Are the log-returns of Italian open-end mutual funds normally distributed? A risk assessment perspective

by Michele Leonardo Bianchi
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ARE THE LOG-RETURNS OF ITALIAN OPEN-END MUTUAL FUNDS NORMALLY DISTRIBUTED? A RISK ASSESSMENT PERSPECTIVE

by Michele Leonardo Bianchi

Abstract

In this paper we conduct an empirical analysis of daily log-returns of Italian open-end mutual funds and their respective benchmarks in the period from February 2007 to June 2013. First, we estimate the classical normal-based model on the log-returns of a large set of funds. Then we compare it with three models allowing for asymmetry and heavy tails. We empirically assess that both the value at risk and the average value at risk are model-dependent and we show that the difference between models should be taken into consideration in the evaluation of risk measures.

JEL Classification: C02, C46, G23.

Keywords: open-end mutual funds, normal distribution, tempered stable distributions, value at risk, average value at risk.

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1 Introduction

The Italian open-end mutual fund market has been analyzed in the literature from various perspectives and an overview that describes the Italian market can be found in Bianchi and Miele [2011]. Cesari and Panetta [2002] made the first comprehensive study of the performance of Italian equity funds and they showed that the funds’ net returns were not significantly different from zero and just enough to compensate for fund fees and risk. Their analysis was conducted on monthly data in the period from 1986 to 1995. Bianchi and Miele [2011], using a modified version of the capital asset pricing model (CAPM), estimated a performance measure for each fund management company in the period from 2003 to 2008, and showed that, for the funds they managed, these companies did not, on average, outperform the benchmarks chosen by the managers. In addition, as expected, the funds’ systematic risk was close to that of the benchmarks. Here, again, the empirical analysis was conducted on monthly data. A review of the factors that led to the growth of Italian investment funds, their history and their contribution to the development of the Italian stock market is found in Coltori [2010]. In addition, Mediobanca publishes a detailed annual statistical survey describing trends and developments in the Italian market (see Mediobanca, Ufficio Studi [2013]).

In this paper, we analyze fund data from a different perspective. We assess the extent to which asymmetric and heavy-tailed distributions are needed to explain the behavior of fund log-returns and to properly evaluate some well-known risk measures. The risk measures we use in this study are value at risk (VaR) and average value at risk (AVaR), the average of VaRs greater than the VaR for a given tail probability. AVaR, also called conditional value-at-risk (CVaR) or expected shortfall, is a superior risk measure to VaR because it satisfies all axioms of a coherent risk measure and it is consistent with preference relations of risk-averse investors (see Rachev et al. [2008]). Both measures have been adopted as standard risk measures in the financial industry and by regulators (European Securities and Markets Authority [2010], Basel Committee on Banking Supervision [May 2012], Banca d’Italia [2013]).

Risk management models require the proper modeling of the return distribution of financial assets. Most of the important models in finance rest on the assumption that randomness is explained through a normal random variable. However, there is ample empirical evidence against the normality assumption, since financial log-returns may be leptokurtic and skewed. Partly in response to those empirical inconsistencies relative to the properties of the normal distribution, a suitable alternative distribution is the family of tempered stable distributions (Rachev et al. [2011]). In this paper, we show that fund log-returns can be modeled by tempered stable (TS) distributions and demonstrate why these distributions should be considered in the evaluation of risk measures. We empirically assess that the normal

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distribution is not flexible enough to explain the dynamics of fund log-returns, whose skewness and fat-tail properties can be captured instead by TS distributions. These facts have an important bearing on the evaluation of widely used risk measures (VaR and AVaR). Similar studies have empirically investigated the performance of different heavy-tailed models in measuring VaR and AVaR in the presence of heavy tails in index and stock log-returns (e.g. Harmantzis et al. [2006], Marinelli et al. [2007], Misiorek and Weron [2012], and Isogai [2014]).

The aim of this paper is to: (1) analyze if fund log-returns deviate from normality; (2) empirically assess that both the VaR and AVaR are model-dependent; (3) compare the fund statistics with those of their benchmarks. To our knowledge this is the first study that analyzes the statistical properties of daily log-returns of Italian open-end funds.

The remainder of the paper is organized as follows. In Section 2 we describe the data used in the empirical study. Section 3 reviews the various tempered stable distributions considered in the empirical study and the ways to computing some widely used risk measures. The historically based estimation together with the main empirical results are discussed in Section 4. Section 5 summarizes the paper’s main conclusions.

2 The data

In our study we examine supervisory and statistical reports sent by management companies to the Bank of Italy for all Italian-law open-end funds. The dataset contains historical balance sheet information and portfolio and financial data (see the Banca d’Italia [2012]). It includes daily data on fund unit and benchmark values, dividend amounts and distribution dates, unit and benchmark conversion ratios (in the case of divisions or mergers) with the conversion dates. Apart from the standard classification (equity, flexible, balanced, bond and money market funds), the funds are divided according to the classification provided by Assogestioni (the Italian mutual fund association), which comprises 42 different categories.

We analyze all the funds (and their respective benchmarks) for which daily unit values and more than one thousand observations are available in the period from February 1, 2007 to June 30, 2013. The 573 funds analyzed (177 equity, 152 flexible, 57 balanced, 175 bond and 12 money market funds) have a median of 1,584 observations. In 389 cases the benchmark is reported (flexible funds may not have a benchmark) and has more than one thousand observations (a median of 1,519). Note that benchmark data are collected only starting from February 1, 2007, and this is the reason why we do not select a wider time window.

Fund and benchmark log-returns are computed by considering daily unit values as well as conversion ratios, in case of divisions or mergers. In the computation of fund log-returns we also consider dividend amounts. Further, in order to compare

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2 In Italy open-end mutual funds are established and managed by asset management companies supervised by the Bank of Italy in cooperation with the Companies and Stock Exchange Commission. In particular, the Bank of Italy receives supervisory and statistical reports regarding all the funds established by Italian management companies (for more detailed information on the characteristics of these data see Banca d’Italia [2012]).
funds with their respective benchmarks, we adjust the benchmark log-returns to take into account the tax treatment of Italian funds until July 1, 2011 (no adjustments are needed after that date).

It should be noted that while in the guidelines of the European Securities and Markets Authority [2010] and the regulations of the Bank of Italy (Banca d’Italia [2013]) the risk of the funds is measured by the VaR of the funds’ portfolios, the present analysis is conducted on the VaR of the log-returns of the fund units. When fund leverage is low, it is possible to assume that these two VaRs are close each other. This is the case for the 573 funds analyzed here: the median ratio of total fund assets to net asset value (leverage) is always lower than 1.02. We do not conduct an analysis at fund portfolio level for three main reasons: (1) we do not have the fund portfolio composition on a daily basis; (2) it may be difficult to find the data of each fund component; (3) to compute the VaR and the AVaR of a portfolio, it is necessary to define a possible non-normal dependence structure and we do not investigate these issues in this paper.

3 The theoretical framework

A formal elegant definition of tempered stable (TS) distributions and processes was proposed in the work of Rosiński [2007]; subsequently, Bianchi et al. [2010b] introduced the class of tempered infinitely divisible (TID) distributions. In this paper we analyze three different parametric examples belonging to these classes: the classical tempered stable (CTS) distribution; the normal tempered stable (NTS) distribution, which can be seen as a normal distribution with tempered stable stochastic time (see Rachev et al. [2011] and Kim et al. [2012]); and the rapidly decreasing tempered stable (RDTS) distribution.

Recall that law \(X\) is said to have a normal distribution with parameters \((\mu, \sigma)\) if the characteristic function of \(X\) is given by

\[
\phi_X(u) = E[\exp(iuX)] = \exp(iu\mu - \frac{1}{2}\sigma^2 u^2),
\]

where \(i\) is the imaginary unit. For each fund, we assume that all the probability laws considered (normal, CTS, NTS, and RDTS) have the same mean \(\mu\) and the same standard deviation \(\sigma\). These statistics correspond to the parameters estimated by fitting the normal distribution to the fund log-returns. In practice this means that only moments greater than two may differ between the distributions considered.

Let \(\alpha, \lambda_+,\) and \(\lambda_-\) be positive constants, and \(\alpha \in (0, 2)\backslash\{1\}\) (in all TS cases, we do not discuss the case \(\alpha = 1\)). Law \(X\) is said to have a CTS distribution with parameters \((\alpha, \lambda_+, \lambda_-, \mu, \sigma)\) if the characteristic function of \(X\) is given by

\[
\phi_X(u) = \exp \left( iu (\mu - CT(1 - \alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1})) 
+ CT(-\alpha)((\lambda_+ - iu)^\alpha - \lambda_+^{\alpha} + (\lambda_- + iu)^\alpha - \lambda_-^{\alpha}) \right)
\]

with

\[
C = \sigma^2 \left( \Gamma(2 - \alpha)(\lambda_+^{a-2} + \lambda_-^{a-2}) \right)^{-1} \quad \text{and} \quad \lambda_\pm = \lambda_\pm / \sigma.
\]
The parameters $\alpha$, $\lambda_+$ and $\lambda_-$ are related to tail weights: $\alpha$ is the main tail parameter, and $\lambda_+$ and $\lambda_-$ control the rate of decay on the positive and negative tails. Additionally, $\lambda_+$ and $\lambda_-$ are also skewness parameters. If $\lambda_+ > \lambda_- (\lambda_+ < \lambda_-)$, then the distribution is skewed to the left (right), and if $\lambda_+ = \lambda_-$, then it is symmetric. Then, $\mu$ and $\sigma$ are location and scale parameter, respectively.

Let $\alpha$, $\theta$ be positive constants, $\beta \in \mathbb{R}$ and $\alpha \in (0,2) \setminus \{1\}$. Law $X$ is said to have a NTS distribution with parameters $(\alpha, \theta, \beta, \mu, \sigma)$ if the characteristic function of $X$ is given by

$$
\phi_X(u) = \exp \left( iu(\mu - \beta') - \frac{2\theta^{1-\alpha/2}}{\alpha} \left( \left( \theta - i(u\beta' + i\gamma^2 u^2) \right)^{\alpha/2} - \theta^{\alpha/2} \right) \right) \quad (3.3)
$$

with

$$
\gamma = \left( \sigma^2 - \beta'^2 \frac{2 - \alpha}{2\theta} \right)^{1/2} \quad \text{and} \quad \beta' = \beta \sigma.
$$

The parameter $\alpha$ controls the tail behavior of the distribution and the parameter $\beta$ is related to the distribution’s skewness. If $\beta < 0$ ($\beta > 0$), then the distribution is skewed to the left (right). Moreover, if ($\beta = 0$), then it is symmetric. Then, $\theta$ and $\sigma$ are scale parameters, and $\mu$ is a location parameter.

Let $\alpha$, $\lambda_+$, $\lambda_-$ be positive constants, and $\alpha \in (0,2) \setminus \{1\}$. Law $X$ is said to have a RDTS distribution with parameters $(\alpha, \lambda_+, \lambda_-, \mu, \sigma)$ if the characteristic function of $X$ is given by

$$
\phi_X(u) = \exp(iu\mu + CG(iu; \alpha, \lambda_+) + CG(-iu; \alpha, \lambda_-)) \quad (3.4)
$$

with

$$
G(x; \alpha, \lambda) = 2^{-\alpha/2-1} x^{\alpha} \left( \Gamma \left( -\frac{\alpha}{2} \right) M \left( -\frac{\alpha}{2}, 1; \frac{\sqrt{2}x}{\lambda} \right)^2 \right. \\
+ \frac{\sqrt{2}x}{\lambda} \Gamma \left( \frac{1 - \alpha}{2} \right) M \left( 1 \frac{\alpha}{2}, 3 \frac{\alpha}{2}; \frac{\sqrt{2}x}{2\lambda} \right)^2 \\
\left. - \frac{\sqrt{2}x}{\lambda} \Gamma \left( \frac{1}{2} - \frac{\alpha}{2} \right) - \Gamma \left( -\frac{\alpha}{2} \right) \right), \quad (3.5)
$$

$$
C = 2^{\alpha/2} \sigma^2 \left( \Gamma(1 - \alpha/2)(\lambda_+^{\alpha-2} + \lambda_-^{\alpha-2}) \right)^{-1}, \quad \lambda_{\pm} = \lambda_+/\sigma.
$$

and where $M(a, c; z)$ is the Kummer’s or confluent hypergeometric function of the first kind as defined in equation (13.1.2) in Abramowitz and Stegun [1974]. An efficient algorithm to compute the characteristic function in equation (3.4) can be constructed (see Gil et al. [2007], and Bianchi et al. [2010a]). The role of the parameters are the same as for the CTS distribution. The tails of a RDTS distribution are lighter than those of a CTS distribution.

Then, by taking into consideration the works of Stoyanov and Racheva-Iotova [2004] and Scherer et al. [2012] it is possible to implement a maximum likelihood estimation method based on the fast Fourier transform (see Bianchi et al. [2013]
Figure 1: Normal, CTS, NTS and RDTS estimation results of a randomly selected fund. On the left, the probability density functions are reported. On the right, a detail of the cumulative distribution functions around the left tail (1% level) is shown. The abscissa (changed in sign) of the point of intersection between the dashed line and the cumulative distribution function is the 1% VaR.

for further details on the algorithm) and to compute the value at risk (VaR) at significant level $\delta$, that is

$$VaR_\delta(X) = -\inf\{x| P(X \leq x) > \delta\} = -F^{-1}_X(\delta)$$

by inverting the cumulative distribution function $F_X$. From Kim et al. [2010] and Kim et al. [2011] it is possible to obtain a closed formula (up to an integration) to compute the average value at risk (AVaR). Recall that the AVaR of a continuous random variable $X$ with finite mean (i.e. $E[X]<\infty$) at tail probability $\delta$ is defined as the average of the VaRs that are greater than the VaR at tail probability $\delta$, that is

$$AVaR_\delta(X) = \frac{1}{\delta} \int_0^\delta VaR_p(X) dp = -E[X | X < -VaR_\delta(X)].$$

Therefore, by construction, AVaR is focused on the losses in the tail that are greater than the corresponding VaR level. We refer to $VaR_{0.01}(X)$ and $AVaR_{0.01}(X)$ as 1% VaR and 1% AVaR, respectively. Note that in all the cases considered, in order to obtain annualized parameters it is necessary to rescale the parameters by a proper function of $\Delta t$ (since we analyze daily data, we have $\Delta t = 1/250$).

4 The empirical study

4.1 Model fit

We conduct the empirical study on the 573 funds described in Section 2. For each fund, the four models (normal, CTS, NTS, and RDTS) are estimated by considering
Figure 2: Normal, CTS, NTS, and RDTS estimation results (above) and percentage differences with respect to the normal model (below): (a) log-likelihood, (b) Kolmogorov-Smirnov distance, (c) 1% VaR and, (d) 1% AVaR. On each boxplot, the central mark is the median, the edges of the box are the 25-th and 75-th percentiles, the whiskers extend to the most extreme data points not considered outliers.

the time series of log-returns. In order to have a visual assessment of the estimation, in Figure 1 we report both the probability density and the cumulative distribution functions of the models analyzed for a randomly selected fund. Examining the probability density function on the left side of the figure, we can assert that the normal model cannot capture the statistical properties of the observed returns, while all TS models do. Then, the probability density functions of the three TS models seem to differ only slightly. On the right side of the figure the behavior of the cumulative distribution functions around the left tail is shown. For each model, by definition, the abscissa (changed in sign) of the point of intersection between the dashed line and the cumulative distribution function is the 1% VaR. Clearly there are differences in the measurement of VaR among the four models, at least for the fund on which the figure is based.

As observed in Section 3, we estimate the parameters of the models using the classical maximum likelihood estimation (MLE) procedure. Besides estimating the parameters, we consider for each model the maximized log-likelihood and the Kolmogorov-Smirnov (KS) distance to identify the superior model. The KS distance compares the empirical cumulative distribution function with that of the reference distribution (see Rachev et al. [2011]). In Figure 2 the boxplots of the log-likelihood and of the KS statistic for each model are shown. A larger (smaller) likelihood (KS statistic) indicates a better fit. The percentage differences\(^3\) with respect to the normal

\[^{3}\text{The percentage difference } \%\Delta \text{ of } b \text{ with respect to } a \text{ is defined as}
\]

\[
\%\Delta = 1 - \frac{b}{a}.
\]
Table 1: Normal, CTS, NTS, and RDTS estimation results. For each fund category, we report the median Kolmogorov-Smirnov distance, the number of funds in each category (n.funds), and the median number of observations (n.obs).

<table>
<thead>
<tr>
<th>Category</th>
<th>Normal</th>
<th>CTS</th>
<th>NTS</th>
<th>RDTS</th>
<th>n.funds</th>
<th>n.obs</th>
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<tbody>
<tr>
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<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>177</td>
<td>1,584</td>
</tr>
<tr>
<td>balanced</td>
<td>0.07</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>57</td>
<td>1,583</td>
</tr>
<tr>
<td>bond</td>
<td>0.10</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>175</td>
<td>1,585</td>
</tr>
<tr>
<td>flexible</td>
<td>0.10</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
<td>152</td>
<td>1,562</td>
</tr>
<tr>
<td>money market</td>
<td>0.22</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
<td>12</td>
<td>1,604</td>
</tr>
<tr>
<td>all funds</td>
<td>0.08</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>573</td>
<td>1,584</td>
</tr>
</tbody>
</table>

The log-likelihood of the normal model is 3 per cent lower than those of the three TS models (median values). In Table 1 for each fund category the median values of the KS distance are reported, together with the number of funds and the median number of observations. It is interesting to note that even for bond funds the TS models show a better performance than the normal model, probably because observed returns are leptokurtic and the normal distribution is not able to explain this shape. For money market funds the estimation is very poor, and the models are all rejected (median values). These empirical finding is probably due to the fact that 12 money market funds analyzed generally have a very low volatility. According to the KS statistic, the CTS model is the best because its median KS statistic is lower than those of all competitor models, and the normal model is the worst one. The CTS model performs particularly well in the case equity funds, even if, as already observed, it is not able to explain the dynamics of money market funds. As shown in Table 2, by looking at the KS distance, the percentage differences between the normal and the TS models are large (75.3, 56.8 and 41.4 per cent in the CTS, the NTS and the RDTS case, respectively) and these differences vary among different fund categories.

Figures 3 reports the behavior of the estimated (annualized) parameters across all analyzed funds. Additionally, the median of the historical average returns ($\mu$) is, 2.9 per cent for bond funds, 1.6 per cent for money market funds, 0.8 per cent for balanced funds, 0.6 per cent for flexible funds, and slightly negative for equity funds (-0.3 per cent). As expected, while the median of the standard deviation ($\sigma$) is large for equity funds (19 per cent), it is small for all other categories (7.2, 5.4, 3.3 and 0.9 per cent for balanced, flexible, bond and money market funds, respectively). Note that the median difference between $\lambda_+$ and $\lambda_-$ is around 1.8 in the CTS case (slightly more than 1.3 in the RDTS case) and the median $\beta$ is $-0.015$ in the NTS case. This means that, in all TS cases, the estimated distribution is asymmetric and the median skew is negative. This property seems to be one of the factor that decreases the estimation error in terms of likelihood function value and KS distance with respect to the normal-based model.
4.2 Risk measures

We estimate the 1% VaR and 1% AVaR for each fund. Then, we compute the percentage difference of these risk measures between the normal and the TS models. As shown in Table 2, the median percentage difference between the 1% VaR computed on the normal distribution and that computed on the CTS distribution is -26.9 (-19.8 and -16.5 in the NTS and RTDS case, respectively). The median difference between the 1% AVaR computed on the normal distribution and that computed on the CTS distribution is -52.5 per cent (-42.7 and -33.8 in the NTS and RTDS case, respectively). For the best-performing model in term of fitting error (i.e. the CTS model) and for all fund categories, both VaR and AVaR are greatest. This means that the normal distribution cannot explain large losses in the recent history of the funds. Recall that the years from 2007 to 2013 were a period of extreme market turmoil. By looking just at those years, the CTS model may overstate the importance of negative tail events. However, similar studies on heavy models have shown that non-normal heavy-tailed distributions are preferable even in periods of calm. As observed by Rachev et al. [2011], the capital charge required for risks managers who use a normal model may not be enough to cover losses in a bear market. Even if non-normal models may be inefficient, in terms of capital charge during periods of calm they could significantly improve the stability of an investment in the case of a financial market collapse. Finally, as shown in Table 2 the percentage differences with respect to the normal model of both VaR and
Table 2: CTS, NTS, and RDTS estimation results for the 573 funds analyzed. For each fund category, the median percentage differences with respect to the normal model of the Kolmogorov-Smirnov distance (1% VaR, and 1% AVaR), the number of funds in each category (n.funds), and the median number of observations are reported (n.obs).

### KS

<table>
<thead>
<tr>
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### 1% VaR

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### 1% AVaR

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<td>-76.20</td>
<td>12</td>
<td>1,604</td>
</tr>
<tr>
<td>all funds</td>
<td>-52.47</td>
<td>-42.74</td>
<td>-33.82</td>
<td>573</td>
<td>1,584</td>
</tr>
</tbody>
</table>

AVaR vary only slightly among fund categories (except for money market funds).

### 4.3 Benchmark analysis

For each model, when possible, we estimate the parameters of the four models on the benchmark log-returns and then compute the KS distance and the percentage difference of the analyzed risk measures between the fund and its benchmark. We have a total of 389 benchmarks. The number of benchmark observations is smaller than those of the funds. The analysis conducted on the benchmark log-returns shows that the CTS model is the best performing model in terms of median KS distance. For the benchmarks of money market funds the estimation is very poor, and the models considered are all rejected (median values). The percentage difference between the 1% VaR of the fund and that of its benchmark depends only slightly on the distributional assumption and its median across all fund categories is slightly more than 2 per cent. This is not the case for the 1% AVaR, since the median percentage difference across all fund categories differs between models. As shown in Table 3, the median percentage difference between the 1% AVaR of the fund and that of its benchmark is 2.3 per cent in the normal case, 1 per cent in the CTS case, 0.2 per cent in the NTS case, and -0.6 per cent in the RDTS. In both the VaR and AVaR cases, these differences vary slightly among different fund categories and models (for money market funds the differences are larger). The percentage difference between the 1% VaR (1% AVaR) of the fund and that of its benchmark is generally positive for equity, bond and flexible funds, and negative for balanced...
Table 3: Normal, CTS, NTS, and RDTS estimation results for the 389 benchmark analyzed. For each fund category, the median Kolmogorov-Smirnov distance and the median percentage differences between the fund and the benchmark 1% VaR (and 1% AVaR), the number of benchmarks in each fund category (n.benchmark), and the median percentage differences between the number of observations of the fund and its respective benchmark are reported (n.diff.obs).

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>CTS</th>
<th>NTS</th>
<th>RDTS</th>
<th>n.benchmark</th>
<th>diff.obs(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KS</td>
<td>equity</td>
<td>0.07</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>balanced</td>
<td>0.08</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>bond</td>
<td>0.08</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>flexible</td>
<td>0.08</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>money market</td>
<td>0.27</td>
<td>0.07</td>
<td>0.12</td>
<td>0.11</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>all funds</td>
<td>0.08</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>389</td>
</tr>
<tr>
<td>1% VaR</td>
<td>equity</td>
<td>2.93</td>
<td>2.09</td>
<td>2.22</td>
<td>2.13</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>balanced</td>
<td>-1.62</td>
<td>-1.79</td>
<td>-2.41</td>
<td>-4.44</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>bond</td>
<td>1.86</td>
<td>4.78</td>
<td>5.27</td>
<td>6.27</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>flexible</td>
<td>3.26</td>
<td>2.61</td>
<td>0.69</td>
<td>-0.20</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>money market</td>
<td>-3.44</td>
<td>38.22</td>
<td>15.81</td>
<td>9.51</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>all funds</td>
<td>2.31</td>
<td>2.51</td>
<td>2.30</td>
<td>2.33</td>
<td>389</td>
</tr>
<tr>
<td>1% AVaR</td>
<td>equity</td>
<td>2.90</td>
<td>1.47</td>
<td>1.49</td>
<td>0.64</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>balanced</td>
<td>-1.71</td>
<td>-5.99</td>
<td>-7.59</td>
<td>-10.59</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>bond</td>
<td>1.75</td>
<td>3.69</td>
<td>2.04</td>
<td>4.26</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>flexible</td>
<td>3.32</td>
<td>1.10</td>
<td>-2.41</td>
<td>-13.34</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>money market</td>
<td>-3.95</td>
<td>22.69</td>
<td>-4.85</td>
<td>-22.46</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>all funds</td>
<td>2.33</td>
<td>0.97</td>
<td>0.21</td>
<td>-0.60</td>
<td>389</td>
</tr>
</tbody>
</table>

funds. These empirical findings show that the risk of the funds considered in this study is slightly higher than the risk of their benchmark (median values) and, as expected, only partially depends on the model selected.

Note that the median percentage difference between the standard deviation of the fund and its benchmark is around 2 per cent. Additionally, we analyze the behavior of the risk measures considered across each fund management company and we observe that the median percentage differences between the risk measures of the fund and those of its benchmark are negative, for some fund management companies, even if the median values computed across all funds are positive. In practice this means that for some companies, the risk of the funds managed is slightly lower than that of their respective benchmarks. This result may depend on the types of fund managed by these companies.

5 Conclusion

We have analyzed the distributional properties of Italian funds’ daily log-returns from a historical time-series perspective by considering continuous-time models allowing for asymmetry and heavy tails. Our first finding is that the models based on the normality assumption do not provide a reliable explanation of the historical distribution of returns. In particular, the empirical evidence indicates that tempered stable models have greater explanatory power in fitting daily log-returns.
compared with standard models based on the normal distribution assumption. For the funds that we analyze and for the time period that we investigate, the CTS model seems to be more satisfactory in daily log-returns analysis, compared with the normal and all other TS models, since it explains the fund’s price historical dynamics better than its competitors.

As far as risk measurement is concerned, our findings indicate that the differences in risk between the funds analyzed and their benchmark are not remarkable. The median risk of funds is slightly higher than that of their respective benchmarks, although for some management companies the opposite is true.

The main finding of the paper is that, in computing funds’ risk measures, we observed that both VaR and AVaR strictly depend on the distributional assumption of the model, in most cases with large differences between models. Since we showed that the skewness and fat-tail properties of fund daily log-returns are important to explain fund price historical dynamics and risk measures are model-dependent, these empirical findings should be taken into consideration for a proper risk assessment. Disregarding these stylized facts can result in models that may incorrectly estimate (in most cases, underestimate) the tail risk of funds.
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