Price pressures in the UK index-linked market: an empirical investigation

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PRICE PRESSURES IN THE UK INDEX-LINKED MARKET: 
AN EMPIRICAL INVESTIGATION

by Gabriele Zinna*

Abstract

We investigate the impact of long term investors' demand for UK index-linked gilts on the term structure of real rates for the 1987-2012 period. This is done by carrying out a structural estimation of the preferred-habitat model of Vayanos and Vila (2009). We use data on long-term investors' holdings of inflation-linked gilts, issuance of index-linked bonds and average maturity to identify the impact of supply/demand imbalances on rates. We find that demand pressure from long-term investors contributed to the decline in longer-term real rates over the 1987-2012 period by compressing bond risk premia. Before 2000, the fall in rates is largely due to the increasing demand pressure exerted by UK pension funds. Foreign institutional investors' demand instead played an important role in the subsequent decade.

JEL Classification: F34, G12, G15.
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Contents

1. Introduction ...................................................................................................................................... 5
2. Model ............................................................................................................................................. 10
3. Preferred-habitat demand ............................................................................................................... 13
4. Data and econometric methodology .............................................................................................. 18
5. Empirical results ............................................................................................................................ 22
   5.1 Parameter estimates and model performance ............................................................................ 23
   5.2 Factor loadings, variance decomposition, and impulse response functions ............................. 23
   5.3 Latent factor dynamics and observable demand ....................................................................... 25
   5.4 Quantifying the impact of long-term investors on real rates .................................................... 26
   5.5 What drives the dynamics of the yield curve? .......................................................................... 28
6. Concluding remarks ....................................................................................................................... 29
A Appendix: proofs ............................................................................................................................ 31
B Appendix: MCMC algorithm ......................................................................................................... 33
References .......................................................................................................................................... 38
Tables and figures .............................................................................................................................. 43

* Bank of Italy, International Relations Directorate.
1 Introduction

‘... [A]nalysts predict that long-dated gilts are likely to remain under pressure amid a lack of supply and continued strong demand for long-dated inflation-protected assets.’

[Risk Magazine, 01 March 2006]

Long-term inflation-linked government bonds are the truly riskless assets for long-term investors (Campbell and Shiller, 1996; Campbell and Viceira, 2001, 2002; Brennan and Xia, 2002; Campbell, Chan and Viceira, 2003; Watcher, 2003; and Campbell, Shiller and Viceira, 2009). The inflation-linked government bond market is, however, not only smaller than the conventional government bond market, but also populated by specialized buy-and-hold investors such as pension funds and life insurers. The functioning of the market is therefore characterized by strong price pressure exerted by these preferred-habitat, or simply long-term, investors (Greenwood and Vayanos, 2010). Moreover, the preferred-habitat investors’ demand is largely inelastic being driven by needs other than risk-return considerations. For example, also as a result of accounting and regulatory reforms, such as the Pensions Acts of 1995 and 2004, UK pension funds’ asset allocation is increasingly driven by their liabilities. Index-linked gilts (UK government bonds) in particular are perceived by many pension funds as the best match asset for their long-term real liabilities. Moreover, the introduction of the UK index-linked gilt market dates back to 1980s, which is much earlier than in other countries (Campbell, Shiller and Viceira, 2009). The UK index-linked gilt market is therefore a natural laboratory to study price pressures induced by institutional investors.

At the same time, the preferred-habitat model of Vayanos and Vila (2009, hereafter VV) is the natural framework to study price pressures in the index-linked government bond market. VV show how demand and supply imbalances can affect bond yields whilst maintaining the no-arbitrage assumption. The model does this by departing from the standard asset pricing literature, in that equilibrium bond prices are determined by the interaction of two types of investors: preferred-habitat (or long-term) investors and arbitrageurs. On the one hand, preferred-habitat investors have strong preferences for specific maturities, and their demand is partly inelastic reflecting investment opportunities outside the bond market. On the other hand, arbitrageurs accommodate the price pressure generated by the preferred-habitat investors’ demand rendering the term structure arbitrage free. Specifically, arbitrageurs bridge the disconnect between the short rate and bond yields and also bring the yields in line with each other, smoothing local demand and supply pressures.

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However, the arbitrageurs’ activity affects the amount of interest-rate risk, or “duration risk”, of their portfolios. For example, to accommodate a decrease in the demand of long-term bonds by preferred-habitat investors, arbitrageurs must absorb more duration risk for which they need to be compensated. Specifically, they require all the bonds in their portfolios to offer higher expected returns, so that prices go down for all bonds, and yields and expected returns go up. That is, due to the presence of risk-averse arbitrageurs, even local demand effects can operate globally by changing the prices of short-rate and demand risk and therefore through bond risk premia. Notably, as longer-term bonds are more sensitive than short-term bonds to changes in duration risk, they are more exposed to price pressures. This also explains why in the event of strong demand pressures the term structure of real rates tends to invert.

The preferred-habitat investors of the VV’s model therefore closely resemble the institutional investors responsible for the price pressures in the UK index-linked gilt market. For this reason, in this paper, we structurally estimate the preferred-habitat model of VV on the term structure of quarterly UK real rates derived from the inflation-linked gilts for the period from 1987 to 2012. The term structure is determined by two factors: the short-term real interest rate and the preferred-habitat demand. In particular, the preferred-habitat demand factor captures the demand pressure exerted by a number of institutional investors, i.e. their net investment relative to the issuance of long-term inflation-linked bonds. Specifically, we consider four types of preferred-habitat investors: pension funds, life insurers, foreign investors and the Bank of England. What is common among these investors is that they tend to base their investment decisions on needs other than risk-return considerations. Moreover, pension funds and life insurers have specific preferences for longer maturities, so that longer-term inflation-linked gilts represent a natural hedge for their liabilities. Anecdotal evidence also suggests that the price pressures exerted by these investors exacerbated around the time important regulatory and accounting reforms were introduced. Some of the buyers of UK index-linked gilts are foreign institutional investors, in light of the depth and liquidity that distinguish the UK index-linked gilt market relative to other inflation-linked government bond markets. Foreign central banks are also notoriously good examples of preferred-habitat investors; their demand for government bonds is largely inelastic as part of their reserve accumulation policy (e.g. Krishnamurthy and Vissing-Jorgensen, 2012). More recently, however, much of the focus has shifted towards central banks in their implementation of the quantitative easing (QE) policies. And the effectiveness of these policies has often been rationalized on the basis of the preferred-habitat theory.

A natural question though is how can we measure the price pressures exerted by these investors? First of all, it is fundamental to capture the interplay between institutional investors’ demand and the issuance by the government. That is, only if the strong demand by preferred-habitat investors is coupled by a lack of supply by the government, we are in the presence of strong demand pressures.
Our measures of demand pressure therefore relate the net purchases of index-linked bonds by institutional investors to the issuance of index-linked gilts. Moreover, in light of the fact that institutional investors have strong preferences for longer-term securities, we conjecture that, when the average maturity of the index-linked gilts outstanding decreases, price pressures by institutional investors intensify. That is, when the average maturity is low, institutional investors compete among each other to grab the few long-term bonds in the hands of arbitrageurs. Of course arbitrageurs, differently from the preferred-habitat investors, display no particular preference for any specific maturity and are therefore able to accommodate this pressure but only to the extent that they are adequately compensated. At times when long-dated gilts are scarce the demand by preferred-habitat investors can only be accommodated by a larger price concession.\(^2\) Arbitrageurs in these circumstances would therefore act as if they were more risk averse. In sum, the price impact of the preferred-habitat investor demand pressure, measured by net investment over issuance, should vary with the average maturity of gilts outstanding.

**Main Results.** A number of interesting results emerge from the analysis. First, the preferred-habitat model fits particularly well the term structure of UK real rates for the entire sample. The short-term real rate (or real policy rate) has a strong impact on shorter-term rates as might be expected. By contrast, the demand factor has a small impact on short-term rates, but an absolutely dominant one for longer maturities. The variance decomposition reveals that shocks to the short rate explain roughly 95 and 75 percent of the 2- and 5-year rates at the one-year horizon, respectively, while their impact on the 20-year rate is negligible. Moreover, the fraction of the variance explained by the short rate decays with the forecasting horizon. In contrast, demand shocks are particularly important for long-term rates and their effect over time is rather persistent. Then the impulse response analysis shows that in response to a 25 basis point shock to the preferred-habitat demand the 5- and 20-year rates fall by 13 and 21 basis points, respectively, on the impact. At the 10 year horizon the 5- and 20-year rates are still 6 and 10 basis points below their pre-shock levels. Taken together these results show that increases in the demand factor drive long-term rates down, producing a persistent impact, whereas movements of the short-rate are less persistent and more relevant for shorter maturity yields. Notably, our results are consistent with the anecdotal evidence that at times of strong demand pressures the term structure of real rates is likely to invert.

Second, the estimated filtered demand factor extracted from the cross section of real rates by means of our structural estimation of the VV’s model trends upward through the whole period, but this trend substantially intensifies in proximity of the regulatory reforms that took place in the mid-
1990s and mid-2000s, and around the implementation of the QE programme. Interestingly, during these episodes of intensified demand, bond risk premia fall substantially, driving the fall in rates. Moreover, we find that our observable measures of pension funds, life insurers and foreign investors’ demand pressure are consistent with the evolution of the estimated preferred-habitat demand factor. Of further interest is that, consistently with our hypothesis, the demand pressure exerted by these investors increases as the average maturity of the inflation-linked gilts outstanding decreases. It is therefore possible to draw a parallel between our result and the empirical evidence in Greenwood and Vayanos (2013), showing that the impact of changes in supply on bond risk premia intensify at times of high arbitrageurs’ risk aversion.

Third, a key advantage of our model specification is that we can quantify the separate impact of demand pressure on real rates generated by each type of institutional investor in turn, and this impact can be estimated consistently across maturities, due to the no-arbitrage assumption. Interestingly, we find that during the 1990-2012 period pension funds’ overall impact on real rates ranges from a minimum of -70 basis points at the 2-year maturity to a maximum of -165 basis points at the 20-year maturity. The impact of life insurers and foreign investors is of similar magnitude ranging roughly from a minimum of -30 basis points at the 2-year maturity to a maximum of -70 basis points at the 20-year maturity. Then, the impact of QE on the 10-year rate is around 100 basis points, being consistent with previous studies (e.g. Joyce et al., 2011). However, the sub-sample analysis reveals that the bulk of the pension funds’ downward pressure on real rates was concentrated in the period around the introduction of the first Pensions Act of 1995. This result is due to the fact that during this period not only pension funds’ purchases of gilts were particularly strong, but also the issuance of inflation-linked bonds was particularly weak and the average maturity of the inflation-linked bonds outstanding was also approaching its sample lows. In contrast, issuance of inflation-linked gilts grew at a faster pace than net purchases of inflation-linked gilts by pension funds around the time the Pensions Act of 2004 was introduced. This may also reflect the fact that pension funds during this period increased their use of inflation derivatives, so that by looking at the net purchases of gilts we are effectively underestimating the total price pressure exerted by pension funds. In contrast, life insurers and foreign investors seem to be responsible for the fall in the long-term real rates observed during the first half of the 2000s.

Related Literature. Our paper is motivated by the extensive anecdotal evidence on institutional price pressures in the UK index-linked market and by the study of Greenwood and Vayanos (2010) that focuses on two episodes of price pressures; the UK Pensions Act of 2004 and the US Treasury buybacks. Notably, they explain these episodes in light of the modern theory of preferred habitat. Our paper complements their analysis by implementing a structural estimation of the preferred-
habitat model of VV that inspires their analysis, and by focusing on a much longer period and on a larger group of institutional investors. In a subsequent study, Greenwood and Vayanos (2013) examine theoretically and empirically how the supply and maturity of the US government debt affect bond yields and expected returns. Interestingly, they find that the data support the predictions of the VV’s model. Their empirical analysis is therefore also based on the VV’s preferred-habitat model, though they do not implement a direct structural estimation of the VV model. Moreover, in their analysis the focus is mainly on the supply effects so that demand effects due to the activity of institutional investors and foreign central banks are not modeled. Hamilton and Wu (2012) are the first to try to include some of the features of the VV’s model within an otherwise standard term-structure model. Notably, their study finds strong empirical evidence in favor of the existence of a duration channel consistent with the preferred-habitat theory. More recently, Li and Wei (2012) also impose some of the structure suggested by the VV’s model within a no-arbitrage term-structure model. Differently from these studies, not only we structurally estimate the VV’s model developing a Bayesian algorithm, but also we look at the UK index-linked market focusing only partly on unconventional policies and more generally on the role of institutional investors.

A large number of studies look at bond yields around specific policy events such as the Operation Twist and the 2000-2002 buybacks by the Treasury (Modigliani and Sutch, 1966; Ross, 1966; Wallace, 1967; Garbade and Rutherford, 2007; and Swanson, 2011), whereas more recently the focus has shifted towards QE programmes in the US (e.g. Ganon et al. 2011; Krishnamurthy and Vissing-Jorgensen, 2011; D’Amico et al. 2012, D’Amico and King 2013) and the UK (Joyce et al. 2011). Our study is also motivated by a number of previous studies on the impact of reserve accumulation by foreign central banks on the level of US interest rates (e.g. Caballero, Farhi and Gourinchas, 2008; among others). Our study tries to bring together these two separate strands of the literature by investigating the price pressures exerted by foreign officials.

Our paper also contributes to the extensive literature on the pricing of inflation-linked bonds (e.g. Barr and Campbell, 1997; Evans, 2003; Buraschi and Jiltsov, 2005; Ang, Bekaert and Wei, 2008; Joyce, Kaminska and Lildholdt, 2012; Pericoli, 2014; and many others). Notably, many of these studies acknowledge the importance of the behavior of institutional investors in determining real rates, however, without directly modeling their behavior. In particular, the behavior of institutional investors is often used to rationalize changes in the liquidity conditions in the TIPS and inflation-linked gilt markets (e.g. D’Amico, Kim and Wei, 2008). Notably, the inelastic demand by pension funds and life insurers, which is the root cause of the price pressures and of the consequent changes

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3This literature composes of a number of theoretical studies on “global imbalances” (e.g. Caballero, 2006; Mendoza, Quadrini and Rios-Rull, 2009; and Caballero and Krishnamurthy, 2009), and a number of empirical studies that quantify the impact of reserve accumulation on US interest rates in a reduced-form fashion (Warnock and Warnock, 2009; Sierra, 2010; and Krishnamurthy and Vissing-Jorgensen, 2012).
in the liquidity conditions, may result not only from regulatory changes, but also from changes in demographics. To this end, our study relates to the study of Favero, Gozluklu and Yang (2013) where the age structure of the population drives the persistent component of interest rates.

Our study also relates to that strand of the literature linking the behavior of institutional investors to asset prices. On the one hand, there is an active emerging policy debate on the impact of non-bank institutional market participants on (bond) risk premia, which can pose severe risks to financial stability (Haldane, 2014; Feroli, Kashyap, Schoenholtz and Shin, 2014). On the other hand, the academic literature has largely focused on the equity market with a particular focus on institutional herding (e.g. Sias, 2004; among others). Surprisingly only little research has been conducted on pension funds despite they hold a significant share of the global market portfolio (Blake et al 2013). Blake, Lehmann and Timmermann (1999) and Blake and Timmermann (2005) are notable exceptions focusing on the UK pension fund industry.

The rest of the paper is organized as follows. Section 2 outlines the main elements of the VV’s preferred-habitat model. Section 3 presents our empirical counterpart to the demand factor in the VV’s model, introducing our measures of preferred-habitat investors’ demand pressure. Section 4 then presents the interest rate data and the econometric methodology, while the empirical findings are discussed in Section 5. Section 6 concludes. In the Appendix we provide a more detailed description of the VV’s model and the building-blocks of the Bayesian algorithm.

2 Model

The model we present in this section builds on the model of VV. This model departs from the standard asset pricing literature, in that bond prices are not determined by a representative agent. Bond prices in equilibrium instead result from the interactions of two types of investors: preferred-habitat investors and arbitrageurs. Preferred-habitat investors have strong preferences for specific maturities. However, the no-arbitrage assumption is maintained by the presence of the arbitrageurs. In their absence, each maturity would constitute a separate market, with its yield being determined by the clientele of investors of that maturity. Arbitrageurs role is essentially twofold: i) they bridge the disconnect between the short rate and bond yields; and ii) they also bring the yields in line with each other smoothing local demand and supply pressures. However, arbitrageurs are risk averse and therefore want to be compensated for rendering the term-structure arbitrage free. In this way, demand pressure can exert an effect on the whole term structure of bond yields. Next, we present a

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\(^4\)A few seminal papers on herding are Scharfstein and Stein (1990), Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Froot, Scharfstein and Stein (1992). Of particular interest is the study by Lakonishok, Schleifer and Vishny (1992) on pension funds.
brief description of the model, while a detailed description is provided in the Appendix.

**Short rate.** The model is set in continuous time and the term structure is therefore represented by a continuum of zero-coupon bonds. The time-$t$ price of a zero coupon bond that pays one dollar at maturity $t+\tau$ is denoted by $P^\tau_t$, and the relative yield by $R_{t,\tau}$. The short rate $r_t$, i.e. the limit of the spot rate $R_{t,\tau}$ for $\tau$ that goes to zero, follows the Gaussian (Ornstein-Uhlenbeck) process:

$$dr_t = \kappa_r (\bar{\tau} - r_t) \, dt + \sigma_r dB_{r,t},$$

(1)

where the unconditional mean, $\bar{\tau}$, the rate of mean reversion, $\kappa_r$, and the standard deviation, $\sigma_r$, are positive constants and $B_{r,t}$ is a Brownian motion.

**Preferred-habitat Investors.** We specify the demand for the bond with maturity $\tau$ as a linear function of the bond’s yield, $R_{t,\tau}$:

$$y_{t,\tau} = \alpha (\tau) \tau (R_{t,\tau} - \beta_t),$$

(2)

where $\alpha (\tau)$ can be a generic function of maturity $\tau$ with the only requirement of taking positive values; however, following VV we adopt the following functional form $\alpha (\tau) = \alpha e^{-\delta \tau}$.\textsuperscript{5} It is worth noting that we will generally refer to $\beta_t$ as to the demand factor, but according to eq. (2) demand $y_{t,\tau}$ increases when $\beta_t$ falls. The aggregate demand factor can result from the combination of several demand factors:

$$\beta_t = \sum_{k=1}^K \theta_k \beta_{t,k},$$

(3)

where $\{\beta_{t,k}\}_{k=1..K}$ denote $K$ demand factors and $\{\theta_k\}_{k=1..K}$ describe the sensitivity of $\beta_t$ to the $K$ factors.\textsuperscript{6} These many factors can capture changes in the hedging needs of preferred-habitat investors (arising because of changes in policies, pension funds’ liabilities or regulation, etc.), or changes in the size or composition of the preferred-habitat investor pool, or changes in the supply of bonds. Therefore, $\beta_t$ captures the part of the demand $y_{t,\tau}$ that is largely inelastic. Moreover, in order to preserve the model tractability, we specify the (stochastic) dynamics of the aggregate demand factor

\textsuperscript{5}See Greenwood and Vayanos (2013) for an alternative specification of eq. (2).

\textsuperscript{6}In the most general form of the preferred-habitat model of VV $y_{t,\tau} = \alpha (\tau) \tau (R_{t,\tau} - \beta_{t,\tau})$ where $\beta_{t,\tau} = \sum_{k=1}^K \theta_k(\tau) \beta_{t,k}$. According to this specification, each of the $K$ demand factors can generate maturity specific effects through $\theta_k(\tau)$, so that $\beta_{t,\tau}$ is maturity specific. In this way, there are local demand effects. However, the lack of maturity-specific data on institutional holdings of gilts does not allows us to model these local effects. Note that the lack of maturity specific loadings, or factors, does not imply that demand shocks impact rates at different maturities equally, as this ultimately depends on each bond risk exposure. A detailed off-model analysis of local supply effects of QE policies on bond-by-bond data is provided, for example, by D’Amico and King (2013).
rather than the individual dynamics of the $K$ demand factors. Precisely, the aggregate demand $\beta_t$ follows the Ornstein-Uhlenbeck process:

$$d\beta_t = \kappa_\beta (\bar{\beta} - \beta_t) \, dt + \sigma_\beta dB_{\beta,t},$$

(4)

where $\bar{\beta}$ is the unconditional mean and $(\kappa_\beta; \sigma_\beta)$ are positive constants and $B_{\beta,t}$ is a Brownian motion.\(^7\)

**Arbitrageurs.** Differently from the earlier preferred-habitat models, such as the seminal paper by Modigliani and Sutch (1966), the presence of arbitrageurs guarantees the presence of no-arbitrage, so that bonds with maturities in close proximity trade at similar prices. However, arbitrageurs are risk averse and therefore demand a compensation in the form of risk premium while trading away arbitrage opportunities. For example, such arbitrage trades can alter the duration risk of arbitrageurs’ portfolios. Formally, the arbitrageurs’ investment strategy follows the following mean-variance portfolio optimization:

$$\max_{\{x_{t,\tau}\}_{\tau \in (0,T]}} \left[ E_t(dW_t) - \frac{a}{2} Var_t(dW_t) \right],$$

(5)

with $a$ denoting arbitrageurs’ risk-aversion coefficient, $x_{t,\tau}$ their dollar investment in the bond with maturity $\tau$ and $W_t$ arbitrageurs time-$t$ wealth. Then, arbitrageurs’ budget constraint is assumed to be:

$$dW_t = \left( W_t - \int_0^T x_{t,\tau} \right) r_t dt + \int_0^T x_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}},$$

(6)

where $P_{t,\tau}$ is the time-$t$ price of the bond with maturity $\tau$ that pays $1$ at time $t+\tau$. Assuming that equilibrium spot rates are affine in the risk factors $r_t$ and $\beta_t$,

$$\tau R_{t,\tau} = A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau),$$

(7)

and imposing equilibrium $x_{t,\tau} = -y_{t,\tau}$, we can solve for the bond sensitivities $A_r(\tau)$ and $A_\beta(\tau)$ and $C(\tau)$ through a system of linear ODEs.

**Bond Risk Premia.** The instantaneous expected return in excess of the risk free rate at any maturity $\tau$ is given by:

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\(^7\)As will become more evident later on, the pricing of the equilibrium interest rates is rather cumbersome in the VV’s model set up when compared to more standard models of the term structure of interest rates, which are already notoriously difficult to estimate. In the presence of a single demand factor the bond pricing consists of solving a system of four non-linear equations. By adding a third factor with its specific dynamics, bond prices would depend on a system of nine non-linear difference equations. By adding a fourth separate factor the tractability of the model is likely to be compromised. In sum, the benefit of improving the model fit, by allowing for separate dynamics of the demand factors, would compromise the possibility of taking the model to the data.
\[ \mu_{t, r} - r_t = A_r (\tau) \lambda_{r,t} + A_\beta (\tau) \lambda_{\beta,t}, \]  

where the prices of the short-rate and demand factors, \( \lambda_{r,t} \) and \( \lambda_{\beta,t} \), respectively are:

\[ \lambda_{r,t} = -a\sigma_r \int_0^T y_{t, \tau} [\sigma_r A_r (\tau) + \rho \sigma_\beta A_\beta (\tau)] d\tau \]  
\[ \lambda_{\beta,t} = -a\sigma_\beta \int_0^T y_{t, \tau} [\sigma_\beta A_\beta (\tau) + \rho \sigma_r A_r (\tau)] d\tau \]

where \( \rho \) is the factor correlation between \( B_{r,t} \) and \( B_{\beta,t} \). Eq. (8) follows solely from imposing the absence of arbitrage, which postulates that expected excess return per unit of each type of risk must be the same for bonds at any maturity. That is, the market prices of risk are the expected excess returns that arbitrageurs require as compensation for taking a marginal unit of short-rate and demand risk. The specific form of the market prices of risk, however, does not follow from the no-arbitrage assumption, it is instead determined by the interaction between arbitrageurs and preferred-habitat investors’ activities. The novelty of the preferred-habitat model therefore pertains to the specification of the prices of risk that directly relate to the excess demand, \( y_{t, \tau} \), for that particular maturity \( \tau \).

In sum, arbitrageurs are willing to accommodate changes in the preferred-habitat investors’ demand only to the extent that the consequent change in the riskiness of their portfolio is adequately compensated. For this reason, demand directly enters the market prices of risk. More fundamentally, local demand effects, by changing the market price of risk that is the same for all bonds, can generate global effects. In this way, even local demand shocks by altering the market prices of risk and consequently the bond risk premia affect the entire term structure of interest rates. In essence, due to the presence of arbitrageurs, demand/supply effects do not operate locally but globally through changes in the prices of risk. Of further interest is that the bonds most heavily affected by the shock are those more sensitive to changes in the market prices of short-rate risk, and therefore may not necessarily coincide with the maturity where the shock was originally located. Bond risk premia are particularly important for longer-term bonds that are therefore particularly responsive to demand shocks.

3 Preferred-habitat Demand

A delicate issue regards the choice of the individual demand factors, in essence our empirical counterpart to eq. (3). In what follows we first describe why pension funds, life insurers and the central bank are likely to be similar to the preferred-habitat investors of the VV’ model. We then present our
measures of demand pressure whereby the demand of these investors is interacted with the supply and the average maturity of the inflation-linked bonds outstanding.

**Preferred-habitat Investors.** Preferred-habitat investors denote not only investors that have preferences for particular maturities, but also those investors that base their investment decisions on considerations other than risk-return. For this reason, natural candidates are long-term investors such as pension funds or life insurers.

As a result of two main regulatory reforms, aimed at enhancing the resilience of the pension fund industry, the asset allocation of pension funds has changed substantially over the past twenty years, making their demand increasingly inelastic. The Pensions Act of 1995 introduced the Minimum Funding Requirement (MFR) to ensure that pension plan members could be paid in full even if the sponsoring firm went bankrupt (Greenwood and Vayanos, 2010). In particular, pension funds were required to set out a minimum level of investment in government bonds. Moreover, the MFR was introduced about the same time of a number of accounting reforms that based some key discount rates to value liabilities on gilt rates. As a result, pension funds have been drawn toward gilts as the natural matching asset for MFR liabilities; there is a reduced risk of failing the test if the asset portfolio reflects the discount rates required to value liabilities (Blake, 2003). These regulations were then abandoned in the early 2000s. Then, the Pensions Act of 2004 established that listed companies were required to measure definite benefit (DB) pension scheme assets and liabilities at fair value, and to recognize deficits and surpluses. In addition, company sponsors were required to address underfunded pension plans and eliminate deficits within 10 years. A Pension Protection Fund (PPF) was also launched, providing compensation for DB scheme members if their employer becomes insolvent and the pension scheme is underfunded. These changes affected DB asset-liability management strategies, increasingly linking asset to the liabilities, and as a result inducing pension funds to hold more fixed income assets. In particular, many LDI strategies involved acquisitions of index-linked products such as inflation-linked government bonds and inflation derivatives.\(^8\) In sum, while prior to the mid-1990s the pension funds’ focus was on the risk-return profile of assets, the regulatory changes, coupled with the 2000s equity crash, shifted their focus to LDI strategies.

Similarly to pension funds, life insurers were subject to a number of regulatory requirements and accounting reforms, and as a result perceived long-term conventional bonds and index-linked gilts as a good hedge against their long-term inflation-indexed liabilities. In particular, the individual capital assessments (ICAS), which were first consulted in 2002 and then put in place in 2004, among other things, prescribed insurers’ liabilities to be discounted using long-term gilt rates (FSA, 2003).\(^9\)

\(^8\)Note that pension fund benefits are fixed and are expected to grow with inflation so that pension liabilities are expected to grow with the price of inflation-linked government bonds.

\(^9\)Short duration general insurance business though is undiscounted, and annuities are discounted using the long-term
Foreign institutional investors may have also invested in the UK index-linked bond market due to its depth and size.\(^\text{10}\) Foreign central banks are also notoriously good examples of preferred-habitat investors; their demand for government bonds is largely inelastic as part of their reserve accumulation policy (e.g. Krishnamurthy and Vissing-Jorgensen, 2012). Another important preferred-habitat investor is the Bank of England that buys gilts as part of its QE policies.

**Measures of demand pressure.** So far we established that good examples of preferred-habitat investors are domestic and international pension funds and life insurers, foreign central banks and the Bank of England. However, the supply and the maturity structure of the government debt is also an important determinant of bond yields and expected returns (D’Amico and King, 2013; and Greenwood and Vayanos, 2013). For this reason, it is fundamental to analyze the interplay between changes in the demand of preferred-habitat investors and in the supply; demand pressures by preferred-habitat investors should intensify (weaken) at times of low (high) issuance of bonds by the government. We therefore standardize the holdings of index-linked bonds by the issuance by the government. Specifically, our measures of demand pressures are: i) the cumulative net investment of pension funds in index-linked bonds over the cumulative issuance of index-linked bonds; ii) the cumulative net investment of life insurers in index-linked bonds over the cumulative issuance of index-linked bonds; iii) foreign investors (including also foreign central banks) holdings of UK gilts over the market value outstanding of gilts; and iv) Bank of England holdings of UK gilts over the market value outstanding of gilts.

Following Greenwood and Vayanos (2010), we use the Official National Statistics data to construct our return-adjusted holdings of index-linked gilts, i.e. the cumulative sum of the net investment, by domestic pension funds and life insurers. We then standardize these measures by the cumulative sum of index-linked issuance by the UK government that is provided by the Debt Management Office (DMO). Thus, changes in pension fund and life insurer demand pressures are not induced by valuation effects due to asset price changes, but they rather reflect the investment decision of pension funds and life insurers.

Ideally, we would like to construct a similar measure of demand pressure for foreign institutional

\(^{10}\)Of course a foreign pension fund for example may not fully hedge the real value of its liabilities by investing in UK index-linked gilts, given the mismatch in the inflation indices used (and the currency risk). A foreign pension fund, however, may still find it attractive to invest in UK index-linked gilts to the extent that its country’s inflation is correlated with the UK RPI index. Also, not all countries have an inflation-linked bond market or even if they do it is not as developed and liquid as the UK inflation-linked gilt market (with the only exception of the Canadian, French and US markets). For this reason, price pressures may be even more substantial in these countries, so that domestic institutional investors may find more convenient to accept the mismatch between the UK inflation and their domestic inflation. For example, there is anecdotal evidence that foreign investors accepted this risk, due to the lack of domestic supply of inflation-linked bonds, and invested in the French index-linked bond market (Garcia and Van Rixtel, 2007). Demand and supply imbalances for inflation-linked bonds in the Euro area were so pervasive that induced the French Treasury to issue a new ten-year bond indexed to the Euro area Harmonised Index of Consumer Prices.
investors. However, the DMO only publishes the amount of gilts held by foreign investors. We therefore do not know how much of these holdings are conventional versus inflation-linked gilts. It is likely that foreign investors mostly hold conventional UK gilts. However, data on the fraction of the inflation-gilt market held by UK domestic pension funds and life insurers (not reported) decreased by roughly 20 percent in the decade from 1999 to 2009. Moreover, Figure 1 shows that the fraction of the gilt market held by foreign investors also increased dramatically over the same period. This evidence is coupled by market intelligence suggesting that foreign investors are largely foreign institutional investors in the UK inflation-linked market. Taken together these facts are consistent with foreign institutional investors increasing their investment in the UK index-linked market over the past decade, or so. But even if this was not the case, i.e. foreign institutional investors only invested in conventional bonds over the period, price pressures at least to some extent should have spilled over from the conventional gilt market to the inflation-linked gilt market. Indeed in normal times it is partly possible to arbitrage away differences between these two markets; an investor can replicate a conventional bond by matching the indexed cash flows and principal of an index-linked bond by executing a series of zero-coupon inflation swaps (Fleckenstain, Longstaff and Lustig 2012). In addition, the only data available on foreign investors is on their holdings of gilts rather than on their net investment, which therefore reflect both valuation and flow effects. For this reason, we standardize the data by the market value of gilts outstanding, which therefore reflect both valuation and flow effects. For this reason, we standardize the Bank of England’s holdings of gilts by the market value of gilts outstanding.\footnote{Note that as part of the QE policies the Bank of England only bought conventional bonds, but by using a similar argument to the one used above for the foreign investors, it is likely that the demand pressures in the conventional gilt market spilled over to the inflation-linked gilt market. Of course this does not imply that we should expect the same impact of QE on nominal and real rates, as QE may impact inflation expectations and local demand effects may also be relevant (Krishnamurthy and Vissing-Jorgensen, 2011). But the duration and scarcity channels should affect both nominal and real rates, through changes in the real term premium. For example, Abrahams, Adrian, Crump and Moench (2013) find that real long-term forward rates moved strongly on days of monetary policy announcements, thus suggesting that QE is largely a real phenomenon.}

Top panels of Figure 2 show the cumulative net investment (green line) and the demand pressure (blue line) of UK pension funds and life insurers. It is apparent that the demand pressure exerted by pension funds reached its peak roughly in 1998, therefore around the time of the implementation of the Pensions Act of 1995 that introduced the MFR, and decreased steadily thereafter. So despite pension funds net investment was generally positive, their demand pressure was concentrated in the second half of the 1990s. Life insurers demand pressure intensified around the two accounting/regulatory changes in 1997 and 2004. In contrast, foreign investors’ demand pressure increased substantially in the past decade.

**Average Maturity.** It is well established that pension funds and life insurers display strong prefer-
ences for longer-term bonds. We therefore conjecture that, when longer-term bonds become scarce, as reflected by the decrease of the average maturity of the inflation-linked gilts outstanding, price pressures by these institutional investors should intensify. In the context of the preferred-habitat model, this enhanced price pressure arises from a higher demand by preferred-habitat investors. The mechanism that leads to an increase in the preferred-habitat demand, i.e. a fall in $\beta_t$, might work as follows. At times of strong price pressure (i.e. when long-term bonds are scarce) preferred-habitat investors should drive out of the market arbitrageurs. That is, because arbitrageurs have no particular preference for specific maturities, they are willing to accommodate the preferred-habitat demand by selling longer-term bonds to the extent that they are adequately compensated. This implies that the price of longer-term bonds should increase enough to induce arbitrageurs to sell longer-term bonds. In the model, for this to happen, bond risk premia should fall reflecting a lower market price of risk.

And, as shown by eqs. (9) and (10), changes in the market price of risk are determined by changes in the demand factor $\beta_t$. As a result, for a given shock to our measure of demand pressure $\beta_t$ should fall by more when the average maturity of the inflation-linked gilts outstanding decreases.\(^{12}\)

Figure 2, bottom-right panel, plots the average maturity of the UK inflation-linked bond outstanding. The average maturity is computed as the weighted sum of the maturities of the inflation-linked bond outstanding. Precisely, the weight for a particular maturity is given by the face value of the inflation-linked bonds outstanding at that maturity over the face value of all the outstanding inflation-linked bonds. The average maturity was roughly 20 years in 1987, reaching its minimum of roughly 13 years around 2001 and increasing since then up to roughly 19 years in 2012. Moreover, the average maturity is strongly negatively correlated with the fraction of the index-linked market held by institutional investors (life insurers and pension funds). In particular, when the average maturity reaches its sample low, ONS data (not reported) show that pension funds and life insurers hold almost the whole of the inflation-linked bond market. Therefore, this is consistent with the view that arbitrageurs are driven out of the market at times when long-dated bonds are scarce. According to our hypothesis, this evidence, coupled with the earlier analysis of pension funds and life insurers’ demand pressures, would suggest that the price pressure exerted by institutional investors reached its peak in the second half of the 90s rather than during the other regulatory change in the mid 2000s. Next, we show how we test formally for these effects.

\(^{12}\)To some extent it is possible to draw a parallel with Greenwood and Vayanos (2013). Precisely, although the risk aversion of the arbitrageurs is constant in the preferred-habitat model, they show empirically that supply factors have stronger effects when arbitrageurs are more risk averse. Similarly, we conjecture that a decrease in the average maturity of the inflation-link bond outstanding has an effect similar to that of an increase in the risk aversion of the arbitrageurs. When the average maturity decreases arbitrageurs are likely to hold fewer long-term bonds in their portfolios and are therefore less willing to sell these bonds to the preferred-habitat investors. On the other hand, preferred-habitat investors are willing to pay a higher price for longer-term bonds at times when they are scarce. In sum, a decreasing maturity of the outstanding bonds is somehow comparable to an increasing risk aversion of the arbitrageurs.
Observable demand. We try to explain the evolution of the preferred-habitat demand by including not only our measures of demand pressures alone, but also by interacting them with the average maturity of the index-linked market, such that:

$$\beta_t = -d_t = \gamma_0 + \kappa Z_t' + \phi (\text{AvgMat}_t \cdot Z_t)' + \zeta QE_t + \gamma_1 \text{AvgMat}_t + \varepsilon_t,$$

where $Z$ includes the demand pressure of life insurers, pension funds and foreign investors; $\text{AvgMat}$ is the average maturity of the inflation-linked gilts outstanding; and $QE$ is the demand pressure exerted by the Bank of England with the QE policies. Note that the preferred-habitat demand in the model is $d_t = -\beta_t$. Our prior is therefore that i) the loadings of the demand pressure terms $\kappa = [\kappa_{LI}, \kappa_{PF}, \kappa_{FO}]$ are negative, i.e. an increase in preferred-habitat demand is associated with an increase in institutional investors’ demand pressure; and ii) the loadings on the interaction terms $\phi = [\phi_{LI}, \phi_{PF}, \phi_{FO}]$ are positive, i.e. the demand pressure exerted by institutional investors increases as the average maturity decreases.

4 Data and Econometric Methodology

Interest Rates. The real rates we use in this paper are the standard Bank of England estimates of zero-coupon real yields. The zero-coupon real yields are extracted from index-linked bond yields by using the method first proposed by Evans (1998) and later on extended by Anderson and Sleath (2001). We use end-of-quarter real rates for the period from 1987Q1 to 2012Q4. The quarterly frequency of the data and the starting date of the sample are dictated by the availability of the data on holdings of inflation-linked bonds by long-term investors.

We use the real rates for the 2-, 5-, 7-, 10-, 15-, and 20-year maturities. The summary statistics, displayed in Table 1, reveal an upward-sloping average term structure ranging from the 2- to the 15-year maturity rates. In contrast, the 20-year average real rate lies in between the 2- and 5-year rates.

Note that our measure of quantitative easing (QE) is included with no interaction term as it only becomes active from 2009 onwards.

We decided to model separately pension funds, life insurers, foreign investors and central banks for a number of reasons. First, while the demand pressure exerted by life insurers and pension funds is modeled in terms of flow effects (i.e. net investment over issuance), the demand pressure of foreign investors and the Bank of England captures both valuation and flow effects (i.e. holdings over amount outstanding). Therefore, it is not possible to aggregate domestic institutional investors with the foreign investors and the Bank of England. Moreover, by modeling separately each type of investor we can then quantify the price impact of each type in turn. More fundamentally, in this way we can capture price pressures determined by the change in the pool of preferred-habitat investors as originally suggested by VV; each investor demand can display a different degree of price elasticity.

We refer to Joyce, Kaminska and Lildholdt (2012) for a detailed description of the real rates.

Over the first years the liquidity of the inflation-linked market is limited. For this reason, for example, Joyce, Kaminska and Lildholdt (2012) start their analysis from 1992. However, the limited liquidity of the market during these years might be of particular interest for our study, as it may reflect demand pressure prices. Moreover, the quarterly frequency of the data induces us to use as many data as possible to have a sufficiently long time series that allows us to estimate the preferred-habitat model using observable demand factors.
The standard deviation of the 2-year rate is substantially higher than the longer-maturity rates, which are of comparable magnitude. The principal component (PC) analysis suggests that two factors explain more than 99.6% of the real yield term structure; the first PC plays a particularly important role for the short maturities, while the second PC loads negatively on short rates and positively on long rates. These two PCs therefore bear the usual interpretation of the level and slope factors, respectively. More fundamentally, the PC analysis supports the use of two factors, consistently with our model set up, to model the term structure of real rates.

The data are displayed in the top panel of Figure 3. It is apparent that real rates experienced a sharp fall over the sample. The substantial fall in rates around 1992 follows the United Kingdom’s withdrawal from the European Exchange Rate Mechanism (Joyce, Kaminska and Lildholdt, 2012). The consequent fall of real market interest rates from mid-1997 to beginning of 1999 was at least partly linked by many observers to the introduction of the Minimum Funding Requirement (MFR) as part of the 1995 Pensions Act, which became effective in April 1997.17 Similarly, the fall in real rates around 2004-2005 was attributed to the purchases of long-term and inflation-index gilts by pension funds in response to the Pensions Act of 2004 (Greenwood and Vayanos, 2010). However, it is also true that the post 2003 fall in rates coincided with a global fall in real rates. Real rates then begun to rise again in the first part of 2006, but this rise ended with the start of the ‘credit crunch’ in the summer of 2007. The subsequent fall in real rates was interrupted by the worsening of the crisis. Then, real rates displayed a short-lived but substantial spike reflecting a shortage of liquidity consequent to Lehman’s default (Campbell, Shiller and Viceira, 2009). Thereafter, as the Bank of England implemented QE policies, real rates fell substantially.

Econometric Methodology. We use a Bayesian approach to estimate the model, specifically we use a Markov Chain Monte Carlo Method similarly to Ang, Dong and Piazzesi (2007), among others. The estimation of standard term-structure models of interest rates is notoriously difficult; the likelihood function is highly dimensional, non-linearly depends on the model parameters, and displays multiple local maxima (Bauer, 2011). These difficulties are even more prominent in our model, where the likelihood function is a highly non-linear function of the parameters. Bayesian estimation, however, allows us to addresses this problem by relying on simple block simulations. Moreover, within a Bayesian setting, it is rather simple to quantify estimation uncertainty. For example, post-estimation calculations, such as term premia and impulse response functions, which are highly non-linear functions of the estimated parameters, can be easily computed using the draws of the MCMC algorithm.

Of interest is also that the estimated dynamic system for the factors underlying the yields is

17Downward pressure from pension fund buying was probably reinforced by the LTCM and Asian crises in Autumn 1997 and 1998, which caused a ‘flight to quality’ into government bonds (Joyce, Kaminska and Lildholdt, 2012).
generally specified as a vector autoregression, which is characterised by highly persistent factors and a relatively small estimation sample size. As a result, maximum likelihood estimates of such models may suffer from small-sample bias, and the estimation inference can therefore be compromised (Bauer, Rudebusch and Wu, 2012). In contrast, a Bayesian method allows us to make no assumption about the order of integration of the variables in the model. This is relevant as the estimation bias may contaminate, among other things, the estimate of longer term bond risk premia, which are based on the forecasts produced by the VAR.

**State-space Representation.** The preferred-habitat model can be naturally casted into a state-space framework. In our setting, the state (or, ‘transition’) equation describes the evolution of the short rate and demand factor under the objective probability measure, while the space (or, ‘measurement’) equation linearly maps these two factors into the observed real rates of selected maturities. The vector of observed yields \( R_t = [R_{t,2}, R_{t,5}, \ldots, R_{t,20}] \) is assumed to be collected with error:

\[
R_{t+\Delta} = z(X_{t+\Delta}, \Phi_1, \Phi_2) + \eta_{t+\Delta} \quad \text{and} \quad \eta_{t+\Delta} \sim N(0, \sigma_\eta I) \tag{12}
\]

where \( z(\cdot) \) is the pricing function of eq. (7) that yields the model implied real rates, which depends on the factors \( X_{t+\Delta} = [r_{t+\Delta}, \beta_{t+\Delta}] \) and on the hyperparameters \( \Phi_1 = [\kappa_r, \tau, \sigma_r, \kappa_\beta, \beta_\beta, \sigma_\beta] \) and \( \Phi_2 = [\alpha, \delta, \alpha] \). The zero mean vector of observation errors, \( \eta_{t+\Delta} \), is i.i.d. across time and yields, and is normally distributed with common variance across different maturities \( \sigma_\eta \).

To estimate the continuous time model from discrete data, we use its Euler discretized version.\(^{18}\) The resulting transitions equations are:

\[
X_{t+\Delta} = F_0(\Phi_1) + F_1(\Phi_1) X_t + u_{t+\Delta} \quad \text{and} \quad u_{t+\Delta} \sim N(0, \Sigma) \tag{13}
\]

The system matrices take the form of:

\[
F_0(\Phi_1) = \begin{bmatrix}
\kappa_r \Delta \\
\kappa_\beta \beta \Delta
\end{bmatrix}
\quad \text{and} \quad
F(\Phi_1) = \begin{bmatrix}
1 - \kappa_r \Delta & 0 \\
0 & 1 - \kappa_\beta \Delta
\end{bmatrix},
\]

and the factors’ variance-covariance matrix is:

\[
\Sigma = \Delta \begin{bmatrix}
\sigma_r^2 & \rho \sigma_r \sigma_\beta \\
\rho \sigma_r \sigma_\beta & \sigma_\beta^2
\end{bmatrix}.
\]

Note that the transition equations only depend on the objective parameters \( \Phi_1 \), whereas the measurement equations also depend on the \( \Phi_2 \) parameters that enter the market prices of risk.

\(^{18}\)The time interval \( \Delta \) equals 1/4 at a quarterly frequency.
To facilitate the estimation, we do not estimate all parameters. Arbitrageurs’ risk aversion, $\alpha$, is not separately identified from the demand elasticity, $\alpha$, as shown in the Appendix A, we therefore estimate the product of the two $\alpha\alpha$. The parameter $\delta$ is fixed to 0.1. Moreover, we fix the unconditional mean of the short rate process to 2% consistently with the standard value assigned to the natural rate (Joyce, Kaminska and Lildholdt, 2012). This parameter essentially tells us where the model expects the short rate to converge in the very long run, rather than were they should be today. The estimation results are robust to the choice of this parameter.

An additional measurement equation that links our observable measures of demand pressures described in Section 4 to the unobservable demand $\beta_{t+\Delta}$ completes the state-space representation.\textsuperscript{19} Precisely, the empirical counterpart to eq. (3) is:

$$\beta_{t+\Delta} = \Phi_3 D_{t+\Delta} + \varepsilon_{t+\Delta} \quad \varepsilon_{t+\Delta} \sim N(0, \sigma_\varepsilon). \quad (14)$$

where $D_t = [\text{AvgMat}_t, Z_t, \text{AvgMat}_t \cdot Z_t, QE_t]$ groups our observable measures of demand/supply imbalances. Precisely, the $Z_t$ vector includes our measures of demand pressure for life insurers, pension funds and foreign investors, while $\text{AvgMat}_t$ denotes the average maturity of inflation-linked gilts outstanding. The interaction term $\text{AvgMat}_t \cdot Z_t$ is calculated as the observation-by-observation product of $\text{AvgMat}_t$ and the individual measures of demand pressures. The parameter vector $\Phi_3 = [\gamma_0, \gamma_1, \kappa, \phi, \zeta]$ composes of a constant, $\gamma_0$; the loading on the average maturity of inflation-linked gilts outstanding, $\gamma_1$; the loadings of the demand pressure terms, $\kappa = [\kappa_{LI}, \kappa_{PF}, \kappa_{FO}]$; the loadings of the interaction terms, $\phi = [\phi_{LI}, \phi_{PF}, \phi_{FO}]$; and the loading on QE, $\zeta$. Finally, $\varepsilon_t$ may reflect other potential determinants of $\beta_{t+\Delta}$ that are not captured by our observables measures of demand pressure, or simply measurement errors on institutional net investment in gilts.\textsuperscript{20}

**Bayesian Estimation.** Bayesian estimation approximates the posterior distribution of parameters and states given the whole set of observations, $p(\Phi, X|Y)$, where $\Phi = [\Phi_1, \Phi_2, \Phi_3]$ denotes the parameters, $X$ denotes the latent states, and $Y = [R, D]$ denotes the data that consist of the market real rates $R = \{R_t\}_{t=1}^T$ and the observed demand variables $D = \{D_t\}_{t=1}^T$. Direct sampling from the posterior distribution $p(\Phi, X|Y)$ is often not feasible due to its high dimensionality or complicated form. The Markov Chain Monte Carlo (MCMC) method solves the problem of simulating from this complicated target distribution by simulating from simpler conditional distributions. Precisely, by

\textsuperscript{19}Additional measurement equations are generally used in term structure models when survey data are included in the estimation to better try to pin down, for example, the objective dynamics of inflation (see for example Wright, 2011).

\textsuperscript{20}According to the Statistical Bulletin of the ONS ‘data from the Pension Funds surveys are of lower quality than equivalent data from other institutional groups because of the difficulties in constructing a suitable sampling frame of pension funds for the surveys.’
applying the Bayes’ rule, the posterior density can be decomposed as follows:

\[ p(\Phi, X|Y) \propto p(Y|\Phi, X)p(X|\Phi)p(\Phi), \] (15)

where \( p(Y|\Phi, X) \) is the likelihood function given the states and the parameters, \( p(X|\Phi) \) is the probability distribution of states conditional on the parameters, and \( p(\Phi) \) is the prior density of the parameters. We can then iteratively draw from the full conditionals \( p(\Phi|X, Y) \) and \( p(X|\Phi, Y) \). The parameter set \( \Phi \) and the state set \( X \) can be further broken into smaller blocks.

We first draw the \( \Phi_1 \) and \( \Phi_2 \) parameters and the measurement error standard deviation \( \sigma_\eta \) conditional on the data and the states. The measurement error \( \sigma_\eta \) has a conjugate prior with inverse Gamma posterior. We can therefore sample directly from its posterior distributions using the Gibbs sampler. In contrast, it is not possible to sample directly from the full conditional posterior distributions of the rest of the \( \Phi_1 \) and \( \Phi_2 \) parameters. For this reason, we implement a series of Random-Walk Metropolis Hastings (RW-MH) steps; we sample a candidate draw from a proposal density, and then accept reject the candidate draw based on an acceptance criterion (see e.g. Johannes and Polson, 2009). Then, the step of drawing the factors conditional on the parameters \( \Phi \) is rather standard, as the linear and Gaussian dynamics of the factors allow us to use the forward-filtering backward sampling proposed by Carter and Kohn (1994). Finally, conditional on the filtered state \( \beta_{t+\Delta} \) we can draw the \( \Phi_3 \) and \( \sigma_\varepsilon \) by using simple Gibbs sampler steps, as none of these parameters enters the bond pricing.

In sum, we implement a hybrid MCMC algorithm that combines the Gibbs sampler with a series of slice RW-MH steps. By repeatedly simulating from the conditional distribution of each block in turn, we get samples of draws. These draws, beyond a burn-in period, are treated as variates from the target posterior distribution. More specifically, we perform 70,000 replications of which the first 30,000 are burned-in, and we save 1 every 20 draws of the last 40,000 replications of the chain so that the draws are independent. The priors used in this study are diffuse, and their distributions are chosen for convenience using a number of earlier papers (e.g., Johannes and Polson, 2009). The algorithm is described in detail in the Appendix.

5 Empirical Results

In this section, we start by presenting parameter estimates and assessing the fit of the model to the term structure of real rates. We then move on to analysing the impact of the short rate and demand factors on the term structure of real rates by looking at the factor loadings, variance decomposition and impulse response functions. Next, we present the estimates of the loadings of the filtered demand factor on our observable measures of demand pressures. Notably, we find strong statistical evidence in
favour of our two hypotheses: i) the demand pressures exerted by life insurers, pension funds, foreign investors and the Bank of England well explain the evolution of the extracted preferred-habitat demand factor; and ii) for a given increase in the observed measure of demand pressure, preferred-habitat investors’ demand increases by more, and therefore the fall in rates is more prominent, when the average maturity of the inflation-linked bonds outstanding is low. Based on these observable measures of demand pressure, we then quantify the price impact determined by each preferred-habitat investor on the term-structure of real rates for the whole period. We conclude the analysis by decomposing the real rates into their expected rate and bond risk premium components.

5.1 Parameter Estimates and Model Performance

Table 2 reports the parameter estimates of the preferred-habitat model. Notably, all parameters are statistically different from zero and estimated with precision, as suggested by the joint inspection of the confidence intervals, numerical standard errors and convergence diagnostics. The short-rate factor mean reverts roughly four times faster than the demand factor, as the mean reversion parameters are 0.42 and 0.09, respectively. Similarly, the standard deviation of the short-rate innovations ($\sigma_r = 0.0227$) is substantially higher than standard deviation of the demand factor innovations ($\sigma_\beta = 0.0051$). The innovations to the two factors display a positive correlation, $\rho = 0.248$. Of interest is also that the risk premium parameter ($\alpha\sigma$) is statistically different from zero, however, it is not possible to identify the risk aversion of the arbitrageurs ($\alpha$) separately from the demand elasticity ($\sigma$) given that we estimate the product of the two. That said, a value different from zero is indicative of market prices of risk also different from zero, and therefore of the presence of time-varying global demand effects.

The standard deviation of the measurement errors, which is common across maturities, is fairly small, $\sigma_\eta = 12.5$ basis points, given that the preferred-habitat model only consists of two factors and has fewer parameters than more standard two-factor term structure models. Visual inspection of the pricing errors, displayed in Figure 3, shows that only in few occasions the pricing errors are in absolute value larger than 20 basis points. Pricing errors are larger during periods of financial turmoil, as around the time of the 1997-1998 Asian crisis and the more recent 2008-2012 crisis. Interestingly pricing errors are particularly small around the periods of the introduction of the Pensions Acts of 1995 and 2004. As a result, the model should deliver accurate estimates of price pressures caused by institutional investors around the times of regulatory changes.

5.2 Factor Loadings, Variance Decomposition and Impulse Response Functions

To understand each factor’s contribution to the yield curve dynamics, we start by examining the estimated loadings of the two factors on the term-structure of yields, which are displayed in Figure
4. Because of the way we have defined the short-rate factor, its loading on the instant maturity yield is normalized to one. It is apparent that the contribution of the short-rate factor decreases with maturity, such that its loadings for maturities longer than ten years are negligible. In contrast, the role of the demand factor is different; it has a small but positive impact on short-maturity yields, but an absolutely dominant one for longer maturities.

The analysis of the loadings, however, is not informative on the persistence of the response of rates to a shock either to the short-rate or demand factor innovations. For this reason, Table 3 presents the variance decomposition of yield levels for various forecasting horizons. The total variance decomposition reveals that shocks to the short-rate explain about 96% and 74% of the total variance of the 2- and 5-year real rates at the 1-year horizon, while it only explains 20% of the 10-year rate, and it has no effect on the 20-year rate. Moreover, the fraction of the variance explained by the short-rate decays with the forecasting horizon. In contrast, demand shocks are particularly important for long-term rates and their effects are rather persistent.

We complete the analysis by presenting the impulse response functions in Figure 5. The 2- and 5-year rates instantaneous increases in response to a 25 basis points increase in the short rate are of roughly 14 and 6 basis points, respectively. These effects are not persistent though, as they half within two years and are negligible at the 10-year horizon. Consistently with the analysis of the factor loadings, the effect of a short-rate shock on the 20-year rate is negligible. The responses of rates to an increase in demand of 25 basis points is remarkably different; on the impact the 5- and 20-year rates fall by 13 and 21 basis points, respectively. Moreover, the impact is rather persistent as at the 10-year horizon the 5- and 20-year rates are still 6 and 10 basis points below their pre-shock levels.

In sum, it emerges from the analysis of the factor loadings, variance decomposition, and impulse response functions that increases in the demand factor drive long-term rates down, producing a persistent impact, whereas movements of the short-rate are less persistent and more relevant for shorter maturity yields. Moreover, our results are consistent with the anecdotal evidence that at times of strong demand pressures the term structure of real rates is likely to invert.

21 The negligible loadings on risk-free rate factor are the result of arbitrageurs’ need to hedge against two different risks. First, an increase in the short rate induces the arbitrageurs to engage in a reverse carry trade, i.e. they short bonds and invest in the short rate. Since the change in expected short rates in response to an increase in the short-rate is much more pronounced at short maturities, arbitrageurs go short more in short term bonds than long term bonds. But this shorting activity leaves them exposed to the risk that the bond prices will move against them, i.e. that bond prices will increase, either because the short rate decreases (like in the one-factor model) or because investor demand increases. At the same time, to hedge the demand risk, arbitrageurs are willing to buy long term bonds, which are most sensitive to this risk. This implies that according to the model, if arbitrageurs’ buying activity of long-maturity bonds (for hedging purposes) dominates their shorting activity (as part of the reverse carry trade), long-term rates could eventually decrease in response to an increase in the short rate.

22 This decomposition are based on standard Cholesky decompositions of the variance of the innovations where the short rate is ordered before the demand factor.
5.3 Latent Factor Dynamics and Observable Demand

In addition to parameter estimates key outputs of the Bayesian algorithm are the estimates of the unobservable factors. Figure 6 presents the filtered estimates of the short-rate (top panel) and demand factor (bottom panel).

**Short-rate Factor.** The short-rate evolution displays important swings, in particular it trends downward in a few occasions such as in the early 1990s, during the 1999-2003 period, and during the 2009-2012 financial turmoil in the aftermath of Lehman’s default. However, due to the lack of short maturity real rates, it is instructive to compare our estimate of the real short rate with a proxy of the real policy rate. Our proxy of the (ex-post) short real rate, displayed in blue, is constructed by subtracting the annual RPI from the end-of-quarter policy rate.\(^{23}\) It is apparent that our estimate of the short rate matches both the level and the evolution of the proxy of the short rate from 1993 onwards. In contrast, prior to 1993 although the evolution of the two is consistent, our estimate of the short rate lies substantially below the proxy of the short rate. Of interest is also that during the recent financial crisis, while the proxy of the short rate trends downward, the estimated short rate displays a temporary blip, possibly reflecting the shortage of liquidity that affected the index-linked bonds.

**Demand Factor.** Figure 6 also reveals that the estimated demand factor trends upward throughout the whole period. In particular, demand increases substantially in proximity of the regulatory reforms and in conjunction with the Bank of England’s QE policies. However, in order to better gauge the importance of the individual measures of demand pressure, we now turn to analyzing the loadings of eq. (11), which are also estimated within the Bayesian algorithm. The loading estimates, displayed in Table 4, show that our observed measures of pension fund, life insurer and foreigner demand pressures load positively on the unobserved demand (i.e. the \(\kappa\) are negative). It also shows that demand pressures generated by these investors increase as the average maturity of index-linked bonds decreases (as shown by the positive estimates of \(\phi\)). In other words, the pressure exerted by these investors increases as the availability of longer-term bonds decreases, which is therefore consistent with our hypothesis presented in Section 3. That is, these investors have strong preferences for longer-term bonds so that their demand pressure is higher at times when there are fewer of these bonds outstanding. It is therefore possible to draw a parallel between our result and the empirical evidence in Greenwood and Vayanos (2013) showing that the impact of changes in supply on bond risk premia intensify at times of high arbitrageurs’ risk aversion.

\(^{23}\)An alternative proxy of the real policy rate consists of using survey expectations of the RPI inflation outturn rather than inflation outturns. However, as pointed out by Joyce, Kaminska and Lildholdt (2011) there are no substantial differences between the two proxies.
The impact of demand pressures on the filtered demand factor extracted from the term structure of real rates by means of the preferred-habitat model is therefore time-varying, depending on the average maturity of the outstanding bonds. Table 5 shows the (average) marginal impact of long-term investors’ demand pressure on the extracted demand factor over the whole sample and separately for several subsamples. Negative coefficients would suggest that our observed measures of demand pressure are consistent with the preferred-habitat demand extracted from the model (i.e. filtered demand factor). In other words, negative coefficients would indicate that pension funds, life insurers and foreign investors well identify the preferred-habitat investors of the VV’s model. Interestingly, we find that ceteris paribus pension funds’ demand pressure has a positive effect on the estimated demand factor during the 1996-2000 and 2001-2005 subsamples, therefore in the periods around the regulatory reforms. Life insurers’ demand pressure has also a positive effect on preferred-habitat demand over an even longer period ranging from 1996 to 2010. Moreover, foreign investors’ demand has positive effect on the preferred-habitat investors’ demand throughout the entire sample. Finally, purchases of the Bank of England well explain the spike in the filtered demand over the 2009-2012 period.

Based on the loadings and the observed measures of demand pressure, we can then construct our observable measure of demand. By doing this we essentially assess to what extent the observed measures of demand can explain the otherwise unobserved demand factor extracted from the term structure of real rates by means of the preferred-habitat model. The blue line in the bottom panel of Figure 6 shows the fitted demand factor based on our observable measures of demand pressures. Notably, the fit is very good.\footnote{The observed and filtered demand factors only temporary diverge in the period soon after the exit of the UK from the European Exchange Rate Mechanism.}

### 5.4 Quantifying the Impact of Long-term Investors on Real Rates

We now turn to quantifying the cumulative impact of demand pressures on the term structure of rates for each institutional investor in turn. We do this by exploiting: i) the demand loadings $A_\beta(\tau)$ that map the demand factor into the rate of maturity $\tau$; ii) the estimated coefficients $\Phi_3$ of eq. (14); iii) and the measures of demand pressure and average maturity. In essence, by using ii) and iii) we project changes in the observed demand measures on the filtered demand factor $\beta$, which is then mapped into the term structure of rates based on i). Precisely, the pressures exerted by pension funds ($i = PF$), life insurers ($i = LI$) and foreign investors ($i = FO$) on the real rate of maturity $\tau$ during the period from $t_0$ to $t_1$ are computed as:

$$\Delta Z_{i_0,t_1} \Rightarrow \Delta R_{\tau_{t_0,t_1}} = \left[\frac{A_\beta(\tau)}{\tau} \times \left(\hat{\kappa}_i + \hat{\phi}_1 \Delta AvgMat_{t_0,t_1}\right)\right] \times \Delta Z_{i_0,t_1}, \quad (16)$$
whereas the effects of the average maturity and QE are calculated simply as:

$$\Delta AvgMat_{t_0, t_1} \Rightarrow \Delta R_{\tau}^{t_0, t_1} = \left[ \frac{A_{\beta} (\tau)}{\tau} \times \gamma_1 \times \Delta AvgMat_{t_0, t_1} \right]$$  \hspace{1cm} (17)

$$\Delta QE_{t_0, t_1} \Rightarrow \Delta R_{\tau}^{t_0, t_1} = \left[ \frac{A_{\beta} (\tau)}{\tau} \times \zeta \times \Delta QE_{t_0, t_1} \right].$$ \hspace{1cm} (18)

Table 6 displays the impact on the term structure of real rates of demand pressures by type of investor for the whole period ranging from 1990 to 2012. We find that the cumulative impact of pension funds ranges from -70 basis points on the 2-year rate to -165 basis points on the 20-year rate. The impact of life insurers and foreigners is of similar magnitude, specifically it ranges from -30 basis points on the 2-year rate to -74 basis points on the 20-year rate. The impact of QE is around 100 basis points on the 10-year rate, therefore is consistent with previous studies on the impact of QE on nominal rates for the UK (e.g. Joyce et al., 2011). The impact of average maturity alone is negligible, which is not surprising given that the average maturity in 2012 is about the same level as of 1990.\(^{25}\)

By using the same methodology it is possible to repeat the analysis over shorter subsamples. The analysis shows that the downward pressure on rates generated by pension funds was concentrated in the period prior to 2000.\(^{26}\) On the one hand, this result supports the anecdotal evidence that in response to the Pensions Act of 1995 pension funds’ purchases of index-linked gilts contributed to the fall in real rates. On the other hand, this result indicates that the increased demand for index-linked gilts by pension funds around the Pensions Act of 2004 does not determine a fall in real rates. This result contrasts with the anecdotal evidence pointing also to the Pensions Act of 2004 as an example of institutional price pressure induced by a regulatory change. However, at least two facts clearly differentiate these two episodes, as shown by Figure 2. First, although pension funds’ net investment in inflation-linked gilts intensified as a result of both regulatory changes, the Pensions Act of 1995 came at the time of weak issuance by the UK Treasury, while the Pensions Act of 2004 was characterised by a higher issuance by the Treasury.\(^{27}\) Second, the average maturity of

\(^{25}\)Note that eq. (17) only captures the partial effect of the average maturity on rates, as it also enters through the interaction terms of eq. (16). In this way, we therefore allocate the time-varying effect of the average maturity to the individual investors’ demand. We do this because our hypothesis is that average maturity matters to extent that alters the pressure exerted by institutional investors’ demand over time. We do not have a prior, however, on the effect of average maturity per se on the bond risk premia and therefore on the demand factor. The reason is that the preferred-habitat model predicts that as the duration of the arbitrageurs’ bond portfolios increases bond risk premia also increase to compensate the arbitrageurs for the increased riskiness of their portfolios. In contrast, if the increased duration of the preferred-habitat portfolios is matched by a decrease in the duration of the arbitrageurs’ portfolios, we should expect bond risk premia to fall. However, our measure of the average maturity refers to the all amount of bonds outstanding, and thus it does not distinguish between the maturity of the arbitrageurs’ portfolios and the maturity of the preferred-habitat investors’ portfolios.

\(^{26}\)We do not report these results for sub-samples of five year to economize on the space.

\(^{27}\)Greenwood and Vayanos (2010) also point to the Pension Act of 2004 as an example of price pressures in the Government bond market. They argue that in response to the Pension Act pension funds increased their exposure to long-term government bonds and reduced that to equities. In addition, they argue that the demand by pension funds of long-dated assets is substantial compared to the supply. Differently from Greenwood and Vayanos (2010), we only
the inflation-linked gilt outstanding was decreasing prior to 2000, while it was increasing thereafter. Taken together these facts explain why pension funds’ demand pressure contributed to the estimated fall in real rates only prior to 2000. There is one caveat though; the use of inflation derivatives by pension funds has increased substantially over the past decade. As a result, our measures of demand pressure are based on purchases of gilts and therefore underestimate the actual demand pressure exerted by pension funds around the time of the Pensions Act of 2004 ad thereafter. But unfortunately adequate data on pension funds’ holdings of inflation derivatives are not available.

In contrast to pension funds’ demand pressure, life insurers’ demand pressure increased substantially in the period leading to the introduction of the ICAS in 2004. Despite during the first half of the 2000s the average maturity of the inflation-linked gilts outstanding was already increasing, the demand of index-linked bonds by life insurers grew at a much faster pace than the issuance. As a result, life insurers’ demand pressure exerted a downward pressure on real rates also during this period of increased supply and increasing average maturity. During this period, however, the downward pressure on rates resulting from foreign investors’ demand was even stronger.

5.5 What Drives the Dynamics of the Yield Curve?

In the preferred-habitat model, demand/supply imbalances generate global effects on the term structure of rates by determining the market price of risk. It is therefore instructive to complete the analysis by looking at bond risk premia. We do this by decomposing real rates into the expected yield and the term premium components.

Figure 7 shows this decomposition for the 5- and 10-year real rates. The fall in the term premium clearly reflects episodes of increased preferred-habitat demand. For example, the 1996-1998 fall in real rates around the introduction of the 1995 Pensions Act is driven by the fall in the term premium that mirrors the increased demand. Of particular interest is also the fall in long-term real rates (right panel) around 2004-2005 that was described as a conundrum by Alan Greenspan, the Federal Reserve Chairman at the time (see Greenspan, 2005). In fact during this period long rates fell despite the policy rate was rising. A number of explanations for the conundrum have been put forward, see for example Backus and Wright (2007) and Joyce, Kaminska and Lildholdt (2012), among others. Figure 7 shows that the fall in long rates was driven by a fall in the term premium rather than by a fall in future real short-term interest rates. Our findings therefore support the hypothesis that real rates were driven lower by imbalances between demand and supply, linked to the preferred-habitat focus on the index-linked gilts, whereas they also include long term conventional gilt. (Note that the UK National Statistics stopped publishing the pension fund holdings of long term gilts.) Our measure of pension fund demand pressure also shows a blip around the time of the Pension Act of 2004, however this blip was short lived. Moreover, Figure 1 shows that the total market share of gilts, both conventional and index-linked, held by domestic institutional investors (though it does not distinguish between pension funds and insurers) strongly decreased after 2004. Thus, a simple analysis of the data also seems to confirm that pension funds demand pressure was concentrated around the Pension Act of 1995 rather than the Pension Act of 2004.
behavior of institutional investors including Asian central banks (though not UK domestic pension funds). Finally, Figure 7 shows that the volatility in long-term rates is largely driven by the term premium whereas the expected yield component is rather stable. In fact, the 5-year expected yield displays more pronounced swings than the 10-year expected yield. Taken together these results suggest that increased preferred-habitat demand is responsible for substantial falls in real rates by pushing down on bond risk premia. And this effect is stronger for longer rates that are riskier than shorter rates.

6 Concluding Remarks

In this paper, we investigate the impact of long-term investors’ demand for UK index-linked gilts on the term structure of real rates for the 1987-2012 period. We do this by carrying out a structural estimation of the preferred-habitat model of Vayanos and Vila (2009, VV). We use data on holdings of inflation-linked gilts by long-term investors, issuance of index-linked bonds and average maturity to identify the impact of supply/demand imbalances on rates. We find that the preferred-habitat model of VV fits the term-structure of real rates particularly well; demand pressures by long-term investors contributed to the decline of longer-term real rates over the 1987-2012 period by compressing bond risk premia. To this end, our analysis complements and refines the findings of previous studies such as Greenwood and Vayanos (2010, 2013), among others.

The impact of UK pension funds on the level of real rates is concentrated in the period prior to 2000. This result partly reflects the fact that, despite pension funds purchased index-linked gilts during the entire sample, net issuance picked up during the past decade. As a result, the impact exerted by pension funds’ demand on rates was attenuated. This fact though seems to contrast with the anecdotal evidence that pension funds exerted downward pressure on rates around the introduction of the Pensions Act of 2004. One potential explanation is that UK pension funds increased their reliance on inflation derivatives as part of their asset and liability management strategies over the past decade. The use of inflation derivatives, which is not captured by our measure of pension fund demand pressure, could also represent a source of pressure on rates. But the lack of data on the use of derivatives does not allow us to capture this additional potential source of demand pressure. In addition, pension funds’ price pressures might materialize in the few very long-term bonds available, producing important local demand-supply effects that are not captured in this study. This is consistent with the anecdotal evidence showing that ‘pension funds call for more long-dated gilts’ to deal with increasing life expectancy (FT, 2011). However, local demand effects can only be identified by using data on investor demand for specific bonds and by modelling maturity specific demand factors. For example, D’Amico and King (2013) provide a detailed analysis of local demand effects in the US
government bond market. In sum, while the focus of our study is on the so-called ‘duration effect’, the analysis of local demand and supply effects in the UK index-linked market remains a fruitful avenue for future research.

There are also important policy implications resulting from our analysis. A number of regulatory initiatives have been developed for insurance companies and pensions funds’ industries (some in response to the recent financial crisis) aimed at enhancing their resilience. These initiatives, however, may not necessarily take into account their impact on the financial system and the real economy as a whole. A key implication of our analysis is that the increased demand for (long-term) risk-free assets induced by these initiatives, if not matched by an increased supply of (long-term) risk-free assets, can determine a fall in rates. The need to expand the market for inflation-linked and ultra-long fixed income securities is a long-established policy guideline in the agenda of public authorities (Group of Ten, 2005; see also CGFS, 2011). Low rates in turn may spur excessive risk taking in financial markets with important consequences for the real economy (Caballero and Krishnamurthy, 2009). This fall in rates determined by institutional demand pressures combined with the use of risk-free rate to value pension funds and insurance companies’ liabilities could create a ‘procyclical’ feedback loop (Haldane, 2014). Specifically, focusing on pension funds, a fall in interest rates mechanically leads to widening of their deficit through an increase in the value of the discounted liabilities. As a consequence, pension funds, worried about further falls in rates, try to lock-in the level of deficits by de-risking their portfolios, i.e. by buying government debt. Increased demand pressure exerted by pension funds in turn may lead to further falls in rates.

In light of the increasing importance of non-bank institutional investors for financial stability, it is fundamental to have better data on the asset allocation of these investors. In particular, data on the use of inflation derivatives by these institutional investors can help quantify their price pressure. Moreover, it seems that foreign investors also played a crucial role over the past decade. However, despite market intelligence suggesting that foreign investors are largely foreign institutional investors, more detailed information on the types of investor, and possibly on the specific gilts held, would constitute a valuable source of information to improve the analysis even further.
Appendix: Proofs

We conjecture equilibrium spot rates that are affine in the risk factors, i.e. the short rate ($r_t$) and the demand factor ($\beta_t$), so that the equilibrium bond price takes the following exponential form

$$P_{t, \tau} = e^{-[A_r(\tau)r_t + A(\tau)\beta_t + C(\tau)]} \quad (A.1)$$

for three functions $A_r(\tau), A(\tau), C(\tau)$ that depend on maturity $\tau$. Applying Ito’s Lemma to A.1 and using the dynamics 1 of $r_t$ and 4 of $\beta_t$, we find that the instantaneous return on the bond with maturity $\tau$ is

$$\frac{dP_{t, \tau}}{P_{t, \tau}} = \mu_{t, \tau}dt - A_r(\tau)\sigma_r dB_{r,t} - A_\beta(\tau)\sigma_\beta dB_{\beta,t}, \quad (A.2)$$

where

$$\mu_{t, \tau} \equiv A'_r(\tau)r_t + A'_\beta(\tau)\beta_t + C'(\tau) - A_r(\tau)\kappa_r(\tau - r_t) - A_\beta(\tau)\kappa_\beta(\beta - \beta_t) \quad (A.3)$$

$$+ \frac{1}{2} A_r(\tau)^2 \sigma_r^2 + \frac{1}{2} A_\beta(\tau)^2 \sigma_\beta^2 + \rho A_r(\tau)A_\beta(\tau)\sigma_r \sigma_\beta$$

is the instantaneous expected return. Substituting (A.2) into the arbitrageurs’ budget constraint (6), we can solve the arbitrageurs’ optimization problem.

Next we show how to derive bond risk premia (or excess returns) of eq. (8). Using (A.2), we can write (6)

$$dW_t = \left[W_t r_t - \int_0^T x_{t, \tau}(\mu_{t, \tau} - r_t) d\tau \right] dt$$

$$- \left[ \int_0^T x_{t, \tau} A_r(\tau) d\tau \right] \sigma_r dB_{r,t} - \left[ \int_0^T x_{t, \tau} A_\beta(\tau) d\tau \right] \sigma_\beta dB_{\beta,t}, \quad (A.4)$$

and (5) as

$$\max_{\{x_{t, \tau}\} \subset (0, \infty)} \int_0^T x_{t, \tau}(\mu_{t, \tau} - r_t) d\tau - \frac{\sigma_r}{\sigma_\beta} \left[ \int_0^T x_{t, \tau} A_r(\tau) d\tau \right]^2 - \frac{\alpha_\beta}{\alpha_r} \left[ \int_0^T x_{t, \tau} A_\beta(\tau) d\tau \right]^2 \quad (A.5)$$

Point-wise maximization of (A.5) yields (8).

Next we derive the factor loadings $A_r(\tau)$ and $A_\beta(\tau)$. By imposing market clearing, so that $x_{t, \tau} = -y_{t, \tau}$, and using $R_t, \tau \equiv -\frac{\log(P_{t, \tau})}{\tau}$ and eqs. (2) and (A.1), we find that

$$x_{t, \tau} = \alpha(\tau) \{ \beta_t \tau - [A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)] \}. \quad (A.6)$$

Substituting $(\mu_{t, \tau}, \lambda_r, \lambda_\beta, x_{t, \tau})$ from (A.3), (9), (10) and (A.6) into (8), we find an affine equation in $(r_t, \beta_t)$. Setting linear terms in $(r_t, \beta_t)$ to zero yields

$$A'_r(\tau) + \kappa_r A_r(\tau) - 1 = A_r(\tau)M_{1,1} + A_\beta(\tau)M_{1,2}, \quad (A.7a)$$

$$A'_\beta(\tau) + \kappa_\beta A_\beta(\tau) = A_r(\tau)M_{2,1} + A_\beta(\tau)M_{2,2} \quad (A.7b)$$
where the matrix $M$ is given by

$$M_{1,1} = -a \sigma_r \int_0^T \alpha(\tau) A_r(\tau) \left[ \sigma_r A_r(\tau) + \rho \sigma_\beta A_\beta(\tau) \right] d\tau,$$

(A.8)

$$M_{1,2} = -a \sigma_\beta \int_0^T \alpha(\tau) A_r(\tau) \left[ \rho \sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau) \right] d\tau,$$

(A.9)

$$M_{2,1} = a \sigma_r \int_0^T \alpha(\tau) \left[ \tau \theta(\tau) - A_\beta(\tau) \right] \left[ \sigma_r A_r(\tau) + \rho \sigma_\beta A_\beta(\tau) \right] d\tau,$$

(A.10)

$$M_{2,2} = a \sigma_\beta \int_0^T \alpha(\tau) \left[ \tau \theta(\tau) - A_\beta(\tau) \right] \left[ \rho \sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau) \right] d\tau.$$

(A.11)

The solution to the system of (A.7a) and (A.7b) is given by equations (A.12) and (A.13):

$$A_r(\tau) = \frac{1 - e^{-\nu_1 \tau}}{\nu_1} + \gamma_r \left( \frac{1 - e^{-\nu_2 \tau}}{\nu_2} - \frac{1 - e^{-\nu_1 \tau}}{\nu_1} \right),$$

(A.12)

$$A_\beta(\tau) = \gamma_\beta \left( \frac{1 - e^{-\nu_2 \tau}}{\nu_2} - \frac{1 - e^{-\nu_1 \tau}}{\nu_1} \right).$$

(A.13)

To determine $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$, we substitute (A.12) and (A.13) into (A.7a) and (A.7b), and identify terms in $\frac{1 - e^{-\nu_1 \tau}}{\nu_1}$ and $\frac{1 - e^{-\nu_2 \tau}}{\nu_2}$. This yields

$$(1 - \gamma_r)(\nu_1 - \kappa_r + M_{1,1}) - \gamma_\beta M_{1,2} = 0,$$

(A.14)

$$\gamma_r(\nu_2 - \kappa_r + M_{1,1}) + \gamma_\beta M_{1,2} = 0,$$

(A.15)

in the case of (A.7a) and

$$\gamma_\beta(\nu_1 - \kappa_\beta + M_{2,2}) - (1 - \gamma_r)M_{2,1} = 0,$$

(A.16)

$$- \gamma_\beta(\nu_2 - \kappa_\beta + M_{2,2}) - \gamma_r M_{2,1} = 0,$$

(A.17)

in the case of (A.7b). Combining (A.14) and (A.15), we find the equivalent equations

$$\nu_1 + \gamma_r(\nu_2 - \nu_1) - \kappa_r + M_{1,1} = 0,$$

(A.18)

$$\gamma_r(1 - \gamma_r)(\nu_1 - \nu_2) - \gamma_\beta M_{1,2} = 0,$$

(A.19)

and combining (A.16) and (A.17), we find the equivalent equations

$$\gamma_\beta(\nu_1 - \nu_2) - M_{2,1} = 0,$$

(A.20)

$$\kappa_\beta - \nu_2 - \gamma_r(\nu_1 - \nu_2) - M_{2,2} = 0.$$

(A.21)

Equations (A.18)-(A.21) are a system of four scalar non-linear equations in the unknowns $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$.

To solve the system of (A.18)-(A.21), we must assume functional forms for $\alpha(\tau), \theta(\tau)$. Many parametrization are possible. A convenient one that we adopt from now on is $\alpha(\tau) \equiv \alpha e^{-\delta \tau}$ and $\theta(\tau) = 1$ (i.e., the demand factor affects all maturities equally in the absence of arbitrageurs). We also set $\alpha = 1$, which is without loss of generality because $\alpha$ matters only through the product $\alpha a$.

Next, we show how to determine the function $C(\tau)$. Setting $x_{t,\tau} = -y_{t,\tau}$ in (9) and (10), and using
\[ R_{t,\tau} \equiv -\frac{\log(P_{t,\tau})}{\tau}, \] (2) and (A.1), we find

\[
\lambda_{r,t} \equiv a\sigma_r^2 \int_0^T \alpha(\tau) [\beta_r \tau - A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau)] A_r(\tau) d\tau 
+ a\sigma_r \rho \sigma_\beta \int_0^T \alpha(\tau) [\beta_r \tau - A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau)] A_\beta(\tau) d\tau, 
\]

\[
\lambda_{\beta,t} \equiv a\sigma_\beta \rho \sigma_r \int_0^T \alpha(\tau) [\beta_r \tau - A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau)] A_r(\tau) d\tau 
+ a\sigma_\beta^2 \int_0^T \alpha(\tau) [\beta_r \tau - A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau)] A_\beta(\tau) d\tau. 
\]

Substituting \( \mu_{t,\tau} \) from (A.3), \( \lambda_{r,t} \) from (A.22), \( \lambda_{\beta,t} \) from (A.23), we find

\[
C'(\tau) - \kappa_r \tau A_r(\tau) + \frac{1}{2} \sigma_r^2 A_r(\tau)^2 + \frac{1}{2} \sigma_\beta^2 A_\beta(\tau)^2 + \rho \sigma_r \sigma_\beta A_r(\tau) A_\beta(\tau) 
= a\sigma_r A_r(\tau) \int_0^T \alpha(\tau) [\beta_r \tau - C(\tau)] [\sigma_r A_r(\tau) + \rho \sigma_\beta A_\beta(\tau)] d\tau 
+ a\sigma_\beta A_\beta(\tau) \int_0^T \alpha(\tau) [\beta_r \tau - C(\tau)] \left[ \rho \sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau) \right] d\tau. 
\]

The solution to (A.24) is

\[
C(\tau) = z_r \int_0^\tau A_r(u) du + z_\beta \int_0^\tau A_\beta(u) du 
- \frac{\sigma_r^2}{2} \int_0^\tau A_r(u)^2 du - \frac{\sigma_\beta^2}{2} \int_0^\tau A_\beta(u)^2 du - \rho \sigma_r \sigma_\beta \int_0^\tau A_r(u) A_\beta(u) du, 
\]

where

\[
z_r \equiv \kappa_\tau \tau - a\sigma_r \int_0^T \alpha(\tau) C(\tau) [\sigma_r A_r(\tau) + \rho \sigma_\beta A_\beta(\tau)] d\tau, 
\]

\[
z_\beta \equiv \kappa_\beta \tau - a\sigma_\beta \int_0^T \alpha(\tau) C(\tau) [\rho \sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)] d\tau. 
\]

Substituting \( C(\tau) \) from (A.25) into (A.26) and (A.27), we can derive \( (z_r, z_\beta) \) as the solution to a linear system of equations.

**B Appendix: MCMC Algorithm**

The pricing of equilibrium interest rates in the preferred-habitat model of Vayanos and Vila (2009) departs from more standard no-arbitrage term structure models of interest rates. As a result, interest rates are highly non-linear functions of the structural parameters. We therefore develop a Bayesian algorithm to estimate our model. Although we are the first to implement the bond pricing of the preferred-habitat model, a number of earlier studies has relied on Bayesian algorithms – Markov
Chain Monte Carlo (MCMC) with Gibbs sampling steps (see Johannes and Polson, 2009, for a general review of MCMC methods in finance) - to estimate term structure models (Ang, Dong and Piazzesi, 2007; Chib and Egarshev, 2009; Bauer, 2011; Feldhutter and Nielsen, 2012; and Li and Zinna, 2013; among others).

We group the parameters of our model as \( \Phi_1 = [\kappa, \tau, \sigma, \beta, \sigma_\beta] \), \( \Phi_2 = [\alpha, \delta] \), \( \Phi_3 = [\gamma_0, \gamma_1, \kappa, \phi, \zeta] \) and then \( \Phi = [\Phi_1, \Phi_2, \Phi_3] \). The yields are observed with error, and \( \sigma_y^2 \) and \( \sigma_\varepsilon^2 \) are the measurement error variance of the real rates and observable demand, respectively. Moreover, \( Y = [R, D] \) denotes the data that consist of the market real rates \( R = \{ R_t \}_{t=1}^T \) and the observed demand variables \( D = \{ D_t \}_{t=1}^T \).

**Likelihood Functions.** The density of the factors is:

\[
\pi(X^T|\Phi_1) \propto \prod |\Sigma|^{-1/2} \exp(-1/2u_t^t\Sigma^{-1}u_t) \quad (B.1)
\]

where \( u_{t+\Delta} = X_{t+\Delta} - F_0(\Phi_1) + F_1(\Phi_1)X_t \) denote the transition equation errors. Note that the errors of the latent factors are correlated, so \( Q \) is a full matrix. Conditional on a realization of the parameters and latent factors the likelihood function of the data is:

\[
L(Y^T|\Phi, P, X^T) \propto \prod |P|^{-1/2} \exp(-1/2v_t^yP^{-1}v_t) \quad (B.2)
\]

where the vector of measurement errors \( v_t = [\eta_{t+\Delta}, \varepsilon_{t+\Delta}] \) composes of the \( M \) real rates pricing errors \( \eta_{t+\Delta} = [\eta_{t+\Delta,2} \cdots \eta_{t+\Delta,20}]^\prime \) where \( \eta_{t+\Delta,\tau} = [Y_{t+\Delta} - f(\Theta, X_{t+\Delta,\tau})] \) for maturities \( \tau = 2, 5, 7, 10, 15, 20 \) and the observed demand measurement errors \( \varepsilon_{t+\Delta} = \beta_{t+\Delta} - \Phi_3D_{t+\Delta} \). The measurement error variance-covariance matrix \( P = \text{diag} (\sigma_{\eta_1}^2, \ldots, \sigma_{\eta_M}^2, \sigma_\varepsilon^2) \) is diagonal and of dimension \( M+1 \times M+1 \).

Finally, the joint posterior distribution of the model parameters and the latent factors is given by:

\[
\pi(\Phi, P, X^T|Y^T) \propto L(Y^T|\Phi, P, X^T)\pi(X^T|\Phi_1)\pi(\Phi), \quad (B.3)
\]
i.e. the product of the likelihood of the observation, the density of the factors and the priors of the parameters \( \pi(\Phi) \).

Next, we present the block-wise Metropolis-Hastings (MH) algorithm within Gibbs sampler that allows us to draw from the full posterior, \( \pi(\Phi, P, X^T|Y^T) \). In principle, we approximate the target density by repeatedly simulating from the conditional distributions of each block in turn. If the conditional distributions were known, this algorithm then consists of a series of Gibbs sampler steps. But in our case most of these conditional distributions are not recognizable, so we replace Gibbs sampler steps with MH steps.

**Step 1: Drawing Latent Factors.** The term structure model is linear and has a Gaussian state-space representation. The measurement and transition equations are linear in the unobserved factors, \( X^T \). And both equations have Gaussian distributed errors. So we use the Carter and Kohn (1994) simulation smoother to obtain a draw from the joint posterior density of the factors, which is:

\[
\pi(X^T|\Phi, P, X^T) \propto \pi(X_T|\Phi, P, X^T) \prod_{t=1}^{T-1} \pi(X_t|X_{t+1}, \Phi, P, X^T). \quad (B.4)
\]

In short, a run of the Kalman filter yields \( \pi(X_T|\Theta, \sigma_\varepsilon^2, X^T) \), and the predicted and smoothed means and variances of the states. By contrast, the simulation smoother provides the updated estimates of
the conditional means and variances that fully determine the remaining densities of equation (B.4) (e.g. Kim and Nelson, 1999).

**Step 2: Drawing Drift Parameters.** The discretized dynamics of the factors follow a VAR process, and VAR parameters have conjugate normal posterior distribution given the factors, $X^T$. But in our model the drift parameters also enter the pricing of the yields, and their conditional posteriors are therefore unknown. We draw the drift parameters of the latent factors using a Random-Walk Metropolis (RWM) algorithm (see Johannes and Polson, 2009).

Let denote with $\phi^{(g)}$ the $(g)^{th}$-draw of the parameter. At the $(g + 1)^{st}$ iteration we draw a candidate parameter $\phi^{(c)}$ from the proposal normal density $\phi^{(c)} = \phi^{(g)} + v_\phi \epsilon$ (B.5)

where $\epsilon \sim N(0, 1)$ and $v_\phi$ is the scaling factor used to tune the acceptance probability around 10-50%. Let define $\Phi_{-\phi}$ as all the $\Phi$ parameters but $\phi$, we accept the candidate draw with probability:

$$p^a = \min \left\{ \frac{L(Y^T | \Phi_{-\phi}, \phi^{(c)}, P, X^T) \pi(X^T | \Theta_{-\phi, 1}, \phi^{(c)}) \pi(\Phi_{-\phi, 1}, \phi^{(c)})}{L(Y^T | \Phi_{-\phi}, \phi^{(g)}, P, X^T) \pi(X^T | \Theta_{-\phi, 1}, \phi^{(g)}) \pi(\Phi_{-\phi, 1}, \phi^{(g)})} \right\}$$ (B.6)

Note that because the proposal density is symmetric it has no impact on the acceptance probability. We perform this RWM step for each of the individual drift parameters ($\beta, \kappa_\beta, \kappa_\tau$).

**Step 3: Drawing the Factors Covariance Matrix (Q).** We now focus on drawing the variance-covariance matrix, $Q$, of the transition equation. The posterior of $Q$ takes the form of:

$$\pi(Q | Y^T) \propto L(Y^T | \Phi, P, X^T) \pi(X^T | \Phi_1) \pi(Q)$$ (B.7)

where $\pi(Q)$ is the prior distribution. By specifying an inverse Wishart prior we can then easily draw from the inverse Wishart proposal distribution:

$$q(Q) = \pi(X^T | \Phi_1) \pi(Q).$$ (B.8)

And the acceptance probability simplifies to:

$$a = \min \left\{ \frac{L(Y^T | \Phi_{-\theta}, \phi^{(c)}, P, X^T)}{L(Y^T | \Phi_{-\theta}, \phi^{(g)}, P, X^T)} \right\}.$$ (B.9)

But we perform the accept/reject step for each individual candidate draw, so for $\phi^{(c)}$ equal to $\rho, \sigma_\tau$ and $\sigma_\beta$, because we would otherwise accept too few draws. For example, we accept/reject $\phi^{(c)} = \sigma_\tau^{(c)}$ conditional on $\sigma_\beta^{(g)}, \rho^{(g)}$ and the rest of parameters. Then, we accept/reject $\phi^{(c)} = \sigma_\beta^{(c)}$ conditional on $\sigma_\tau^{(g+1)}, \rho^{(g)}$ and so forth.

**Step 4: Drawing Arbitrageurs Risk Aversion times Excess Demand Elasticity ($a_\alpha$).** Arbitrageurs’ risk aversion, $a$, and excess demand elasticity, $\alpha$, are not separately identified, so that we estimate $a_\alpha$. The estimation of $a_\alpha$ is similar in spirit to the market price of risk parameters in traditional no-arbitrage models. These parameters are notably difficult to estimate as they only enter the measurement equation (bond pricing). We again use a RWM algorithm, but the acceptance probability in this case simplifies to:
\[ a = \min \left\{ \frac{L(Y^T | \Phi_{-a}, a \alpha^{(c)}, P, X^T)}{L(Y^T | \Phi_{-a}, a \alpha^{(g)}, P, X^T)}, 1 \right\}. \] (B.10)

because \( a \) do not enter the transition equations.

**Step 5: Drawing Measurement Error Variance (\( \sigma_n^2 \)).** We simply use a Gibbs sampler to draw the variance of the pricing errors. Conditional on the the other parameters, \( \Phi \), the factors and the observed yields, we get the measurement errors, \( \eta \). And because we assume a common variance for all the maturities, we implicitly pool the \( n \) vectors of residuals into a single series. So the inverse Gamma distribution becomes the natural prior for the variance, \( \sigma_n^2 \).

**Step 6: Drawing the Observable Demand Parameters** (\( \Phi_3 = [\gamma_0, \gamma_1, \kappa, \phi, \zeta] \) and \( \sigma_x^2 \)). Conditional on the \( g \)-th draw of the demand factor, \( \beta^{(g)} \), drawing the conditional mean \( \Phi_3 \) and variance \( \sigma_x^2 \) simply consists of implementing the linear regression algorithm (Koop, 2003).

**Priors.** We set the priors such that they are proper but only little informative. The priors on the transition equation covariance matrix is inverse Wishart, and the one on the measurement error variance is inverse Gamma. The rest of the parameters have normal or, in a few cases, truncated normal distributions. For example, we impose arbitrageur’s risk aversion to be positive, and also the mean reversion parameters to be positive, so that the factors are stationary. We discard the draws that do not fall within the desired region, and we keep drawing a proposal parameter until it respects the constraint. But to avoid that the chain gets stuck we specify a maximum number of draws, otherwise we retain the old draw. Note that after few iterations the draws lie away from the boundaries.

**Implementations Details and Convergence Check.** We perform 70,000 replications, of which the first 30,000 are "burned" to insure convergence of the chain to the ergodic distribution. We save 1 every 20 draws of the last 40,000 replications of the Markov chain to limit the autocorrelation of the draws.

The RWM algorithm converges for an acceptance level of accepted draws around 20-40% (Johannes and Polson (2009)). If the variance is too high we will reject nearly every draw, and the opposite is true for a variance that is too low. In order to reach reasonable acceptance ratios we follow the method of Feldhutter (2008). The variance is tuned over the first half of the burn-in period and we check the acceptance ratio every 100 draws. If we accepted more than 50 draws over the last 100, we double the standard deviation. If, instead, we accepted less than 10 draws we halve the standard deviation.

In order to check the convergence of the Markov chain we carried on several exercises. We implemented a preliminary Maximum Likelihood (ML) estimation of the model. Chib and Ergashev (2009) show that a ML estimation of the model may efficiently help the Bayesian algorithm, in particular by tuning the priors and proposal densities. We simply use the ML estimates to initialize the parameters and, above all, the unobserved factors. But we have also estimated the model from many initial values, and the results do not change.

Moreover, the posterior distributions of the parameters are unimodal. We also use two convergence diagnostics: the numerical standard error (NSE), and the convergence diagnostic (CD) of Geweke (1992). The NSE is a widely used measure of the approximation error. A good estimate of NSE has to compensate for the correlation in the draws (Koop, 2003). The second diagnostic,

\[ \text{To compute the NSE and CD we use the codes of James P. LeSage.} \]
CD, relies on the idea that an estimate of the parameter based on the first half of the draws must be essentially the same to an estimate based on the last half. If this was not the case, then either the number of replications is too small, or the effect of the starting value has not vanished. Table (3) presents the posterior results, and the convergence diagnostics. And the convergence diagnostics support convergence of the chain.

29 Following Koop (2003), the middle set of 50 percent of the draws is dropped to have the first and second set of draws to be independent.

30 CD is distributed as standard normal, thus values of CD less than 1.96, in absolute value, support the convergence of the Markov chain Monte Carlo.
References


Bauer, M. D., 2011, Term premia and the news, mimeo, University of California, San Diego.


### Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Yield Summary Statistics</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td>2.10</td>
<td>2.37</td>
<td>2.43</td>
<td>2.50</td>
<td>2.56</td>
<td>2.26</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td></td>
<td>1.77</td>
<td>1.31</td>
<td>1.28</td>
<td>1.26</td>
<td>1.29</td>
<td>1.24</td>
</tr>
<tr>
<td>AC</td>
<td></td>
<td>0.89</td>
<td>0.93</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>5.11</td>
<td>4.17</td>
<td>4.32</td>
<td>4.47</td>
<td>4.63</td>
<td>4.31</td>
</tr>
<tr>
<td>Min</td>
<td></td>
<td>-2.23</td>
<td>-1.24</td>
<td>-1.04</td>
<td>-0.59</td>
<td>-0.29</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Yield Loadings on PCs</th>
<th>ExVar</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td></td>
<td>88.5</td>
<td>0.41</td>
<td>0.43</td>
<td>0.42</td>
<td>0.41</td>
<td>0.41</td>
<td>0.36</td>
</tr>
<tr>
<td>PC2</td>
<td></td>
<td>10.9</td>
<td>-0.82</td>
<td>-0.11</td>
<td>0.06</td>
<td>0.21</td>
<td>0.33</td>
<td>0.39</td>
</tr>
<tr>
<td>PC3</td>
<td></td>
<td>0.6</td>
<td>-0.35</td>
<td>0.47</td>
<td>0.35</td>
<td>0.18</td>
<td>-0.06</td>
<td>-0.70</td>
</tr>
<tr>
<td>PC4</td>
<td></td>
<td>0.0</td>
<td>0.16</td>
<td>-0.49</td>
<td>-0.15</td>
<td>0.26</td>
<td>0.65</td>
<td>-0.47</td>
</tr>
</tbody>
</table>

Top panel (Yield Summary Statistics) presents mean, standard deviation, autocorrelation, maximum and minimum values of the yields for the 2-, 5-, 7-, 10-, 15- and 20-yr maturities over the period from 1987Q1 to 2012Q4. Bottom panel (Yield Loadings) presents the principal component (PC) loadings for the term structure of real yields from Jan-2004 to Nov-2012. Column EV shows the proportion of the total variance explained by each PC. Note that the 2-, 5-, and 20-yr real rate series are discontinued, so that the principal component analysis is performed over those quarters for which all rate are available.
Table 2: **Parameter Estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>mean</th>
<th>lb</th>
<th>ub</th>
<th>nse</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_r$</td>
<td>0.42</td>
<td>0.37</td>
<td>0.46</td>
<td>0.0011</td>
<td>0.96</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa_\beta$</td>
<td>0.09</td>
<td>0.07</td>
<td>0.12</td>
<td>0.0005</td>
<td>0.33</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>1.75</td>
<td>1.61</td>
<td>1.89</td>
<td>0.0034</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>2.27</td>
<td>2.12</td>
<td>2.42</td>
<td>0.0035</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>0.51</td>
<td>0.47</td>
<td>0.54</td>
<td>0.0008</td>
<td>0.40</td>
</tr>
<tr>
<td>$\rho$</td>
<td>24.8</td>
<td>16.79</td>
<td>33.05</td>
<td>0.1828</td>
<td>0.26</td>
</tr>
<tr>
<td>$a\alpha$</td>
<td>45.3</td>
<td>41.4</td>
<td>49.1</td>
<td>0.0945</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>12.55</td>
<td>11.78</td>
<td>13.13</td>
<td>0.0216</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The table presents the posterior mean, the one-standard deviation credible intervals, the numerical standard errors (nse), and the absolute value of the convergence diagnostic (CD), as in Geweke (1992), for the estimated parameters. These estimates result from the Bayesian estimation, described in Section 4, based on monthly UK real rates from 1987Q1 to 2012Q2 for the 2-, 5-, 7-, 10-, 15- and 20-yr maturities.
Table 3: **Variance Decomposition**

<table>
<thead>
<tr>
<th>Short-rate</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2yr 5yr 10yr 20yr</td>
</tr>
<tr>
<td>1</td>
<td>96.3 73.8 22.9 0.3</td>
</tr>
<tr>
<td>2</td>
<td>95.7 69.9 19.5 0.2</td>
</tr>
<tr>
<td>3</td>
<td>95.0 66.3 16.9 0.2</td>
</tr>
<tr>
<td>4</td>
<td>94.4 63.1 15.0 0.2</td>
</tr>
<tr>
<td>5</td>
<td>93.8 60.4 13.5 0.2</td>
</tr>
<tr>
<td>6</td>
<td>93.3 58.1 12.4 0.1</td>
</tr>
<tr>
<td>7</td>
<td>92.9 56.1 11.6 0.1</td>
</tr>
<tr>
<td>8</td>
<td>92.5 54.5 10.9 0.1</td>
</tr>
<tr>
<td>9</td>
<td>92.1 53.2 10.4 0.1</td>
</tr>
<tr>
<td>10</td>
<td>91.9 52.1 9.9 0.1</td>
</tr>
</tbody>
</table>

The table reports the variance decomposition of the forecast variance (in percentage) for rates of 2, 5, 10, and 20 year maturity. All the variance decompositions are computed using the posterior mean of the parameters listed in Table 2.
Table 4: Demand Loadings

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>lb</th>
<th>ub</th>
<th>nse</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.368</td>
<td>0.329</td>
<td>0.408</td>
<td>0.0009</td>
<td>0.62</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.020</td>
<td>-0.022</td>
<td>-0.017</td>
<td>0.0001</td>
<td>0.56</td>
</tr>
<tr>
<td>$\kappa_{LI}$</td>
<td>-0.433</td>
<td>-0.499</td>
<td>-0.364</td>
<td>0.0015</td>
<td>1.05</td>
</tr>
<tr>
<td>$\phi_{LI}$</td>
<td>0.027</td>
<td>0.023</td>
<td>0.032</td>
<td>0.0001</td>
<td>1.17</td>
</tr>
<tr>
<td>$\kappa_{PF}$</td>
<td>-0.262</td>
<td>-0.302</td>
<td>-0.221</td>
<td>0.0009</td>
<td>1.46</td>
</tr>
<tr>
<td>$\phi_{PF}$</td>
<td>0.018</td>
<td>0.015</td>
<td>0.020</td>
<td>0.0001</td>
<td>1.44</td>
</tr>
<tr>
<td>$\kappa_{FO}$</td>
<td>-0.844</td>
<td>-0.973</td>
<td>-0.717</td>
<td>0.0029</td>
<td>0.42</td>
</tr>
<tr>
<td>$\phi_{FO}$</td>
<td>0.041</td>
<td>0.033</td>
<td>0.049</td>
<td>0.0002</td>
<td>0.43</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-0.056</td>
<td>-0.075</td>
<td>-0.037</td>
<td>0.0004</td>
<td>0.96</td>
</tr>
</tbody>
</table>

This table presents posterior means (mean), one-standard deviation credible intervals (lb,ub), numerical standard errors (nse) and convergence diagnostic (CD) for the parameter of the following supply equation:

$$\beta_t = -d_t = \gamma_0 + \kappa Z'_t + \phi (AvgMat_t' Z_t) + \zeta QE_t + \gamma_1 AvgMat_t + \varepsilon_t$$

where $\beta_t$ is the excess supply as described in Section 2. The vector $Z_t$ includes the main terms: 1) Life insurers’ demand pressure; 2) Pension funds’ demand pressure; and 3) Foreign investors’ demand pressure. The interaction terms $AvgMat_t' Z_t$ are constructed as the simple observation-by-observation product of average maturity of the inflation-linked bonds outstanding and the individual measures of demand pressure $Z_{t,i}$. The vector $\kappa = [\kappa_{LI}, \kappa_{PF}, \kappa_{FO}]$ stacks the loadings of the main terms, and $\phi = [\phi_{LI}, \phi_{PF}, \phi_{FO}]$ the loadings of the interaction terms. Note that QE is included with no interaction term, as it only becomes active from 2009 and therefore only few quarterly observations are available.
<table>
<thead>
<tr>
<th>Sample (1990-2012)</th>
<th>Sub-samples (1990-95), (1996-00), (2001-05), (2006-10), (2009-12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_i)</td>
<td>(0.005) (0.003) (0.009) (0.006) (0.003) (0.001)</td>
</tr>
<tr>
<td>(\kappa_{LI}^{c})</td>
<td>([-0.016] ([0.044] ([-0.035] ([-0.081] ([-0.028] ([0.031]</td>
</tr>
<tr>
<td>(\kappa_{PF}^{c})</td>
<td>([0.01] ([0.049] ([-0.003] ([-0.033] ([0.002] ([0.04]</td>
</tr>
<tr>
<td>(\kappa_{FI}^{c})</td>
<td>([-0.215] ([-0.124] ([-0.244] ([-0.314] ([-0.234] ([-0.145]</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>(-0.235] ([-0.145] ([-0.267] ([-0.346] ([-0.256] ([-0.164]</td>
</tr>
</tbody>
</table>

| \([-0.195\] \([-0.145\] \([-0.222\] \([-0.281\] \([-0.213\] \([-0.126\] |

This table presents the conditional loadings of the measures of demand pressure and average maturities. The
original model takes the form of:

\[ \beta_t = -d_t = \gamma_0 + \kappa Z^t_i + \phi (\text{AvgMat}_t \cdot Z^t_i) + \zeta QE_t + \gamma_1 \text{AvgMat}_t + \varepsilon_t \]

where \(\beta_t\) is the excess supply as described in Section 2. The vector \(Z_t\) includes the main terms: 1) Life insur-
ers’ demand pressure; 2) Pension funds’ demand pressure; and 3) Foreign investors’ demand pressure. The
interaction terms \(\text{AvgMat}_t \cdot Z^t_i\) are constructed as the simple observation-by-observation product of average
maturity of the inflation-linked bonds outstanding and the individual measures of demand pressure \(Z^t_i\). The
vector \(\kappa = [\kappa_{LI}, \kappa_{PF}, \kappa_{FO}]\) stacks the loadings of the main terms, and \(\phi = [\phi_{LI}, \phi_{PF}, \phi_{FO}]\) the loadings
of the interaction terms. Note that QE is included with no interaction term, as it only becomes active from
2009 and therefore only few quarterly observations are available. The conditional loadings for \(i=LI, PF, FO\)
are computed as:

\[
\kappa_{i,t_0,t_1}^c = \kappa_i + \phi_i \overline{\text{AvgMat}}_{t_0,t_1}
\]

\[
\gamma_{1,t_0,t_1}^c = \gamma_1 + \phi \overline{Z}^t_{t_0,t_1}
\]

where \(\overline{Z}_{t_0,t_1}\) and \(\overline{\text{AvgMat}}_{t_0,t_1}\) denote the period average from \(t_0\) to \(t_1\). As a result, \(\kappa_{i,t_0,t_1}^c\) is the average
effect of \(Z_i\) on \(\beta\) during the period from \(t_0\) to \(t_1\), and \(\gamma_{1,t_0,t_1}^c\) is the average effect of \(\text{AvgMat}\) on \(\beta\).
<table>
<thead>
<tr>
<th>AvgMat</th>
<th>Life</th>
<th>Pens</th>
<th>Foreign</th>
<th>QE</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2yr</td>
<td>-0.9</td>
<td>-30.8</td>
<td>-68.6</td>
<td>-30.7</td>
<td>-46.3</td>
</tr>
<tr>
<td></td>
<td>[-1.0;-0.8]</td>
<td>[-78.9;-58.2]</td>
<td>[-47.7;-14.1]</td>
<td>[-62.6;-30.4]</td>
<td>[-190.2;-164.4]</td>
</tr>
<tr>
<td>5yr</td>
<td>-1.5</td>
<td>-50.7</td>
<td>-112.7</td>
<td>-50.3</td>
<td>-76.1</td>
</tr>
<tr>
<td></td>
<td>[-1.7;-1.3]</td>
<td>[-64.5;-36.3]</td>
<td>[-129.6;-96.3]</td>
<td>[-78.3;-23.3]</td>
<td>[-102.2;-50.1]</td>
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<tr>
<td>7yr</td>
<td>-1.7</td>
<td>-57.2</td>
<td>-127.2</td>
<td>-56.8</td>
<td>-85.9</td>
</tr>
<tr>
<td></td>
<td>[-1.9;-1.5]</td>
<td>[-72.7;-41.0]</td>
<td>[-146.2;-108.6]</td>
<td>[-88.6;-26.4]</td>
<td>[-115.5;-56.7]</td>
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<tr>
<td>10yr</td>
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<td>-95.0</td>
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<tr>
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<td>[-2.1;-1.7]</td>
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</tr>
<tr>
<td>15yr</td>
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<td>-69.6</td>
<td>-154.7</td>
<td>-69.1</td>
<td>-104.4</td>
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<tr>
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<td>[-2.4;-1.9]</td>
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<td>[-177.7;-131.7]</td>
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</tr>
<tr>
<td>20yr</td>
<td>-2.2</td>
<td>-74.1</td>
<td>-164.8</td>
<td>-73.6</td>
<td>-111.3</td>
</tr>
<tr>
<td></td>
<td>[-2.5;-2.0]</td>
<td>[-94.6;-53.1]</td>
<td>[-189.2;-140.1]</td>
<td>[-114.8;-34.1]</td>
<td>[-149.8;-73.3]</td>
</tr>
</tbody>
</table>

The table presents the impact of the average maturity and several measures of demand pressures on real rates of 2-, 5-, 7-, 10-, 15- and 20-year maturities for the period from 1990Q1 to 2012Q3. The vector $Z_i$ includes the main terms as defined in Section 3: 1) Life insurers’ demand pressure (Life); 2) Pension funds’ demand pressure (Pens); and 3) Foreign investors’ demand pressure (Foreign). The interaction terms are calculated as the simple observation-by-observation product of average maturity (AvgMat) of the inflation-linked bonds outstanding. For a generic measure of demand pressure $Z_i$ for $i=\text{LI, PF, FO}$ we quantify its total impact on the yield at maturity $\tau$ for the period that goes from time $t_0 = 1990Q1$ to $t_1 = 2012$, such as:

$$\Delta Z_{t_0,t_1}^{li} \Rightarrow \Delta R_{t_0,t_1}^{\tau} = \left[\frac{A_\beta(\tau)}{\tau} \times \left(\hat{\kappa}_i + \hat{\phi}_i \Delta AvgMat^{t_0,t_1}\right) \times \Delta Z_{t_0,t_1}^{li}\right],$$

whereas the effects of the average maturity (AvgMat) and QE are calculated simply as:

$$\Delta AvgMat^{t_0,t_1} \Rightarrow \Delta R_{t_0,t_1}^{\tau} = \left[\frac{A_\beta(\tau)}{\tau} \times \gamma_1 \times \Delta AvgMat^{t_0,t_1}\right],$$

$$\Delta QE^{t_0,t_1} \Rightarrow \Delta R_{t_0,t_1}^{\tau} = \left[\frac{A_\beta(\tau)}{\tau} \times \zeta \times \Delta QE^{t_0,t_1}\right].$$

Note that the loadings $\gamma_1, \hat{\kappa}_i, \hat{\phi}_i$ and $\zeta$ are presented in Table 4. Column (Total) denotes the sum of the effects of the different components. One-standard deviation credible intervals are reported in squared brackets.
Figure 1: Distribution of Gilt Holdings. The figure shows the holdings of gilts by type of investor divided by central government liabilities (market value). Data span the 1987Q1-2012Q2 period and are in percentage. Other non financial includes private non-financial companies, local government and public corporations. These data are provided by the Office of National Statistics (ONS) and the Bank of England. Data available on the UK Debt Management Office website: http://www.dmo.gov.uk/index.aspx?page=Gilts/Data.
Figure 2: Measures of Demand Pressure and Average Maturity. Green lines (top panels) present the cumulative net purchases (CNP) of index-linked gilts (in billion) by Pension Funds (left panel) and Life Insurers (right panel). The starting value represents the stock of index-linked gilts held as of 1987Q1. Blue lines show the measures of demand pressure (CDP) computed as the cumulative net purchases divided by the cumulative issuance of long-term index-linked gilts. The starting value is the ratio of the stock of index-linked gilts held over the market value of long-term index-linked gilts as of 1987Q1. Bottom left panel shows the holdings of gilts by foreign investors divided by central government liabilities (CDP), which consists of the sum of Foreign Central Banks and Other Foreign in Figure 1. Bottom right panel shows the average maturity of index-linked gilts outstanding, whereby the remaining maturity of every index-linked bond outstanding is weighted by the face value of the bond. Sources: UK Office of National Statistics (MQ5), Debt Management Office and author’s calculations.
Figure 3: Market Rates, Model Implied Rates and Pricing Errors. Top panel presents the term structure of real rates for the 2-, 5-, 7-, 10-, 15- and 20-yr maturities spanning the period from 1987Q1 to 2012Q2. Bottom panel presents the pricing errors, which are computed as observed rates minus model-implied rates. Model-implied rates are computed using parameter estimates and smoothed estimates of the factors resulting from the MCMC estimation described in Section 4.
Figure 4 Factor Loadings. The figure shows the loadings of yields at different maturities (x-axis) on the short rate ($r_t$) and demand factor ($\beta_t$).
**Figure 5 Impulse Response Functions.** Top panels plot the impulse responses of 2-, 5-, and 20-yr rates to a 25 bps shock to the short-rate. Bottom panels plot the responses to a demand shock, i.e. a -25 bps shock to $\beta_t$. One standard deviation confidence intervals in red.
**Figure 6: Estimated Factors.** Smoothed factors with one-standard deviation credible intervals. Top panel shows in red the short-term real interest rate (first unobservable factor, $r_t$), and a proxy of the policy real rate in blue. Bottom panel presents the smoothed demand factor ($d_t = -\beta_t$) in red and the projected demand factor in blue, which is based on the predicted demand factor resulting from

$$\hat{\beta}_t = \Phi_3 D_t$$

where $D_t=\left[\text{cons}, \text{AvgMat}_t, \text{AvgMat}_t \cdot Z_t, \text{QE}_t\right]$ groups our observable measures of demand/supply imbalances. Precisely, the $Z_t$ vector includes our measures of demand pressure for life insurers, pension funds and foreign investors, while $\text{AvgMat}_t$ denotes the average maturity of inflation-linked gilts outstanding. The interaction term $\text{AvgMat}_t \cdot Z_t$ is calculated as the observation-by-observation product of $\text{AvgMat}_t$ and the individual measures of demand pressures. The parameter vector $\Phi_3 = [\gamma_0, \gamma_1, \kappa, \phi, \zeta]$ composes of a constant, $\gamma_0$; the loading on the average maturity of inflation-linked gilts outstanding, $\gamma_1$; the loadings of the demand pressure terms, $\kappa = [\kappa_{LI}, \kappa_{PF}, \kappa_{FO}]$; the loadings of the interaction terms, $\phi = [\phi_{LI}, \phi_{PF}, \phi_{FO}]$; and the loading on QE, $\zeta$. 

54
Figure 7. Real Interest Rate Decomposition. This figure presents the decomposition of the five- and ten-year model implied rates into the term premium and the average expected short rate over a five and ten year horizon.
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