BANCA D'ITALIA
EUROSISTEMA

Temi di Discussione

(Working Papers)

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June 2014

Number 962

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ISSN 1594-7939 (print)
ISSN 2281-3950 (online)

Printed by the Printing and Publishing Division of the Bank of Italy

COOPERATIVE R&D NETWORKS AMONG FIRMS AND PUBLIC RESEARCH INSTITUTIONS

by Marco Marinucci*

Abstract

This paper provides theoretical background to the increasing R&D cooperation among firms and public research institutions. We find that R&D spillovers may impede cooperation among firms or research institutions even when the cost of forming a link is negligible. Further, the presence of heterogeneous players results in different concepts of network regularity but also increases the number of possible pairwise stable networks. Consequently, stronger concepts of stability are needed to study networks in which players are not homogeneous.

JEL Classification: C70, L14, O30
Keywords: networks, innovation, R&D cooperation, spillovers.

Contents

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1 Introduction [1]

Since the half of the 1970s there has been a steady growth of R&D cooperative agreements among firms. As pointed out by Cabral [2000], in the U.S.A., the R&D partnerships increased from 750 in the 1970s to more that 20,000 in the 1987-1992 years. Such R&D alliances produced an increasing impact on firms' innovation capacity and their competitiveness as well.[2]

On the other hand, the R&D activity made by public research institutions (PRI), namely public research labs and universities, played an essential role for many innovations (Rosenberg & Nelson [1994], Mansfield [1995], Cohen et al. [2002]). In the last years, firms formed an increasing number of agreements also with such PRIs (Poyago-Theotoky et al. [2002], Capellari [2011]). Take for example the European framework Programme,[3] where thousands of firms work together with PRIs in order to develop new technologies as well as products; or the development of the Mpeg-2 video standard, which involved firms like Canon and Columbia University as well.[4]

As shown in the empirical literature, there are two main reasons that lead firms to cooperate with PRIs. On one hand, the cooperation with PRIs allow firms to exploit public funds. On the other hand, firms are increasingly diversifying their R&D alliance portfolio in order to increase their innovative capacity and keep their market competitiveness high. In this strategy setting, it is shown (Veugelers & Cassiman [2002] [2005]) that firms cooperate with PRIs in order to access skills and resources that competitors do not have.

From a theoretical point of view, the R&D cooperation is actually one of the

[1]This paper is drawn from my Ph.D thesis at Université Catholique de Louvain, Louvain La Neuve, Belgium. I would like to thank my supervisors Paul Belleflamme and Vincent Vannetelbosch and the other members of the jury Jan Bouckaert, Frederic Deroian, Wouter Vergote and Xavier Wauthy for their insightful comments and suggestions. Many thanks also to the anonymous referees and the participants of the ARET seminar (Rome) and the DEAMS seminar (Trieste). The scientific responsibility is assumed by the author. The opinions expressed in this paper are those of the author and do not necessarily represent those of the Bank of Italy.

[2]See also Roijakkers & Hagedoorn [2006], who show that the R&D partnering in the pharmaceutical biotechnology sector passed from only few alliances among 30 firms in the 1970s to thousands of partnerships among more than 600 firms in the 1990s.

[3]The Framework Programme is promoted by the European Commission since 1984. Its aim is to foster the cooperation among European firms and PRIs. The Framework Programme increased its relevance to pursue the so called Lisbon strategy. As a matter of fact, it is the main tool for the development of the European Research Area. For more information about the Framework Programme see the EU Commission documents [2006a] [2006b].

[4]For further details about Mpeg-2 see Shapiro [2001] and the following wikipedia website *http://en.wikipedia.org/wiki/MPEG-2*.

most active fields: since the seminal papers by Katz [1986] and d'Aspremont & Jacquemin [1988] many scholars paid attention to the causes and the incentives that lead two, or more, firms to form an R&D alliance.[5] However, to the best of our knowledge, there is no theoretical contribution that explains why firms and PRIs cooperate in R&D.

Our first aim is therefore to fill this gap by providing a theoretical background to the increasing R&D cooperation among firms and public research institutions. Secondly, we show that, even when the cost of forming a link is nil, firms may not want to form partnerships with PRIs because they fear the spillover benefit that can be enjoyed by their competitors.

From a policy point of view, this suggests that subsidizing the formation of an R&D partnership among firms and PRIs can be ineffective when R&D spillovers are sufficiently high.

Our contribution is set in the network game literature, which properly describes the strategic decision of a player to relate with another one by forming a " link". Even though it is quite recent,[6] the network game approach spread quickly thanks to its interesting applications in the field of Industrial Organization.[7] Among them, one of the most important topics is the analysis of R&D networks.

As a matter of fact, in the network game models players form links among ach other with the aim of creating a "profitable relationship". In R&D the networks players are firms, a link is a profitable R&D partnership, and consequently, the whole network represents the existing R&D partnerships among the firms.

The most notable papers on R&D networks are Goyal & Moraga [2001] and Goyal & Joshi [2003]. The former considers a three stage game wherein firms have to decide respectively at each stage a) their R&D partnerships, b) their R&D effort, and c) the quantity to produce in the final market (Cournot competition). Their main finding is that the complete network is always stable but it is not always socially optimal when the competition in the final market is strong.

On the other hand, Goyal & Joshi [2003] adopt a simpler two-stage game model where firms have first to decide which links to form and then they compete either *à la Cournot* or *à la Bertrand*. The aim is to study how the two different market regimes and the cost of forming a link affect the stability of the networks. One of the main objectives of this paper is to extend the picture by including

[5]See Marinucci [2012] and Caloghirou et al. [2003] and references therein.

[6]Actually, this class of games has been defined by Myerson [1991], the first paper that analyzes the equilibrium solution of network games is Jackson & Wolinsky [1996].

[7]See Jackson [2005] for a survey of the literature.

another kind of players, namely the public research institutions. The model is a two-stage game where in the first stage players (i.e. firms and PRIs) can form an R&D cooperative link in order to increase their amount of R&D. In the second stage firms compete *à la Cournot* while PRIs receive their public funds according to the amount of R&D produced.

This last assumption (public funding related to the research output) is justified by the increasing debate about the universities funding system,[8] where it is suggested that universities should be financed according to their research performance rather than their number of students and/or personnel. Therefore, since in the near future PRIs will likely compete according to their R&D performance, it seems quite reasonable to consider PRIs willing to cooperate in R&D in order to increase their R&D "production". This is empirically confirmed by an increasing amount of partnering collaboration among PRIs, as shown by Deiaco et al. [2010], who study some cooperative agreements among European and North American universities.

For both types of players, firms and PRIs, the benefits enjoyed from the cooperation with an agent of their own type markedly differ from those enjoyed from the cooperation with an agent of the other type.[9] Therefore, we suppose in our model that for each player the benefit of making a link with a firm and the benefit of a link with a PRI are independent of one another. Moreover, the creation of a link between two players has a (positive) spillover effect to their competitors. The existence of R&D spillovers between firms is widely known both theoretically (d'Aspremont & Jacquemin [1988]) and empirically (Griliches [1992]). However, it has been recently shown (e.g. Arnold [2004] Siune et al.[2004]) that R&D spillovers are one of the main motivations that drives firms to avoid cooperation with PRIs.

The analysis of R&D networks with heterogeneous agents is not new in the literature. Based on Goyal & Moraga [2001], Zikos [2008] analyses the interaction among the private and the public sectors when subsidies to the R&D cooperation are possible. In his model there are three firms (one public and two private firms) that compete in a Cournot setting. In contrast to Goyal & Moraga [2001], the author finds that the complete network (i.e. where each firm collaborates with all others) is uniquely stable and socially efficient.

On the other hand, Zirulia [2005] extends the contribution of Goyal & Moraga [2001] by considering two technologically different group of firms. Looking at the case of four firms the results are mixed. For weak stability concepts and a

[8] See for example the OECD [2007a] [2007b] report about higher education institutions.

[9] For example, partnerships with firms are more helpful in applied (hence marketable) research projects while cooperations with PRIs are more useful to get more basic research.

degree of technological differentiation, the complete network is the unique to be stable and, in some cases, it is also efficient. However, stronger stability concepts make the complete network rarely stable whereas dominant fringe network comes out as the unique stable network.

Differently from Zikos [2008] and Zirulia [2005], who base their analysis on Goyal & Moraga [2001], we propose a model more in line with Goyal & Joshi [2003] insofar as we do not endogenize the R&D effort level. Even though this approach has the drawback of not capturing the relationship between the R&D effort and the incentive to form a link, it has the advantage of allowing us to study networks with an indefinite number of firms and PRIs. This is even more important for two reasons. First, because the above mentioned papers as well as more recent contributions (Zu et al. [2011] and Kesavayuth and Zikos [2013]) still consider networks with three players. Second, differently from Kranton & Minehart [2001], who were actually the first to study network with heterogeneous players, we allow that links can be formed not only with players of the other group, but also with players of the same type.

As shown in the paper, the analysis of an indefinite number of players when they are heterogeneous has non-trivial implications. First, we find that the regularity concept of a network can be slightly different from the one used in "simple" networks (i.e. networks with homogeneous players). In its "classic" definition, a network is regular when all the players have the same number of links. On the other hand, when we look at "mixed" networks (i.e. where there are at least two types of players) the regularity can be seen in terms of a) the number of links held by the player of the same type and/or b) the number of links that each player has toward the players of different kind. This implies the existence of a "relative" and an "absolute" form of regularity that differ from the classic regularity concept. In particular, a network is "relative regular" when players of the same type have the same number of link among each other and/or with the players of the other type. On the other hand, a network is "absolute regular" whenever the number of links among homogeneous players and the connections between heterogeneous players are the same. Given these definitions,[10] we find that a) the relative regularity does not usually match the classic regularity while b) the absolute one is a particular case of the latter.

Second, even though we define the conditions under which a relative and absolute regular networks are pairwise stable (PWS), we show that the presence of heterogeneous players greatly extends the number of possible pairwise stable networks with respect to the case with homogeneous players. Thus, the pairwise

[10] A formal definition of "relative" and "absolute" regularity will be provided in Section 5.

stability is such a weak concept that it is necessary to use stronger equilibrium refinements to study networks with heterogeneous agents. However, it is worth mentioning that our model is a first attempt to define the problems related to the study of mixed networks so that the investigation of such refinements of the pairwise stability goes beyond our purpose.[11]

In the next section we provide some basic features of the model, then we analyze simple networks, namely networks where only homogeneous players are involved. We first look at the network of firms (Section 2), then we consider the network of PRIs (Section 4). In Section 5 we study the network with both type of players, focusing on players' incentives to form an alliance with a partner of different type. Some concluding remarks are provided in the last section.

2 The Model

2.1 Modelling Networks

Before presenting the model we introduce the necessary notation to describe the network game. Let us consider $N \geq 3$ players which can form a link (i.e. a relationship) among them. It is possible to describe the existence of a link among players i and j through a binary variable ij such that

$$ij = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are linked} \\ \\ 0 & \text{otherwise.} \end{cases}$$

A network g is therefore a list[12] which defines the existing relationships among all N players. In particular, we say that $ij \in g \Leftrightarrow ij = 1$. To simplify the notation, $g + ij$ means that the link ij is added to the network g. Similarly, $g - ij$ corresponds to the network g without the link ij. Let $N_i(g)$ denote the set of players that are linked with i, then $\eta_i(g) = |N_i(g)|$ represents the number of partners held by i in the network g.

When every player i has the same number of links such that $\eta_i(g) = k$, then g is a regular network of degree k. Among the regular networks, the most studied are the empty network g^0, where no player has a link with another ($\eta_i(g^0) = 0$) and the complete network g^c, where each player is linked to all the others ($\eta_i(g^c) = N - 1$). Apart form the regular networks, it is worth mentioning the

[11] A first, interesting analysis of networks with N heterogeneous players was given by Galeotti et al. [2006] who study the Nash equilibrium networks, but as already said, stronger stability concepts has to be analysed in the next future.

[12] Formally the network g is a $N \times N$ matrix whose elements are the binary variable defined above. See Jackson [2010] for a survey on the applications of network games in Economics.

star network g^*, where $N-1$ players are connected only with a "hub" player. In order to study the networks' equilibria, we use the *pairwise stability* (PWS), a network stability concept introduced by Jackson & Wolinsky [1996]. Formally, a network g is pairwise stable when the following conditions are satisfied

1. $\forall ij \in g \;\; \pi_i(g) \geq \pi_i(g-ij) \quad \text{and} \quad \pi_j(g) \geq \pi_j(g-ij)$
2. $\forall ij \notin g \;\; \text{if} \;\; \pi_i(g+ij) > \pi_i(g) \Rightarrow \pi_j(g+ij) < \pi_j(g)$

where $\pi_i(g)$ is the payoff that the player i gets in the network g. Intuitively, the two conditions state that, starting from a network g, there is neither a firm who wants to severe a link (condition 1) nor a couple of firms that want to form a new one (condition 2). The notation

$$g_7 \overset{\{i,j\}}{\longrightarrow} g_8 \;\; ; \;\; g_8 \overset{\{k\}}{\nrightarrow} g_7$$

means that players i and j prefers to form a new link and deviate from a network g_7 toward g_8; whereas player k prefers not to delete a link and move from g_8 to g_7.

Finally, to avoid useless complication, we use the term "player" to mention either a firm or a PRI while a "competitor" of i is another player of the same type.

2.2 Timing and Assumptions

As already said, our aim is to model an R&D network where players (i.e. firms and PRIs) can form a link. Given the complexity of the topic, we simplify the model by assuming that for each player, the choice of making a link with a firm is independent from the decision of forming a link with a PRI. In other words, we assume the following:

Assumption 1 *For each player, the R&D benefit of a link with a firm and the benefit from a link with a PRI are additive.*

Such assumption is not so strong as it seems. First, this additivity assumption allows the marginal benefit to be independent on the network formation path.[13] Moreover, several studies[14] prove that firms' cooperate with PRIs (universities

[13]For example, suppose that network g_k can be reached by adding a link either to network g_a or g_b. Then, without additivity, firm i's benefit to form a new link $g_k + ij$ will depend on the fact that g_k is reached from g_a rather than g_b.

[14]See for example Veugelers & Cassiman [2005] and Belderbos et al. [2003]. See also Caloghirou et al. [2001] for a deeper analysis on the cooperation among firms and universities in the Framework Programme.

above all) to get skills and know-how that competitors cannot provide: firms' R&D alliances with PRIs are usually more focused on a basic research, whereas R&D partnerships with competitors are of a more applied nature. On the other hand, PRIs are willing to cooperate with firms in order to get benefits that other PRI cannot supply.[15] Therefore, additivity reflects not only a theoretical need but also the empirical evidence that cooperation between heterogeneous agent is due to the research of skills that cannot be provided by players of the same type.

Concerning the cost of making a link (i.e. a partnership), we suppose that the following condition holds:

Assumption 2 *There is no direct cost of forming a cooperative link.*

The rationale behind this assumption is twofold. First, network games are suitable to "weak" alliances, namely alliances where partners share information and/or do not need to provide significant amount of resources to make the cooperation possible. Moreover, we aim to show that players may not want to cooperate even when the cost of making a link is nil or negligible because there are other (indirect) costs related to the R&D cooperation.

Some of these costs are pointed out by Mc Kinsey [2002] where it is shown that cooperative alliances are difficult to be managed: partnerships lack coordination, do not have a common strategy and, as a consequence, they are not in line with the global strategy of the firm. Such difficulty, together with the usual problems of forming an alliance with a partner (moral hazard, adverse selection, etc.), may reduce the benefit enjoyed by a firm as its number of partnerships increases.

Assumption 3 *The marginal benefit of forming a new alliance decreases with the number of partnerships already held.*

We will provide a formal definition of this assumption in the next section, however it is worth mentioning that it corresponds to suppose that, for each player, the payoff function is concave with respect to the number of links that this player has.

A second type of costs related to the link formation are the R&D spillovers that player's competitors enjoy from the new partnership. As a matter of fact, a new alliance increases the information leakage of the R&D activity made by the partners in favor of their competitors. Usually the literature addresses this issue by assuming that the information flow goes only to players that are indirectly linked with one/both partners (e.g. Deroian [2008], König et al. [2008]).

[15] For example, the possibility to get access to private labs, organize internship periods for students and researchers, the prestige of cooperating with firms etc.

However, there are some cases where spillovers concerns are considered serious independently of the existing links. For example, Arnold [2004] shows that one of the most important reasons that lead firms not to enter the European Framework Programme is the fear that their participation to the program would cause information leakages in favor of their competitors. This concern was motivated *independently* of the fact that such competitors were in the program (i.e., somehow linked with some partner) or they were not. Therefore, since players may not want to form a link with a player because it would benefit all the other competitors, it is reasonable to assume the validity of the following statement:

Assumption 4 *The creation of a link between two players generates a spillover benefit in favor of all the competitors of the two players.*

Given these assumptions, the analysis of the model can be split into two parts. In the first one we focus our attention to a player decision to form a link with a player of the same type. This can be formalized as a "simple network" where only the homogeneous players are involved. In the second part, we focus on the players' decision to form a link with player of the other type. Finally we consider the "mixed" network as a whole, where both firms and PRIs are related.
Since in the rest of the paper we are dealing with different type of players we always indicate i and j as firms such that $i, j \in \{F\}$ s.t. $i \neq j$, where {F} stands for the set of F firms involved in the network. Similarly, PRIs u and v are PRIs such that $u, v \in \{P\}$; $u \neq v$, where {P} is the set of P public research institutions involved in the network.

3 Simple Oligopoly Network

In order to understand the firms' decision to form a link with a competitor we study the following simple network. Let us consider a Cournot oligopoly where $F \geq 3$ firms face a linear demand $a - Q$ and linear costs of production.[16] We assume that firms can reduce their costs by forming R&D alliances. Each partnership can be seen as a link in an R&D network g. Therefore, according to the network structure, each firm i will enjoy a particular *benefit function* $b_i(g)$ such that its cost function is

$$c_i(g) = c - b_i(g); \tag{1}$$

[16]Note that we assume that $F \geq 3$ because we are considering a network with homogeneous agents. In case of a network with heterogeneous agents, where there are both F firms and P PRIs it is possible that $F < 3$ under the condition that $F + P \geq 3$, i.e. there are at least three players in the network.

that is, the cost reduction enjoyed by the firm is a function of the structure of R&D links (i.e. the network) formed in the industry. Once the alliances are formed and firms observe the resulting cooperative network, firms compete *á la Cournot* in the final market. Given the cost functions determined by the first-stage alliances, the optimal quantity produced by each firm is

$$q_i(g) = \frac{a - Fc_i(g) + \sum_{j \neq i} c_j(g)}{F + 1},$$

which can be rewritten as

$$q_i(g) = \frac{a - c + Fb_i(g) - \sum_{j \neq i} b_j(g)}{F + 1}. \tag{2}$$

It is assumed that the latter quantity is positive for all network g (i.e., that an interior Cournot-Nash equilibrium obtains whatever the structure of first-stage R&D alliances). The gross profit is $\pi_i(g) = q_i^2(g)$, which is also the net profit as we assume that there is no cost of forming links and of performing R&D. Therefore, a firm i will form a link with another firm j if and only if it increases $q_i(g)$. Formally, a new link ij is profitable for a firm i whenever

$$\pi_i(g+ij) - \pi_i(g) \geq 0 \Leftrightarrow F[b_i(g+ij) - b_i(g)] - \sum_{j \neq i} [b_j(g+ij) - b_j(g)] \geq 0. \tag{3}$$

To make the latter condition more explicit, we posit the following specific benefit function

$$b_i(g) = \underbrace{\sum_{f=0}^{F_i} \phi^f}_{R\&D \ \ benefit} + \underbrace{\frac{\sum_{j \neq i}(F_j - ij)}{2} \phi'}_{Spillover \ \ benefit} \qquad 0 \leq \phi' \leq \phi \leq 1 \tag{4}$$

where F_i is the number of partnerships held by the firm i in the network g.[17] The first term in the right hand side (RHS) of the equation is the R&D benefit that firm i gets from its partners and it reflects the decreasing returns to scale assumption mentioned in the previous section: as the number of alliances increases, the firms' organization is less able to exploit all the R&D results and/or is less capable of managing new alliances.

The second term of the RHS is the spillover benefit ϕ' enjoyed by firm i from all the cooperative links where it is not involved. We assume it is linear in order to reflect the rationale behind Assumption 4. As a matter of fact, linearity implies that the spillover benefits depend neither on the number of links nor on the

[17] Actually we should write $F_i(g)$. However, we omit the term in brackets to avoid useless complications.

network shape. Thus, it suits very well the evidence that some firms do not make links because their R&D spillover concerns are so serious independently of whether and how competitors are connected within the network.
Now, assume that firms i and j want to form a new link. The marginal benefit they get from this new partnership depends on the number of links they already have with the other players. In particular

$$b_i(g+ij) - b_i(g) = \phi^{F_i+1} \;\; ; \;\; b_j(g+ij) - b_j(g) = \phi^{F_j+1}.$$

On the other hand, all the other competitors will get the following spillover benefit

$$b_k(g+ij) - b_k(g) = \phi' \quad k \neq i, j.$$

Using Equations (3) and (4), it is straightforward to see the validity of the following statement.

Lemma 1 *Given a network g where the link ij does not exist, firm i wants to form the new link ij if and only if*

$$\pi_i(g+ij) \geq \pi_i(g) \Leftrightarrow F \;\; \phi^{F_i+1} \;\; \geq \;\; \phi^{F_j+1} + (F-2) \;\; \phi'. \tag{5}$$

This means that firm i will form a link only when the benefit is greater than the one enjoyed by all the other competitors (partner included).
Thanks to Lemma 1 and the definition of pairwise stability, it is possible to characterize the conditions under which a regular network is pairwise stable.[18]

Lemma 2 *Let g^k be a regular networks of degree $k \in [0, N)$. Then,*

1. *a network g^k is pairwise stable if and only if*
$$\frac{F-1}{F-2}\phi^k \geq \phi' \quad ; \quad \frac{F-1}{F-2}\phi^{k+1} \leq \phi';$$
2. *the complete network g_c is pairwise stable if and only if*
$$\phi' \leq \frac{F-1}{F-2}\phi^{F-1};$$
3. *the empty network g^0 is never pairwise stable.*

Proof: See Appendix A

Note that the complete network is less likely to be stable as the spillover level and/or the number of firms involved in the network are higher. Using Lemmas 1 and 2, we can now establish our first result:

[18]Recall that a network is regular when all the players have the same number of links.

Proposition 1 *There always exists a non-empty regular pairwise stable network.*

Proof: The proof is straightforward. First, the empty network cannot be PWS because of the last point of Lemma 2. Moreover, since $0 \leq \phi' \leq \phi$, there always exists a value $k \in (0, F-1)$ such that either the first part of Lemma 2

$$\frac{F-1}{F-2}\phi^k \geq \phi' \geq \frac{F-1}{F-2}\phi^{k+1}$$

or its second condition

$$\phi' \leq \frac{F-1}{F-2}\phi^{F-1}$$

is valid. **Q.E.D.**

Concerning the analysis of non-regular networks, it is possible to prove that the following result is valid.

Proposition 2 *The star network g^* is never pairwise stable.*

Proof: See Appendix B.

Intuitively, this result is due to the decreasing returns to scale of the benefit function, which makes the benefit of the hub firm the lowest among the ones enjoyed by its competitors. Therefore, when the hub player finds it convenient to keep all its links, then each couple of non-hub players have the incentive to form a new link.[19]

An example with 4 firms In order to better understand the single oligopoly network, suppose there are $F = 4$ firms, so that the possible network structures are the ones depicted in Figure 1. To check the PWS of a network we proceed with the following steps. For each network we first define the players' payoff, which corresponds to the square of the quantity produced by each firm. Then, we use these payoffs to check whether there exists an incentive to deviate from each network (Lemma 1).
For example, in network g_2 firms' benefit are

$$b_1(g_2) = 1 + \phi + \phi^2 \;\; ; \quad b_2(g_2) = 1 + \phi + \phi' \;\; ;$$

$$b_3(g_2) = 1 + \phi + \phi' \;\; ; \quad b_4(g_2) = 1 + 2\phi'.$$

[19]Obviously, Proposition 2 holds only if we are excluding the trivial case where $\phi = \phi' = 0$.

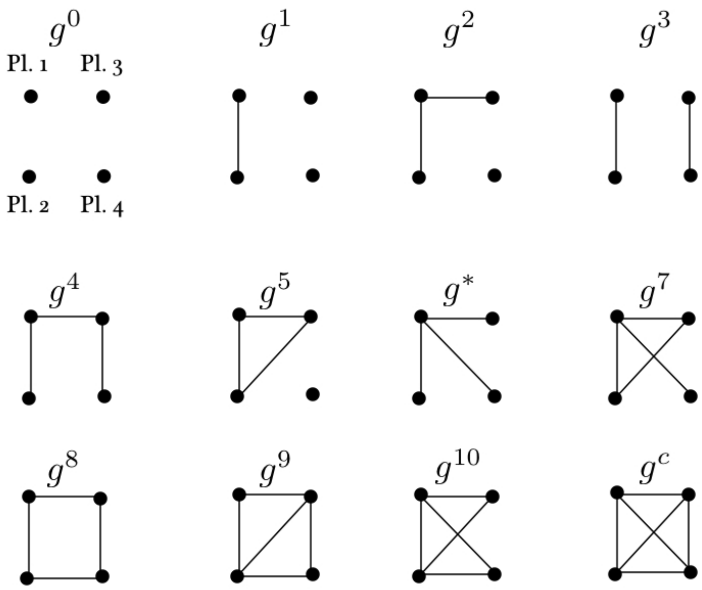

Figure 1: Main network structures with four players (i.e. firms).

According to Equation (2) firms' profit are

$$\pi_1(g_2) = \left(\frac{a-c+1+2\phi+4\phi^2-4\phi'}{5}\right)^2 \quad ;$$

$$\pi_2(g_2) = \left(\frac{a-c+1+2\phi-\phi^2+\phi'}{5}\right)^2 ;$$

$$\pi_3(g_2) = \left(\frac{a-c+1+2\phi-\phi^2+\phi'}{5}\right)^2 \quad ;$$

$$\pi_4(g_2) = \left(\frac{a-c+1-3\phi-\phi^2+6\phi'}{5}\right)^2 .$$

Using this approach for all the other networks, the pairwise stability can be checked by seeing whether no one is better off by creating/severing a link. Continuing with the example, the network g_2 is PWS if

$$g_1 \to g_2 \quad \text{and} \quad g_2 \nrightarrow g_4 \ ; \quad g_2 \nrightarrow g_5 \ ; \quad g_2 \nrightarrow g^*$$

namely when firms 1 and 2 want to move to g_2 and no one else want to build up another link after reaching g_2. Now, g_2 is reached if

$$\pi_1(g_1) < \pi_1(g_2) \Rightarrow 3\phi - 2\phi' \leq 2\phi + 4\phi^2 - 4\phi' \Rightarrow -\phi + 4\phi^2 \geq 2\phi'$$

and

$$\pi_2(g_1) \leq \pi_2(g_2) \Rightarrow 3\phi' - 2\phi \leq 2\phi - \phi^2 + \phi' \Rightarrow 4\phi - \phi^2 \geq 2\phi'$$

A condition that is always satisfied because ϕ is lager of both ϕ^2 and ϕ' On the other hand, starting from g_2, firms 3 and 4 will move to g_4 if and only if

$$\pi_3(g_4) \geq \pi_3(g_4) \Leftrightarrow \phi + 3\phi^2 - \phi' \geq 2\phi - \phi^2 + \phi' \Rightarrow -\phi + 4\phi^2 \geq 2\phi'$$

and

$$\pi_4(g_2) \geq \pi_4(g_4) \Leftrightarrow -3\phi - \phi^2 + 6\phi'^2 \geq \phi - 2\phi^2 + 4\phi'$$

Since both conditions $\pi_1(g_1) < \pi_1(g_2)$ and $\pi_3(g_2) \geq \pi_3(g_4)$ are identical, then

$$g_1 \overset{\{1,2\}}{\longrightarrow} g_2 \Rightarrow g_2 \overset{\{2,4\}}{\longrightarrow} g_4.$$

Therefore, when there is the incentive for two firms to reach g_2 there exists another couple of firms which have an incentive to create a new link and deviate from g_2. Hence network g_2 is never PWS.

Using the same method for all the networks, we find that in a simple R&D network with 4 firms:

- g^c is PWS $\Leftrightarrow 0 < 2\phi' \leq 3\phi^3$
- g^8 is PWS $\Leftrightarrow 3\phi^3 < 2\phi' \leq 3\phi^2$
- g^5 is PWS $\Leftrightarrow -\phi + 4\phi^3 < 2\phi' \leq 3\phi^2$
- g^3 is PWS $\Leftrightarrow 3\phi^2 < 2\phi' \leq 2\phi$

which implies that, in our example, there always exists a unique PWS regular network ans, under some conditions, a PWS dominant group network.

4 Simple Public Research Institutions Network

In this section we study a network with only P public research institutions. First, the payoff function depends on the R&D level produced by each PRI. Consequently, whatever the nature of the PRI (R&D lab/university), the cooperation among PRIs can be an effective strategy to increase the partnering PRIs R&D level with the aim of gaining a larger amount of government funds. Formally, we suppose the following benefit function for a PRI:

$$b_u(g) = \underbrace{\sum_{r=0}^{P_u} \rho^r}_{R\&D\ \ benefit} + \underbrace{\frac{\sum_{v \neq u}(P_v - uv)}{2}\ \rho'}_{Spillover\ \ benefit} \qquad 0 \leq \rho' \leq \rho \leq 1$$

$$(6)$$

where P_u represents the number of partnerships held by u with the other PRIs in g. As in the previous section, we assume the existence of two benefits: a) The R&D benefit, ρ, which captures the benefit for a PRI to form a link with another PRI and b) the spillover benefit ρ', that comes out from all the links where u is not involved.

Therefore, the benefit function $b_u(g)$ can be seen as the effective R&D amount that u obtains from network g. As in the simple oligopoly network case, we assume that the R&D benefit follows a decreasing returns to scale regime because the management of a new alliance becomes more difficult as the number of cooperations increase. On the opposite, the spillover benefit is linear because it does not imply any kind of "congestion" effect to the free riders.

The second step is to define the payoff function of PRIs. We start from the observation that the allocation of the government funds toward PRIs are increasingly based on their R&D level. In particular, as mentioned in the introduction, policy makers are increasingly allocating their funds according to the relative performance produced by each PRI (OECD [2007a] [2007b]). Consequently, it is likely that a PRI is in competition with the others in order to get an amount of public funds as large as possible.[20]

The creation of a link among the PRIs u and v influences the payoff of both partners in two ways. First, it increases the R&D level of each partner. Second, the new link makes also the free riders stronger because of the spillover effects. In order to measure these two contrasting effects we adopt the following PRI payoff function

$$R_u(g) = \frac{b_u(g)}{\sum_{p=1}^{P} b_p(g)}, \tag{7}$$

which is simply the share of the effective R&D level enjoyed by u with respect to the other PRIs in the network g. It is worth mentioning that this payoff function can be seen as the payoff of a Tullock game where players can previously form cooperative links.[21] Given this payoff function, a PRI u will form a link only

[20] Moreover, the access to more funds is also a way for the PRI to increase its *prestige* and its ranking with respect to the other institutions.

[21] The relationship among network theory and "contest" games is not new in the literature, even though it is very recent. For example, Tellone and Vergote [2011] analyze how cooperative network links can influence the outcome of a Tullock game. In their paper the authors prove that, when the marginal benefit of forming a link is either linear or increasing in the number of links already held by the partners, the complete network and the exclusive group network can be PWS at the same time. Even though we do not formalize our results for any network, we show that their results can also be valid when the benefit of forming a link is decreasing with respect of the number of links.

when its share increases. Formally, for each $uv \notin g$

$$R_u(g+uv) \geq R_u(g) \Leftrightarrow \frac{b_u(g+uv)}{\sum_{p=1}^{P} b_p(g+uv)} \geq \frac{b_u(g)}{\sum_{p=1}^{P} b_p(g)}. \tag{8}$$

A simple application of Equation (6) on Equation (8) will lead to the following profitability condition

$$\rho^{P_u+1} \geq \frac{b_u(g)}{\sum_{v\neq u} b_v(g)}[\rho^{P_v+1} + (P-2)\rho^{'}].$$

Therefore, differently from the simple oligopoly network, the profitability condition for two PRIs to cooperate depends on the network they want to deviate from. We focus now on the pairwise stability of regular networks.

PWS of regular networks Let us consider a regular network g^k of degree $k \in [0, N-1]$. Since the benefit is the same for all the PRIs, their payoffs in a regular network g^k are always $1/P$. Given this framework, it is possible to characterize the pairwise stability of regular networks among PRIs.

Lemma 3 *Let g^k be a regular network of degree $k \in [0, N-1)$ and $k+1$ respectively. Then*

1. *a network g^k is pairwise stable if and only if*

$$\rho^k \geq \rho' \quad ; \quad \rho^{k+1} \leq \rho' \quad ; \quad 0 \leq \rho' \leq \rho \leq 1;$$

2. *the complete network g^c is pairwise stable if and only if*

$$\rho' \leq \rho^{P-1};$$

Lemma 4 *The empty network g_0 is never pairwise stable*

Proof: See Appendix C

Note that like in the simple oligopoly case, the R&D spillover effect has a negative effect on the PWS of the most linked networks. Moreover, the complete network is less likely to be stable as P increases. Finally, since $0 \leq \rho' \leq \rho \leq 1$, this lemma shows also the existence and the uniqueness of a PWS regular network.

Proposition 3 *There always exists a non-empty regular pairwise stable network.*

An intuitive proof is the following: suppose that it is profitable to deviate from g^k by adding a new link ($\rho^{k+1} \geq \rho'$), then there is a path of networks that lead to the (next) regular network g^{k+1} where either it is still profitable to deviate ($\rho^{k+2} \geq \rho'$) or it is not. This process continues until either we reach a regular network where there is no incentive to add a link or we end up in the complete network.[22]

PWS of non-regular networks Looking at non regular networks, it is possible to show the following result:

Proposition 4 *The star network g^* is never pairwise stable.*

Proof: See Appendix D

We now provide an example that shows how we study the PWS of networks where there are only PRIs.

Example: 3 PRIs To give an intuition of the PRIs' decision to form a link among each other, let us consider the case of a network with three public research institutions. The possible networks are described in Figure 2. Similarly to the

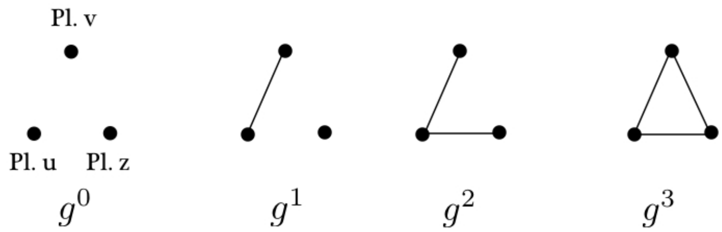

Figure 2: Main network structures with three players (i.e. firms).

previous example, we check the network stability by using Lemma 3, namely the conditions under which a network is pairwise stable. For example, given the benefits shown in Table 1, we can verify the PWS of g^2 in the following way

[22]Moreover, the incentive to deviate by adding a link to g^k is equal to the incentive to keep a link from g^{k+1}. Formally, the former condition is

$$R_u(g+uv) \geq R_u(g^k) \Leftrightarrow \frac{b^k+\rho^{k+1}}{Pb^k+2\rho^{k+1}+(P-2)\rho'} \geq \frac{1}{P}$$

while no one severes a link from g^{k+1} if and only if

$$R_u(g^{k+1}-uv) \geq R_u(g^{k+1}) \Leftrightarrow \frac{b^k-\rho^{k+1}}{Pb^k-2\rho^{k+1}-(P-2)\rho'} \leq \frac{1}{P}$$

Simple calculations show that both conditions hold if and only if $\rho^{k+1} \geq \rho'$.

First, note that in order to have g^2 pairwise stable, we need that the hub u has

Network / Benefits	b_u	b_v	b_z
g^0	1	1	1
g^1	$1+\rho$	$1+\rho$	$1+\rho'$
g^2	$1+\rho+\rho^2$	$1+\rho+\rho'$	$1+\rho+\rho'$
g^3	$1+\rho+\rho^2+\rho'$	$1+\rho+\rho^2+\rho'$	$1+\rho+\rho^2+\rho'$

Table 1: PRI payoffs according to the network structures.

no incentive to delete its links, namely

$$\frac{1+\rho+\rho^2}{3(1+\rho)+\rho^2+2\rho'} \geq \frac{1+\rho}{3+2\rho+\rho'} \Leftrightarrow \rho' \leq \frac{\rho^2(1+\rho)-\rho}{1+\rho-\rho^2}$$

On the other hand, v and z will not form a link and deviate to g^3 only when

$$\frac{1+\rho+\rho'}{3(1+\rho)+\rho^2+2\rho'} \geq \frac{1}{3} \Leftrightarrow \rho' \geq \rho^2$$

However, since $\frac{\rho^2(1+\rho)-\rho}{1+\rho-\rho^2} < \rho^2$, these two conditions are always incompatible, meaning that the star network g^2 is never PWS. Making a similar analysis for all the remaining networks, we can conclude that in a network of 3 PRIs, the following results hold:

- the empty and star networks are never PWS;
- the exclusive group network g^1 is PWS whenever[23] $\rho' \geq \frac{\rho^2(1+\rho)-\rho}{1+\rho-\rho^2}$;
- the complete network g^3 is PWS if and only if $\rho' \leq \rho^2$;
- networks g^1 and g^3 are both PWS when $\frac{\rho^2(1+\rho)-\rho}{1+\rho-\rho^2} \leq \rho' \leq \rho^2$.

It is straightforward to see that this example reflects the results found in the general part.

5 Mixed Network

5.1 Link among Firms and PRIs

In this section we study the situation where firms and universities can form links among themselves. Our first aim is to define a benefit function $b_i(g)$ that allows us to define when it is profitable for a firm to cooperate with a PRI, namely when $\pi_i(g+iu) \geq \pi_i(g)$ holds. Then we look at the reverse case.

[23]Note that whenever $0 < \rho \leq (\sqrt{5}-1)/2$, the PWS condition is always satisfied because its right hand side becomes negative.

Firm's strategy Before proceeding, it is worth reminding that, in line with some stylized facts, we suppose that the benefit of two heterogeneous players are additive (Assumption 1). As a consequence, we define the benefit function enjoyed by a firm i as follows

$$b_i(g) = \underbrace{\sum_{f=0}^{F_i} \phi^f + \frac{\sum_{j\neq i}(F_j - ij)}{2}\phi'}_{Equation\ \ (4)} + \sum_{e=0}^{P_i} \psi^e + \sum_{j\neq i} P_j \;\; \psi' \tag{9}$$

$$\text{s. t. } 0 \le \phi' \le \phi \le 1 \;\; ; \;\; 0 \le \psi' \le \psi \le 1$$

which is simply the sum of the benefit that i gets from a) the link with its F_i competitors (Equation 4) and b) the partnerships with P_i PRIs. Note that in $b_i(g)$, we do not consider the benefit that i gets from the links among PRIs because they have the same effect on all firms such that, for the sake of simplicity, it can be ignored (for the same reason we will ignore the effect of the links among firms on the benefit of a PRI $b_u(g)$).

Let us focus now on the second part of the right hand side in the last equation, which is actually the added value in the analysis of networks between heterogeneous agents

$$b_i(g) = \ldots + \underbrace{\sum_{e=0}^{P_i} \psi^e}_{R\&D\ \ benefit} + \underbrace{\sum_{j\neq i} P_j \;\; \psi'}_{Spillover\ \ benefit} \qquad\qquad 0 \le \psi' \le \psi \le 1$$

As can be seen, we formalize the benefit of firms from R&D partnerships with PRI following the approach already used in the simple oligopoly networks: ψ and ψ' are parameters used to describe respectively the benefit enjoyed by firm i from an alliance with PRI u and the spillover effect from the "mixed" partnerships where i is not involved. Moreover, even though the benefit for firm i to create an alliance with a PRI is a higher level of R&D, such benefit is lower when the number of partnerships already held by the firm is larger. On the other hand, the new link also provides a free riding benefit to its competitors. Therefore, like in the simple oligopoly network, we assume the presence of a congestion effect in the number of partnerships and a linear spillover benefit.

Given this framework, a firm i that competes in a Cournot oligopoly with F firms, will form a link with a PRI u only when

$$\pi_i(g + iu) \ge \pi_i(g) \Leftrightarrow F[b_i(g + iu) - b_i(g)] - \sum_{j\neq i}[b_i(g + iu) - b_i(g)] \ge 0$$

which is similar to the profitability condition of a link among two firms (Equation (1)). Using the definition of $b_i(g)$ described in Equation (9) we get

$$\pi_i(g + iu) \ge \pi_i(g) \Leftrightarrow F\psi^{P_i+1} - (F - 1)\psi' \ge 0$$

Note that, since $\psi \geq \psi'$, firm i is always willing to form at least one link with a PRI.

PRI Looking at the PRIs incentive to cooperate with a firm, we use adopt the same approach the benefit enjoyed by u in a mixed network g is

$$b_u(g) = \underbrace{\sum_{r=0}^{P_u} \rho^r + \frac{\sum_{v \neq u}(P_v - uv)}{2} \rho'}_{Equation\ (6)} + \sum_{s=0}^{F_u} \sigma^s + \sum_{v \neq u} F_v \ \sigma' \tag{10}$$

$$\text{s. t.} \quad u, v \in \{P\} \ ; \ 0 \leq \rho' \leq \rho \leq 1 \ ; \ 0 \leq \sigma' \leq \sigma \leq 1$$

where F_u (F_v) are the number of firms which are (not) allied with u. Note that, similarly to firm's analysis, $b_u(g)$ corresponds to the sum of the benefits that PRI u gets from cooperating with P_u PRIs (Equation 6) and F_u PRIs. Considering the second part of Equation (10)

$$b_u(g) = \ldots + \underbrace{\sum_{s=0}^{F_u} \sigma^s}_{R\&D\ benefit} + \underbrace{\sum_{w \neq u} F_w \ \sigma'}_{Spillover\ benefit} \qquad 0 \leq \sigma' \leq \sigma \leq 1,$$

we can see that also in this case we assume the existence of both congestion and linear spillover effect.

Like the simple PRI network, the payoff function of the PRI is the relative amount of the effective R&D level enjoyed by u with respect to the other PRIs (Equation (7)). Therefore, u will find it profitable to cooperate with i only when

$$R_u(g + ui) \geq R_u(g) \Leftrightarrow \frac{b_u(g + ui)}{\sum_{p=1}^{P} b_p(g + ui)} \geq \frac{b_u(g)}{\sum_{p=1}^{P} b_p(g)},$$

which lead to the following profitability condition when we apply Equation (10) on the payoff functions:

$$\sigma^{F_u + 1} \geq (P - 1) \ \frac{b_u(g)}{\sum_{v \neq u} b_v(g)} \ \sigma'. \tag{11}$$

Similarly to the previous section, it is straightforward to see that each PRI has the incentive to form at least one link with a firm.[24]

Summarizing, given a network g with F firms and P public research institutions,

[24] In fact, let us consider a network where no PRI is linked with firms. Then, u will form a link with i only when

$$\frac{1 + \sigma}{1 + \sigma + (P - 1)\sigma'} \geq \frac{1}{P} \Rightarrow (P - 1)(1 + \sigma) \geq (P - 1)\sigma' \Rightarrow (1 + \sigma) \geq \sigma'$$

which is always satisfied because $0 \leq \sigma' \leq \sigma \leq 1$.

a new link between a firm i and a PRI i is formed if and only if the following conditions hold:

$$\psi^{P_i+1} - \frac{F-1}{F}\ \psi' \geq 0; \tag{12}$$

$$\sigma^{F_u+1} - \frac{b_u(g)(P-1)}{\sum_{v\neq u} b_v(g)}\ \sigma' \geq 0 \tag{13}$$

Given these conditions about the network formation among heterogeneous agents, we are now able to consider a whole network where both firms and PRIs are involved.

5.2 Pairwise Stability

Regular Networks A first step to study the pairwise stability is to look at regular networks, namely networks where players have the same number of links. However, since there are two types of players, it is possible to consider several "degrees" of regularity.

In order to explain this point it is worth extending the notation used so far. First, the set of players linked with i can be split according to the type of partner such that the set of players linked with i, $N_i(g)$, is given by the union between the set of firms linked with i and the set of PRIs linked with i. Formally $N_i(g) = F_i(g) \cup P_i(g)$. As a consequence, the number of partners held by firm i is given by the sum of the partnering firms of i plus the PRIs cooperating with i, i.e. $\eta_i(g) = \eta_i^F(g) + \eta_i^P(g)$. In a similar vein, it is possible to define the neighbourhood of the PRI u as $N_u(g) = F_u(g) \cup P_u(g)$ such that its number of links is given by the following sum: $\eta_u(g) = \eta_u^F(g) + \eta_u^P(g)$.

Given this notation, we now define the following types of regularity.

Definition 1 (Regularity) *Let $\hat{g}$ be a network with F firms and P public research institutions, then*

1. *the network $\hat{g}$ is relative regular if at least one of the following conditions holds:*
   (a) $\eta_i^F(\hat{g}) = \eta_j^F(\hat{g}); \quad \eta_h^N \neq \eta_k^N; \quad \forall i,j \in F; \forall h,k \in N; \quad N = F,\ P$
   (b) $\eta_u^P(\hat{g}) = \eta_v^P(\hat{g}); \quad \eta_h^N \neq \eta_k^N; \quad \forall u,v \in P; \forall h,k \in N; \quad N = F,\ P$
   (c) i. $\eta_i^P(\hat{g}) = \eta_j^P(\hat{g}); \quad \eta_h^N \neq \eta_k^N; \quad \forall i,j \in F; \forall h,k \in N; \quad N = F,\ P$
       ii. $\eta_u^F(\hat{g}) = \eta_v^F(\hat{g}); \quad \eta_h^N \neq \eta_k^N; \quad \forall u,v \in P; \forall h,k \in N; \quad N = F,\ P$
2. *the network $\hat{g}$ is absolute regular if* $\eta_h^N(\hat{g}) = \eta_k^N(\hat{g}) \quad ; \quad \forall h,k \in N$

In a more intuitive way, the regularity of a network is absolute when all the players have the same number of links toward both types of agents whereas it is relative when firms and/or PRIs have an identical number of links toward either their competitors or the players of the other type.
Note that both absolute and relative regularity of networks do not necessarily coincide with the "classic" regularity concept used in homogeneous networks. To prove this point it is sufficient to see Figures 3, 4, and 5 where the usual regularity of homogeneous network is compared with our relative and absolute regularity respectively. As can be seen, the absolute regularity is a particular case of the classic regularity in that it imposes for each player x not only that $\eta_x(g) = \eta_x^F(g) + \eta_x^P(g)$ to be constant but also that $\eta_x^F(g) = \eta_x^P(g) = \bar{\eta}$. On

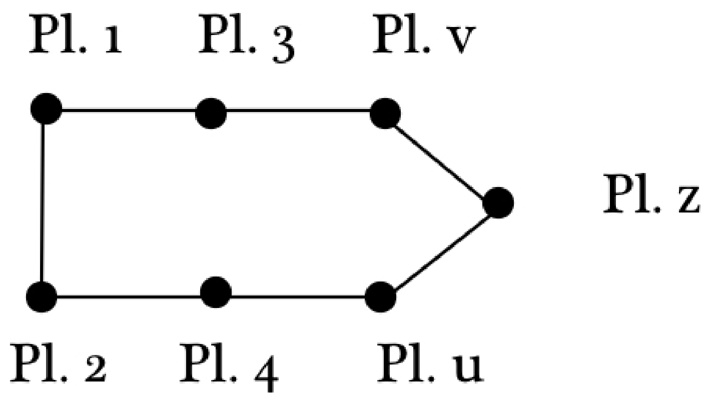

Figure 3: Classic regular network among 4 firms and 3 PRIs where $\eta_h = 2$ $\forall h \in N$

the other hand, the relative regularity requires that $\eta_x^F(g)$ or $\eta_x^P(g)$ must be constant for at least one type of players while classic regularity can occur also when $\eta_x^F(g)$ and $\eta_x^P(g)$ differs among the players as long as $\eta_x(g)$ is constant for all the players.
Another important remark is related to the link formation conditions among heterogeneous players (Equations (12) and (13)).

Lemma 5 *Let us consider a network such that*

$$\psi^{\hat{p}} \geq \frac{F-1}{F} \ \psi' \geq \psi^{\hat{p}+1} \ ; \ \sigma^{\hat{f}} \geq \ \sigma' \geq \sigma^{\hat{f}+1}. \tag{14}$$

namely where $\hat{p} \leq P$ and $\hat{f} \leq F$ are the maximum number of links that firms and PRI want to hold respectively with the other type of player. Then,

1. *the number of links among heterogeneous agents is $\lambda = \min(\hat{p} \cdot F; \hat{f} \cdot P)$*

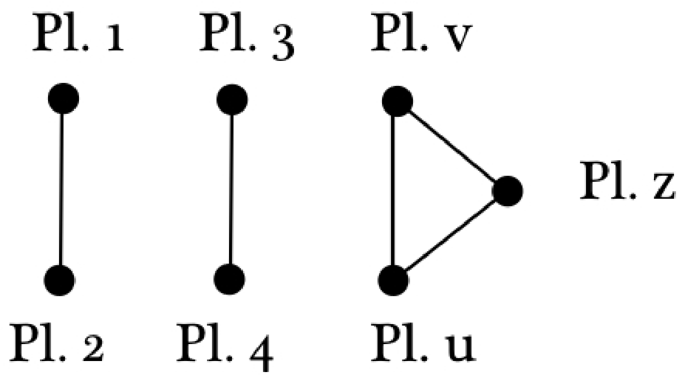

Figure 4: Relative regular network among 4 firms and 3 PRIs such that $\eta_i = \eta_i^F = 1$ and $\eta_u = \eta_u^P = 2$ $\forall i \in F$ $\forall u \in P$ $\forall i \in F$ $\forall u \in P$.

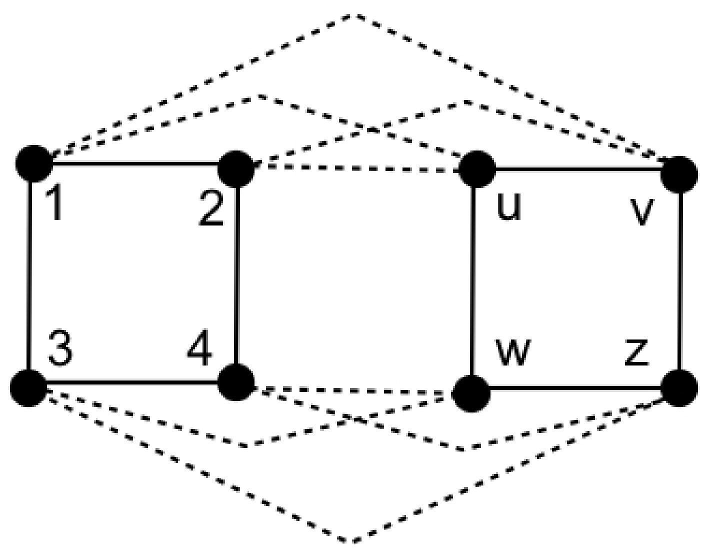

Figure 5: Absolute regular network among 4 firms and 4 PRIs where $\eta_h^N = 2$ $\forall h \in N$ and $N = F, P$. Plain and dotted lines are respectively links among homogeneous and heterogeneous players.

2. *the number of links that firms and PRIs can form are respectively*

$$\eta_i^P \leq \hat{p} \;\; ; \quad \eta_u^F \leq \hat{f} \quad \forall i \in F; \forall u \in P$$

with strict inequality for at least one player whenever $\hat{p} \cdot F \neq \hat{f} \cdot P$

3. *if* $\hat{p} \cdot F = \hat{f} \cdot P \quad \forall i, j \in F \; \forall u, v \in P$, *then there exists a network* $\hat{g}$ *where* $\eta_i^P(\hat{g}) = \eta_j^P(\hat{g})$ *and* $\eta_u^F(\hat{g}) = \eta_v^F(\hat{g})$

Proof: See Appendix E.
Given this minor finding it is possible to get the following result

Proposition 5 *There always exists a PWS relative regular network where*

1. $\eta_i^F(\hat{g}) = \eta_j^F(\hat{g}) = \kappa \quad ; \quad \forall i, j \in F$
2. $\eta_u^P(\hat{g}) = \eta_v^P(\hat{g}) = \omega \quad ; \quad \forall u, v \in P$
3. $\eta_i^P \leq \hat{p} \;\; ; \quad \eta_u^F \leq \hat{f} \;\; ; \quad \forall i \in F; \forall u \in P;$
with strict inequality for at least one player whenever $\hat{p} \cdot F \neq \hat{f} \cdot P$.

Moreover, if $\kappa = \omega = \eta_i^P = \eta_u^F$, *then the PWS network is also absolute regular.*

Proof: The proof is quite straightforward. First, according to Assumption 1, the decision of a player to link with a firm is independent of its decision to form a link with a PRI. Therefore, the pairwise stability of the relative regular network comes out from the PWS of the a) simple oligopoly networks (Section 3); b) simple PRIs networks (Section 4) and c) the links among heterogeneous players.
Proposition 1 proves the existence of PWS regular network among firms of degree $\kappa \in [1, F-1]$, while Proposition 3 proves the existence of a PWS regular network among PRIs of degree $\omega \in [1, P-1]$. The PWS of the links among heterogeneous players is ensured by Lemma 5 because there is always a network that satisfies Equation (14) such that the number of heterogeneous links is $\lambda = \min(\hat{p} \cdot F; \hat{f} \cdot P)$. If we consider a regular network that embeds all these pairwise stability conditions, the proof is done. **Q.E.D.**

Even though we prove the existence of a PWS mixed network, the presence of heterogeneous agents has the drawback of dramatically increasing the number of the possible PWS equilibria. Thus, the pairwise stability is too weak a concept to refine a sufficient amount of mixed networks. To prove this point, we now introduce the following example of mixed network with 4 firms and 4 PRIs.

Example: 4 firms and 4 PRI's Let us study a relative regular network where

1. $\frac{F-1}{F-2}\ \phi \ \geq \ \phi' \ \geq \ \frac{F-1}{F-2}\ \phi^2 \Rightarrow$ firms form at most $\kappa = 1$ links among themselves;

2. $\rho^2 \ \geq \ \rho' \ \geq \ \rho^3 \Rightarrow$ PRIs want to form $\omega = 2$ links among themselves;

3. $\sigma^2 \ \geq \ \sigma' \ \geq \ \sigma^3 \Rightarrow$ PRIs want to create at most 2 links with firms.

4. $\frac{F}{F-1}\ \psi^4 \ \geq \ \psi' \Rightarrow$ firms are always willing to form a partnership with PRIs; Thus the number of links among firms and PRIs is $\lambda = 2 \cdot 4 = 8$.

Then the network in Figure 6 is PWS. However, it is possible to prove that

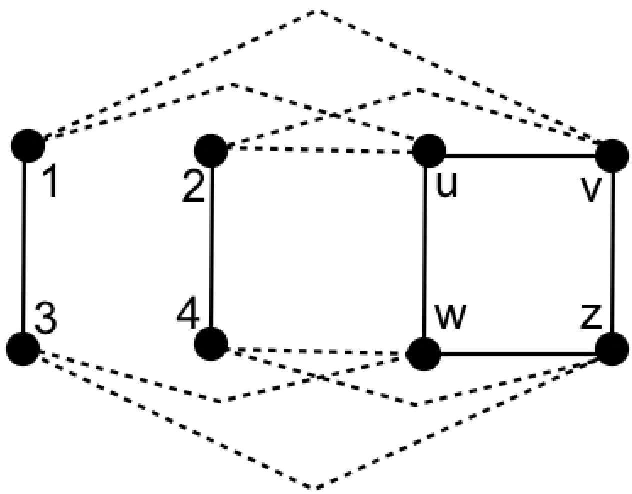

Figure 6: Example of PWS relative regular network with 4 firms and 4 PRIs. Plain and dotted lines are respectively links among homogeneous and heterogeneous players.

under the same conditions the network in Figure 7 is also PWS. On the other hand, the network depicted as in Figure 6 is PWS also when we modify the last PWS as follows:

- $\frac{F}{F-1}\ \psi^3 \ \geq \ \psi' \geq \frac{F}{F-1}\ \psi^4$ firms want to form at most three links with with PRIs; meaning that $\lambda = 8$ is still valid.

This example shows how a) the same PWS conditions are valid for both (relative) regular networks; b) if a regular network is PWS when λ is equal to (say)

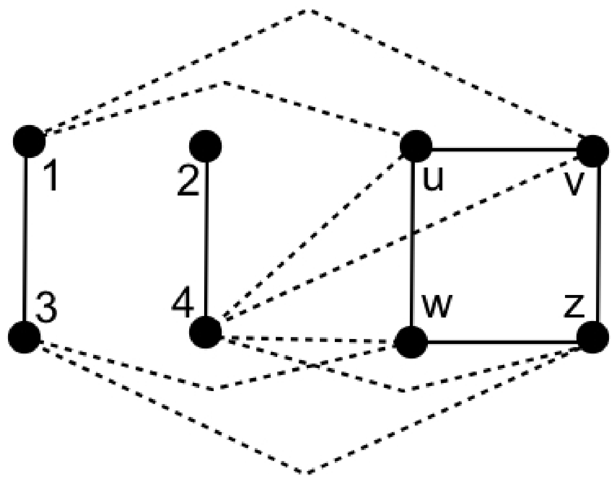

Figure 7: Example of PWS non regular network with 4 firms and 4 PRIs

three, then any other network that ensures $\lambda = 3$ is *coeteris paribus* also PWS. Therefore, a higher number of PWS networks is mainly due to the strategic interaction among heterogeneous players which multiplies the conditions under which the links among heterogeneous players are stable.

5.3 Equilibrium analysis

So far, we have shown the existence of a PWS regular network among firms and PRIs throughout the definition of two new regularity concepts (i.e. relative and absolute regularity). Summarizing, for all the networks that satisfy Proposition 5 we have that

1. firms want to create κ links among each other:

$$\phi^{\kappa} \geq \frac{F-2}{F-1} \ \phi' \geq \phi^{\kappa+1} \tag{15}$$

2. PRIs want to form ω links among each other:

$$\sigma^{\omega} \geq \sigma' \geq \sigma^{\omega+1} \tag{16}$$

3. firms want to be linked with $\hat{p}$ PRIs:

$$\psi^{\hat{p}} \geq \frac{F-1}{F} \ \psi' \geq \psi^{\hat{p}+1} \tag{17}$$

4. PRIs are willing to form $\hat{f}$ finks with firms:

$$\sigma^{\hat{f}} \geq \sigma' \geq \sigma^{\hat{f}+1} \tag{18}$$

such that the last two conditions lead to the formation of $\lambda = \min(\hat{f}, \hat{p})$ links among the two type of players. Note that the four conditions are more difficult to satisfy as the spillover provided to the player competitors is higher. Now, it is worth studying which factors influence the formation of a PWS regular network rather than another. A first, straightforward result comes from the conditions just mentioned above:

Corollary 1 *Suppose that Proposition 5 holds for a network g. Then, if the number of firms F increases, PWS requires a smaller number of links among firms and a smaller number of links between firms and PRIs.*

This outcome is rather intuitive insofar as if F increases the creation of a new partnership will provide a spillover benefit to a larger number of competitors, hence a lower incentive to create links (Equations 15 and 17). Note that such effect is due to the Cournot competition that occurs in the final market: for PRIs, Equations 16 and 18 do not depend on P.

A second issue is when firms prefer to link among each other rather than with PRIs, namely when Condition 15 is more/less restrictive than Condition 17. From this point of view, it is quite reasonable to assume that the cost reduction stemming from the first link with another firm is larger than from the first link with a PRI (i.e. $\phi > \psi$). This assumption is reasonable if we think that PRIs are more involved in basic research activity (less suitable to firms' production strategies), and because firms and PRIs may lack coordination in their R&D partnership because of their different objectives (commercialization vs. publication of R&D results). Given that $\phi > \psi$, the following statement is valid:

Proposition 6 *Firms will cooperate more among each other than with PRIs if, for each firm, a new link with a partnering firm provides the same/lower amount of spillover benefit to its own competitors than a new link with a PRI. Formally, if* $\frac{F-1}{F}\psi' \geq \frac{F-2}{F-1}\phi'$ *then* $\kappa \geq \hat{p}$

Proof: Since $0 \leq \psi < \phi \leq 1$, then $\phi^x > \psi^x$ for all $x \in [0, \infty)$. Therefore, Condition 15 defines a larger interval than Condition 17:

$$\phi^\kappa \geq \frac{F-2}{F-1}\ \phi' \geq \phi^{\kappa+1}$$

$$\psi^{\hat{p}} \geq \frac{F-1}{F}\psi' \geq \psi^{\hat{p}+1}$$

whenever $\kappa = \hat{p} = x$. In fact, such a case leads to two possible scenarios:

I) $\phi^x \geq \psi^x \geq \phi^{x+1} \geq \psi^{x+1}$

II) $\phi^x \geq \phi^{x+1} \geq \psi^x \geq \psi^{x+1}$

Suppose now that Condition 2 is valid for $\hat{p} = x$. Then $\frac{F-1}{F}\psi' = \frac{F-2}{F-1}\phi'$ implies that in the first scenario either

$$\phi^x \geq \psi^x \geq \frac{F-1}{F}\psi' = \frac{F-2}{F-1}\phi' \geq \phi^{x+1} \geq \psi^{x+1}$$

or

$$\phi^x \geq \psi^x \geq \phi^{x+1} \geq \frac{F-1}{F}\psi' = \frac{F-2}{F-1}\phi' \geq \psi^{x+1},$$

while in the second scenario we have

$$\phi^x \geq \phi^{x+1} \geq \psi^x \geq \frac{F-1}{F}\psi' = \frac{F-2}{F-1}\phi' \geq \psi^{x+1}.$$

Only in the former case we have that the PWS condition for a link between firms is valid for $\kappa = \hat{p} = x$. In the other two cases, PWS conditions require that $\kappa > \hat{p} = x$. This result is also valid when $\frac{F-1}{F}\psi' > \frac{F-2}{F-1}\phi'$ because in such a case $\frac{F-1}{F}\psi'$ requires a PWS such that $\hat{p} \leq x$. **Q.E.D.**

Keeping in mind that $\phi > \psi$, the first inequality of Proposition 6

$$\frac{F-1}{F}\ \psi' \geq \frac{F-2}{F-1}\ \phi' \Rightarrow \psi' \geq \frac{F(F-2)}{(F-1)^2}\ \phi' \tag{19}$$

implies that firms can be more inclined to form a link among each other (rather than with a PRI) even when the spillover parameter of forming a link with a PRI ψ is smaller than the respective spillover for a link among firms (i.e. ϕ).[25] Therefore, this result is in line with the stylized fact mentioned in Section 2.2, where we pointed out that firms sometimes do not cooperate with PRIs because the R&D spillover level where so serious to overcome the potential benefits of the R&D alliance.

A third interesting point is to see under which conditions a firm has higher/lower incentives than a PRI to cooperate in a firm-PRI partnership. However, since we cannot say *a priori* if "mixed" alliances provide larger benefits to the PRIs

[25] In fact, since $\frac{F(F-2)}{(F-1)^2} < 1$, then it is possible that $\phi' \geq \psi' \geq \frac{F(F-2)}{(F-1)^2}\ \phi'$.

rather than to the firms,[26] we can only answer this question on a case-by-case analysis.
However, suppose that $\sigma = \psi$, namely that the benefit of the first link among a firm and a PRI is the same for both types of players; then Conditions 17 and 18 lead to the following result:

Proposition 7 *If a firm and a PRI would get the same benefit from creating a first link among each other and the spillover level provided to firm's competitors is not smaller than the spillover given to PRI's contenders, then the firm will be at most equally inclined to cooperate than its PRI partner. Formally,*

$$\psi = \sigma \ \ and \ \ \frac{F-1}{F}\psi' \geq \sigma' \Rightarrow \hat{p} \leq \hat{f}.$$

Proof: See Appendix F.
A corollary to Proposition 7 is that the lower propensity of the firm to cooperate with a PRI is valid not only when both get the same benefit from the first link created (i.e. $\psi = \sigma$), but also when the firm gets a smaller one, namely that $\psi \leq \sigma$.[27] However, other cases (like $\psi < \sigma$ and $\frac{F-1}{F}\psi' \leq \sigma'$) cannot be disentangled without a case-by-case analysis. It is quite evident that the interaction between firms and PRI needs further investigation in order to properly define which hypothesis about the benefits and spillovers are more realistic.
Before concluding, it is worth mentioning the existence of a possible contrast between the equilibrium network and the most socially efficient. This is due to the fact that a new link in our model increases the performance (i.e. reduces the marginal costs of all the firms and increases the R&D produced by all the PRIs) of both partners and free riders, leading to a higher production of research (for PRIs) and final good (for firms). As a consequence the complete network is the most desirable from a welfare point of view but, as we found in the previous section, the equilibrium network may be different.

6 Concluding Remarks

In the last years there has been an increasing debate about the social benefits of R&D cooperation among firms. However, little attention is paid to the coop-

[26]For example, the advantage could be lower for a PRI than for a firm in a cooperation between a well-known research centre and a start-up. The reverse case occurs in a partnership between a multinational enterprise and a small university.

[27]Note also that, since $\frac{F-1}{F} < 1$, Proposition 7 implies that

$$\psi = \sigma \ \text{ and } \ \psi' = \sigma' \Rightarrow \hat{p} \geq \hat{f}$$

eration among firms and public research institutions (i.e. public R&D labs and universities) whose role is nowadays crucial for the firms' innovative capacity. In this paper we aim to cover this gap by providing a theoretical background to the increasing R&D cooperation among firms and PRIs.

Another objective is to show that firms may not want to form R&D partnerships even when the cost of forming a link is null because the benefits they get from the R&D alliance can be lower than the R&D spillover benefits that go to their competitors.

We show that the incentive for the players to form a partnership is decreasing with the spillover effect such that the networks with a higher number of links become less likely to be stable as spillover concerns are higher.

We get the realistic result that the PWS of a complete network is more difficult to hold also when the number of players increases because the more competitors there are, the larger the spillover benefits that a player provides them with when forming a new alliance. Furthermore, both the empty and the star networks are never PWS. The former because there is always an incentive to form at least one collaborative link when there are no link cost, the latter because, if the hub player has the incentive to hold its links, then a couple of peripheral players have the incentive to form a new partnership. Finally, from a methodological point of view, we find that different types of players interacting in a single network has non trivial implications in the pairwise stability analysis. In fact, in order to determine which network can be pairwise stable, the interaction of heterogeneous agents requires a concept of network regularity different from the classic one. This leads us to define the concepts of relative and absolute regularity of a network, which allow us to overcome the problems of the PWS analysis that would have been met by using the classic regularity notion.

Nevertheless, the analysis is not immune to some important drawbacks: as already mentioned, we do not endogenize the R&D effort, an issue which can be relevant to observe in the presence of R&D spillovers. Second, as shown in the example of Section 5.2, the presence of heterogeneous players dramatically increases the number of PWS networks such that refining the pairwise stability concept becomes necessary. From this point of view, the use of strong stability and/or farsighted stability may be two possible solutions to simplify the scenario of the possible network equilibria. Another possible extension of the model is to generalize the simple oligopoly network by overcoming the linear Cournot oligopoly in favor of a generic function that relates profits with the firms' linking strategy.

A Proof of Lemma 2

Proof of 1: Applying Lemma 1 to network g_k, a firm k will keep the links already held if the following condition holds

$$\pi_i(g) \geq \pi_i(g - kl) \Leftrightarrow F \;\; \phi^k \;\; \geq \;\; \phi^k + (F-2) \;\; \phi' \quad \forall k, l \in \{N\}$$

which corresponds to the first (PWS) condition. On the other hand, two firms will not deviate from g_k by adding the link ij only when

$$\pi_i(g + ij) \leq \pi_i(g) \Leftrightarrow F \;\; \phi^{k+1} \;\; \leq \;\; \phi^{k+1} + (F-2) \;\; \phi' \quad \forall i, j \in \{N\}$$

Isolating ϕ' lead us to our second condition. **Q.E.D.**

Proof of 2: Let $g^c - ij$ be a network which differs from the complete one for a missing link among i and j. If these two firms form the link then they get

$$b_i(g + ij) - b_i(g) = b_j(g + ij) - b_j(g) = \phi^{F-1}$$

Since the other firms get a spillover benefit ϕ', the profitability condition (Equation (3)) for the two partners is

$$\pi_i(g+ij) \geq \pi_i(g) \Leftrightarrow F \;\; \phi^{F-1} \;\; \geq \;\; \phi^{F-1} + (F-2) \;\; \phi' \Leftrightarrow \phi' \;\; \leq \;\; \frac{F-1}{F-2} \;\; \phi^{F-1}$$

Proof of 3: Applying Lemma 2 when $k = 0$ we get that g^0 is PWS if and only if

$$\frac{F-1}{F-2} \geq \phi' \geq \frac{F-1}{F-2}\phi$$

Since $\frac{F-1}{F-2}\phi > \phi > \phi'$ then the condition on the right is violated because $0 \leq \phi' \leq \phi \leq 1$ by assumption. **Q.E.D.**

B Proof of Proposition 2

Let g^* a star network where h is the hub firm i.e. the firm to which the other $F-1$ firms are linked. To prove the proposition we have to show that starting from g^* either a firm wants to delete a link or a couple of firms want to form another link.

Part a). Let us consider the network $g^* - hj$ i.e. a star network without the link hj. If firms h and j want to form such link, their benefits are respectively

$$b_h(g^*) - b_h(g^* - hj) = \phi^{F-1} \;\; ; \;\; b_j(g^*) - b_j(g^* - hj) = \phi$$

while the other firms will get

$$b_k(g^*) - b_k(g^* - hj) = \phi' \qquad k \neq h, j.$$

The hub firm will form a link if and only if

$$\pi_h(g^*) \geq \pi_h(g^* - hj) \Leftrightarrow F\phi^{F-1} - \phi - (F-2)\phi' \geq 0 \Leftrightarrow \phi' \leq \frac{F\ \phi^{F-1} - \phi}{F-2}, \tag{20}$$

while the profitability condition for the firm j is

$$\pi_j(g^*) \geq \pi_j(g^* - hj) \Leftrightarrow F\ \phi - \phi^{F-1} - (F-2)\ \phi' \geq 0,$$

which is clearly satisfied.[28] Therefore, when condition (20) does not hold, the star network is not pairwise stable because the hub firm has the incentive to severe one link.

Part b). Now, starting from g^*, we have to see under which condition two firms want to form a new link ij. In this case partners' benefits are

$$b_i(g^* + ij) - b_i(g^*) = b_j(g^* + ij) - b_j(g^*) = \phi^2 \qquad i, j \neq h$$

while the other firms will get the usual benefit

$$b_k(g^*) - b_k(g^* - hj) = \phi' \qquad k \neq h, j.$$

The firm i will form the new link whenever

$$(F-1)\phi^2 - (F-2)\phi' \geq 0 \Leftrightarrow \phi' \leq \frac{F-1}{F-2}\ \phi^2.$$

Therefore, a new link will not be added to the star network if and only if

$$\phi' \geq \frac{F-1}{F-2}\ \phi^2. \tag{21}$$

Now to prove the proposition 2, we have to show that the conditions for the pairwise stability (20) and (21) cannot hold at the same time. Note that the two conditions lead to the following range

$$\frac{F-1}{F-2}\ \phi^2 \leq \phi' \leq \frac{F\ \phi^{F-1} - \phi}{F-2}$$

which implies that

$$\frac{F-1}{F-2}\ \phi^2 \leq \frac{F\ \phi^{F-1} - \phi}{F-2} \Rightarrow \phi \leq F\ \phi^{F-1} - (F-1)\ \phi^2 \tag{22}$$

First, note that for $F = 3$, the condition (22) becomes $\phi \leq \phi^2$, which is never valid. Moreover, the RHS of the condition (22) is decreasing in F. In fact

$$(F+1)\ \phi^F - (F)\ \phi^2 \leq F\ \phi^{F-1} - (F-1)\ \phi^2 \Rightarrow$$

$$F\underbrace{(\phi^F - \phi^{F-1})}_{<0} + \phi^F \leq \phi^F \leq \phi^2$$

Therefore, the pairwise stability conditions for a star network lead to a range which is always empty. **Q.E.D.**

[28]In fact, it is easy to see that $(F-1)\phi \geq \phi^{F-1} - (F-2)\phi'$ because ϕ is larger than both ϕ^{F-1} and ϕ'.

C Proof of Lemma 3

Let us consider a regular network of degree $k \in [0, n-1)$ such that the benefit enjoyed by each player is b^k, then no firm will have an incentive to delete a link from g^k if

$$\frac{b^k - \rho^k}{2(b^k - \rho^k) + (P-2)(b^k - \rho')} \leq \frac{1}{P}$$

hence

$$P(b^k - \rho^k) \leq 2(b^k - \rho^k) + (P-2)(b^k - \rho')$$

Rearranging terms and simplifying we obtain

$$(P-2)(b^k - \rho^k) \leq (P-2)(b^k - \rho')$$

which leads to our first condition of part 1

$$\rho' \leq \rho^k.$$

Note that, this condition holds also for a complete network. Therefore Part 2 of the lemma is a straightforward application of this condition as $k = P - 1$

On the other hand two firms will not have an incentive to form a link if

$$\frac{b^k + \rho^{k+1}}{2(b^k + \rho^{k+1}) + (P-2)(b^k + \rho')} \leq \frac{1}{P}.$$

Rearranging the terms we get

$$P(b^k + \rho^{k+1}) \leq 2(b^k + \rho^{k+1}) + (P-2)(b^k + \rho').$$

Simplifying and rearranging some terms

$$(P-2)(b^k + \rho^{k+1}) \leq (P-2)(b^k + \rho')$$

which implies that

$$\rho^{k+1} \leq \rho',$$

which corresponds to the second condition of Part 1. Finally, it is straightforward to see that the empty network cannot be PWS because $\rho^{k+1} \geq \rho'$ when $k = 0$.

D Proof of Proposition 4

The proof is similar to previous one. Before continuing we first find the benefit functions for all the networks needed to prove the PWS of the star network which are $g^* - uh$, g^* and $g^* + uv$.

Network $g^* - uh$ Let consider the network $g^* - uh$ where u is an isolated PRI and h is the hub. Then, we have that

$$b_u = 1 + (P-2)\rho' \; ; \; b_z = 1 + \rho + (P-3)\rho' \; ; \; b_h = \sum_{r=0}^{P-2} \rho^r,$$

which implies that the overall network benefit is

$$B(g^* - uh) = \sum_{p=1}^{P} b_p = P + (P-2)\rho + (P-2)^2\rho' + \sum_{r=1}^{P-2} \rho^r.$$

Network g^* On the other hand in the star network

$$b_u = b_z = 1 + \rho + (P-2)\rho' \; ; \; b_h = \sum_{r=0}^{P-1} \rho^r,$$

which implies that the overall network benefit is

$$B(g^*) = \sum_{p=1}^{P} b_p = P + (P-1)\rho + (P-1)(P-2)\rho' + \sum_{r=1}^{P-1} \rho^r.$$

Network $g^* + uv$ Finally, in the network $g^* + uv$ where two non-hub PRI are linked we have

$$b_u = b_v = 1 + \rho + \rho^2 + (P-2)\rho' \; ; \; b_z = 1 + \rho + (P-1)\rho' \; ; \; b_h = \sum_{r=0}^{P-1} \rho^r + \rho',$$

which implies that the overall network benefit is

$$B(g^* + uv) = \sum_{p=1}^{P} b_p = P + (P-1)\rho + 2\rho^2 + P(P-2)\rho' + \sum_{r=1}^{P-1} \rho^r.$$

Pairwise Stability from severing a link To find the PWS of g^* we first check under which conditions the hub and the PRI u are better off in the star network rather than in the network $g^* - uh$. Using the equation (8), PRI u will find profitable to cooperate whenever

$$\frac{1 + \rho + (P-2)\rho'}{B(g^*)} \geq \frac{1 + (P-2)\rho'}{B(g^* - uh)} \Rightarrow$$

$$\rho \; B(g^* - uh) \geq [1 + (P-2)\rho'] \, [B(g^*) - B(g^* - uh)] \, .$$

while the hub PRI is better off when

$$\frac{\sum_{r=0}^{P-1} \rho^r}{B(g^*)} \geq \frac{\sum_{r=0}^{P-2} \rho^r}{B(g^* - uh)} \Rightarrow \rho^{P-1} B(g^* - uh) \geq \sum_{r=0}^{P-2} \rho^r \, [B(g^*) - B(g^* - uh)] \, .$$

Since the profitability condition is more restrictive for the hub player we will restrict our analysis on it. Now, given that $[B(g^* - B(g^* - uh))] = \rho + \rho^{P-1} + (P-2)\rho'$ we have that

$$\rho^{P-1} \left[P + (P-2)\rho + (P-2)^2 \rho' + \sum_{r=1}^{P-2} \rho^r \right] \geq \sum_{r=0}^{P-2} \rho^r \left[\rho + \rho^{P-1} + (P-2)\rho' \right],$$

hence

$$\rho^{P-1} \left[P - 1 + (P-2)\rho + (P-2)^2 \rho' + \sum_{r=0}^{P-2} \rho^r \right] \geq \sum_{r=0}^{P-2} \rho^r \left[\rho + \rho^{P-1} + (P-2)\rho' \right]$$

Simplifying some terms

$$\rho^{P-1} \left[P - 1 + (P-2)\rho + (P-2)^2 \rho' \right] \geq \sum_{r=0}^{P-2} \rho^r \left[\rho + (P-2)\rho' \right]$$

and extending the left hand side

$$(P-1)\rho^{P-1} + (P-2)\rho^{P-1} \left[\rho + (P-2)\rho' \right] \geq \left[\rho + (P-2)\rho' \right] + \sum_{r=1}^{P-2} \rho^r \left[\rho + (P-2)\rho' \right],$$

we obtain

$$\left[(P-1)\rho^{P-1} - \rho - (P-2)\rho' \right] \geq \left[\rho + (P-2)\rho' \right] \left[\sum_{r=1}^{P-2} \rho^r - (P-2)\rho^{P-1} \right]$$

or

$$\left[\rho^{P-1} - \rho - (P-2)(\rho^{P-1} - \rho') \right] \geq \left[\rho + (P-2)\rho' \right] \left[\sum_{r=1}^{P-2} \rho^r - (P-2)\rho^{P-1} \right]$$

Now, since the RHS is always positive a necessary but not sufficient condition to have h better off in the star network is that the LHS is positive. This is true if and only if

$$\rho^{P-1} - \rho' \geq \Rightarrow \rho' \leq \rho^{P-1} \tag{23}$$

Let us keep this condition in mind and continue to the other possible deviation, namely the creation of a new link from the star network.

Pairwise Stability from a new link formation Now, suppose that we are in g^*, two non-hub PRIs u and v will **not** form a link if

$$\frac{1+\rho+(P-2)\rho'+\rho^2}{B(g^*+uv)} \leq \frac{1+\rho+(P-2)\rho'}{B(g^*)} \Rightarrow$$

$$\rho^2 B(g^*) \leq [1+\rho+(P-2)\rho'][B(g^*+uv)-B(g^*)]$$

Substituting the definition of $B(g^*)$ and $B(g^*+uv)$ we get

$$\rho^2\left[P+(P-1)\rho+(P-1)(P-2)\rho'+\sum_{r=1}^{P-1}\rho^r\right] \leq [1+\rho+(P-2)\rho'][2\rho^2+(P-2)\rho']$$

Putting +/- 1 in the term within brackets of the left hand side we have

$$\rho^2\left[P-1+(P-1)\rho+(P-1)(P-2)\rho'+\sum_{r=0}^{P-1}\rho^r\right] \leq [1+\rho+(P-2)\rho'][2\rho^2+(P-2)\rho']$$

allowing us to rearrange the terms in the following way

$$\rho^2(P-1)[1+\rho+(P-2)\rho']+\rho^2\sum_{r=0}^{P-1}\rho^r \leq [1+\rho+(P-2)\rho'][2\rho^2+(P-2)\rho']$$

Now, let us rewrite the condition such that

$$[1+\rho+(P-2)\rho'][(P-2)\rho^2-(P-2)\rho']+\rho^2\sum_{r=0}^{P-1}\rho^r-\rho^2[1+\rho+(P-2)\rho'] \leq 0$$

hence

$$[1+\rho+(P-2)\rho'](P-2)(\rho^2-\rho')+\rho^2\left[\sum_{r=0}^{P-1}\rho^r-(1+\rho+(P-2)\rho')\right] \leq 0$$

Now, a necessary but not sufficient condition to satisfy the inequality is one of the terms to be negative. The first term is negative if and only if $\rho^2 \leq \rho'$ while the second is negative only when

$$\sum_{r=0}^{P-1}\rho^r-(1+\rho+(P-2)\rho') \leq 0 \Rightarrow \sum_{r=2}^{P-1}\rho^r \leq (P-2)\rho' \Rightarrow$$

$$\rho' \geq \frac{\sum_{r=2}^{P-1}\rho^r}{P-2} \tag{24}$$

Therefore, since $\rho^2 \geq \frac{\sum_{r=2}^{P-1}\rho^r}{P-2}$, the latter condition is less demanding to have at least one of the two terms with negative values.
Now if we merge Equations (23) and (24),

$$\rho^{P-1} \geq \rho' \geq \frac{\sum_{r=2}^{P-1}\rho^r}{P-2}$$

we can see that they are incompatible insofar as $(P-2)\rho^{P-1} \leq \sum_{r=2}^{P-1}\rho^r$. Therefore, since the two necessary conditions to have the PWS of a star network cannot coexist, the star network is never pairwise stable. **Q.E.D.**

E Proof of Lemma 5

Note first that, for any $0 \leq \psi' \leq \psi \leq 1$ and $0 \leq \sigma' \leq \sigma \leq 1$, there always exist values of $p \in [1, P]$ and $f \in [1, F]$ that satisfy the two conditions stated in Equation (14).

Concerning the first point, since $\hat{p}$ and $\hat{f}$ are the maximum number of links that firms and PRI want to form among each other, then the number of mixed links that all the firms and PRIs want to form is respectively $\hat{p} \cdot F$; and $\hat{f} \cdot P$. Therefore, the number of links actually formed in the network is given by the lowest number of mixed links "demanded" by each type of players.

The second part is a straightforward consequence of Equation (14). The strict inequality condition comes out from the fact that $\hat{p} \cdot F \neq \hat{f} \cdot P$ occurs either because$\hat{p} \cdot F < \hat{f} \cdot P$ or $\hat{p} \cdot F > \hat{f} \cdot P$, which implies that there is at least one player will not be able to create all the desired links (a PRI in the former case, a firm in the second).

On the other hand $\hat{p} \cdot F = \hat{f} \cdot P$ means that all the players are able to create all their desired links. Hence $\eta_i^P(\hat{g}) = \hat{p}$ and $\eta_u^F(\hat{g}) = \hat{f} \quad ; \forall i \in F$; hs $\forall u \in P$. **Q.E.D.**

F Proof of Proposition 7

Before proceeding with the proof let us keep in mind that, since $\hat{p}$ and $\hat{f}$ are integers, it is not possible to have

$$\sigma^{\hat{f}} > \sigma^{\hat{p}} \geq \sigma^{\hat{f}+1} > \sigma^{\hat{p}+1}$$

because it would mean that $\hat{p} \geq \hat{f} + 1 > \hat{p} + 1$.

Therefore, if $\hat{\mathbf{f}} > \hat{\mathbf{p}}$, then

$$\sigma^{\hat{f}} > \sigma^{\hat{f}+1} \geq \sigma^{\hat{p}} > \sigma^{\hat{p}+1} \tag{25}$$

while $\hat{\mathbf{f}} = \hat{\mathbf{p}}$ trivially implies that

$$\sigma^{\hat{f}} = \sigma^{\hat{p}} > \sigma^{\hat{f}+1} = \sigma^{\hat{p}+1} \tag{26}$$

Now, we can rewrite Condition 18 as a function of ψ (that is equal to σ)

$$\psi^{\hat{f}} \geq \sigma' \geq \psi^{\hat{f}+1}$$

keeping in mind Condition 17

$$\psi^{\hat{p}} \geq \frac{F-1}{F} \ \psi' \geq \psi^{\hat{p}+1}$$

Since $\sigma' \leq \frac{F-1}{F}\ \psi'$, it is straightforward to see that, when Condition 18 holds either

$$\psi^{\hat{f}} \geq \frac{F-1}{F}\ \psi' \geq \sigma' \geq \psi^{\hat{f}+1}$$

or

$$\frac{F-1}{F}\ \psi' > \psi^{\hat{f}} \geq \sigma' \geq \psi^{\hat{f}+1}$$

The latter case implies that $\hat{f} > \hat{p}$ because otherwise we cannot satisfy Condition 17 without violating Equation (25). In the former case $\hat{f} = \hat{p}$ because, according to Equation (26), Condition 17 would be otherwise violated. **Q.E.D.**

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