An estimated DSGE model with search and matching frictions in the credit market

by Danilo Liberati
Temi di discussione
(Working papers)

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Editorial Assistants: Roberto Marano, Nicoletta Olivanti.

ISSN 1594-7939 (print)
ISSN 2281-3950 (online)

Designed and printed by the Printing and Publishing Division of the Banca d’Italia.
AN ESTIMATED DSGE MODEL  
WITH SEARCH AND MATCHING FRICTIONS IN THE CREDIT MARKET

by Danilo Liberati*

Abstract

Financial frictions have become fundamental for studying the business cycle and credit market dynamics. This work adds to the existing literature by introducing a search and matching scheme in the financial market into a cash in advance New Keynesian DSGE theoretical model. We provide an alternative explanation of the degree of incompleteness in the pass-through from policy rate to loan rates depending on credit market tightness, the search costs sustained by banks, and the relative powers of the agents in loan interest rate bargaining. The model is able to reproduce the countercyclical behaviour of the credit spread with respect to a positive technology shock. It also proposes a scenario in which a credit shock hits the economy. The model is estimated by using the Bayesian procedures. Finally, since there is still some disagreement about the theoretical mechanism by which the interest rate on loans is derived, we survey and compare these theoretical devices with that proposed by this paper.

JEL Classification: C78, E13, E43, E44.
Keywords: interest rate pass-through, credit spread, search and matching, credit market frictions, Bayesian techniques.
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1 Introduction

The study of the business cycle has always been one of the main focus in the economic literature. The labor market analysis, through the introduction of different kinds of imperfections and rigidities into New Keynesian (NK) Dynamic Stochastic General Equilibrium (DSGE) models with sticky prices and monopolistic competitive markets, is now a standard in this framework. Further, in the recent years, the introduction of the banking sector in the basic model was growing. This extension is receiving an increased attention since it helps to describe in a better way the dynamics of the main financial variables and their relationships with the real economy. In particular, the study of the (in)completeness of the (lending and deposit) interest rate pass-through (PT from now on) to change of the policy rate and the behavior of the credit spread (CS from now on) over the cycle are two features on which the literature is still facing.

Despite the large empirical evidence on the interest rate PT some disagreement still exists about the theoretical mechanism by which the sluggishness in the interest rate on loans is derived. We improve this research line by providing a new theoretical tool to study the (in)completeness of PT and the cyclical behavior of the CS. We introduce search and matching frictions in the credit market into the standard New Keynesian DSGE model without wage rigidities. After showing how credit matching frictions help to highlight the limited response of the loan rate charged by commercial banks when a Central Bank modifies the policy rate and the countercyclical behavior of the credit spread, we estimate and compare different models generating incomplete PT to determine the one with the greatest empirical evidence consistent with data.

We are not the first to analyze the role of search and matching technology in the credit market. Along the lines of Diamond (1990), Becsi et al. (2005, 2013) introduce search and matching frictions in a credit market where borrowers and lenders try to establish a credit relationship and bargain over the interest rate consistent with the optimal loan contract. den Haan et al. (2003) study the lenders-borrowers relationships in a search and matching framework where the agents contract the liquidity allocation along with the entrepreneur’s effort choice. Vesala (2007) applies the matching frictions by an “urn ball” process to the financial markets in an economy with asymmetric information. Finally Beaubrun-Diant and Tripier (2013), study the cyclical behavior of the credit spread by using search and matching frictions in the credit market in a partial equilibrium analysis.

Differently from previous cited works we disregard the heterogeneity of agents and the moral hazard problem. Nevertheless, we also depart from the previous contributions in other several aspects. First, whereas in the den Haan et al. (2003)’s model the anticipation of the funds, in terms of goods, is determined by a liquidity allocation rule, we adopt a cash in advance (CIA) setup which requires banks to advance the funds necessary to pay for a variable wage bill depending on the real wage and employment. Households choose consumption, employment and the level of deposits, and the banking sector maximizes its profits with respect to the number of vacancies to post and the lines of credit to offer. As in Becsi et al. (2005) we improve on the standard pairwise matching model by Diamond (1990) by allowing for the endogenous entry of firms in the steady state equilibrium. Furthermore we consider a general equilibrium approach whereas Beaubrun-Diant and Tripier (2013), use a partial equilibrium analysis. Finally, differently from all previous works, in our model the bargained interest rate on loans depends on the policy (deposit) rate: this allows us to study how the monetary policy affects the determination of the banking lending rate and the dynamics of the economy.

This work firstly provides a new explanation of the sluggish adjustment of the banking lending rates to modifications of the policy rate and of the countercyclical dynamics of the CS based on the search costs in the credit market and the bargaining mechanism which determines the interest rate on loans. Moreover the Bayesian estimation and comparison provide a positive evidence in...
favor of a model where the banking lending rate depends on its past value, on the policy rate and on credit search and matching frictions.

The work is structured as follows. In the next section we report the main macroeconomic implications related to an incomplete interest rate pass-through and to the countercyclical behavior of the credit spread. In section 3 we describe the model economy. In section 4 we discuss our estimation strategy. In section 5 we present the dynamic properties of the model and our results on interest rate PT and CS. The comparison of alternative models generating incompleteness of interest rate PT is done in section 6. Finally section Section 7 concludes.

2 Macroeconomics and credit frictions

Incompleteness of the interest rate PT is confirmed by several empirical contributions. Differences exist across countries (see Karagiannis et al., 2010) and since episodes of financial crisis may alter the speed and the degree of the response of the banking lending rates to changes of the policy rate, non linear VAR/VECM models are often used. In particular, by using a static VECM model IMF (2008) shows that in the short run “the initial impact of changes in the policy rate takes more than two or three months to take effect.” Furthermore, forecasts of interest rates can be useful to determine the PT (see Baumerjee et al., 2010). Finally, a partial adjustment of the banking lending rates may accommodate variations in credit demand by providing insurance to firms against liquidity shocks but potentially increasing their overall riskiness as well as the business cycle volatility (see Burgasteller and Sharler, 2010).

One of the most common explanations refers to the bank’s collusive behavior and the concentration in the financial market (Sander and Kleimeier, 2004). In particular Van Leuvensteijn et al. (2013) show that the competitive pressure is greater in the loan market than in the deposits market such that banks under competition compensate the reduction of the revenues in the loan market by lowering the deposit rates. Explanations of the incompleteness of the interest rate PT also rely to the presence in the credit market of agency costs à la Stiglitz and Weiss (1981) and customer switching costs à la Klemperer (1987). The former depending on the imperfect information which characterizes the financial market can provide credit rationing effects; the other one refers to learning and transactions costs or any type of cost imposed by firms. Further, fixed adjustment costs or menu costs can explain the sluggishness of the retail rates (Hannan and Berger, 1991; Hofmann and Mizen, 2004). Close customer relationships developed over time (Berger and Udell, 1992; Gambacorta and Mistrulli, 2014) can be relevant: banks with close relationships to their customers may hold interest rates relatively constant despite variations of the policy rate. Hannan and Berger (1991) also propose the so-called customer reaction hypothesis linked to different clients’ reaction with respect to an upward or downward price change and to the degree of the bargaining power of borrowers.

The previous arguments are also often used to justify the behavior of the CS which is one of the main indicator of the business cycle dynamics and volatility. Chen (1991) and Fama and French (1989) show as the difference between the average yields on BAA rated and AAA-rated corporate bonds rises during recessions and fall during business cycle booms. Then, several authors argue that credit spread can be used as leading indicator of the business cycle both in the short term (Stock and Watson, 1989) and for long maturity (Guha and Hiris, 2002). Gilchrist et al. (2009) show as shock to corporate credit spreads account for a significant fraction of the variance in U.S. economic activity.

Furthermore, it is useful to note as movements in interest rate spreads can also be associated to the literature concerning the banking markups. Hence, Dueker and Thornton (1997), by employing
a model with switching costs by which baking industry has some kind of market power, show for
the period 1973-1993 in U.S. a countercyclical behavior of the loan spread defined as the difference
between the commercial paper rate (prime lending rate) and the Treasury bill rate (180-day certifi-
cates of deposit rate). Corvoisier and Gropp (2002), by using yearly data from 1995 and 1999 for
11 Euro Area countries finds a countercyclical movement of the difference between the contractual
loan and deposit rates. Andreasen et al. (2013) study the persistence of the average deposit and
loan rate in a RBC model with maturity transformation and multi-period loans. They find that
the former is less persistent than the second one, confirming the U.S. empirical evidence by using
corporate bond data. In this model the stickiness of the loan rate depends on the duration of the
loan contract for which the loan rate is fixed; on the other hand depositors face a floating rate each
period. A recent contribution by Olivero (2010) proposes a two country, two good RBC model with
complete asset market and noncompetitive financial intermediation to study the transmission of
the productivity shock in an open economy. She finds a main role of the countercyclical behavior of
the loan margin in explaining the cross-country dynamics of the principal macroeconomic variables.
Moreover she confirms the countercyclical behavior of the banking interest rate spread in U.S. by
employing different methods.

After the recent financial crises several authors argue as the CS and interest rate PT can affect
the transmission of exogenous shocks in presence of credit frictions. Curdia and Woodford (2009,
2010) introduce financial frictions into a standard NK DSGE model. They consider different shocks
in an economy in which spreads may exist. In line with Lown and Morgan (2002) they find that
the CS is compressed during monetary policy tightening. So, differently from the works cited in
the previous section, this means that the loan spread has a procyclical behavior even though the
interest rate PT is incomplete. Furthermore, they find that a modified interest rate rule that reacts
to the contemporaneous variation of the credit spread can improve upon the standard Taylor rule
by reducing distortions caused by some kind of disturbances as variations of the risk of bad loans.
However, the optimal degree of adjustment is not the same for all shocks. Further, it is smaller
than that proposed by McCulley and Toloui (2008) (100% of the spread’s change) and depends on
the degree of persistence of the disturbances.

The cost channel of monetary policy may be affected by the degree of incompleteness of the
interest rate pass-through: In New Keynesian DSGE models if the cost channel exists any exogenous
shock to the economy generates the stabilization trade-off for the monetary policy. Hence a limited
PT also has macroeconomic implications: the dynamics of the retail rates by affecting the cost
channel may amplify or moderate of output and inflation fluctuations. Scharler (2008) investigates
on the previous point by using a New Keynesian DSGE model; she finds that a more incomplete PT
in the long run reduces the output volatility at the cost of higher inflation volatility with respect
to a cost push shock. Hence, the incompleteness exacerbates the typical trade-off of the monetary
policy that results inefficient. A recent investigation on optimal monetary policy under imperfect
interest rate pass-through is proposed by Kobayashi (2008). He finds that the interest rate PT
creates fluctuations in the average loan rate which, by the cost channel, determines an inefficient
allocation of worked hours. Hence, a Central Bank has to stabilize the loan rate when it determines
the optimal monetary policy. The previous stabilization involves in a more inertial response of the
policy rate in presence of technology and preference shock and in a more sharp response when an
exogenous shock directly push up the banking lending rates.

Hülsewig et al. (2009) also explore the effects of the cost channel on inflation’s dynamics that
arise from the presence of an incomplete PT. They employ the Calvo setup for the banking lending
rate which allow them to study both the short and the long run PT of a monetary policy shock to
a interest rate on loans: a larger fraction of banks which charge the last period rate on loans and
a less competitiveness in the banking sector imply a more incompleteness of the interest rate PT.
The cost channel of the monetary policy contributes to reproduce a delayed and inertial dynamics of the price inflation; the incomplete interest rate pass-through attenuates the cost channel effect. A similar attenuation effect is found by Gerali et al. (2010) in a New Keynesian DSGE model with financial frictions and quadratic costs for adjusting retail rates under imperfect competition in the banking sector and incomplete interest rate PT of policy rates to retail rates. Güntner (2011) finds that the degree of monopolistic competition affects the PT of policy rates to banking lending rates in the short run. Specifically, when a monetary policy shock hits the economy, a less competition among banks provides a financial accelerator effect by changes of deposit rates and an attenuator effect by changes of the banking lending rates reducing the efficiency of the monetary policy.

Finally, the interest rate pass-through can be relevant for the determinacy of the model’s equilibrium. Kwapil and Scharler (2006) show that the standard Taylor principle is not sufficient to avoid fluctuations due sunspot shocks: in presence of incomplete interest rate PT the monetary policy rate have to rise more than the increase that would occur in the case of perfect PT otherwise a no determinate equilibrium may be possible. Further, they also show as a limited pass-through can help to stabilize the fluctuations arising from fundamental shocks. This is especially true when the interest rate PT is incomplete in the long run and in bank-based financial systems (Kwapil and Sharler, 2010).

3 The Model Setup

We introduce search and matching frictions in the financial market into a cash in advance New Keynesian DSGE model with sticky prices. The economy is composed by four sets of agents: households, firms, banks and a monetary authority. In order to pay the wage bill and produce, firms that do not possess capital must obtain loans from banks. Hence, before production begins, wholesale competitive firms search for lines of credit posted by banks, \( V_B^t \), which also collect deposits, \( D_t \), from households. Then banks can match with wholesalers. Each realized match provides the firm with the funds necessary to pay wages to the workers whose nominal value is \( P_t w_t N_t \) where \( P_t \) is the price index of the economy, \( w_t \) is the real wage and \( N_t \) represents the household members employed. After wages are paid production occurs. Monopolistic competitive retail firms transform wholesale homogeneous goods into differentiated retail goods which are sold to households. At the end of the period, banks receive from firms the principal plus interest on loans; households receive profit income from financial intermediaries and firms, and the principal plus interest on deposits. A fraction of the existing financial relationships between wholesale firms and banks is separated in each period according to an exogenous separation rate. The firms that do separate from banks obtain loans also in the next period. The monetary authority sets the rate of interest according to a rule to be specified below.

3.1 Matching

In the credit market search frictions prevent some firms from obtaining the lines of credit necessary to borrow funds, and some banks from filling all their posted lines of credit vacancies. Banks choose the number of credit vacancies, \( V_B^t \), they want to post, whereas the demand for lines of credit is represented by the number of wholesale firms searching for a bank, \( s_F^t \). In order to describe the matching process, we employ a Cobb-Douglas function for the matches in the credit market: \( H_t = \varsigma_t (V_B^t)^\xi (s_F^t)^{1-\xi} \), where \( \varsigma_t \) represents a credit market efficiency shock whose stochastic stationary first-order autoregressive process is \( \varsigma_t = \varsigma_{t-1} e^{\xi t} \) with \( e^{\xi t} \sim N(0, \sigma^2) \). Hence, it is possible to specify the credit market probabilities in a form which will be useful when analyzing steady states and
log-linearizing the model: \( p_t^B = H_t/s_t^F = \varphi_t(V_t^B)^{\xi}(s_t^F)^{-\xi} \) and \( q_t^B = H_t/V_t^B = \varphi_t(V_t^B)^{1-\xi}(s_t^F)^{1-\xi} \).

The credit market tightness is \( \theta_t^C \equiv s_t^F/V_t^B \). It follows that: \( p_t^B = q_t^B/\theta_t^C \). If credit market tightness increases (because the number of firms searching for a line of credit goes up, or because the number of credit vacancies posted by banks falls), the probability that a firm matches with a line of credit posted by a bank, \( p_t^B \), diminishes, whereas the probability that a credit vacancy is filled, \( q_t^B \), increases. Moreover, it is possible to define the inverse of credit market tightness as an index of the liquidity of the credit market (Wasmer and Weil, 2004).

As in the search and matching models in the labor market, the elasticities with respect to searchers and vacancies in the credit market measure externality effects. In particular: \( \xi \) represents the positive externality (the liquidity market effect) caused by banks on searching firms; \( \xi - 1 \) is the negative effect of the banks on the other financial intermediaries; \( -\xi \) represents the congestion effect determined by the firms having a credit relationship with a bank on the firms which do not have a financial relation; \( 1 - \xi \) measures the positive externality from firms searching for a line of credit to banks.

### 3.2 Households

There exists a continuum of households of mass one maximizing the expected discounted value of their utility. The preferences of the representative household are defined over a composite consumption good, consisting of the differentiated goods produced by retail firms, and leisure. The household enters each period with a given amount of nominal cash holding \( M_t \) and buy retail goods using the money endowments and the wage income \( (P_tw_tN_t) \) net of nominal deposits with banks \( D_t \). It follows that \( M_t + P_tw_tN_t - D_t \) is spent to purchase consumption goods from retail firms, of value \( P_tC_t \).

As in Dixit and Stiglitz (1977), it is \( C_t = \left( \int_0^1 C_{it}^{1-\varepsilon} dt \right)^{\frac{1}{1-\varepsilon}} \), where \( C_{it} = (P_{it}^F)^{-\varepsilon}C_t \) and \( \varepsilon > 1 \) is the parameter governing the elasticity of individual goods, which are indexed by \( i \). The cost of one unit of the consumption basket is given by the aggregation of the prices of the differentiated products, \( P_t = \left( \int_0^1 P_{it}^{1-\varepsilon} dt \right)^{\frac{1}{1-\varepsilon}} \). Hence, the purchase of consumption goods is subject to the CIA constraint: \( P_tC_t \leq M_t + P_tw_tN_t - D_t \).10 At the end of the period households receive firms’ and banks’ profits, denoted by \( \Pi_t^F \) and \( \Pi_t^B \), and obtain the reimbursement of their deposits plus the interest on them: \( R_t^P D_t = (1+r_t^P)D_t \). It follows that the amount of money carried over to the following period is: \( M_{t+1} = M_t + P_tw_tN_t - D_t - P_tC_t + \Pi_t^F + \Pi_t^B + R_t^P D_t \).11 By substituting the CIA constraint into this equation we get: \( M_{t+1} = \Pi_t^F + \Pi_t^B + R_t^P D_t \). Calculating this equation a period backward and substituting the result into the CIA constraint gives: \( P_tC_t = P_tw_tN_t + \Pi_{t-1}^F + \Pi_{t-1}^B - D_t + R^P_{t-1} D_{t-1} \), which can be expressed in real terms as:

\[
C_t = w_tN_t + \frac{\Pi_{t-1}^F}{P_t} + \frac{\Pi_{t-1}^B}{P_t} - \frac{D_t}{P_t} + \frac{R^P_{t-1} D_{t-1}}{P_t} \tag{1}
\]

This equation states that consumption and savings are financed by real labor income \( w_tN_t \), the sum generated by previous period deposits, \( \frac{R^P_{t-1} D_{t-1}}{P_t} \), and profits from banks and retailers, \( \frac{\Pi_{t-1}^F + \Pi_{t-1}^B}{P_t} \). The representative household hence solves the problem:

\[
J_t^H = \max \left[ \varphi_t U(C_t, N_t) + \beta E_t J_{t+1}^H \right] \\
s.t. \tag{1}
\]

\[ J_t^H = \max \left[ \varphi_t U(C_t, N_t) + \beta E_t J_{t+1}^H \right] \\
\]
where $\beta$ is the household’s subjective discount factor and $\varphi_t$ is a preference shock on the wedge between consumption and leisure whose stochastic process is $\varphi_t = \varphi_{t-1} e^{\epsilon_t}$ with $\epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2_\varphi)$.

A CRRA specification for $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \varphi_t \frac{N_t^{1+\varphi}}{1+\varphi}$ provides the first order conditions which lead to the standard Euler equation of the baseline New Keynesian model and to the definition of the real wage equal to the marginal rate of substitution between consumption and leisure:

$$\lambda_t = R^D_t \beta E_t \frac{P_t}{P_{t+1}} \lambda_{t+1}$$

(2)

$$w_t = \varphi_t \bar{\varphi} \frac{N_t^\varphi}{C_t^{-\sigma}}$$

(3)

where $\lambda_t = \varphi_t C_t^{-\sigma}$ is the marginal utility of consumption, $\bar{\varphi}$ is a constant term, and $\varphi_t$ is a preference shock on leisure whose stochastic process is $\varphi_t = \varphi_{t-1} e^{\epsilon_t}$ with $\epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2_\varphi)$. The unemployment is $U_t = 1 - N_t$.

### 3.3 Wholesale firms

There exists a continuum of wholesale firms in the unit interval producing homogeneous goods in a competitive sector. The production function of the representative wholesale firm is:

$$Y_t^w = A_t N_t^\alpha$$

(4)

where $A_t$ is a productivity shock with unit mean, $E_t(A_t) = 1$, and whose stochastic stationary first-order autoregressive process is $A_t = A_{t-1} e^{\epsilon_A_t}$ with $\epsilon_A_t \overset{i.i.d.}{\sim} N(0, \sigma^2_A)$. The representative firm must determine the labor demand. Its profit maximization problem is:

$$\max \frac{Y_t^w}{\mu_t} - w_t R^L_t N_t$$

s.t. (4)

where $\mu_t$ is the mark-up of the retail sector over the price of the wholesale good, $P_t/P^w_t$, and the costs depend on the repayment of the loans received by banks (the wage bill granted to households plus the interest on loans). The first order condition with respect to the employment yields:

$$\frac{1}{\mu_t} = \frac{w_t R^L_t}{mpl_t}$$

(5)

where $mpl_t = \alpha A_t N_t^{\alpha-1}$ is the labor marginal productivity. Given the competitiveness of the wholesale sector, the mark-up $\frac{1}{\mu_t}$ is equal to the real marginal cost paid by the retail firms to buy the homogeneous good. It is thus $\frac{1}{\mu_t} = m_{ct}$, i.e., the firms’ real marginal cost is equal to the usual ratio between the labor cost, $w_t R^L_t$, and the labor marginal productivity, $mpl_t$.

### 3.4 Retail firms

Retail firms purchase the goods produced by the wholesale sector and transform them into the differentiated products purchased by households. Each firm, which is a monopolist in its sector sets prices according to the Calvo (1983) rule, adjusting its price with probability $1 - \omega$. We assume that credit vacancy posting is “produced” at no cost by a specialized firm. Vacancies are costs for banks and proceeds for the specialized firm which enter aggregate profits that can be spent by
households on the basis of the consumption demand for the individual good. This allows us to write \( C_{it} = Y_{it} \), or in aggregate terms, \( C_t = Y_t \). Then, in a symmetric equilibrium, all firms set the price \( P_t^* = P_t \), so as to maximize the expected lifetime profits subject to the demand:

\[
\max E_t \sum_{l=0}^{\infty} \omega^l \beta^l \left[ \frac{C_{t+l}}{P_{t+l}} \right] Y_{it+l}
\]

s.t. \( Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t \)

This provides the price equation:

\[
\frac{P_t^*}{P_t} = \Theta \frac{E_t \sum_{l=0}^{\infty} \omega^l \beta^l m_{t+l} \left( \frac{P_{t+l}}{P_t} \right)^{\varepsilon} C_{t+l}^{1-\sigma}}{E_t \sum_{l=0}^{\infty} \omega^l \beta^l C_{t+l}^{1-\sigma}}
\]

where: \( \Theta = \varepsilon \). Under flexible prices, equation (6) reduces to the standard Blanchard and Kiyotaki (1987) equation:

\[
\frac{P_t^*}{P_t} = \Theta m_{t}
\]

From (6) the usual (log-linearized) New Keynesian Phillips curve (NKPC) is obtained:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{m}_t + \hat{\psi}_t
\]

where \( \kappa = \frac{(1-\beta\omega)(1-\omega)}{\omega} \) and \( \hat{\psi}_t = \rho^\psi \hat{\psi}_{t-1} + \epsilon_t^\psi \) with \( \epsilon_t^\psi \sim N \left( 0, \sigma_\psi^2 \right) \) is the log-linearized version of the cost push shock process \( \psi_t = \psi_t^\psi \epsilon_t^\psi \). The symbol “hat” denotes the percentage deviation of a variable from its steady state value. The NKPC (8) can be expressed in terms of the output gap \( x_t = \bar{Y}_t - \hat{Y}_t^q \) where \( \hat{Y}_t^q = \frac{1+\phi}{[1+\phi+\alpha(\sigma-1)]} \hat{A}_t \) represents the quasi flexible equilibrium output:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa (\hat{R}_t^L + \hat{\psi}_t) + \kappa \tau x_t + \hat{\psi}_t
\]

where \( \tau = \frac{1+\phi+\alpha(\sigma-1)}{\alpha} \).

### 3.5 Banks

Each match in the credit market provides firms with the funds necessary to pay the wage bill to the households. Production then starts, the proceeds from sales allow firms to repay the loans and to pay the charged interest to the bank. In the following period, if a separation does not occur, each of these firms will continue to have their wage bill financed by banks. The exogenous separation rate in the credit market is denoted \( \rho^B \in [0,1] \). We make the timing assumption that the new matches in the credit market are transformed into lines of credit immediately. Then the lines of credit financing firms evolve according to:

\[
L_t^N = (1-\rho^B)L_{t-1}^N + q^B V_t^B
\]

Assuming that the atomistic wholesale firms have unit mass, the previous equation contributes to determine the fraction of those searching for credit:

\[
s_t^F = 1 - (1-\rho^B)L_{t-1}^N
\]
At the beginning of the period banks receive deposits and money injection, \(X_t = M_{t+1} - M_t\), from households and monetary authority, respectively.\(^{15}\) At the same time they use this cash to lend loans and pay the vacancies’ cost.\(^{16}\) Then, in each period, the following equilibrium condition must holds:

\[
\frac{D_t}{P_t} + \frac{X_t}{P_t} = \frac{L_t}{P_t} + k^B V_t^B
\]

where \(k^B\) is the real cost of posting a credit vacancy and \(\frac{L_t}{P_t} = w_t N_t L_t^N\) represents the loans financing the wage bill of the firms having a line of credit. At the end of period banks repay the depositors:

\[
R_t^D \frac{D_t}{P_t} = R_t^D \left( w_t N_t L_t^N + k^B V_t^B - \frac{X_t}{P_t} \right)
\]

Given the net revenues, \(\frac{R_t^L L_t}{P_t} - \frac{R_t^D D_t}{P_t}\), the optimal value function of the representative bank is:

\[
J_t^B = \max \left[ (R_t^L - R_t^D)w_t N_t L_t^N - R_t^D k^B V_t^B + R_t^D \frac{X_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}^B \right]
\]

(12)

The bank chooses \(V_t^B\) by maximizing (12) subject to (10). Its decision yields:

\[
\frac{k^B R_t^D}{q_t^B} - (R_t^L - R_t^D)w_t N_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}^B}{\partial L_t^N}
\]

(13)

By using the envelope theorem we obtain:

\[
\frac{\partial J_t^B}{\partial L_{t-1}^N} = (1 - \rho^B)(R_t^L - R_t^D)w_t N_t + \beta(1 - \rho^B)E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}^B}{\partial L_t^N}
\]

(14)

Combining equations (13) and (14) we get what we interpret as the “credit creating condition”:

\[
\frac{k^B R_t^D}{q_t^B} = (R_t^L - R_t^D)w_t N_t + (1 - \rho^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{k^B R_{t+1}^D}{q_{t+1}^B}
\]

(15)

The condition to offer a new line of credit depends on the bank’s discounted stream of earnings and of savings on credit vacancy posting. In particular, the expected cost of financing a matched firm, \(\frac{k^B R_t^D}{q_t^B}\), is equal to the marginal profits that bank obtains from the loan advanced to a matched firm plus the expected saving the following period of not having to create a new match. Note that if \(k^B = 0\), then it must be \(R_t^L = R_t^D\).

### 3.6 Loan interest rate bargaining

The rate of interest on loans is negotiated by banks and firms by a Nash bargaining. The value of an unfilled credit vacancy is:

\[
B_t^u = -k^B R_t^D + q_t^B B_t^m + (1 - q_t^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} B_{t+1}^u
\]

(16)

The value of an unfilled credit vacancy is provided by the cost incurrence \(k^B R_t^D\) and by the bank current and (discounted) future values, \(B_t^m\) and \(B_{t+1}^u\) which a bank gets if a credit match is obtained (with probability \(q_t^B\)) or not (with probability \(1 - q_t^B\)), respectively. Banks open vacancies until it
is profitable to do so. Given the free entry condition \( B^u_t = 0 \) \( \forall t \), the value of a filled credit vacancy is \( B^m_t = \frac{k^BR^D}{q^B_t} \) and the bank’s surplus, \( S^B_t = B^m_t - B^u_t \), is:

\[
S^B_t = \frac{k^BR^D}{q^B_t} \tag{17}
\]

or, from condition (15), \( S^B_t = (R^L_t - R^D_t)w_tN_t + (1 - \rho^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{k^BR^D}{q^B_{t+1}} \).

The surplus of a firm is the difference between the value of the firm if a match is obtained, \( F^m_t \), and if it is not obtained, \( F^u_t \). \( F^m_t \) is equal to the current profits of the firms plus the expected value in the following period. In period \( t + 1 \) the firm will have a value equal to \( F^m_{t+1} \) if it does not experience a separation with the bank \( (1 - \rho^B) \) or if a separation occurs and it finds a new match in the credit market \( (\rho^B p^B_{t+1}) \). The firm’s value will be equal to \( F^u_{t+1} \) if it does not realize a match in the credit market after a separation, \( \rho^B (1 - p^B_{t+1}) \). If a firm in period \( t \) does not find a match with a bank it does not obtain the funds necessary to start production, its current profits are zero and its current value depends only on its expected value: if in the period \( t + 1 \) it finds a match the value will be \( F^m_{t+1} \); it will be \( F^u_t \) otherwise. So the values of the generic firm are:

\[
F^m_t = \frac{Y^w}{\mu_t} - w_tR^L_t N_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \rho^B + \rho^B p^B_{t+1})F^m_{t+1} + \rho^B (1 - p^B_{t+1})F^u_{t+1} \right] \tag{18}
\]

\[
F^u_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ p^B_{t+1}F^m_{t+1} + (1 - p^B_{t+1})F^u_{t+1} \right] \tag{19}
\]

and the firm’ surplus is:

\[
S^F_t = F^m_t - F^u_t = \frac{Y^w}{\mu_t} - w_tR^L_t N_t + (1 - \rho^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - p^B_{t+1})S^F_{t+1} \tag{20}
\]

The loan interest rate is determined by the maximization of the Nash product:

\[
\max(S^F_t) = (S^B_t)^{1-z} \tag{21}
\]

where \( z \) represents the bargaining power of firms. The optimal condition is:

\[
(1 - z)\gamma^B_t S^F_t + z\gamma^F_t S^B_t = 0 \tag{22}
\]

where \( \gamma^B_t = \frac{\partial S^B_t}{\partial R^D_t} = w_1N_t \) and \( \gamma^F_t = \frac{\partial S^F_t}{\partial R^L_t} = -w_1N_t \) are the marginal effects of the loan interest rate on the surplus of the agents. Then the optimal condition is reduced to \( (1 - z)S^F_t = zS^B_t \). By using the definition of the bank’s surplus it is possible to write:

\[
S^F_t = \frac{z}{(1 - z)} \frac{k^BR^D}{q^B_t}. \tag{23}
\]

By using the credit creating condition (15) and the definitions of agents’ surplus (17) and (20) it is possible to obtain the bargained loan interest rate:

\[
R^L_t = \frac{(1 - z)Y^w}{w_1N_t \mu_t} + \frac{z}{w_1N_t} \left[ R^D_t w_1N_t - (1 - \rho^B)\beta E_t \frac{\lambda_{t+1} k^B R^D_{t+1}}{q^B_{t+1}} \right] \tag{24}
\]

The interest rate on loans turns out to be a weighted average of the firm’s revenues, on one side, and the rate of interest on deposits net of the banks’ future expected present saving from maintaining a credit relation with a firm, on the other side. The weights are the relative bargaining powers of the agents. If \( z = 1 \) firms are able to obtain a loan interest rate equal to the deposit interest rate net the bank’ saving; if \( z = 0 \) banks are able to set a loan interest equal to the firm’s marginal profit.
3.7 Interest rate pass-through and credit spread

The absence of asymmetric information, default probabilities and bankruptcy costs implies that the definition of the interest rate spread used in this work is different from those of external financial premium and of corporate credit spread.\textsuperscript{17} In particular, we define the credit spread as

\[ SP_t = R^L_t - R^D_t \]

where its log linearized version is \( \hat{SP}_t = \frac{R^L_t - R^D_t}{\hat{SP}} \). So, it is evident that the behavior of the CS depends on both dynamics and steady state values of the loan and policy rates. The interest rate pass-through is defined as the percentage deviation of the interest rate on loans from its steady state value, \( \hat{R}^L_t \), minus that of the policy rate from own steady state, \( \hat{R}^D_t \). Hence we have a complete, incomplete or more than complete interest rate PT if \( \frac{\partial \hat{R}^L_t}{\partial \hat{R}^D_t} = 0 \) is equal, less or greater than 1, respectively. Moreover by using equations (2) and (5), the log-linearized version of equation (24) can be written in the following way:

\[
\hat{R}^L_t = \Lambda_1 \hat{R}^D_t + \Lambda_2 \left[ E_t \hat{\theta}^C_{t+1} - E_t \hat{R}^D_{t+1} + \left( \hat{w}_t + \hat{N}_t - E_t \hat{\pi}_{t+1} \right) \right] + \hat{v}_t
\]

(25)

where the stochastic term \( \hat{v}_t = \rho^\nu \hat{v}_{t-1} + \epsilon_t^\nu \) with \( \epsilon_t^\nu \sim N(0, \sigma^2) \) denotes the log-linearized version of the stochastic stationary first-order autoregressive process of the exogenous shock \( v_t = v_{t-1}^\nu e^\nu_t \) appended to the bargained interest rate on loans equation to estimate the model.\textsuperscript{18} The term \( \Lambda_1 = \frac{\alpha}{R^\nu(\alpha-1+z)} \hat{R}^D_t + \frac{(1-\rho^\nu)p^B_k q^B}{wN} \) represents the direct PT from the policy rate, \( \hat{R}^D_t \), to the retail banking lending rate, \( \hat{R}^L_t \), whereas the coefficient \( \Lambda_2 = \frac{\alpha z}{R^\nu(\alpha-1+z)} \frac{(1-\rho^\nu)p^B_k q^B}{wN} \) represents the indirect PT due to the credit frictions depending on the expected credit market tightness \( E_t \hat{\theta}^C_{t+1} \), the expected policy rate \( E_t \hat{R}^D_{t+1} \), and on the nominal value of the loan (equal to the wage bill) lent to the generic wholesale firm. It is useful to note that the steady state banking lending interest rate can be written as \( R^L = \Upsilon R^D \) where \( \Upsilon = \frac{z \left\{ \frac{1}{1-\frac{\alpha}{\eta}} + \frac{\eta}{\eta} \right\}}{\frac{1}{1-\frac{\alpha}{\eta}} + \frac{\eta}{\frac{\alpha}{\eta} \frac{1}{1-\frac{\alpha}{\eta}}} \} \) is the mark-up over the steady state policy rate.\textsuperscript{19}

By observing equation (25) we get the following points:

**Proposition 1** If banks have no bargaining power, i.e. \( z = 1 \), then \( \Upsilon = 1 \) and \( R^L = R^D \), the direct pass-through is more than complete, i.e. \( \Lambda_1 > 1 \), and the degree of the interest rate pass-through depends on the credit frictions measured by \( \Lambda_2 \).

When banks have no bargaining power, firms are able to obtain a banking lending interest rate which differs from the policy rate only for the presence of the banks’ saving. Hence, in this case, the posting costs are paid period by period from banks and they do not affect the steady state interest rate on loans.

**Proposition 2** If the posting cost \( k^B = 0 \) the indirect pass-through is \( \Lambda_2 = 0 \) and \( z = 1 \). Then, the direct pass-through \( \Lambda_1 = 1 \) and the pass-through is perfectly complete and \( \hat{R}^L_t = \hat{R}^D_t \).

When the posting activity is free there are not expected saving in the following period: search and matching credit frictions do not matter and the banking lending rate is the result of a more simplified weighted average of the current revenues of the firms and the policy rate. The degree of the interest rate pass-through depends on the value of the bargaining power \( z \). Moreover, when \( k^B = 0 \), in order to have consistency among equations (15), (25) and the steady state version of the loan rate, \( R^L = \Upsilon R^D \), it must be \( z = 1 \).
Proof.  
Equation (15) is $k^B R^D = (R^L - R^D) w_1 N_t + (1 - \rho^B) \beta E_1 \frac{\lambda_{t+1}^L k^B R^D_{t+1}}{q_{t+1}^B}$. Then, if $k^B = 0$ we have: $R^L_t = R^D_t$.  
Equation (25) is $\hat{R}^L_t = \Lambda_1 \hat{R}^D_t + \Lambda_2 \left[ E_t \theta^C_{t+1} - E_t \hat{R}^D_{t+1} + \left( \hat{w}_t + \hat{N}_t - E_t \hat{\pi}_{t+1} \right) \right]$. Then, if $k^B = 0$ we have: $\hat{R}^L_t = \Lambda_1 \hat{R}^D_t$. In order to obtain $R^L = R^D$ and $\hat{R}^L = \hat{R}^D$, it must be $\Lambda_1 \frac{\alpha z}{(\alpha - 1 + z) \Omega} = 1$ where $\Omega = z(1 + \Omega) \left[ 1 - \frac{(1 - z)}{1} \right]$ and $\Omega = \left\{ \frac{(1 - \rho^B) \beta p^B}{1 - (1 - \rho^B) \beta} \right\} > 0$ does not depend on $z$. Then, by simplifying we have:

$$\frac{\alpha z \left[ 1 - \frac{(1 - z)}{\alpha} + z \Omega \right]}{(\alpha - 1 + z) z (1 + \Omega)} = 1 \implies (\alpha - 1 + z) + \alpha z \Omega = (\alpha - 1 + z) (1 + \Omega) \implies \alpha z = (\alpha - 1 + z) \implies (\alpha - 1) z = (\alpha - 1) \implies z = 1$$

It is useful to observe that $\Lambda_1 < 1 \rightarrow z > 1$ but this condition is impossible by definition because $z$ lives in a uniform interval. On the other hand $\Lambda_1 > 1 \rightarrow z < 1$ but this condition implies $R^L \neq R^D$ and $\hat{R}^L \neq \hat{R}^D$. So, the only possibility is $z = 1$.

From the steady state version of the loan rate, $R^L = \Upsilon R^D$, given $\alpha \neq 0$ to obtain $R^L = R^D$ we must have $\Upsilon = z(1 + \Omega) \left[ 1 - \frac{(1 - z)}{\alpha} + z \Omega \right]^{-1} = 1$. The only way is $z = 1$. In fact in this case $\Upsilon$ reduces to $(1 + \Omega) [1 + \Omega]^{-1} = 1$.

Differently from the standard literature on search and matching with a bargaining mechanism, and following Becsi et al. (2005), we assume a steady state equilibrium condition for the endogenous entry of firms. We suppose that the unmatched value of firms is not zero, but that it can be thought of as the firm’s reservation value which is associated to a flow entry cost. This allows us to determine the endogenous credit line finding rate of firms.20 Firms will search for a line of credit as long as their unmatched value exceeds their entry cost, $\bar{\sigma}$. By contrast, firms do not enter the credit market if the entry cost is larger than the unmatched value. Hence, the entry competition among the atomistic wholesale firms implies, in equilibrium, the steady state condition $F^u = \bar{\sigma}$. By using the steady state version of equations (18) and (19), and by imposing $F^u = \bar{\sigma}$, we obtain the zero profit condition:

$$p^B = \frac{\bar{\sigma}(1 - \beta)(1 - \beta + \beta p^B)}{\beta \left( \frac{\hat{Y} w}{\mu} - w R^L N \right) - (1 - \beta)(1 - \rho^B) \bar{\sigma}} \quad (26)$$

Consistently with the Becsi et al. (2005) findings, this condition states that the credit line finding rate satisfying the zero profit condition increases with the entry cost, $\bar{\sigma}$, the interest rate on loans, $R^L$, and the exogenous separation rate, $\rho^B$, and decreases with the productivity level, $A$.21 Given the inverse relationship between the net profits of firms and their probability of obtaining a line of credit, it is possible to highlight the congestion effect in the credit market. As the firms’ (expected) profits rise, more firms look for a line of credit in the financial market, the probability to find a line of credit falls and the (extra)profits are reset.22 Furthermore we can observe that:

**Proposition 3** Being $a$ and $b$ two different scenarios, if $k^B \rightarrow 0$ then $q^B \rightarrow 0$ and $\frac{k^B}{k^B} = \frac{q^B}{q^B}$ such that the expected posting cost $\frac{k^B}{q^B}$ is always constant. Then variations of the degree of the interest
rate pass-through depend on changes of the expected posting cost, i.e. \( \Delta \left( \frac{k^B}{q^B} \right) \). Moreover since the expected firms entry cost can be rewritten as \( \rho = k^B R^D (1 - z)(1 - \beta) \), variations of \( k^B \) do not modify the ratio \( \frac{\rho}{q^B} \).

**Proof.** The steady state version of equation (15) yields \( q^B = \frac{k^B R^D (1 - (1 - \rho))}{(R^L - R^D)w_N} \). Given \( \rho^B \), \( N \) and \( \beta \), and since \( R^L \), \( R^D \) and \( w \) are not affected by changes of \( k^B \), if in the scenario \( a \) \( k^B = k^B \) yields \( q^B = q^B \), and in the scenario \( b \) \( k^B = k^B \) yields \( q^B = q^B \), then we always get \( \frac{k^B}{q^a} = \frac{k^B}{q^b} = \frac{k^B}{R^D (1 - (1 - \rho))} \) constant. Moreover, given \( F^u = \tau \), the steady state version of the equation (19) provides \( S^F = \frac{1 - \beta \tau}{\rho^B q^B} \) whereas that of equation (23) is \( S^F = \frac{z}{1 - z} \frac{k^B R^D}{q^B} \). By equating the previous two definitions we get \( \frac{\rho}{q^B} = \frac{k^B R^D (1 - z)(1 - \beta)}{q^B z\beta} \).

The previous propositions highlights as the bargaining power \( z \) and the expected posting cost \( \frac{k^B}{q^B} \) are the determinants of the degree of the pass-through in the short run as well as of the value of the steady state banking lending rate.

### 3.8 Monetary authorities

A central bank employs the following (log-linearized) monetary rule to set the policy rate, which we assume for simplicity equal to the rate on deposits:

\[
\tilde{R}_t^D = \rho^R \tilde{R}_{t-1}^D + (1 - \rho^R) [\delta_\pi \hat{\pi}_t + \delta_x x_t] + \hat{\nu}_t
\]

where \( \rho^R \) is the degree of interest rate smoothing and \( \delta_\pi \) and \( \delta_x \) are the weights assigned to the inflation \( \hat{\pi}_t \) and output gap \( x_t \), respectively. The stochastic term \( \hat{\nu}_t = \rho^\nu \hat{\nu}_{t-1} + \epsilon_t \) with \( \epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2_\epsilon) \) denotes the log-linearized version of the stochastic stationary first-order autoregressive process of the monetary policy shock \( \nu_t = \nu^\nu_{t-1} \epsilon_t \).

### 3.9 Market clearing

The aggregate resource constraint (ARC) is derived by starting the definition of the aggregate money carried over to the following period in real terms. In particular, being \( \int L^N_{jt} dj = L^N_t \) the total number of financed firms (those having a relationship with a bank) then the total labor demand is \( L^N_t N_t \). Furthermore, by considering the total firms’ profits \( \frac{\Pi^F}{P_t} \) as the sum of those obtained by retail and specialized firms \( \left( \frac{\Pi^R}{P_t} \text{ and } \frac{\Pi^{SP}}{P_t} \right. \) respectively \) the ARC becomes:

\[
\frac{X_t}{P_t} = w_t N_t L^N_t - \frac{D_t}{P_t} - C_t + R^D_t D_t \frac{P_t}{P_t} + \frac{\Pi^R}{P_t} + \frac{\Pi^B}{P_t} + \frac{\Pi^{SP}}{P_t}
\]

The aggregate banks’ balance sheet and profit function are \( \frac{D_t}{P_t} + \frac{X_t}{P_t} = w_t N_t L^N_t + k^B V^B_t \) and \( \frac{\Pi^B}{P_t} = R^L_t w_t N_t L^N_t - R^D_t w_t N_t L^N_t - R^D_t k^B V^B_t + R^D_t X_t \frac{P_t}{P_t} \) respectively. By replacing them into equation
(28) and by remembering that $\frac{\Pi^{SP}_t}{P_t} = k^BV_t^B$ we have:

$$C_t = \frac{\Pi^R_t}{P_t} + R^L_t w_t N_t L^N_t$$

(29)

The total real aggregate wholesale production is $\int L_t^N Y_t^w \frac{P^w_t}{P_t} dj = Y_t^w \frac{P^w_t}{P_t} \int L_t^N dj = L_t^N Y_t^w \frac{P^w_t}{P_t}$. Then the real profits of retail firms are $\frac{\Pi^R_t}{P_t} = Y^d_t - L_t^N Y_t^w \frac{P^w_t}{P_t}$ where $Y^d_t$ is the aggregate demand for goods. Further the equilibrium in the good market implies $Y^d_t = Y_t$. Then the ARC becomes:

$$C_t = Y_t - L_t^N Y_t^w \frac{P^w_t}{P_t} + R^L_t w_t N_t L^N_t$$

(30)

Since its competitiveness the wholesale sector do zero profits. Then it must be: $L_t^N Y_t^w \frac{P^w_t}{P_t} = L_t^N R^L_t w_t N_t$. By replacing the latter condition into (30) we finally have:

$$C_t = Y_t$$

(31)

Furthermore the wholesale production is linked to the aggregate demand by the following expression:

$$Y_t^w = Y_t f_t$$

(32)

where $f_t = \int \left( \frac{P_t}{P_t} \right)^{-\varepsilon} di$ is a factor of price dispersion. As shown by Galí (2008) this factor is equal to zero up to a first order approximation when we linearized the model in a neighborhood of zero inflation steady state.

### 4 Estimation

The model is estimated by Bayesian methods. In this section, we first discuss the data, the calibrated parameters and the priors, and then we report the parameter estimates. In particular, we estimate the structural credit market and agents’ preferences parameters driving the model dynamics and allowing to compute the steady state version of the model. Further we employ a sensitivity analysis in order to identify the stability domain of the model.

#### 4.1 Data

We use 7 observables for the U.S.: real GDP, employment, real wage, inflation, the federal funds rate, the weighted average effective loan rate and an index of the credit market tightness. For a description of the data, see the appendix A.1. The sample period is 1997:Q2 - 2013:Q2. We use the logarithmic transformation for the quarterly interest rates, i.e. $\log(1 + \frac{r^j_t}{100})$ where $j = D, L$ whereas the credit market tightness index is demeaned and the inflation rate is computed as the quarter on quarter log difference of nominal prices. All remaining data are transformed by employing the logarithmic first difference operator. Figure 1 plots the transformed data.
4.2 Calibrated Parameters

In this section we set the values of the calibrated parameters of the model. Table 1 reports them. Due to the large consensus on the quarterly value of the discount factor by the economic literature, we impose $\beta$ equal to 0.996 so as to obtain a quarterly real steady state rate on deposits $R^D = 1.0035$ (de Walque et al., 2010). Further, we assume a logarithmic form for the utility function over consumption ($\sigma = 1$). As widely accepted by the literature on the subject matter we calibrate the sticky price parameter $\omega = 0.8$. The previous impositions imply that the coefficient attached to the real marginal costs into the equation (8) is $\kappa = \frac{(1-\beta \omega)(1-\omega)}{\omega} = 0.0507$. In line with the empirical observations we calibrate the elasticity of output to employment, $\alpha$, equal to 0.66. Finally, according to Ravenna and Walsh (2008) we set the steady state employment $N$ equal to 0.95, and the elasticity of substitution of the individual goods $\varepsilon$ equal to 6 such that the mark-up of the retail sector over the price of the wholesale good is 20 per cent ($\mu = 1.2$).

<table>
<thead>
<tr>
<th>Calibrated Parameter</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\omega$</th>
<th>$\alpha$</th>
<th>$N$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
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<td>1</td>
<td>0.8</td>
<td>0.66</td>
<td>0.95</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1: Calibrated Parameters.

4.3 Priors

In this section we declare the prior distributions of the remaining deep parameters of the model. The shape of the distributions is chosen according to the standard practice: for parameters defined in a $[0-1]$ interval we assume the beta distribution whereas for parameters which can take values over the whole support $\mathbb{R}$ we adopt the normal distribution. For parameters assuming values over
the \([0 - \infty]\) interval we assume the gamma distribution. Finally, the reference distribution for the structural shocks is the inverted gamma which is defined over the range \(\mathbb{R}^+\).

For the monetary policy rule parameters we employ values widely used by the literature. Hence, for the coefficients attached to the expected inflation and output gap terms, \(\delta_\pi\) and \(\delta_\omega\) respectively, we assume a normal distribution with prior mean 2.0 and 0.1 and a standard error equal to 1.00 and 0.05 respectively; for the autoregressive coefficient \(\rho^B\) defining the degree of the interest rate smoothness we assume a beta distribution with prior mean 0.5 and standard error 0.25. The inverse labor supply Frish elasticity parameter \(\phi\) is assumed normal distributed with prior mean equal to 0.5 and standard error equal to 0.1. For the all persistence parameters of the autoregressive stochastic processes of the exogenous shocks, we assume a beta distribution with prior mean of 0.5 and standard deviation of 0.1.

Concerning the credit market parameters we have not references on possible prior values\(^{24}\). Then, we adopt a gamma distribution for the credit vacancy posting cost with mean value equal to its labor market counterpart\(^{25}\), i.e. \(k^B = 0.1\) and standard error equal to 0.05. The prior on the firm’s entry cost \(\pi\) is harder to set, so we assume a rather widespread gamma distribution with a mean of 25 and a standard deviation of 12.5. For the matching function elasticity \(\xi\) and for the firms’ bargaining power \(\rho\) we assume an uninformative position by considering a beta distribution with prior mean equal to 0.5 and standard error equal to 0.25 for both. Finally, for the separation rate \(\rho^B\) we adopt the strategy of setting its value between the minimal (0.07) and the maximum (0.02) values of the bankruptcy rate calibrated by Dell’Ariccia and Garibaldi (1998). As a consequence the separation rate in the credit market is assumed beta distributed with prior mean 0.05 and standard error equal to 0.025.

Finally, for all standard deviation of the exogenous shocks we use the inverted gamma distribution as prior distribution with mean equal to 0.01 with two degrees of freedom\(^{26}\).

4.4 Sensitivity Analysis: Mapping Stability

In this section we identify the stability domain of the model. A Monte Carlo simulation is performed in order to detect what parameters mostly drive the model into a specific region.

According to Ratto (2008) we consider two regions: an acceptable stable region \(G\) satisfying the standard Blanchard-Kahn rank condition and an unacceptable region \(\overline{G}\) caused by the instability and indeterminacy of the model. Hence, in order to explore all the prior space we sample uniformly from the prior distributions defined above and we categorize each parameter into the two alternative regions. The sample is generated using a Sobol’s quasi Monte Carlo sequence of dimension \(N = 2048\). Hence, we get two subsets, \((\varpi_s/G)\) of size \(n\) and \((\varpi_s/\overline{G})\) of size \(\overline{n}\), representing draws from the unknown probability density functions \(f_n(\varpi_s/G)\) and \(f_{\overline{n}}(\varpi_s/\overline{G})\) where \(\varpi\) is the vector of the parameters, \(s\) is the parameter’s index and \(n + \overline{n} = N\). Finally the identification of the parameters (and relative values) driving in the (un)acceptable region is defined by the comparison of the previous density functions by the two-sided Smirnov-Kolmogorov test:

\[
d_{n,\overline{n}} = \sup ||F_n(\varpi_s/G) - F_{\overline{n}}(\varpi_s/\overline{G})||
\]

where \(F_n(\varpi_s/G)\) and \(F_{\overline{n}}(\varpi_s/\overline{G})\) are the cumulative distribution functions (cdf) of the generic parameter \(\varpi_s\). Given the null hypothesis \(f_n(\varpi_s/G) = f_{\overline{n}}(\varpi_s/\overline{G})\) and the significance level at which it is rejected, if for a parameter \(\varpi_s\) the two distributions are significantly different (a larger \(d_{n,\overline{n}}\)), it is possible define the parameter as a key driver of the model behavior as well the values of the parameter space leading in one region or in other one. Alternatively, if the distance between the distributions is not significant, \(\varpi_s\) is not important for the model’s dynamics and its values can belong, indifferently, either to \(G\) or \(\overline{G}\).
From the Monte Carlo filtering procedure we get that the 46 per cent of the prior support is stable. The remaining part gives indeterminacy (51.3 percent) and instability (2.7 percent). By running the Smirnov-Kolmogorov test we can highlight that indeterminacy is essentially driven by $\delta_\pi$. In particular, by comparing the cdf of the sample producing indeterminacy with the cdf of the original prior sample we find that small values of $\delta_\pi$ drive to indeterminacy. Table 2 reports the detailed results of the Smirnov-Kolmogorov tests.

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td>$\phi$</td>
<td>0.1170</td>
<td>0.0504</td>
<td>0.1550</td>
<td>$\rho^R$</td>
<td>0.0482</td>
<td>0.0333</td>
<td>0.2340</td>
</tr>
<tr>
<td>$\delta_\pi$</td>
<td>0.5780</td>
<td>0.3020</td>
<td>0.4290</td>
<td>$\rho^A$</td>
<td>0.0247</td>
<td>0.0104</td>
<td>0.0841</td>
</tr>
<tr>
<td>$\delta_x$</td>
<td>0.0571</td>
<td>0.0295</td>
<td>0.0105</td>
<td>$\rho^\nu$</td>
<td>0.0276</td>
<td>0.0126</td>
<td>0.1070</td>
</tr>
<tr>
<td>$\rho^B$</td>
<td>0.0711</td>
<td>0.0368</td>
<td>0.1240</td>
<td>$\rho^\varsigma$</td>
<td>0.0207</td>
<td>0.0102</td>
<td>0.1320</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.0264</td>
<td>0.0186</td>
<td>0.1620</td>
<td>$\rho^\varphi$</td>
<td>0.0262</td>
<td>0.0129</td>
<td>0.0912</td>
</tr>
<tr>
<td>$z$</td>
<td>0.0426</td>
<td>0.0543</td>
<td>0.7190</td>
<td>$\rho^\vartheta$</td>
<td>0.0261</td>
<td>0.0146</td>
<td>0.0878</td>
</tr>
<tr>
<td>$k_B$</td>
<td>0.0307</td>
<td>0.0126</td>
<td>0.0856</td>
<td>$\rho^\varphi$</td>
<td>0.0331</td>
<td>0.0149</td>
<td>0.1520</td>
</tr>
<tr>
<td>$\bar{\tau}$</td>
<td>0.1440</td>
<td>0.0935</td>
<td>0.4800</td>
<td>$\rho^\varphi$</td>
<td>0.0272</td>
<td>0.0177</td>
<td>0.0873</td>
</tr>
</tbody>
</table>

Table 2: Smirnov-Kolmogorov statistics in driving stability, indeterminacy and instability.

4.5 Posterior estimates

Table 3 summarizes the posterior mode and the posterior mean for the model’s parameters. The panel also shows the 90 percent probability intervals for the model parameters and the relative prior assumptions. Draws from the posterior distributions are obtained by running the random walk version of the Metropolis-Hastings algorithm. We ran ten parallel chains, each with a length of 100,000 replications.\(^{27}\)

The posterior mean estimates are generally close to the respective modal values. The estimation of the separation rate $\rho^B$ confirms the high mismatch between the financial and productive sectors of the economy experienced in the recent years. The importance of search and matching credit market parameters is stressed by Dell’Ariccia and Garibaldi (1998) which show as the bargaining power of banks and the speeds at which new loans become available and at which banks recall existing loans are fundamental in explaining the dynamic relationship between aggregate banking lending and interest rate changes. Our estimate of the banks’ bargaining power reports a low value in line to that found by Petrosky-Nadeau and Wasmer (2011) in a model with good, labor and financial frictions to accommodate a targeted share of the financial sector in GDP but much lower than the value estimated by Petrosky-Nadeau and Wasmer (2013) by using a “trembling hand” calibration method in a model with search and matching frictions in labor and credit markets.

The posting cost $k_B$ and the matching elasticity $\xi$ are larger and lower than the values found by Petrosky-Nadeau and Wasmer (2013) respectively. The estimates of the monetary policy parameters and of the inverse labor supply Frish elasticity parameter are in line with the literature on the subject matter.\(^{28}\)

5 Dynamic properties of the model

In order to focus on the issues of interest rate pass-through and credit spread as well as of the role of the credit market frictions on the real economy, in this work we focus on the dynamics of the model.
### Table 3: Posterior Estimates: Structural and Shock Process Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean (Std. Dev.)</td>
</tr>
<tr>
<td>φ</td>
<td>N</td>
<td>0.500 (0.250)</td>
</tr>
<tr>
<td>δπ</td>
<td>N</td>
<td>2.000 (1.900)</td>
</tr>
<tr>
<td>δx</td>
<td>N</td>
<td>0.100 (0.050)</td>
</tr>
<tr>
<td>ρR</td>
<td>B</td>
<td>0.500 (0.250)</td>
</tr>
<tr>
<td>ρB</td>
<td>B</td>
<td>0.050 (0.025)</td>
</tr>
<tr>
<td>ξ</td>
<td>B</td>
<td>0.500 (0.250)</td>
</tr>
<tr>
<td>z</td>
<td>B</td>
<td>0.500 (0.250)</td>
</tr>
<tr>
<td>kB</td>
<td>G</td>
<td>0.100 (0.050)</td>
</tr>
<tr>
<td>τ</td>
<td>G</td>
<td>25.00 (12.50)</td>
</tr>
<tr>
<td>ρA</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρν</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρς</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρφ</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρθ</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρψ</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>ρυ</td>
<td>B</td>
<td>0.500 (0.100)</td>
</tr>
<tr>
<td>σA</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
<tr>
<td>σν</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
<tr>
<td>σς</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
<tr>
<td>σφ</td>
<td>IG</td>
<td>0.010 (2)</td>
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<tr>
<td>σθ</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
<tr>
<td>σψ</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
<tr>
<td>συ</td>
<td>IG</td>
<td>0.010 (2)</td>
</tr>
</tbody>
</table>
variables with respect to a negative interest rate shock and a positive technology shock. Finally we propose an exercise in which the economy is hit by an exogenous credit market shock.

5.1 Monetary policy shocks and the interest rate pass-through

Figure 2 shows the Bayesian impulse response functions (IRFs) regarding the main macroeconomic and credit market variables. With nominal rigidities on good prices, a monetary easing implies a decrease of the policy rate that produces a lower real interest rate. This reduction determines a substitution effect between current and future consumption: households increase current spending and both the output and the output gap rise. The increase in the demand of goods implies a greater labor demand by firms: employment and wages increase and the fall in the marginal product of labor leads to higher real marginal costs. By the NKPC, inflation goes up.

The increase in the wage bill expands the borrowing demand to the banks and their expected profits rise. Banks try to increase the number of firms to finance by opening new lines of credit and their vacancy posting jumps up in a significant way. Then, the matching process works: not all credit vacancies are filled so that the increases of new credit matches and lines of credit are less significant than that of the credit supply. By equation (11) the number of firms searching for a line of credit falls starting from the next period. As a consequence in the credit market there is a less congestion effect: the credit line finding rate increases, the credit vacancy filling rate falls and the credit market tightness decreases.

In order to highlight the response of the banking lending rate we can observe equation (24) which shows that banks aspire to capture a value equal to the marginal revenues of the firms, whereas firms want set a loan rate lower than the policy rate by trying to appropriate of the banks' saving. Then, the determination of the cost of credit depends on the dynamics and the steady state effects of the model’s variables as well as on the distribution of the joint surplus by the relative bargaining power of the agents. Hence, on one hand, the fall in the labor marginal productivity...
and the increase in real wages contribute to the negative dynamics of the firms’ revenues and, on the other hand, the decrease in the credit market tightness contributes to a reduction of the banks’ saving. Nevertheless the dampening effect due to the estimated value of the banks’ bargaining power attenuates the previous dynamics such that the loan rate decreases less than the policy rate and firms are willing to accept a higher banking lending rate than they would like. This is shown in figure 2. In terms of equation (25), given the estimated values of the model’s parameters it is possible to observe that the coefficient $\Lambda_1$ is greater than 1. This means that the incompleteness of the PT depends on the term $\Lambda_2$ which measures the indirect effect due to the presence of credit frictions.

5.1.1 Bargaining power of banks and interest rate pass-through

In this section we analyze the scenario in which the banking sector has a different market power in the negotiation of the loan interest rate. We simulate the model when $z$ decreases whereas all other deep parameters remain equal to their estimated values. IRFs remain qualitatively unchanged. Then we focus on the degree of the interest rate PT. When banks have more bargaining power they have more possibilities to appropriate of firms’ profits, so their expected profits increase boosting the credit vacancy posting, the financial matches and the lines of credit. Equation (11) determines a lower reduction in the number of firms searching for a line of credit. Hence, the credit vacancy filling rate decreases more, the credit line finding rate rises more and the credit market tightness from the firms point of view falls more than the benchmark model. This changes feed back, by equation (24), on the determination of the loan interest rate, which decreases less than the benchmark model implying a more incomplete interest rate PT (figure 3). Finally it is possible to observe as small variations of $z$ determines large modifications of the loan rate response that may even lead to its overshooting. The opposite chain of effects is produced by an increase in the firms’ bargaining power.

**Proposition 4** When $0 < z < 1$ and $k^B > 0$, the greater is the banks’ bargaining power, $(1−z)$, the more incomplete is the interest rate pass-through.

5.2 Technology shocks and the credit spread

Figure 4 shows the Bayesian IRFs of the main macroeconomic and credit market variables with respect to a positive TFP shock. An increase in productivity implies an increase in the supply of goods by firms. Moreover, as stressed by Galí and Rabanal (2004) in presence of staggered prices and weak accommodation of the policy rule to the TFP shock, the potential output increases more than the actual output. This implies a not optimal decrease of aggregate price level and an increase in aggregate demand which is less than proportional to that of the productivity. Then the aggregate demand will be satisfied with lower employment (productivity employment puzzle), in line with the evidence proposed by a recent strand of the literature (Basu et al., 2004). The effect on the real wage depends on the relative strength of the income and substitution effects as well as on the set of model parameters: by the households’ first order condition (3), the real wage increases allowing for a positive response without making use of nominal or real wage rigidities. Although the fall of the employment, the increase in real wage is such that the wage bill soars. The previous effect together the increased goods’ supply does grow the banks’ expected profits. As a consequence, there is a more intensive credit vacancy posting activity: financial matches and lines of credit boost. This increase forces the number of firms searching for a line of credit to fall.
In the credit market there is a lower congestion effect from the point of view of firms: the credit vacancy filling rate goes down, the credit line finding rate goes up and the credit market tightness drops determining a diminishing of the banking lending rate, that together the increase of the labor marginal productivity, lowers the real marginal costs and the inflation. The latter decrease together the fall of the output gap lower the policy rate by the NKPC.

The credit market frictions contribute to produce countercyclical dynamics of the spread between the banking lending rate and the policy (deposit) rate which are similarly to those observed by the literature (see figure 5). In particular, when a TFP shock hits the economy, the procyclical effect due to the increase of the banks’ profits is more than offset by the countercyclical effect due to the modification of the external opportunities of firms implying greater ease of finding funds.

In order to clarify the previous considerations we can rewrite the CS by using equation (24):

\[
SP_t = \frac{(1 - z)}{w_t N_t} \left( \frac{Y^w}{\mu_t} - R^D_t \right) - \frac{z(1 - \rho^B)}{w_t N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{k^B R^D_{t+1}}{\theta^C_{t+1}}
\]

The previous equation points out that the credit spread depends on two terms: the first one is procyclical, the other one is countercyclical. On one hand, as in Beaubrun-Diant and Tripier (2013), the term \( \frac{(1 - z)}{w_t N_t} \left( \frac{Y^w}{\mu_t} - R^D_t \right) \) increases when a positive TFP shock hits the economy: even though the wage bill rises, the fall in real marginal costs, the increase of output and the reduction
of the policy rate improve the firms’ revenues. Hence, for a given $\rho^B$, when banks have a positive bargaining power ($z < 1$) they can obtain a higher interest rate on loans such that the CS rises. On the other hand, as described above, a positive technological shock improves the expected profits of banks generating a less congestion effect in the credit market for firms. Hence, by observing the term $z(1 - \rho^B)\beta E_t \lambda_{t+1} N_t + \theta_{t+1}$, even though the wage bill and the real interest rate rise the fall of the credit market tightness implies more external opportunities for firms and a lower expected time to obtain a line of credit. Then firms can negotiate a lower banking lending rate tightening the CS. The overall countercyclical dynamics of the credit spread, which also depends on its steady state value $SP$, indicates that the countercyclical effect depending on the search and matching frictions is stronger than those procyclical due to the profits of firms and to the policy rate.

### 5.2.1 Bargaining power of banks and credit spread

In this section we analyze the scenario in which the banking sector has a more bargaining power in the negotiation of the loan interest rate in order to understand how changes the cyclical behavior of the CS. As in section (5.1.1) the magnitude of the IRFs of the main macroeconomic and credit market variables do not change significantly. Then modification of the behavior of the CS is attributable to the variation of the sharing of the total surplus of the credit match.

A more bargaining power of banks amplifies the magnitude of the procyclical effect analyzed in the previous section by improving the share of firms’ revenues of which the lenders may appropriate. Further it mitigates the countercyclical role of the credit market frictions. However, all these effects are smaller than the benchmark model because a decrease of $z$, by augmenting the steady state banking lending rate $R^L$, lowers the scale factor $\frac{1}{SP}$. Hence, as shown by figure 5 the final effect is a more reduction of the credit spread compared to the benchmark model. Moreover, it is useful to note as, differently to the policy rate shock case, small variations of $z$ implies small changes in
Figure 5: Credit spread with respect to a technology shock

the CS.

**Proposition 5** When $0 < z < 1$ and $k^B > 0$, in presence of exogenous positive productivity shock, the greater is the banks’ bargaining power, $(1 - z)$, the more countercyclical is the credit spread.

5.3 The credit shock

In this section we study the Bayesian IRFs produced by the model when a credit efficiency shock hits the credit market (figure 6). An increase in the efficacy of the credit market can be thought as an improvement of the structural and technology reforms of the financial market. As Becsi et al. (2005) argue, a positive shock to $\zeta_t$ may represents a better efficiency of the financial intermediation arising from being to identify easier banking lending and borrowing opportunities. At the same time this kind of shock can be interpreted as a cost push shock which affects the real marginal costs and inflation, by the credit market tightness channel.

The first consequence of this shock is an increase in the number of financial matches. Then, by equation (10), the number of lines of credit rises. This improvement determines a fall in the number of firms searching for a line of credit: the credit line finding rate goes up. At the same time, the increase of the lines of credit determines a rise in the banks’ expected profits (see equation
Figure 6: IRFs with respect to a credit shock

12) and so a more intensive vacancy posting activity which increases, however, less than financial matches. As a consequence, the credit vacancy filling rate rises less than the credit line finding rate so that the credit market tightness falls, the interest rate on loans decreases, real marginal costs and inflation go down. Since the inverse of the credit market tightness can be interpreted as an index of the credit market liquidity (Wasmer and Weil, 2004), according to Becsi et al. (2005), we obtain that an improvement in the matching efficiency generates a liquidity increase. By the monetary rule, the policy rate decreases such that the real interest rate falls: current and future spending (output and output gap) rises and drops, respectively. The increase in the demand for goods requires more labor input: employment and the real wages rise. Finally it is useful to note as the initial rise of the inflation due to the increase of the financial matches is more than offset by the decrease of the inflation due to the fall of the credit market tightness.

6 Model Comparison: changing the banking sector setup

In this section we assume different setup of the banking sector leading to different definitions of the interest rate on loans. The other parts of the model are the same.

6.1 The price mark-up assumption

In order to show a simple way to insert a price mark up in the relationship between the interest rate on loans and the policy rate, we follow Chowdhury et al. (2006). Instead of providing an explicit microfoundation of the financial imperfection by which the effects of the policy rate on the banking lending rate can be amplified, they consider a continuously differentiable function \( \Psi(R^L_t) \in (0, 1) \), that summarizes the effects of the policy rate on the return of the loans. Then, the optimal value
function of the representative bank is:

\[
J_t^B = \max \left( R_t^L \left[ 1 - \Psi(R_t^D) \right] \frac{L_t}{P_t} - R_t^D \frac{D_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}^B \right)
\]

\[
s.t. \quad L_t = D_t + X_t
\]

Its decision yields:

\[
R_t^L \left[ 1 - \Psi(R_t^D) \right] = R_t^D
\]

(34)

The steady state version of equation (34) is \( R_t^L = \frac{1}{1 - \Psi^0} R_t^D \); its log-linearized version is:

\[
\hat{R}_t^L = (1 + \Psi R_t^D) \hat{R}_t^D + \hat{\nu}_t
\]

(35)

where \( \Psi_R = \frac{\Psi' R_t^D}{1 - \Psi} \). Hence, \( 1 + \Psi_R \) can be smaller or larger than one, depending on the sign of the term \( \Psi_R \). \(^{33}\) When we estimate the model we use the functional form \( \Psi_t = \left[ 1 - \Psi_0 \left( R_t^D \right)^\kappa \right] \) with \( 0 < \Psi_0 < 1 \) and \( \kappa > 0 \). Then \( \Psi_t' = -\Psi_0 \kappa \left( R_t^D \right)^{\kappa-1} \) whereas the steady state interest rate on loans becomes \( R_t^L = \frac{1}{\Psi_0} \left( R_t^D \right)^{1-\kappa} \). Then, \( \Psi_R = -\kappa \) and \( \hat{R}_t^L = (1 - \kappa) \hat{R}_t^D + \hat{\nu}_t \).

\[\textbf{6.2 The interest rate smoothing assumption}\]

In order to insert smoothness in the loan rate dynamics we follow Kaufmann and Scharler (2009) by assuming that banks are able to create loans using deposits by a function depending on the current and past values of the banking lending rate. In particular, financial intermediaries solve the following problem:

\[
J_t^B = \max \left( R_t^L \frac{L_t}{P_t} - R_t^D \frac{D_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}^B \right)
\]

\[
s.t. \quad L_t = \Sigma_0 \left[ \frac{R_t^L}{\left( R_{t-1}^L \right)^{\zeta_0}} \right] \left( D_t + X_t \right)
\]

where \( \Sigma_0 > 0 \) and \( \zeta_1 > 0 \). Their decision yields:

\[
(R_t^L)^{1+\zeta_1} \Sigma_0 (R_{t-1}^L)^{-\zeta_0 \zeta_1} = R_t^D
\]

(36)

The steady state version of equation (36) is \( R_t^L = \Sigma_1 \left( R_t^D \right)^{\Sigma_2} \) where \( \Sigma_1 = \left( \frac{1}{\Sigma_0} \right)^{\Sigma_2} \) and \( \Sigma_2 = \frac{1}{1 + (1 - \zeta_0) \zeta_1} \). Furthermore, the log-linearized version of equation (36) provides:

\[
\hat{R}_t^L = \frac{1}{1 + \zeta_1} \hat{R}_t^D + \frac{\zeta_0 \zeta_1}{1 + \zeta_1} \hat{R}_{t-1}^L + \hat{\nu}_t
\]

(37)

The previous equation states that the banking lending rate depends on the policy rate as well as on the persistence factor due its past value. It is useful to note that if \( \zeta_0 = 0 \) we obtain the Chowdhury et al. (2006)’s version (35) whereas for \( \zeta_1 = 0 \) we obtain the simple case with no mark-up.
6.3 Monopolistic competition and Calvo’s rule

In this section, following Hülsewig et al. (2009) we assume that each wholesale firm holds a loan portfolio diversified over all types of loans \( k \in [0, 1] \) offered by a banking sector which advances loans under a monopolistic competition’s regime. They are aggregated in the following way:

\[
    L_t = \left[ \int_0^1 L_{kt} \frac{\eta - 1}{\eta} dk \right]^\frac{\eta}{\eta - 1}
\]

where \( \eta > 1 \) represents the elasticity of substitution between the different types of loans \( k \). Each firm minimizes the reimbursement of the loans demanded to the each monopolist bank \( k \), \( L_{kt} \), plus the related banking lending rate \( R_{kt}^L \), subject to the demand for loans of the individual firm (38):

\[
    \min \int_0^1 L_{kt} R_{kt}^L \\
    s.t. \quad (38)
\]

The first order condition with respect to \( L_{kt} \) yields:

\[
    L_{kt} = \left( \frac{R_{kt}^L}{R_t^L} \right)^{-\eta} L_t
\]

where the gross loan interest rate is given by the aggregation of the banking lending rates of loan types \( k \):

\[
    R_t^L = \left[ \int_0^1 (R_{kt}^L)^{1-\eta} dk \right]^{\frac{1}{1-\eta}}
\]

Further in this setting we assume that banks face Calvo’s frictions when they set their interest rate on loans. Hence, only a fraction of the banks, \( 1 - \chi \), can be adjust their prices each period \( t \) whereas the fraction \( \chi \) maintain the gross banking lending rate unchanged. Then the aggregate interest rate on loans can be rewritten as:

\[
    R_t^L = (1-\chi) (R_t^{L*})^{1-\eta} + \chi (R_t^{L*})^{1-\eta} \left[ \int_0^1 (R_{kt}^L)^{1-\eta} dk \right]^{\frac{1}{1-\eta}}
\]

Given each bank \( k \) holds the balance sheet \( L_{kt} = D_{kt} + X_{kt} \), in a symmetric equilibrium all banks set the price, \( R_t^{L*} = R_{kt}^L \), so as to maximize the expected lifetime profits subject to the loan demand of the firms:

\[
    \max \quad E_t \sum_{l=0}^{\infty} \chi^l \beta^l \lambda_{t+l} \left( R_t^{L*} - R_{t+l}^D \right) \frac{L_{kt+l}}{P_{t+l}} \\
    s.t. \quad (39)
\]

This provides the gross interest rate on loans equation identical for all banks:

\[
    R_t^{L*} = \Xi E_t \sum_{l=0}^{\infty} \chi^l \beta^l \lambda_{t+l} R_{t+l}^D (R_{t+l}^L)^{\eta} (L_{t+l}/P_{t+l}) \\
    = \Xi R_t^D
\]

where: \( \Xi = \frac{\eta}{\eta-1} \). Under flexible prices equation (41) implies that the optimal interest rate on loans is a mark-up over the policy rate as already seen in the previous section:

\[
    R_t^{L*} = \Xi R_t^D
\]
Furthermore the steady state version of equation (41) is \( R^L = \Xi R^D \). By log linearizing the equations (40) and (41) we obtain:

\[
\hat{R}_t^L = \frac{\beta \chi}{1 + \beta \chi^2} E_t \hat{R}_{t+1}^L + \frac{\chi}{1 + \beta \chi^2} \hat{R}_{t-1}^L + \frac{(1 - \beta \chi)(1 - \chi)}{1 + \beta \chi^2} \hat{R}_t^D + \hat{\upsilon}_t
\]

(43)

The previous condition states that the interest rate on loans depends on its backward and future values and on the policy rate. Moreover if \( \chi = 0 \) we obtain the equality between the banking lending and the policy rates.

### 6.4 Bayesian selection

In this section we provide the estimation of the models described above. In particular we use the Bayesian Monte Carlo method which allows an empirical performance comparison of the models according to the information arising from the marginal distributions of the models.

#### 6.4.1 Bayesian comparison

The Bayes rule allows to get the posterior distribution of the model parameters conditioning on the prior assumptions on the vector parameters \( \kappa \in K \), the model \( M_j \) and the sample information \( Y_t = \{y_t\}_{t=1}^T \):

\[
P(\kappa/Y_T, M_j) = \frac{P(Y_T/\kappa, M_j)P(\kappa, M_j)}{P(Y_T, M_j)}
\]

(44)

where \( P(\kappa, M_j) \) is the prior distribution, \( P(Y_T/\kappa, M_j) \) represents the conditional distribution and \( P(\kappa/Y_T, M_j) \) is the posterior density. The latter distribution is computed by the numerical integration which, operationally, is obtained by the Kalman smoother in order to approximate the conditional distribution and by the Metropolis Hastings algorithm to implement the Monte Carlo integration.

Furthermore, Bayesian procedures can be used in order to compare alternative models. This aim is achieved by comparing the Bayes factor, i.e. the ratio between the probabilities of having observed the data conditional to two different models. Hence, by considering the Bayes theorem, assuming that all alternative models are true, the posterior density in terms of two models is:

\[
P(M_j, Y_T) = \frac{P(Y_T/M_j)P(M_j)}{P(Y_T/M_j)P(M_j) + P(Y_T/M_s)P(M_s)}
\]

(45)

where \( P(Y_T/M_j) = \int P(Y_T/\kappa_j, M_j)P(\kappa_j, M_j)d\kappa_j \) is the marginal density and \( j \neq s \). By considering the ratio between the posterior distributions of two models we get the posterior odds ratio, \( PO_{j,s} \), which coincides with the Bayes factor, \( B_{j,s} \), when we have no prior model preferences, i.e. \( \frac{P(M_j)}{P(M_s)} = 1 \):

\[
PO_{j,s} = \frac{P(M_j, Y_T)}{P(M_s, Y_T)} = \frac{P(Y_T/M_j)}{P(Y_T/M_s)} = B_{j,s}
\]

(46)

The Bayes factor is an index which indicates both the acceptance (or not) of a model and its relative evidence compared to another one. Following Schorfheide (2000) we compute the posterior (marginal) log-likelihood of the models by employing the Laplace approximation method. In order to select the model which is more supported by the empirical evidence we employ the Jeffrey (1961)’s method which scale the evidences provided by the log-Bayes factors of different models.35
6.4.2 Credit priors

In the following we compare the models which can generate an incompleteness of the interest rate PT. Then, we do not consider the pure credit economy scenario where, given the assumptions, the incompleteness in the transmission of the monetary policy cannot exist. Instead we study the other four scenarios and we denote the model with the mark-up assumption as model A, the model with the smoothness factor as model B, the model in which there is monopolistic competition in the banking sector and where the pricing is subject to a Calvo’s rule as model C, and the model which incorporates search and matching frictions in the credit market as model D. In order to correctly compare the models we use same observables as data entry: then we exclude the credit market index which is a specific observable for the model D and we do not consider the credit shock. The common calibrated parameters and the common priors are those described in sections 4.2 and 4.3 respectively. Also the priors on the credit market parameters of model D are those reported before. Concerning the credit market parameters of the other models we have not many references on possible prior values. Kaufmann and Scharler (2009) estimate for the U.S. $\frac{1}{1+\zeta_1} = 0.95$ and $\frac{\zeta_0\zeta_1}{1+\zeta_1} = 0.03$ from which it is possible to get $\zeta_1 = 0.0526$ and $\zeta_0 = 0.6$, and by the steady state version of equation (36), $\Sigma_0 = 0.98$. Hence, we decide to use the previous estimates as prior means with $\zeta_1$ and $\Sigma_0$ gamma distributed with standard errors equal to 0.0263 and 0.49 respectively and $\zeta_0$ normal distributed with standard error equal to 0.3. Furthermore, by using the identity principle of polynomials (see the appendix A.3.) between the steady state versions of the banking lending rate by Kaufmann and Scharler (2009) when $\zeta_0 = 0$ and by Chowdhury et al. (2006), we are able to derive the values of the parameters which we use as prior means for the functional form of $\Psi_t$ when the price mark-up assumption is done. In particular, we assume a gamma distribution for $\kappa$ and $\Psi_0$, with prior mean equal to 0.05 and 0.97 and standard error equal to 0.25 and 0.485, respectively. Under the scenario in which there is monopolistic competition in the banking sector for the Calvo’s parameter describing the share of banks that can adjust their banking lending rates, $\chi$, we assume a uninformative position by employing a beta distribution with prior mean equal to 0.5 and standard error equal to 0.25. Furthermore, for the elasticity of substitution between different loans $\eta$, we assume a gamma distribution with mean 6 and standard error 3 subject to the requirement $\eta > 1$. The appendix A.2 shows the priors and the posterior estimates of all models.

6.4.3 Bayes factors

From a qualitative point of view the Bayesian IRFs of the alternative models are similar to those of the model with credit search frictions whereas from a quantitative point of view differences are minimal when we observe the interest rate PT and are slightly different when we focus on the CS (the model D shows a more countercyclical dynamics). In this section we compare the models on the basis of the Bayesian model selection procedure described above. Considering the Laplace approximation, the posterior log-likelihoods of models A, B, C and D are 1288.15, 1289.06, 1287.12 and 1285.46 respectively. The Bayes factors are reported in table 4.

Results indicate, according to Jeffrey (1961)’s scale of equivalence, an evidence in favor of model B. Hence, a model in which the banking lending rate depends on its past value as well as on the policy rate seems to be more supported than other models emphasizing the role assumed by the past value of the banking lending rate in shaping the interest rate dynamics. Then, we modify the search and matching framework by introducing a backward looking social norm for the banking lending rate in a fashion similar to Christoffel and Linzert (2010) and Hall (2005) for the real wage. We assume that the actual loan rate is equal to a weighted average of its past loan rate and the
equilibrium (bargained) loan rate, $\hat{R}_{t}^{LN}$, defined by equation (25):

$$\hat{R}_{t}^{L} = (1 - \varrho)\hat{R}_{t}^{LN} + \varrho \hat{R}_{t-1}^{L}$$

where $\varrho$ denotes the degree of the loan rate rigidity. Then we estimate the new model that we label as $E$ by considering the same priors assumed for the model $D$ and by employing a beta distribution with prior mean 0.5 and standard error 0.25 for the degree of loan rate rigidity $\varrho$. Further the steady state version of the model $E$ coincides with that of the model $D$ given $R_{t}^{L} = R_{t}^{LN}$.

Table 4: Bayes Factors ($B_{j,s} = e^{[\log P(Y_{t}/M_{j}) - \log P(Y_{t}/M_{s})]}$).

<table>
<thead>
<tr>
<th>Model</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$e^{-0.91}$</td>
<td>$e^{1.03}$</td>
<td>$e^{2.69}$</td>
<td>$e^{-2.47}$</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$e^{0.91}$</td>
<td>$e^{-1.94}$</td>
<td>$e^{3.60}$</td>
<td>$e^{-1.56}$</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>$e^{-1.03}$</td>
<td>$e^{-1.94}$</td>
<td>$e^{1.66}$</td>
<td>$e^{-3.50}$</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$e^{-2.69}$</td>
<td>$e^{-3.60}$</td>
<td>$e^{-1.66}$</td>
<td>$e^{-5.16}$</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$e^{2.47}$</td>
<td>$e^{1.56}$</td>
<td>$e^{3.50}$</td>
<td>$e^{5.16}$</td>
<td>$e^{-5.16}$</td>
</tr>
</tbody>
</table>

Figure 7: Interest rate pass-through (monetary shock) and credit spread (TFP shock) under loan rate rigidity.

The Bayesian estimation provides $\varrho = 0.60$ showing a discrete dependence of the actual loan rate on its past value. Figure 7 shows the comparison of posterior IRFs of models $D$ and $E$. In the top
panel we can observe that when a negative interest rate shock hits the economy the interest rate pass-through becomes more incomplete compared to the benchmark model whereas the bottom panel shows that in presence of a positive TFP shock the credit spread is slightly procyclical. The first result depends on the loan rate stickiness that adds persistence and on the not optimal adjustment of the banking lending rate when search and matching frictions are present in the credit market. The second finding is determined by a more weight of the procyclical component due to the increase of the banks’ profits by the rise of the share of which banks can appropriate given the more firms’ revenues. Furthermore, the posterior log-likelihood of model $E$ is 1290.62. The new Bayes factors reported in table 4 allow us to highlight a decisive evidence in favor of the new model where the loan rate depends, on one hand, on the price bargained by firms and banks on the basis of their bargaining powers, and on the other hand, on its past value.

7 Conclusions

The recent crises has shown that the reduction of the monetary policy rate by major Central Banks has not been completely transmitted to the bank retail rates. In particular, the incompleteness of this pass-through was more prominent in the loan market. So, understanding the causes of this phenomena becomes relevant in order to address the monetary policy strategy. This work helps to do it.

We have introduced search and matching frictions in the credit market into a cash in advance New Keynesian DSGE theoretical model with sticky prices and no wage rigidities. The model is estimated by using the Bayesian methods. Given prior assumptions on the model’s parameters in line with the empirical literature, in order to run the estimation we use seven observed series in the sample 1997:Q2-2013:Q2. Further, we identify the stability domain of the model by the two-sided Smirnov-Kolmogorov test.

The dynamic properties of our model are consistent with the main cyclical evidence reported in the NK DSGE literature, but the model provides some main new findings. First, it is able to highlight an incomplete pass-through of policy rate changes to the interest rate on loans. Second, when the model is hit by a positive technology shock, in line with the empirical evidence, it provides the countercyclical behavior of the credit spread.

The previous findings depend on the search and matching frictions in the credit market and on the bargaining mechanism over the loan interest rate, which affect the responses of the main real macroeconomic variables to exogenous shocks. In particular a more banks’ bargaining power exacerbates the incompleteness of the adjustment of the loan rate to variation of the policy rate and provides a more countercyclical behavior of the credit spread with respect to a positive technology shock. Moreover, by using the Bayesian techniques, we provide the estimated values of some structural parameters of the credit market useful to the study of the financial markets. A simple exercise in which the economy is hit by an exogenous credit shock confirms the importance of the channel represented by the credit market tightness in the determination of a banking lending rate affecting the market liquidity and the dynamics of the main real macroeconomic variables.

Finally by using the Bayesian procedures we compare the model with other framework which differ only for the building of the banking sector. The comparison provides a positive evidence in favor of a model in which the banking lending rate depends on its past value, on policy rate and on credit search and matching frictions.
Notes

1. We wish to thank P. Benigno, E. Castelnuovo, L. Cavallari, G. Ciccarone, F. Giuliani, F. Mattesini, S. Neri, F. M. Signoretti, M. Tancioni and the participants in a seminar held at the Bank of Italy on 21st December 2011.

2. See also Section 2.

3. As compared to the pairwise matching framework by Mortensen and Pissarides (1994), the "urn ball" process allows for the possibility that entrepreneurs have multiple simultaneous contacts with different financiers. The price formation is hence provided by an auction rather than a bargaining mechanism.

4. Recent works employ models embedding search and matching frictions in several interdependent markets. Search frictions in both the credit and labor market are analyzed by Wasmer and Weil (2004), Nicoletti and Pierrard (2006), Ernst and Semmler (2010) and Petrosky-Nadeau and Wasmer (2013); a first attempt to study the simultaneous interdependence of good, labor and credit market under search and matching frictions is provided by Petrosky-Nadeau and Wasmer (2011).

5. In the den Haan et al. (2003)'s model there is only a good that can be consumed or invested through lenders, whereas Becsi et al. (2005) assume lenders as a rudimetal moneyholders which are a fusion of households and financial intermediaries.


7. See, e.g., Fuertes et al. (2010) and Humala (2005) for U.K and Argentinian data respectively.

8. See, e.g., Christiano et al. (2005) and Ravenna and Walsh (2006).

9. Goodfriend and McCallum (2007)’s model is able to highlight a banking attenuator effect if the monetary shock has a very large volatility. Similarly to Gerali et al. (2010), the imperfect competition in the financial intermediation sector produces attenuation in Andrés and Arce (2012) and Aslam and Santoro (2008)’s models.

10. The CIA constraint is always binding because the nominal interest rate is positive and agents choose their asset (deposit) holdings after observing the current shock but before entering the good market (Lucas, 1982).

11. The firms’ profits are the sum of those of retail and specialized firms. See section (3.9).

12. For all models of section (6) the equilibrium condition $C_t = Y_t$ holds.

13. The detailed derivation of the model is provided in a technical appendix available from the author upon request.

14. Empirical evidence suggests that break-ups are relatively insensitive under “relationship lending” (see, e.g., Bolton et al., 2013): then we decide to use exogenous separation rate. On the other hand, some authors assume endogenous break-ups because they help to determine a countercyclical dynamics of the net interest rate margin (Beaubrun-Diant and Tripier, 2013). Money injection represents a transfer from the Central Bank to the banks as in Kobayashi (2008).

15. It is useful to remark as the payment of the vacancies produced by the specialized firm is made at the beginning of the period because vacancies are needed to search firms.

16. The definition of external finance premium is different from that of corporate credit spread. The former is the wedge between the rate of return on capital and the risk-free rate; the latter one is the difference between the contractual loan interest rate and the risk-free rate. See Levin et al. (2004) for more details.

17. The same shock is appended to the banking lending rate equation of the models reported in section 6.
19. When \( z = 0 \) the steady state value of the loan rate is zero. This is a limit case due to the solution of the sequential steady state of the model. However, it is always possible to demonstrate that there exists \( 0 \leq \tilde{z} \leq 1 \) such that for \( z > \tilde{z} \) we have \( R_L > 0 \), \( \frac{\partial R_L}{\partial z} < 0 \) and \( \frac{\partial^2 R_L}{\partial z^2} > 0 \).

20. Becsi et al. (2005) use the flow entry cost to tie down the number of firms in equilibrium. In the present work, instead, this issue is implemented only to find a steady state version of the credit line finding rate, \( p^B \), depending on the entry cost \( \tilde{c} \) and on the other deep parameters. As a matter of fact our firms’ population is normalized to 1 and the number of firms searching for a relationship with a bank depends on the state variable \( L^N_t \). Similar results are obtained by directly fixing the credit finding probability.

21. It is useful to note that a larger separation rate increases the credit line finding rate if \( \beta \left( \frac{\mu}{\bar{\mu}} - wR_LN \right) > (1 - \beta)(1 - \beta + \beta^2) \tau \) holds.

22. Equations (24) and (26) show that the loan rate and the credit finding probability are increasing and decreasing in firms’ profits respectively. As explained, this is the result of the search and matching mechanism. But these effects are not in line with the implications of the “balance sheet channel” according to which profitable firms would arguably enjoy low rates and higher matching likelihood. A way to reconcile the two views is making endogenous the parameter \( z \) such that the larger the firms’ profits the lower the banks’ bargaining power \( (1 - z) \). This effect could partially offset the “search and matching channel”. This extension is left to future research.

23. The complete explanation of the relationship between the expected entry cost and the expected posting cost is available from the author upon request.

24. The only work reporting values for the search and matching credit market parameters is Petrosky-Nadeau and Wasmer (2013). They employ a trembling hand calibration method by an iterating perturbation of the set of parameters by a random shock drawn from a normal distribution in the space of the parameters to calibrate.

25. The values of posting cost employed by the literature on search and matching frictions in the labor market range between 0.01 (Hairault, 2002; Walsh, 2005) and 0.213 (Shimer, 2005). About the credit market, Petrosky-Nadeau and Wasmer (2013) use a monthly parametrization but in a previous working paper version of the work (2012) they consider a quarterly calibration: in this case the credit vacancies’ values range between 0.045 and 0.080.

26. In order to simulate the model we add some measurement equations linking the log-levels of the variables with their differences. We assume these equations contain a constant term that we estimate. For these terms, in the estimation phase, we assume a normal distribution and we adopt the strategy to set the prior mean equal to the sample mean of the time series to which the constant refers, and a standard error which implies a prior pseudo-\( t \)-value (the ratio between the prior mean and the prior standard deviation) equal to 2. Table 3 does not report the constants’ estimates. More details are available from the authors upon request.

27. The fraction of drops of the initial parameters vector is set at 20%. The calibration of the scale factor provides acceptance rates around 37 percent for the ten blocks. For the application of the Bayesian estimation and of the sensitivity analysis we employ the latest stable version (4.2.1) of the open-source software Dynare.

28. The estimation values reported in table 3 imply the following realistic and acceptable steady state values: \( p^B = 0.02 \), \( q^B = 0.29 \), \( R_L = 1.0773 \), \( R_D = 1.0035 \), \( SP = 0.738 \), \( w = 0.52 \), \( L^N = 0.19 \) and \( s^F = 0.83 \).

29. Since the number of searching of firms at time \( t \) depends on the lines of credit at time \( t - 1 \), the impact response of \( s^F_t \) is always zero.

30. They consider a model where the Taylor rule depends on the level of the output.
31. Beaubrun-Diant and Tripier (2013), show another countercyclical effect due to the modification of the idiosyncratic productivity reservations of the agents. They conclude for an overall countercyclical effect of the credit spread.

32. It is useful to note that the usual dynamics of the matching probabilities which depend on the sign of the credit market tightness change by the magnitude of the credit efficiency shock and the value of the externality $\xi$. As a matter of fact the matching probabilities can be rewritten as $\hat{p}_t^B = \zeta_t - \xi \hat{\theta}_t^C$ and $\hat{q}_t^B = \zeta_t + (1 - \xi) \hat{\theta}_t^C$. In our case since the increase of the $\zeta_t$ is greater than the decrease of $(1 - \xi) \hat{\theta}_t^C$ the dynamics of the credit vacancy filling rate is increasing and not decreasing.

33. Chowdhury et al. (2006) insert in the profit function of the financial intermediaries a managing cost of the loans that allows to obtain a negative value of the term $\Psi_R$.

34. See the appendix A.2 for the computation of the lending rate equation of the all models.

35. Model $j$ is supported if $B_{j,s} \geq 1$. On the other hand, there are a slight, moderate, decisive or strong evidence against the model $j$ if $10^{-\frac{1}{2}} \leq B_{j,s} < 1$, $10^{-1} \leq B_{j,s} < 10^{-\frac{3}{2}}$, $10^{-2} \leq B_{j,s} < 10^{-1}$ and $B_{j,s} < 10^{-2}$, respectively.
References


A Appendix

A.1 Data, Sources and Bayesian estimates


Credit market tightness: Quarterly Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Small Firms, Board of Governors of the Federal Reserve System and Federal Reserve Economic Data of Saint Luis.

The quarterly net percentage of Domestic Respondents Tightening Standards for commercial and industrial loans small firms - the series that we interpret as the tightness index of the credit market - is provided by the Senior Loan Officer Opinion Survey on Bank Lending Practices from 1990:Q2. This index reports the evaluation of the U.S. banks about the conditions of the credit market according to the more or less tight banks’ standards applied to the small firms. Hence the tightness of the credit market is evaluated from the point of view of the firms. The survey is based on the responses from 57 domestic banks and 23 U.S. branches and agencies of foreign banks. The small business’ definition by the U.S. Small Business Administration varies by industry, ranging from fewer than 100 employees (e.g. for the wholesale trade) and fewer than 1500 workers (e.g. for the telecommunications). Hence, even when considering only those with fewer than 100 employees (including nonemployer firms) we are able to represent about 99.5% of the total U.S. firms. Then the previous tightness index can be considered a good proxy of the thickness in the credit market for all U.S. businesses.

We compute the average of the monthly flows to transform the monthly time series into quarterly data. In line with Fernández-Villaverde (2010) we compute the real wages by deflating the relative nominal series by the consumer price index. The inflation rate is computed as the quarter on quarter log differences in the Consumer Price Index. Finally, the series of the real GDP, the employment and the real wages are made stationary by the first-difference transformation.
Table A.1: Posterior Estimates: Structural and Shock Process Parameters.
A.2 The Lending Rate Equations

In the following the log-linearization and the steady state versions of the lending rate equation of models presented in section 6 are provided.

A.2.1 The mark-up case

The log-linearization of equation (34) provides:

\[ R^L \hat{R}^L_t - R^L \Psi(R^D) \hat{R}^L_t - R^L \Psi'(R^D) R^D \hat{R}^D_t - R^D \hat{R}^D_t = 0 \]

By rearranging we have:

\[ R^L \left[ 1 - \Psi(R^D) \right] \hat{R}^L_t = \left[ \hat{R}^D_t - \Psi'(R^D) R^D \right] \hat{R}^D_t \]

by remembering that \( R^L \left[ 1 - \Psi(R^D) \right] = R^D \) we finally have:

\[ \hat{R}^L_t = (1 + \Psi_R) \hat{R}^D_t \]

where \( \Psi_R = \frac{\Psi'(R^D)}{1-\Psi} \). The previous equation is the (35) of the text.

A.2.2 The persistence factor case

From equation (36) we have:

\[ \hat{R}^L_t + \zeta_1 \left( \hat{R}^L_t - \zeta_0 \hat{R}^L_{t-1} \right) - \hat{R}^D_t = 0 \]

Then:

\[ (1 + \zeta_1) \hat{R}^L_t - \zeta_1 \zeta_0 \hat{R}^L_{t-1} = \hat{R}^D_t \]

Finally we have:

\[ \hat{R}^L_t = \frac{1}{1 + \zeta_1} \hat{R}^D_t + \frac{\zeta_1 \zeta_0}{1 + \zeta_1} \hat{R}^L_{t-1} \]

which is (37) of the text.

A.2.3 The monopolistic competition case

From equation (40) we have:

\[ 1 = (1 - \chi) \left( \frac{R^{L*}}{R^L_t} \right)^{1-\eta} + \chi \left( \frac{R^{L*}_{t-1}}{R^L_t} \right)^{1-\eta} \]

The log-linearization of the previous equation yields:

\[ 0 = (1 - \chi)(1 - \eta) \left( \frac{R^{L*}}{R^L_t} \right)^{-\eta} \left( \hat{R}^{L*}_t - \hat{R}^L_t \right) - \chi(1 - \eta) \left( \frac{R^L}{R^L_t} \right)^{-\eta} \left( \hat{R}^L_t - \hat{R}^L_{t-1} \right) \]

Since \( R^{L*} = R^L \), by solving with respect to \( \hat{R}^L_t \) we obtain:

\[ \hat{R}^{L*}_t = \frac{1}{1 - \chi} \hat{R}^L_t - \frac{\chi}{1 - \chi} \hat{R}^L_{t-1} \quad (A.1) \]
By replacing (A.4) into (A.5) we obtain:

\[ R_t^L s_i \sum_{t=0}^{\infty} \chi^t \beta^t C_{t+i}^{-\sigma} \frac{(R_t^L)^\eta_{t+i}}{P_{t+i}} = \Xi E_t \sum_{t=0}^{\infty} \chi^t \beta^t C_{t+i}^{-\sigma} R_{t+i}^D (R_t^L)^\eta_{t+i} \frac{L_{t+i}}{P_{t+i}} \]

The log-linearization of the left side of the previous equation yields:

\[ \frac{L}{P} \frac{(R_t^L)^{1+\eta C^{-\sigma}}}{1-\chi \beta} \hat{R}_t^L + \frac{L}{P} \frac{(R_t^L)^{1+\eta C^{-\sigma}}}{1-\chi \beta} \sum_{t=0}^{\infty} \chi^t \beta^t \left( E_t \hat{L}_{t+i} - E_t \hat{P}_{t+i} + \eta E_t \hat{R}_{t+i}^L - \sigma \hat{E}_t C_{t+i} \right) \]

By remembering that \( \Xi R^D = R^L \) the right side provides:

\[ \frac{L}{P} (R_t^L)^{1+\eta C^{-\sigma}} \sum_{t=0}^{\infty} \chi^t \beta^t \left( E_t \hat{L}_{t+i} - E_t \hat{P}_{t+i} + \eta E_t \hat{R}_{t+i}^L - \sigma E_t \hat{C}_{t+i} + E_t \hat{R}_{t+i}^D \right) \]

By equating and simplifying the two sides we have:

\[ \hat{R}_t^L = (1-\chi \beta) \sum_{t=0}^{\infty} \chi^t \beta^t E_t \hat{R}_{t+i}^D \] (A.2)

By iteration solution equation (A.2) can be written as:

\[ \hat{R}_t^L = (1-\chi \beta) \hat{R}_t^D + \chi \beta \hat{R}_{t+1}^L \] (A.3)

By using equation (A.1) into (A.2) we have:

\[ \frac{1}{1-\chi} \hat{R}_t^L - \frac{\chi}{1-\chi} \hat{R}_{t-1}^L = (1-\chi \beta) \hat{R}_t^D + \frac{\chi \beta}{1-\chi} \hat{R}_{t+1}^L - \frac{\chi^2 \beta}{1-\chi} \hat{R}_t^L \]

By solving with respect to the \( \hat{R}_t^L \) we obtain:

\[ \hat{R}_t^L = \frac{\beta \chi}{1+\beta \chi^2} E_t \hat{R}_{t+1}^L + \frac{\chi}{1+\beta \chi^2} \hat{R}_{t-1}^L + \frac{(1-\beta \chi)(1-\chi)}{1+\beta \chi^2} \hat{R}_t^D \]

which is equation (43) of the text.

**A.2.4 The search and matching case**

The first order condition and the envelope theorem of the bank’s problem in the search and matching framework are:

for \( V_t^B \):

\[ \frac{k^B R_{t}^D}{q_t^B} - (R_t^L - R_t^D) w_t N_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}^B}{\partial L_t^N} \] (A.4)

for \( L_t^N \):

\[ \frac{\partial J_t^B}{\partial L_t^N} = (1-\rho) (R_t^L - R_t^D) w_t N_t + \beta (1-\rho^B) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial J_{t+1}^B}{\partial L_t^N} \] (A.5)

By replacing (A.4) into (A.5) we obtain:

\[ \frac{\partial J_t^B}{\partial L_t^N} = (1-\rho^B) \frac{k^B R_{t+1}^D}{q_t^B} \] (A.6)
By updating equation (A.6) and then by replacing into (A.4), we then get:

\[
\frac{k^B R^D_t}{q^B_t} = (R^L_t - R^D_t)w_t N_t + (1 - \rho^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{k^B R^D_{t+1}}{q^B_{t+1}}
\]

which is equation (15) of the text. The surpluses of a bank and of a firm are:

\[
S^B_t = B^m_t - B^u_t \quad \text{(A.7)}
\]

and

\[
S^F_t = F^m_t - F^u_t \quad \text{(A.8)}
\]

The value of a unfilled credit vacancy is:

\[
B^u_t = -k^B R^D_t + q^B_t B^m_t + (1 - q^B_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} B^u_{t+1} \quad \text{(A.9)}
\]

Since the free entry condition, \(B^u_t = 0 \forall t\), the value of a filled credit vacancy is equal to the bank surplus:

\[
B^m_t = S^B_t = \frac{k^B R^D_t}{q^B_t} \quad \text{(A.10)}
\]

or by remembering the equation (15) \(S^B_t = (R^L_t - R^D_t)w_t N_t + (1 - \rho^B)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{k^B R^D_{t+1}}{q^B_{t+1}}.\)

The values of a filled and unfilled credit vacancy are respectively:

\[
F^m_t = \frac{Y^w_t}{\mu_t} - w_t R^L_t N_t + \beta E_t \frac{\lambda^{t+1}}{\lambda_t} \left[ (1 - \rho^B + \rho^B p^{B}_{t+1})F^m_{t+1} + \rho^B (1 - p^{B}_{t+1})F^u_{t+1} \right] \quad \text{(A.11)}
\]

\[
F^u_t = \beta E_t \frac{\lambda^{t+1}}{\lambda_t} \left[ p^{B}_{t+1} F^m_{t+1} + (1 - p^{B}_{t+1})F^u_{t+1} \right] \quad \text{(A.12)}
\]

Then the surplus of the firm is:

\[
S^F_t = F^m_t - F^u_t = \frac{Y^w_t}{\mu_t} - w_t R^L_t N_t + (1 - \rho^B)\beta E_t \frac{\lambda^{t+1}}{\lambda_t} (1 - p^{B}_{t+1})S^F_{t+1} \quad \text{(A.13)}
\]

The optimal condition of the problem (21) of the text is:

\[
(1 - z)\gamma^B_t S^F_t + z\gamma^F_t S^B_t = 0 \quad \text{(A.14)}
\]

where \(\gamma^B_t = \frac{\partial S^B_t}{\partial R^L_t} = w_t N_t \) and \(\gamma^F_t = \frac{\partial S^F_t}{\partial R^L_t} = -w_t N_t.\) From the previous equation we have:

\[
S^F_t = \frac{z}{(1 - z)} \frac{k^B R^D_t}{q^B_t}. \quad \text{(A.15)}
\]

By replacing equation (A.15) one period ahead in the firms’ surplus definition (A.13) we obtain:

\[
S^F_t = \frac{Y^w_t}{\mu_t} - w_t R^L_t N_t + (1 - \rho^B) \frac{z}{(1 - z)} \beta E_t \frac{\lambda^{t+1}}{\lambda_t} (1 - p^{B}_{t+1}) \frac{k^B R^D_{t+1}}{q^B_{t+1}} \quad \text{(A.16)}
\]
Moreover, they estimate
\[ \frac{(1 - z)Y_t^w}{\mu_t} - (1 - z)w_t R_t^L N_t + (1 - \rho B) z \beta E_t \frac{\lambda_{t+1}}{\lambda_t} k^B R_{t+1}^D \frac{q_{t+1}^B}{q_t^B} - (1 - \rho B) z \beta E_t \frac{\lambda_{t+1}}{\lambda_t} k^B R_{t+1}^D \frac{q_{t+1}^C}{q_t^C} = \frac{z}{k^B R_t^D} \]

By using the the credit creating condition and by solving with respect to the interest rate on loans we obtain:

\[ R_t^L = \frac{(1 - z) Y_t^w}{\mu_t} + \frac{z}{w_t N_t} \left[ R_t^D w_t N_t - (1 - \rho B) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} k^B R_{t+1}^D \frac{q_{t+1}^C}{q_t^C} \right] \]

which is equation (24) of the text. By using equations (2) and (5) of the text, the previous equation can be written in the following way:

\[ R_t^L = \frac{(1 - z)}{\alpha} R_t^L + z R_t^D - \frac{z(1 - \rho B)}{w_t N_t} \frac{1}{R_t^D} E_t \frac{P_{t+1}}{P_t} \frac{k^B R_{t+1}^D}{\theta_{t+1}^C} \]

Then, the log-linearization of the previous equation is:

\[ R^L \hat{r}_t^L = \frac{(1 - z)}{\alpha} R_t^L \hat{r}_t^L + z R_t^D \hat{r}_t^D + \]
\[ - \frac{z(1 - \rho B)}{w N} R_t^D \frac{k^B}{R^B} \frac{p^B}{q^C} \left[ E_t \hat{P}_{t+1} - \hat{P}_t - \hat{w}_t - \hat{N}_t - \hat{R}_t^D + E_t \hat{R}_{t+1}^D - E_t \hat{\theta}_{t+1}^C \right] \]

By solving with respect to \( \hat{r}_t^L \) we have:

\[ \hat{r}_t^L = \frac{\alpha z}{R^L (\alpha - 1 + z)} \left[ R^D + \frac{(1 - \rho B) \beta p^B k^B}{w N q^B} \right] \hat{r}_t^D + \]
\[ + \frac{\alpha z (1 - \rho B) p^B k^B}{R^L (\alpha - 1 + z) w N q^B} \left[ E_t \hat{\theta}_{t+1}^C - E_t \hat{R}_{t+1}^D + \left( \hat{w}_t + \hat{N}_t - E_t \hat{\theta}_{t+1}^C \right) \right] \]

which is equation (25) of the text.

### A.3 The identity principle of polynomials

Kaufmann and Scharler (2009) estimate \( \frac{1}{1 + \zeta_1} = 0.95 \) from which it is possible to get \( \zeta_1 = 0.053 \).

Moreover, they estimate \( \frac{\zeta_0 \zeta_1}{1 + \zeta_1} = 0.03 \) which implies \( \zeta_0 = 0.6 \).

The steady state version of equation (36) is:

\[ R^L = \left( \frac{1}{\Sigma_0} \right)^{\frac{1}{1+\zeta_0}} \left( R^D \right)^{\frac{1}{1+\zeta_0}} \]

By computing the sample mean of \( r_t^L \) and \( r_t^D \) we can compute \( \Sigma_0 = \frac{1 + \bar{r}_t^D}{(1 + \bar{r}_t^L)^{1+\zeta_0}} = 0.9806 \).

Further, if we assume \( \zeta_0 = 0 \) we obtain a formulation consistent with Chowdhury et al. (2006):

\[ R^L = \left( \frac{1}{\Sigma_0} \right)^{\frac{1}{1+\zeta_1}} \left( R^D \right)^{\frac{1}{1+\zeta_1}} \quad (A.17) \]
In this case we are able to compute \( \Sigma_0 = \frac{1 + \bar{r}_D}{(1 + \bar{r}_L)^{1 + \zeta_1}} = 0.9755 \). Furthermore, equation (A.17) can be rewritten as \( R^L = a (R^D)^b \) where \( a = \left( \frac{1}{\Sigma_0} \right)^{\frac{1}{1 + \zeta_1}} \) and \( b = \frac{1}{1 + \zeta_1} \).

The steady state version of equation (34) is:

\[
R^L = \frac{1}{\Psi_0} (R^D)^{1 - \kappa} \tag{A.18}
\]

The previous equation can be rewritten as \( R^L = c (R^D)^d \) where \( c = \frac{1}{\Psi_0} \) and \( d = 1 - \kappa \).

Equations (A.17) and (A.18) have the same form. Hence, by the identity principle of polynomials, it has to be \( a = c \) and \( b = d \), or \( \left( \frac{1}{\Sigma_0} \right)^{\frac{1}{1 + \zeta_1}} = \frac{1}{\Psi_0} \) and \( \frac{1}{1 + \zeta_1} = 1 - \kappa \). Hence, given the values of \( \Sigma_0 \) and \( \zeta_1 \) we are able to compute \( \Psi_0 = 0.9767 \) and \( \kappa = 0.05 \).
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