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QUANTILE AGGREGATION OF DENSITY FORECASTS

by Fabio Busetti*

Abstract

Quantile aggregation (or 'Vincentization') is a simple and intuitive way of combining probability distributions, originally proposed by S. B. Vincent in 1912. In certain cases, such as under Gaussianity, the Vincentized distribution belongs to the same family as that of the individual distributions and can be obtained by averaging the individual parameters. This paper compares the properties of quantile aggregation with those of the forecast combination schemes normally adopted in the econometric forecasting literature, based on linear or logarithmic averages of the individual densities. In general we find that: (i) larger differences among the combination schemes occur when there are biases in the individual forecasts, in which case quantile aggregation seems preferable overall; (ii) the choice of the combination weights is important in determining the performance of the various methods. Monte Carlo simulation experiments indicate that the properties of quantile aggregation fall between those of the linear and the logarithmic pool, and that quantile averaging is particularly useful for combining forecast distributions with large differences in location. An empirical illustration is provided with density forecasts from time series and econometric models for Italian GDP.

JEL Classification: C53, E17.
Keywords: fan charts, macroeconomic forecasts, model combinations.

Contents

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1 Introduction

Economic forecasts are increasingly reported as point estimates supplemented by confidence bands or selected quantiles of the predictive distributions, in order to provide measures of uncertainty and risks around the central outcome. Indeed a common practice for central banks is to present forecasts of inflation and output in the form of 'fan charts' that describe in probabilistic terms the evolution of these variables along the forecast horizon. The (subjective) assessment of the likelihood of alternative macroeconomic scenarios is accounted using skewed density forecasts, that reflect higher probability for events in either tail of the distribution. For example, the Bank of England publishes fan charts for inflation since 1996; see Britton et al. (1998).

The econometric literature on forecast evaluation has been progressively extended to density forecasts in order to gauge the relative performance of different prediction models in terms of their distributions; see, inter alia, Diebold et al. (1998), Corradi and Swanson (2003, 2006), Mitchell and Hall (2005), Amisano and Giacomini (2007). In parallel, the idea of forecast combination, initiated by the classical paper of Bates and Granger (1969), has been applied to predictive distributions. The basic tools have been borrowed from the statistics literature on aggregation of subjective distribution functions, where the the task is to form an 'opinion pool'; cf. Genest and Zidek (1996) for a review. Econometric studies have focussed on linear or logarithmic weighting of the individual densities, where the weights may be data-driven reflecting the past performance of different models. Some examples are Wallis (2005), Hall and Mitchell (2007), Mitchell and Wallis (2010), Geweke and Amisano (2011), Fawcett et al. (2013). Kacha and Ravazzolo (2010) provide an empirical comparison of the linear versus the logarithmic opinion pool of several forecasting models of inflation. A comprehensive review of recent developments in density forecasting is Hall and Mitchell (2009).

This paper considers combining forecast distributions by quantile aggregation (or 'Vincentization'). This simple and intuitive approach, that consists in averaging the quantiles of the individual distributions, was originally proposed in Vincent (1912). Ratcliff (1979) and Thomas and Ross (1980) show that in certain cases, such as under Gaussianity, the Vincentized distribution belongs to the same family as that of the individual distributions and it can be obtained by averaging the individual parameters. A somewhat related approach is Granger et al. (1989), where individual quantiles are modelled separately and, for each of them, a linear combination of the forecasts is taken.

The properties of quantile aggregation are here compared with those of the linear and the logarithmic opnion pool. We find that larger differences among the combination schemes occur when there are biases in the individual forecasts, in which case quantile aggregation seems overall preferable. The choice of the combination weights is important in determining the relative performance of the methods. Monte Carlo simulation experiments of forecasting with partially misspecified time series models indicate that the properties of quantile aggregation are in between those of the linear and the logarithmic pool and that quantile averaging appears useful for combining forecast distributions with large differences in location.

The paper proceeds as follows. Section 2 defines quantile aggregation vis a vis the linear and the logarithmic opinion pools and it recalls the main issues on density forecast evaluation. Results for the special case of Gaussian distributions are contained in section 3. Section 4 sets several Monte Carlo experiments to evaluate the properties of forecast density combinations in the context of simple time series models. An empirical illustration with time series and econometric models for Italian GDP is given in section 5. Section 6 provides concluding remarks.

2 Quantile aggregation and other density forecast combinations

Let $f_{it}(y_t)$ be forecast densities for a scalar variable y_t and denote by $F_{it}(y_t)$ the corresponding cumulative distribution functions, for $i = 1, 2, ..., n$. For a set of non-negative weights ω_i such that $\sum_{i=1}^{n} \omega_i = 1$, the combined distribution defined by quantile aggregation (or 'Vincentization') is given by

$$F^{-1}_{\mathrm{vin},t}(\alpha) = \sum_{i=1}^{n} \omega_i F^{-1}_{it}(\alpha), \quad 0 < \alpha \leq 1, \tag{1}$$

where $F^{-1}_i(\alpha) = \inf\{y : \ F_i(y) \geq \alpha\}$ are the quantile functions of the individual forecast distributions. Quantile averaging was originally proposed in Vincent (1912), hence it is sometimes called 'Vincentization'.

Ratcliff (1979) and Thomas and Ross (1980) proved the following theorem on the properties of Vincentization for 'location scale' families of distributions; see also Genest (1992).

Theorem. *Let the individual distributions $F_{it}(y)$ be of the form $F_{it}(y) = H\left((y - \lambda_i)/\gamma_i\right)$, where λ_i is a centering parameter, γ_i is the scale and H is some distribution function, $i = 1, ..., n$. Then the Vincentized distribution is given by $F_{\mathrm{vin},t}(y) = H\left((y - \overline{\lambda})/\overline{\gamma}\right)$ with $\overline{\lambda} = \sum_{i=1}^{n} \omega_i \lambda_i$, $\overline{\gamma} = \sum_{i=1}^{n} \omega_i \gamma_i$.*

Under the conditions of this theorem (which include the Gaussian, Cauchy, exponential and logistic random variables) the Vincentized distribution is given simply by averaging the parameters of the individual distributions.

When not available in closed form, the Vincentized density can be obtained by numerical approximation as the derivative of the inverse of the quantile function (1). Note that quantile aggregation cannot be straightforwardly generalized to the multivariate case.

As regards other density forecast combination methods, the 'linear opinion pool', proposed by Stone (1961), is defined as

$$f_{\text{lin},t}(y_t) = \sum_{i=1}^{n} \omega_i f_{it}(y_t).$$

Here the combined distribution is a linear mixture distribution, which in general can be multi-modal, even under Gaussianity of the individual forecasts.

The 'logarithmic opinion pool' is instead given by

$$f_{\log,t}(y) = \frac{\prod_{i=1}^{n} f_{it}(y_t)^{\omega_i}}{\int \prod_{i=1}^{n} f_{it}(y_t)^{\omega_i} dy}.$$

Compared with the linear opinion pool, the combined distribution is typically unimodal and less dispersed. As in the case of Vincentization, it is closed under Gaussianity of the individual distributions.[1]

The advantages of forecast combinations are well understood for the case of point forecasts, where combinations are showed to work well in several empirical studies; cf. Timmermann (2006). Briefly, a 'portfolio diversification' argument as well as providing insurance against misspecified models and structural breaks are among the main reasons behind the success of combining point forecasts.

Less studies have investigated the advantages of combining predictive distributions. Importantly, Kascha and Ravazzolo (2010) show that a density forecast combination is at least as good as the worst model in terms of distributional accuracy, where the metrics is the average probability of observing the realized values (the 'log-score', defined in section 2.2 below).

However, a density forecast combination is not necessarily superior to individual models when evaluated under the MSE loss function (that is typical

[1] One drawback of the logarithmic opinion pool is that it gives probability of zero to events that have zero probability under any of the individual distributions.

for point forecasts). As argued by Hall and Mitchell (2009), '*density forecast combinations will in general increase the combined variance. However, this increase in uncertainty need not be deleterious ...*'.

For density forecasts the benefits deriving from the diversification argument are less clear, since (negative) correlations are not taken into account. The following example clarifies this issue. Let e_1 and e_2 be two forecast errors from competing models, with mean zero, variance equal to σ_1^2 and σ_2^2 respectively, and covariance equal to $\rho\sigma_1\sigma_2$; $\rho \in [-1, 1]$ is their correlation. Without loss of generality assume that $\sigma_2^2 = k^2\sigma_1^2$ with $0 < k \leq 1$. The mean square error of an equal weight point forecast combination is $MSE_{EWPF} = \frac{1}{4}\left(1 + 2\rho k + k^2\right)\sigma_1^2$ while, under the linear opinion pool, that of the density combination is $MSE_{EWDF} = \frac{1}{2}\left(1 + k^2\right)\sigma_1^2$. Thus $MSE_{EWDF} \geq MSE_{EWPF}$ where the equality holds only when both ρ and k are 1. In addition, while the MSE of the density forecast combination does not depend on ρ, the lower ρ the lower is MSE_{EWPF}.

2.1 The aggregation weights

The properties of forecast combinations to some extent depend on the weighting scheme adopted. Ideally, the weights should reflect the past performance of the different models and be time-varying, i.e. computed recursively at each point in time using all observations available. In practice equal weights forecasts are often adopted; empirically these are found difficult to beat. Timmermann (2006) extensively discusses the issue of setting the combination weights, mainly in the context of point forecasts. He argues that there is some weak empirical evidence that time-varying weights work better. In the context of density forecasts, Kascha and Ravazzolo (2010) do not find significant improvements of using time-varying over equal weights.

A simple metrics for setting weights is to use the prediction mean square error of different models $(PMSE_i)$,

$$\omega_i = \frac{1/PMSE_i}{\sum_{i=1}^{n} 1/PMSE_i}.$$

These *inverse MSE weights* are widely used for point forecasts; they would be optimal if the forecasts were independent[2]; see Bates and Granger (1969).

In the context of density forecasts, models are usually compared in terms of their average (log) predictive density, the so-called log-score, $S_i = \frac{1}{T}\log\sum_{t\in\Upsilon} f_{it}\left(y_t\right)$,

[2] In empirical works cross correlation among forecasts is rarely taken into account for setting combination weights. One reason may be that estimates of the correlation structure tend to be very imprecise.

where the average is over some sample Υ. In particular, the forecast density f_i is seen as a better approximation of the true distribution than f_j if the (out-of sample) log score is higher, $S_i > S_j$, i.e. if it gives higher probability to the events that really occurred; see e.g. Mitchell and Hall (2005). Amisano and Giacomini (2007) gives a formal test of equal forecast performance based on the difference in the log score of the two models.

The *log-score weights* are defined as

$$\omega_i = \frac{\exp(S_i)}{\sum_{i=1}^{n} \exp(S_i)}.$$

In a Bayesian framework these weights are related to the models' posterior probabilities[3].

3 Properties under Gaussianity of the individual distributions

Simple formulas apply under Gaussianity. Let the mean and variance of the individual distributions be μ_i and σ_i^2, respectively. Then: (i) the Vincentized distribution is Gaussian with mean $\mu_{\text{vin}} = \sum_i \omega_i \mu_i$ and variance $\sigma_{\text{vin}}^2 = \sum_i \omega_i \sigma_i^2$; (ii) the linear pool is in general non-Gaussian with mean $\mu_{\text{lin}} = \sum_i \omega_i \mu_i$ and variance $\sigma_{\text{lin}}^2 = \sum_i \omega_i \sigma_i^2 + \sum_i \omega_i \left(\mu_i - \mu_{\text{lin}}\right)^2$; (iii) the logarithmic pool is Gaussian with mean $\mu_{\log} = \left(\sum_i \omega_i / \sigma_i^2\right)^{-1} \sum_i \mu_i \omega_i / \sigma_i^2$ and variance $\sigma_{\log}^2 = \left(\sum_i \omega_i / \sigma_i^2\right)^{-1}$.

Note that the linear opinion pool has the same mean but higher variance than quantile aggregation. The logarithmic opinion pool on the other hand rescales the weights such that it is relatively 'closer' to the individual distribution with smaller variance. Finally if the individual Gaussian distributions have the same mean, then $\mu_{\text{vin}} = \mu_{\text{lin}} = \mu_{\log}$ and $\sigma_{\text{vin}}^2 = \sigma_{\text{lin}}^2 \leq \sigma_{\log}^2$; here the linear opinion pool is Gaussian and it coincides with quantile aggregation.

As an example, figure 1 shows the result of the three combination methods where the individual density functions are a N(0,1) and N(2,0.5). The linear pool gives rise to a bimodal distribution. Compared with quantile aggregation (that has the same mean as the linear pool), the logarithmic pool tends more towards the individual distribution with lower variance.

[3] Hall-Mitchell (2007) suggest using 'optimal log-score weights', defined as those that maximize the log-score of the combined distribution under the linear opinion pool; see also Conflitti, de Mol and Giannone (2013).

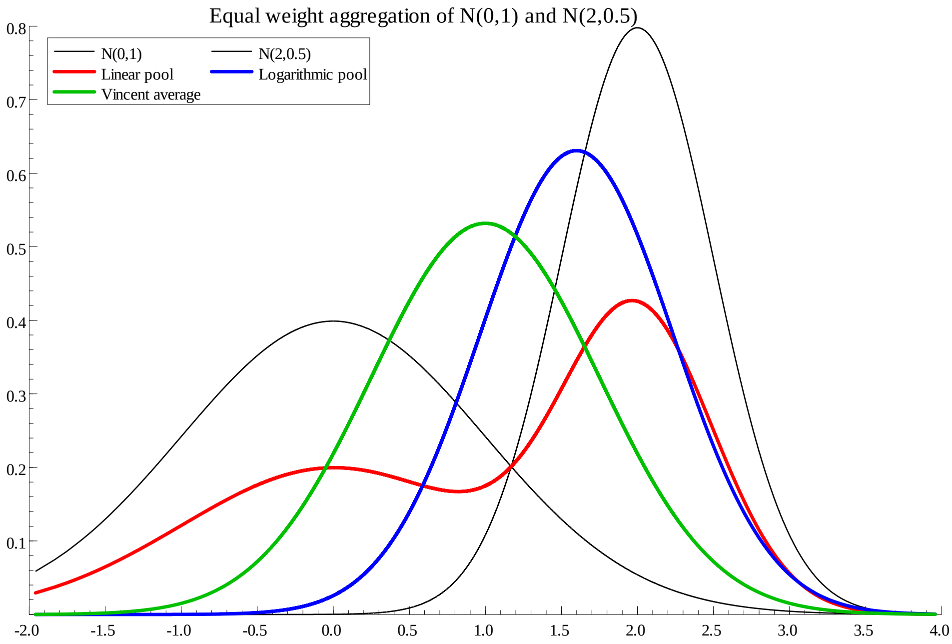

Figure 1: Aggregation of Gaussian densities

3.1 The bias-variance tradeoff

The main message of Figure 1 is that large differences may arise among the three combination methods if the individual distributions are not centered around the same mean. However for the MSE loss function, typical of point forecasts, a prediction bias may be compensated by lower variance, leaving the mean square error unchanged.

Here we investigate the combination of Gaussian density forecasts that are equally good in terms of the PMSE metrics, but have different biases. We find that under such a bias-variance tradeoff the aggregation method may matter.

Let y_0 follow a standard normal distribution f_0. We have two competing forecast densities: a mean unbiased y_1 drawn from $f_1 \sim N(0, \sigma_1^2)$ and a mean biased with smaller variance y_2 drawn from $f_2 \sim N(\mu_2, \sigma_1^2 - \mu_2^2)$, independent from f_1. As the PMSE of the two forecasts is the same, the equal weight combination is the same as the combination with inverse MSE weights.

Figure 2 compares the three different density forecast combinations against the true $N(0, 1)$ as the bias of the second forecast increases (the x-axis contains values of μ_2^2/MSE between 0.1 and 0.9). The metrics for comparison is the Kullback-Leibler information criterion, or KLIC; thus the lower the

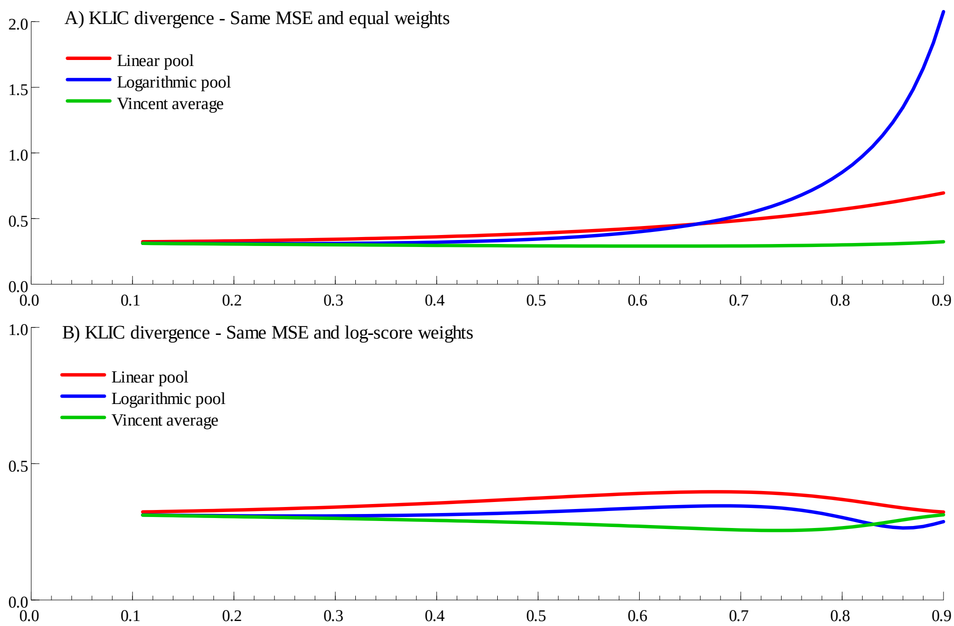

Figure 2: Combined density forecasts under a bias-variance tradeoff

better. Both inverse MSE weights and log-score weights are used.

For small biases ($\mu_2^2/\sigma_1^2 < 0.4$) all combined distributions behave very similarly. The Vincent average seems overall the better option as it maintains similar properties irrespectively of the bias. The logarithmic opinion pool provides a bad approximation to the correct distribution unless log-score weights are used.

4 Monte Carlo comparison of the combination methods for simple time series models

This section compares the density forecasts obtained with the different combination methods for simple time serie models and data generating processes. The first experiment is the same one considered in Mitchell and Wallis (2010), where the data are generated by an AR(2) process and the forecasting models to be combined contain only one lag of the dependent variable. In a second experiment the data process is an AR(1) process with GARCH disturbances, while the forecasts are generated ignoring either the autoregressing component or the time-varying conditional volatility.

The main metric over which forecast densities are compared is the KLIC distance from the true distribution, defined as $KLIC_i = E[log\text{f}_0(y)$ - logf(y)], $where\text{f}_0$ is the true density and f its forecast. Results are also reported for a Wald-type test of the null hypothesis of equal forecast distributions (as in Amisano and Giacomini, 2007), based on the statistic

$$t_{ij} = \frac{\frac{1}{T}\sum_t (\log f_{it}(y_t) - \log f_{jt}(y_t))}{\widehat{\sigma}_{ij}/\sqrt{T}}, \tag{2}$$

where $f_{it}(.)$, $f_{jt}(.)$ are the competing forecasts densities, $\widehat{\sigma}^2_{ij}$ is an appropriate estimate of the asymptotic variance of $\log f_{it}(y_t) - \log f_{jt}(y_t)$, and T is the forecast sample size. Under the null hypothesis, t_{ij} converge in distribution to a $N(0,1)$.

We report results based on 2000 Monte Carlo simulations and a sample size $T = 150$, as in Mitchell and Wallis (2010).

4.1 An AR(2) data generating process

We assume that the data are generated by the AR(2) process

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t, \qquad \varepsilon_t \sim NIID(0,\ \sigma^2_\varepsilon).$$

The 'ideal forecast'of y_t, distributed as $F_{0t} = N\left(a_1 y_{t-1} + a_2 y_{t-2},\ \sigma^2_\varepsilon\right)$, is first compared with two individual forecasts obtained from misspecified models where y_t is regressed on either y_{t-1} or y_{t-2} only. These are distributed as $F_{1t} = N\left(\rho_1 y_{t-1},\ \sigma^2_y(1-\rho^2_1)\right)$ and $F_{2t} = N\left(\rho_2 y_{t-2},\ \sigma^2_y(1-\rho^2_1)\right)$, where $\sigma^2_y = \sigma^2_\varepsilon/(1 - a_1\rho_1 - a_2\rho_2)$, $\rho_1 = a_1/(1-a_2)$, $\rho_2 = a_1\rho_1 + a_2$.The individual forecasts are then aggregated with equal weights according to three schemes considered in this paper, yielding the combined distributions $F_{\text{lin},t}$, $F_{\text{log},t}$ and $F_{\text{vin},t}$ respectively.

The resulting density forecasts are compared against the ideal forecast in the table 1, where AR_1 and AR_2 denote the individual forecasts f_{1t} and f_{2t}, respectively. For each distribution we report the KLIC distance against the true density f_{0t} (the lower the better) and the percentage rejections of the test (2) of equality with f_{0t} at the 5% significance level.

We consider three configurations of the autoregressive parameters of the data generating process: (1) $a_1 = 1.5$, $a_2 = -0.6$, (2) $a_1 = 0.15$, $a_2 = 0.2$, (3) $a_1 = -0.5$, $a_2 = 0.3$. The corresponding first and second order autocorrelation coefficients are: (1) $\rho_1 = 0.94$, $\rho_2 = 0.80$, (2) $\rho_1 = 0.19$, $\rho_2 = 0.23$, (3) $\rho_1 = -0.71$, $\rho_2 = 0.66$.

The first three rows of the table contain nearly the same numbers as those reported by Mitchell and Wallis (2010), to which we add the results for the logarithm opinion pool and the vincentization aggregation methods. In case (1) where data are very persistent ($\rho_1 = 0.94$), the AR_1 model achieves the lowest KLIC distance from the true distribution. Among the combined forecasts, the logarithmic opinion pool is the preferable aggregation scheme although it remains significantly worse than the AR_1. In case (2) and (3) the three aggregation methods delivers similar results (with a slightly inferior performance of the linear opinion pool), yielding a better outcome than the individual forecasts.

Table 1. Comparison of density forecasts for an AR(2) data generating process with unbiased forecasts.

Forecast	Case (1)		Case (2)		Case (3)	
	KLIC	*Test*	*KLIC*	*Test*	*KLIC*	*Test*
AR_1	22.5	99	2.1	24	4.8	47
AR_2	75.6	100	1.2	16	12.1	95
Lin. Pool	42.9	100	0.7	10	3.5	53
Log. Pool	36.3	100	0.7	8	1.9	26
Vincentization	49.5	100	0.6	8	2.2	28

In table 1 all forecasts are unbiased, while we have seen in section 3 that greater differences may occur when we allow for a bias. To this extent we also consider a forecast obtained from the true data generating process but evaluated under an asymmetric loss function of the 'linex' type, which delivers a (constant) forecast bias; see e.g. Christoffersen and Diebold (1997). The performance of the three aggregation schemes is evaluated when this biased forecast is combined with the AR_1 model. Table 2 reports the results for the parametrization labelled 'case (1)' of the AR(2) data generating process with $\sigma_\varepsilon^2 = 1$, for a forecast bias equal to 0.5, 1 and 2. We report the KLIC distances from the true distribution for both equal weight and inverse MSE weights combinations, denoted $KLIC_0$ e $KLIC_1$ respectively.

When the bias is relatively small, less or equal to 1, the performance of the unbiased AR_1 forecasting model is worse than that of any of the three combination schemes. Overall Vincentization appears to be the preferable aggregation scheme: it is significantly better when the bias is larger, while being not much different from the log opinion pool otherwise. In particular, for the case of equal weight aggregation, the test of equal forecast distributions between Vincentization and the logarithmic pool (not shown in the table) has rejection rates of the null hypothesis of 25, 79 and 97% when the bias is equal to 0.5, 1.0 and 2.0 respectively. Using inverse MSE weights (columns $KLIC_1$) may improve significantly the accuracy of the combined distributions, but in this case it does not change the relative rankings of the three methods.

Table 2. Comparison of density forecasts for an AR(2) data generating process, for case (1) with biased forecasts.

Forecast	$bias = 0.5$		$bias = 1.0$		$bias = 2.0$	
	$KLIC_0$	$KLIC_1$	$KLIC_0$	$KLIC_1$	$KLIC_0$	$KLIC_1$
AR_1	22.5	22.5	22.5	22.5	22.5	22.5
Asymmetric loss	13.0	13.0	51.1	51.1	202.0	202.0
Lin. Pool	10.9	10.5	22.1	21.1	51.9	31.9
Log. Pool	8.6	8.4	20.3	18.4	66.6	28.1
Vincentization	9.6	9.1	17.1	16.3	46.8	22.9

4.2 Time-varying volatility

Here we assume that the data are generated by the AR(1)-GARCH(1,1) process

$$\begin{aligned} y_t &= \rho y_{t-1} + \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim NIID(0,1), \\ \sigma_t^2 &= \gamma + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2. \end{aligned}$$

We compare the properties of combining, with equal weights, the forecast densities of two misspecified models: an AR(1) with constant conditional variance and a GARCH(1,1) with constant conditional mean, denoted as f_{1t} and f_{2t} respectively. For each individual distribution and forecast combination Table 3 reports the KLIC distance from the true density f_{0t} and the percentage rejections of the test (2) of equality with f_{0t}, at the 5% significance level. We report results only for a typical parametrization of the GARCH ($\alpha = 0.04$, $\beta = 0.95$) and for two cases of high and low persistence, $\rho = 0.75$ and $\rho = 0.25$ respectively.

For the case of highly persistent data ($\rho = 0.75$) the AR(1) model with constant variance provides the most accurate approximation of the density forecast to the true distribution, rejecting the null hypothesis of equality only 20% of the times. Among the combined distributions, the logarithmic pool has comparably better properties than the linear pool and the Vincent average, the latter two behaving similarly. On the other hand, when the data persistence is lower all combined distributions behave comparably better than each individual forecasts, with the Vincent average being only slightly superior.

Table 3. Comparison of density forecasts for an AR(1)-GARCH(1,1) data generating process

Forecast	$\rho = 0.75$		$\rho = 0.25$	
	KLIC	*Test*	*KLIC*	*Test*
AR	2.6	20	2.6	20
GARCH	31.4	98	2.9	30
Lin. Pool	11.2	86	1.1	15
Log. Pool	5.8	51	1.2	13
Vincentization	12.9	85	0.9	13

5 An empirical illustration

As an example we consider combining forecasts of Italian GDP from two simple models: (1) an autoregression of order 4; (2) a three variables VARX model for GDP, inflation and long-term interest rate, with two lags of the endogenous variables and additional exogenous regressors for foreign demand, oil prices and the short term interest rate.

The VARX model can be viewed as a rough approximation of the macroeconometric models typically used for producing conditional forecasts, with specific assumptions on the future paths of foreign variables and of the monetary policy rate. If the assumptions turn out to be more or less correct, then the conditional forecasts can be much more accurate than the unconditional ones. For example, in our case the (in-sample) variance of the 4-step ahead prediction errors of percentage GDP growth is 1.9 for the AR model and 0.8 for the VARX.

The models are estimated on quarterly data for the period 1986-2006, i.e. before the 'Great Recession' of 2008-09 and the sovereign debt crisis. Figure 3 provides in-sample and out-of-sample point predictions of the two models for the four-step ahead percentage growth rate of GDP, $y_{t+4|t} = 100(\log GDP_{t+4} - \log GDP_t)$, together with the top and lower percentiles of the equal-weights combinations of the forecast densities.

The two (out-of-sample) crises are clear outliers with respect to the reported confidence bands, calculated on the basis of the in-sample fit.

Further insights regarding the properties of the forecast distributions are contained in figure 4. The larger difference between the various combination methods occur in the tail of the distributions. The outer percentiles of the linear (logarithmic) opinion pool are closer to those of the less (more) precise model, with the Vincent average being somewhat in between. The figure also reports for each point in time the forecast of the probability that GDP

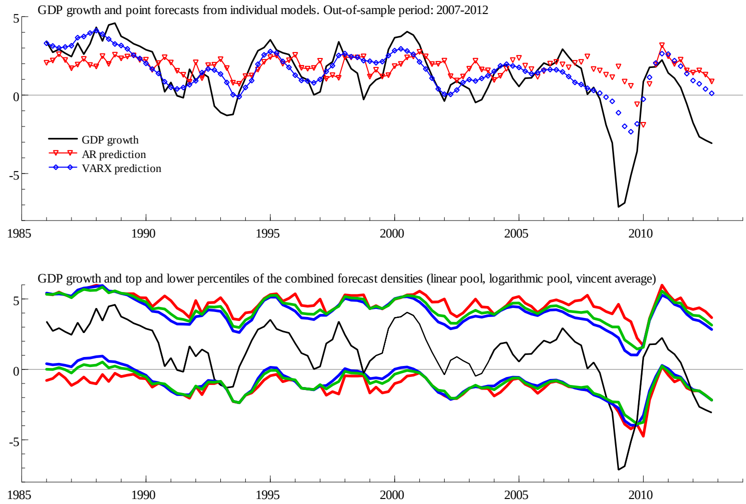

Figure 3: Individual point forecasts and confidence bands of the combined distributions

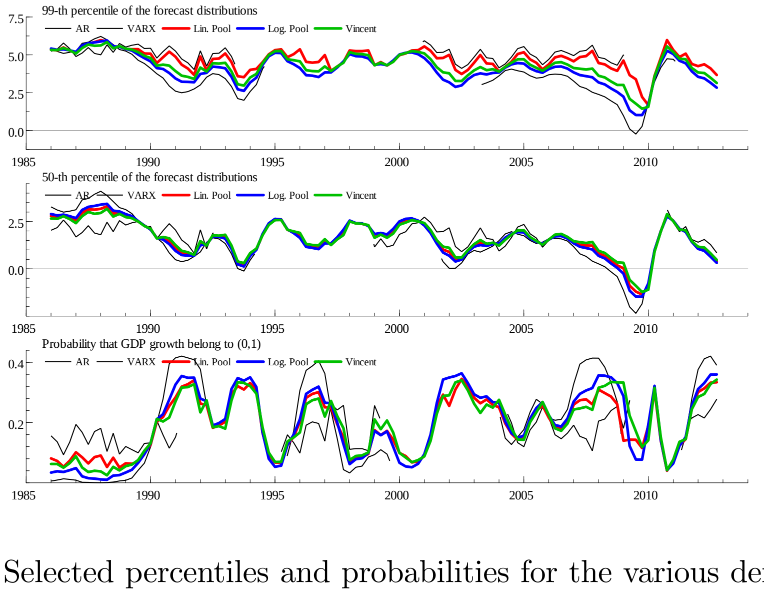

Figure 4: Selected percentiles and probabilities for the various density forecasts

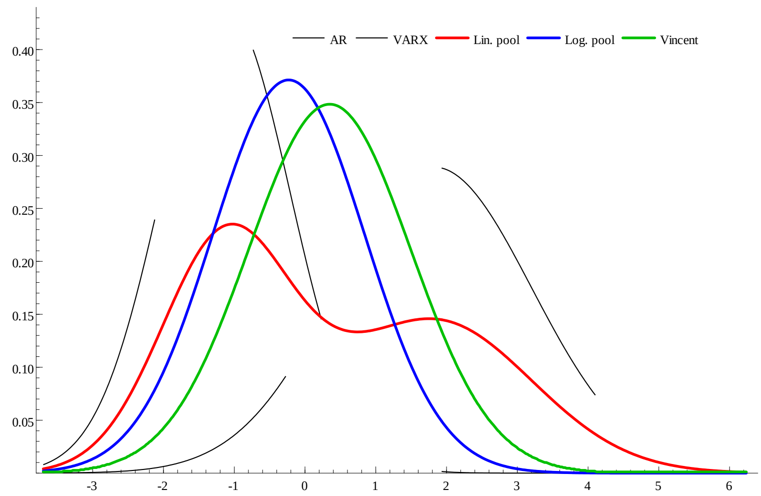

Figure 5: Density forecasts of 4-th step ahead predictions for GDP growth in 2009q1

growth is between 0 and 1 percent in the corrisponding quarter of next year. The individual models can imply quite different probability statements of future GDP growth, whereas the three combined distributions provide very similar answers in most cases.

Figure 5 contains the various densities for the 4-th step ahead GDP prediction for the first quarter of 2009, in the middle of the great recession. Here the individual models provide quite diverging predictions (partly because of the assumption that the VARX uses the realized values of the exogenous regressors), resulting in marked differences in the combined densities. The result is qualitatively similar to the example provided figure 1, where the logarithmic opinion pool is relatively closer to the less dispersed individual distribution.

The test of equal forecast distributions, run over the estimation sample, rejects the null hypothesis in all cases except for the comparison of the linear combination pool with the Vincent average. Outside the estimation sample the combined distribution obtained by Vincentization is overall preferable in the sense that it achieves the highest log-score; the test of equal forecast distributions however does not reject the null hypothesis. The apparent low power of the test may be related to the small number of observations out-of-sample.

6 Concluding remarks

This paper has compared quantile aggregation against the linear and the logarithimic opinion pool as methods for combining density forecasts. The three methods imply larger differences in the combined distribution if there are non-negligible biases in the individual forecasts. Overall the properties of quantile aggregation are in between those of the linear and the logarithmic pool. Quantile averaging appears particularly useful for combining forecast distributions with large differences in location. Finally, the choice of weights is important in determining the properties of the combined distributions and calls for further research.

References

[1] Amisano, G. and R. Giacomini (2007), Comparing density forecasts via weighted likelihood ratio tests, *Journal of Business and Economic Statistics*, 25, 177–190.

[2] Bates, J.M. and C.W.J. Granger (1969), Combination of forecasts, *Operational Research Quarterly*, 20, 451–468.

[3] Britton, E., Fisher, P. and J. Whitley (1998), "The Inflation Report Projections: Understanding the Fan Chart", *Bank of England Quarterly Bulletin*, 38, 30-37.

[4] Clements, M.P. (2004), Evaluating the Bank of England density forecasts of inflation, *Economic Journal*, 114, 844–866.

[5] Conflitti, C., de Mol, C. and D. Giannone (2012), Optimal Combination of Survey Forecasts, CEPR Discussion Papers.

[6] Corradi V. and N.R. Swanson (2003), Bootstrap conditional distribution tests in the presence of dynamic misspecification, *Journal of Econometrics*, 133, 779–806.

[7] Corradi V. and N.R. Swanson (2006), Predictive density evaluation, In *Handbook of Economic Forecasting*, Elliot G, Granger CWJ, Timmermann A (eds). Elsevier: Amsterdam; 197–284.

[8] Diebold, F.X, Gunther T. and A.S. Tay (1998), Evaluating density forecasts with applications to finance and management, *International Economic Review*, 39, 863–883.

[9] Fawcett, N., Kapetanios, G., Mitchell, J. and S. Price (2013), Generalised density forecast combinations, Bank of England Working Paper.

[10] Genest, C. (1992), Vincentization revisited, *The Annals of Statistics*, 20, 1137-1142.

[11] Genest, C. and J.V. Zidek (1986), Combining probability distributions: a critique and an annotated bibliography, *Statistical Science*, 1, 114-148.

[12] Geweke, J. and G. Amisano (2011), Optimal prediction pools, *Journal of Econometrics*, 164, 130-141.

[13] Granger, C.W.J., H. White and M. Kamstra (1989), Interval forecasting: an analysis based upon ARCH-quantile estimators, *Journal of Econometrics*, 40, 87-96.

[14] Hall, S.G. and J. Mitchell (2007), Combining density forecasts, *International Journal of Forecasting*, 23, 1–13.

[15] Hall, S.G. and J. Mitchell (2009), Recent developments in density forecasting, In *Palgrave Handbook of Econometrics, Volume 2: Applied Econometrics*, Mills TC, Patterson K (eds), MacMillan.

[16] Hendry, D.F. and M.P. Clements (2004), Pooling of forecasts, *Econometrics Journal*, 7, 1–31.

[17] Jore, A.S., Mitchell, J. and S.P. Vahey (2010), Combining forecast densities from VARs with uncertain instabilities, *Journal of Applied Econometrics*, 25, 621-634.

[18] Kascha, C. and F. Ravazzolo (2010), Combining inflation density forecasts, *Journal of Forecasting*, 29, 231-250.

[19] Mitchell, J. and S.G. Hall (2005), Evaluating, comparing and combining density forecasts using the KLIC with an application to the Bank of England and NIESER 'fan' charts of infl ation, *Oxford Bulletin of Economics and Statistics*, 67, 995–1033.

[20] Mitchell, J. and K.F. Wallis (2010), Evaluating density forecasts: forecast combinations, model mixtures, calibration and sharpness, *Journal of Applied Econometrics.*

[21] Ratcliff, R. (1979), Group reaction time distributions and an analysis of distribution statistics, *Psychological Bulletin*, 86, 446-461.

[22] Stone, M. (1961), The opinion pool, *Annals of Mathematical Statistics*, 32, 1339–1342.

[23] Thomas, E.A.C. and B.H. Ross (1980), On appropriate procedures for combining probability distributions within the same family, *Journal of Mathematical Psychology*, 21, 136-152.

[24] Timmermann A. (2006), Forecast combinations, In *Handbook of Economic Forecasting*, Elliot G, Granger CWJ, Timmermann A (eds). Elsevier: Amsterdam; 135–196.

[25] Vincent, S.B. (1912), The function of the viborissae in the behavior of the white rat. *Behavioral Monographs*, 1.

[26] Wallis, K.F. (2005), Combining density and interval forecasts: a modest proposal, *Oxford Bulletin of Economics and Statistics*, 67, 983–994.

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2012

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2014

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