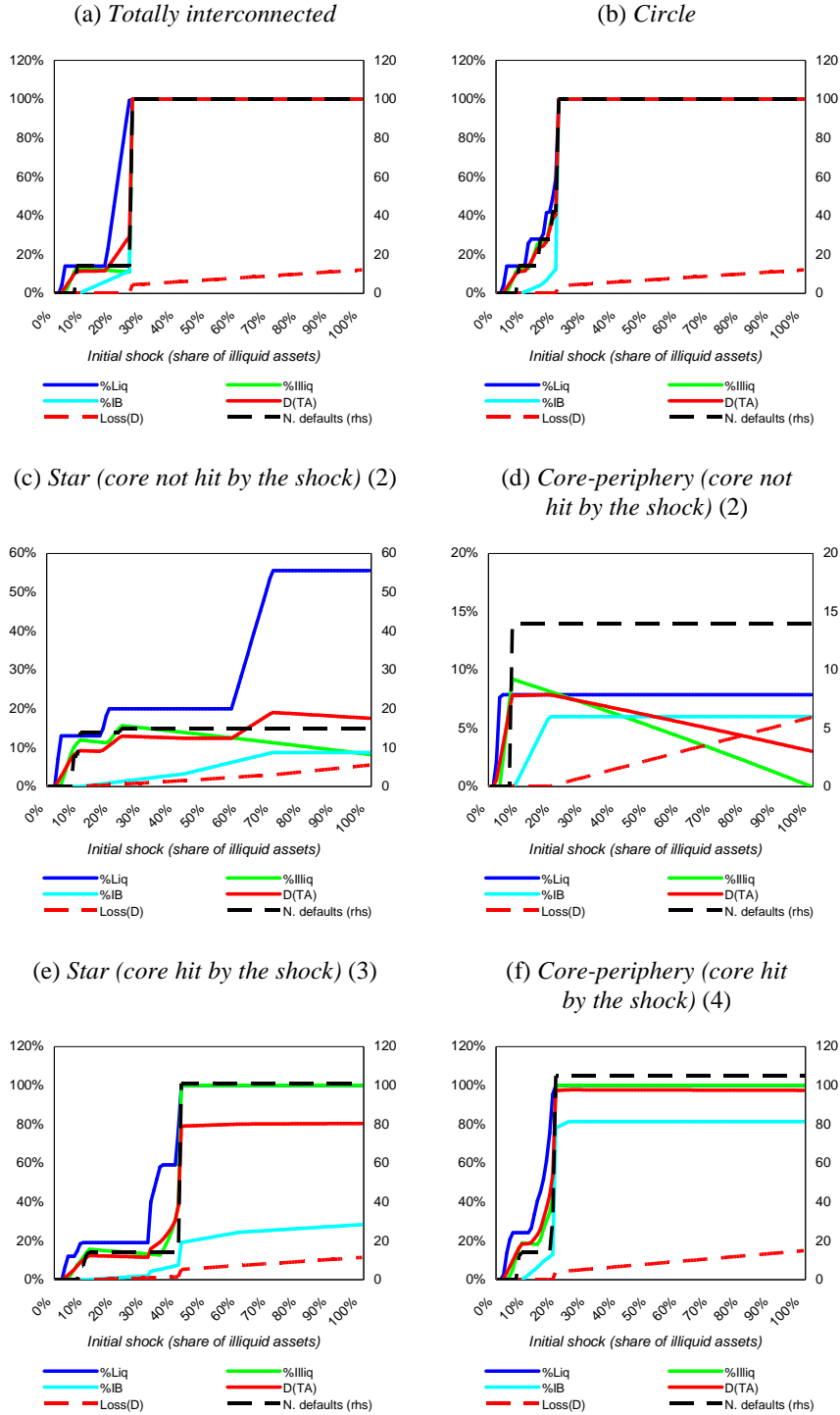
The first simulation exercise we present is a standard contagion analysis, performed on the four different stylized topologies described above. In particular, we fix beforehand a set of banks that are hit by an exogenous shock and calculate the impact metrics letting the magnitude of the shock vary. More precisely, the chosen banks are affected by an initial shock eroding a share S of the illiquid asset endowment of each of them, with S ranging between 0 and 100 per cent (for simplicity, S is equal for all banks hit by the exogenous shock). The results are shown in Figure 2.

The first overall conclusion conveyed by the results is the existence of “phase transition” points in each configuration, i.e. threshold values of the initial shock above which the number of defaults shifts from virtually zero to the number of banks hit by the shock, and then to the total number of banks in the system. While exacerbated by the simplicity of the model and by the chosen set-up (homogeneous banks), a regime-switching behavior based on different values of the relevant parameters may be found also in more realistic network models (see e.g. Newman, 2003) and should be taken into account when assessing the resilience of real financial networks.

Considering more in detail the four topologies analyzed, Figure 2 displays a few remarkable differences. As shown by panels (a) and (b), the total interconnected topology and the circle configuration display similar outcomes with respect to all metrics, although the circle starts yielding to the default of all banks at a lower level of the initial shock (roughly 20 per cent, instead of 30). Thus, compared with the totally interconnected topology, it displays a narrower range of shocks leading to non-catastrophic contagion.¹⁴ On the other hand, the transition between the two regimes (either only the banks hit by the shock or all banks default) is more gradual, due to the heterogeneous structure of the in-

¹⁴This conclusion does not hold if there is a clustering of the banks hit by the shock. In this case (not shown in Figure 2), the circle configuration may be especially apt to dampen balance-sheet contagion effects, due to the scanty interconnected structure of interbank exposures.

Figure 2: Contagion analysis and magnitude of the initial shock (1)



(1) On the x axis the share S of illiquid assets canceled by the initial shock for each affected bank is reported, while the y axis refers to the metrics defined in Section 4.1. The number of defaults ($N. defaults$) is on the right-hand scale. The main parameters of the model are set as follows: $\bar{r} = 4\%$; $p_{min} = 0.9$. – (2) The banks hit by the initial shock are uniformly chosen among the peripheral ones. – (3) The core of the star is affected, in addition to thirteen peripheral banks. – (4) Two core banks are affected, in addition to twelve uniformly chosen peripheral banks.

terbank exposures.¹⁵ In the case of the star and core-periphery configurations, where heterogeneity between bank balance-sheets matters by definition, the results depend on how the initial shock is targeted. If it only affects peripheral banks (Figure 2, panels (c) and (d)), these topologies prove to be particularly resilient, as contagion effects are not able to trigger additional defaults (apart from those of the banks hit by the shock). On the other hand, as one or more core banks are involved in the set of shocked institutions, the results become very similar to those recorded for the totally interconnected and circle topologies, with the only difference that, even in the regime in which the whole banking system defaults, the interbank assets are not completely wiped out (about 30 and 80 per cent of interbank loans are not paid back in the star and core-periphery topologies, respectively). Moreover, while the cut-off is similar for the core-periphery and circle configurations (an initial loss of about 20 per cent of illiquid assets of the banks hit by the shock), it is far larger for the star topology (above 40 per cent), suggesting that the overall resilience of a core-periphery type of network depends substantially on the relative dimension of the core, as compared with the periphery.

In conclusion, the results of this exercise indicate that a star or core-periphery topology might be better than other stylized network configurations in order to dampen contagion effects, as implied also by current regulatory effort to bring securities markets transactions to be cleared by central counterparties (CCPs). However, the resilience of these topologies may deteriorate significantly when they are subject to targeted shocks hitting the core of the network, especially if the dimension of the core is huge compared with that of peripheral banks. This calls for additional care by supervisors in controlling risk management practices at core banks (or CCPs, extrapolating the results to the case of securities markets). Moreover, it may justify the introduction of additional capital buffers for core institutions, as envisaged by the Financial Stability Board (FSB) for systemically important financial institutions (SIFIs). On the other hand, by the same argument one could advocate lower capital buffers for peripheral banks, compared with banks having similar asset size but organized according to different network structures. In other words, the “interconnectedness” of a bank is not equivalent to the size of its total interbank assets and liabilities, but is intrinsically determined

¹⁵Recall from Section 4.1 that the heterogeneity stems here only from the structure of the unweighted adjacency matrix (i.e., the matrix of interbank liabilities obtained from L replacing all non-zero elements with 1), as all links have equal weights (all banks have equal interbank assets/liabilities).

by the actual decomposition of those aggregate figures at the level of bilateral exposures (i.e., the topology of the network).

4.3 Capital injections: An allocation problem

While in the previous exercise the set of banks hit by the initial shock was fixed and the magnitude of the shock varied, we now let the pool of shocked banks vary as well; we will then show how this exercise can be interpreted in terms of a problem of optimal recapitalization of a banking system subject to a Government budget constraints.

Assume that the system is hit by a shock which cancels overall a percentage M of the total illiquid assets; we want to analyze to what extent the final impact depends on how the shock is distributed among several banks. To this end we split the shock in equal proportions among an increasing number n of banks ($n = 2, \dots, 100$). In particular, we choose the banks to be hit by the shock in such a way that the serial numbers identifying the n banks are equally spaced over the range $1, \dots, 100$ (e.g., if $n = 5$, we can shock banks 1, 21, 41, 61 and 81) and, in the case of the star and core-periphery topologies, no core banks are hit by the initial shock.¹⁶

The results are shown in Figure 3, where the impact in terms of number of defaults (panels (a1), (b1), (c1) and (d1)) and change in total asset value (Δ_{TA} ; panels (a2), (b2), (c2) and (d2)) are plotted against the percentage M of initial shock (x axis; $M = 1\%, \dots, 4\%$) and the number n of banks hit by the shock (y axis; $N = 2, \dots, 100$), for all four stylized network configurations.

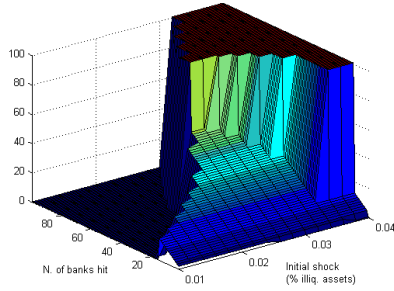
A common pattern emerges from these plots, indicating a regime-shifting behavior. Irrespective of the topology and the chosen distress metric (either number of defaults or change in total asset value), there exists a threshold value \overline{M} of the initial shock with the following characteristics:

- If $M \leq \overline{M}$, there exists a threshold $\overline{n}(M)$ such that the impact of contagion is increasing (or non monotone, in the case of the circle topology) in the number n of banks hit by the shock if $n \leq \overline{n}$, decreasing otherwise (in particular, the number of defaults abruptly goes down to zero). This means that, if the initial aggregate shock is not too big, it may be better born by

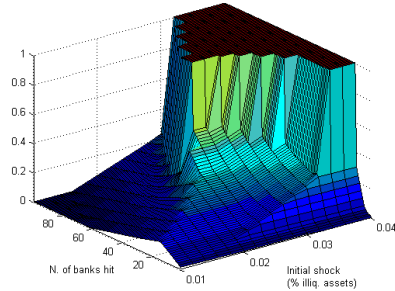
¹⁶This is consistent with the goal of this exercise, i.e. an optimal recapitalization problem: we saw in the previous section that when the core takes a loss, the impact on the star and core-periphery networks is more substantial than if only peripheral banks are hit; in this case, then, the choice of how to allocate capital may be more straightforward.

Figure 3: Contagion analysis and distribution of the initial shock (1)

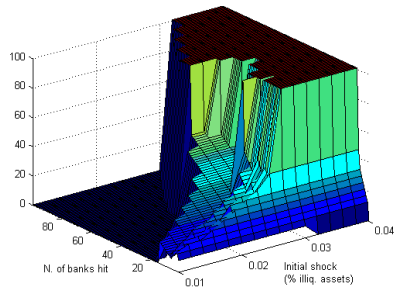
(a1) *Totally interconnected – N. defaults*



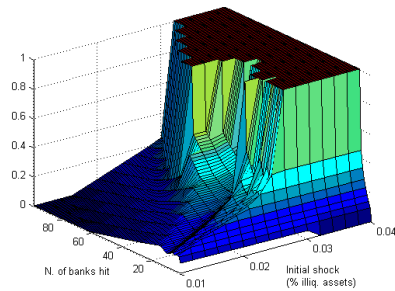
(a2) *Totally interconnected – Loss_{TA}*



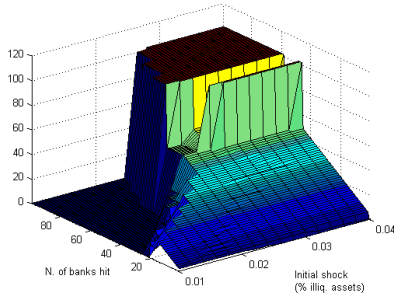
(b1) *Circle – N. defaults*



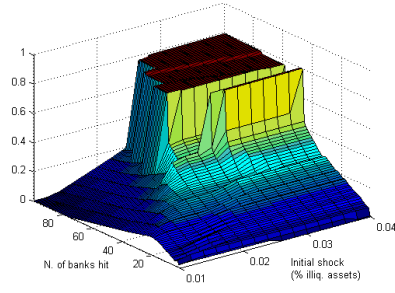
(b2) *Circle – Loss_{TA}*



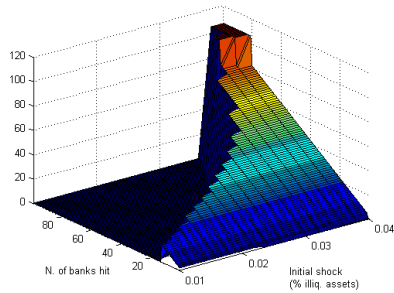
(c1) *Star – N. defaults*



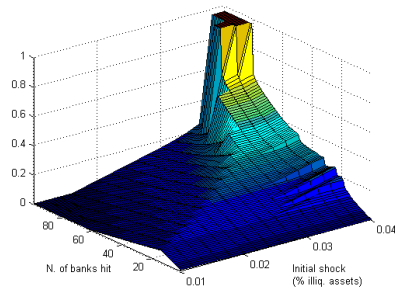
(c2) *Star – Loss_{TA}*



(d1) *Core-periphery – N. defaults*



(d2) *Core-periphery – Loss_{TA}*



(1) On the x axis, the aggregate magnitude M of the initial shock is reported ($M = 1\%, \dots, 4\%$ of aggregate illiquid asset endowment); on the y axis, the number n of banks hit by the initial shock ($n = 2, \dots, 100$); on the z axis, either the number of defaults ($N.defaults$) or the change in the value of assets (Δ_{TA}), as defined in Section 4.1. The main parameters of the model are set as follows: $\bar{r} = 4\%$; $p_{min} = 0.9$. The banks hit by the initial shock are uniformly chosen in the system (among the peripheral ones only, in the star and core-periphery configurations).

the system if it is split among a sufficient number of banks, as in this case the initial loss per bank can be easily absorbed selling liquid assets (or an amount of illiquid assets not so substantial as would be necessary to trigger a fire sale).

- If $M > \bar{M}$, there exists a threshold $\bar{n}(M)$ such that the impact of contagion is increasing (or non monotone, in the case of the circle and star topology) in the number n of banks hit by the shock if $n \leq \bar{n}$, constantly equal to the maximum level of distress reached at $n = \bar{n}$ otherwise (all banks default). In this case, the magnitude of the initial shock is so large that its impact is less material if the shock is split among a few banks than if it is spread over the whole system. Indeed, in the latter case the initial loss faced by each bank is sufficiently big to force the bank to sell all of its assets without being able to comply with the regulatory ratio.

While the first part of the previous result can be intuitively grasped in terms of “diversification” (the more spread out the shock, the less substantial its impact), the second part indicates that, if the aggregate shock is big enough, dilution of the shock among many banks is not beneficial for the network. This remark suggests that, when coping with systemic risk, not only the magnitude of the shock should be taken into account, but also how it is distributed across the system, even in the stylized case of homogeneous banks.

The policy implications of this conclusion may be better rephrased if we interpret the exercise reported in Figure 3 as a problem of optimal recapitalization of the banking system subject to a budget constraint. We assume that a huge shock hits all the banks and the Government wants to inject new capital in the system (or force banks to raise additional capital),¹⁷ but it is able or willing to do so only up to a certain point: here M is the percentage of pre-shock illiquid asset endowment which remains unfilled after the recapitalization. In this context, how should the Government allocate its constrained resources in order to restore the initial conditions of the banking network? Arguably, given that bank balance-sheets in our stylized networks are homogeneous, it should simply inject equal quantities of capital in each bank. However, Figure 3 proves that, whenever the initial shock is greater than a given threshold value \bar{M} , such a uniform recapitalization would not be optimal. A uniform recapitalization of all banks

¹⁷To be consistent with the model, assume that the capital injections take place immediately after the shock, before marking-to-market the illiquid asset.

except a few of them would do better, both in terms of number of defaults and total assets' value deterioration.

4.4 The incremental effect of fire sales

The results of the previous exercises did not allow to discern the relative importance of the two different channels of contagion considered in the model. To this end, we perform two contagion analysis exercises with the same initial shock and parameters, except that in one of them we consider all assets as liquid (this simply amounts to set $p_{min} = 1$).¹⁸ In this case, direct balance-sheet exposures are the only possible channel of contagion, as no fire sale effects can emerge. Different values of the metrics described in Section 4.1 across the two exercises are then a proxy of the incremental effect obtained assuming that one asset is illiquid.

In particular, Figure 4 shows how the change in total assets' value Δ_{TA} changes as an initial shock of fixed magnitude is split among an increasing number of banks (similar to the exercise presented in Section 4.4, but with the aggregate magnitude M of the initial shock alternatively set equal to two different given values, for the sake of simplicity). In each panel, the red line refers to the case with fire sales, while the blue line represents the case with liquid assets only.

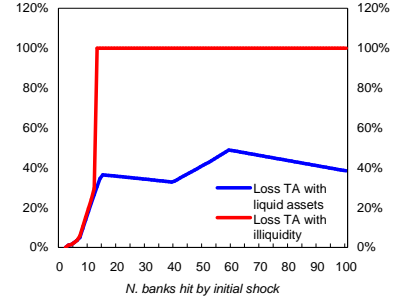
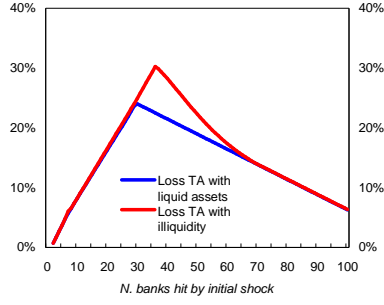
The results of these simulations suggest that the effect of asset fire sales adds to the direct balance-sheet exposure channel of contagion in an incremental way and then gradually fades away as the number n of banks hit by the initial contagion tends to 100 (the total number of banks in the system – the number of peripheral banks, in the star and core-periphery topologies). Indeed, in each panel of Figure 4 three main intervals of values of n may be broadly discerned, that merge with one another in a continuous way:

- For n sufficiently small (independently of the value of M), the shock is confined to a few banks in the system and can not trigger enough illiquid assets sales to generate a sizable price deterioration. In this case, direct balance-sheet exposures (loans on the interbank market) are the main channel of contagion and the final impact of the exogenous shock can hardly be distinguished in the cases with or without illiquid assets.
- As n takes on “intermediate” values (the exact range depends on the net-

¹⁸This analysis is a particular case of the sensitivity analysis presented in Section 4.5, but is presented separately in order to explicitly focus on the effect of fire sales per se, independently on the degree of illiquidity assumed.

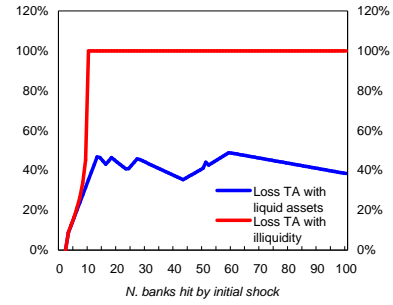
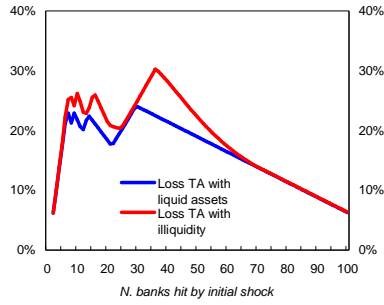
Figure 4: Incremental effect of illiquidity (1)

(a1) *Totally interconnected* – $M = 2\%$ (a2) *Totally interconnected* – $M = 4\%$



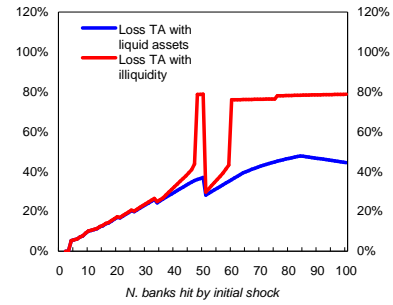
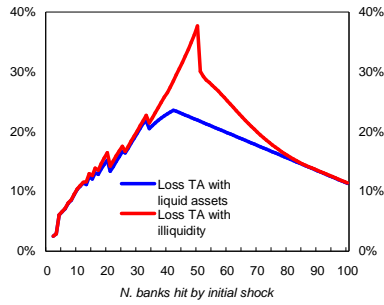
(b1) *Circle* – $M = 2\%$

(b2) *Circle* – $M = 4\%$



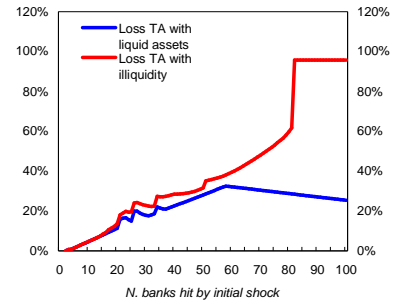
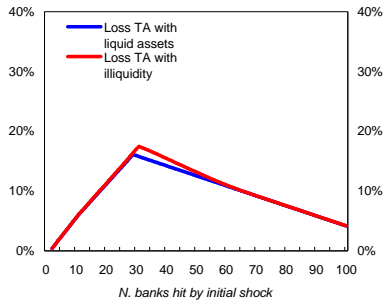
(c1) *Star* – $M = 2\%$

(c2) *Star* – $M = 4\%$



(d1) *Core-periphery* – $M = 2\%$

(d2) *Core-periphery* – $M = 4\%$



(1) On the x axis, the number n of banks hit by the initial shock ($n = 2, \dots, 100$) is reported (the aggregate magnitude M of the initial shock equals either 2% or 4% of the aggregate illiquid asset endowment); on the y axis, the change in the value of assets is reported, as defined in Section 4.1. The main parameters of the model are set as follows: $\bar{r} = 4\%$; $p_{min} = 0.9$. The banks hit by the initial shock are uniformly chosen in the system (among the peripheral ones only, in the star and core-periphery configurations).

work topology and the value of M), more banks have to liquidate their illiquid asset endowment, generating a material effect on price. As a consequence, the final impact on the total assets' value is significantly different from that recorded in the case with liquid assets only. However, as n is in this range, the difference is mainly in levels, while in each panel the two lines show qualitatively similar contours overall.

- Finally, as n takes sufficiently high values, depending on the magnitude M of the initial shock there may be two completely different outcomes. If M is low (2% of the aggregate illiquid asset endowments in Figure 4, panels (a1), (b1), (c1), (d1)), then the exogenous shock is split among a sufficiently high number of banks. Therefore, the shock faced by each bank is relatively small and the regulatory leverage ratio can be met without selling substantial amounts of illiquid asset. In this case, the final impact (as measured by Δ_{TA}) displays negligible differences (if any) in the case with the illiquid asset and the case with liquid assets only. On the contrary, if M is high (4% in the figure, panels (a2), (b2), (c2), (d2)), all banks in the system¹⁹ eventually lose their assets in full due to the effect of fire sales (differently from the case with no liquid assets).²⁰

4.5 Sensitivity analysis: Relative effectiveness of different policies

Finally, we present the results of an analysis of sensitivity of the model to its key parameters, i.e. the leverage ratio \bar{r} that the banks have to comply with and the minimum price p_{min} attainable in case all banks sell their illiquid asset endowments in full. This is equivalent to investigating how the resilience of the banking system depends on its initial capitalization level and on the degree of illiquidity of the asset with non-zero price elasticity. In addition, we assess how the functional form of the inverse demand function (exponential or polynomial) may affect the contagion analysis results.

The relevance of this exercise is given by its policy implications. In the simplified world described by the model, a policymaker could act on two levers to make the banking system more resilient: (i) require banks to hold more liquid

¹⁹Peripheral banks only, in the case of the star or core-periphery topologies.

²⁰For higher values of M (not shown in Figure 4), the shock may be huge enough to trigger the default of all banks also in the case with liquid assets only.

assets in their portfolios; (ii) ask for higher capital levels. We will show that the two instruments may not be perfect substitutes, especially for some network topologies. Moreover, we will show that the relative importance of the two policy instruments may also depend on the characteristics of the initial shock and on the properties of the demand of illiquid assets.

The results of this simulation exercise are shown in Figure 5: we let \bar{r} range between 2% and 5% (x axis) and p_{min} over the interval $[0.2, 1.0]$ (y axis), and plot against them the deleveraging of liquid assets ($\%Liq$; panels (a1), (b1) and (c1)) and the change of assets' value (Δ_{TA} ; panels (a2), (b2) and (c2)).²¹ As expected, the final impact of the shock (i.e., the role played by contagion) depends heavily on how the parameters are initialized: an initial shock of fixed magnitude can either trigger no deleveraging (if both assets are liquid and the regulatory leverage ratio is low) or force all banks to sell the liquid assets in their portfolios and eventually default.

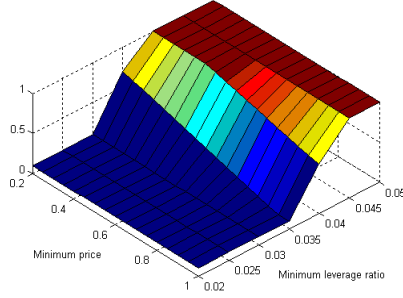
However, the sensitivity analysis also shows how the relative importance of the two parameters in triggering contagion depends on the topology considered, something that could not be easily predicted *ex ante*. In particular, in the totally interconnected configuration (Figure 5, panels (a1), (a2)) the level of the regulatory ratio influences the final impact of the shock significantly more than the minimum price does. Irrespective of how low (high) the price of the illiquid asset can be, no deleveraging occurs (all banks default, respectively) if the regulatory ratio is sufficiently low (high). In the circle topology (panels (b1), (b2)), on the contrary, the elasticity of the price of the illiquid asset is the leading parameter. No matter how highly capitalized the banking system is (i.e., how low the regulatory ratio is, compared with the actual leverage of the banks), if the asset is sufficiently illiquid, all banks will eventually go bankrupt. Finally, the star topology (panels (c1), (c2)) is somewhere in the middle between these two extreme situations. Indeed, there is no clear hierarchy between the two parameters: low leverage (i.e., a high value of the ratio r) and high liquidity of the assets are equally relevant for the soundness of the banking system.

The previous remarks give an insight on the relative importance of the two instruments which can be used by a policymaker to make this stylized banking system more resilient (require banks to hold more liquid assets in their portfolios,

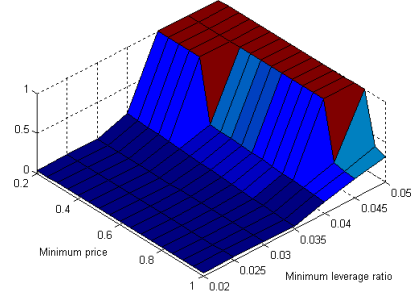
²¹In order to better visualize the qualitative behavior of the model as a function of the parameters, the exercise reported in Figure 5 is performed assigning shocks of different magnitude to each stylized network configuration (results for the core-periphery topology are similar to those of the star topology and are omitted).

Figure 5: Sensitivity to regulatory ratio and illiquidity (1)

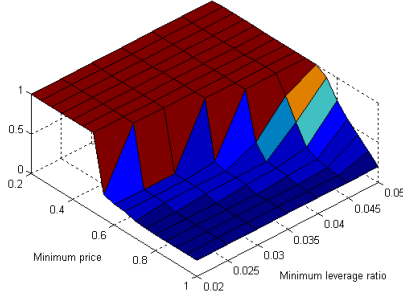
(a1) *Totally interconnected* – %Liq



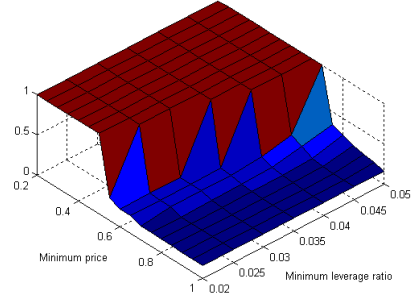
(a2) *Totally interconnected* – $Loss_{TA}$



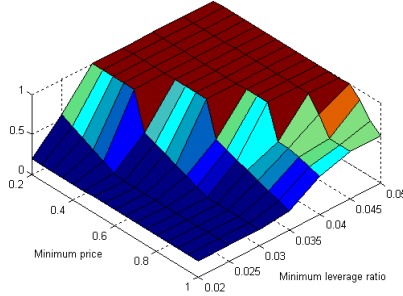
(b1) *Circle* – %Liq



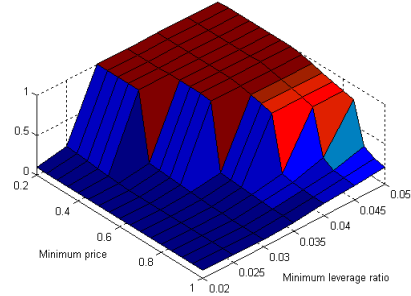
(b2) *Circle* – $Loss_{TA}$



(c1) *Star* – %Liq



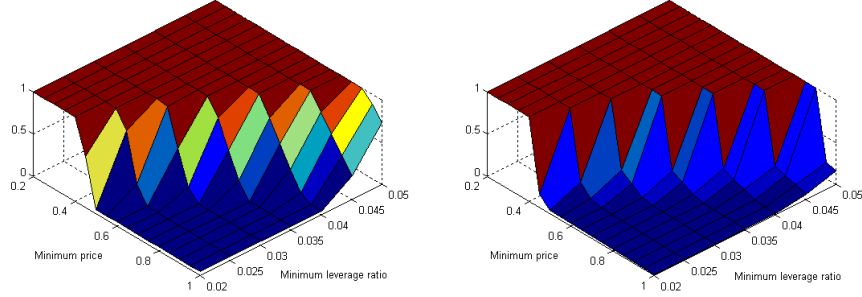
(c2) *Star* – $Loss_{TA}$



(1) On the x axis, the regulatory leverage ratio \bar{r} is reported; on the y axis, the minimum price (p_{min}) that can be attained by the illiquid asset; on the z axis, either the deleveraging of liquid assets (%Liq) or the change of the value of assets (Δ_{TA}), as defined in Section 4.1. The initial shock consists of the wipe-out of illiquid assets of n banks ($n = 10$ in the totally interconnected topology; $n = 5$ in the circle; $n = 14$ in the star). The banks hit by the initial shock are uniformly chosen in the system (among the peripheral ones only, in the star topology).

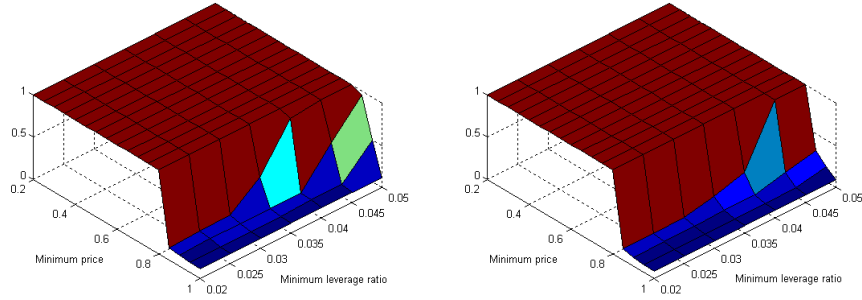
Figure 6: Sensitivity to regulatory ratio and illiquidity in the case of an exponential inverse demand function (1)

(a1) *Totally interconnected – %Liq* (a2) *Totally interconnected – Loss_{TA}*



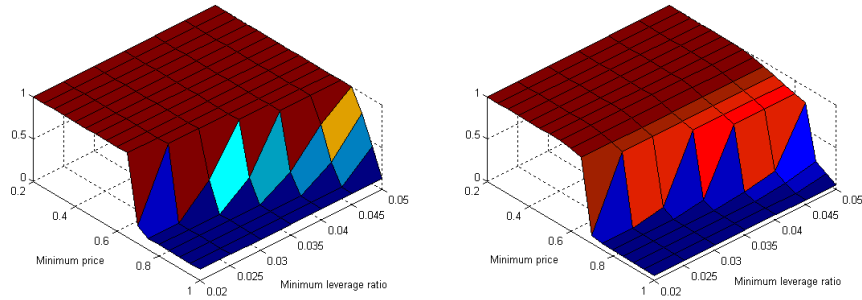
(b1) *Circle – %Liq*

(b2) *Circle – Loss_{TA}*



(c1) *Star – %Liq*

(c2) *Star – Loss_{TA}*



(1) On the x axis, the regulatory leverage ratio \bar{r} is reported; on the y axis, the minimum price (p_{min}) that can be attained by the illiquid asset; on the z axis, either the deleveraging of liquid assets ($\%Liq$) or the change of the value of assets (Δ_{TA}), as defined in Section 4.1. The initial shock consists of the complete wipe-out of illiquid assets of n banks ($n = 5$ in the totally interconnected topology; $n = 3$ in the circle; $n = 7$ in the star). The banks hit by the initial shock are uniformly chosen in the system (among the peripheral ones only, in the star topology).

or ask for higher capital levels). We have seen that, depending on the topology of the banking network, the two instruments may not be perfect substitutes. With respect to the specification adopted in the present exercise, while they are substitutes in the star configuration, in the totally interconnected topology it may be useless to try to improve the soundness of the banking system simply increasing the liquidity of banks' assets without acting on capitalization levels (the converse holds for the circle topology).

Finally, the simulations show that the relative substitutability of the two policy instruments may also depend on the characteristics of the initial shock and on the functional form of the inverse demand function.

Obviously, an exponential function implies a more sizable impact of fire sales than a polynomial function does. Therefore, when performing the previous sensitivity analysis under the assumption that the inverse demand function has an exponential form, we have to lower the magnitude of the initial shock substantially in order to obtain results that are qualitatively similar to (or at least comparable with) those discussed above. The results of this exercise with an exponential function are presented in Figure 6, where the magnitude of the initial shock is roughly half of that assumed in Figure 5. It is apparent that the relative importance of the minimum price over the regulatory leverage ratio is greater in the exponential case, thus shifting the balance between the two policy instruments considered. Although the results for the circle topology are basically unchanged (the supremacy of liquidity conditions is confirmed), the totally interconnected topology now displays a higher degree of substitutability between the two instruments (similar to the star configuration in Figure 5), while the behavior of the star topology with respect to changes in the two parameters of the model becomes closer to that of the circle configuration.

4.6 A more complex and realistic model: scale-free inter-bank network

In the previous paragraphs we analyzed the results of a set of simulation exercises performed on extremely stylized networks models. This approach is useful to highlight the role of the network topology in the flow of contagion, but it may oversimplify the results and lead to a lack of reliability. In this paragraph we test the model on a network structure which bears a greater resemblance to actual banking systems, thus getting a deeper insight in the contagion dynamics.

Following Georg (2013), we focus on a scale-free network configuration, i.e. a network in which the degree distribution is asymptotically described by a power law. Empirical evidence suggests that power laws are very common in natural and social systems, due to their capability to reflect complex behaviour and self-organization. In particular, we study a Barabasi and Albert model (Barabasi and Albert, 1999), which is characterized by low average path length, high clustering and equilibrium power-law distributions.

The build-up of the banking network is a two-step process:

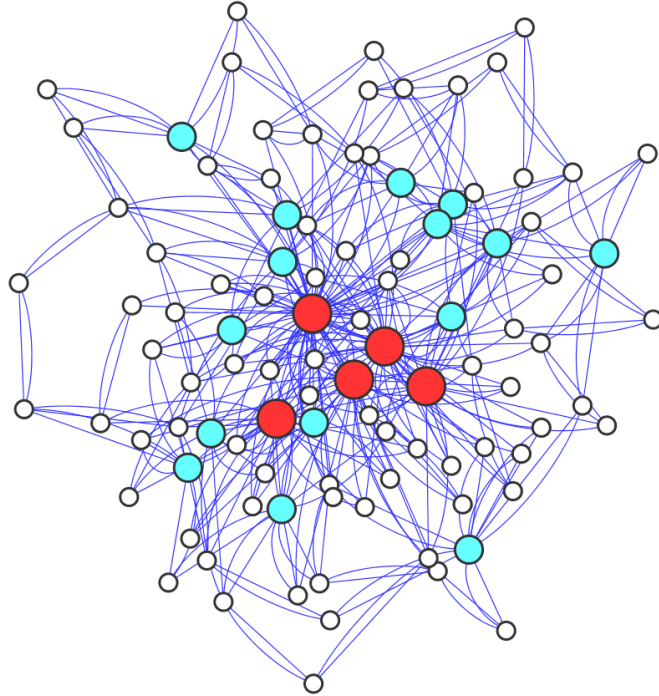
- first of all, we construct a Barabasi and Albert undirected network by implementing a preferential attachment algorithm. The procedure starts with N_0 nodes connected in a complete graph. At each step a node is added and connected to d distinct nodes chosen among the pre-existing ones with a probability proportional to their current degree. The procedure ends when the system reaches N nodes.²² As shown in Georg (2013), this algorithm leads to a configuration in which a small number of banks plays the role of hubs of the network;
- secondly, we determine the balance sheet for each node and the value of the interbank loans. Namely, the nodes are ordered according to their degree and divided into three subsets: core (identified as the most interconnected 5 per cent of the network), semicore (the subsequent 15 per cent of the system) and periphery (the remaining 80 per cent). So far this classification reflects only the degree of interconnectedness of the nodes; we set the balance sheets for the different nodes such that the biggest banks are also the most connected. Then, for each core bank the total asset value is set equal to a predetermined value T , while the banks in the semicore and the periphery are assigned a total asset value equal to $T/2$ and $T/10$, respectively. We assume that for each bank interbank liabilities account for 20 per cent of the magnitude of the balance sheet and that they are evenly distributed among their counterparties; this determines the value of all the entries in the interbank claims matrix L .²³ Finally, we assume that equity equals 5 per cent of total assets (and the completion is given by deposits)

²²In our case, we set $N_0 = 2$, $d = 2$ and $N = 100$.

²³Actually, we assume that for each bank the interbank assets cannot exceed 30 per cent of total assets. If a bank violates this condition, we lower the interbank loans of all of its borrowers by a proportional amount, so that the sum of its interbank assets lowers to 30 per cent. Then the proportion of interbank liabilities can be somewhat lower than 20 per cent of the total liabilities.

and that liquid assets account for 30 per cent of non-interbank assets, the remainder being illiquid assets.

Figure 7: Scale free network (1)



(1) Node colors identify the core of the network (red), the semicore (light blue) and the periphery (white).

The above procedure leads to a network in which we can distinguish three subgroups (core C, semicore SC and periphery P), characterized by increasing size and interconnectedness; Figure 7 shows a realization of this network. Note that although we start with an undirected graph, by construction the interbank matrix L built in the second step is not symmetric.

In the simulations we follow a Monte Carlo approach: we replicate the above construction $S = 1000$ times and we apply CFS model to each of the resulting networks. This approach aims at wiping out the effects due to the specific features of one realization of the Barabasi and Albert model, thus focusing on the patterns that are common to this class of scale-free networks. The Monte Carlo procedure allows us to derive a mean value and a distribution of the most relevant metrics. In this setting we are interested in particular in the following quantities:

- the number of defaulting banks, i.e. banks such that $r_i < \bar{r}$ at the end of the contagion dynamics;
- percentage change in the marked-to-market value of total assets,

$$\Delta_{TA} = 1 - \frac{\sum_{j=1}^N (c_j - t_j + p \cdot (e_j - s_j) + \Pi'x)}{\sum_{j=1}^N (c_j + p_0 \cdot e_j + \Pi'L)};$$

- the share of liquid assets sold by the banks in the system,

$$\%LiqTot = \frac{\sum_{i=1}^N t_i}{\sum_{i=1}^N c_i};$$

- the share of illiquid assets sold by the banks in the system,

$$\%IlliqTot = \frac{\sum_{i=1}^N s_i}{\sum_{i=1}^N e_i};$$

- the share of interbank liabilities which are not payed back,

$$\%IBTot = 1 - \frac{\sum_{i=1}^N x_i}{\sum_{i=1}^N L_i}.$$

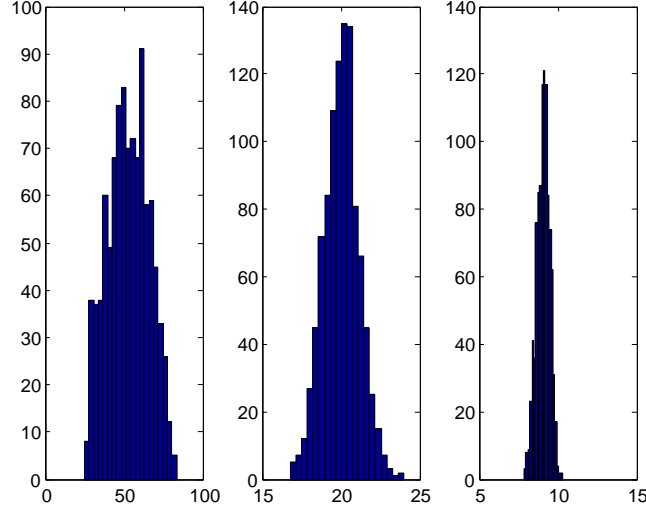
In this simulation exercise the extent and the target of the initial shock were also modeled through a Monte Carlo approach. We considered three stylized cases: (a) an idiosyncratic shock on two core banks; (b) an idiosyncratic shock on three semicore banks; (c) an idiosyncratic shock on five peripheral banks. In every scenario a shock on bank i wipes out a share $1 - \alpha$ of its initial endowment of illiquid assets e_i , thus leading to the after-shock level $e_i^0 = \alpha e_i$. The parameter α is randomly generated from a beta distribution: we draw $S' = 1000$ values for α , apply CFS model for each α and compute the resulting network metrics. The network metrics are then averaged across the S' -dimensional sample.²⁴ We firstly model a moderate shock by generating α from a $\beta(2, 2)$ distribution (in particular, mean = 0.5), then we simulate a bigger shock through a $\beta(3, 5)$ distribution (mean = 0.625).

The results are shown in figures 8-11, where the histograms represent the distribution of some relevant metrics across the S realizations of Barabasi and

²⁴Note that this procedure is replicated $S = 1000$ times, i.e. for each realization of the stochastic scale-free network.

Albert network. For each simulated network, the value of the corresponding metrics is the result of an average across S' generated values of the initial shock parameter α .

Figure 8: Number of defaulting banks (1)



(1) The histograms show the frequency of the number of defaulting banks across the S realizations of the network; the three figures correspond to an idiosyncratic shock on the core, the semicore and the periphery.

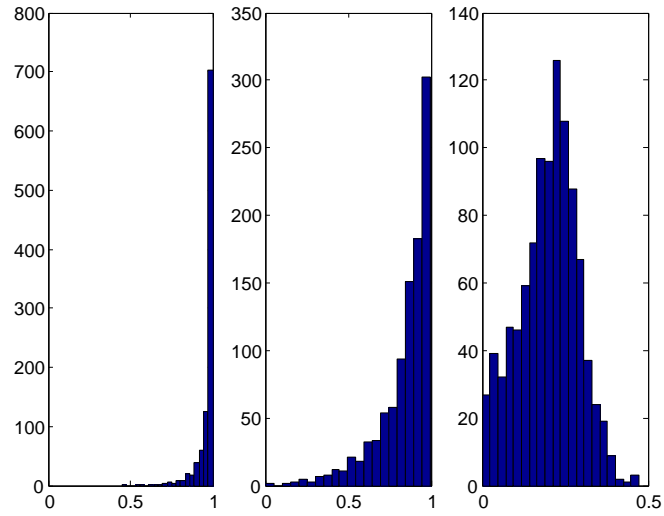
The impact of a shock on the core is clearly the most disruptive, whereas stressing the semicore of the network can give a wide range of outcomes, depending on the specific network topology. When an idiosyncratic shock hits the periphery the cascade effect is limited in terms of number of defaulting banks and share of interbank loans which is not repaid; nevertheless, the banks in the network may need to sell a high percentage of their liquid assets to deal with the effects of the shock.

5 An extension to multiple illiquid assets

The CFS model assumes a stylized representation of a bank's balance-sheet, in which there are only two types of assets (apart from the interbank loans), i.e. a totally liquid asset and an illiquid one. A natural generalization of the model that would make it more realistic consists of expanding the range of available assets.²⁵

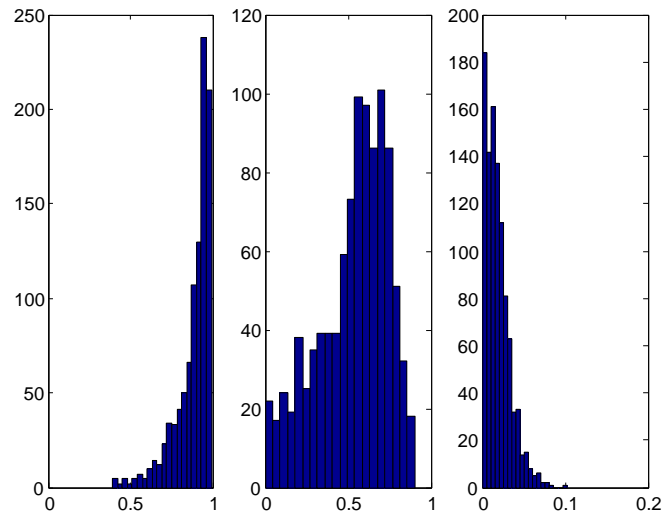
²⁵Gauthier et al. (2012) partly move in this direction, introducing heterogeneity in prices of different banks' illiquid assets. However, they model this as a fluctuation around the fundamental price derived in the CFS framework, due to heterogeneous levels of riskiness of individual

Figure 9: Deleveraging of liquid assets (1)



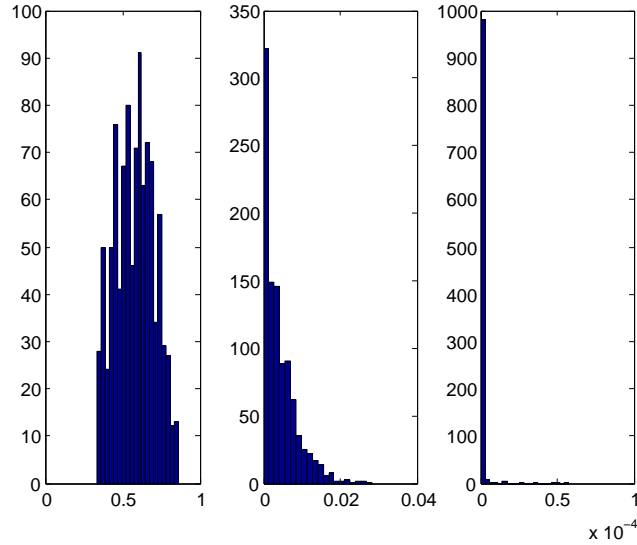
(1) The histograms show the frequency across the S realizations of the network of the value of the deleveraging of liquid assets; the three cases correspond to an idiosyncratic shock on the core, the semicore and the periphery.

Figure 10: Deleveraging of illiquid assets (1)



(1) The histograms show the frequency across the S realizations of the network of the value of the deleveraging of illiquid assets; the three cases correspond to an idiosyncratic shock on the core, the semicore and the periphery.

Figure 11: Share of non-repaid interbank loans (1)



(1) The histograms show the frequency across the S realizations of the network of the share of interbank loans which is not paid back; the three cases correspond to an idiosyncratic shock on the core, the semicore and the periphery.

In the context of the CFS model, there are only two features that can characterize an asset: (i) its degree of liquidity (described by the inverse demand function); (ii) how it is distributed across banks' portfolios. The interaction between these two factors determines to what extent a bank can contribute to or is affected by price deterioration in the case of fire sales. Analogously, when considering several assets, we can discriminate among them assuming different degrees of liquidity and/or different endowments across banks.

The question then arises as to how a bank should liquidate its assets when forced to deleverage in order to comply with the regulatory ratio (for a given interbank payments vector x). In the single illiquid asset case considered in the CFS model, the authors assumed that the liquidation strategy was the same for all banks: sell as many units of the liquid assets as needed to restore the minimum leverage ratio; if selling the whole endowment of liquid asset does not suffice, then start selling as many units of the illiquid asset as necessary. While this behavioral rule is economically sensible, it does not have analytical underpinnings in the CFS model and is unclear how to generalize it to the multiple assets case that we are considering now.²⁶ Even though we could agree that a sensible strategy

banks. On the contrary, we want to model assets that may differ per se in their degree of liquidity and have the same value for different banks holding them.

²⁶In a similar context, Geertsema (2014) assumes that the liquidation is performed on a *pro rata* basis, advocating this choice based on the claim that the results are reasonably sim-

consists of selling more liquid assets first, it is debatable how liquidity should be defined in this context. For instance, at least two strategies can be devised (each prevalently hinging on one of the two aspects that characterize illiquid assets) that may lead to different liquidation decisions: (i) selling first the asset whose price deteriorates less rapidly (e.g., in the case of the exponential inverse demand function in equation (4), selling the asset with a smaller α first); (ii) selling the asset whose minimum price is higher (this depends on the aggregate endowment of illiquid assets of each class, in addition to the parameter α that identifies that class). Notice that, in the case of a liquid asset and an illiquid one only, both strategies reduce to the behavioral rule assumed in the CFS model. In the general case, however, there may be cases in which both of them are sub-optimal, as shown in the following. Thus, the extension of the CFS model to the multiple assets case may add economic insight to the model.

To answer the question on liquidation strategies, we provide a simple microfoundation based on the assumption that each bank is willing to pick any liquidation strategy that maximizes the final value of its equity (equivalent to maximize total assets' value in this simplified setting). In Section 5.1 we present the problem and derive some analytical conditions. In Section 5.2 we show the results of a numerical example.

5.1 Microfoundation of liquidation strategies

In this section we propose a possible way to extend the algorithm described in Section 3.2 to the case of multiple assets. The notation remains broadly unchanged, except that now there are $M + 1$ assets in the economy and we denote by:²⁷

- e the *matrix of endowments* – e_{ik} (or e_i^k , as we will write it) is the quantity of asset j that bank i is initially endowed with, $i = 1, \dots, N$, $k = 0, \dots, M$;
- s the *matrix of liquidation strategies* – s_{ik} (or s_i^k) is the quantity of asset j sold by bank i to try to comply with the regulatory leverage ratio, $i = 1, \dots, N$, $k = 0, \dots, M$;

ilar to those obtain with a sequential liquidation, a strategy consistently employed by many practitioners. We show in the following that both these strategies may be sub-optimal.

²⁷For simplicity, one can think of asset 0 as the liquid asset, but in the framework that we are describing it is neither necessary to assume that there is a perfectly liquid asset, nor that it is unique. Therefore, we no longer use a different variable (c or t , respectively) to denote the endowment or selling of the liquid asset.

- p_0 and p_{min} the vectors of after-shock prices and minimum attainable prices, respectively – we assume $0 < p_{min}^k \leq p_0^k \leq 1$ for $k = 0, \dots, M$;
- $g_k : [0, \sum_{i=1}^N e_i^k] \rightarrow [p_{min}^k, p_0^k]$ the *inverse demand function* of asset $k = 0, \dots, M$ after a shock σ – we assume it to be either constant (if asset k is completely liquid) or continuous and decreasing.²⁸

We also denote by e_i, s_i the endowment and strategy vectors of bank i and by e_{-i}, s_{-i} the correspondent matrices of all other banks. Similarly, e^k and s^k will denote the vectors of quantities of asset k that each bank is endowed with or decides to sell according to its liquidation strategy. Furthermore, when focusing on bank i 's problem, we will write $g_k(s_i^k, s_{-i}^k)$ instead of $g_k(\sum_{j=1}^N s_j^k)$.

Recall that at each step h of the algorithm, two sub-algorithms have to be implemented, that in the current notation read: (i) given the vector of interbank payments x^{h-1} , find an equilibrium matrix of liquidation strategies s^h and a corresponding price vector p^h ($p_k^h = g_k(s^k)$);²⁹ (ii) given p^h and s^h , compute the new interbank clearing vector x^h with the adapted version of the fictitious default algorithm by Eisenberg and Noe (2001) described in Section 3.2. The second sub-algorithm is not affected by the extension to the multiple assets case. Therefore, we can focus on the first sub-algorithm only: given a clearing vector x , find an equilibrium liquidation strategy s and the associated price vector p (we drop the h indexes for ease of notation).

A standard way to model banks' liquidation strategies is to assume that they try to maximize the final value of their equity. However, this simple assumption brings two issues along. First, a priori the optimal strategy may not be unique. Second, we have to model in which way prices are updated by the bank when evaluating the final value of equity.

As far as the former issue is concerned, one can complement this simple optimization problem with an auxiliary optimization or behavioral rule that discriminates among the optimal solutions found according to a given criterion. As for the latter issue, in the CFS model the behavioral rule adopted does not leave

²⁸The strict monotonicity assumption (satisfied both by the exponential inverse demand function used in the CFS model and by the quadratic function used in most simulations of Section 4) makes the proofs in this section easier. Although it rules out the interesting case of an asset that is liquid up to a certain level of sales and then becomes illiquid, this case can be easily approximated by a decreasing function with arbitrary small (in absolute value) derivative in the range of sales in which the asset is liquid.

²⁹While in Section 3.2 at each iteration bank i 's strategy s_i was determined on the basis of the equilibrium price, here it is more convenient to reason in the opposite way, letting the equilibrium price be determined by the interaction of banks' optimal strategies.

room for any choice on the part of the liquidating bank. As a consequence, it is ultimately immaterial whether the bank considers the price impact of its selling or not. However, in the present context the price impact of different liquidation strategies matters and should be taken into account when computing the final value of assets. Therefore, we make the following assumption.³⁰

Assumption 5.1. *Every bank i , given the strategies s_{-i} of other banks, picks its optimal strategy \bar{s}_i in the solution set of the problem*

$$\begin{aligned} \max_{s_i : s_i^k \in [0, e_i^k]} \quad & \max \left\{ 0, \sum_{k=0}^M (e_i^k - s_i^k) g_k(s_i^k, s_{-i}^k) + (\Pi'x)_i - d_i - L_i \right\} \\ \text{sub} \quad & \bar{r} \left(\sum_{k=0}^M (e_i^k - s_i^k) g_k(s_i^k, s_{-i}^k) + (\Pi'x)_i \right) \leq \\ & \leq \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + (\Pi'x)_i - d_i - L_i. \end{aligned} \quad (22)$$

The strategy adopted is then

$$s_i = \begin{cases} e_i & \text{if } x_i < L_i, \text{ or problem (22) has no solution} \\ \bar{s}_i & \text{if } x_i = L_i. \end{cases} \quad (23)$$

Taking into account that the minimum ratio requirement does not allow the equity value to become negative and that the previous constraint is always binding at an optimal solution, we can prove that problem (22) can be simplified and replaced with problem (24).

Lemma 5.2. *If, given s_{-i} , the strategy $s_i = (0, \dots, 0)'$ is feasible, then it is the unique optimal solution to problem (22). Otherwise, under suitable constraint*

³⁰Actually, to ensure that the algorithm presented in Section 3.2 converges, we need an increasing sequence of liquidation strategy vectors $\{s_i^{(h)}\}$, $i = 1, \dots, N$. Therefore, the box constraints $[0, e_i^k]$ in the optimization problem should be replaced by $[(s_i^k)^{(h)}, e_i^k]$. This would not change the results that follow, but would make the notation more cumbersome. Analogously, we solve the algorithm iteratively, considering in the optimization problem of bank i at step h the liquidation strategies of other banks at step $h - 1$, $s_{-i}^{(h-1)}$, but we drop the indexes for simplicity.

qualification conditions, that problem is equivalent to the following

$$\begin{aligned} & \max_{s_i : s_i^k \in [0, e_i^k]} \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) \\ & \text{sub } \bar{r} f_i(s_i, s_{-i}) - \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + d_i + L_i - (1 - \bar{r})(\Pi' x)_i = 0, \end{aligned} \quad (24)$$

where

$$f_i(s_i, s_{-i}) = \sum_{k=0}^M (e_i^k - s_i^k) g_k(s_i^k, s_{-i}^k).$$

Proof. Obviously, the objective function in problem (22) can be replaced by $f_i(s_i, s_{-i})$. Rearranging the constraint, we obtain the equivalent problem

$$\begin{aligned} & \max_{s_i : s_i^k \in [0, e_i^k]} f_i(s_i, s_{-i}) \\ & \text{sub } \bar{r} f_i(s_i, s_{-i}) - \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + d_i + L_i - (1 - \bar{r})(\Pi' x)_i \leq 0. \end{aligned} \quad (25)$$

First of all, as $g_k(\cdot, s_{-i}^k)$ is decreasing and positive, then

$$\frac{\partial f_i}{\partial s_i^k}(s_i, s_{-i}) = (e_i^k - s_i^k) \frac{\partial g_k}{\partial s_i^k}(s_i^k, s_{-i}^k) - g_k(s_i^k, s_{-i}^k) < 0. \quad (26)$$

Therefore, whenever the strategy $s_i = (0, \dots, 0)'$ is feasible, it is the unique solution to the problem.

Suppose then that it is not feasible and let s_i be an optimal solution with $s_i^m > 0$ for some $m \in \{0, \dots, M\}$. The Lagrangian of problem (25) reads

$$\mathcal{L}_i = f_i(s_i, s_{-i}) + \sum_{k=0}^M \lambda_k (e_i^k - s_i^k) - \mu h_i(s_i, s_{-i})$$

with

$$h_i(s_i, s_{-i}) = \bar{r} f_i(s_i, s_{-i}) - \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + d_i + L_i - (1 - \bar{r})(\Pi' x)_i \quad (27)$$

and $\mu \geq 0$, $\lambda_k \geq 0$ for $k = 0, \dots, M$. If $\mu = 0$, first order conditions would imply

$$\frac{\partial \mathcal{L}_i}{\partial s_i^m} = \frac{\partial f_i}{\partial s_i^m}(s_i, s_{-i}) - \lambda_m = 0,$$

and, by equation (26),

$$0 \leq \lambda_m = \frac{\partial f_i}{\partial s_i^m}(s_i, s_{-i}) < 0,$$

a contradiction. Therefore, the constraint is always binding at a non-zero solution and can be replaced by the equality constraint. From the new constraint we obtain

$$f_i(s_i, s_{-i}) = \frac{1}{\bar{r}} \left(\sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + (1 - \bar{r})(\Pi'x)_i - d_i - L_i \right).$$

Substituting for the objective function in problem (25) and dropping the constant terms, we obtain problem (24). ■

Remark 5.3. In the following we will assume that the constraint qualification conditions required for Lemma 5.2 to hold are satisfied. For instance, if the inverse demand functions g_k are concave and C^2 (e.g., quadratic, as we assumed in Section 4), it is easily verified that the function h_i is (strictly) convex. Therefore all constraints are convex (the box constraints are affine) and Slater qualification condition only requires that there exist a feasible point at which the constraints are not binding. Notice that, under these assumptions, KKT conditions for problem (24) with the equality constraint $h_i = 0$ (and h_i is defined as in equation (27)) replaced by $h_i \leq 0$ are necessary and sufficient conditions for a point to be a local maximum, as all constraints are convex and the objective function is concave (it can be easily shown along the same lines of Lemma 5.2 that the inequality constraint $h_i \leq 0$ is binding at any optimal solution, so that the solution set of problem (24) remains the same if we replace $h_i = 0$ with $h_i \leq 0$).

As a consequence of Lemma 5.2, in the remainder of this section we will focus on problem (24), instead of problem (22). In particular, we derive from the first order condition a few equations that allow us to better understand how optimal liquidation strategies relate to the heuristic strategies discussed above. First of all, we prove that, whenever a bank is endowed with a liquid asset, it is not optimal to sell any amount of illiquid assets, unless the liquid asset has been sold in full. This provides an analytical foundation to the liquidation strategy assumed in the CFS model.

Proposition 5.4. *Let asset ℓ be liquid ($g_\ell(s_i^\ell, s_{-i}^\ell) = 1$ for all s^ℓ) and asset m be illiquid (g_m be decreasing). Any strategy s_i such that $s_i^\ell < e_i^\ell$ and $s_i^m > 0$ is not optimal for bank i .*

Proof. The Lagrangian of problem (24) is

$$\mathcal{L}_i = \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) + \sum_{k=0}^M \lambda_k (e_i^k - s_i^k) - \mu h_i(s_i, s_{-i}), \quad \mu \in \mathbb{R}.$$

If $s_i^k > 0$, the first order condition related to s_i^k reads

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial s_i^k} = 0 &\Leftrightarrow e_i^k \frac{\partial g_k}{\partial s_i^k}(s_i, s_{-i}) - \lambda_k - \mu \left(\bar{r} \frac{\partial f_i}{\partial s_i^k}(s_i, s_{-i}) - e_i^k \frac{\partial g_k}{\partial s_i^k}(s_i^k, s_{-i}^k) \right) = 0 \\ &\Leftrightarrow e_i^k \frac{\partial g_k}{\partial s_i^k}(s_i, s_{-i}) - \lambda_k + \mu \left(((1 - \bar{r})e_i^k + \bar{r}s_i^k) \frac{\partial g_k}{\partial s_i^k}(s_i^k, s_{-i}^k) + \bar{r}g_k(s_i^k, s_{-i}^k) \right) = 0. \end{aligned} \quad (28)$$

For asset ℓ , equation (28) simply reads $\lambda_\ell = \mu \bar{r}$. Therefore, if $s_i^\ell < e_i^\ell$, one has $\lambda_\ell = 0$ and, as a consequence, $\mu = 0$. On the other hand, if $s_i^m > 0$ equation (28) with $\mu = 0$ implies

$$0 \leq \lambda_m = e_i^m \frac{\partial g_m}{\partial s_i^m}(s_i, s_{-i}) < 0, \quad (29)$$

a contradiction. ■

Consider now under what circumstances it may be optimal for bank i to sell a positive amount of two different illiquid assets ($s_i^\ell > 0$ and $s_i^m > 0$). It follows from inequalities (29), arguing by contradiction, that $\mu \neq 0$. Then we can solve equation (28) for μ with $k = \ell, m$. Equating the two expressions obtained, we then have

$$\frac{e_i^\ell \frac{\partial g_\ell}{\partial s_i^\ell}(s_i^\ell, s_{-i}^\ell) - \lambda_\ell}{\bar{r} \frac{\partial f_i}{\partial s_i^\ell}(s_i, s_{-i}) - e_i^\ell \frac{\partial g_\ell}{\partial s_i^\ell}(s_i^\ell, s_{-i}^\ell)} = \frac{e_i^m \frac{\partial g_m}{\partial s_i^m}(s_i^m, s_{-i}^m) - \lambda_m}{\bar{r} \frac{\partial f_i}{\partial s_i^m}(s_i, s_{-i}) - e_i^m \frac{\partial g_m}{\partial s_i^m}(s_i^m, s_{-i}^m)}.$$

As a consequence, we obtain the following necessary (and sufficient, under suitable assumptions; see Remark 5.3) condition for an interior solution in the assets ℓ and m , i.e. a liquidation strategy s_i such that $s_i^\ell \in (0, e_i^\ell)$ and $s_i^m \in (0, e_i^m)$:

$$\frac{e_i^\ell \frac{\partial g_\ell}{\partial s_i^\ell}(s_i^\ell, s_{-i}^\ell)}{e_i^m \frac{\partial g_m}{\partial s_i^m}(s_i^m, s_{-i}^m)} = \frac{\bar{r}g_\ell(s_i^\ell, s_{-i}^\ell) + ((1 - \bar{r})e_i^\ell + \bar{r}s_i^\ell) \frac{\partial g_\ell}{\partial s_i^\ell}(s_i^\ell, s_{-i}^\ell)}{\bar{r}g_m(s_i^m, s_{-i}^m) + ((1 - \bar{r})e_i^m + \bar{r}s_i^m) \frac{\partial g_m}{\partial s_i^m}(s_i^m, s_{-i}^m)}. \quad (30)$$

The previous condition shows that the optimality of a liquidation strategy that is an interior point in (at least) two assets depends on the optimal strategies

of other banks and all the parameters of the model: the inverse demand function and its derivative, the initial endowments of bank i and the minimum leverage ratio set by the regulator. On the other hand, the same equality may be used to derive sufficient conditions under which an interior solution may not be optimal. An example is provided in the following remark, under additional assumptions on the inverse demand functions.

Remark 5.5. (a) In the exponential case³¹

$$g_k(s_i^k, s_{-i}^k) = \exp\left(-\alpha_k \sum_{j=1}^N s_j^k\right), \quad \alpha_k > 0, \quad k = \ell, m,$$

used in the CFS model for the single-asset case, condition (30) simplifies to

$$\frac{\alpha_\ell e_i^\ell}{\alpha_m e_i^m} = \frac{\alpha_\ell((1 - \bar{r})e_i^\ell + \bar{r}s_i^\ell) - \bar{r}}{\alpha_m((1 - \bar{r})e_i^m + \bar{r}s_i^m) - \bar{r}},$$

from which, after a few algebraic manipulations,

$$\frac{\alpha_\ell e_i^\ell}{\alpha_m e_i^m} = \frac{1 - \alpha_\ell s_i^\ell}{1 - \alpha_m s_i^m}. \quad (31)$$

This necessary condition has the remarkable property that it does not depend on the liquidation strategies adopted by other banks (and it does not depend on the regulatory ratio \bar{r} either).

Moreover, it clearly shows that an optimal liquidation strategy should not be based only on the minimum price that an asset k can attain, or on the velocity with which its price deteriorates (as measured by the derivative of the inverse demand function, or its semi-elasticity α). The quantity that matters is instead the product $\alpha_k e_i^k$ of the semi-elasticity times the endowment of asset k initially held by bank i , i.e. a transformation of the maximum price deterioration that bank i could cause by selling the whole of its endowment of asset k .

Finally, notice that equation (31) can be used to derived sufficient conditions under which an interior strategy in assets ℓ and m can not be optimal.

³¹In this remark and Remark 5.6, we are assuming that when no asset sales have occurred, all prices are normalized to one, as in the CFS model.

Indeed, since

$$1 - \alpha_\ell e_i^\ell \leq \frac{1 - \alpha_\ell s_i^\ell}{1 - \alpha_m s_i^m} \leq \frac{1}{1 - \alpha_m e_i^m}, \quad \forall s_i^\ell \in [0, e_i^\ell], s_i^m \in [0, e_i^m],$$

if

$$\frac{1}{1 - \alpha_m e_i^m} < \frac{\alpha_\ell e_i^\ell}{\alpha_m e_i^m}, \quad \text{or} \quad \frac{\alpha_\ell e_i^\ell}{\alpha_m e_i^m} < 1 - \alpha_\ell e_i^\ell,$$

then equation (31) can not hold.

(b) Equation (30) simplifies also in the case of affine inverse demand functions

$$g_k(s_i^k, s_{-i}^k) = 1 - \alpha_k \left(\sum_{j=1}^N s_j^k \right), \quad \alpha_k > 0, \quad k = \ell, m,$$

reading

$$\frac{\alpha_\ell e_i^\ell}{\alpha_m e_i^m} = \frac{1 - \alpha_\ell \left(s_i^\ell + \sum_{j=1}^N s_j^\ell \right)}{1 - \alpha_m \left(s_i^m + \sum_{j=1}^N s_j^m \right)}.$$

While in this case the strategy of bank i depends on the strategies of other players, comments similar to those made in the exponential case apply.

Since it may be the case that no interior solution is optimal, it is useful to derive optimality conditions that allow to compare boundary solutions as well. This can be done evaluating the objective function of problem (24) directly. Indeed, consider two feasible points for that problem that differ only in the two components ℓ and m , namely s_i and \hat{s}_i such that $s_i^\ell \neq \hat{s}_i^\ell$, $s_i^m \neq \hat{s}_i^m$ and $s_i^k = \hat{s}_i^k$ for all $k \neq \ell, m$. By direct evaluation of the objective function, s_i is a better strategy than \hat{s}_i if and only if

$$\begin{aligned} \sum_{k=0}^M e_i^k g_k(s_i^k, s_{-i}^k) &> \sum_{k=0}^M e_i^k g_k(\hat{s}_i^k, s_{-i}^k) \Leftrightarrow \sum_{k=0}^M e_i^k (g_k(s_i^k, s_{-i}^k) - g_k(\hat{s}_i^k, s_{-i}^k)) > 0 \\ \Leftrightarrow e_i^\ell (g_\ell(s_i^\ell, s_{-i}^\ell) - g_\ell(\hat{s}_i^\ell, s_{-i}^\ell)) &+ e_i^m (g_m(s_i^m, s_{-i}^m) - g_m(\hat{s}_i^m, s_{-i}^m)) > 0. \end{aligned} \quad (32)$$

In case of two boundary solutions with $s_i^\ell > 0$, $s_i^m = 0$, $\hat{s}_i^\ell = 0$ and $\hat{s}_i^m > 0$, it is easy to pick the better one based on condition (32), that now reads

$$\frac{g_m(0, s_{-i}^m) - g_m(\hat{s}_i^m, s_{-i}^m)}{g_\ell(0, s_{-i}^\ell) - g_\ell(s_i^\ell, s_{-i}^\ell)} > \frac{e_i^\ell}{e_i^m}. \quad (33)$$

Remark 5.6. In the case of affine inverse demand functions, condition (33) simplifies to

$$\frac{\alpha_m e_i^m}{\alpha_\ell e_i^\ell} > \frac{s_i^\ell}{s_i^m}.$$

This condition goes in the same direction as Remark 5.5, confirming the role played by the maximum potential impact of bank i 's sales of each illiquid asset when discriminating among different strategies.

5.2 Numerical results

We finally present the results of two simulation exercises in order to illustrate the main points discussed in Section 5.1 on the optimality of different liquidation strategies.

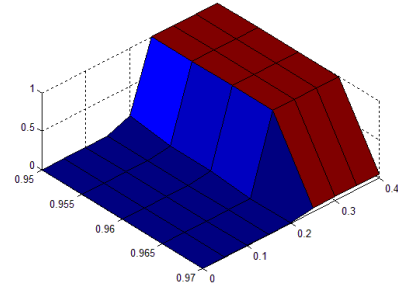
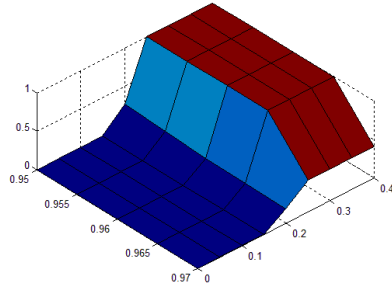
In the first example, we consider a totally interconnected banking system of 10 homogeneous banks. The liability side of their balance-sheet is as in Table 1. On the asset side, each bank is endowed with 120 units of asset 1 and with 50 units of asset 2 (interbank loans amount to 30). We then perform contagion analysis assuming an initial shock that cancels an equal share S of bank 1's endowments of both assets. Keeping the minimum price of asset 2 constant, we let the minimum price of asset 1 and the share S vary. The results are presented in Figure 12.

Focusing on the shares of each asset liquidated by banks that are not hit by the initial shock, we can distinguish three main strategies, depending on the values of the two parameters:

- (i) both shares equal zero, meaning that contagion effects are not strong enough to trigger asset sales by other banks – this happens when the initial shock is small in magnitude;
- (ii) both shares are equal to one, i.e. contagion effects are so strong that force all banks to completely deleverage and, eventually, default – this case covers the region in which the shock is higher and asset 1 is less liquid;
- (iii) both shares are in the interval $(0, 1)$, i.e. an interior point strategy is optimal – this is the case only in two small regions of the parameter space that we are showing, corresponding either to an intermediate value of the initial shock, or a higher degree of liquidity of asset 1. Notice that, in the first case, while the ratio of the quantity of asset liquidated to the initial endowment decreases slowly for asset 1 as its liquidity improves, the decrease is exponential for asset 2. This suggests that, as liquidity conditions

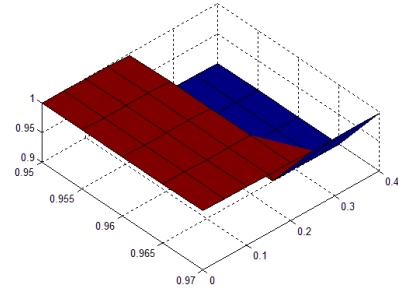
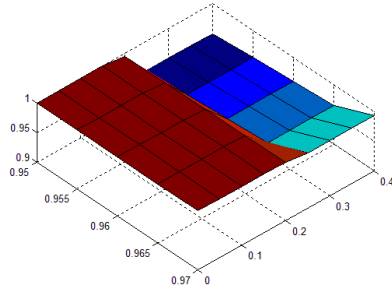
Figure 12: Contagion analysis with two illiquid assets (1)

(a1) *Share of asset 1 sold by bank i* (a2) *Share of asset 2 sold by bank i*



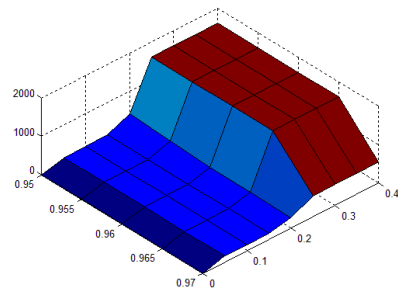
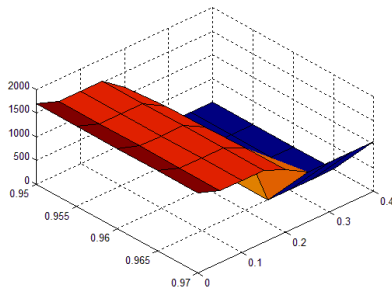
(b1) *Final price of asset 1*

(b2) *Final price of asset 2*



(c1) *Final aggregate asset value*

(c2) *Final loss on non-interbank liabilities*



(1) On the x axis the share S of assets of bank 1 canceled by the initial shock is reported. On the y axis, the minimum price that asset 1 can attain. On the z axis, different metrics are plotted: the share of asset 1 and asset 2 sold by banks that are not hit by the initial shock, in panels (a1) and (a2); the price of the assets after contagion has taken place, in panels (b1) and (b2); the final value of total assets, panel (c1); the amount of senior debt that banks are not able to pay back, panel (c2). The main parameters of the model are set as follows: $\bar{r} = 4\%$; $p_{min}(2) = 0.9$.

of asset 1 improve, its preeminence in optimal liquidity strategies increases (at a more than linear pace).

These outcomes are in line with the claim of Section 5.1 that an optimal liquidation strategy can not be chosen in advance, as it depends crucially on the characteristics of the initial shock and of banks' portfolios. That is, from the point of view of a given bank, liquidity as measured by price elasticity or by the minimum attainable price may not be the only relevant parameter on the basis of which a liquidation strategy is planned (unless one perfectly liquid asset is available).

In this connection, another finding that is worth pointing out is that, in the same region of parameter values that yield interior point strategies, the final price of the first asset is lower than the price of the second one. Although in principle asset 1 is more liquid than asset 2 (as indicated by the minimum price that they could attain if all banks completely liquidated their endowments), it may end up with a greater price deterioration due to fire sales. At least in some cases, this might be a socially undesirable outcome, as prices could then provide incorrect signals concerning the intrinsic value of assets.

Another example of how all relevant parameters interact in the choice of the optimal liquidation strategy may be provided in a slightly more complicated setting, in which there are three illiquid assets and each bank holds two of them only in its portfolio. In particular, asset 1 (more liquid) represents a share L of each bank's total assets, while only banks 1 to 5 are endowed with asset 2 (intermediate level of liquidity) and banks 6 to 10 hold asset 3 (less liquid) each. Asset 2 and 3 account for a share M of total assets of the banks holding them. Given an exogenous shock to the holdings of asset 2 in bank 1's portfolio, we want to analyze how the liquidation strategies of other banks are affected as L and M change, while keeping the sum $L + M$ constant (as a consequence, the amount of interbank loans of each bank is constant too).³²

The results of Table 2 show that the asset hit by the initial shock to bank 1 is never sold by other banks (banks i in the table), unless their selling of asset 1 is not enough to restore the minimum leverage ratio required by the regulator. This is an instance of sequential selling, justified by the significant deterioration of asset 2's price in the aftermath of the initial shock. On the other hand, the behavior of banks 6 to 10 (banks j in the table) is more complex. Unless they default or do not have to deleverage, their optimal strategy is always at some interior point.

³²Balance-sheet dimensions and network topology are the same as in the previous exercise.

In particular, the share of assets 1 and 3 that they have to sell is increasing in M (i.e., the ratio of asset 3 – more illiquid – to total assets). However, they grow at a different pace, with the share of asset 1 sold that increases considerably faster as bank’s portfolio becomes less liquid.

Table 2: Optimal liquidation strategies (1)

L	M	Bank i , asset 1	Bank i , asset 2	Bank j , asset 1	Bank j , asset 3
45.0%	40.0%	0.0%	0.0%	0.0%	0.0%
40.0%	45.0%	5.5%	0.0%	0.0%	0.0%
37.5%	47.5%	12.6%	0.0%	0.0%	2.4%
35.0%	50.0%	22.9%	0.0%	5.1%	5.5%
32.5%	52.5%	41.8%	0.0%	17.8%	9.0%
30.0%	55.0%	100%	100%	100%	100%

(1) The first two columns report values of the shares L and M of a bank’s assets that are of type 1 and 2 (or 3), respectively. On the other columns, we report optimal liquidation strategies of representative banks from the two groups: bank i stands for banks 2 to 5; bank j stands for banks 6 to 10. The parameters are set as follows: $\bar{r} = 4\%$; $p_{min}(1) = 0.95$; $p_{min}(2) = 0.92$; $p_{min}(3) = 0.90$.

6 Conclusion

In this paper we build on the model by Cifuentes et al. (2005a,b), a pioneering reference for contagion analysis in banking networks with market liquidity effects due to asset fire sales, in order to study how different network topologies respond to exogenous shocks (either systemic or idiosyncratic). First, we provide detailed analytical foundations to the key insights of that model, proving under fairly general assumptions the existence of an equilibrium (in principle there might be several equilibria, but only one of them can actually occur in the model) and the convergence of the algorithm to compute it. Then, to answer our research

question (to what extent the resilience of an interbank network depends on its topology), we run a set of simulation exercises testing how some stylized networks are affected by different types of shocks. Finally, we extend the original model to include the case where banks hold in their portfolios multiple illiquid assets, with different degrees of illiquidity.

The main conclusions and policy implications can be summarized as follows. First, consistent with the relevant literature, our results confirm that the resilience of the banking system may depend heavily on the network topology. From a policy perspective, this might justify that macroprudential authorities have access to granular data on interbank exposures.³³ Second, we show that different network configurations may be affected by changes in relevant market and regulatory parameters (assets' liquidity and minimum leverage ratio, respectively) in different ways. The search for an optimal balance between different policy instruments that can be used to cope with systemic risk would benefit from a deeper knowledge of the sensitivity of the banking network at hand on the key parameters. Finally, in a more realistic setting bank assets show heterogeneous degrees of illiquidity (as measured by price elasticity); in this context, there exist several channels of contagion (or, more precisely, interdependence) and banks that take losses and are forced to partially deleverage have to choose what liquidation strategy to adopt. To this end, we show that optimal liquidation strategies (from an equity maximization perspective) may be in disagreement with common behavioral strategies. For a bank that has to deleverage, the liquidity of an asset is not an absolute quality of the asset, but relates to both the composition of the bank's portfolio and the liquidation strategies adopted by other banks.

Future research extending this paper could extend the analysis of the multiple asset setting and include the additional channel of contagion represented by funding liquidity.

³³For policy purposes, it is also worth mentioning that the impact of an exogenous shock of given magnitude depends on how it is distributed among the banks in the system (even in the case of homogeneous banks) and the structure of this dependence varies for different network configurations. This has implications, for instance, for the optimal allocation of limited resources for banks' recapitalizations.

References

- Acemoglu, D., O. Asuman, and A. Tahbaz-Salehi (2015a), “Systemic risk and stability in financial networks.” *American Economic Review*, 105, 564–608.
- Acemoglu, D., O. Asuman, and A. Tahbaz-Salehi (2015b), “Systemic risk in endogenous financial networks.” *Columbia Business School Research Paper*.
- Adrian, T. and H.S. Shin (2010), “Liquidity and leverage.” *Journal of Financial Intermediation*, 19, 418–437.
- Allen, F. and A. Babus (2009), “Networks in finance.” *in: The Network Challenge: Strategy, Profit and Risk in an Interlinked World*, Kleindorfer, P. and Wind, Y., Pearson Prentice Hall.
- Allen, F., A. Babus, and E. Carletti (2012), “Financial connections and systemic risk.” *Advances in Network Analysis and its Applications*.
- Allen, F. and D. Gale (2000), “Financial contagion.” *Journal of Political Economy*, 1–33.
- Amini, H. and A. Minca (2012), “Mathematical modeling of systemic risk.” *in: Advances in Network Analysis and its Applications*, Kranakis, E. (ed.), Springer, 3–26.
- Babus, A. (2015), “The formation of financial networks.” *RAND Journal of Economics*.
- Barabasi, A.L. and R. Albert (1999), “Emergence of scaling in random networks.” *Science*, 286, 509–512.
- Battison, S., G. Di Iasio, L. Infante, and F. Pierobon (2015), “Capital and contagion in financial networks.” *IFC Bulletins chapters*, *in: Bank for International Settlements (ed.), Indicators to support monetary and financial stability analysis: data sources and statistical methodologies*.
- Brunnermeier, M. and L. Pedersen (2009), “Market liquidity and funding liquidity.” *Review of Financial Studies*, 22, 2201–2238.
- Caballero, R.J. and A. Simsek (2013), “Fire sales in a model of complexity.” *The Journal of Finance*.

- Cabrales, A., P. Gottardo, and F. Vega-Redondo (2014), “Risk-sharing and contagion in networks.” *FEDEA Working Papers*, 18.
- Cifuentes, R., G. Ferrucci, and H.S. Shin (2005a), “Liquidity risk and contagion.” *Bank of England Working papers*.
- Cifuentes, R., G. Ferrucci, and H.S. Shin (2005b), “Liquidity risk and contagion.” *Journal of the European Economic Association*.
- Craig, B. and G. von Peter (2014), “Interbank tiering and money center banks.” *Journal of Financial Intermediation*, 322–347.
- Eisenberg, L. and T.H. Noe (2001), “Systemic risk in financial systems.” *Management Science*.
- Elliot, M., B. Golub, and M.O. Jackson (2014), “Financial networks and contagion.” *American Economic Review*, 104, 3115–3153.
- Farboodi, M. (2014), “Intermediation and voluntary exposure to counterparty risk.” *Working Paper*.
- Freixas, X., B. Parigi, and J.C. Rochet (2000), “Systemic risk, interbank relations and liquidity provision by the central bank.” *Journal of Money, Credit and Banking*, 32, 611 – 638.
- Fricke, D. and T. Lux (2015), “Core-periphery structure in the overnight money market: Evidence from the e-mid trading platform.” *Computational Economics*, 359–395.
- Gai, P. (2013), “Systemic risk: The dynamics of modern financial systems.” *Oxford University Press*.
- Gai, P., A. Haldane, and S. Kapadia (2011), “Complexity, concentration and contagion.” *Journal of Monetary Economics*, 58, 453 – 470.
- Gai, P. and S. Kapadia (2010), “Contagion in financial networks.” *Bank of England Working Paper*, 383.
- Gauthier, C., A. Lehar, and M. Souissi (2012), “Macroprudential capital requirements and systemic risk.” *Journal of Financial Intermediation*.
- Geertsema, P. (2014), “Fire sales and contagion in financial assets.” *electronic copy available at <http://ssrn.com/abstract=2537171>*.

- Georg, C. (2013), “The effect of the interbank network structure on contagion and common shocks.” *Journal of Banking and Finance*, 37.
- in ‘t Veld, D. and I. van Lelyveld (2014), “Finding the core: Network structure in interbank markets.” *Journal of Banking and Finance*, 27–40.
- Jackson, M.O. (2010), “Social and economic networks.” *Princeton University Press*.
- Jotikasthira, C., C. Lundblad, and T. Ramadorai (2012), “Asset fire sales and purchases and the international transmission of funding shocks.” *The Journal of Finance*.
- Mistrulli, P. (2011), “Assessing financial contagion in the interbank market: maximum entropy versus observed interbank lending patterns.” *Journal of Banking and Finance*, 35, 1114–1127.
- Newman, M.E.J. (2003), “The structure and function of complex networks.” *SIAM Review*.
- Newman, M.E.J. (2010), “Networks: An introduction.” *Oxford University Press*.
- Roukny, T., H. Bersini, H. Pirotte, G. Caldarelli, and S. Battiston (2013), “Default cascades in complex networks: topology and systemic risk.” *Scientific Reports*, 3.
- Shleifer, A. and R. Vishny (1992), “Liquidation values and debt capacity: a market equilibrium approach.” *Journal of Finance*, 47, 1343–1366.
- Shleifer, A. and R. Vishny (2011), “Fire sales in finance and macroeconomics.” *Journal of Economic Perspectives*, 25, 29–48.
- United States Treasury (2009), “Public-private investment program.” *Press release*.
- Upper, C. (2011), “Simulation methods to assess the danger of contagion in interbank markets.” *Journal of Financial Stability*, 7, 111–125.
- Zawadowski, A. (2013), “Entangled financial systems.” *The Review of Financial Studies*, 26, 1291–1323.

Appendix

In Section 3 we use extensively Tarski's fixed point theorem, that we recall here for convenience of the reader (see e.g. Carter, 2001).

Theorem A.1. (Tarski's fixed point theorem, 1955) *Let X be a non-empty complete lattice. If $f : X \rightarrow X$ is non-decreasing, then the set of fixed points of f is a non-empty complete lattice. In particular, f has a greatest fixed point \bar{x} and a least fixed point \underline{x} .*

We also prove two useful results that we need in Section 3.

Lemma A.2. *Let X be a compact metric space and a complete lattice, and let $f : X \rightarrow X$ be non-decreasing and continuous. Denote by \bar{m} the least upper bound of X and by \bar{x} the greatest fixed point of f . Then*

$$f^{(n)}(\bar{m}) \xrightarrow{n} \bar{x}.$$

Proof. By definition of \bar{m} and \bar{x} , as f is non-decreasing,

$$\bar{x} = f(\bar{x}) \leq f(\bar{m}).$$

By iterated application of f (which is non-decreasing) to the first and last term of the previous chain of inequalities,

$$\bar{x} \leq f^{(n)}(\bar{m}). \tag{A.1}$$

Since X is compact, we can assume without loss of generality that the sequence $(f^{(n)}(\bar{m}))_n$ converges to a limit x^* . By continuity of f ,

$$x^* = \lim_n f^{(n)}(\bar{m}) = f\left(\lim_n f^{(n-1)}(\bar{m})\right) = f(x^*),$$

i.e. x^* is a fixed point of f . Taking limits on both sides of (A.1), we obtain $\bar{x} \leq x^*$, from which $\bar{x} = x^*$, by definition of \bar{x} . ■

Lemma A.3. *Let X be a complete lattice and $f, g : X \rightarrow X$. If f is non-decreasing and $f \leq g$ on X , then*

$$f^{(n)}(x) \leq g^{(n)}(x), \quad \forall x \in X, n \in \mathbb{N} \setminus \{0\}.$$

Proof. We prove the result by induction. If $n = 1$, then $f^{(n)}(x) \leq g^{(n)}(x)$ by hypothesis. Assume that the previous inequality holds for an arbitrary $n \geq 1$. Then, for all $x \in X$,

$$f^{(n+1)}(x) = f(f^{(n)}(x)) \leq f(g^{(n)}(x)) \leq g(g^{(n)}(x)) = g^{(n+1)}(x),$$

where the two inequalities follow from the monotonicity of f and the hypothesis that $f \leq g$, respectively. ■

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- F. Busetti and J. MARCUCCI, *Comparing forecast accuracy: a Monte Carlo investigation*, International Journal of Forecasting, v. 29, 1, pp. 13-27, **TD No. 723 (September 2009)**.
- D. DOTTORI, S. I-LING and F. ESTEVAN, *Reshaping the schooling system: The role of immigration*, Journal of Economic Theory, v. 148, 5, pp. 2124-2149, **TD No. 726 (October 2009)**.
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- F. NUCCI and M. RIGGI, *Performance pay and changes in U.S. labor market dynamics*, Journal of Economic Dynamics and Control, v. 37, 12, pp. 2796-2813, **TD No. 800 (March 2011)**.
- G. CAPPELLETTI, G. GUAZZAROTTI and P. TOMMASINO, *What determines annuity demand at retirement?*, The Geneva Papers on Risk and Insurance – Issues and Practice, pp. 1-26, **TD No. 805 (April 2011)**.
- A. ACCETTURO e L. INFANTE, *Skills or Culture? An analysis of the decision to work by immigrant women in Italy*, IZA Journal of Migration, v. 2, 2, pp. 1-21, **TD No. 815 (July 2011)**.
- A. DE SOCIO, *Squeezing liquidity in a "lemons market" or asking liquidity "on tap"*, Journal of Banking and Finance, v. 27, 5, pp. 1340-1358, **TD No. 819 (September 2011)**.
- S. GOMES, P. JACQUINOT, M. MOHR and M. PISANI, *Structural reforms and macroeconomic performance in the euro area countries: a model-based assessment*, International Finance, v. 16, 1, pp. 23-44, **TD No. 830 (October 2011)**.
- G. BARONE and G. DE BLASIO, *Electoral rules and voter turnout*, International Review of Law and Economics, v. 36, 1, pp. 25-35, **TD No. 833 (November 2011)**.
- O. BLANCHARD and M. RIGGI, *Why are the 2000s so different from the 1970s? A structural interpretation of changes in the macroeconomic effects of oil prices*, Journal of the European Economic Association, v. 11, 5, pp. 1032-1052, **TD No. 835 (November 2011)**.
- R. CRISTADORO and D. MARCONI, *Household savings in China*, in G. Gomel, D. Marconi, I. Musu, B. Quintieri (eds), *The Chinese Economy: Recent Trends and Policy Issues*, Springer-Verlag, Berlin, **TD No. 838 (November 2011)**.
- A. ANZUINI, M. J. LOMBARDI and P. PAGANO, *The impact of monetary policy shocks on commodity prices*, International Journal of Central Banking, v. 9, 3, pp. 119-144, **TD No. 851 (February 2012)**.
- R. GAMBACORTA and M. IANNARIO, *Measuring job satisfaction with CUB models*, Labour, v. 27, 2, pp. 198-224, **TD No. 852 (February 2012)**.
- G. ASCARI and T. ROPELE, *Disinflation effects in a medium-scale new keynesian model: money supply rule versus interest rate rule*, European Economic Review, v. 61, pp. 77-100, **TD No. 867 (April 2012)**.
- E. BERETTA and S. DEL PRETE, *Banking consolidation and bank-firm credit relationships: the role of geographical features and relationship characteristics*, Review of Economics and Institutions, v. 4, 3, pp. 1-46, **TD No. 901 (February 2013)**.
- M. ANDINI, G. DE BLASIO, G. DURANTON and W. STRANGE, *Marshallian labor market pooling: evidence from Italy*, Regional Science and Urban Economics, v. 43, 6, pp.1008-1022, **TD No. 922 (July 2013)**.
- G. SBRANA and A. SILVESTRINI, *Forecasting aggregate demand: analytical comparison of top-down and bottom-up approaches in a multivariate exponential smoothing framework*, International Journal of Production Economics, v. 146, 1, pp. 185-98, **TD No. 929 (September 2013)**.
- A. FILIPPIN, C. V. FIORIO and E. VIVIANO, *The effect of tax enforcement on tax morale*, European Journal of Political Economy, v. 32, pp. 320-331, **TD No. 937 (October 2013)**.

- G. M. TOMAT, *Revisiting poverty and welfare dominance*, *Economia pubblica*, v. 44, 2, 125-149, **TD No. 651 (December 2007)**.
- M. TABOGA, *The riskiness of corporate bonds*, *Journal of Money, Credit and Banking*, v.46, 4, pp. 693-713, **TD No. 730 (October 2009)**.
- G. MICUCCI and P. ROSSI, *Il ruolo delle tecnologie di prestito nella ristrutturazione dei debiti delle imprese in crisi*, in A. Zazzaro (a cura di), *Le banche e il credito alle imprese durante la crisi*, Bologna, Il Mulino, **TD No. 763 (June 2010)**.
- F. D'AMURI, *Gli effetti della legge 133/2008 sulle assenze per malattia nel settore pubblico*, *Rivista di politica economica*, v. 105, 1, pp. 301-321, **TD No. 787 (January 2011)**.
- R. BRONZINI and E. IACHINI, *Are incentives for R&D effective? Evidence from a regression discontinuity approach*, *American Economic Journal : Economic Policy*, v. 6, 4, pp. 100-134, **TD No. 791 (February 2011)**.
- P. ANGELINI, S. NERI and F. PANETTA, *The interaction between capital requirements and monetary policy*, *Journal of Money, Credit and Banking*, v. 46, 6, pp. 1073-1112, **TD No. 801 (March 2011)**.
- M. BRAGA, M. PACCAGNELLA and M. PELLIZZARI, *Evaluating students' evaluations of professors*, *Economics of Education Review*, v. 41, pp. 71-88, **TD No. 825 (October 2011)**.
- M. FRANCESE and R. MARZIA, *Is there Room for containing healthcare costs? An analysis of regional spending differentials in Italy*, *The European Journal of Health Economics*, v. 15, 2, pp. 117-132, **TD No. 828 (October 2011)**.
- L. GAMBACORTA and P. E. MISTRULLI, *Bank heterogeneity and interest rate setting: what lessons have we learned since Lehman Brothers?*, *Journal of Money, Credit and Banking*, v. 46, 4, pp. 753-778, **TD No. 829 (October 2011)**.
- M. PERICOLI, *Real term structure and inflation compensation in the euro area*, *International Journal of Central Banking*, v. 10, 1, pp. 1-42, **TD No. 841 (January 2012)**.
- E. GENNARI and G. MESSINA, *How sticky are local expenditures in Italy? Assessing the relevance of the flypaper effect through municipal data*, *International Tax and Public Finance*, v. 21, 2, pp. 324-344, **TD No. 844 (January 2012)**.
- V. DI GACINTO, M. GOMELLINI, G. MICUCCI and M. PAGNINI, *Mapping local productivity advantages in Italy: industrial districts, cities or both?*, *Journal of Economic Geography*, v. 14, pp. 365-394, **TD No. 850 (January 2012)**.
- A. ACCETTURO, F. MANARESI, S. MOCETTI and E. OLIVIERI, *Don't Stand so close to me: the urban impact of immigration*, *Regional Science and Urban Economics*, v. 45, pp. 45-56, **TD No. 866 (April 2012)**.
- M. PORQUEDDU and F. VENDITTI, *Do food commodity prices have asymmetric effects on euro area inflation*, *Studies in Nonlinear Dynamics and Econometrics*, v. 18, 4, pp. 419-443, **TD No. 878 (September 2012)**.
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- F. D'AMURI and G. PERI, *Immigration, jobs and employment protection: evidence from Europe before and during the Great Recession*, *Journal of the European Economic Association*, v. 12, 2, pp. 432-464, **TD No. 886 (October 2012)**.
- M. TABOGA, *What is a prime bank? A euribor-OIS spread perspective*, *International Finance*, v. 17, 1, pp. 51-75, **TD No. 895 (January 2013)**.
- G. CANNONE and D. FANTINO, *Evaluating the efficacy of european regional funds for R&D*, *Rassegna italiana di valutazione*, v. 58, pp. 165-196, **TD No. 902 (February 2013)**.
- L. GAMBACORTA and F. M. SIGNORETTI, *Should monetary policy lean against the wind? An analysis based on a DSGE model with banking*, *Journal of Economic Dynamics and Control*, v. 43, pp. 146-74, **TD No. 921 (July 2013)**.
- M. BARIGOZZI, CONTI A.M. and M. LUCIANI, *Do euro area countries respond asymmetrically to the common monetary policy?*, *Oxford Bulletin of Economics and Statistics*, v. 76, 5, pp. 693-714, **TD No. 923 (July 2013)**.
- U. ALBERTAZZI and M. BOTTERO, *Foreign bank lending: evidence from the global financial crisis*, *Journal of International Economics*, v. 92, 1, pp. 22-35, **TD No. 926 (July 2013)**.

- R. DE BONIS and A. SILVESTRINI, *The Italian financial cycle: 1861-2011*, *Cliometrica*, v.8, 3, pp. 301-334, **TD No. 936 (October 2013)**.
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- D. PIANESELLI and A. ZAGHINI, *The cost of firms' debt financing and the global financial crisis*, *Finance Research Letters*, v. 11, 2, pp. 74-83, **TD No. 950 (February 2014)**.
- J. LI and G. ZINNA, *On bank credit risk: systemic or bank-specific? Evidence from the US and UK*, *Journal of Financial and Quantitative Analysis*, v. 49, 5/6, pp. 1403-1442, **TD No. 951 (February 2015)**.
- A. ZAGHINI, *Bank bonds: size, systemic relevance and the sovereign*, *International Finance*, v. 17, 2, pp. 161-183, **TD No. 966 (July 2014)**.
- G. SBRANA and A. SILVESTRINI, *Random switching exponential smoothing and inventory forecasting*, *International Journal of Production Economics*, v. 156, 1, pp. 283-294, **TD No. 971 (October 2014)**.
- M. SILVIA, *Does issuing equity help R&D activity? Evidence from unlisted Italian high-tech manufacturing firms*, *Economics of Innovation and New Technology*, v. 23, 8, pp. 825-854, **TD No. 978 (October 2014)**.

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- M. BUGAMELLI, S. FABIANI and E. SETTE, *The age of the dragon: the effect of imports from China on firm-level prices*, *Journal of Money, Credit and Banking*, v. 47, 6, pp. 1091-1118, **TD No. 737 (January 2010)**.
- R. BRONZINI, *The effects of extensive and intensive margins of FDI on domestic employment: microeconomic evidence from Italy*, *B.E. Journal of Economic Analysis & Policy*, v. 15, 4, pp. 2079-2109, **TD No. 769 (July 2010)**.
- G. BULLIGAN, M. MARCELLINO and F. VENDITTI, *Forecasting economic activity with targeted predictors*, *International Journal of Forecasting*, v. 31, 1, pp. 188-206, **TD No. 847 (February 2012)**.
- A. CIARLONE, *House price cycles in emerging economies*, *Studies in Economics and Finance*, v. 32, 1, **TD No. 863 (May 2012)**.
- D. FANTINO, A. MORI and D. SCALISE, *Collaboration between firms and universities in Italy: the role of a firm's proximity to top-rated departments*, *Rivista Italiana degli economisti*, v. 1, 2, pp. 219-251, **TD No. 884 (October 2012)**.
- D. DEPALO, R. GIORDANO and E. PAPAPETROU, *Public-private wage differentials in euro area countries: evidence from quantile decomposition analysis*, *Empirical Economics*, v. 49, 3, pp. 985-1115, **TD No. 907 (April 2013)**.
- G. BARONE and G. NARCISO, *Organized crime and business subsidies: Where does the money go?*, *Journal of Urban Economics*, v. 86, pp. 98-110, **TD No. 916 (June 2013)**.
- P. ALESSANDRI and B. NELSON, *Simple banking: profitability and the yield curve*, *Journal of Money, Credit and Banking*, v. 47, 1, pp. 143-175, **TD No. 945 (January 2014)**.
- M. TANELI and B. OHL, *Information acquisition and learning from prices over the business cycle*, *Journal of Economic Theory*, 158 B, pp. 585-633, **TD No. 946 (January 2014)**.
- R. AABERGE and A. BRANDOLINI, *Multidimensional poverty and inequality*, in A. B. Atkinson and F. Bourguignon (eds.), *Handbook of Income Distribution*, Volume 2A, Amsterdam, Elsevier, **TD No. 976 (October 2014)**.
- V. CUCINIELLO and F. M. SIGNORETTI, *Large banks, loan rate markup and monetary policy*, *International Journal of Central Banking*, v. 11, 3, pp. 141-177, **TD No. 987 (November 2014)**.
- M. FRATZSCHER, D. RIMEC, L. SARNOB and G. ZINNA, *The scapegoat theory of exchange rates: the first tests*, *Journal of Monetary Economics*, v. 70, 1, pp. 1-21, **TD No. 991 (November 2014)**.
- A. NOTARPIETRO and S. SIVIERO, *Optimal monetary policy rules and house prices: the role of financial frictions*, *Journal of Money, Credit and Banking*, v. 47, S1, pp. 383-410, **TD No. 993 (November 2014)**.
- R. ANTONIETTI, R. BRONZINI and G. CAINELLI, *Inward greenfield FDI and innovation*, *Economia e Politica Industriale*, v. 42, 1, pp. 93-116, **TD No. 1006 (March 2015)**.
- T. CESARONI, *Procyclicality of credit rating systems: how to manage it*, *Journal of Economics and Business*, v. 82, pp. 62-83, **TD No. 1034 (October 2015)**.
- M. RIGGI and F. VENDITTI, *The time varying effect of oil price shocks on euro-area exports*, *Journal of Economic Dynamics and Control*, v. 59, pp. 75-94, **TD No. 1035 (October 2015)**.

FORTHCOMING

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- R. BRONZINI and P. PISELLI, *The impact of R&D subsidies on firm innovation*, *Research Policy*, **TD No. 960 (April 2014)**.
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- L. BURLON and M. VILALTA-BUFI, *A new look at technical progress and early retirement*, *IZA Journal of Labor Policy*, **TD No. 963 (June 2014)**.
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- A. CALZA and A. ZAGHINI, *Shoe-leather costs in the euro area and the foreign demand for euro banknotes*, *International Journal of Central Banking*, **TD No. 1039 (December 2015)**.