Temi di Discussione

(Working Papers)

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and productivity crises

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SOVEREIGN DEBT AND RESERVES
WITH LIQUIDITY AND PRODUCTIVITY CRISES

by Flavia Corneli* and Emanuele Tarantino**

Abstract

During the recent financial crisis developing countries have continued to accumulate both sovereign reserves and debt. To account for this empirical fact, we model the optimal portfolio choice of a country that is subject to liquidity and productivity shocks. We determine the equilibrium level of debt and its cost through a contracting game between a country and international lenders. Although raising debt increases the sovereign exposure to liquidity and productivity crises, the simultaneous accumulation of reserves can mitigate the negative effects of such crises. This mechanism rationalizes the complementarity between debt and reserves.

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1 Introduction[1]

During the recent financial crisis several developing countries have kept accumulating both sovereign debt and reserves. These include Chile, Colombia, Malaysia, South Africa and Turkey, among others (Figure 1). The return on reserves is significantly lower than the cost of sovereign debt (Rodrik, 2006), thus this piece of evidence has posed a puzzle (Alfaro and Kanczuk, 2009; Obstfeld et al., 2010): why don't countries repay debt instead of raising reserves? We develop a static model of optimal portfolio choice of a country that is subject to liquidity and productivity shocks. We find that, although raising debt increases the country's exposure to liquidity and productivity crises, the contemporaneous accumulation of reserves mitigates the consequences of these crises on sovereign welfare. We therefore show that, in the sovereign's optimal strategy, reserves can complement debt.
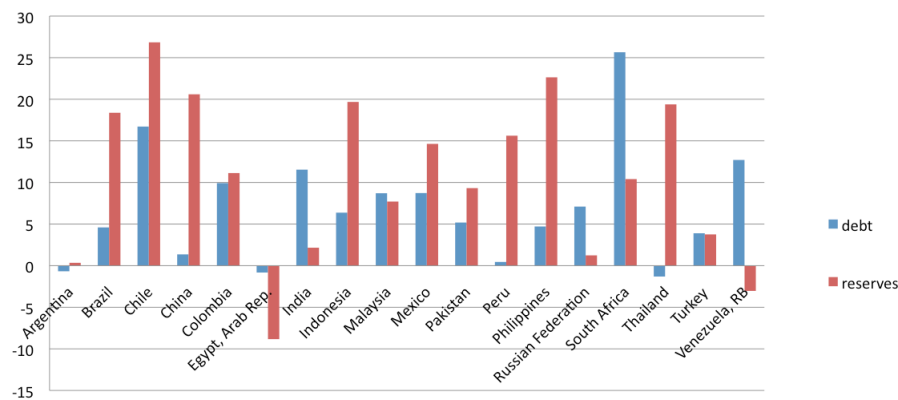

Figure 1: External Debt and Reserve Accumulation — Average % Variation 2008-2011. Source: World Bank WDI.

In principle, reserves and debt can be substitute means to address liquidity crises. By decreasing its level of debt, a country reduces its exposure to liquidity shocks. By accumulating reserves, the country holds resources that can be liquidated and injected if a liquidity shock occurs. Yet, Rodrik (2006) and Mohd Daud and Podivinsky (2011) show that the accumulation of reserves is not accompanied by a reduction of debt. Moreover, Dominguez et al. (2012), Bussière et al. (2014), and Broner et al. (2013a), among others, document that during the recent crisis countries depleted their sovereign reserves and then rapidly replenished them. This evidence suggests that reserves and debt can be in a relationship of complementarity.

We analyze the problem of sovereign reserves and debt accumulation in a model with liquidity and productivity shocks that draws on Bolton and Jeanne (2007). A risk-neutral country borrows

from international lenders and decides on the investment of these resources into a liquid (reserves) and an illiquid (production technology) asset. Debt and reserves are chosen by the country, instead international lenders set the interest rate on debt. The value of sovereign output depends on the productivity of the country's technology and the share of borrowed resources that the sovereign invests in public expenditure; that is, those resources that are not invested in reserves. The interaction between the sovereign and international lenders is shaped by two frictions. First, the country cannot commit not to default on debt (Aguiar and Amador, 2015). Second, the country does not have access to additional borrowing in the event of a liquidity shock.[2] By these two frictions, we introduce asset incompleteness in our model.[3]

A simple trade-off determines the equilibrium value of reserves. On the one hand, they distract resources from the production technology. On the other hand, they can be injected in the event of a liquidity crisis to avoid default. Moreover, consistently with the literature and prevalent anecdotal evidence, they cannot be seized by investors should the country decide to default. The trade-off behind the accumulation of debt is as follows. By increasing its borrowing the country holds more resources for investment. However, issuing debt raises the country's exposure to liquidity and productivity crises. Indeed, debt increases both the likelihood that the country is hit by a liquidity shock and the probability that the due repayment to lenders falls short of realized output, thereby rendering more likely a default on the productivity shock.

To establish the relationship between reserves and debt, we solve for two distinct set-ups. First, we consider a benchmark economy without liquidity shock and characterize the conditions determining the equilibrium value of debt and reserves. In this environment, the motive for reserve accumulation is only related to the country's genuine interest in maximizing the return of its portfolio. Then, we introduce the possibility that a liquidity shock hits the country. Following Cole and Kehoe (2000), this shock occurs with a probability that increases in the level of sovereign debt. Moreover, the country can only use its reserves to avoid default following the liquidity shock. Thus, we ask: will the country borrow less than in the benchmark economy, so to reduce the likelihood that the liquidity shock occurs? Alternatively, will the country borrow more and at the same time accumulate reserves to use them should the liquidity shock occur?

In our model of portfolio allocation, a relationship of complementarity arises at equilibrium between debt and reserves: an increase in the value of debt spurs the country to raise its reserve holdings. The country increases its borrowing so to have additional resources for investment. Although a larger stock of debt renders the occurrence of the liquidity shock and the default on the productivity shock more likely, the accumulation of reserves allows the country to mitigate the consequences of these shocks on country's welfare. At equilibrium, lenders respond to an

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[2]In an extension, we relax this assumption and show that our main results remain the same.

[3]Asset incompleteness is crucial to disentangle the relationship between the sovereign decision to default and the cost of sovereign debt (Arellano, 2008): with noncontingent assets, risk-neutral competitive lenders incorporate the probability of default in the premium set on the debt contract.

increase in the probability of default caused by larger sovereign debt and reserves by raising the cost of debt.

We conduct numerical simulations that illustrate our theoretical finding that debt and reserve are complements to the country. We show that, using levels of the parameters taken from the literature and consistent with empirical evidence, when subject to the risk of a liquidity shock it can be optimal for the country to increase, at the same time, its stocks of debt and reserves with respect to a benchmark economy without liquidity shock. What are the implications of these results for country's output? We find that, at equilibrium, a higher level of reserves is associated with higher expected output. This outcome is consistent with Benigno and Fornaro (2012) and Dominguez et al. (2012), who find a positive relationship between reserve accumulation and GDP growth. In these numerical exercises, we obtain values of the variables of interest that are close to those featured by emerging and developing economies. In particular, reserves amount to about 30% of expected GDP, slightly above 26%, the IMF estimate for emerging and developing countries for 2011. Moreover, the equilibrium level of debt is equal to about 55% of GDP, somewhat larger than the IMF estimate of about 36%. Finally, the equilibrium spread set by international lenders is slightly below the estimate of 5.44% for EMBI in Borri and Verdelhan (2011), and close to the 9-year excess return of 4.6% obtained by Broner et al. (2013b).

We extend our theoretical model by relaxing the assumption that foreign lenders cannot supply additional funds in the event of a liquidity shock. There, we allow the country to issue more debt when hit by a liquidity shock. We find that an equilibrium exists in which the country issues high-return debt and uses those resources to overcome the crisis; moreover, we show that the complementarity between debt and reserves is preserved at equilibrium.

These results offer new insights to the literature that studies the relationship between sovereign reserves and debt. In our setup as in the extant papers, there may be circumstances where the reduction in debt and the accumulation of reserves are substitute means to deal with the prospect of a liquidity crisis. The country might reduce the amount of external debt and, thus, its exposure to a liquidity crisis. Or, it could increase the amount of reserves to inject the needed resources when the shock occurs. We show that, at the equilibrium of a model of sovereign portfolio allocation, reserves and debt can be in a relationship of complementarity: by raising both reserves and debt the country can, at the same time, increase its total resources for investment in the production technology and use its liquidity in the event of a crisis. The novel element of our analysis compared to previous studies is that reserves play a dual role. They act as a consumption smoothing asset when the sovereign decides to default, thereby offering post-default resources for consumption (as in, e.g., Alfaro and Kanczuk, 2009; Bianchi et al. 2013; Jeanne and Rancière, 2011; among others). Moreover, they can be directly used in order to avoid default, should the liquidity shock occur.

Our model is related to Alfaro and Kanczuk (2009), who study the interaction between debt

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issuance and reserve holdings for a country that can be hit by sudden stops. They propose a small open endowment economy model to show that a standard quantitative setup cannot account for the significant amount of reserves that countries hold. Building on Alfaro and Kanczuk (2009), Bianchi et al. (2013) allow the country to choose the maturity of its debt structure and find that, in equilibrium, the country decides to hold a positive amount of reserves since they represent an insurance not only against default, but also against future increases in the borrowing cost. Our work complements the one of Bianchi et al. (2013) in that we abstract from the analysis of optimal debt maturity and sovereign risk aversion and focus instead on a risk-neutral country's investment decision, in which reserves also represents an alternative and imperfect substitute for output.

Moreover, the present work is related to Aizenman and Marion (2004). In a two-period model, they study the relationship between debt and reserves and find that the optimal borrowing choice increases with reserves. They then introduce political uncertainty and find that political instability and corruption reduce the optimal size of reserves and increase the one of external debt, altering the relationship between reserves and debt. They however abstract from the role of reserves in the event of sudden stops and the analysis of the relationship between reserves and the cost of debt.

We also contribute to two other strands of the literature. The first one studies the optimal contractual arrangements in the presence of commitment problems and non-contingent contracts. In analogy to Aguiar and Gopinath (2006), Arellano (2008) and Yue (2010), we show that default arises at equilibrium after an adverse shock occurs, consistently with prevalent empirical evidence. However, while these papers study the relationship between default risk and output, consumption and foreign debt, we look at the interaction between default risk, sovereign debt and reserves, and the implications of this relationship for output and the cost of debt.

The second related strand studies the role of sovereign reserves as a buffer against liquidity shocks.[4] Jeanne and Rancière (2011) model reserves as an insurance contract that allows countries to smooth the absorption of the output costs caused by a capital outflow. They find a closed form expression for the optimal amount of sovereign reserves that is able to account for the average level of reserves accumulated by emerging economies since 1980. Differently from our setup, in their model debt level is exogenous and the country is risk-averse. We show that even a risk neutral country can decide to hold positive amounts of reserves at equilibrium for reasons related to the management of its portfolio.[5]

In the next section, we present our main model. In Section 3, we discuss the equilibrium

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[4]Obstfeld et al. (2010) and Calvo et al. (2012) provide evidence supporting this motive behind reserve accumulation. See Bussière et al., 2014, for a survey on this subject.

[5]See Carroll and Jeanne (2009) and Guerrieri and Lorenzoni (2011) for models of precautionary savings. Extending our model to account for country's risk-aversion would most likely lead to an even higher equilibrium level of reserves, all the rest being equal (Corneli and Tarantino, 2011).

analysis and illustrate the qualitative features of the model's equilibrium. In Section 4, we perform numerical simulations of our theoretical setting and, in Section 5, we analyze an extension in which foreign lenders can intervene with fresh capital in the event of a liquidity crisis. Section 6 concludes.

2 The model

Consider a sovereign country with access to foreign borrowing. At stage zero, the country decides the the face valued of debt D it wants to borrow from foreign lenders and its allocation. The lenders set the cost of D. At stage one, if the country is hit by a liquidity shock it decides whether to default or inject the resources needed to redeem the shock and bring the production technology to completion; otherwise the game proceeds to stage three. At stage three, the productivity shock realizes and at stage four the sovereign can again choose whether to default.

Figures 2 illustrates the timing of the game.[6] We solve the model by backward induction and the equilibrium concept we employ is the Sub-game Perfect Nash Equilibrium (SPNE) in pure strategies. In what follows, we present in detail how we model each relevant node and the main ingredients of the game.

$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
Lending Game δ	Liquidity Shock $\eta(D), E$	Default Decision	Productivity Shock $z \sim F(z)$	Default Decision
Investment Game D, R				
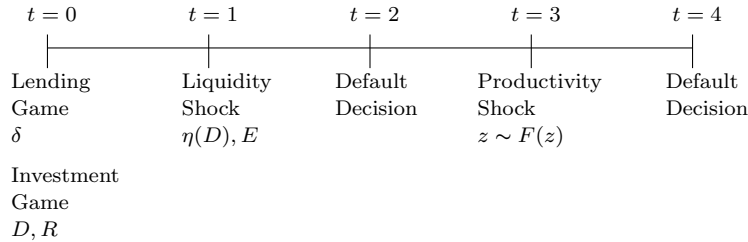
Figure 2: Timeline

2.1 The lending game

Our world is populated by a continuum of atomistic and risk-neutral foreign lenders (indexed by $i \in I$). The total mass of lenders is large, ensuring that perfect competition prevails and lenders do not extract any rent; also, lenders have unlimited access to funds at the riskless interest rate (that, for simplicity, we assume to be zero).

The sovereign borrows from a subset of mass 1 of lenders. As in Bolton and Jeanne (2007), lenders participate in a bidding game following the sovereign's announcement of a fund-raising goal D, which represents the face value of country's debt. Lenders move first by each simultaneously submitting a bid. The sovereign then decides which bid(s) to accept.

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[6]Figure 9 illustrates the full game-tree.

The lenders' payoff is equal to the value of the repayment D discounted by the probability that the sovereign repays in full. At the bidding stage of the game each lender i makes an offer on the rate of return, $r(i)$, that insures break even. That is, lender i solves a problem of the following sort:

$$
\begin{aligned}
D &= D(1 + r(i))Prob\{\text{The Country is Solvent}\} \\
1/(1 + r(i)) &= Prob\{\text{The Country is Solvent}\} \\
\delta(i) &= Prob\{\text{The Country is Solvent}\}.
\end{aligned}
$$

Thus, throughout the model we will denote by $\delta(i) = 1/(1 + r(i)) \in (0, 1)$ the inverse of the rate of return $r(i)$ that lender i asks in exchange for a loan D. Accordingly, the Nash equilibrium of the lending game is defined by a set of bids $(\delta(i))_{i \in I}$ such that, for all i, bid $\delta(i)$ maximizes the lender i's payoff taking all the other bids $\delta(j)$, with $j \neq i$, as given. At equilibrium the sovereign squeezes all the surplus from the lending relationship and (randomly) selects among a set of identical bids, $\delta(j) = \delta(i) = \delta$, so we can focus on a representative sovereign-lender pair.

2.2 The investment game and the productivity shock

The country decides the amount of debt to borrow from the lender (D) and how to invest these funds in public expenditure, g, and/or reserves, R. Specifically, the sovereign budget constraint is given by

$$
\delta D = g + R \iff g = \delta D - R. \tag{1}
$$

This implies that, once D and δ are determined, the country's decision on the value of R pins down the resources invested in g. The country is risk-neutral. It allocates its resources by maximizing the expected value its portfolio, $E(W)$, which corresponds to the sovereign expected welfare. $E(W)$ is given by the amount of liquid resources, R, and the expected value of the production technology, Y, after repaying debt, D.

Reserves yield the risk-less interest rate and act as a safe and liquid technology that can be carried over from stage zero to the end of the game. The production technology is the risky and illiquid asset: it requires capital, that is public expenditure (g), as sole input and is subject to a productivity shock, z. The receipts generated by the production technology materialize only at stage four, after the country has decided on the allocation of its resources and the uncertainty over the productivity shock resolves.

The production function that determines the value of the illiquid asset, Y, is linearly affected

by the productivity shock. Thus, using (1):

$$Y(z, g) = zY(g) = zY(\delta, D, R),$$

where z is a random variable drawn from a continuous density function $f(z)$, and $F(z)$ is the cdf induced by $f(z)$. Throughout the paper we assume that z follows a continuous uniform distribution with positive density over $[1-c, 1+c]$. The function $Y(\delta, D, R)$ is twice differentiable, with $Y' \geq 0$ and $Y'' < 0$, is increasing in δ and D (as $\partial Y/\partial \delta = DY' \geq 0$ and $\partial Y/\partial D = \delta Y' \geq 0$), decreasing in R ($\partial Y/\partial R = -Y' \leq 0$) and satisfies standard Inada Conditions ($\lim_{g \to 0} Y = 0$, $\lim_{g \to 0} Y' = \infty$ and $\lim_{g \to \infty} Y' = 0$).

2.3 The liquidity shock

The country is exposed to the risk that a liquidity crisis occurs at stage one, before the production technology's receipts materialize. Following Chang and Velasco (2000), the liquidity shock hits the country with probability $\eta(D) \in (0, 1)$, and its consequences are that the production technology needs a further injection of resources $E \in [0, \delta D]$ at stage two in order to be completed, with $\lim_{D \to 0} \eta(D) = 0$, $\lim_{D \to \bar{D}} \eta(D) = 1$, and $\eta', \eta'' > 0$ for all $D \in [0, \bar{D}]$.

This formalization captures, in a reduced form, the definition of sudden stop as a large and abrupt decline of capital inflows (as in, e.g., Caballero and Panageas, 2007).[7] The liquidity shock of our model can be interpreted as a withdrawal of private capital that causes disruptions in the production technology unless additional resources are injected. Rodrik (2006) and Obstfeld et al. (2010) document that to overcome capital outflows countries need to sink resources for about 10% of their GDP. Moreover, assuming that the probability of the shock increases with debt is consistent with the evidence on the negative relationship between indebtedness and the likelihood of a liquidity crisis (Cole and Kehoe, 2000).

In the main model, if the liquidity shock occurs the country can only use its reserves to inject E, while it cannot dismantle the capital invested in the production technology or borrow additional resources from lenders. This is consistent with Rodrik (2006), who finds that for countries with larger holdings of reserves it is easier to face the consequences of capital outflows. Conversely, if it decides not to tackle the shock, the country defaults on the production technology and retains reserves. In Section 5, we will relax this assumption and look at the case in which the country can borrow E from financial markets to intervene in case of a liquidity crisis.

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[7]We would obtain analogous results by assuming that an exogenous fraction of debt D needs to be reimbursed at stage two.

2.4 The decision to default

There are two stages at which the country may choose to default. The first is after the realization of the liquidity shock (stage two). The second is after the realization of the productivity shock, when uncertainty over the value of output realizes (stage four). The main cost of default is that the country loses the entire value of the production technology (as in, e.g., Bolton and Jeanne, 2007).[8] Instead, even after defaulting the country keeps its reserves (e.g., Aizenman and Marion, 2004; Alfaro and Kanczuk 2009). In what follows, we discuss the determinants of the sovereign decision to default.

The country defaults on the productivity shock whenever the revenue generated by the production technology is lower than the face value of debt, or $zY \leq D$. Hence, at equilibrium this decision depends on the realized level of the productivity shock z, but also on the outcome of the investment and lending games.

To analyze the choice to default after the liquidity shock, throughout the paper we distinguish between two sub-games. If the country plays sub-game N, it chooses to default if hit by the liquidity shock. Instead, if it plays sub-game F, the country chooses to inject the needed resources (E) in the event of a shock. We find this a useful distinction because it allows us to easily characterize all possible branches of the game-tree.[9] Country's decision to play sub-game F must be credible, that is, it must be feasible and time-consistent. Feasibility requires that at the investment game the country accumulates a level of reserves larger than E (resource constraint). Time consistency requires that the decision not to default is sub-game perfect: if the country's payoff from the continuation-game following the shock is less than E then it will prefer to default on the shock and keep its reserves (time-consistency constraint).

3 Model solution

In order to disentangle the role of reserves in our framework, we first solve the model in absence of the liquidity shock. Then, we reintroduce the liquidity shock and study whether the country injects the needed resources should the shock happen. All formal proofs are relegated to the appendix.

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[8]See Alfaro and Kanczuk (2005) for a discussion on output losses in the event of default and Gennaioli et al. (2014) for a model of costly sovereign defaults.

[9]Figure 10 illustrates the branches of the game-tree corresponding to the two sub-games F and N.

3.1 Benchmark economy without liquidity shock

The country defaults at the final stage if:

$$zY(\delta, D, R) - D + R \leq R \iff z \leq \bar{z}(\delta, D, R) \equiv \frac{D}{Y(\delta, D, R)}. \tag{2}$$

The value of $\bar{z}$ in (2) corresponds to the threshold of the productivity shock above which the country repays its creditors and keeps the residual. Instead, if z falls below $\bar{z}$, the country defaults on debt and loses the entire value of output.

At stage zero, the country decides on the value of D and how much of δD it invests in reserves R. Specifically, it solves the following maximization problem:

$$\max_{D, R \in [0, \delta D]} E(W(\delta, D, R)) = R + \int_{\bar{z}(\delta, D, R)}^{\infty} (zY(\delta, D, R) - D)dF(z), \tag{3}$$

under the condition that the value of R is feasible, that is, $R \in [0, \delta D]$. In turn the lender sets the optimal value of δ to break-even in expectation:

$$\delta D = D \int_{\bar{z}(\delta, D, R)}^{\infty} dF(z) \iff \delta = 1 - F(\bar{z}(\delta, D, R)). \tag{4}$$

Proposition 1 characterizes the equilibrium results in the benchmark without liquidity shock.

PROPOSITION 1. *The sovereign country chooses D^* and R^* in the viable range to solve the following first-order conditions*

$$\int_{\bar{z}(\delta, D, R)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) = 0 \tag{5}$$

$$1 - Y'(\delta, D, R) \int_{\bar{z}(\delta, R, D)}^{\infty} zdF(z) = 0. \tag{6}$$

Given the vector of equilibrium values of debt and reserves that solve (5) and (6), it exists a unique equilibrium discount factor $\delta^* \in (0, 1)$ that solves the lender zero-profit condition.

The first-order conditions in Proposition 1 have a natural interpretation. Specifically, in expression (5) the equilibrium value of debt solves the trade-off between the increase in the expected marginal return of the production technology due to an additional unit of debt (equal to $\delta Y' \geq 0$) and the marginal cost in terms of a higher repayment due at maturity (equal to $1 - F(\bar{z})$). Expression (6) determines the country's investment in reserves: the equilibrium value of R equates their marginal rate of return (equal to 1) to the marginal rate of return of the production technology. This second term is given by the marginal productivity of capital (Y')

discounted by the probability that the realization of the productivity shock is larger than $\bar{z}$, because otherwise the country defaults and loses the entire value of output.

In Section 4, we carry out numerical simulations using a Cobb-Douglas production function. In that context, two parameters influence the equilibrium value of reserves resulting from (6): the income share of capital and the variability of the production technology. All else being equal, an increase in the income share of capital increases the marginal rate of return of the production technology, thereby decreasing the amount of resources invested in reserves. Analogously, the marginal rate of return of the production technology increases if the support of the production technology widens. The reason is that the country disregards the lower tail of the distribution of z. This again implies that the reserves the country accumulates should decrease, as they become relatively less valuable.

In the appendix, we give the conditions for the unicity of the equilibrium of the sovereign investment game as resulting from conditions (5) and (6), and we check that they are satisfied in our numerical simulations. Finally, the deal signed at the initial stage between the lender and the sovereign can be implemented by a debt contract in which the country receives $\delta^* D^*$ at stage zero against the promise to repay D^* at stage four. Hence, the lender earns $(D^* - \delta^* D^*)$ (or, equivalently, $D^* r^*/(1 + r^*)$, with $r^*/(1 + r^*) = 1 - \delta^*$) provided the country repays in full, zero otherwise. In the following section, we will introduce the liquidity shock in our model's economy and study how it shapes sovereign borrowing and investment decisions.

3.2   Economy with liquidity shock

In this section, a liquidity shock can hit the country at stage one. We study whether, relative to the benchmark economy in Section 3.1, the country reduces the amount of accumulated debt, so as to lower the probability that the liquidity shock occurs, or it increases the amount of reserves, so as to alleviate the consequences of the shock on its welfare.

If the liquidity crisis does not occur at stage one, at stage four the country defaults after the realization of the productivity shock if $z \leq \bar{z}$, where $\bar{z}$ is as defined in (2). Conversely, if the liquidity shock occurs at stage one, we distinguish between two cases, depending on whether the country injects (sub-game F) or not (sub-game N) the resources needed to overcome the shock (E).

Sub-game N At stage two the sovereign defaults after the liquidity shock, thus, with probability η it loses output but keeps the accumulated liquidity (R). The lender anticipates country's decision and it sets δ to break-even in expectation:

$$\delta D = D(1 - \eta(D)) \int_{\bar{z}(\delta, D, R)}^{\infty} dF(z) \iff \delta = (1 - \eta(D))(1 - F(\bar{z}(\delta, D, R))). \tag{7}$$

The country's choice on debt (D) and reserves (R) is obtained by solving the following maximization problem:

$$\max_{D,R\in[0,\delta D]} E(W_N(\delta, D, R)) = R + (1 - \eta(D)) \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z). \tag{8}$$

On the one hand, the decision to default after the liquidity shock implies that the sovereign will not inject E to overcome the crisis. On the other hand, it will obtain the net expected payoff generated by the production technology only with probability $1 - \eta$ (that is, if the shock does not take place).

Sub-game F At stage four the country defaults after the realization of the productivity shock if $z \leq \bar{z}$, as in (2). At stage two, the country liquidates a portion of its reserves to inject E. A necessary condition for this to happen is that the country has accumulated enough liquidity, or $R \geq E$ (*resource constraint*).

The decision to redeem the shock must also be sub-game perfect. Given the sovereign choice of D and R and the discount factor (δ) offered by the lender, the country must have an incentive to continue with the production technology instead of defaulting strategically at stage two, otherwise the decision to inject E would not be time-consistent. That is, the following *time-consistency constraint* has to hold:

$$R - E + \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) \geq R$$

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) \geq E.$$

At stage zero, the lender solves the following zero-profit condition:

$$\delta D = D \int_{\bar{z}(\delta,D,R)}^{\infty} dF(z) \iff \delta = 1 - F(\bar{z}(\delta, D, R)). \tag{9}$$

This formulation of the lender's problem takes into account that the sovereign does not default after the liquidity shock occurs and always brings the production technology to completion. The country decides on the accumulation of D and the value of R by solving

$$\max_{D,R\in[E,\delta D]} E(W_F(\delta, D, R)) = R + \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - \eta(D)E, \tag{10}$$

under the time-consistency constraint. The first term in (10) reflects the fact that the country is certain about R, because reserves are not lost in the event of default. The second term in (10) is the expected value of the production technology net of the face value of debt D: this

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second term is positive by construction, because the expected value of the receipts is truncated downwards by $\bar{z}$. The third term in (10) corresponds to the expected cost borne by the country in the event of a liquidity shock.

3.3 Equilibrium definition

The equilibrium of the game with liquidity shock is defined by the vector $\{\delta, D, R\}$ such that the country maximizes its expected welfare and the lender breaks even in expectation. If the country does not default on the liquidity shock (that is, when sub-game F is played), the country's actions must also be feasible and sub-game perfect, insofar as both the time-consistency and resource constraints must be satisfied.

DEFINITION 1. *Denote by $\{\delta_F^*, D_F^*, R_F^*\}$ the vector that characterizes the equilibrium of the game in which the country injects E should the liquidity shock occur (sub-game F), and by $\{\delta_N^*, D_N^*, R_N^*\}$ the vector that characterizes the equilibrium of the game in which the country defaults should the liquidity shock occur (sub-game N).*

At $\{\delta_F^*, D_F^*, R_F^*\}$, the resource constraint

$$R_F^* \geq E,$$

and the time-consistency constraint

$$\int_{\bar{z}(\delta_F^*, D_F^*, R_F^*)}^{\infty} (zY(\delta_F^*, D_F^*, R_F^*) - D)dF(z) \geq E$$

must be satisfied.

At the SPNE of the game, the country chooses to play sub-game F if, and only if, condition $E(W(\delta_F^*, D_F^*, R_F^*)) \geq E(W(\delta_N^*, D_N^*, R_N^*))$ is satisfied and the relevant constraints hold true. Otherwise, the SPNE of the game features the choice of sub-game N.

We first determine the conditions for the existence and unicity of $\{\delta_N^*, D_N^*, R_N^*\}$ and $\{\delta_F^*, D_F^*, R_F^*\}$. Then, we assess the strategic relationship between the country's and lender's choice variables. The country's final decision depends on the economy's parameters. Therefore, we analyze by means of numerical simulations the country's choice between sub-game F and sub-game N and its consequences on the level of sovereign reserves, debt, output and welfare.

3.4 Equilibrium analysis

Sub-game N   To begin with, we study the country's decision to accumulate debt and reserves, and the lender choice of δ, the discount factor, in the scenario featuring country's default in the event of a liquidity shock (sub-game N).

PROPOSITION 2. *In the sub-game in which the country defaults following the liquidity shock, the sovereign country chooses D_N^* and R_N^* in the viable range to solve the following first order conditions*

$$(1 - \eta(D)) \int_{\bar{z}(\delta,D,R)}^{\infty} (z\delta Y'(\delta, D, R) - 1)) \, dF(z) - \eta'(D) \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D) dF(z) = 0 \, (11)$$

$$1 - (1 - \eta(D))Y'(\delta, D, R) \int_{\bar{z}(\delta,D,R)}^{\infty} z \, dF(z) = 0. \tag{12}$$

Given the pair of equilibrium values of debt and reserves that solve (11) and (12), it exists a unique equilibrium discount factor $\delta_N^* \in (0,1)$ that solves the lender's zero-profit condition (7).

The basic trade-offs that shape the solution to the investment and lending problems in the benchmark economy also influence the equilibrium results in Proposition 2. The crucial difference is that the country also needs to take into account that the accumulation of a higher amount of debt increases the likelihood that a liquidity shock occurs. Consequently, the value of the LHS of the first-order condition in (11) is lower than the one of the corresponding first-order condition in the benchmark, (5), dictating a lower level of debt, *ceteris paribus*. By the same token, the possibility that a liquidity shock occurs implies that the LHS of the first order condition in (12) is larger than the one in the benchmark (6). Hence, the country has an incentive to accumulate a higher level of reserves than in the benchmark, *ceteris paribus*.[10]

The main takeaway of Proposition 2 is that the accumulation of sovereign reserves might arise at equilibrium independently of the decision to inject resources following a liquidity shock. The reason is that reserves are a liquid asset that the country retains in case of default.

In the following corollary, we derive the relationship between agents' choice variables at equilibrium in sub-game N.

COROLLARY 1. *If Y' is sufficiently small at $\{\delta_N^*, D_N^*, R_N^*\}$, we find that:*

$$(i) \quad d\delta_N^*/dD < 0, \quad d\delta_N^*/dR \leq 0, \quad (ii) \quad sign\{dR_N^*/dD\} = sign\{dD_N^*/dR\} > 0.$$

Moreover,

$$(iii) \quad dR_N^*/d\delta > 0, \quad dD_N^*/d\delta < 0.$$

The equilibrium values of debt and the discount factor are in an inverse relationship; that is, as D increases the equilibrium value of δ decreases. Intuitively, as the country gets more indebted, the probability of default following the productivity crisis rises. Repayment concerns

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induce the lender to reduce the value of the discount factor. We also find that an increase in the amount of reserves triggers a decrease in the value of the discount factor: as the country accumulates more liquid assets, the expected value of output decreases and so does what the country can pledge to the lender.

We also find that there is a direct relationship between debt and reserves, showing that these variables are in a relationship of complementarity. Reserves cannot be taken over by the lender, thus they mitigate the adverse consequences of a default for country's welfare. Thus, the sovereign raises the level of debt to have more resources for investment. Moreover, since larger debt renders default more likely, it hoards more reserves to retain them in default states.

Finally, we show that if the discount factor increases, also the value of reserves increases at equilibrium, because the country has more resources to invest in this asset. Conversely, an increase in the discount factor reduces the amount of debt at equilibrium. Intuitively, an increase in the discount factor raises the size of the country's total resources, so the sovereign can decrease the amount of debt and reduce the probability of the liquidity shock without sacrificing the scale of the investment in the production technology.

The sufficient condition under which we derive the results in Corollary 1 is that Y' is sufficiently small at equilibrium. If Y' is small, the opportunity cost of accumulating reserves is small and this, of course, stimulates the country to raise reserves, *ceteris paribus*. However, in the proof of Corollary 1 we show that most of our results (and in particular the ones on the relationship between R and D) goes through under the alternative sufficient condition that the equilibrium level of reserves is small enough.

Sub-game F We now turn to the equilibrium analysisis when the country does not default following the liquidity shock (sub-game F).

PROPOSITION 3. *If the country does not default following the liquidity shock, then we find that:*

1. *In the unconstrained equilibrium, the sovereign country chooses D_F^* and R_F^* in the viable range to solve the following first-order conditions:*

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (z\delta Y'(\delta,D,R) - 1)dF(z) - \eta'(D)E = 0, \tag{13}$$

$$1 - \int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta,D,R)dF(z) = 0. \tag{14}$$

2. *If the resource constraint is binding, at the equilibrium the optimal level of reserves is $R_F^* = E$.*

3. *If the time-consistency constraint is binding, at the equilibrium the optimal level of reserves R_F^* is determined by the following condition*

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta,D,R) - D)dF(z) = E.$$

The value of D_F^* in the two classes of constrained equilibrium results from the corresponding first order condition. Finally, given D_F^* and R_F^*, it exists a unique equilibrium discount factor $\delta_F^* \in (0,1)$ that solves the lender zero-profit condition.

In the unconstrained case, the country sets the levels of D and R as prescribed by (13) and (14), respectively. The optimal value of D is determined by two conflicting forces: on the one hand, a higher level of D raises the expected revenue of the production technology; on the other hand, if D increases both the repayment due to the lender and the probability that the country is hit by the liquidity shock increase. When deciding on the value of reserves, the country solves the same trade-off as in the benchmark economy: it is optimal to increase the amount of reserves as long as the marginal gain from one additional unit of resources invested in liquid assets (equal to 1) outweighs the expected marginal gain from investing the same amount in the production technology. Note that the LHS of the first-order condition that determines R in the unconstrained scenario, (14), is smaller than the corresponding one in sub-game N, (12). The reason is that, in sub-game N, the country can bring the production technology to completion only if it is exempted by the liquidity shock. This reduces the marginal return of the production technology and boosts investment in reserves, *ceteris paribus*.[11]

If the country's choice is constrained by either the resource or the time-consistency constraint, then the equilibrium level of reserves is determined by the binding condition and the level of debt is set to solve the corresponding first-order condition. Moreover, we find that there is no pure-strategy Nash equilibrium at which both the resource and the time-consistency constraints are binding. If the resource constraint is binding, the country would like to accumulate a lower level of reserves than E, but the presence of the resource constraint fixes R exactly at E. In other words, the resource constraint sets a value of R that is larger than in an unconstrained optimum. The time-consistency constraint works in the opposite direction. Since the expected output after repaying the lender in the event of no default (that is, the LHS of the time-consistency constraint) is decreasing in R,[12] a binding time-consistency constraint implies that although the country would like to accumulate a larger level of reserves, it has to reduce R in order to satisfy the constraint with an equality. Thus, the time-consistency constraint pins down a higher value of

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[11]As for Propositions 1 and 2, the conditions for the unicity of the equilibrium of the sovereign investment game are in the appendix. We will verify that they are satisfied when we carry out numerical simulations with specific functional forms.

[12]We show this in the proof of Proposition 3.

R than in an unconstrained optimum. Overall, these considerations imply that an equilibrium in which both constraints are simultaneously binding cannot exist.

In the following corollary, we analyze the relationship between model's choice variables in sub-game F at equilibrium.

COROLLARY 2. *If Y' is sufficiently small at $\{\delta_F^*, D_F^*, R_F^*\}$, we find that:*

$$(i) \quad d\delta_F^*/dD < 0, \quad d\delta_F^*/dR \leq 0, \quad (ii) \quad sign\{dR_F^*/dD\} = sign\{dD_F^*/dR\} > 0.$$

Moreover,
$$(iii) \quad dR_F^*/d\delta > 0, \quad dD_F^*/d\delta < 0.$$

These results hinge on the same insights developed after Corollary 1, with the difference that in sub-game F, the country's incentive to accumulate reserves is augmented by the need to inject E in the event of a liquidity crisis. This implies that the relationship of complementarity between reserves and debt is even stronger here than in sub-game N. As discussed in the Introduction, these results offer new insights on the role of reserves and the relationship between reserves and debt accumulation.

4   Numerical simulations

In this section, we present numerical simulations that illustrate the results of the theoretical framework.[13] We first investigate how the choice of reserves, debt, and the cost of debt change as the variability of the production technology increases; that is, as the investment in the production technology becomes relatively riskier than reserves. In a second set of simulations, we study how portfolio allocation and the cost of debt change as the value of E, the amount of resources that needs to be injected to overcome a liquidity shock, rises. We assume throughout that the production function is a Cobb-Douglas of the following type:

$$zY(\delta, D, R) = z(\delta D - R)^\alpha.$$

We maintain the assumption that the productivity shock is distributed as a uniform random variable:
$$z \sim U(1, c^2/3), z \in [1-c; 1+c].$$

Moreover, the probability of the liquidity shock, $\eta(D) \in (0, 1)$, is modeled as an exponential function,
$$\eta(D) = 2^{D/\bar{D}} - 1,$$

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[13]For the sake of the exposition, we focus on the cases in which an equilibrium in pure strategies exists and is well-defined.

and fulfills the properties laid out in Section 2.3 (namely, $\lim_{D\to 0}\eta(D)=0$, $\lim_{D\to\bar{D}}\eta(D)=1$, $\eta', \eta'' > 0$ for all $D \in (0, \bar{D})$).

Income share of capital	α	0.3
Resource injection (as % of GDP)	E	10%
Liquidity shock parameter	$\bar{D}$	4
Support of the productivity process	c	$[1, 2.5]$

Table 1: Calibration Parameters

Our calibration is parsimonious, we need to set only four parameters to deliver a numerical simulation of our theoretical framework. Table 1 reports the values of the four parameters that determine the behavior of the model variables in the first set of simulations, i.e., the one in which we let the variance of the productivity shock vary. The value of the income share of capital (α) is taken from the literature. The amount of resources that needs to be injected in the event of a liquidity shock (E) amounts to 10% of expected GDP, consistently with the estimates in Rodrik (2006) and Obstfeld et al. (2009). The liquidity shock parameter cannot be estimated from the data. We therefore choose it as to obtain that, in equilibrium, the probability of a liquidity crisis is about 10%, which is the unconditional probability of a sudden stop in Jeanne and Rancière (2011).[14] Finally, the parameter determining the support of the productivity process, and its variance, ranges between 1 and 2.5. We make it vary to study how country's and lender's choices are affected by an increase in the variability of the production technology.

The simulations in Figure 3 plot our variables of interest as the variance of the underlying productivity process rises. In particular, the figure reports the rate of return r chosen by the lender, together with the equilibrium level of debt and reserves in the benchmark without liquidity shock, and in the economy with liquidity crises. Finally, the lower-right panel plots the value of expected welfare when the country plays sub-games F and N.[15]

Although ours is not a fully quantitative model, the simple parametrization in Table 1 allows us to match the value of our variables of interest to that featured by emerging and developing countries. Reserves amount to about 30% of expected GDP, slightly above 26%, the IMF estimate for emerging and developing countries in 2011. The level of debt is about 55% of GDP, somewhat higher than the IMF estimate of 36%. Finally, the rate of return $r = 1/\delta - 1$ set by the lender oscillates around 1.6:[16] this is consistent with the rate of return generated by a 10-year

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[14]In our model, this probability is determined by the equilibrium level of debt chosen by the country. Note that, in these simulations, the value of $\eta(D)$ does not change by much as the value of $\bar{D}$ varies.

[15]The economy without liquidity shock represents a benchmark that the sovereign cannot implement. For this reason, we do not report the welfare of the country in that case. Note also that, at the value of the parameters in Table 1, the resource and time-consistency constraints never bind.

[16]Recall that the discount factor δ is equal to $1/(1+r)$. That is, in our model δ is inversely correlated with the cost of debt: an increase in the value of δ set by the lender reflects an increase in the probability that the lender expects the country to be solvent and stands for a decrease in the cost of debt.

bond capitalized at an annual interest rate of about 5%. Since we set the risk free interest rate to zero, the rate of return coincides with the spread set on sovereign debt. Thus, the spread in our simulations is slightly below the estimate of 5.44% for EMBI in Borri and Verdelhan (2011) and close to the 9-year excess return of 4.6% obtained by Broner et al. (2013b).

In this first set of simulations, the country does not default following a liquidity shock at equilibrium (that is, it plays sub-game F). The lender anticipates the country's decision and sets a lower rate of return r than in the case with default on the liquidity shock (sub-game N). Moreover, the country accumulates larger stocks of debt and reserves than in the benchmark economy, thus showing the relationship of complementarity between debt and reserves. Note that the equilibrium value of reserves is well above E, the amount of resources needed in the event of a liquidity shock. The relationship of complementarity arises also when looking at how reserves and debt vary with the variance of the productivity process. Indeed, we find that as the value of the variance increases, the country raises its holdings of both debt and reserves. It also turns out that, as debt and reserves increase, the rate of return set by the lender rises, reflecting the higher probability of default on the shocks. Taken together, these simulations confirm the results in Corollary 2.

Figure 4 shows that the value of expected output increases in the variance of the production technology. Conditionally on the country not defaulting following the productivity shock, the expected value of output $E(Y|z \geq \bar{z})$ increases because the sovereign enjoys the upper tail of the distribution of the productivity shock, whereas it disregards the lower tail. Moreover, $E(Y)$, the expected value of output, increases in the variance of z because country's total resources (δD) and the amount of resources invested in the production technology ($\delta D - R$) increase with the variance of the productivity process. Higher levels of reserves are associated with higher expected output, an outcome that is consistent with Benigno and Fornaro (2012) and Dominguez et al. (2012). Finally, the output of the country when it uses its reserves to redeem the liquidity shock (sub-game F) is larger than if it defaults following that shock (sub-game N). This is because, by using reserves to inject the needed resources following a liquidity shock, the country does not lose its output.

Figure 5 plots the variables of interest for different values of E, the resources needed to overcome the liquidity crisis. Specifically, in this figure we let E vary from 0.02 to 0.12, that is, from around 4% to 24% of country's expected GDP. The value of c, the parameter that determines the variance of the productivity process, is set at 2.25, the other parameters are as in Table 1. Note that E influences only the country's choices in sub-game F, the reason is that in the benchmark economy and when the country chooses to default following the liquidity shock (sub-game N), the expressions determining the optimal levels of reserves, debt and the cost of debt are independent of E.

For low values of E, the country decides not to default if the liquidity shock occurs (that is,

it plays sub-game F at equilibrium). Instead, when E is particularly large, then the country decides to default on the liquidity shock (sub-game N). As E rises above a threshold, the time-consistency constraint binds and the levels of debt and reserves are as in the constrained equilibrium of Proposition 3. In particular, the constrained level of reserves is lower than when the time-consistency constraint does not bind. In these simulations as in those of Figure 3, debt and reserves are in relationship of complementarity. The country accumulates more debt and reserves than in the benchmark economy. At the same time, debt and reserves increase as the amount of liquid resources E needed in the event of a liquidity shock rises.

5   Foreign lenders' intervention

In the main model we assume that, if the liquidity shock occurs, the country can inject the resources E and complete the production technology only by using its reserves. In other words, the lender cannot intervene in the event of a liquidity crisis. In this extension, we relax this friction and assume that, if the liquidity crisis happens at stage one, the country can issue new debt to cover E. We interpret this as a form of foreign lenders' intervention to tackle the shock.

We change the baseline model of Section 2 by introducing a new lending game in stage one. More specifically, before taking the decision to default on the liquidity shock, the country approaches the lender announcing a fund raising goal of $\hat{D}$ to cover E. In turn, the lender decides on the discount factor $\hat{\delta}$. Figure 6 illustrates the new timing of the game.

The objective of this section is to assess the impact of stage-one lenders' intervention on stage zero lending and investment decisions. Following Section 3.2, if the liquidity crisis does not occur at stage one, at stage four the country defaults after the realization of the productivity shock if $z \leq \bar{z}$, where $\bar{z}$ is as defined in (2). Conversely, if the liquidity shock occurs at stage one, we distinguish between three cases. In the first, the country uses its reserves to inject the resources needed to overcome the shock (sub-game F). In the second, it issues additional debt $\hat{D}$ to cover E (sub-game FL). In the third, it chooses to default (sub-game N). The first and the third cases have been analyzed in Propositions 2 and 3. In what follows, we describe how we solve for the equilibrium of sub-game FL.

Sub-game FL At stage one, the country sets $\hat{D}$ by maximizing its continuation payoff post-liquidity shock. The lender responds by setting the value of $\hat{\delta}$ that insures break-even in expectation.

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The new stage-zero maximization problem follows:

$$
\max_{D,R\in[0,\delta D]} E(W_{FL}(\delta, D, R)) = R + \eta(D)\left(\int_{\hat{z}(\delta,D,R,\hat{D})}^{\infty} (zY(\delta, D, R) - (D + \hat{D}))dF(z) + \hat{\delta}\hat{D} - E \right)
$$
$$
+ (1 - \eta(D)) \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z),
$$

where the value of $\hat{D}$ and $\hat{\delta}$ are determined by the stage-one lending game.

This formulation of the investment game takes into account that if the liquidity crisis does not occur at stage one, at stage four the country defaults if $z \leq \bar{z}$ as in (2). Conversely, if the liquidity shock takes place at stage one, the threshold value of z that triggers country's default at stage four is $\hat{z} = (D + \hat{D})/Y(\delta, D, R)$ (see the proof of Proposition 4 for details), with $\hat{z} > \bar{z}$, *ceteris paribus*. Due to stage-one lenders' intervention, the country uses $\hat{\delta}\hat{D}$ to inject the needed resources in the event of a shock and these resources cannot be invested in the production technology. This raises the likelihood of a default following the productivity shock.

The country's stage-zero investment problem is not constrained by the resource and time-consistency constraints. The resource constraint is redundant because the country can use the new credit line to inject the needed resources. The time-consistency constraint is redundant because when deciding on $\hat{D}$, the country maximizes its continuation payoff post-liquidity shock, implying that the time-consistency constraint always holds true.

Finally, at stage zero the lender sets δ to solve the zero profit condition:

$$
\delta D = D\left((1 - \eta(D)) \int_{\bar{z}(\delta,D,R)}^{\infty} dF(z) + \eta(D) \int_{\hat{z}(\delta,D,R,\hat{D})}^{\infty} dF(z) \right)
$$
$$
\delta = (1 - \eta(D))(1 - F(\bar{z}(\delta, D, R))) + \eta(D)(1 - F(\hat{z}(\delta, D, R, \hat{D}))).
$$

Proposition 4 summarizes the equilibrium features of the stage-one lending game in sub-game FL.

PROPOSITION 4. *At stage one, the country borrows* $\hat{D}^* = E/\hat{\delta}^*$. *Given* $\hat{D}^*$, *it exists a unique value of* $\hat{\delta} \in (0, 1)$ *that solves the lender's zero-profit condition,* $\hat{\delta}^*$.

The sovereign decisions on D and R differ from those obtained under sub-games F and N in Section 3.4, because in the continuation game the sovereign will obtain additional credit after the liquidity shock occurs. To illustrate the results of the investment and lending games under sub-game FL and the country's equilibrium choice, we perform a numerical simulation. We use the same value of the parameters and the functional forms as in Section 4. Our objective is to assess whether, at equilibrium, the country chooses to borrow from the foreign lenders' should the liquidity shock occur, and the consequences of this choice on the values of debt, reserves, discount factor (or, equivalently, the rate of return) and welfare.

Figure 7 reports the same equilibrium outcomes obtained under sub-game F as in Figure 3 (that is, without stage-one intervention), and adds the equilibrium levels of rate of return, debt and reserves when the country can issue new debt in the event of a liquidity crisis.[17] The lower-right panel shows that, with this parametrization, the country prefers to issue fresh debt if hit by a liquidity shock instead of depleting its reserves. The upper-left panel shows the rates of return charged when the country can decide to borrow in stage one: the rate of return set at stage zero is slightly lower than the one charged without foreign lenders' intervention. Instead, the stage-one rate of return is higher than the stage-zero rate, because stage-one lenders anticipate that the probability of default after the realization of the production technology increases with foreign lenders' intervention. Finally, the level of debt and reserves chosen by the sovereign at stage zero is lower than without stage-one lenders' intervention.

Corroborating the conclusions obtained without lenders' intervention, Figure 7 shows that even when the country can obtain additional credit at stage one debt and reserves appear to be in a relationship of complementarity. Indeed, a comparison with Figure 3 shows that their value increases with respect to the benchmark economy as well as when the variability of the production technology increases. The lower-right panel illustrates the welfare implications of foreign lenders' intervention, and shows that the country will always choose to borrow from foreign lenders in stage one instead of making use of its reserves to overcome a liquidity crisis. When it can resort to lenders at stage one, the country will keep reserves in the event of a liquidity shock, while it will pay back its larger debt only if the value of the production technology is large enough.

This exercise shows that the country accumulates reserves independently from their use in the event of a liquidity crisis. Indeed, even though reserves are no longer necessary to overcome a liquidity crisis, they are still a valuable asset since the country can retain them in case of default. Consequently, the optimal portfolio allocation still prescribes a positive amount of investment in reserves.

Figure 8 shows the overall cost of debt with and without the possibility of foreign lenders' intervention in stage one. We show that the sum of rate of returns $r = 1/\delta - 1$ and $\hat{r} = 1/\hat{\delta} - 1$ weighted by the borrowed amounts D and $\hat{D}$ is larger than the rate of return in sub-game F. The combined effect of the accumulated level of reserves and the higher probability of default implies that the total cost of debt set by lenders is higher when the country can borrow from foreign lenders following a liquidity crisis, compared to a world in which the country has to liquidate its reserves to overcome a liquidity shock. This shows that when liquidity crises occur, the possibility to obtain additional credit does not necessarily ease the pressure on the sovereign spreads.

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[17]The outcomes obtained under sub-game N are the same as in Figure 3, with and without foreign lenders' intervention, and are not reported because under the parameterization in Table 1 sub-game N is never chosen at equilibrium.

6 Conclusions

We develop a model of optimal portfolio choice of a risk neutral country that is subject to liquidity and productivity shocks, and study sovereign optimal choice of debt and reserves. In our model, on the one hand reserves distract resources from the production technology, on the other hand, they have two important upsides for the country: first, they can be used in the event of a liquidity crisis; second, they cannot be seized by investors should the country decide to default.

Our main result is that, in the sovereign's optimal strategy, reserves and debt are in a relationship of complementarity. Although raising debt increases the country's exposure to liquidity and productivity crises, the contemporaneous accumulation of reserves mitigates the consequences of these crises on sovereign welfare. The numerical simulations of our theoretical model account for a number of stylized facts arising from data on emerging economies: the accumulation of reserves is not necessarily accompanied by a reduction of debt. Moreover, sovereign debt cost increases with the level of debt. Finally, the value of output is positively related with the amount of sovereign reserves.

The rapid increase in the emerging economies' sovereign reserves has been often considered one of the elements that has fueled global imbalances and that could destabilize international capital markets by distorting asset prices. In our model, sovereign decision to hoard reserves is the result of an optimal policy strategy. Therefore, the analysis in this paper prescribes that, if the level of sovereign reserves is judged to be excessively high, the international community should act on the underlying economic conditions that shape a country's investment decisions.

In our future research, we would like to develop a dynamic model along the lines of, e.g., Arellano and Ramanarayanan (2012), Bianchi et al. (2013) and Aguiar and Amador (2013) to study the optimal maturity structure that maximizes the returns of country's portfolio allocation. Also, we would like to build a growth model with debt and reserves to determine whether the negative relationship between sovereign indebtedness and growth arising from the data (Reinhart and Rogoff, 2011) is affected by the accumulation of reserves.

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Appendix: Proofs

Proof of Proposition 1. First note that the country and the lender solve respective problems simultaneously, so each takes the choice of the other as given.

We begin by analyzing the problem of the sovereign country. The sovereign country solves the investment game in (3):

$$\max_{D,R\in[0,\delta D]} E(W(\delta, D, R)) = R + \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z). \tag{1}$$

The optimal value of reserves and debt, D^* and R^*, results from the following two first-order conditions with respect to D and R, respectively:

$$E(W(\delta, D, R))_D = \int_{\bar{z}(\delta,R,D)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) = 0 \tag{2}$$

$$E(W(\delta, D, R))_R = 1 - Y'(\delta, D, R) \int_{\bar{z}(\delta,R,D)}^{\infty} zdF(z) = 0, \tag{3}$$

Note that $R = \delta D$ is not an equilibrium because, for any given value of δ and D, $Y'(0) = \infty$ implies that (3) never holds when all resources are spent in reserves. For (2) and (3), we denote the first derivative of country's expected welfare with respect to a variable x as $E(W(\delta, D, R))_x$. The notation for the second order derivatives is analogous. Specifically, the second order conditions for (3) are:[18]

$$E(W)_{DD} = \delta Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{1}{Y}\left(\frac{Y - \delta DY'}{Y}\right)^2 \tag{4}$$

$$E(W)_{RR} = Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{1}{Y}\left(\frac{DY'}{Y}\right)^2 \tag{5}$$

$$E(W)_{DR} = E(W)_{RD} = -\delta Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{DY'}{Y^3}(Y - \delta DY'), \tag{6}$$

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[18]For the ease of the exposition, in the following we drop functional notation.

and the sufficient conditions for unicity require that $E(W)_{DD} < 0$ and $E(W)_{DD}E(W)_{RR} > (E(W)_{DR})^2$ at the equilibrium.

We now turn to the analysis of the lender problem. The equilibrium value of $\delta \in (0,1)$, denoted δ^*, that solves lender's zero-profit condition exists and is unique if, and only if, $1 - F(\bar{z})$ in (4) satisfies the following four conditions.

1. The first condition insures that, as δ approaches zero, $(1 - F(\bar{z}))$ takes a finite value. Formally,
$$\exists \lim_{\delta \to 0}(1 - F(\bar{z})) < \infty.$$

   To show that this condition holds, first note that as $\delta \to 0$ the value of $\bar{z}(\delta, D, R)$ goes to infinity. Indeed, a nil value of δ implies that the country has no resources to invest and $\lim_{\delta \to 0} Y(\delta, D, R) = 0$. Consequently, $F(\bar{z}(\delta, D, R))$ goes to 1 and

$$\lim_{\delta \to 0}(1 - F(\bar{z})) = 0.$$

2. The second condition insures that, as δ approaches one, the value of $(1 - F(\bar{z}))$ is smaller than one, or
$$\lim_{\delta \to 1}(1 - F(\bar{z})) < 1.$$

   This condition is clearly satisfied: as $\delta \to 1$ the value of $\bar{z}(\delta, D, R)$ is equal to $\frac{D}{Y(D-R)}$, and the value of $F(\bar{z})$ taken at $\bar{z} = \frac{D}{Y(D-R)}$ is strictly below one.

3. The third condition concerns the curvature of $(1 - F(\bar{z}))$. Specifically, the necessary (but not sufficient) condition for uniqueness requires the concavity of $(1 - F(\bar{z}(\delta, D, R)))$ with respect to δ.

   We first compute the first derivative of $(1 - F(\bar{z}))$ with respect of δ, and find it is strictly increasing:

$$\begin{aligned}\frac{\partial(1 - F(\bar{z}))}{\partial\delta} &= -\frac{\partial F(\bar{z})}{\partial z}\frac{d\bar{z}}{d\delta} = \\ &= f(\bar{z})\frac{D^2 Y'}{Y^2} > 0.\end{aligned}$$

   The second derivative with respect to δ follows:

$$\frac{\partial^2(1 - F(\bar{z}))}{\partial\delta^2} = \frac{df(\bar{z})}{d\delta}\frac{D^2 Y'}{Y^2} + D^3 f(\bar{z})\left(\frac{Y''}{Y^2} - \frac{2(Y')^2}{Y^4}\right) < 0. \quad (7)$$

   The first term is nil because z follows a uniform distribution, so $\partial f(\bar{z})/\partial\delta = 0$. To establish the sign of (7) we note that the term in curly brackets is negative, thus rendering the all expression negative.

30

4. The final and necessary condition insuring that an interior solution different from the trivial one $\delta = 0$ exists and is unique requires that

$$\lim_{\delta \to 0} \partial(1 - F(\bar{z}))/\partial\delta > 1.$$

Taking limits, we find that:

$$\lim_{\delta \to 0} \frac{\partial(1 - F(\bar{z}))}{\partial\delta} = \lim_{\delta \to 0} \frac{D^2 Y'}{Y^2} = \infty. \tag{8}$$

Indeed, since a nile value of δ implies that the country has not resources to invest,

$$\lim_{\delta \to 0} \frac{1}{Y^2} = \infty$$

and, by the Inada conditions,

$$\lim_{\delta \to 0} Y' = \infty.$$

The result in (8) also uses the assumption that z follows a uniform distribution, implying that $\partial f(\bar{z})/\partial\delta = 0$.[19]

We conclude that, under our assumptions, it exists a unique value of δ, denoted δ^*, that satisfies the lender zero-profit condition in (4). Q.E.D.

Proof of Proposition 2. As remarked in the proof of Proposition 1, since the country and the lender choices take place simultaneously, each takes the choice of the other as given. The sovereign country solves the investment game in (8):

$$\max_{D,R \in [0,\delta D]} E(W_N(\delta, D, R)) = R + (1 - \eta(D)) \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z). \tag{9}$$

The optimal value of reserves and debt, D_N^* and R_N^*, results from the the following two first-order conditions with respect to D and R, respectively:

$$\begin{aligned} E(W_N(\delta, D, R))_D &= -\eta'(D) \int_{\bar{z}(\delta,R,D)}^{\infty} (zY(\delta, D, R) - D)dF(z) \\ &\quad + (1 - \eta(D)) \int_{\bar{z}(\delta,R,D)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) = 0 \end{aligned} \tag{10}$$

$$E(W_N(\delta, D, R))_R = 1 - (1 - \eta(D))Y'(\delta, D, R) \int_{\bar{z}(\delta,R,D)}^{\infty} zdF(z) = 0. \tag{11}$$

---

[19]Of course, we are implicitly restricting our attention to the cases in which $\bar{z}(\delta, D, R)$ lies in the support for z, $[1 - c, 1 + c]$, since otherwise the lender would never serve the country.

As in the proof of Proposition 1, $R = \delta D$ cannot be an equilibrium because, for any given value of δ and D, $Y'(0) = \infty$ implies that (11) would never hold true when all resources are spent in reserves. The second order derivatives follow:[20]

$$
\begin{aligned}
E(W_N)_{DD} &= -\eta'' \int_{\bar{z}}^{\infty} (zY - D)dF(z) - 2\eta' \int_{\bar{z}}^{\infty} (z\delta Y' - 1)dF(z) \\
&\quad + (1 - \eta)\left(\delta^2 Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{1}{Y}\left(\frac{Y - \delta DY'}{Y} \right)^2 \right), \quad (12)
\end{aligned}
$$

$$
E(W_N)_{RR} = (1 - \eta)\left(Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{1}{Y}\left(\frac{DY'}{Y} \right)^2 \right), \quad (13)
$$

$$
\begin{aligned}
E(W_N)_{DR} = E(W_N)_{RD} &= \eta'Y' \int_{\bar{z}}^{\infty} zdF(z) \\
&\quad + (1 - \eta)\left(-\delta Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{DY'}{Y^3}(Y - \delta DY') \right). \quad (14)
\end{aligned}
$$

The sufficient conditions for unicity dictate that $E(W_N)_{DD} < 0$ and $E(W_N)_{DD}E(W_N)_{RR} > (E(W_N)_{DR})^2$. Throughout our analysis, we assume that these conditions hold true and verify that they indeed are satisfied in our numerical simulations.

As far as the lender problem in (7) is concerned, the analysis follows the same steps as in the proof of Proposition 1, because $\eta(D)$ does not depend on δ. Therefore, it exists also in this setup a unique value of δ, denoted δ_N^*, that satisfies the lender zero-profit condition in (7) for a given couple of D and R that solves (10) and (11). Q.E.D.

Proof of Corollary 1. We begin with the results in (i). The lender zero-profit condition in the case of sub-game N is as in (7). In the proof of Proposition 2 we argue that it exists a unique value of δ_N^* that solves (7), that is

$$
ZP_N(\delta_N^*, D, R) = \delta_N^* - (1 - \eta(D))(1 - F(\bar{z}(\delta_N^*, D, R))) = 0. \quad (15)
$$

Applying the Implicit Function Theorem, we find that

$$
\frac{d\delta_N^*}{dR} = -\frac{ZP_N(\delta_N^*, D, R)_R}{ZP_N(\delta_N^*, D, R)_\delta} \leq 0 \quad (16)
$$

$$
\frac{d\delta_N^*}{dD} = -\frac{ZP_N(\delta_N^*, D, R)_D}{ZP_N(\delta_N^*, D, R)_\delta} < 0. \quad (17)
$$

Indeed, by definition of δ_N^*, $ZP_N(\delta_N^*, D, R)_\delta > 0$: at δ_N^* a marginal increase of δ raises the difference between the RHS and the LHS in (7). In what follows, to ease the exposition we drop

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[20]In the rest of this proof we drop functional notation.

functional notation. The derivative of (15) with respect to R is

$$ZP_N(\delta_N^*, D, R)_R = (1 - \eta)f(\bar{z})\frac{DY'}{Y^2} \geq 0, \tag{18}$$

implying that the sign of (16) is clearly positive. As for the derivative of (15) with respect to D, this is equal to

$$ZP_N(\delta_N^*, D, R)_D = \frac{f(\bar{z})(1 - \eta)}{Y}\frac{Y - \delta_N^* DY'}{Y} + \eta'(1 - F(\bar{z})) > 0 \tag{19}$$

if Y' is small or R is small. Indeed, the second term in (19) is clearly positive, whereas the sign of the first term depends on the sign of

$$\frac{Y - \delta_N^* DY'}{Y},$$

and

$$\frac{Y - \delta DY'}{Y} \geq 0 \iff \frac{Y}{\delta D} \geq Y' \geq 0, \tag{20}$$

which holds true if Y' is sufficiently small. Alternatively, due to the concavity of the production function Y, (20) is positive if reserves are small.

We now turn to the results in (ii) and (iii). The equilibrium value of D in sub-game N, D_N^*, results from (11). By the Implicit Function Theorem, we obtain that:

$$\frac{dD_N^*}{dR} = -\frac{E(W_N(\delta, D_N^*, R))_{DR}}{E(W_N(\delta, D_N^*, R))_{DD}} > 0 \tag{21}$$

$$\frac{dD_N^*}{d\delta} = -\frac{E(W_N(\delta, D_N^*, R))_{D\delta}}{E(W_N(\delta, D_N^*, R))_{DD}} < 0. \tag{22}$$

To establish the signs of these conditions, first recall that the condition for the unicity of the equilibrium of the investment game requires that $E(W(\delta, D, R))_{DD} < 0$ at the equilibrium (see the proof of Proposition 2). From the proof of Proposition 2, we also know that $E(W_N(\delta, D, R))_{DR}$ is equal to

$$E(W_N(\delta, D_N^*, R))_{DR} = \int_{\bar{z}}^{\infty} (\eta'Y' - (1 - \eta)\delta Y'')z dF(z) + (1 - \eta)\frac{D_N^* Y'}{Y^3}(Y - \delta D_N^* Y'). \tag{23}$$

The first term in (23) is clearly positive ($Y'' < 0$ and $\eta' > 0$). The second term is positive if $Y - \delta DY' > 0$, which, as discussed above (see (20)), is true if either Y' or R are small at equilibrium. This implies that, under the same conditions, (23), and (21), are positive.

To determine the sign of (22) we need to discuss the sign of $E(W(\delta, D_N^*, R))_{D\delta}$, which is

given by

$$
\begin{aligned}
E(W_N(\delta, D_N^*, R))_{D\delta} &= D_N^* \int_{\bar{z}}^{\infty} (\delta(1-\eta)Y'' - \eta'Y')z dF(z) + \\
&\qquad (1-\eta)\left(\int_{\bar{z}}^{\infty} zY'dF(z) - (\bar{z}\delta Y' - 1)\frac{\partial \bar{z}}{\partial D} \right).
\end{aligned} \tag{24}
$$

The first term is clearly negative (for $Y'' < 0$ and $\eta' > 0$, $Y' > 0$). As for the second term, the first part is small if Y' is sufficiently small. As for the second part, we show that the value of $\bar{z}\delta Y' - 1$ is negligible. Indeed, (10) implies that, at D_N^*, the following holds true:

$$
\begin{aligned}
\frac{\eta'}{1-\eta} \int_{\bar{z}}^{\infty} (zY - D_N^*)dF(z) &= \int_{\bar{z}}^{\infty} (z\delta Y' - 1)dF(z) \\
&= (\bar{z}\delta Y' - 1) + \int_{\bar{z}+\zeta}^{\infty} (z\delta Y' - 1)dF(z) \tag{25}
\end{aligned}
$$

with $\zeta > 0$, small. Since, by construction, $\bar{z}Y - D = 0$, (25) can be rewritten as

$$
(\bar{z}\delta Y' - 1) = \frac{1}{1-\eta}\left(\eta' \int_{\bar{z}+\zeta}^{\infty} (zY - D_N^*)dF(z) - (1-\eta)\int_{\bar{z}+\zeta}^{\infty} (z\delta Y' - 1)dF(z) \right) \approx 0. \tag{26}
$$

For ζ small, the term in squared brackets corresponds to (10) and thus it is (approximatively) equal to zero at equilibrium. All this implies that (24) and (22) are negative.

Finally, using the first-order condition in (12) and applying the Implicit Function Theorem, we have that, at R_N^*:

$$
\begin{aligned}
\frac{dR_N^*}{dD} &= -\frac{E(W_N(\delta, D, R_N^*))_{RD}}{E(W_N(\delta, D, R_N^*))_{RR}} > 0 \tag{27} \\
\frac{dR_N^*}{d\delta} &= -\frac{E(W_N(\delta, D, R_N^*))_{R\delta}}{E(W_N(\delta, D, R_N^*))_{RR}} > 0. \tag{28}
\end{aligned}
$$

By the condition for the unicity of the equilibrium of the investment game $E(W_N(\delta, D, R_N^*))_{RR} < 0$. Moreover, $E(W_N(\delta, D, R))_{RD} = E(W_N(\delta, D, R))_{DR}$ implies that the sign of (27) is the same as the sign of (21). To pin down the sign of (28), recall that $E(W_N(\delta, D, R_N^*))_{R\delta}$ is equal to

$$
E(W_N(\delta, D, R_N^*))_{R\delta} = (1-\eta)D\left(-Y'' \int_{\bar{z}}^{\infty} zdF(z) + D^2 \left(\frac{Y'}{Y}\right)^2 \right). \tag{29}
$$

The first part of the term in squared brackets is positive, whereas the second part is small if Y' is sufficiently small. Hence, the all expression is positive and also (28) is positive under the same condition. Q.E.D.

Proof of Proposition 3. Note that the since country and lender moves take place simultaneously,

each takes the choice of the other as given. The sovereign country solves the investment game in (10):

$$\max_{D,R\in[0,\delta D]} E(W_F(\delta, D, R)) = R + \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - \eta(D)E. \tag{30}$$

Under the time-consistency and resource constraints

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) \geq E \tag{31}$$

$$R \geq E, \tag{32}$$

and, of course, under $R \leq \delta D$. We solve the problem by maximizing the following expression, where we denote the Lagrange multipliers to the time-consistency and resource constraints by λ and μ, respectively (we will show that the third constraint is always satisfied at equilibrium):

$$\begin{aligned}
\mathcal{L}(D, R, \lambda, \mu) &= R + \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - \eta(D)E \\
&\quad + \lambda\left(\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - E\right) - \mu(R - E).
\end{aligned}$$

The relative Kuhn-Tucker conditions are given below:

$$\mathcal{L}(D, R, \lambda, \mu)_R = 1 - (1 + \lambda)\int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta, D, R)dF(z) - \mu = 0$$

$$\mathcal{L}(D, R, \lambda, \mu)_D = (1 + \lambda)\int_{\bar{z}(\delta,D,R)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) - \eta'(D)E = 0$$

$$\mathcal{L}(D, R, \lambda, \mu)_\lambda = \int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - E = 0$$

$$\mathcal{L}(D, R, \lambda, \mu)_\mu = R - E = 0$$

$$\lambda\left(\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - E\right) = 0$$

$$\mu(R - E) = 0$$

$$\mu \geq 0, \lambda \geq 0$$

There are four cases to be discussed.

Case A: $\lambda = \mu = 0$. In this case the two constraints are both slack. Then, the optimal levels of

debt and reserves are determined by, respectively:

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (z\delta Y'(\delta,D,R) - 1)dF(z) - \eta'(D)E = 0 \tag{33}$$

$$1 - \int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta,D,R)dF(z) = 0, \tag{34}$$

which correspond to the conditions in Proposition 3. Note that, since $Y' \to \infty$ as $R \to \delta D$, $R = \delta D$ cannot be an equilibrium.

Case B: $\lambda > 0$, $\mu = 0$. The time-consistency constraint is binding, whereas the resource constraint is slack (meaning that $R > E$ at this candidate equilibrium). The optimal levels of debt and reserves and the Lagrange multiplier λ are determined by the following three conditions:

$$(1 + \lambda) \int_{\bar{z}(\delta,D,R)}^{\infty} (z\delta Y'(\delta,D,R) - 1)dF(z) - \eta'(D)E = 0, \tag{35}$$

$$1 - (1 + \lambda) \int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta,D,R)dF(z) = 0, \tag{36}$$

$$\int_{\bar{z}(\delta,D,R)}^{\infty} (zY(\delta,D,R) - D)dF(z) - E = 0. \tag{37}$$

The optimal value of reserves is computed by solving the time-consistency constraint for R, however it cannot be such to deplete all resources (i.e., $R = \delta D$) because in this case $Y(0) = 0$ and (37) would never hold. The value of the time-consistency constraint multiplier is obtained by solving (36) for λ,

$$\lambda = \frac{1 - \int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta,D,R)dF(z)}{\int_{\bar{z}(\delta,D,R)}^{\infty} zY'(\delta,D,R)dF(z)}, \tag{38}$$

and D results from (35) after plugging λ from (38). Note that reaction function for $R = R(\delta,D)$ that solves (37) is lower than the one that solves (34), *ceteris paribus*. This follows from two considerations: first, the time-consistency constraint is decreasing in R and, second, the time-consistency constraint is slack in **Case A**. Moreover, since $\lambda > 0$, (35) is larger than (33), so the reaction function for $D = D(\delta,R)$ that solves (35) is larger than the one that solves (33), *ceteris paribus*.

Case C: $\lambda = 0$, $\mu > 0$. The time-consistency constraint is slack, whereas the resource constraint is binding. Then, the optimal level of debt, reserves and the Lagrange multiplier μ are determined

by the following conditions:

$$\int_{\bar{z}(\delta, D, R)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) - \eta'(D)E = 0, \tag{39}$$

$$1 - \int_{\bar{z}(\delta, D, R)}^{\infty} zY'(\delta, D, R)dF(z) - \mu = 0, \tag{40}$$

$$R = E \tag{41}$$

Thus the equilibrium value of reserves is $R = E < \delta D$, the optimal value of D is computed using (39) and μ results from (40). In this case, the expression for the reaction function $D = D(\delta, R)$ that solves (39) is equal to the one that solves (33), as (33)=(39). Instead, the expression of $R = R(\delta, R)$ that solves (40) is lower than in **Case A**, *ceteris paribus*. The reason is that for a given D, in **Case A** the value of $R(\delta, D)$ that solves (34) must be such that

$$1 - \int_{\bar{z}}^{\infty} zY'(\delta, D, R)dF(z) = 0 < \mu.$$

Case D: $\lambda > 0$, $\mu > 0$. In this case, both the time-consistency constraint and the resource constraint are binding. The candidate equilibrium values of D, R, λ and μ are given by the following four conditions:

$$1 - (1 + \lambda) \int_{\bar{z}(\delta, D, R)}^{\infty} zY'(\delta, D, R)dF(z) - \mu = 0, \tag{42}$$

$$(1 + \lambda) \int_{\bar{z}(\delta, D, R)}^{\infty} (z\delta Y'(\delta, D, R) - 1)dF(z) - \eta'(D)E = 0, \tag{43}$$

$$\int_{\bar{z}(\delta, D, R)}^{\infty} (zY(\delta, D, R) - D)dF(z) - E = 0, \tag{44}$$

$$R = E. \tag{45}$$

However, notice that

$$\lim_{R \to E} \int_{\bar{z}(\delta, D, R)}^{\infty} (zY(\delta, D, R) - D)dF(z) > E \quad \forall E \in [0, \delta D),$$

implying that the time-consistency constraint is always violated at $R = E$. To show this, we first note that

$$\frac{\partial}{\partial R} \int_{\bar{z}(\delta, D, R)}^{\infty} (zY(\delta, D, R) - D)dF(z) = -Y'(\delta, D, R) \int_{\bar{z}(\delta, D, R)}^{\infty} zdF(z) < 0,$$

that is, the LHS of the time-consistency constraint is decreasing in R. Second, if $E = 0$ we have

that, by construction, $\int_{\bar{z}(\delta,D,E)}^{\infty}(zY(\delta,D,E)-D)dF(z)>0$. Instead, if $E\to\delta D$,

$$\int_{\bar{z}(\delta,D,E)}^{\infty}(zY(\delta,D,E)-D)dF(z)-E\to 0,$$

because the country would be investing all available resources in reserves, implying that the revenue generated by the production technology would be nil, the discount factor set by the lender equal to zero and $\lim_{E\to\delta D}E\to 0$. All this implies that the time-consistency and resource constraints cannot both be binding, and the candidate equilibrium associated to **Case D** cannot be an equilibrium of the investment game.

In the following, we derive the second order derivatives of the maximand in (30):[21]

$$E(W_F)_{DD} = \delta^2 Y'' \int_{\bar{z}}^{\infty} zdF(z) - \eta'' E + \frac{1}{Y}\left(\frac{Y-\delta DY'}{Y}\right)^2, \qquad (46)$$

$$E(W_F)_{RR} = Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{1}{Y}\left(\frac{DY'}{Y}\right)^2, \qquad (47)$$

$$E(W_F)_{DR} = E(W_F)_{RD} = -\delta Y'' \int_{\bar{z}}^{\infty} zdF(z) + \frac{DY'}{Y^3}(Y-\delta DY'). \qquad (48)$$

The usual sufficient conditions for unicity prescribe that $E(W_F)_{DD}<0$ and $E(W_F)_{DD}E(W_F)_{RR}>(E(W_F)_{DR})^2$. Throughout our analysis, we assume that these conditions hold true and we verify that they indeed are satisfied in our numerical simulations.

Finally, the analysis of the lender problem in (9) follows the same steps as in the proof of Proposition 1. Therefore, it exists also in this framework a unique value of δ, denoted δ_F^*, that satisfies the lender zero-profit condition in (7) for a given couple of equilibrium values of D and R. Q.E.D.

Proof of Corollary 2. We begin with the conditions in (i). The lender zero-profit condition in the case of sub-game F is as in (9). In the proof of Proposition 2 we show that it exists a unique value of δ_F^* that solves (9), that is

$$ZP_F(\delta_F^*,D,R) = \delta_F^* - (1 - F(\bar{z}(\delta_F^*,D,R))). \qquad (49)$$

By the Implicit Function Theorem we find that

$$\frac{d\delta_F^*}{dR} = -\frac{ZP_F(\delta_F^*,D,R)_R}{ZP_F(\delta_F^*,D,R)_\delta} \leq 0 \qquad (50)$$

$$\frac{d\delta_F^*}{dD} = -\frac{ZP_F(\delta_F^*,D,R)_D}{ZP_F(\delta_F^*,D,R)_\delta} < 0. \qquad (51)$$

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[21]Note that, for the ease of the exposition, we are dropping functional notation.

By definition of δ_F^*, $ZP_F(\delta_F^*, D, R)_\delta > 0$, because, at δ_F^*, a marginal increase of δ raises the difference between the RHS and the LHS in (9).[22] Moreover,

$$ZP_F(\delta_F^*, D, R)_R = f(\bar{z})\frac{DY'}{Y^2} \geq 0, \tag{52}$$

thus the sign of (50) is clearly positive. Also

$$ZP_F(\delta_F^*, D, R)_D = f(\bar{z})Y\frac{Y - \delta DY'}{Y^2} > 0 \tag{53}$$

if Y' small. Indeed, the sign of (53) depends on the sign of $(Y - \delta DY')/Y$, which, as shown in the proof of Corollary 1 (see (20) and the ensuing discussion), is positive if Y' is sufficiently small or, due to the concavity of the production function Y, when reserves are small at equilibrium.

We now turn to the derivation of the results in (ii) and (iii). The equilibrium value of D in the unconstrained case of sub-game F results from (13)=(33). Using the Implicit Function Theorem, we obtain that:

$$\frac{dD_F^*}{dR} = -\frac{E(W_F(\delta, D_F^*, R))_{DR}}{E(W_F(\delta, D_F^*, R))_{DD}} > 0 \tag{54}$$

$$\frac{dD_F^*}{d\delta} = -\frac{E(W_F(\delta, D_F^*, R))_{D\delta}}{E(W_F(\delta, D_F^*, R))_{DD}} < 0. \tag{55}$$

To begin with, the condition for the unicity of the equilibrium of the investment game implies that $E(W_F)_{DD} < 0$ at the equilibrium (see the proof of Proposition 3). From the proof of Proposition 3 we also know that $E(W_F(\delta, D, R))_{DR}$ is given by

$$E(W_F(\delta, D_F^*, R))_{DR} = -\delta Y'' \int_{\bar{z}}^{\infty} z dF(z) + \frac{D_F^* Y'}{Y^3}(Y - \delta D_F^* Y'). \tag{56}$$

The first term in (56) is clearly positive ($Y'' < 0$). The second term is positive if $Y - \delta D_F^* Y' > 0$, which, as discussed above, is positive if Y' is small or R is small enough. This implies that (21) is positive under the same conditions.

To determine the sign of (55) we need to discuss the sign of $E(W_F)_{D\delta}$, which is given by

$$E(W_F(\delta, D_F^*, R))_{D\delta} = D_F^* \int_{\bar{z}}^{\infty} \delta Y'' z dF(z) + \int_{\bar{z}}^{\infty} z Y' dF(z) - (\bar{z}\delta Y' - 1)\frac{\partial \bar{z}}{\partial D}. \tag{57}$$

The first term is clearly negative ($Y'' < 0$). As for the second term, it tends to zero if Y' is sufficiently small. Finally, we show below that the value of $\bar{z}\delta Y' - 1$ is negligible. Recall that

___

[22]In what follows, to ease the exposition we drop functional notation.

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(33) implies that, at the equilibrium, the following holds true:

$$\int_{\bar{z}}^{\infty} (z\delta Y' - 1)dF(z) = \eta' E$$

$$(\bar{z}\delta Y' - 1) + \int_{\bar{z}+\zeta}^{\infty} (z\delta Y' - 1)dF(z) = \eta' E$$

$$(\bar{z}\delta Y' - 1) = \left(\eta' E - \int_{\bar{z}+\zeta}^{\infty} (z\delta Y' - 1)dF(z)\right) \approx 0, \qquad (58)$$

with ζ positive but small. The last equality in (58) follows from the fact that, by (33), the term in squared brackets goes to zero for ζ negligible.

Finally, using (34) and applying the Implicit Function Theorem:

$$\frac{dR_F^*}{dD} = -\frac{E(W_F(\delta, D, R_F^*))_{RD}}{E(W_F(\delta, D, R_F^*))_{RR}} > 0 \qquad (59)$$

$$\frac{dR_F^*}{d\delta} = -\frac{E(W_F(\delta, D, R_F^*))_{R\delta}}{E(W_F(\delta, D, R_F^*))_{RR}} > 0. \qquad (60)$$

The unicity of the equilibrium of the investment game requires that $E(W_F(\delta, D, R_F^*))_{RR} < 0$. Then, since $E(W_F(\delta, D, R_F^*))_{RD} = E(W_F(\delta, D, R_F^*))_{DR}$ the sign of (59) is the same as the sign of (54). Finally, $E(W_F(\delta, D, R_F^*))_{R\delta}$ is equal to

$$E(W_F(\delta, D, R_F^*))_{R\delta} = -Y''D \int_{\bar{z}}^{\infty} z dF(z) + D^2 \left(\frac{Y'}{Y}\right)^2. \qquad (61)$$

The first term is clearly positive, whereas the second term is positive and goes to zero if $Y'(\cdot)$ is small. This condition insures that the all expression is positive and also (60) is positive.   Q.E.D.

Proof of Proposition 4. We first consider the impact of foreign lenders' intervention when the country decides to inject resources after the shock occurs.

If the liquidity shock occurs at stage one, at stage four the country defaults if

$$zY(\delta, D, R) - (D + \hat{D}) \leq 0 \iff z \leq \frac{D + \hat{D}}{Y(\delta, D, R)} \equiv \hat{z}(\delta, D, R, \hat{D}).$$

Since the country uses $\hat{\delta}\hat{D}$ to inject the needed resources in the event of a shock and these resources cannot be invested in the production technology, the threshold that prescribes default at stage four does not depend on $\hat{\delta}$, but only on $\hat{D}$.

At stage one, the country chooses $\hat{D}$ to maximize its continuation payoff (post-liquidity

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shock):[23]

$$\max_{\hat{D}} \quad R + \int_{\hat{z}}^{\infty} (zY - (D + \hat{D}))dF(z) + \hat{\delta}\hat{D} - E \tag{62}$$

subject to

$$\hat{\delta}\hat{D} - E \le 0. \tag{63}$$

The constraint in (63) implies that foreign lenders' intervention covers no more than the nominal value of the shock (E). In turn, the lender sets $\hat{\delta}$ to solve

$$\hat{\delta}\hat{D} = \hat{D} \int_{\hat{z}}^{\infty} dF(z) \iff \hat{\delta} = 1 - F(\hat{z}).$$

The solution of stage one lender's problem follows the same steps as in the proof of Proposition 1. This means that it exists a unique value of $\hat{\delta} \in (0, 1)$, denoted $\hat{\delta}^*$, that solves the lender zero-profit condition for a given value of $\hat{D}$.

The first-order condition of the country's borrowing problem in (62) is equal to:

$$\hat{\delta} - (1 - F(\hat{z})) = 0 \iff \hat{\delta} = (1 - F(\hat{z})). \tag{64}$$

Note that (64) coincides with the zero-profit condition that determines $\hat{\delta}^*$. Thus, the only equilibrium of the game features the country borrowing $\hat{D}^* = E/\hat{\delta}^*$, that is, as much as the lender is willing to provide at the discount factor $\hat{\delta}$ that solves lender's zero-profit condition.

We now turn to the analysis of the case in which the country defaults after the liquidity shock. Foreign lenders' intervention is unfeasible in these circumstances. Indeed, at stage one, the country sets $\hat{D}$ to maximize its continuation payoff, which is equal to $R + \hat{\delta}\hat{D}$: given that the country defaults on the occurrence of the liquidity shock, the revenue of the production technology is nil at stage four. International lenders anticipate this and thus refuse to provide liquidity. Q.E.D.

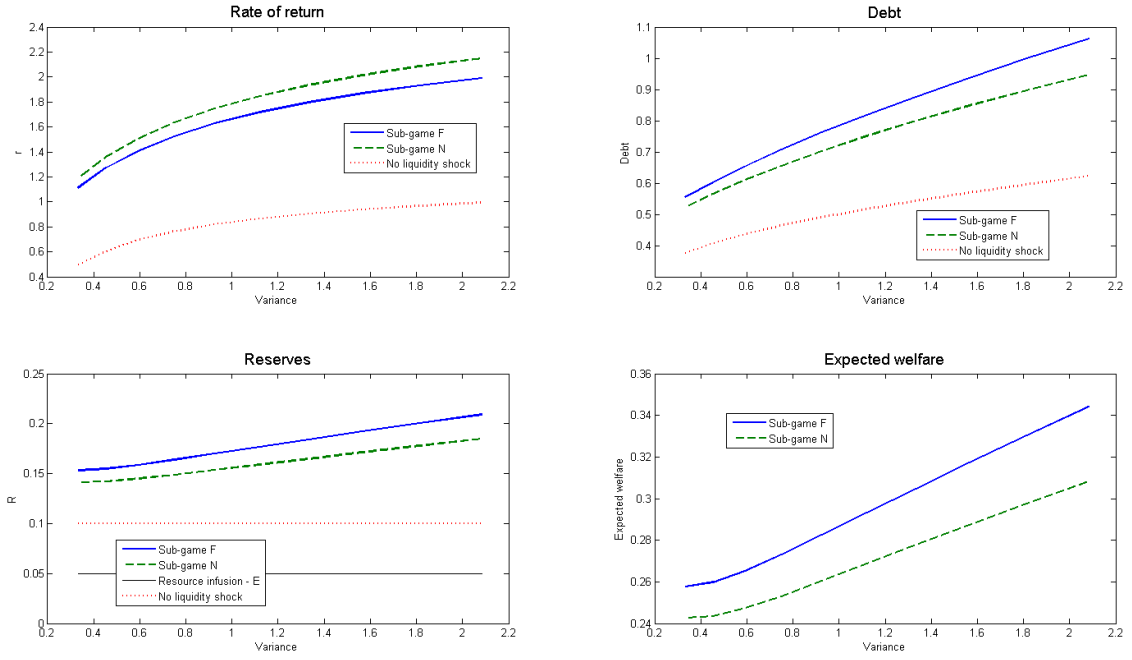[23]In what follows, to ease the exposition we drop functional notation.

Figures

Figure 3: Numerical simulations with respect to the variance of the productivity shock ($c^2/3$).
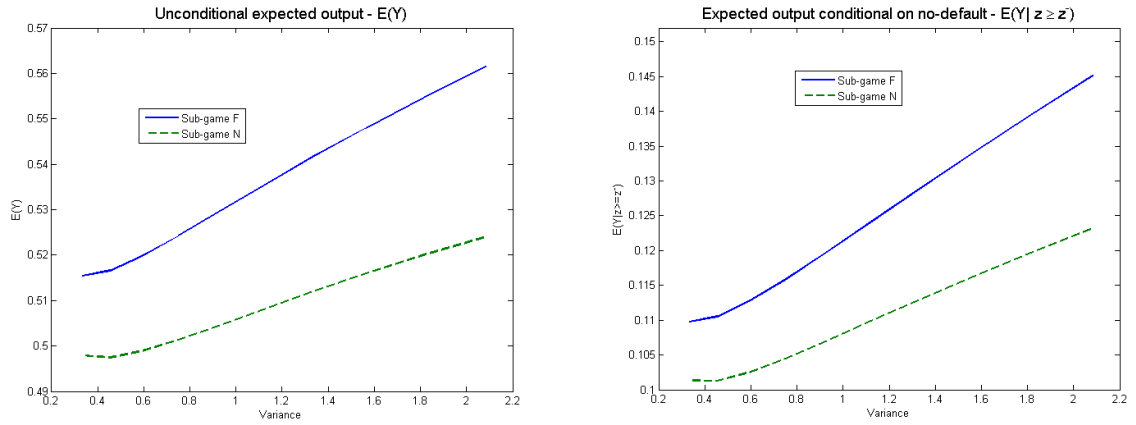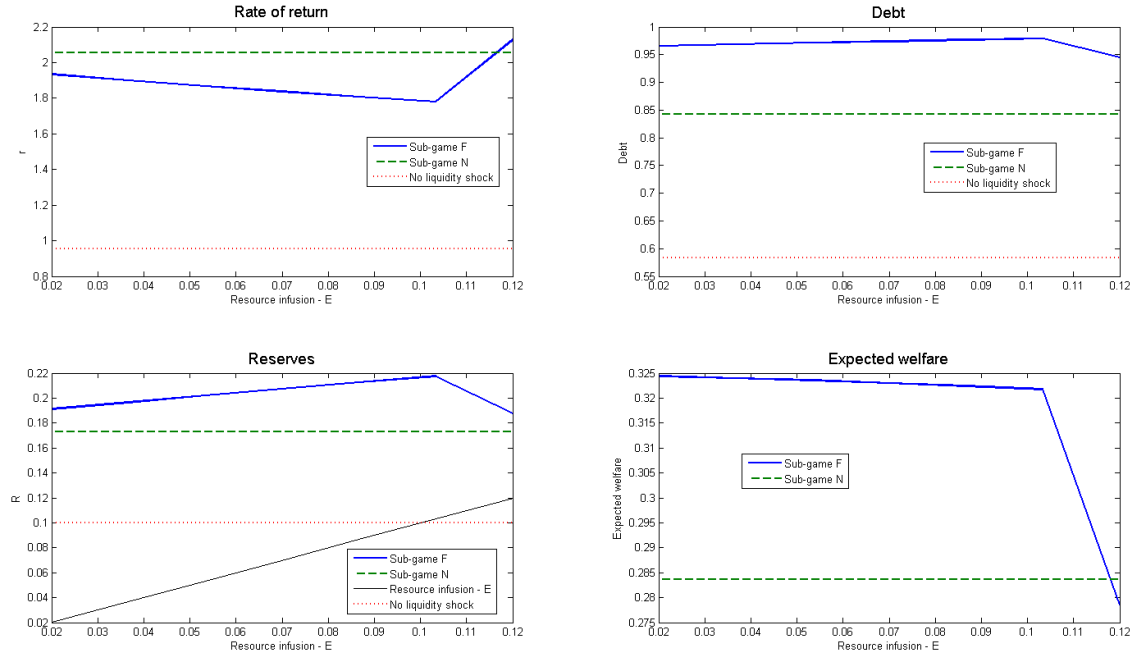

Figure 4: Numerical simulations — Value of output.

Figure 5: Numerical simulations with respect to the resources needed in the event of a liquidity shock (E).
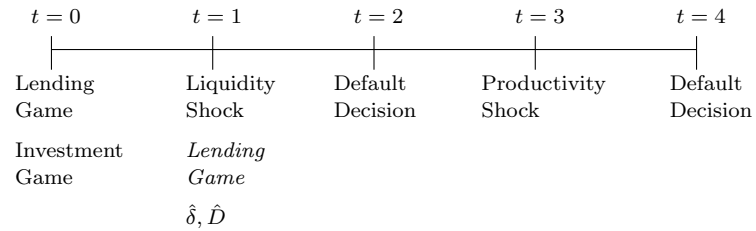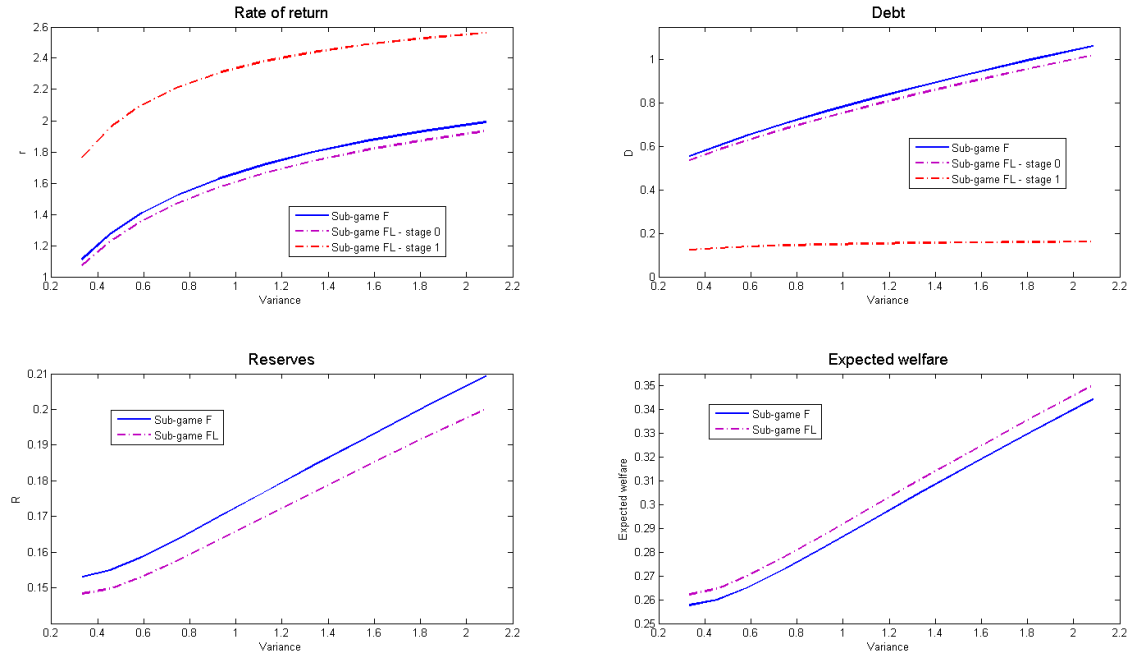

Figure 6: Timeline with foreign lenders' intervention

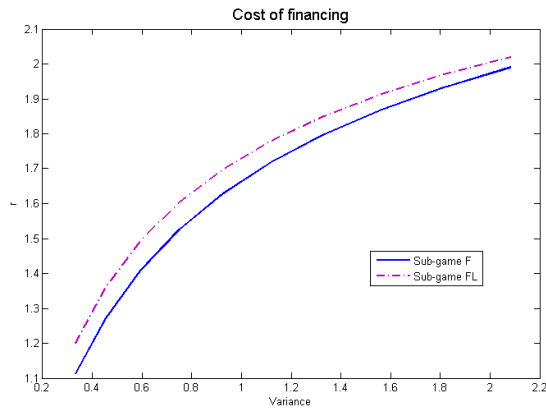**Figure 7:** Numerical simulations with foreign lenders' intervention.

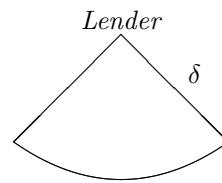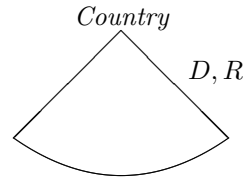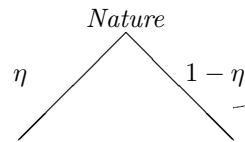**Figure 8:** Cost of debt with foreign lenders' intervention.

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t=0 **- Lending game**   *Lender*
                                                          δ

t=0 **- Investment game**   *Country*
                                                          D, R

t=1 **- Liquidity shock**   *Nature*
                                          η $1 - \eta$

Liquidity shock

t=2 **- Default decision**   *Country*
                            No Default   *Default*
                                                                  $\{R\}$
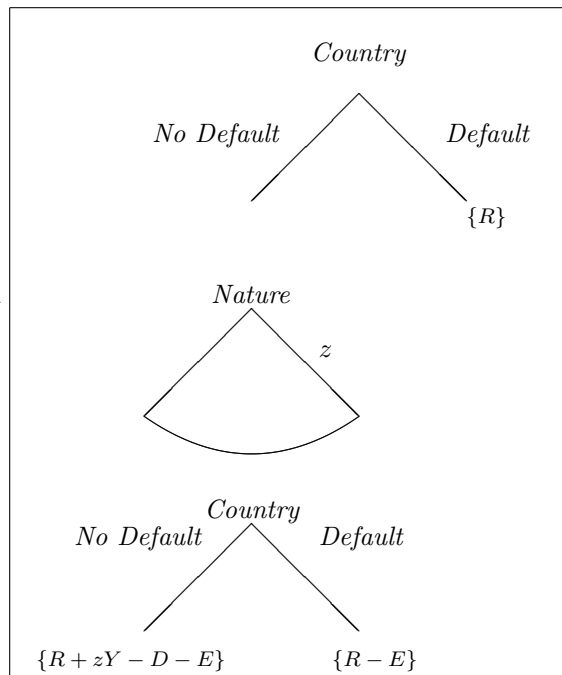
t=3 **- Productivity shock**   *Nature*   *Nature*
                                              z z
No liquidity shock

t=4 **- Default decision**   *Country*   *Country*
                          No Default   *Default*   *No Default*   *Default*

                  $\{R + zY - D - E\}$ $\{R - E\}$ $\{R + zY - D\}$ $\{R\}$
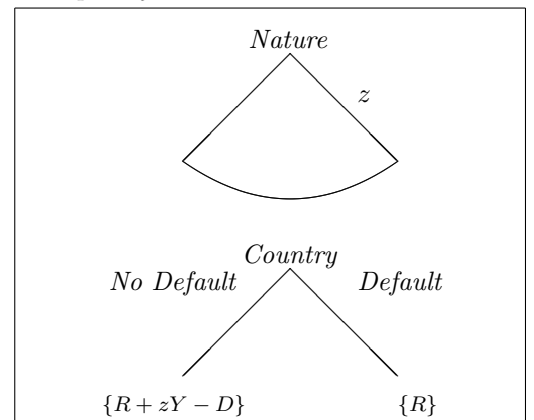
Figure 9: Game-tree and country's payoffs

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Liquidity shock

t=2 - **Default decision**

Country

No Default   *Default*

{R}

t=3 - **Productivity shock**

Nature

z

Sub-game N

t=4 - **Default decision**

Country

No Default   *Default*

{R + zY − D − E}   {R − E}
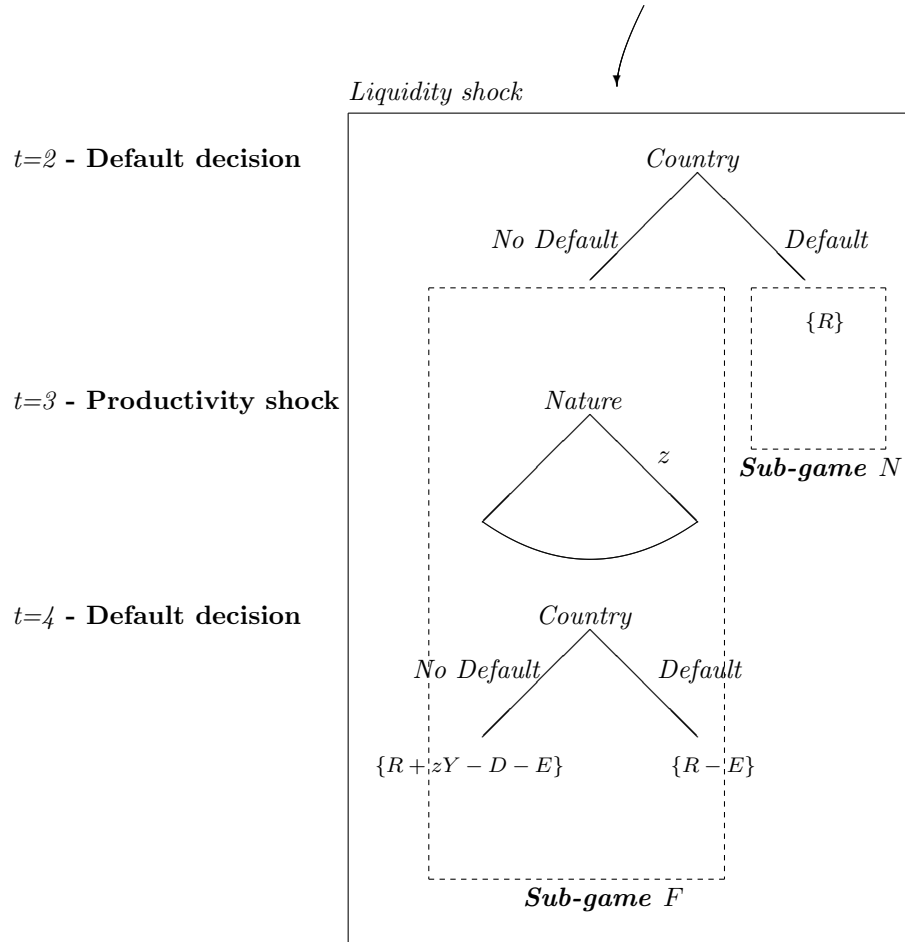
Sub-game F

Figure 10: Economy with liquidity shock, sub-games N and F

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