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a general equilibrium analysis of market entry decisions

by Sara Formai

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HETEROGENEOUS FIRMS AND CREDIT FRICTIONS: A GENERAL EQUILIBRIUM ANALYSIS OF MARKET ENTRY DECISIONS

by Sara Formai*

Abstract

This paper develops a general equilibrium model of international trade with heterogeneous firms and imperfect credit markets. To finance the costs for product innovation and domestic and foreign market entry, firms must raise external capital. The model underscores the importance of considering a general equilibrium setting in order to characterize fully the misallocations of resources that stem from the existence of credit frictions. These have important implications for firms' entry decisions in the different markets and for the welfare effects of imperfect financial institutions. Allowing for liquidity-constrained firms and imperfect credit markets alters, and in some cases reverses, some of the main results from the literature on heterogeneous firms. In particular, the model predicts that trade liberalization does not necessarily lead to an increase in average productivity and consumers' welfare.

JEL Classification: F12, F36, G20.

Keywords: consumer welfare, credit frictions, heterogeneous firms, market entry, trade liberalization.

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1 Introduction ¹

A growing body of work studies how inefficient institutions create distortions in the allocation of resources across domestic and exporting firms and how these distortions affect trade patterns and gains. There has been particular interest in the linkages between financial institutions, firms' production decisions and international trade. The rapid and sharp decline in international trade that followed the 2007-2009 global financial crisis has shifted the analysis towards of the short-run response of the economic activity to credit market shocks.

This paper takes a step back, and looks at the steady-state effects of financial frictions in a setting where firms' domestic and foreign market entry decisions combine to shape the long-run allocation of resources within an open economy. It describes how firms' choices, average productivity and the number of producers interact in general equilibrium and how via these channels credit frictions affect consumers' welfare and the role of played by trade costs. The focus is thus on the role that financial frictions can have on firms' long-run entry opportunities and on the effects of worsening credit market conditions that go beyond the short-run responses to a negative shock in credit supply.

Establishing a business entails substantial ex-ante sunk costs in both the initial stage of product development and in the later stages of domestic and foreign markets' entry. It is natural to think that the ability of a financial system to provide firms with these innovation and entry investments may play an important role in determining the number and type of firms active on the markets and, ultimately, consumer welfare. One key result of my analysis is that financial frictions create rents that divert resources away from innovation activities, limit the access of firms to credit and constrain entry decisions. Moreover, I find that since they are bigger in term of total sales and profits, exporting firms can have an advantage in terms of access to credit and this shifts resources from innovation to foreign market entry. I thus show that credit frictions reduce the number of entrants and the competition in the market, and more low-productivity firms can survive. The market can thus be characterized by a low number of big and inefficient firms and this has negative effects on welfare. I also show that credit frictions interact with trade costs in such a way that trade liberalization does not necessarily lead to higher average productivity and higher individual welfare.

Formally, I introduce credit market frictions in a heterogeneous firm model in the spirit of Melitz (2003). The model features two symmetric economies where monopolistically competitive firms differ in their productivity levels. Before knowing their productivity, liquidity-constrained firms must raise capital to pay for the sunk costs needed for innovation and market entry, both in the domestic and in the foreign market. Capital is provided by a competitive credit market where lenders face imperfect protections. In other words, I assume that there is a probability $1 - \lambda > 0$ that the borrower can avoid paying the per-period debt reimbursement without

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incurring any sanction. First, I solve for the optimal debit contract that maximizes the firm's expected value from variety introduction and satisfies the incentive compatibility constraint of the firm and the participation constraint of the lender. Then, I solve for the aggregate variables that define the general equilibrium characterized by an infinite mass of potential entrants. In particular, I solve for the minimum productivity levels that grant access to the domestic and foreign market, for the number of active firms in the economy and for individual welfare.

In general equilibrium, when both domestic firms and exporters face imperfect credit markets and the number of firms is endogenously derived from a free entry condition, credit frictions (inversely measured by the parameter λ) imply two main forms of resource misallocation. First, because of the possibility of opportunistic behavior, firms must be granted resources that would be otherwise allocated to innovation and entry investments. It follows that the creditor's pledgeable profits are only a fraction λ of the return of the investment in fixed costs. As a result, the total credit awarded in the economy is suboptimal, and so is the total number of entrants. This reduces competition among incumbents allowing low-efficiency firms to survive. Credit frictions thus imply a lower average productivity. A smaller number of firms and a lower average productivity have a negative effects on individual utility: consumer welfare decreases as credit frictions increase. Second, the fact that domestic profits can be de facto used to back up the loan to pay for foreign market entry makes exporters less likely to be credit constrained, at least for not overly low levels of financial development.

A second important set of results that stems from the general equilibrium properties of the model concerns the effects of decreasing trade costs, modeled as iceberg shipping costs to the foreign market. When these costs fall, export activities become more profitable, increasing the number of exporting firms. On the other hand, the increased demand for limited resources lowers the overall number of domestic firms that can be sustained in equilibrium. Depending on which effect dominates, trade liberalization can either increase or decrease average productivity and consumer welfare. In particular, I show that the lower the substitutability among varieties and the more firms are concentrated in the lower tail of the productivity distribution, the more likely it is that a reduction in trade costs will lead to a decrease in the total number of varieties available and lower competition among firms. It follows that the minimum and average productivity of domestic firms decrease, and so does the welfare of consumers that have a smaller number of more expensive varieties. This result should not be surprising given the general principle that reductions in the extent of one distortion do not necessarily lead to an increase in welfare when there are other distortions in the economy (see Bhagwati(1969)). In my paper, the distortions introduced by trade and credit frictions are such that reducing one of the two increases the effects the other one introduces into the system.

The main contributions of this paper are to the recent but fast growing literature on financial institutions and trade. First of all, it provides this literature with a tractable full-fledged general equilibrium model. This allows taking into account interactions among variables that have important implications for firms' selection and consumers' welfare. For instance, Manova (2008) develops a model with credit-constrained heterogeneous firms and imperfect financial

institutions. The model presents several attractive features: many countries with different levels of financial development and many sectors with different levels of financial needs and collateralizable assets. It rationalizes in a parsimonious way some empirical findings over a panel of country and industry-level data. Using credit to the private sector over GDP as a measure for financial development, the author finds that once controlled for sector- and country-fixed effects, financially developed countries export relatively higher volumes in sectors that require more external funding and in sectors with fewer collateralizable assets. The model presented in Manova (2008) is solved in a partial equilibrium setting and credit frictions by assumption affect only foreign market investments.² In this setting, credit constraints reinforce the way in which firms with higher productivity select into exporting, but there is no link between domestic and foreign market entry decisions. It follows that the distortions on the number of firms and on average productivity are not taken into account.

Two general equilibrium trade models with heterogeneous firms and liquidity constraints to entry decisions are developed in Chaney (2005) and Muûls (2008). In the first, developed in order to explain the lack of sensitivity of exports to exchange rate fluctuations, only firms with sufficient liquidity, either as an endowment or as profits from domestic activities, are able to export. As in the present setting, this allows for the possibility that, thanks to domestic market profits, foreign market entry is not necessarily constrained by the lack of liquidity. On the other hand, the number of firms is exogenously given. Muûls (2008) also presents a general equilibrium model but with a specification of financial constraints similar to Manova (2008). As in Chaney (2005), the number of potential entrants is fixed and this does not allow the mechanism linking firms' competition and firms' selection that is key in my analysis to be taken into account. Other interesting general equilibrium contributions are those by Suwantaradon (2008) and Wang (2011). These papers consider a dynamic framework in which some firms can accumulate liquidity over time and partially overcome the credit constraints. The first of the two again takes as given the mass of entrants while the second restricts the effects of imperfect financial markets on exporting activities only. Moreover, none of these papers studies the interaction between trade costs and financial frictions.

Another important difference between this paper and the majority of the contributions in the literature on credit constraint and trade is that my analysis focuses on the role of financial institutions, meant as an exogenous characteristic of the environment in which all firms operate. Many of the papers in this literature, including those listed above, focus more on the role of idiosyncratic firms' characteristics in shaping credit constraints, such as firms' own liquidity (Chaney (2005) and Muûls (2008)), net worth (Wang (2011) and Suwantaradon (2008)) and tangibility of the collaterals (Manova (2008) and Muûls (2008)). These choices are motivated by

²This approach is justified by the idea that exporting activities entail a higher degree of risk and uncertainty that causes the financing to be more difficult than for domestic activities. My setup could be easily extended to account for exogenously higher frictions to fund foreign market entry. Everything else equal, this would reduce the probability of becoming an exporter. On the other hand, the result that domestic profits can reduce the liquidity constraints faced by exporter would not disappear and this is an interesting general equilibrium effect that acts in favor of the number of exporting firms and, once more, against the resources available for a higher number of active firms.

the increasing availability of firm-level data on both exporting status and financial health and the evidence that the two tend to be positively correlated (see Manova (2008), Muûls (2008), Wang (2011) and Suwantaradon (2008), Berman and Héricourt (2010), Bellone et al. (2010)). In this respect, my work can be considered as being complementary and as opening up a new avenue of empirical investigation.

This paper also makes an important contribution to the more mature literature on trade and heterogeneous firms first introduced by Melitz (2003). My analysis shows that the conclusion that trade liberalization increases average productivity and consumer welfare can be reversed when other frictions, besides trade frictions, are introduced in the model. In Melitz (2003), and in many of the contributions that followed (for instance Bernard et al. (2007)), trade liberalization has three main effects: it increases the number of foreign varieties, it increases average productivity and it reduces the number of domestic firms. The first two effects increase consumer welfare and always dominate.³In my analysis, the increase in the number of exported varieties and the fall in the number of domestic varieties can be such that the second dominates and the total number of varieties decreases allowing for less productive firms to enter the domestic market. When this happens consumer welfare drops. When trade costs fall, the profits from exports increase so the minimum productivity level required for foreign market entry goes down. In Melitz (2003), this unambiguously increases the expected profits from exports and decreases the expected profits from domestic sales. This result is achieved by pushing up the minimum productivity required for domestic market entry. In the present setting, this mechanism can be totally reversed. This happens because, from the lender's point of view, the expected net return from foreign market entry can be negative if covered by positive expected net profits on the domestic market. When this is the case and the minimum productivity for foreign market entry goes down, the lender's expected profits from financing export activities can decrease, requiring the expected profits from domestic activities to increase. This happens if the minimum productivity level for domestic market entry goes down.

The rest of the paper is organized as follows. Section 2 describes the general setting of the model. Section 3 introduces credit frictions, describes the optimal contract between the firm and the creditor and solves the model for both a closed and an open economy. Section 4 presents the equilibrium properties and the main results of the paper. Section 5 concludes.

2 Set Up of the Model

The economy consists of two symmetric countries, Home and Foreign. The only factor of production is labor, and the population is of size L . The wage rate w is the same in both economies by symmetry and we set $w = 1$, treating labor as the numeraire. There is a single consumption-

³This prediction has been already questioned (for instance by Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010) and Formai (2010)). They show that by introducing an engine of growth into the Melitz (2003) model, trade liberalization can permanently delay productivity growth in the short run and make consumers worse off in the long run. In other words, they add a fourth effect from lower trade costs that also reduces the number of varieties in steady states and can offset the positive effects driving Melitz's result.

good sector that produces a continuum of differentiated varieties. Each firm operating in this sector supplies one of these goods under Dixit-Stiglitz monopolistic competition. Production is characterized by increasing returns to scale. To produce, a firm must first incur a sunk innovation investment. Only after having incurred this fixed cost, the firm receives a patent to exclusively produce the new variety and learns its productivity. The productivity parameter is drawn from a probability distribution, so firms have different variable production costs. After having learned its productivity, each firm decides whether or not to incur the additional fixed costs needed to start selling its variety. The domestic and the foreign market requires separate entry costs. Firms need to draw a sufficiently high productivity to enter the domestic market and an even more favorable draw to enter the export market.

The main assumption in this paper is that firms face liquidity constraints and need to borrow in order to finance both the innovation and the market entry sunk costs. Credit markets do not provide perfect protection to lenders. I assume that in each period the firm can avoid the repayment specified by the debt contract. In this case, there is only a probability of less than one that a court will enforce the payment and punish the insolvent firm.

2.1 Demand

The representative consumer is endowed with one unit of labor and has a C.E.S. utility function given by

$$U = \left[\int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \quad (1)$$

where $q(\omega)$ is the consumer's quantity consumed of a variety ω , and Ω is the set of varieties domestically available with constant elasticity of substitution given by $\sigma \equiv 1/(1 - \rho) > 1$. The aggregate price index defined over prices of varieties $p(\omega)$ is given by

$$P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

Given the individual expenditure e , the individual demand function for each variety ω is given by:

$$q(\omega) = e \frac{p(\omega)^{-\sigma}}{P^{1-\sigma}}, \forall \omega \in \Omega. \quad (2)$$

2.2 Production

There is a continuum of firms, each producing a different variety ω . Firm technology is characterized by a cost function that exhibits constant marginal costs. Labour used in production is a linear function of output $Q : l = Q/\varphi$, where φ is the productivity level and l is the labor employed by the firm. As in Melitz (2003), firms have different productivity levels φ , which they draw from a known and common distribution right after variety innovation. All producers face a residual demand with constant elasticity σ , and thus choose the same profit maximizing

markup $\sigma/(\sigma - 1) = 1/\rho$. This yields to the pricing rule

$$p(\varphi) = \frac{1}{\rho\varphi}. \quad (3)$$

For a firm with productivity φ , the profit, gross of any fixed cost, is

$$\pi(\varphi) = p(\varphi)Q(\varphi) - l(\varphi) = r(\varphi)/\sigma$$

where $r(\varphi)$ is the firm's revenue and $Q(\varphi)$ is the demand for the firm's variety and is given by (2), where individual consumer expenditure e is replaced by the economy-wide consumer expenditure $E = eL$. Using the expressions for the demand and the price (3), I can rewrite

$$r(\varphi) = E (P\rho\varphi)^{\sigma-1} \quad (4)$$

$$\pi(\varphi) = \frac{E}{\sigma} (P\rho\varphi)^{\sigma-1}. \quad (5)$$

Notice that the ratio of any two firms' revenues and profits can be written as a function of their productivity only:

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \frac{\pi(\varphi_1)}{\pi(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2} \right)^{\sigma-1}. \quad (6)$$

2.3 Variety Introduction and Market Entry

In each economy there is an unbounded pool of identical prospective entrants into the differentiated good industry. In order to enter, each firm must first discover a new variety and obtain a patent to exclusively produce it. This requires an investment of F_E units of domestic labor. Only when this investment is sunk, the firm gets to know its productivity level φ . This is drawn from a common probability density function $g(\varphi)$ with support $(0, \infty)$ and corresponding cumulative distribution function $G(\varphi)$. Melitz (2003) worked with a general probability distribution of firm productivity φ , but the model becomes considerably more tractable analytically if a Pareto distribution is assumed. Moreover, the empirical literature on firm productivity distribution suggests that a Pareto distribution is a reasonable approximation.⁴ In what follows I thus assume that $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi} \right)^a$ for all $\varphi \geq \varphi_m$, where $\varphi_m > 0$ is the minimum productivity value and a is a shape parameter that indexes the dispersion of productivity draws. As a increases, the relative number of low-productivity firms increases, and the productivity distribution is more concentrated at these low productivity levels. In order for the average productivity to be finite, I must assume $a > \sigma - 1$.

Once productivity is known, firms decide whether to exit immediately or to start production for either the domestic market or for both the domestic and the foreign market. Entry in the

⁴Del Gatto, Mion and Ottaviano (2006) estimate the distribution of total factor productivity using firm-level data for a panel of 11 EU countries and 18 manufacturing sectors. They find that the Pareto distribution provides a very good fit for firm productivity across sectors and countries. Moreover, the assumption of Pareto distribution for productivity induces distribution of firm size that is also Pareto which fits firm-level as well (see Helpman et al. (2004), Eaton et al. (2011) and Axtell (2001)).

domestic market requires an up-front investment of F_D units of domestic labor and entry in the foreign market an additional investment of F_X units of domestic labor. These costs can be thought of as the investments needed to promote the new variety and customize it to suit the taste of domestic and foreign consumers respectively. Given these fixed costs, only if the productivity level φ is high enough, it is profitable to enter a market and produce for it. If the firm does produce, it then faces a constant probability δ in every period of a bad shock that would force it to exit.

The equilibrium will be characterized by a mass M of firms in the differentiated good sector and a probability density function $\mu(\varphi)$ of the productivity levels of the active firms defined over a subset of $(0, \infty)$.

3 Liquidity Constraints and Credit Market Frictions

I now introduce two crucial assumptions. First, I assume that potential entrepreneurs face liquidity constraints. This means that in order to incur any investment they need to borrow on the financial market. For the sake simplicity, I assume that entrepreneurs are ex-ante identical and that they all have zero initial wealth. As a result, before learning their productivity level, firms need to borrow the entire sum required for product development and variety introduction. I thus introduce a competitive credit market, where firms can borrow the sum needed to pay for the fixed costs F_E , F_D and F_X . To do this, each firm must sign a debt contract with a deep-pocket investor. At the end of any period of time, once revenues are realized and workers have been paid, the borrower is expected to pay the lender the sum established by the debt contract they had previously signed.

The second assumption is that the credit market is not perfect. I assume that parties can sign full contingent contracts but that there is imperfect lender protection. In other words, when the parties meet, the firm's productivity is still unknown but they can sign a contract establishing the future conduct, contingent on any possible realization of φ . Nevertheless, at the end of each period, the borrower can choose to hide profits and avoid the per period repayment due to the investor. If this happens, there is only a probability $\lambda \in (0, 1)$ of a court enforcing the payment and punishing the borrower with a fine that exhausts all its per period profits. In other words, λ measures the efficiency of the financial market. If $\lambda = 1$, then there are no frictions on the credit market, and the entrepreneur's liquidity constraints become irrelevant. In this case the model reduces to the standard Melitz (2003) setting, with the same steady state equilibrium properties.

There is another assumption that is standard in the definition of a private ownership equilibrium (see Mas-Colell et al. (1995)) and that is worth stating explicitly here. That is consumers ultimately own firms and their initial endowment constitutes, not only of labour, but also of a claim to firms' profits. As a result, their total wealth derives from wages and firms' profits, if any. Ownership shares are assumed to be equal across all consumers.

3.1 Timing

Consider the case of a firm that decides to invent a new variety of the differentiated good. This requires the up-front innovation cost $F_E w$ and, eventually, the further entry costs F_D and F_X for the domestic and foreign markets respectively. Since the firm is liquidity constrained it needs to find an investor. Lenders behave competitively in the sense that the loan, if any, makes zero profits. That is, I assume that several prospective lenders will compete to issue the loans to the firms. I also assume that the contract is renegation-proof, in the sense that no one can be forced to stay in the relationship if in any period his expected profits are negative (see Laffont and Martimort (2002)).⁵ In exchange for the amount F_E that the lender finances immediately, the debt contract sets a fixed ex ante transfer $K \geq 0$ from the lender to the borrower and a plan of action for each possible realization of the productivity φ . The plan of action establishes the entry rules and, in the event of entry, the per-period (incentive compatible) repayment $f(\varphi)$ due by the firm as a per-period debt reimbursement. The entry rules are given by $i(\varphi) \in \{0, 1\}$ and $i_x(\varphi) \in \{0, 1\}$ for the domestic and foreign markets respectively. If $i(\varphi) = 1$ then the firm enters the domestic market, if $i(\varphi) = 0$ then there is no entry. The same is true for foreign market entry rule: if $i_x(\varphi) = 1$ then the firm enters the foreign market, otherwise it stays out. As the contract has been signed, the firm incurs the cost F_E and discovers its productivity level: if its productivity is such that entry costs are not financed, the firm is forced to exit immediately, and the lender loses the initial investment F_E . If its entry is financed, in either one or both markets, the firm starts production and revenues are realized. Once workers have been paid the wage $w = 1$, the firm decides whether or not to pay the per-period debt reimbursements $f(\varphi)$. In the event of there being no repayment, with probability λ a court will enforce the reimbursement and impose a fee equal to the remaining firm's profits.⁶ At the beginning of the next period, the firm faces the exogenous shock that leads to exit with probability δ . If the firm survives, the game is repeated from the production stage. Figure 1 summarizes the timing of the events.

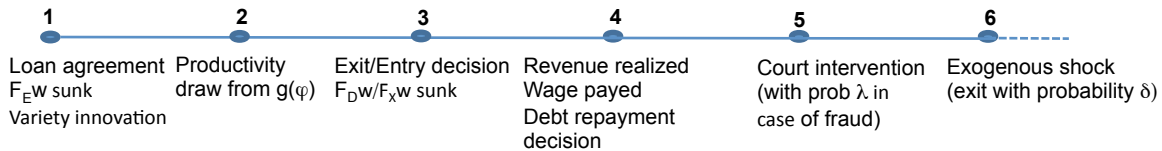


Figure 1: Timing

⁵Do not confuse renegation-proofness with renegotiation-proofness. The first concept, used in this paper, requires that no one can commit staying in the relationship if, at any stage, their individual continuation value is lower than their outside option. Renegotiation-proofness means imposing that the contract is such that the coalition of the firm and the creditor cannot improve the joint pay-off by renegotiating it.

⁶The fee can be interpreted as a reimbursement of the trial expenses and of the cost of the court's work that would otherwise have been borne by society.

3.2 The Optimal Debt Contract in a Closed Economy

Starting from the closed economy case, where the only entry decision concerns the domestic market and the fixed cost F_D , I will solve for an equilibrium characterized by unrestricted entry of profit maximizing firms and by perfect competition on the credit market. I split the problem into two steps. The first step is to derive the optimal debt contract that the firm will offer as a take-it or leave-it offer to the potential investor. The second step is to derive the general equilibrium conditions and to solve for a steady state equilibrium in which all the endogenous aggregate variables remain constant.

Each firm will choose the optimal contract by solving:

$$\max_{i(\varphi) \in \{0,1\}, f(\varphi), K} v_E \equiv \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi(\varphi) - f(\varphi)) \right] i(\varphi) g(\varphi) d\varphi + K \quad (\text{P})$$

subject to

$$K \geq 0, \quad (\text{LC})$$

$$\pi(\varphi) - f(\varphi) \geq (1-\lambda)\pi(\varphi) \text{ for all } \varphi \text{ such that } i(\varphi) = 1, \quad (\text{IC})$$

$$v_L \equiv \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) - F_D \right] i(\varphi) g(\varphi) d\varphi - F_E - K \geq 0, \quad (\text{PC})$$

$$\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) \geq F_D \text{ for all } \varphi \text{ such that } i(\varphi) = 1. \quad (\text{RP})$$

The objective function of the firm is given by the present discounted value of future profits net of the per-period repayment $f(\varphi)$, and in addition to the ex ante transfer K . Since upon entry each firm's productivity level φ does not change over time, this implies that the per-period profit will also remain constant as defined by (5). The discounting is given by the exogenous death rate δ . The profits are earned only when entry occurs, and this happens when $i(\varphi) = 1$. The requirement that K is non-negative derives from the liquidity constraint of the borrower (LC), which is a key assumption in the analysis. At stage 4 in Figure 1, φ is known, the firm has received the second loan F_D , and it has entered the market and started production. Once revenues are realized and wages paid, the firm is left with profits $\pi(\varphi)$. The firm will behave if the profits $\pi(\varphi)$, net of the per period repayment $f(\varphi)$, exceed the expected gain from avoiding the payment and risking, with probability λ , being fined by a court. This implies the incentive compatibility constraint (IC). The lender's participation constraint (PC) requires the expected value of the contract in period 1 v_L to be non-negative. This happens when the present discounted value of the per period repayments, net of the entry cost $F_D w$, exceeds the the initial investment F_E and the transfer K . Entry occurs according to the entry rule, when $i(\varphi) = 1$. The last constraint is the renegation-proof condition (RP). It requires the lender's expected payoff to be positive also at stage 3 in Figure 1, once the productivity is known and the initial investment F_E is sunk. The lender finds it profitable to invest the further amount F_D , providing this does not exceed the present discounted value of the flow of future incentive

compatible repayments.

The appendix spells out all the steps that allow me to solve for the optimal contract. First note that the firm can increase v_E by raising the term $T \equiv K - \int_{\varphi_m}^{\infty} [\sum_{t=0}^{\infty} (1 - \delta)^t f(\varphi)] i(\varphi) g(\varphi) d\varphi$ until the (PC) binds ($v_L = 0$). As a result in equilibrium it must hold that

$$K = \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1 - \delta)^t f(\varphi) \right] i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - F_E \geq 0. \quad (7)$$

It follows that $v_E = \int_{\varphi_m}^{\infty} \frac{\pi(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - F_E$, which does not depend on the way the transfer T is split between the ex-ante transfer K and the ex-post per-period repayments $f(\varphi)$. Accordingly, any pair $(K, f(\varphi))$ such that (7) holds together with the (IC) and (RP) requirements that $\delta F_D \leq f(\varphi) \leq \lambda \pi(\varphi)$ is a valid solution to (P). This implies that the optimal contract is not unique. Nevertheless, if for the generic firm with productivity φ (IC) and (RP) can be slack, this is not the case for the marginal firm entering the market. In order to meet the lender's (RP) condition, this firm has the incentive to pay a per-period repayment $f(\varphi)$ as high as the (IC) allows for. In other words, a binding (IC) and a binding (RP) solve for the entry rule $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*$, where φ^* represents the entry-productivity cut-off such that:

$$\lambda \pi(\varphi^*) = \delta F_D. \quad (8)$$

Any firm with productivity $\varphi \geq \varphi^*$ will obtain the financing needed to enter the market. If $\varphi < \varphi^*$, then the firm is forced to exit immediately. Without credit frictions ($\lambda = 1$), the above condition becomes $\pi(\varphi^*) = \delta F_D$, which is equivalent to the standard zero profit condition in Melitz (2003). When $\lambda < 1$, the profitability of the marginal firm is higher than the minimum productivity levels that would satisfy the efficiency requirement $\pi(\varphi) \geq \delta F_D$. The intuition is that higher profits are needed to give firms the right incentives to comply with the debt contract. To summarize:

Proposition 1. *Any optimal contract satisfies:*

I.

$$i(\varphi) = \begin{cases} 1 & \text{if } \varphi \geq \varphi^*, \\ 0 & \text{otherwise} \end{cases}$$

where φ^* is such that $\pi(\varphi^*) = \frac{\delta F_D}{\lambda}$;

II. $(K, f(\varphi))$ are such that

$$K = \int_{\varphi^*}^{\infty} \frac{f(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*)) F_D - F_E \geq 0 \quad (9)$$

and

$$\begin{cases} f(\varphi) = \lambda \pi(\varphi) & \text{if } \varphi = \varphi^*, \\ \delta F_D < f(\varphi) \leq \lambda \pi(\varphi) & \text{if } \varphi > \varphi^*. \end{cases}$$

Moreover

$$v_E = \int_{\varphi^*}^{\infty} \frac{\pi(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*)) F_D - F_E.$$

Proof: See Appendix \square

3.2.1 Equilibrium in a Closed Economy

In the frictionless set-up with unrestricted entry, the notion of equilibrium requires that v_E cannot be positive since the mass of prospective entrants is unbounded (see also Melitz (2003)). Since no firm would want to enter the market when $v_E < 0$, $v_E = 0$ represents the free entry condition. With liquidity constrained firms and credit frictions, things become more complicated. The equilibrium notion provided wants to capture the idea (much as in any other paper) that no firm left outside of the market should find entry strictly profitable ($v_E > 0$) and strictly feasible ($K > 0$) at the same time. Unlike from a frictionless set-up where profitability and feasibility would coincide, here I need to introduce the second condition that refers to the ability of firms to find a creditor. I will require that in equilibrium at least one of the above conditions fails. First of all $v_E < 0$ is ruled out as an equilibrium outcome. Now consider $v_E = 0$. Note that the maximum value which can be set for K is the one for $f(\varphi) = \lambda\pi(\varphi)$ for all φ such that $\varphi \geq \varphi^*$. I thus define $K_{max} \equiv \int_{\varphi^*}^{\infty} \frac{\lambda\pi(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*)) F_D - F_E < v_E$. It follows that $v_E = 0$ implies $K_{max} < 0$, meaning that even if the firm were to pledge as much future income as possible (credit markets are imperfect so the firm cannot pledge all future income), the creditors would still require an upfront payment to end up even. Given the liquidity constrain (9), $K \geq 0$ is required. By definition, also K_{max} must be non-negative and $v_E = 0$ is thus ruled out as an equilibrium outcome. I conclude that in equilibrium it must be that $v_E > 0$. It follows that the unbounded mass of prospective entrants must find entry strictly not feasible in equilibrium, i.e. the maximum possible K must be equal to zero. In my model the free entry condition (henceforth FEC) is thus given by $K_{max} = 0$ and can be simplified to obtain

$$\lambda\bar{\pi} = \delta \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right). \quad (10)$$

where $\bar{\pi} \equiv [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^{\infty} \pi(\varphi) g(\varphi) d\varphi$ are the expected ex-ante profits. Condition (10) has an intuitive economic interpretation. Firms will be able to obtain credit and enter the market until the maximum lender's expected pledgeable profits, given by λ times the flow of profits for the average firm, equal the expected total cost for variety introduction. This is given by the fixed cost of variety innovation F_E , multiplied by the number of attempts needed to develop a successful variety, $\frac{1}{1 - G(\varphi^*)}$, plus the fixed cost for entry on the domestic market, F_D . The existence of credit market frictions thus creates positive rents for the firms equal to $v_E = (1 - G(\varphi^*)) \frac{(1 - \lambda)\bar{\pi}}{\delta}$ which reduce the resources invested by lenders in innovation and entry. By assumption, these rents are split uniformly among consumers, according to their ownership shares.

The distribution of firms' productivity conditional on entry is given by

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^*, \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

implying that $\bar{\pi}$ is also the average profit of firms that are in the market. I now define the weighted average productivity

$$\begin{aligned} \tilde{\varphi} &\equiv \left[\int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \\ &= \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^\infty \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}. \end{aligned} \quad (12)$$

As shown by Melitz (2003), this measure is extremely useful since it summarizes the information in the density $\mu(\varphi)$ that is relevant for all aggregate variables. I can use $\tilde{\varphi}$ to write aggregate revenues and variable profits as $R = Mr(\tilde{\varphi})$ and $\Pi = M\pi(\tilde{\varphi})$ respectively. It follows that $\bar{r} = R/M = r(\tilde{\varphi})$ and $\bar{\pi} = \Pi/M = \pi(\tilde{\varphi})$ represent both the average revenue and profit, as well as the revenue and profit of the firm with a productivity level equal to the average productivity $\tilde{\varphi}$. It is convenient to rewrite the profits of the marginal entry firm $\pi(\varphi^*)$ in terms of the average profits, which in the appendix are shown to equal the profits of the firm with the weighted average productivity ($\bar{\pi} = \pi(\tilde{\varphi})$). According to (6), $\pi(\varphi^*) = \bar{\pi} \left[\frac{\varphi^*}{\tilde{\varphi}} \right]^{\sigma-1}$ and I can rewrite (8) as

$$\lambda \bar{\pi} = \delta F_D \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1}. \quad (13)$$

In line with the existing literature, I define the above condition as the zero profit condition (ZPC). The equilibrium values for the average profits $\bar{\pi}$ and for the productivity cutoff φ^* are pinned down by the system given by (13) and (10). Note that the two equations can also be interpreted as a system in the productivity cutoff φ^* and in the new variable $\bar{z} \equiv \lambda \bar{\pi}$, that is the maximum pledgeable profits. This is the average revenue of the agent, in this case the lender, who commits to the innovation and entry investments. For $\lambda < 1$, clearly $\bar{z} < \bar{\pi}$. When φ is Pareto distributed on $[\varphi_m, \infty)$ with shape parameter a , the system can be rewritten as:

$$\begin{aligned} (ZPC) \quad \lambda \bar{\pi} &= \frac{\delta F_D a}{a - \sigma + 1} \\ (FEC) \quad \lambda \bar{\pi} &= \delta \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right). \end{aligned} \quad (14)$$

Compared to the frictionless case ($\lambda = 1$), with $\lambda < 1$ the ZPC and the FEC must associate to each cutoff value a higher average profit for the maximum pledgeable income to be in line with the ex ante and the ex post investment costs.

Given the equilibrium values $\bar{\pi}_{cc}$ and φ_{cc}^* , the equilibrium values for the other endogenous variables are pinned down as follows: in a stationary equilibrium the number of total entrants

M_e in each period is such that the number of successful entrants exactly replace the mass of dying firms: $[1 - G(\varphi_{cc}^*)]M_e = \delta M$. New entrants employ a total amount of labor equal to the number of workers needed for product innovation plus the number of workers used to pay for domestic market entry: $L_I \equiv M_e F_E + M_e [1 - G(\varphi^*)]F_D$. Production workers, L_P , are such that $\Pi = R - L_P$. The labor market clearing condition is then given by $L = L_I + L_P$. In the closed economy the value of production equals the value of consumption, $R = E$, where E is the economy aggregate expenditure. Finally, $\Pi = M\bar{\pi}$, $\Pi = R/\sigma$ (see appendix) and the expression for the price index $P = M^{1/(1-\sigma)}p(\tilde{\varphi})$ complete the characterization of the equilibrium.

Proposition 2. *The equilibrium exists and is unique. Furthermore some firms are credit constrained ($\pi(\varphi^*) > \delta F_D$) and φ^* , $\bar{\pi}$ and M satisfy:*

$$\frac{\partial \bar{\pi}}{\partial \lambda} < 0, \quad \frac{\partial \varphi^*}{\partial \lambda} = 0, \quad \frac{\partial M}{\partial \lambda} > 0.$$

Proof: See Appendix \square

The above proposition states that credit frictions prevent some firms from entering the domestic market, even when it would be profitable enough to do so. When firms can avoid repaying their debt, profits must be sufficiently high for the risk of a court intervention to give the right incentive to behave. Knowing that, lenders will finance the entry investment F_D only to firms with profits $\pi(\varphi) \geq \frac{\delta F_D}{\lambda} > \delta F_D$. As a result, average profits are higher than in a frictionless setup and they are increasing in credit frictions ($\lambda \downarrow$). Higher equilibrium average profits are not due to higher average productivity $\tilde{\varphi}(\varphi^*)$, but to a lower number of active firms M . According to proposition 2, φ^* is constant in λ , while M is increasing in λ and takes on a maximum value when there are no credit frictions ($\lambda = 1$).⁷ The intuition for this result is the following: when credit frictions increase ($\lambda \downarrow$), the profits of the marginal firm, and so the average profits $\bar{\pi}$, have to increase for the entry investment F_D to be profitable from the lender's point of view (the ZPC moves upwards). $\bar{\pi}$ can increase either via an increase of the productivity cutoff φ^* or a decrease in the number of firms which leads to an increase in $\pi(\varphi)$ for all $\varphi \geq \varphi^*$. In the first case ($\varphi^* \uparrow$), the chance of extracting a good productivity level would decrease and the expected cost of entry would go up ($\varphi^* \uparrow \Rightarrow r.h.s. \text{ of FEC } \uparrow$). According to the FEC, this would require that average profits $\bar{\pi}$ increase even further (*l.h.s. of FEC*). With $\bar{\pi}$ increased more than in proportion to the initial change in λ , the ZPC would be violated and this cannot represent the new equilibrium. This implies that $\bar{\pi}$ can increase only as a result of a lower mass M of firms. The FEC moves upward but in such a way that in equilibrium φ^* is unchanged. Another way of interpreting this result is to look at the system (14) as a system in φ^* and in the pledgeable income \bar{z} . Then, λ does not directly affect either the investment for domestic market entry (*r.h.s. of ZPC*) or the expected cost of innovation (*r.h.s. of FEC*). As a result both \bar{z} and φ^* are constant and only $\bar{\pi}$ varies according to $\bar{\pi} = \bar{z}/\lambda$. It is interesting to note that none of the above results depend on the Pareto parametrization of the productivity distribution function.

To summarize, a tighter credit market restricts the number of firms obtaining loans and this

⁷ φ^* is the same as in a frictionless setup.

increases the marginal and average profits. All other things being equal, a less efficient credit market does not affect the average productivity $\tilde{\varphi}$, but allows only a smaller number of bigger firms (in terms of average sales \bar{r}) to access credit ($\lambda \downarrow \Rightarrow \bar{\pi} \uparrow \Rightarrow M \downarrow$ and $\bar{r} = \bar{\pi}/\sigma \uparrow$).

Lemma 2.1. *In the closed-economy equilibrium with credit frictions, per-capita welfare U is given by*

$$U = \sigma \rho \varphi^* \left[\frac{L}{\delta F_D} \right]^{\frac{1}{\sigma-1}} \lambda^{\frac{1}{\sigma-1}} \left[\frac{1}{\sigma - 1 + \lambda} \right]^{\frac{\sigma}{\sigma-1}}$$

and $\frac{\partial U}{\partial \lambda} > 0$.

Proof: See Appendix \square

According to the above lemma, when credit frictions become less severe ($\lambda \uparrow$), per-capita welfare increases. This effect is the result of the increase in the number of varieties that, due to the love for variety assumption, pushes down the price index P and increases real expenditure for the differentiated goods q_1 ($\lambda \uparrow \Rightarrow M \uparrow \Rightarrow M^{1/(\sigma-1)} \uparrow \Rightarrow P \downarrow \Rightarrow U \uparrow$).

3.3 The Optimal Debt Contract in an Open Economy

I now introduce the possibility of becoming an exporter. To do so after entry firms must make the sunk investment of F_X units of labor. International trade also requires additional variable costs, modeled in the standard iceberg formulation: $\tau > 1$ unit of a good must be shipped in order for 1 unit to reach the destination. Domestic prices, revenues and profits are still given by (3), (4) and (5) and will henceforth be indexed by the subscript “D”. The subscript “X” will instead refer to the foreign market. Exporting firms will set higher prices on the foreign market that reflect the higher marginal cost due to τ : $p_X(\varphi) = \tau/\varphi\rho$. This price implies revenues $r_X(\varphi) = E \left(P \frac{\rho \varphi}{\tau} \right)^{\sigma-1}$ and profits $\pi_X(\varphi) = \frac{E}{\sigma} \left(P \frac{\rho \varphi}{\tau} \right)^{\sigma-1}$, where E and P are not indexed by country due to the symmetry assumption.

The credit contract will now specify, subject to the realization of φ , whether the firm should start production and, in this case, whether entry in the domestic market only or entry in both the domestic and the foreign market is financed.⁸ Accordingly, the contract defines the level of the debt per-period repayments. I now denote with $i(\varphi) \in \{0, 1\}$ and with $i_x(\varphi) \in \{0, 1\}$ the entry rule for the domestic and the foreign market respectively, with $f(\varphi)$ the reimbursement for firms engaged in the domestic market only and with $f'(\varphi)$ the reimbursement for firms engaged in both the domestic and the foreign market. In this second case, the reimbursement also covers

⁸I focus only on the case where exporting firms also serve the domestic market.

the additional investment F_X . Each firm will choose the optimal contract by solving:

$$\begin{aligned} \max_{i(\varphi), i_x(\varphi), f(\varphi), f'(\varphi), K} v_E \equiv & \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi_D(\varphi) - f(\varphi)) \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi \\ & + \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi)) \right] i_x(\varphi) g(\varphi) d\varphi + K \end{aligned} \quad (\text{P}')$$

subject to

$$K \geq 0, \quad (\text{LC})$$

$$\pi_D(\varphi) - f(\varphi) \geq (1-\lambda)\pi_D(\varphi), \text{ for all } \varphi \text{ such that } i(\varphi) = 1 \text{ and } i_x(\varphi) = 0 \quad (\text{IC})$$

$$\pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi) \geq (1-\lambda)(\pi_D(\varphi) + \pi_X(\varphi)), \quad (\text{IC}')$$

for all φ such that $i(\varphi) = 1$ and $i_x(\varphi) = 1$

$$\begin{aligned} v_L \equiv & \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) - F_D \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi \\ & + \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f'(\varphi) - (F_D + F_X) \right] i_x(\varphi) g(\varphi) d\varphi - F_E - K \geq 0, \end{aligned} \quad (\text{PC})$$

$$\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) \geq F_D, \text{ for all } \varphi \text{ such that } i(\varphi) = 1 \text{ and } i_x(\varphi) = 0 \quad (\text{RP})$$

$$\sum_{t=0}^{\infty} (1-\delta)^t f'(\varphi) \geq (F_D + F_X), \quad (\text{RP}')$$

for all φ such that $i(\varphi) = 1$ and $i_x(\varphi) = 1$

The objective function of the firm is given by the expected discounted value of net future profits plus the ex ante transfer K . When only domestic entry occurs, net profits are given by $\pi_D(\varphi) - f(\varphi)$, and, given the assumption that no firm exports without also serving the domestic market, this happens when $i(\varphi) - i_x(\varphi) = 1$. When entry in both markets occurs, then net profits are given by $\pi_D + \pi_X(\varphi) - f'(\varphi)$, and this happens when $i_x(\varphi) = 1$. Again, the requirement that K is non-negative derives from the liquidity constraint of the borrower (LC). At stage 4 in Figure 1, the domestic firm with known productivity φ faces the same incentive compatible constraint (IC) described above. The firm with known productivity φ that produces for both markets pays the per-period reimbursement $f'(\varphi)$ if the domestic and foreign market profits, net of the payment $f'(\varphi)$, exceed the expected profits from cheating. This condition is expressed by the second incentive compatibility constraint (IC'). The lender's participation constraint (PC) requires the expected value of the contract in period 1 to be non-negative. This happens when the present discounted value of per-period repayments, net of the entry costs, exceeds the initial investment $F_E w$ and the transfer K . Entry occurs in either one or both markets according to the entry rules $i(\varphi)$ and $i_x(\varphi)$. The last two constraints are the lender's renegation-proof conditions, requiring the lender's continuation payoff at stage 3 to be positive. The lender finds it profitable to invest further in market entry, if the fixed costs do not exceed

the present discounted value of the flow of future incentive compatible repayments. Again, I distinguish between entry in the domestic market only (RP) and entry in both the domestic and foreign market (RP').

As before, the firm will increase the term $T' \equiv K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{f'(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi$ until (PC) becomes binding. As a result, in equilibrium

$$K = \int_{\varphi_m}^{\infty} \left[\frac{f(\varphi)}{\delta} - F_D \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \left[\frac{f'(\varphi)}{\delta} - F_D - F_X \right] i_x(\varphi) g(\varphi) d\varphi - F_E \geq 0. \quad (15)$$

As for the closed economy, the optimal contract is not unique. Any triplet $(K, f(\varphi), f'(\varphi))$ such that (15) holds together with (IC), (RP), (IC') and (RP') is a valid solution to (P). The way the transfer T is split between the ex-ante transfer K and the ex-post per-period repayments does not affect the optimal value v_E . Although the (IC) and the (IC') do not have to be binding for the generic firm with productivity φ , they will be for the marginal firms entering the domestic and the foreign market respectively. Setting $f(\varphi)$ and $f'(\varphi)$ as high as the (IC) and the (IC') allow for, makes it easier to meet the lender renegation proof constraint. For the marginal firms it must be that $f(\varphi) = \lambda\pi(\varphi)$ and that $f'(\varphi) = \lambda[\pi_D(\varphi) + \pi_X(\varphi)]$. Plugging these values into the renegation-proof conditions, they become

$$\begin{aligned} \pi_D(\varphi) &\geq \frac{\delta F_D}{\lambda} \\ \pi_D(\varphi) + \pi_X(\varphi) &\geq \frac{\delta[F_D + F_X]}{\lambda}. \end{aligned} \quad (16)$$

$F_D/F_X < \tau^{\sigma-1}$ ensures that no firm becomes an exporter without also serving the domestic market. Condition (16) implicitly states that firms can use the profits gained on the domestic market to finance the repayment of the exporting fixed costs. In other words, domestic entry subsidizes entry on the foreign market.⁹ On the other hand, no firm will be willing to enter the foreign market if the present discounted profits from exporting do not exceed the entry cost F_X (efficiency condition). It follows that $\varphi^* = \{\varphi : \pi_D(\varphi) = \frac{\delta F_D}{\lambda}\}$ always identifies the domestic market productivity cutoff, while the lowest productivity level of exporting firms φ_x^* is given by the largest between $\inf\{\varphi : \pi_D(\varphi) + \pi_X(\varphi) > \frac{\delta[F_D + F_X]}{\lambda}\}$ and the efficient cutoff defined the efficiency condition, $\inf\{\varphi : \pi_X(\varphi) > \delta F_X\}$. Which of the two cases prevails in defining the equilibrium cutoff φ_x^* depends on λ .

Proposition 3. *The foreign market cutoff is defined by $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \delta F_X\}$ if $\lambda \geq \hat{\lambda}$ and by $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})}\}$ if $\lambda < \hat{\lambda}$, where $\hat{\lambda} \equiv \frac{F_D/F_X + 1}{1 + \tau^{\sigma-1}}$.*

Proof: See Appendix \square

When $\lambda < 1$, domestic firms are always credit-constrained, meaning that there are firms that would find it profitable to enter the domestic market, meaning $\frac{\pi_D(\varphi)}{\delta} \geq F_D$, but, due to imper-

⁹To see this, condition (16) can be rewritten as $\pi_X(\varphi) + [\pi_D(\varphi) - \frac{\delta F_D}{\lambda}] \geq \frac{\delta F_X}{\lambda}$.

fect creditor protection, are not able to borrow the sum $F_D w$. Conversely, proposition 3 implies that exporters are credit-constrained only for low values of λ . A firm could obtain the credit needed for both domestic and foreign market entry even when foreign market entry itself is not optimal, meaning $\frac{\pi_X(\varphi)}{\delta} < F_X$. Entry in the foreign market would then imply net losses and this possibility is excluded by the contract; entry in the export takes place only if $\frac{\pi_X(\varphi)}{\delta} \geq F_X$. This happens because when the repayment of the two investments is joint the risk of losing both $\pi_D(\varphi)$ and $\pi_X(\varphi)$ can make it easier to meet condition (16) than to meet the efficient condition $\frac{\pi_X(\varphi)}{\delta} \geq F_X$. Based on proposition 3, the worse financial institutions (low λ), the higher the cost of domestic market entry (high F_D), the lower the cost of foreign market entry (low F_X) and the lower the variable trade costs (low τ), the more likely it is for exporting decisions to be constrained. The effect of λ is clear: the higher the credit frictions, the greater is the incentive to misbehave. Higher profits are thus needed to ensure the repayment. The effect of the other three parameters derives from the fact that the net domestic profits $\pi_D(\varphi) - F_D$ can be seen as extra resources to finance foreign market entry. The higher F_D , the lower these net profits and so the higher $\pi_X(\varphi)$ must be. As $\pi_D(\varphi) = \tau^{\sigma-1} \pi_X(\varphi)$, the lower τ is, the lower the value of $\pi_D(\varphi)$ associated with a given value of $\pi_X(\varphi)$. If domestic net profits are lower, then higher foreign market profits are needed to compensate. A higher F_X always increases the productivity needed for foreign market entry. On the other hand, this effect could be stronger without credit frictions than with credit frictions. The reason is that in the first case the fixed cost is covered by foreign market profits only, while in the second case it is covered by both the foreign market profits and the net domestic profits.

The entry rules are thus given by $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*$ and by $i_x(\varphi) = 1$ if and only if $\varphi \geq \varphi_x^*$. The next proposition summarizes these results.

Proposition 4. *Any optimal contract satisfies:*

I.

$$i(\varphi) = \begin{cases} 1 & \text{if } \varphi \geq \varphi^*, \\ 0 & \text{otherwise} \end{cases}$$

where $\varphi^* = \{\varphi : \pi_D(\varphi) = \frac{\delta F_D}{\lambda}\};$

II.

$$i_x(\varphi) = \begin{cases} 1 & \text{if } \varphi \geq \varphi_x^*, \\ 0 & \text{otherwise} \end{cases}$$

where $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \delta F_X\}$ if $\lambda \geq \hat{\lambda}$ and by $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})}\}$ if $\lambda < \hat{\lambda}$;

III. $(K, f(\varphi), f'(\varphi))$ are such that

$$K = \int_{\varphi^*}^{\varphi_x^*} \left[\frac{f(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_{cc-x^*}}^{\infty} \left[\frac{f'(\varphi)}{\delta} - (F_D + F_X) \right] g(\varphi) d\varphi - F_E w \geq 0, \quad (17)$$

$$\begin{cases} f(\varphi) = \lambda \pi_D(\varphi) & \text{if } \varphi = \varphi^* \\ \delta F_D \leq f(\varphi) \leq \lambda \pi_D(\varphi) & \text{if } \varphi^* < \varphi < \varphi_x^*. \end{cases}$$

and

$$\begin{cases} f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi)) & \text{if } \varphi = \varphi_x^*, \\ \delta(F_D + F_X) \leq f'(\varphi) \leq \lambda(\pi_D(\varphi) + \pi_X(\varphi)) & \text{if } \varphi > \varphi_x^*. \end{cases}$$

Moreover

$$v_E = \int_{\varphi^*}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi - F_E.$$

Proof: See Appendix \square

3.3.1 Equilibrium in an Open Economy

In equilibrium no firm left outside of the market should find entry strictly profitable ($v_E > 0$) and strictly feasible ($K > 0$) at the same time. I will require that in equilibrium at least one of the above conditions fails. The maximum value for K is the one for $f(\varphi) = \lambda\pi_D(\varphi)$ for all $\varphi \in [\varphi^*, \varphi_x^*)$ and $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ for all $\varphi \geq \varphi_x^*$. Plugging this value into (17), I obtain

$$K_{max} = \int_{\varphi^*}^{\infty} \left[\frac{\lambda\pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\lambda\pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi - F_E < v_E. \quad (18)$$

As for the closed economy, free entry cannot lead either to $v_E = 0$ or to $v_E < 0$. The unbounded mass of prospective entrants together with the firms liquidity constraint require that in equilibrium $K_{max} = 0$.¹⁰ This is the FEC, and can be simplified to obtain:

$$\lambda\bar{\pi} = \delta \left[\frac{F_E}{1 - G(\varphi^*)} + F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \right]. \quad (19)$$

In equilibrium firms will be able to obtain credit and enter the market until the maximum lender's expected pledgeable profits $\lambda\bar{\pi}$ equal the expected total cost for variety introduction. This is given by the fixed cost of variety innovation F_E , multiplied by the number of attempts needed to develop a successful variety, $\frac{1}{1 - G(\varphi^*)}$, plus the fixed cost for entry on the domestic market, F_D , plus the fixed cost for entry on the foreign market F_X , multiplied by the probability $[1 - G(\varphi_x^*)]/[1 - G(\varphi^*)]$. The existence of credit market friction creates positive rents equal to $v_E = (1 - G(\varphi^*)) \frac{(1 - \lambda)\bar{\pi}}{\delta}$ which reduce the resources invested in innovation and entry. These rents are split uniformly among consumers, according to their ownership shares.

As before, the density function of productivity levels for incumbent firms, $\mu(\varphi)$, is determined by the ex-ante distribution of productivity levels, conditional on a successful entry (see (11)). The ex-ante probability of becoming an exporter is given by $p_x = [1 - G(\varphi_x^*)]/[1 - G(\varphi^*)]$ and $M_X = p_x M$ is the mass of exporting firms. Using the same weighted average function defined in (12), let $\tilde{\varphi} = \tilde{\varphi}(\varphi^*)$ and $\tilde{\varphi}_x = \tilde{\varphi}(\varphi_x^*)$ denote the average productivity of, respectively, all firms and exporting firms only. The overall average of combined revenues and profits earned

¹⁰For a more complete argument, see the section on the closed economy.

on both markets is given by:

$$\bar{r} = r_D(\tilde{\varphi}) + p_x r_X(\tilde{\varphi}_x)$$

and

$$\bar{\pi} = \pi_D(\tilde{\varphi}) + p_x \pi_X(\tilde{\varphi}_x) \quad (20)$$

where $r_D(\tilde{\varphi})$ and $r_X(\tilde{\varphi}_x)$, $\pi_D(\tilde{\varphi})$ and $\pi_X(\tilde{\varphi}_x)$ are respectively the average revenue and profits from domestic sales and exports. Using (6) to express the profits of the marginal firms in terms of the average profits together with (20), I can write the zero profit condition in terms of the overall average profits as

$$\bar{\pi} = \begin{cases} \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1} + p_x \frac{\delta(F_D+F_X)}{\lambda(1+\tau^{\sigma-1})} \left[\frac{\tilde{\varphi}_x}{\varphi_x^*} \right]^{\sigma-1} & \text{if } 0 < \lambda < \hat{\lambda} \\ \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1} + p_x \delta F_X \left[\frac{\tilde{\varphi}_x}{\varphi_x^*} \right]^{\sigma-1} & \text{if } \hat{\lambda} \leq \lambda \leq 1. \end{cases} \quad (21)$$

It is also useful to derive the ratio between the two productivity cutoffs:

$$\frac{\varphi^*}{\varphi_x^*} = \begin{cases} \tau^{-1} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \tau^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } \hat{\lambda} \leq \lambda \leq 1. \end{cases} \quad (22)$$

The equilibrium values for the average profits $\bar{\pi}$ and for the productivity cutoff φ^* are pinned down by the system given by the conditions (19) and (21). When φ is Pareto distributed on $[\varphi_m, \infty)$ with shape parameter a , the system becomes

$$\begin{aligned} (FEC) \quad \lambda \bar{\pi} &= \delta F_D \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right] \\ (ZPC) \quad \lambda \bar{\pi} &= \frac{\delta a F_D}{a - \sigma + 1} \left[1 + \tau^{-a} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] \end{aligned} \quad (23)$$

when $0 < \lambda < \hat{\lambda}$ and

$$\begin{aligned} (FEC) \quad \lambda \bar{\pi} &= \delta F_D \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^a \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] \\ (ZPC) \quad \lambda \bar{\pi} &= \frac{\delta a F_D}{a - \sigma + 1} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] \end{aligned} \quad (24)$$

when $\hat{\lambda} \leq \lambda \leq 1$. As for the closed economy, the above conditions can also be interpreted as a system in the cutoff φ^* and in the maximum expected pledgeable income of the investor $\lambda \bar{\pi}$, which is ultimately the basis for evaluating the profitability of the overall investment.

Given the equilibrium values $\bar{\pi}$ and φ^* , from (22) I can solve for φ_x^* . Aggregation implies $\Pi = M \bar{\pi}$ and $R = M \bar{r}$, with $\Pi = R/\sigma$. Stationarity requires $\delta M = [1 - G(\varphi^*)] M_e$. The demand

for labor satisfies $\Pi = R - L_P$ for production workers and $L_I = M_e F_E + M_e [1 - G(\varphi^*)] F_D + M_e [1 - G(\varphi_x^*)] F_X$ for workers employed in product innovation and market entry. The labor market clearing condition is thus given by $L = L_I + L_P$. In an open economy, balanced trade ensures $R = E$. M is given by $\Pi/\bar{\pi}$ and M_X by $M \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$. Given the total number of varieties available in each market $\bar{M} \equiv M + M_X$, the overall average productivity weighted by the market shares of domestic and exporting firms is:

$$\bar{\varphi} \equiv \left\{ \frac{1}{\bar{M}} [M \bar{\varphi}^{\sigma-1} + M_X (\tau^{-1} \bar{\varphi}_x)^{\sigma-1}] \right\}^{\frac{1}{\sigma-1}}.$$

Since $\bar{\varphi}$ also represents the weighted average productivity of all domestic and foreign firms competing on the same market, the price index is given by $P = \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi}) = \bar{M}^{\frac{1}{1-\sigma}} \frac{w}{\rho \bar{\varphi}}$. In the appendix I also derive the total value of firms' rents $V_E \equiv M_e v_E = (1 - \lambda)\Pi$. These are resources that are misallocated as a result of credit market frictions. To avoid opportunistic behavior, firms must be granted resources that would otherwise be allocated to innovation and entry investments.

Proposition 5. *The equilibrium exists and is unique.*

Proof: See Appendix \square

4 Steady-State Properties of the Model

In this section I describe the properties of the open economy equilibrium with respect to the credit market frictions λ and the trade frictions τ . In particular, I study how the endogenous variables of the model change when λ and τ vary symmetrically in the two countries. The following analyses rely on a comparison of steady-state equilibria and should therefore be interpreted as capturing the long-run consequences of changes in the economic environment.

Proposition 6. *φ^* is continuous and not decreasing in $\lambda \in (0, 1]$. In particular*

$$\varphi^* = \begin{cases} \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{\frac{a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{-1} - 1 \right] \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} & \text{if } \hat{\lambda} \leq \lambda \leq 1. \end{cases}$$

and

$$\frac{\partial \varphi^*}{\partial \lambda} \begin{cases} = 0 & \text{if } 0 < \lambda < \hat{\lambda}, \\ > 0 & \text{if } \hat{\lambda} \leq \lambda < 1. \end{cases}$$

Moreover, the mass of firms M is continuous and increasing in $\lambda \in (0, 1]$ and the relative number of exporting firms $M_X/M = \left(\frac{\varphi^*}{\varphi_x^*} \right)^a$ is continuous and non-increasing in $\lambda \in (0, 1]$.

Proof: See Appendix \square

Proposition 6 contains one of the main results and it is surprising. It states that for $\lambda < 1$ the productivity threshold, and accordingly the average productivity $\tilde{\varphi} = \left[\frac{a}{a-\sigma+1} \right]^{\frac{1}{\sigma-1}} \varphi^*$, is constant in λ when frictions are high ($\lambda < \hat{\lambda}$), while it increases with λ when credit market frictions are low ($\lambda \geq \hat{\lambda}$). It follows that φ^* is always lower than it would be in a frictionless setting ($\lambda = 1$). Because of general equilibrium effects and because of the interaction between domestic and foreign market entry, the minimum firm productivity does not increase when credit frictions increase, although this implies that it is more difficult to obtain credit. According to the proposition, credit frictions also imply too few firms and relatively too many exporters.

In the open economy, credit market frictions introduce two sources of inefficiency: first, as for the closed economy, credit frictions allow opportunistic behavior that generates rents for the firm ($v_E > 0$) and diverts resources away from innovation. This restricts entry and M is too low compared to the case $\lambda = 1$. Second, the use of domestic profits to subsidize foreign market entry (see (16)), distorts the incentives for domestic market entry. The best way to see this is to look at the FEC and at the conditions defining φ^* and φ_x^* , expressed in terms of the pledgeable profits $z_D(\varphi) \equiv \lambda \pi_D(\varphi)$ and $z_X(\varphi) \equiv \lambda \pi_X(\varphi)$. By setting (18) equal to zero, I obtain:

$$\int_{\varphi^*}^{\infty} \left[\frac{z_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{z_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi = F_E \quad (25)$$

while the productivity cutoffs are given by:

$$z_D(\varphi^*) = \delta F_D \quad (26)$$

and, from proposition 3, by

$$z_X(\varphi_x^*) = \begin{cases} \frac{\delta(F_D + F_X)}{1 + \tau^{\sigma-1}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \lambda \delta F_X & \text{if } \hat{\lambda} \leq \lambda \leq 1 \end{cases} \quad (27)$$

From (27) it follows that, for values of φ close to the threshold φ_x^* , $z_X(\varphi) < \delta F_X w$. This implies that from the creditor's standpoint, low productivity exporters generate losses (the first elements of the second integral in (25) are negative). To compensate for these losses, creditors will grant loans to a lower number of firms with a higher probability of generating domestic market profits (M and φ^* are inefficiently low). When credit frictions are low ($\lambda > \bar{\lambda}$), foreign market entry is not constrained and a decrease in λ directly affects the creditor's pledgeable profits from the marginal exporter ($\lambda \downarrow \Rightarrow z_X(\varphi_x^*) \downarrow$). As a result, the expected lender's return from export activities (second integral in (25)) decreases and the subsidization from expected domestic profits must increase further. This happens if both φ^* and M decrease: the term $\int_{\varphi^*}^{\infty} \left[\frac{z_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi$ can increase either if φ^* goes down or if $z_D(\varphi)$ goes up for all $\varphi \geq \varphi^*$. Since from (26) $z_D(\varphi^*)$ is constant in λ and decreasing in φ^* , they will both move. If $z_D(\varphi)$ goes up for all $\varphi \geq \varphi^*$, then $z_X(\varphi)$ also increases for all $\varphi \geq \varphi_x^*$. Combining this result with the fact that $z_X(\varphi_x^*)$ decreases, means that φ_x^* must also decrease. In particular $\frac{\varphi_x^*}{\varphi^*} = p_x$ goes up: since only domestic market entry is credit constrained, a decrease in λ increases the relative

profitability of foreign activity with respect to domestic activity. If $z_D(\varphi)$ and $z_X(\varphi)$ increase for all active firms when λ goes down, then the profits $\pi_D(\varphi)$ and $\pi_X(\varphi)$ must increase. As a result the mass M of firms must become smaller ($\lambda \downarrow \Rightarrow \varphi^* \downarrow, \varphi_x^* \downarrow$ and $M \downarrow$).

When credit frictions are high ($0 < \lambda < \hat{\lambda}$) the only effect of λ is on the firm's rent v_E . None of the three conditions listed above depends on λ . It follows that $\varphi^*, \varphi_x^*, z_D(\varphi^*)$ and $z_X(\varphi_x^*)$ are constant in λ . As both domestic and foreign market entry are restricted, their relative profitability from the lender's point of view does not depend on λ and $\frac{\varphi^*}{\varphi_x^*}$ is also unaffected by λ (see (22)). Given the definitions of $z_D(\varphi)$ and $z_X(\varphi)$, when λ varies, $\pi_D(\varphi)$ and $\pi_X(\varphi)$ have to vary accordingly for all $\varphi \geq \varphi^*$ in order for $z_D(\varphi^*)$ and $z_X(\varphi_x^*)$ to remain constant. For instance, when credit frictions increase ($\lambda \downarrow$), it must be the case that for every firm profits go up ($\pi_D(\varphi) \uparrow$ and $\pi_X(\varphi) \uparrow$). This is possible only if the mass of firms becomes smaller ($M \downarrow$).

The bottom line of proposition 6 is that credit frictions imply a misallocation of resources from domestic to exporting activities and an inefficient selection of entrants: as a result the number of incumbents and the average productivity level are too low while there is a overly high number of exporters relative to the total number of firms. The model's predictions can thus account for the existence of many low productive firms in economies characterized by a high degree of openness (in terms of either p_x or in terms of relative export sales). This scenario, which characterized many emerging economies (see Hsieh and Klenow (2009) for evidence on China and India), cannot be explained by Melitz's traditional framework, where the competition for production factors induced by a high degree of openness tends to force low productive firms out of the market.

Proposition 7. *For $\lambda \in (0, 1)$, $\partial\varphi^*/\partial\tau < 0$ if a is sufficiently small (close to $\sigma - 1$) and $\partial\varphi^*/\partial\tau > 0$ otherwise (if a is sufficiently large). The mass of firms M is always increasing in τ while M_X/M is decreasing in τ .*

Proof: See Appendix \square

The equilibrium properties for M and M_X/M are the same as in Melitz (2003) and in the related heterogeneous firms' literature. Lower trade costs ($\tau \downarrow$) imply higher expected ex ante returns from exports and this induces more firms to become exporters ($M_X/M \uparrow$) and to invest in introducing new products. This increases the demand for both production and innovation workers and, given the limited number of resources L in the economy, less firms can be sustained in equilibrium ($M \downarrow$). In Melitz (2003), this increased competition among firms forces the least productive firms to exit and the domestic productivity cut-off is always increasing as τ decreases. When there are credit market frictions, this is no longer always true and the effect of τ on φ^* depends on the productivity distribution of firms in the differentiated goods sector. According to proposition 7, trade liberalization induces the exit of the least productive firms ($\partial\varphi^*/\partial\tau < 0$) only when the parameter a is sufficiently low. This parameter characterizes the shape of the Pareto distribution of the productivity levels. In particular, the smaller a is, the larger the proportion of very-high-productivity firms. The explanation for this result is in the wedge that credit frictions create between the total expected profits from market entry and the

maximum expected profits that can be pledged to the lender. For instance, a decrease in τ increases both expected profits $\bar{\pi}_x$ and the expected ex ante cost of entry, as the probability of investing in foreign market entry p_x also goes up. The first effect always dominates and the net expected return from exports increases. On the other hand, while bearing the full increase in the expected costs, the lender internalizes only a fraction $\lambda < 1$ of the increase in the expected profits. From the lender's point of view, the net effect can thus be negative and the value of the second integral in (25) can be increasing in τ . For the free entry condition to hold, the lender's net expected profit from domestic market entry must increase. This requires φ^* to fall and M to increase. Whether the increase in expected pledgeable profits is higher or not than the increase in the expected cost depends on the parameter a . In particular, when a is small, the distribution of productivity is more concentrated at high levels and the positive effect of a decreasing τ on the expected pledgeable profits $\lambda\bar{\pi}_x$ tends to be higher and to dominate the negative effect on the expected costs. It follows that $\partial\varphi^*/\partial\tau < 0$.

Lemma 7.1. *In the open economy equilibrium with credit frictions, per-capita welfare U is given by*

$$U = \frac{\sigma\rho\varphi^*}{\sigma - 1 + \lambda} \left(\frac{\lambda L}{\delta F_D(\sigma - 1 + \lambda)} \right)^{\frac{1}{\sigma-1}}.$$

Moreover $\frac{\partial U}{\partial \lambda} > 0$, $\partial U/\partial\tau < 0$ when a is sufficiently small and $\partial U/\partial\tau > 0$ otherwise (when a is sufficiently large).

Proof: See Appendix \square

According to proposition 6, the number of domestic varieties M is increasing in λ , and the domestic productivity φ^* , which determines the level of average productivity, is non-decreasing in λ . Because of these effects, lemma 7.1 claims that consumer welfare, which is increasing in the average productivity and in the number of varieties available to consumers, is also increasing in λ . The lemma also claims that the effect of trade liberalization on consumers' welfare depends on the exogenous parameter a . The effect of τ on U ultimately depends on the domestic productivity cut-off φ^* and, according to proposition 7, trade liberalization induces the productivity cut-off to decrease when a is sufficiently large and to increase when a is sufficiently small. This result contrasts with Melitz (2003) which predicts that trade liberalization has unambiguously positive effects on consumers' welfare. The explanation is in the more general proposition that reductions in the degree of one distortion are not necessarily welfare increasing if there is another distortion in the system (Bhagwati (1969)). Here, the trade friction ($\tau > 1$) and the credit market inefficiency ($\lambda < 1$) distort resources in opposite directions: the first against and the second in favor of exporting activities. A decline in one of the two distortions magnifies the negative effect imposed by the other, and the total effect on welfare is ambiguous.

5 Concluding Remarks

This paper presents a rich, though tractable, trade model that studies the steady-state equilibrium effects of imperfect credit markets on the allocation of resources between domestic and exporting firms. The model incorporates heterogeneous firms and credit market frictions in the form of imperfect creditors' protection.

I show the importance of considering a general equilibrium setting in order to fully characterize the misallocations of resources that derive from the existence of credit frictions. First, the possibility of opportunistic behavior generates rents for firms that reduce the resources employed in product innovation and market entry. As a result the total number of firms is sub-optimal; this reduces competition among incumbents allowing low-efficiency firms to survive. Credit frictions thus imply a lower average productivity level. A lower number of firms and a lower average productivity level have negative effects on individual utility: consumer welfare decreases as the credit frictions increase. The second form of resource misallocation goes from domestic to foreign firms. The fact that domestic profits can be de facto used to back-up the loan to pay for foreign market entry makes exporters less likely to be credit constrained.

The paper also shows that allowing for liquidity-constrained firms and imperfect credit markets alters, and in some cases reverses, some of main results from the heterogeneous firms literature. The model predicts that trade liberalization does not necessarily lead to an increase in average productivity and consumers' welfare. In particular, I show that trade liberalization increases consumer welfare only when the distribution of firm productivity is more concentrated towards high values.

Considering a general equilibrium setting with an endogenous mass of firms, this model fills an important gap in the recent literature on trade and financial development. Moreover, given its tractability, the model could be easily extended in some relevant directions. First of all, allowing for higher frictions of export financing would reconcile the model with the prediction present in other contributions that export activities are more dependent on external capital. Second, allowing for asymmetric credit frictions between countries, it would be possible to study the effects of financial developments on the patterns of trade. These are possible directions for future research.

A Technical Appendix

A.1 Proof of proposition 1: the optimal contract in the closed economy

To solve for the optimal contract, I start from the fact that the firm value

$$\begin{aligned}
v_E &= \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi(\varphi) - f(\varphi)) \right] i(\varphi) g(\varphi) d\varphi + K \\
&= \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t \pi(\varphi) \right] i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) \right] i(\varphi) g(\varphi) d\varphi + K \\
&= \int_{\varphi_m}^{\infty} \frac{\pi(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi + K
\end{aligned}$$

is increasing in the term $T \equiv K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi$. Since I am solving for an equilibrium with perfect competition among potential lenders, the firm will increase T until the (PC) binds. $v_L = 0$ implies:

$$\begin{aligned}
\int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) - F_D \right] i(\varphi) g(\varphi) d\varphi - F_E - K &= 0 \\
\int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) - F_D \right] i(\varphi) g(\varphi) d\varphi - F_E &= K.
\end{aligned}$$

Using the definition of T , the above condition can also be written as:

$$\begin{aligned}
- \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - F_E &= K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi \\
- \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - F_E &= T.
\end{aligned} \tag{28}$$

Plugging this value in v_E , I obtain

$$\begin{aligned}
v_E &= \int_{\varphi_m}^{\infty} \frac{\pi(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi + T \\
&= \int_{\varphi_m}^{\infty} \frac{\pi(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - F_E.
\end{aligned}$$

Note that for $v_L = 0$ the firm's objective function does not depend on K and $f(\varphi)$ separately, but on T only. It follows that, so far as (LC), (IC) and (RP) hold, (28) can be achieved by arbitrarily varying both K and $f(\varphi)$. This introduces a degree of freedom in determining the optimal contract. On the other hand, this is not the case for the firm with the minimum productivity level needed to obtain the loan $F_D w$ and start production. In fact, in order to meet the lender's (RP) condition, this firm has the incentive to pay a per-period repayment $f(\varphi)$ as high as the (IC) allows for. In other words, a binding (IC) and a binding (RP) solve for the entry rule: $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*$, where φ^* represents the entry-productivity

cut-off. A binding (IC) implies:

$$\begin{aligned}\pi(\varphi) - f(\varphi) &= (1 - \lambda)\pi(\varphi) \\ \pi(\varphi) - f(\varphi) &= \pi(\varphi) - \lambda\pi(\varphi) \\ f(\varphi) &= \lambda\pi(\varphi).\end{aligned}$$

Using this in the (RP) condition, φ^* is such that

$$\begin{aligned}\sum_{t=0}^{\infty} (1 - \delta)^t f(\varphi^*) &= F_D \\ \frac{\lambda\pi(\varphi^*)}{\delta} &= F_D \\ \lambda\pi(\varphi^*) &= \delta F_D.\end{aligned}$$

Once the entry rule has been solved for, the firm's problem reduces to:

$$\max_{f(\varphi), K} v_E = \int_{\varphi^*}^{\infty} \frac{\pi(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*))F_D - F_E \quad (\text{P})$$

subject to

$$K = \int_{\varphi^*}^{\infty} \frac{f(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*))F_D - F_E \geq 0 \quad (\text{LC+PC})$$

$$\delta F_D \leq f(\varphi) \leq \lambda\pi(\varphi) \text{ for all } \varphi \text{ such that } \varphi \geq \varphi^* \quad (\text{IC+RP})$$

that, as argued above, does not have a unique solution for the pair $(K, f(\varphi))$.

A.2 The FEC in the closed economy

FEC In the text I argue that the FEC is given by

$$K_{max} \equiv \int_{\varphi^*}^{\infty} \frac{\lambda\pi(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*))F_D - F_E = 0.$$

Given $\bar{\pi} \equiv [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^{\infty} \pi(\varphi) g(\varphi) d\varphi$, I can rewrite it as

$$\begin{aligned}\frac{\lambda(1 - G(\varphi^*))}{\delta(1 - G(\varphi^*))} \int_{\varphi^*}^{\infty} \pi(\varphi) g(\varphi) d\varphi &= [(1 - G(\varphi^*))F_D + F_E] \\ \frac{\lambda(1 - G(\varphi^*))\bar{\pi}}{\delta} &= [(1 - G(\varphi^*))F_D + F_E] \\ \lambda\bar{\pi} &= \delta \left[\frac{F_E}{1 - G(\varphi^*)} + F_D \right]\end{aligned}$$

In the same way, I can rewrite v_E as

$$\begin{aligned}
v_E &= \int_{\varphi^*}^{\infty} \frac{\pi(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*)) F_D - F_E \\
&= \frac{(1 - G(\varphi^*)) \bar{\pi}}{\delta} - (1 - G(\varphi^*)) F_D - F_E \\
&= \frac{(1 - G(\varphi^*)) \bar{\pi}}{\delta} - \frac{\lambda(1 - G(\varphi^*)) \bar{\pi}}{\delta} \\
&= (1 - G(\varphi^*)) \frac{(1 - \lambda) \bar{\pi}}{\delta} > 0.
\end{aligned}$$

A.3 Aggregate quantities in the closed economy

Given that there is a mass M of firms in equilibrium, $\mu(\varphi)$ is the distribution of productivity of active firms defined on a subset of $(0, \infty)$ and using $r(\varphi) = E(P\rho\varphi)^{\sigma-1}$ the aggregate revenue is given by

$$\begin{aligned}
R &= \int_0^{\infty} r(\varphi) M \mu(\varphi) d\varphi \\
&= M \int_0^{\infty} E(P\rho\varphi)^{\sigma-1} \mu(\varphi) d\varphi \\
&= M E(P\rho)^{\sigma-1} \left[\int_0^{\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right] \\
&= M E(P\rho)^{\sigma-1} \tilde{\varphi}^{\sigma-1} \\
&= M E(P\rho\tilde{\varphi})^{\sigma-1} \\
&= M r(\tilde{\varphi})
\end{aligned}$$

where $\tilde{\varphi} \equiv \tilde{\varphi}(\varphi^*) = \left[\int_0^{\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$. In the same way, using $\pi(\varphi) = \frac{E}{\sigma} \left(P \frac{\rho}{\varphi} \right)^{\sigma-1}$, I obtain

$$\begin{aligned}
\Pi &= \int_0^{\infty} \pi(\varphi) M \mu(\varphi) d\varphi \\
&= M \int_0^{\infty} \frac{E}{\sigma} (P\rho\varphi)^{\sigma-1} \mu(\varphi) d\varphi \\
&= M \frac{E}{\sigma} (P\rho)^{\sigma-1} \left[\int_0^{\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right] \\
&= M \frac{E}{\sigma} (P\rho)^{\sigma-1} \tilde{\varphi}^{\sigma-1} \\
&= M \frac{E}{\sigma} (P\rho\tilde{\varphi})^{\sigma-1} \\
&= M \pi(\tilde{\varphi}) \\
&= M \bar{\pi}.
\end{aligned}$$

Moreover,

$$\Pi = M \frac{E}{\sigma} (P \rho \tilde{\varphi})^{\sigma-1} = \frac{R}{\sigma}.$$

The price index for the consumption bundle can be written as

$$\begin{aligned} P &= \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \\ &= \left[\int_0^\infty p(\varphi)^{1-\sigma} M \mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

Now, using $p(\varphi) = 1/\rho\varphi$, I obtain

$$\begin{aligned} P &= \left[\int_0^\infty \left(\frac{1}{\rho\varphi} \right)^{1-\sigma} M \mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= M^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left[\int_0^\infty \left(\frac{1}{\varphi} \right)^{1-\sigma} \mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= M^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left[\int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{-\frac{1}{\sigma-1}} \\ &= M^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}} \\ &= M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}). \end{aligned}$$

A.4 The ZPC in the closed economy

First of all, define $\tilde{\varphi} \equiv \tilde{\varphi}(\varphi^*) = \left[\int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$ as the weighted average productivity, and note that the average profits can be rewritten as the profits of the firm with productivity $\tilde{\varphi}$:

$$\begin{aligned} \bar{\pi} &\equiv [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^\infty \pi(\varphi) g(\varphi) d\varphi \\ &= [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^\infty \frac{E}{\sigma} (P \rho \varphi)^{\sigma-1} g(\varphi) d\varphi \\ &= E (P \rho)^{\sigma-1} [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^\infty \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{E}{\sigma} (P \rho)^{\sigma-1} \int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \\ &= \frac{E}{\sigma} (P \rho)^{\sigma-1} \tilde{\varphi}^{\sigma-1} \\ &= \frac{E}{\sigma} (P \rho \tilde{\varphi})^{\sigma-1} \\ &= \pi(\tilde{\varphi}). \end{aligned}$$

Given (6), $\pi(\varphi^*) = \pi(\tilde{\varphi}) \left[\frac{\varphi^*}{\tilde{\varphi}} \right]^{\sigma-1} = \bar{\pi} \left[\frac{\varphi^*}{\tilde{\varphi}} \right]^{\sigma-1}$. This, together with the condition for the productivity cutoff $\varphi^* = \{\varphi : \pi(\varphi^*) = \frac{\delta F_D}{\lambda}\}$, means I can derive the zero profit condition as

$$\begin{aligned} \lambda \bar{\pi} \left[\frac{\varphi^*}{\tilde{\varphi}} \right]^{\sigma-1} &= \delta F_D \\ \lambda \bar{\pi} &= \delta F_D \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1}. \end{aligned}$$

A.5 The FEC and the ZPC under the Pareto distribution assumption. Closed economy

Given the Pareto cumulative distribution function $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi} \right)^a$, the FEC can be rewritten as:

$$\begin{aligned} \lambda \bar{\pi} &= \delta w \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right) \\ \lambda \bar{\pi} &= \delta w \left(\frac{F_E}{1 - 1 + \left(\frac{\varphi_m}{\varphi^*} \right)^a} + F_D \right) \\ \lambda \bar{\pi} &= \delta w \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right). \end{aligned}$$

The probability density function of the Pareto distribution is given by:

$$\begin{aligned} g(\varphi) &= G'(\varphi) \\ &= -(-a)(\varphi)^{-a-1}(\varphi_m)^a \\ &= \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi} \right)^a. \end{aligned}$$

Given $a > \sigma - 1$, I can re-write the average productivity as:

$$\begin{aligned}
\tilde{\varphi} \equiv \tilde{\varphi}(\varphi^*) &= \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1 - 1 + \left(\frac{\varphi_m}{\varphi^*}\right)^a} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi}\right)^a d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*}{\varphi_m}\right)^a a \varphi_m^a \int_{\varphi^*}^{\infty} \varphi^{\sigma-1-1-a} d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*)^a}{\sigma - 1 - a} \varphi^{\sigma-1-a} \Big|_{\varphi^*}^{\infty} \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*)^a}{\sigma - 1 - a} (0 - (\varphi^*)^{\sigma-1-a}) \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \varphi^*.
\end{aligned}$$

Using this expression in (13), the ZPC becomes:

$$\begin{aligned}
\lambda \bar{\pi} &= \delta F_D \left[\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\sigma-1} \\
\lambda \bar{\pi} &= \delta F_D \left[\frac{\left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \varphi^*}{\varphi^*} \right]^{\sigma-1} \\
\lambda \bar{\pi} &= \frac{\delta F_D a}{a - \sigma + 1}.
\end{aligned}$$

A.6 Proof of proposition 2: the existence and uniqueness of equilibrium in a closed economy with imperfect creditor protection

If in the system

$$\begin{aligned}
(ZPC) \quad \bar{\pi} &= \frac{\delta F_D a}{\lambda(a - \sigma + 1)} \\
(FEC) \quad \bar{\pi} &= \frac{\delta}{\lambda} \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right)
\end{aligned}$$

φ^* is replaced by φ and $\bar{\pi}$ is replaced by π , then these equations can be graphed in (φ, π) space with FEC increasing in φ with $\pi = \frac{\delta}{\lambda}(F_E + F_D)$ at $\varphi = \varphi_m$ and ZPC constant for all $\varphi \geq \varphi_m$ (see Figure 2). The FEC cuts the ZPC line only once from below when F_E is sufficiently small:

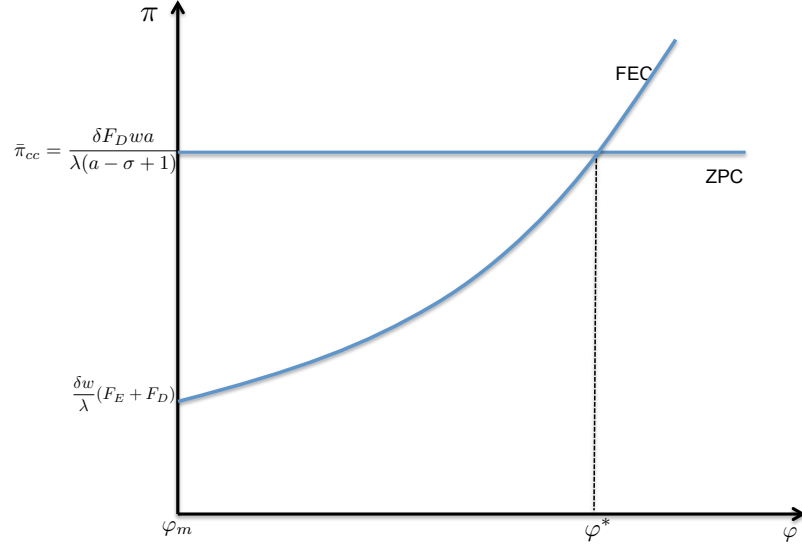


Figure 2: Determination of Equilibrium φ^* and $\bar{\pi}$

$$\begin{aligned}
 \frac{\delta F_D a}{\lambda(a - \sigma + 1)} &> \frac{\delta}{\lambda}(F_E + F_D) \\
 \frac{F_D a}{a - \sigma + 1} &> F_E + F_D \\
 \frac{a}{a - \sigma + 1} &> \frac{F_E}{F_D} + 1 \\
 \frac{a - a + \sigma - 1}{a - \sigma + 1} &> \frac{F_E}{F_D} \\
 \frac{\sigma - 1}{a - \sigma + 1} &> \frac{F_E}{F_D}.
 \end{aligned}$$

In this case $\bar{\pi} = \frac{\delta F_D a}{\lambda(a - \sigma + 1)} > \bar{\pi}$ and $\varphi^* > \varphi_m$, and I can solve for φ^* :

$$\begin{aligned}
 \frac{\delta F_D a}{\lambda(a - \sigma + 1)} &= \frac{\delta}{\lambda} \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right) \\
 \frac{F_D a}{a - \sigma + 1} &= F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \\
 \frac{a}{a - \sigma + 1} &= \frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + 1 \\
 \frac{a - a + \sigma - 1}{a - \sigma + 1} \frac{F_D}{F_E} &= \left(\frac{\varphi^*}{\varphi_m} \right)^a \\
 \varphi_m \left[\frac{\sigma - 1}{a - \sigma + 1} \frac{F_D}{F_E} \right]^{\frac{1}{a}} &= \varphi^*.
 \end{aligned}$$

The fact that the ZPC does not depend on φ^* follows from the Pareto parametrization of the productivity distribution function. Nevertheless, as shown in Melitz (2003), for the system above to have a unique solution a sufficient condition is that $g(\varphi)\varphi/[1 - G(\varphi)]$ be increasing to infinity on $(0, +\infty)$. This is a property of many common families of distributions.

As shown above, φ^* uniquely determines $\tilde{\varphi}(\varphi^*) = \left[\frac{a}{a-\sigma+1} \right]^{\frac{1}{\sigma-1}} \varphi^*$. The FEC also implies

$$\begin{aligned}\bar{\pi} &= \frac{\delta}{\lambda} \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right) \\ M\bar{\pi} &= \frac{M\delta}{\lambda} \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right) \\ \Pi &= \frac{M\delta}{\lambda} \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right).\end{aligned}$$

Given $[1 - G(\varphi^*)]M_e = \delta M$, I can derive the total wages of innovation workers $wL_I = L_I$ as follows:

$$\begin{aligned}L_I &= M_e F_E + M_e [1 - G(\varphi^*)] F_D \\ L_I &= \frac{\delta M}{1 - G(\varphi^*)} [F_E + (1 - G(\varphi^*)) F_D] \\ L_I &= \delta M \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right).\end{aligned}$$

Combining the expressions for Π and L_I with $R = \sigma\Pi$, I obtain $L_I = \lambda\Pi = \frac{\lambda R}{\sigma}$. The wages to production workers in the differentiated sector are given by $wL_P = L_P = R - \Pi = R(\sigma - 1)/\sigma$. Now, using $R = E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E :

$$\begin{aligned}L &= L_I + L_P \\ L &= L_I + L_P \\ L &= \frac{\lambda R}{\sigma} + \frac{R(\sigma - 1)}{\sigma} \\ L &= \frac{\lambda E}{\sigma} + \frac{E(\sigma - 1)}{\sigma} \\ L &= E \frac{\lambda + \sigma - 1}{\sigma} \\ E &= \frac{\sigma L}{\sigma - 1 + \lambda}\end{aligned}$$

where $\frac{\sigma}{\sigma-1+\lambda} > 1$. The aggregate expenditure exceeds thus the labor income wL because this is augmented by the extra rents $V_E \equiv M_e v_E$ that belongs to consumers as owners of the firms.

Using $M_e = \delta M / (1 - G(\varphi^*))$, $\Pi = M\bar{\pi}$ and the expression derived above for v_E , I obtain:

$$\begin{aligned}
V_E &= M_e(1 - G(\varphi^*)) \frac{(1 - \lambda)\bar{\pi}}{\delta} \\
&= \frac{\delta M}{1 - G(\varphi^*)} (1 - G(\varphi^*)) \frac{(1 - \lambda)\bar{\pi}}{\delta} \\
&= M(1 - \lambda)\bar{\pi} \\
&= (1 - \lambda)\Pi.
\end{aligned}$$

Given $\Pi = \frac{R}{\sigma} = \frac{E}{\sigma}$, the expression above becomes $V_E = \frac{(1 - \lambda)E}{\sigma}$. Adding this to $L = \frac{E(\sigma - 1 + \lambda)}{\sigma}$, I get

$$V_E + L = \frac{(1 - \lambda)E}{\sigma} + \frac{E(\sigma - 1 + \lambda)}{\sigma} = \frac{(1 - \lambda)E}{\sigma} + \frac{E\sigma}{\sigma} - \frac{(1 - \lambda)E}{\sigma} = E.$$

Now, given the identity $\bar{\pi} \equiv \frac{\Pi}{M}$ and the equilibrium values for E and $\bar{\pi}$, the number of firms is pinned down by

$$\begin{aligned}
M &= \frac{\Pi}{\bar{\pi}} \\
&= \frac{E}{\sigma\bar{\pi}} \\
&= \frac{L}{(\sigma - 1 + \lambda)\bar{\pi}} \\
&= \frac{L}{(\sigma - 1 + \lambda)} \left[\frac{\delta F_D a}{\lambda(a - \sigma + 1)} \right]^{-1} \\
&= \frac{L\lambda(a - \sigma + 1)}{(\sigma - 1 + \lambda)\delta F_D a}.
\end{aligned}$$

Taking the derivative with respect to λ , I obtain

$$\begin{aligned}
\frac{\partial M}{\partial \lambda} &= \frac{L(a - \sigma + 1)}{\delta F_D a} \frac{\sigma - 1 + \lambda - \lambda}{(\sigma - 1 + \lambda)^2} \\
&= \frac{L}{\delta F_D a} \frac{(a - \sigma + 1)(\sigma - 1)}{(\sigma - 1 + \lambda)^2} > 0.
\end{aligned}$$

Once M is known, the equilibrium price index is also known and given by $P = M^{1/(1 - \sigma)} p(\tilde{\varphi})$.

Given $p(\tilde{\varphi}) = \frac{1}{\rho\tilde{\varphi}} = \frac{1}{\rho\varphi^*} \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{1 - \sigma}}$, this is equal to:

$$P = \left[\frac{L\lambda(a - \sigma + 1)}{(\sigma - 1 + \lambda)\delta F_D a} \right]^{\frac{1}{1 - \sigma}} \frac{1}{\rho\varphi^*} \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{1 - \sigma}} = \left[\frac{L\lambda}{(\sigma - 1 + \lambda)\delta F_D} \right]^{\frac{1}{1 - \sigma}} \frac{1}{\rho\varphi^*}.$$

A.7 Proof of Lemma 2.1: Individual welfare U

As a measure of per-capita welfare, I consider the individual utility U in (1). Given the expressions for $q(\omega)$, this is given by

$$\begin{aligned}
U &= \left[\int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}} \\
&= \frac{e}{P^{1-\sigma}} \left[\int_{\omega \in \Omega} p(\omega)^{-\sigma\rho} d\omega \right]^{\frac{1}{\rho}} \\
&= \frac{e}{P^{1-\sigma}} \left[\int_{\omega \in \Omega} p(\omega)^{-\sigma \frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \\
&= \frac{e}{P^{1-\sigma}} \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{-\frac{\sigma}{1-\sigma}} \\
&= \frac{e}{P^{1-\sigma}} P^{-\sigma} \\
&= \frac{e}{P}
\end{aligned}$$

Using the closed form solutions for $e = E/L = \frac{\sigma}{\sigma-1+\lambda}$ and P , welfare becomes:

$$\begin{aligned}
U &= \frac{\sigma}{\sigma-1+\lambda} \rho \varphi^* \left[\frac{L\lambda}{(\sigma-1+\lambda)\delta F_D} \right]^{\frac{1}{\sigma-1}} \\
&= \sigma \rho \varphi^* \left[\frac{L}{\delta F_D} \right]^{\frac{1}{\sigma-1}} \lambda^{\frac{1}{\sigma-1}} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}}.
\end{aligned}$$

Given that φ^* is constant in λ , in order to study the sign of $\frac{\partial U}{\partial \lambda}$ I can focus on the term $\Lambda \equiv \lambda^{\frac{1}{\sigma-1}} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}}$:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial \lambda} &= \frac{1}{\sigma-1} \lambda^{\frac{1}{\sigma-1}-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}} + \lambda^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}-1} \frac{-1}{[\sigma-1+\lambda]^2} \\
&= \frac{1}{\sigma-1} \lambda^{\frac{1}{\sigma-1}-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}} \left[1 - \frac{\lambda\sigma}{\sigma-1+\lambda} \right] \\
&= \frac{1}{\sigma-1} \lambda^{\frac{1}{\sigma-1}-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}} \frac{\sigma-1+\lambda-\lambda\sigma}{\sigma-1+\lambda} \\
&= \frac{1}{\sigma-1} \lambda^{\frac{1}{\sigma-1}-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}} \frac{\sigma(1-\lambda)-(1-\lambda)}{\sigma-1+\lambda} \\
&= \frac{1}{\sigma-1} \lambda^{\frac{1}{\sigma-1}-1} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{\sigma}{\sigma-1}} \frac{(\sigma-1)(1-\lambda)}{\sigma-1+\lambda}.
\end{aligned}$$

Since $\lambda < 1$ and $\sigma > 1$, all terms in the expression above are positive and $\frac{\partial U}{\partial \lambda} > 0$.

A.8 Sorting condition between domestic firms and exporters

Consider the marginal firms, so those firms whose (IC) and (IC') are binding. In case of a firm that produces only for the domestic market $f(\varphi) = \lambda\pi_D(\varphi)$ and the lender renegation proof condition requires

$$\pi_D(\varphi) \geq \frac{\delta F_D}{\lambda}.$$

Analogously, if there were firms producing for the foreign market only, it should hold that

$$\pi_X(\varphi) \geq \frac{\delta F_X}{\lambda}. \quad (29)$$

I now derive the condition under which, whenever the above condition is met, the firm always finds a creditor willing to also finance the entry on the domestic market. As shown in the text, with $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ this requires

$$\pi_D(\varphi) + \pi_X(\varphi) \geq \frac{\delta[F_D + F_X]}{\lambda}.$$

First, using $\pi_D(\varphi) = \tau^{\sigma-1}\pi_X(\varphi)$, I can rewrite the above condition as:

$$\begin{aligned} \pi_X(\varphi)(1 + \tau^{\sigma-1}) &\geq \frac{\delta[F_D + F_X]}{\lambda} \\ \pi_X(\varphi) &\geq \frac{\delta[F_D + F_X]}{\lambda(1 + \tau^{\sigma-1})}. \end{aligned} \quad (30)$$

Then, I find the condition such that, whenever (29) is met, (30) is also met. In other words, I find the condition for $\frac{\delta F_X}{\lambda} > \frac{\delta[F_D + F_X]}{\lambda(1 + \tau^{\sigma-1})}$:

$$\begin{aligned} F_X &> \frac{F_D + F_X}{1 + \tau^{\sigma-1}} \\ F_X + F_X\tau^{\sigma-1} &> F_D + F_X \\ F_D &< F_X\tau^{\sigma-1} \\ \frac{F_D}{F_X} &< \tau^{\sigma-1}. \end{aligned}$$

A.9 Proof of proposition 3: the foreign market cutoff φ_x^*

In a frictionless world, it is optimal for a firm to enter a market whenever the flow of future profits exceeds the fixed entry cost, meaning when $\frac{\pi_D(\varphi)}{\delta} \geq F_D$ and when $\frac{\pi_X(\varphi)}{\delta} \geq F_X$, for the domestic and foreign market respectively. As shown in the text, when there is imperfect creditor protection ($\lambda < 1$), the firm incentive compatibility constraint and the lender incentive rationality constraint imply that firms will be able to borrow $F_D w$ only when $\frac{\pi_D(\varphi)}{\delta} > \frac{F_D}{\lambda}$. Since $\frac{F_D}{\lambda} > F_D$, this means that there are firms that are sufficiently productive to enter in the perfect credit market case, but that are not productive enough to borrow $F_D w$ in the imperfect credit market case. For the export decision, the incentive compatibility constraint and the

incentive rationality constraint imply that a firm will be able to borrow both F_D and F_E when $\pi_D(\varphi) + \pi_X(\varphi) \geq \frac{\delta[F_D+F_X]}{\lambda}$. Using $\pi_D(\varphi) = \tau^{\sigma-1}\pi_X(\varphi)$, I can rewrite it as $\frac{\pi_X(\varphi)}{\delta} \geq \frac{[F_D+F_X]}{\lambda(1+\tau^{\sigma-1})}$. This condition can be weaker than the one under the assumption of no-credit market frictions. This happens when:

$$\begin{aligned} \frac{[F_D + F_X]}{\lambda(1 + \tau^{\sigma-1})} &< F_X \\ \frac{F_D + F_X}{\lambda(1 + \tau^{\sigma-1})} &< F_X \\ \frac{F_D + F_X}{F_X} &< \lambda(1 + \tau^{\sigma-1}) \\ \lambda &> \hat{\lambda} \equiv \frac{F_D/F_X + 1}{1 + \tau^{\sigma-1}}. \end{aligned} \tag{31}$$

Under this condition, creditors would be willing to finance foreign market entry even when this is unprofitable from the firm's point of view. As a result, only firms with productivity φ such that $\frac{\pi_X(\varphi)}{\delta} \geq F_X$ will indeed enter. When $\lambda \geq \hat{\lambda}$ the foreign market cutoff is thus given by $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \delta F_X\}$. If instead $\lambda < \hat{\lambda}$, then there are firms that would enter the foreign market in the absence of credit frictions but are prevented from doing so and $\varphi_x^* = \{\varphi : \pi_X(\varphi) = \frac{\delta(F_D+F_X)}{\lambda(1+\tau^{\sigma-1})}\}$. Looking at (31), exporters are more likely to be credit constrained the higher the right hand side ($F_D \uparrow$, $F_X \downarrow$ and $\tau \downarrow$) and the lower the left hand side ($\lambda \downarrow$).

A.10 Proof of proposition 4: the optimal contract in the open economy

To solve for the optimal contract, I start from the fact that the firm value

$$\begin{aligned} v_E &= \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi_D(\varphi) - f(\varphi)) \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi \\ &+ \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t (\pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi)) \right] i_x(\varphi) g(\varphi) d\varphi + K \\ &= \int_{\varphi_m}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - \frac{f(\varphi)}{\delta} \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi \\ &+ \int_{\varphi_m}^{\infty} \left[\frac{\pi_D(\varphi) + \pi_X(\varphi)}{\delta} - \frac{f'(\varphi)}{\delta} \right] i_x(\varphi) g(\varphi) d\varphi + K \\ &= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi) + \pi_X(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi \\ &+ K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{f'(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi \end{aligned}$$

is increasing in the term $T' \equiv K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{f'(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi$. Since I am solving for an equilibrium with perfect competition among potential lenders, the firm will

increase T' till the (PC) binds. $v_L = 0$ implies that

$$\begin{aligned} & \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) - F_D \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi \\ & + \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f'(\varphi) - (F_D + F_X) \right] i_x(\varphi) g(\varphi) d\varphi - F_E - K = 0 \\ & \int_{\varphi_m}^{\infty} \left[\frac{f(\varphi)}{\delta} - F_D \right] (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \left[\frac{f'(\varphi)}{\delta} - (F_D + F_X) \right] i_x(\varphi) g(\varphi) d\varphi - F_E = K. \end{aligned}$$

Rearranging the above condition, I get

$$\begin{aligned} & - \int_{\varphi_m}^{\infty} F_D (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} (F_D + F_X) i_x(\varphi) g(\varphi) d\varphi - F_E = \\ & K - \int_{\varphi_m}^{\infty} \frac{f(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{f'(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi. \end{aligned}$$

Using the definition of T' , I obtain

$$\begin{aligned} & - \int_{\varphi_m}^{\infty} F_D (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} (F_D + F_X) i_x(\varphi) g(\varphi) d\varphi - F_E = T' \\ & - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_X i_x(\varphi) g(\varphi) d\varphi - F_E = T'. \end{aligned} \tag{32}$$

Plugging this value into v_E , I obtain

$$\begin{aligned} v_E &= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} (i(\varphi) - i_x(\varphi)) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi) + \pi_X(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi + T' \\ &= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi \\ &+ \int_{\varphi_m}^{\infty} \frac{\pi_X(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_X i_x(\varphi) g(\varphi) d\varphi - F_E \\ &= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_X(\varphi)}{\delta} i_x(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D i(\varphi) g(\varphi) d\varphi \\ &- \int_{\varphi_m}^{\infty} F_X i_x(\varphi) g(\varphi) d\varphi - F_E \\ &= \int_{\varphi_m}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D \right] i(\varphi) g(\varphi) d\varphi + \int_{\varphi_m}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X \right] i_x(\varphi) g(\varphi) d\varphi - F_E. \end{aligned}$$

Note that for $v_L = 0$ the firm's objective function does not depend on K , $f(\varphi)$ and $f'(\varphi)$ separately, but on T' only. It follows that, so far as (LC), (IC), (RP), (IC') and (RP') hold, the condition (32) on T' can be achieved by arbitrarily varying K , $f(\varphi)$ and $f'(\varphi)$. This introduces a degree of freedom in determining the optimal contract and the (IC) and the (IC') do not have to be binding. On the other hand, the firms with the minimum productivity level needed

to obtain the loans F_D and F_X have the incentive to make per-period repayments as high as the (IC) and the (IC') allow for. This makes it easier to meet the lender's (RP) and (RP') conditions. In other words, the (IC) and the (IC') will be binding for the marginal firms:

$$\begin{aligned}\pi_D(\varphi) - f(\varphi) &= (1 - \lambda)\pi_D(\varphi) \\ \pi_D(\varphi) - f(\varphi) &= \pi_D(\varphi) - \lambda\pi_D(\varphi) \\ f(\varphi) &= \lambda\pi_D(\varphi)\end{aligned}$$

for the domestic firm and

$$\begin{aligned}\pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi) &= (1 - \lambda)(\pi_D(\varphi) + \pi_X(\varphi)) \\ \pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi) &= \pi_D(\varphi) + \pi_X(\varphi) - \lambda(\pi_D(\varphi) + \pi_X(\varphi)) \\ f'(\varphi) &= \lambda(\pi_D(\varphi) + \pi_X(\varphi))\end{aligned}$$

for the exporting firm. Using these values in (RP) and (RP'), I obtain

$$\begin{aligned}\sum_{t=0}^{\infty} (1 - \delta)^t f(\varphi) &\geq F_D \\ \frac{\lambda\pi_D(\varphi)}{\delta} &\geq F_D \\ \pi_D(\varphi) &\geq \frac{\delta F_D}{\lambda}.\end{aligned}$$

$$\begin{aligned}\sum_{t=0}^{\infty} (1 - \delta)^t f'(\varphi) &\geq F_D + F_X \\ \frac{\lambda(\pi_D(\varphi) + \pi_X(\varphi))}{\delta} &\geq F_D + F_X \\ \pi_D(\varphi) + \pi_X(\varphi) &\geq \frac{\delta(F_D + F_X)}{\lambda}.\end{aligned}\tag{33}$$

The entry rule for the domestic market is given by $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*$, where φ^* is such that $\pi_D(\varphi^*) = \frac{\delta F_D}{\lambda}$. To solve for the entry rule on the export market, condition (33) must be compared with the first best condition $\pi_X(\varphi) \geq \delta F_X$. If the latter is not met, exporting induces net losses and no firms will be willing to enter the foreign market although the (RP') is satisfied. It follows that $i_x(\varphi) = 1$ if and only if $\varphi \geq \varphi_x^*$, where φ_x^* is given by the larger between $\inf\{\varphi : \pi_D(\varphi) + \pi_X(\varphi) > \frac{\delta(F_D + F_X)}{\lambda}\}$ and the frictionless case cutoff $\inf\{\varphi : \pi_X(\varphi) > \delta F_X\}$ (see

proposition 3). Given the optimal entry rules, the firm's problem reduces thus to:

$$\max_{K, f(\varphi), f'(\varphi)} v_E = \int_{\varphi^*}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi - F_E \quad (\text{P})$$

subject to

$$K = \int_{\varphi^*}^{\varphi_x^*} \left[\frac{f(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{f'(\varphi)}{\delta} - (F_D + F_X) \right] g(\varphi) d\varphi - F_E \geq 0 \quad (\text{LC+PC})$$

$$\delta F_D \leq f(\varphi) \leq \lambda \pi_D(\varphi) \text{ for all } \varphi \in [\varphi^*, \varphi_x^*] \quad (\text{IC+RP})$$

$$\delta(F_D + F_X) \leq f'(\varphi) \leq \lambda(\pi_D(\varphi) + \pi_X(\varphi)) \text{ for all } \varphi \geq \varphi_x^* \quad (\text{IC'+RP'})$$

that, as argued above, does not have a unique solution for the triplet $(K, f(\varphi), f'(\varphi))$.

A.11 Aggregate quantities in the open economy

In the open economy, the price index for the consumption bundle includes all domestic varieties plus the varieties exported by the foreign country. The probability density function of productivity levels on the subset $[\varphi^*, \infty)$ is given by $g(\varphi)/[1 - G(\varphi^*)]$. Because of symmetry, I can write P as:

$$\begin{aligned} P &= \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \\ &= \left[\int_{\varphi^*}^{\infty} p_D(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} p_X(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

Now, using $M_X = p_x M$, $p_x = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$, $p_D(\varphi) = w/\rho\varphi$ and $p_X(\varphi) = w\tau/\rho\varphi$, I obtain

$$\begin{aligned} P &= \left[\int_{\varphi^*}^{\infty} p_D(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} p_X(\varphi)^{1-\sigma} M_X \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= \frac{1}{\rho} \left[M \int_{\varphi^*}^{\infty} \left(\frac{1}{\varphi} \right)^{1-\sigma} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \left(\frac{\tau}{\varphi} \right)^{1-\sigma} \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= \frac{1}{\rho} \left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi + M_X \tau^{1-\sigma} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= \frac{1}{\rho} [M \tilde{\varphi}^{\sigma-1} + M_X \tau^{1-\sigma} \tilde{\varphi}_x^{\sigma-1}]^{\frac{1}{1-\sigma}} \\ &= \frac{1}{\rho} \left[\frac{\bar{M}}{\bar{M}} (M \tilde{\varphi}^{\sigma-1} + M_X (\tau^{-1} \tilde{\varphi}_x)^{\sigma-1}) \right]^{\frac{1}{1-\sigma}} \\ &= \bar{M}^{\frac{1}{1-\sigma}} \frac{1}{\rho \bar{\varphi}} \\ &= \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi}) \end{aligned}$$

where $\bar{M} \equiv M + M_X$, $\tilde{\varphi} \equiv \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$, $\tilde{\varphi}_x \equiv \left[\frac{1}{1-G(\varphi_x^*)} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$ and $\bar{\varphi} \equiv \left\{ \frac{1}{\bar{M}} [M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \right\}^{\frac{1}{\sigma-1}}$. Analogously, using $r_D(\varphi) = E(P\rho\varphi)^{\sigma-1}$ and $r_X(\varphi) = E\left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1}$, the aggregate revenue is given by

$$\begin{aligned}
R &= \int_{\varphi^*}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} r_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} r_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} E(P\rho\varphi)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} E\left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= E(P\rho)^{\sigma-1} \left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \tau^{1-\sigma} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \right] \\
&= E(P\rho)^{\sigma-1} [M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \\
&= ME(P\rho\tilde{\varphi})^{\sigma-1} + M_X E\left(P\frac{\rho\tilde{\varphi}_x}{\tau}\right)^{\sigma-1} \\
&= Mr_D(\tilde{\varphi}) + M_X r_X(\tilde{\varphi}_x).
\end{aligned}$$

Dividing by M , the average revenue \bar{r} is given by:

$$\begin{aligned}
\bar{r} &= \frac{R}{M} = r_D(\tilde{\varphi}) + \frac{M_X}{M} r_X(\tilde{\varphi}_x) \\
&= r_D(\tilde{\varphi}) + p_x r_X(\tilde{\varphi}_x).
\end{aligned}$$

Alternatively, using the definition of $\bar{\varphi}$, I obtain

$$\begin{aligned}
R &= E\left(P\frac{\rho}{w}\right)^{\sigma-1} [M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \\
&= \bar{M} E\left(P\frac{\rho}{w}\right)^{\sigma-1} \left[\frac{1}{\bar{M}} (M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}) \right] \\
&= \bar{M} E\left(P\frac{\rho\bar{\varphi}}{w}\right)^{\sigma-1} \\
&= \bar{M} r_D(\bar{\varphi}).
\end{aligned}$$

Given $\pi_D(\varphi) = \frac{E}{\sigma} (P\rho\varphi)^{\sigma-1}$ and $\pi_X(\varphi) = \frac{E}{\sigma} \left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1}$, following the same steps as above I can derive

$$\begin{aligned}
\Pi &= \int_{\varphi^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \pi_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \pi_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} \frac{E}{\sigma} (P\rho\varphi)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \frac{E}{\sigma} \left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \frac{E}{\sigma} (P\rho)^{\sigma-1} \left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \tau^{1-\sigma} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \right] \\
&= \frac{E}{\sigma} (P\rho)^{\sigma-1} [M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \\
&= M \frac{E}{\sigma} (P\rho\tilde{\varphi})^{\sigma-1} + M_X \frac{E}{\sigma} \left(P\frac{\rho\tilde{\varphi}_x}{\tau}\right)^{\sigma-1} \\
&= M\pi_D(\tilde{\varphi}) + M_X\pi_X(\tilde{\varphi}_x).
\end{aligned}$$

Dividing by M , the average profits are given by:

$$\bar{\pi} = \frac{\Pi}{M} = \pi_D(\tilde{\varphi}) + p_x \pi_X(\tilde{\varphi}_x).$$

Finally, since $\pi_D(\varphi) = \frac{E}{\sigma} (P\rho\varphi)^{\sigma-1} = \frac{r_D(\varphi)}{\sigma}$ and $\pi_X(\varphi) = \frac{E}{\sigma} \left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1} = \frac{r_X(\varphi)}{\sigma}$ for all φ , $\Pi = M\pi_D(\tilde{\varphi}) + M_X\pi_X(\tilde{\varphi}_x) = M\frac{r_D(\tilde{\varphi})}{\sigma} + M_X\frac{r_X(\tilde{\varphi}_x)}{\sigma} = \frac{R}{\sigma}$.

A.12 The ZPC in the open economy

Using (6), I can express the profits of the marginal firms in terms of the average profits: $\pi_D(\varphi^*) = \pi_D(\tilde{\varphi}) \left[\frac{\varphi^*}{\tilde{\varphi}}\right]^{\sigma-1}$ and $\pi_X(\varphi_x^*) = \pi_X(\tilde{\varphi}_x) \left[\frac{\varphi_x^*}{\tilde{\varphi}_x}\right]^{\sigma-1}$. It then follows:

$$\begin{aligned}
\pi_D(\varphi^*) &= \frac{\delta F_D}{\lambda} \Leftrightarrow \pi_D(\tilde{\varphi}) = \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1} \\
\pi_X(\varphi_x^*) &= \delta F_X \Leftrightarrow \pi_X(\tilde{\varphi}_x) = \delta F_X \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1} \\
\pi_X(\varphi_x^*) &= \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})} \Leftrightarrow \pi_X(\tilde{\varphi}_x) = \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})} \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1}
\end{aligned}$$

where the second row refers to the case $\lambda \geq \hat{\lambda}$ and the third row to the case $\lambda < \hat{\lambda}$. Together with (20), the above conditions allow me to express the zero profit condition in terms of the overall average profits:

$$\bar{\pi} = \begin{cases} \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1} + p_x \delta F_X \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1} & \text{if } \lambda \geq \hat{\lambda}, \\ \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1} + p_x \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})} \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1} & \text{if } \lambda < \hat{\lambda}. \end{cases}$$

A.13 The ratio φ^*/φ_x^*

When $\lambda < \hat{\lambda}$, the domestic market cutoff is defined by $\pi_D(\varphi^*) = \frac{\delta F_D}{\lambda}$ and the foreign market cutoff by $\pi_X(\varphi_x^*) = \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})}$. Combining the two conditions, I get:

$$\begin{aligned} \frac{\pi_D(\varphi^*)}{\pi_X(\varphi_x^*)} &= \frac{\frac{\delta F_D}{\lambda}}{\frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})}} \\ \frac{\frac{E}{\sigma} \left(P \frac{\rho \varphi^*}{w} \right)^{\sigma-1}}{\frac{E}{\sigma} \left(P \frac{\rho \varphi_x^*}{\tau} \right)^{\sigma-1}} &= \frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \\ \left(\frac{\varphi^* \tau}{\varphi_x^*} \right)^{\sigma-1} &= \frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \\ \frac{\varphi^*}{\varphi_x^*} &= \tau^{-1} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{1}{\sigma-1}}, \end{aligned}$$

which does not depend on λ . Notice that the term $\tau^{-1}(1 + \tau^{\sigma-1})^{\frac{1}{\sigma-1}}$, can be rewritten as

$$\tau^{-\frac{\sigma-1}{\sigma-1}}(1 + \tau^{\sigma-1})^{\frac{1}{\sigma-1}} = (\tau^{-(\sigma-1)} + 1)^{\frac{1}{\sigma-1}}$$

which is decreasing in τ . It follows that also φ^*/φ_x^* is also decreasing in τ .

When $\lambda \geq \hat{\lambda}$, the domestic market cutoff is again defined by $\pi_D(\varphi^*) = \frac{\delta F_D}{\lambda}$ and the foreign market cutoff by $\pi_X(\varphi_x^*) = \delta F_X$. Combining the two conditions I get:

$$\begin{aligned} \frac{\pi_D(\varphi^*)}{\pi_X(\varphi_x^*)} &= \frac{\frac{\delta F_D}{\lambda}}{\delta F_X} \\ \frac{\frac{E}{\sigma} (P \rho \varphi^*)^{\sigma-1}}{\frac{E}{\sigma} \left(P \frac{\rho \varphi_x^*}{\tau} \right)^{\sigma-1}} &= \frac{F_D}{\lambda F_X} \\ \left(\frac{\varphi^* \tau}{\varphi_x^*} \right)^{\sigma-1} &= \frac{F_D}{\lambda F_X} \\ \frac{\varphi^*}{\varphi_x^*} &= \tau^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

It immediately follows that φ^*/φ_x^* is now decreasing in both λ and τ . Moreover when $\lambda = 1$ then the value is the same as in the frictionless setup, while when $\lambda = \hat{\lambda} \equiv \frac{F_D/F_X + 1}{1 + \tau^{\sigma-1}}$ the two expressions computed above have the same value.

A.14 The FEC in the open economy

In the text, I claim that entry will continue till $K_{max} = 0$. K_{max} is achieved when $f(\varphi) = \lambda \pi_D(\varphi)$ for all $\varphi^* \leq \varphi < \varphi_x^*$ and $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ for all $\varphi \geq \varphi_x^*$. Plugging these

values into (17), I obtain:

$$\begin{aligned} K_{max} &= \int_{\varphi^*}^{\varphi_x^*} \left[\frac{\lambda \pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\lambda(\pi_D(\varphi) + \pi_X(\varphi))}{\delta} - (F_D + F_X) \right] g(\varphi) d\varphi - F_E \\ &= \int_{\varphi^*}^{\infty} \left[\frac{\lambda \pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\lambda \pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi - F_E. \end{aligned}$$

The FEC is thus given by

$$\begin{aligned} &\int_{\varphi^*}^{\infty} \left[\frac{\lambda \pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\lambda \pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi = F_E \\ &\int_{\varphi^*}^{\infty} \frac{\lambda \pi_D(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi^*)) F_D + \int_{\varphi_x^*}^{\infty} \frac{\lambda \pi_X(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi_x^*)) F_X = F_E. \end{aligned}$$

Using the definitions $\pi_D(\tilde{\varphi}) \equiv [1 - G(\varphi^*)]^{-1} \int_{\varphi^*}^{\infty} \pi_D(\varphi) g(\varphi) d\varphi$, $\pi_X(\tilde{\varphi}_x) \equiv [1 - G(\varphi_x^*)]^{-1} \int_{\varphi_x^*}^{\infty} \pi_X(\varphi) g(\varphi) d\varphi$ and $\bar{\pi} \equiv \pi_D(\tilde{\varphi}) + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \pi_X(\tilde{\varphi}_x)$, I can rewrite the above condition as:

$$\begin{aligned} &\frac{\lambda \pi_D(\tilde{\varphi})}{\delta} (1 - G(\varphi^*)) + \frac{\lambda \pi_X(\tilde{\varphi}_x)}{\delta} (1 - G(\varphi_x^*)) = F_D (1 - G(\varphi^*)) + F_X (1 - G(\varphi_x^*)) + F_E \\ &\frac{\lambda \pi_D(\tilde{\varphi})}{\delta} + \frac{\lambda \pi_X(\tilde{\varphi}_x)}{\delta} \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} = F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \\ &\lambda \bar{\pi} = \delta F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)}. \end{aligned}$$

The same definitions used above can be used to find the equilibrium value of

$$\begin{aligned} v_E &= \int_{\varphi^*}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D \right] g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X \right] g(\varphi) d\varphi - F_E \\ &= \frac{\pi_D(\tilde{\varphi})}{\delta} (1 - G(\varphi^*)) + \frac{\pi_X(\tilde{\varphi}_x)}{\delta} (1 - G(\varphi_x^*)) - [F_D (1 - G(\varphi^*)) + F_X (1 - G(\varphi_x^*)) + F_E] \\ &= (1 - G(\varphi^*)) \left\{ \frac{\pi_D(\tilde{\varphi})}{\delta} + \frac{\pi_X(\tilde{\varphi}_x)}{\delta} \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} - \left[F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \right] \right\} \\ &= \frac{1 - G(\varphi^*)}{\delta} \left\{ \bar{\pi} - \delta \left[F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \right] \right\}. \end{aligned}$$

Given the FEC, this becomes

$$\begin{aligned} v_E &= \frac{1 - G(\varphi^*)}{\delta} [\bar{\pi} - \lambda \bar{\pi}] \\ &= (1 - G(\varphi^*)) \frac{(1 - \lambda) \bar{\pi}}{\delta}. \end{aligned}$$

A.15 The FEC and the ZPC under the Pareto distribution assumption. Open economy

The expressions for $\tilde{\varphi}$ and $\tilde{\varphi}_x$ under the Pareto distribution assumption are given by:

$$\begin{aligned}
\tilde{\varphi} &\equiv \tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1 - 1 + \left(\frac{\varphi_m}{\varphi^*} \right)^a} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi} \right)^a d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*}{\varphi_m} \right)^a a \varphi_m^a \int_{\varphi^*}^{\infty} \varphi^{\sigma-1-1-a} d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*)^a}{\sigma - 1 - a} \varphi^{\sigma-1-a} \Big|_{\varphi^*}^{\infty} \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \varphi^*
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\varphi}_x &\equiv \tilde{\varphi}(\varphi_x^*) = \left[\frac{1}{1 - G(\varphi_x^*)} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1 - 1 + \left(\frac{\varphi_m}{\varphi_x^*} \right)^a} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi} \right)^a d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi_x^*}{\varphi_m} \right)^a a \varphi_m^a \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1-1-a} d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi_x^*)^a}{\sigma - 1 - a} \varphi^{\sigma-1-a} \Big|_{\varphi_x^*}^{\infty} \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \varphi_x^*.
\end{aligned}$$

As for the ZPC, plugging these equations into (21), together with $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi}\right)^a$, I obtain

$$\begin{aligned}
\bar{\pi} &= \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1} + p_x \delta F_X \left[\frac{\tilde{\varphi}_x}{\varphi_x^*} \right]^{\sigma-1} \\
&= \frac{\delta F_D}{\lambda} \left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi^*}{\varphi^*} \right]^{\sigma-1} + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \delta F_X \left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi_x^*}{\varphi_x^*} \right]^{\sigma-1} \\
&= \frac{\delta F_D a}{\lambda(a - \sigma + 1)} + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \frac{\delta F_X a}{a - \sigma + 1} \\
&= \frac{\delta F_D a}{\lambda(a - \sigma + 1)} + \frac{\left(\frac{\varphi_m}{\varphi^*}\right)^a}{\left(\frac{\varphi_m}{\varphi^*}\right)^a} \frac{\delta F_X a}{a - \sigma + 1} \\
&= \frac{\delta a}{a - \sigma + 1} \left[\frac{F_D}{\lambda} + \left(\frac{\varphi^*}{\varphi_x^*}\right)^a F_X \right]
\end{aligned}$$

when $\lambda \geq \hat{\lambda}$ and

$$\begin{aligned}
\bar{\pi} &= \frac{\delta F_D}{\lambda} \left[\frac{\tilde{\varphi}}{\varphi^*} \right]^{\sigma-1} + p_x \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})} \left[\frac{\tilde{\varphi}_x}{\varphi_x^*} \right]^{\sigma-1} \\
&= \frac{\delta F_D}{\lambda} \left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi^*}{\varphi^*} \right]^{\sigma-1} + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \frac{\delta(F_D + F_X)}{\lambda(1 + \tau^{\sigma-1})} \left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi_x^*}{\varphi_x^*} \right]^{\sigma-1} \\
&= \frac{\delta F_D a}{\lambda(a - \sigma + 1)} + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} \frac{\delta(F_D + F_X) a}{\lambda(1 + \tau^{\sigma-1})(a - \sigma + 1)} \\
&= \frac{\delta F_D a}{\lambda(a - \sigma + 1)} + \frac{\left(\frac{\varphi_m}{\varphi^*}\right)^a}{\left(\frac{\varphi_m}{\varphi^*}\right)^a} \frac{\delta(F_D + F_X) a}{\lambda(1 + \tau^{\sigma-1})(a - \sigma + 1)} \\
&= \frac{\delta a}{\lambda(a - \sigma + 1)} \left[F_D + \left(\frac{\varphi^*}{\varphi_x^*}\right)^a \frac{F_D + F_X}{1 + \tau^{\sigma-1}} \right]
\end{aligned}$$

when $\lambda < \hat{\lambda}$. Now given (22), the above conditions further simplify to:

$$\begin{aligned}
\bar{\pi} &= \frac{\delta a}{a - \sigma + 1} \left\{ \frac{F_D}{\lambda} + \left[\tau \left(\frac{\lambda F_X}{F_D} \right)^{\frac{1}{\sigma-1}} \right]^{-a} F_X \right\} \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a}{\sigma-1}} \frac{\lambda F_X}{F_D} \right] \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a}{\sigma-1} - 1} \right] \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \right]
\end{aligned}$$

and

$$\begin{aligned}
\bar{\pi} &= \frac{\delta a}{\lambda(a - \sigma + 1)} \left\{ F_D + \left[\tau \left(\frac{F_D + F_X}{F_D(1 + \tau^{\sigma-1})} \right)^{\frac{1}{\sigma-1}} \right]^{-a} \frac{F_D + F_X}{1 + \tau^{1-\sigma}} \right\} \\
&= \frac{\delta a}{\lambda(a - \sigma + 1)} \left\{ F_D + \tau^{-a} \left(\frac{F_D + F_X}{F_D(1 + \tau^{\sigma-1})} \right)^{\frac{-a}{\sigma-1}} F_D \frac{F_D + F_X}{F_D(1 + \tau^{1-\sigma})} \right\} \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left\{ 1 + \tau^{-a} \left(\frac{F_D + F_X}{F_D(1 + \tau^{\sigma-1})} \right)^{\frac{-a}{\sigma-1}} \frac{F_D + F_X}{F_D(1 + \tau^{1-\sigma})} \right\} \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left\{ 1 + \tau^{-a} \left(\frac{F_D + F_X}{F_D(1 + \tau^{\sigma-1})} \right)^{\frac{-a}{\sigma-1} + 1} \right\} \\
&= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left\{ 1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \right\}
\end{aligned}$$

respectively.

As for the FEC

$$\begin{aligned}
\bar{\pi} &= \frac{\delta}{\lambda} \left[\frac{F_E}{1 - G(\varphi^*)} + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + F_D \right] \\
&= \frac{\delta}{\lambda} \left[\frac{F_E}{\left(\frac{\varphi_m}{\varphi^*} \right)^a} + F_X \frac{\left(\frac{\varphi_m}{\varphi^*} \right)^a}{\left(\frac{\varphi_m}{\varphi^*} \right)^a} + F_D \right] \\
&= \frac{\delta}{\lambda} \left[F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_X \left(\frac{\varphi^*}{\varphi_x^*} \right)^a + F_D \right].
\end{aligned}$$

Again, from (22) I obtain:

$$\begin{aligned}
\bar{\pi} &= \frac{\delta}{\lambda} \left[F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_X \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a}{\sigma-1}} + F_D \right] \\
&= \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{\frac{-a}{\sigma-1}} \frac{F_X}{F_D} \left(\frac{F_D}{F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right] \\
&= \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{\frac{-a}{\sigma-1} - 1 + 1} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} + 1 \right] \\
&= \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} + 1 \right]
\end{aligned}$$

when $\lambda \geq \hat{\lambda}$ and

$$\begin{aligned}\bar{\pi} &= \frac{\delta}{\lambda} \left[F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_X \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + F_D \right] \\ &= \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right]\end{aligned}$$

when $\lambda < \hat{\lambda}$.

A.16 Proof of proposition 5: the existence and uniqueness of equilibrium in an open economy with imperfect creditor protection

First, I consider (23) when $0 < \lambda < \hat{\lambda}$:

$$\begin{aligned}(FEC) \quad \bar{\pi} &= \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right] \\ (ZPC) \quad \bar{\pi} &= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \right].\end{aligned}$$

If φ^* is replaced by φ and $\bar{\pi}$ is replaced by π , I can graph FEC and ZPC in (φ, π) space. The FEC is increasing in φ with $\pi = \frac{\delta a F_D}{\lambda} \left[\frac{F_E}{F_D} + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right]$ at $\varphi = \varphi_m$ and the ZPC is constant in φ for all $\varphi \geq \varphi_m$. Considering only the case where not all firms start production, implies:

$$\begin{aligned}\frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \right] &> \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right] \\ \frac{a}{a - \sigma + 1} \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \right] &> \frac{F_E}{F_D} + \left[1 + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} \right].\end{aligned}$$

Rearranging, I obtain

$$\begin{aligned}\frac{a}{a - \sigma + 1} - 1 + \frac{a \tau^{-a}}{a - \sigma + 1} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} - \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} &> \frac{F_E}{F_D} \\ \frac{\sigma - 1}{a - \sigma + 1} + \frac{a \tau^{-a}}{a - \sigma + 1} \left(\frac{F_D}{F_X} \frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} - \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D}{F_X} \frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} &> \frac{F_E}{F_D} \\ \frac{\sigma - 1}{a - \sigma + 1} + \frac{a \tau^{-a}}{a - \sigma + 1} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} - \tau^{-a} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} &> \frac{F_E}{F_D} \\ \frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma - 1}} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a - \sigma + 1} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{-1} - 1 \right] &> \frac{F_E}{F_D}\end{aligned}\tag{34}$$

Under this assumption, which holds when F_E is sufficiently small, the FEC cuts the ZPC line only once from below and $\varphi^* > \varphi_m$ and $\bar{\pi} = \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$.

Next, I consider (24) when $\hat{\lambda} \leq \lambda < 1$:

$$(FEC) \quad \bar{\pi} = \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right]$$

$$(ZPC) \quad \bar{\pi} = \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right].$$

If φ^* is replaced by φ and $\bar{\pi}$ is replaced by π , I can graph FEC and ZPC in (φ, π) space. FEC is increasing in φ with $\pi = \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} + \left(\tau \lambda^{\frac{1}{\sigma-1}} \right)^{-a} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right]$ at $\varphi = \varphi_m$ and ZPC is constant in φ for all $\varphi \geq \varphi_m$. As before, I will consider only the case in which not all firms are efficient enough to start production. This happens when:

$$\begin{aligned} \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] &> \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] \\ \frac{a}{a-\sigma+1} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] &> \frac{F_E}{F_D} + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \\ \frac{a}{a-\sigma+1} - 1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) &> \frac{F_E}{F_D} \\ \frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) &> \frac{F_E}{F_D}. \end{aligned} \quad (35)$$

Under this assumption, which holds when F_E is sufficiently small, the FEC cuts the ZPC only once from below and $\varphi^* > \varphi_m$ and $\bar{\pi} = \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$.

Once φ^* is known, (22) uniquely pins down φ_x^* . I can then solve for $\tilde{\varphi} = \left[\frac{a}{a-\sigma+1} \right]^{\frac{1}{\sigma-1}} \varphi^*$ and for $\tilde{\varphi}_x = \left[\frac{a}{a-\sigma+1} \right]^{\frac{1}{\sigma-1}} \varphi_x^*$.

The FEC (19) also implies that

$$\begin{aligned} \lambda \bar{\pi} &= \delta \left(F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \right) \\ \bar{\pi} M &= \frac{\delta M}{\lambda} \left(F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \right) \\ \Pi &= \frac{\delta M}{\lambda} \left(F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} + \frac{F_E}{1 - G(\varphi^*)} \right). \end{aligned}$$

Given $[1 - G(\varphi^*)]M_e = \delta M$ and $M_X = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)} M$, I can derive the total demand of innovation

workers L_I as follows:

$$\begin{aligned}
L_I &= M_e[1 - G(\varphi^*)]F_D + M_e[1 - G(\varphi_x^*)]F_X + M_e F_E \\
L_I &= \frac{\delta M}{1 - G(\varphi^*)}[(1 - G(\varphi^*))F_D + (1 - G(\varphi_x^*))F_X + F_E] \\
L_I &= \delta M \left(F_D + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}F_X + \frac{F_E}{1 - G(\varphi^*)} \right).
\end{aligned}$$

Combining the expressions derived above together with $R = \sigma\Pi$, I obtain $L_I = \lambda\Pi = \frac{\lambda R}{\sigma}$. The demand of production workers is given by $L_P = R - \Pi = R(\sigma - 1)/\sigma$. Now, using $R = E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E :

$$\begin{aligned}
L &= L_I + L_P \\
L &= \frac{\lambda R}{\sigma} + \frac{R(\sigma - 1)}{\sigma} \\
L &= \frac{\lambda E}{\sigma} + \frac{E(\sigma - 1)}{\sigma} \\
\sigma L &= \lambda E + E\sigma - E \\
\sigma L &= E(\sigma - 1 + \lambda) \\
\frac{\sigma L}{\sigma - 1 + \lambda} &= E > L.
\end{aligned}$$

The difference between the aggregate expenditure E and the labor income $wL = L$ is given by the rents gained by new entrants as a result of credit market frictions. The rents $V_E \equiv M_e v_E$ are divided among the consumers who own shares in firms' profits. Using $M_e = \delta M/(1 - G(\varphi^*))$, $\Pi = M\bar{\pi}$ and the expression derived above for v_E , I obtain:

$$\begin{aligned}
V_E &= M_e(1 - G(\varphi^*))\frac{(1 - \lambda)\bar{\pi}}{\delta} \\
&= \frac{\delta M}{1 - G(\varphi^*)}(1 - G(\varphi^*))\frac{(1 - \lambda)\bar{\pi}}{\delta} \\
&= M(1 - \lambda)\bar{\pi} \\
&= (1 - \lambda)\Pi.
\end{aligned}$$

Given $\Pi = \frac{R}{\sigma} = \frac{E}{\sigma}$, the expression above becomes $V_E = \frac{(1 - \lambda)E}{\sigma}$. Adding this to $L = \frac{E(\sigma - 1 + \lambda)}{\sigma}$, I obtain

$$\frac{(1 - \lambda)E}{\sigma} + \frac{E(\sigma - 1 + \lambda)}{\sigma} = \frac{(1 - \lambda)E}{\sigma} + \frac{E\sigma}{\sigma} - \frac{(1 - \lambda)E}{\sigma} = E.$$

Given the identity $\bar{\pi} \equiv \frac{\Pi}{M}$, the number of firms in equilibrium is given by $M = \frac{\Pi}{\bar{\pi}} = \frac{L}{(\sigma - (1 - \lambda))\bar{\pi}}$. Once M is known, I can solve for $M_X = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}M$ and then for $\bar{M} \equiv M + M_X$. I can thus pin down the equilibrium values for $\bar{\varphi} \equiv \left\{ \frac{1}{\bar{M}}[M\tilde{\varphi}^{\sigma-1} + M_x(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \right\}^{\frac{1}{\sigma-1}}$ and for $P = \bar{M}^{1/(1-\sigma)}p_D(\bar{\varphi}) = \bar{M}^{1/(1-\sigma)}\frac{w}{\rho\bar{\varphi}}$.

A.17 Proof of proposition 6: steady-state analysis with respect to λ

From (23), when $0 < \lambda < \hat{\lambda}$, the closed-form solution for φ^* is given by:

$$\begin{aligned}
\frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 \right] &= \frac{\delta a F_D}{\lambda(a - \sigma + 1)} \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \right] \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} + 1 &= \frac{a}{a - \sigma + 1} \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1} - 1} \right] \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{a - a + \sigma - 1}{a - \sigma + 1} + \frac{a}{a - \sigma + 1} \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1} - 1} - \tau^{-a} \frac{F_X}{F_D} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a - \sigma + 1} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{-1} - \frac{F_X}{F_D} \right] \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{F_D}{F_X} \frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a - \sigma + 1} \left(\frac{F_D}{F_X} \frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{-1} - \frac{F_X}{F_D} \right] \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a - \sigma + 1} \frac{F_X}{F_D} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{-1} - \frac{F_X}{F_D} \right] \\
\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a}{\sigma-1} - 1} \left[\frac{a}{a - \sigma + 1} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{-1} - 1 \right] \\
\varphi^* &= \varphi_m \left\{ \left[\frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \left(\frac{a}{a - \sigma + 1} \frac{F_D/F_X + 1}{1 + \tau^{\sigma-1}} - 1 \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}}.
\end{aligned}$$

The threshold is thus constant in λ . Note that, because of condition (34), $\varphi^* > \varphi_m > 0$.

Moreover, given $\hat{\lambda} \equiv \frac{F_D/F_X + 1}{1 + \tau^{\sigma-1}}$, the above can be rewritten as

$$\begin{aligned}
\varphi^* &= \varphi_m \left\{ \left[\frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \hat{\lambda}^{\frac{-a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \left(\frac{a \hat{\lambda}}{a - \sigma + 1} - 1 \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \\
&= \varphi_m \left\{ \left[\frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \hat{\lambda}^{\frac{-a}{\sigma-1} + 1} \left(\frac{F_D}{F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \left(\frac{a}{a - \sigma + 1} - \frac{1}{\hat{\lambda}} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \\
&= \varphi_m \left\{ \left[\frac{\sigma - 1}{a - \sigma + 1} + \tau^{-a} \left(\frac{F_D}{\hat{\lambda} F_X} \right)^{\frac{a - \sigma + 1}{\sigma-1}} \left(\frac{a}{a - \sigma + 1} - \frac{1}{\hat{\lambda}} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \tag{36}
\end{aligned}$$

When $\hat{\lambda} \leq \lambda < 1$, the closed form solution for φ^* is given by solving (24):

$$\begin{aligned} \frac{\delta F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] &= \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] \\ \frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 &= \frac{a}{a-\sigma+1} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right] \\ \frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 &= \frac{a}{a-\sigma+1} + \frac{a\tau^{-a}}{a-\sigma+1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \end{aligned}$$

Rearranging, I obtain

$$\begin{aligned} \frac{F_E}{F_D} \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \frac{a-a+\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \\ \left(\frac{\varphi^*}{\varphi_m} \right)^a &= \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \\ \varphi^* &= \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}}. \end{aligned}$$

Again, from (35), it follows that $\varphi^* > \varphi_m > 0$. Moreover, note that when $\lambda = \hat{\lambda}$, φ^* has the same value as in (36). To study the sign of $\partial\varphi^*/\partial\lambda$, I can focus on the partial derivative of the term $\lambda^{\frac{-a+\sigma-1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) = \lambda^{\frac{-a}{\sigma-1}} \left(\frac{a\lambda}{a-\sigma+1} - 1 \right)$. This is equal to:

$$\begin{aligned} & -\frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}-1} \left(\frac{a\lambda}{a-\sigma+1} - 1 \right) + \lambda^{\frac{-a}{\sigma-1}} \frac{a}{a-\sigma+1} = \\ & -\frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}} \frac{a}{a-\sigma+1} + \frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}-1} + \lambda^{\frac{-a}{\sigma-1}} \frac{a}{a-\sigma+1} = \\ & \lambda^{\frac{-a}{\sigma-1}} \frac{a}{a-\sigma+1} \left(1 - \frac{a}{\sigma-1} \right) + \frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}-1} = \\ & \lambda^{\frac{-a}{\sigma-1}} \frac{a}{a-\sigma+1} \left(-\frac{a-\sigma+1}{\sigma-1} \right) + \frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}-1} = \\ & -\lambda^{\frac{-a}{\sigma-1}} \frac{a}{\sigma-1} + \frac{a}{\sigma-1} \lambda^{\frac{-a}{\sigma-1}-1} = \\ & \lambda^{\frac{-a}{\sigma-1}} \frac{a}{\sigma-1} \left(\frac{1}{\lambda} - 1 \right) > 0. \end{aligned}$$

In the proof of proposition 5, I derived $M = \frac{\Pi}{\bar{\pi}} = \frac{L}{(\sigma-1+\lambda)\bar{\pi}}$. Given the equilibrium value of $\bar{\pi}$, I can solve for the closed form solution. When $0 < \lambda < \hat{\lambda}$, $\bar{\pi} = \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$

and

$$\begin{aligned}
M &= \frac{L}{(\sigma - 1 + \lambda)\bar{\pi}} \\
&= \frac{L}{(\sigma - 1 + \lambda)} \frac{\lambda(a - \sigma + 1)}{\delta a F_D \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\
&= \frac{L\lambda(a - \sigma + 1)}{\delta a F_D(\sigma - 1 + \lambda) \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]}. \tag{37}
\end{aligned}$$

The sign of $\partial M / \partial \lambda$ is pinned down by the sign of the partial derivative of the term $\frac{\lambda}{\sigma-1+\lambda}$ with respect to λ . The derivative is given by

$$\frac{\sigma - 1 + \lambda - \lambda}{[\sigma - 1 + \lambda]^2} = \frac{\sigma - 1}{[\sigma - 1 + \lambda]^2} > 0.$$

When $\hat{\lambda} \leq \lambda < 1$, $\bar{\pi} = \frac{\delta a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$ and

$$\begin{aligned}
M &= \frac{L}{(\sigma - 1 + \lambda)\bar{\pi}} \\
&= \frac{wL}{(\sigma - 1 + \lambda)} \frac{\lambda(a - \sigma + 1)}{\delta a F_D \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\
&= \frac{L\lambda(a - \sigma + 1)}{\delta a F_D(\sigma - 1 + \lambda) \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]}. \tag{38}
\end{aligned}$$

As before, when λ increases, M increases because of the effect of the term $\frac{\lambda}{\sigma-1+\lambda}$. Moreover, M increases also because of the additional lambda within the squared brackets ($\lambda \uparrow \Rightarrow$ denominator $\downarrow \Rightarrow M \uparrow$, given the assumption $a > \sigma - 1$). As a result $\partial M / \partial \lambda > 0$. Finally, note that setting $\lambda = \hat{\lambda}$ in (37) yields

$$\begin{aligned}
M &= \frac{L\hat{\lambda}(a - \sigma + 1)}{\delta a F_D(\sigma - 1 + \hat{\lambda}) \left[1 + \tau^{-a} \left(\frac{F_D(1 + \tau^{\sigma-1})}{F_D + F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\
&= \frac{L\hat{\lambda}(a - \sigma + 1)}{\delta a F_D(\sigma - 1 + \hat{\lambda}) \left[1 + \tau^{-a} \left(\frac{F_D}{F_X} \frac{1 + \tau^{\sigma-1}}{F_D/F_X + 1} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\
&= \frac{L\hat{\lambda}(a - \sigma + 1)}{\delta a F_D(\sigma - 1 + \hat{\lambda}) \left[1 + \tau^{-a} \left(\frac{F_D}{\hat{\lambda} F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]}
\end{aligned}$$

which is the same value as in (38) for $\lambda = \hat{\lambda}$.

Given $M_X = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}M$, I can solve for the share of exporting firms as:

$$\frac{M_x}{M} = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)} = \frac{\left(\frac{\varphi_m}{\varphi_x^*}\right)^a}{\left(\frac{\varphi_m}{\varphi^*}\right)^a} = \left(\frac{\varphi^*}{\varphi_x^*}\right)^a.$$

From

$$\frac{\varphi^*}{\varphi_x^*} = \begin{cases} \tau^{-1} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \tau^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } \hat{\lambda} \leq \lambda \leq 1. \end{cases}$$

it immediately follows that $\frac{\partial(M_X/M)}{\partial\lambda} \leq 0$.

A.18 Proof of proposition 7: steady-state analysis with respect to τ

Given proposition 6, to study the sign of $\partial\varphi^*/\partial\tau$ when $0 < \lambda < \hat{\lambda}$, I can focus on the term

$$\begin{aligned} \Gamma &\equiv \tau^{-a}(1+\tau^{\sigma-1})^{\frac{a}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{-1} - 1 \right] \\ &= \tau^{-(\sigma-1)\frac{a}{\sigma-1}}(1+\tau^{\sigma-1})^{\frac{a}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{-1} - 1 \right] \\ &= \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right] \\ &= \left(\tau^{-(\sigma-1)} + 1 \right)^{\frac{a}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right]. \end{aligned}$$

Taking the partial derivative with respect to τ , I obtain:

$$\begin{aligned}
\frac{\partial \Gamma}{\partial \tau} &= -\frac{a}{\sigma-1} \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}-1} (\sigma-1)\tau^{-(\sigma-1)-1} \left[\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right] \\
&\quad + \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}} \frac{a(1+F_D/F_X) - (\sigma-1)\tau^{\sigma-2}}{(a-\sigma+1)(1+\tau^{\sigma-1})^2} \\
&= -a \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}} \left[\frac{\tau^{\sigma-1}\tau^{-\sigma}}{1+\tau^{\sigma-1}} \left(\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right) + \frac{1+F_D/F_X}{a-\sigma+1} \frac{(\sigma-1)\tau^{\sigma-1}\tau^{-1}}{(1+\tau^{\sigma-1})(1+\tau^{\sigma-1})} \right] \\
&= -a \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}} \left[\frac{\tau^{\sigma-1}\tau^{-\sigma+1-1}}{1+\tau^{\sigma-1}} \left(\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right) + \frac{1+F_D/F_X}{a-\sigma+1} \frac{(\sigma-1)\tau^{-1}}{1+\tau^{\sigma-1}} \frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \right] \\
&= -a \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}-1} \tau^{-1} \left[\tau^{-(\sigma-1)} \left(\frac{a}{a-\sigma+1} \frac{1+F_D/F_X}{1+\tau^{\sigma-1}} - 1 \right) + \frac{1+F_D/F_X}{a-\sigma+1} \frac{(\sigma-1)}{1+\tau^{\sigma-1}} \right] \\
&= -a\tau^{-1} \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left[\frac{a(1+F_D/F_X)}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}} - \frac{1}{\tau^{\sigma-1}} + \frac{(1+F_D/F_X)(\sigma-1)}{(a-\sigma+1)(1+\tau^{\sigma-1})} \right] \\
&= -a\tau^{-1} \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a-\sigma+1}{\sigma-1}} \frac{a + aF_D/F_X - (a-\sigma+1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1}}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}} \\
&= -a\tau^{-1} \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a-\sigma+1}{\sigma-1}} \frac{a + aF_D/F_X - a - a\tau^{\sigma-1} + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1}}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}} \\
&= -a\tau^{-1} \left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}} \right)^{\frac{a}{\sigma-1}-1} \left[\frac{-a(\tau^{\sigma-1} - F_D/F_X) + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1}}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}} \right].
\end{aligned}$$

Since by assumption $\tau^{\sigma-1} > F_D/F_X$, the sign of the term in squared brackets is ambiguous and depends on the values taken by the exogenous parameters. In particular it is positive when a is sufficiently small and it is negative when a is sufficiently large. First of all, note that the term is continuous and decreasing in $a \in (\sigma-1, \infty)$. For $a = \sigma-1$ the numerator becomes

$$\begin{aligned}
&-a(\tau^{\sigma-1} - F_D/F_X) + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} = \\
&-(\sigma-1)(\tau^{\sigma-1} - F_D/F_X) + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} = \\
&(\sigma-1)[- \tau^{\sigma-1} + F_D/F_X + 1 + \tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}] = \\
&(\sigma-1)[F_D/F_X + 1 + (1+F_D/F_X)\tau^{\sigma-1}] > 0
\end{aligned}$$

which implies that for a sufficiently close to $\sigma-1$, the sign of the term in square brackets is positive. It follows that for a sufficiently small $\partial\varphi^*/\partial\tau < 0$. On the other hand, whenever

$$\begin{aligned}
a(\tau^{\sigma-1} - F_D/F_X) &> (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} \\
a(\tau^{\sigma-1} - F_D/F_X) &> (\sigma-1)[1 + \tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}] \\
a &> (\sigma-1) \frac{1 + \tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}}{\tau^{\sigma-1} - F_D/F_X} > \sigma-1
\end{aligned}$$

the numerator is negative. This implies that for a sufficiently large the term in square brackets

is also negative and $\partial\varphi^*/\partial\tau > 0$.

Given (37), the sign of $\partial M/\partial\tau$ is determined by the sign of the partial derivative with respect to τ of the term $\tau^{-a}(1 + \tau^{\sigma-1})^{\frac{a-\sigma+1}{\sigma-1}}$. This can be re-written as:

$$\tau^{-(\sigma-1)\frac{a}{\sigma-1}}(1 + \tau^{\sigma-1})^{\frac{a}{\sigma-1}-1} = (\tau^{-(\sigma-1)} + 1)^{\frac{a}{\sigma-1}}(1 + \tau^{\sigma-1})^{-1}.$$

Notice that τ appears only with a negative exponent, meaning that the term above is decreasing in τ . Since the term is in the denominator of M , it follows that $\partial M/\partial\tau > 0$.

When $\hat{\lambda} \leq \lambda < 1$, the sign of $\partial\varphi^*/\partial\tau$ depends on the sign of the term in round brackets $\frac{a}{a-\sigma+1} - \frac{1}{\lambda}$ (see the expression in proposition 6), which is multiplied by τ^{-a} . When

$$\begin{aligned} \frac{a}{a-\sigma+1} - \frac{1}{\lambda} &> 0 \\ a\lambda - a + \sigma - 1 &> 0 \\ -a(1-\lambda) &> 1-\sigma \\ a &< \frac{\sigma-1}{1-\lambda} \end{aligned}$$

then $\partial\varphi^*/\partial\tau < 0$. For a sufficiently small, $\partial\varphi^*/\partial\tau < 0$, while for a sufficiently high ($a > (\sigma-1)/(1-\lambda)$), $\partial\varphi^*/\partial\tau > 0$. Since τ appears only at the denominator of (38) with a negative exponent, it follows immediately that $\partial M/\partial\tau > 0$.

Given $M_X/M = \left(\frac{\varphi^*}{\varphi_x^*}\right)^a$, the proof that $\frac{\partial(M_X/M)}{\partial\tau} < 0$ follows from the results derived above for $\frac{\varphi^*}{\varphi_x^*}$ (see A.13).

A.19 Proof of Lemma 7.1: Individual welfare U

Welfare is given by:

$$U = \frac{e}{P} = \frac{E}{LP} = \frac{R}{LP} = \frac{R}{L\bar{M}^{1/(1-\sigma)}p_D(\bar{\varphi})} = \frac{R\rho\bar{\varphi}}{L\bar{M}^{1/(1-\sigma)}w}.$$

Remember that I can always write $\frac{r_D(\bar{\varphi})}{r_D(\varphi^*)} = \left(\frac{\bar{\varphi}}{\varphi^*}\right)^{\sigma-1}$, where $r_D(\bar{\varphi}) = R/\bar{M}$ and, from the renegation proof condition for the domestic investment F_Dw , $r_D(\varphi^*) = \sigma\pi_D(\varphi^*) = \frac{\sigma\delta F_Dw}{\lambda}$. It follows that

$$\begin{aligned} \left(\frac{\bar{\varphi}}{\varphi^*}\right)^{\sigma-1} &= \frac{R\lambda}{\bar{M}\sigma\delta F_Dw} \\ \bar{\varphi} &= \varphi^* \left(\frac{R\lambda}{\bar{M}\sigma\delta F_Dw}\right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

Using this in the expression for U , I obtain

$$\begin{aligned}
U &= \frac{R\rho\varphi^*}{LM^{1/(1-\sigma)}} \left(\frac{R\lambda}{M\sigma\delta F_D} \right)^{\frac{1}{\sigma-1}} \\
&= \frac{E\rho\varphi^*}{L} \left(\frac{E\lambda}{\sigma\delta F_D} \right)^{\frac{1}{\sigma-1}} \\
&= \frac{\frac{\sigma L}{\sigma-1+\lambda}\rho\varphi^*}{L} \left(\frac{\lambda\frac{\sigma L}{\sigma-1+\lambda}}{\sigma\delta F_D} \right)^{\frac{1}{\sigma-1}} \\
&= \frac{\sigma\rho\varphi^*}{\sigma-1+\lambda} \left(\frac{\lambda L}{\delta F_D(\sigma-1+\lambda)} \right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

When $0 < \lambda < \hat{\lambda}$, φ^* is constant in λ and the sign of $\frac{\partial U}{\partial \lambda}$ is determined by the sign of the partial derivative of the term $\lambda^{\frac{1}{\sigma-1}} \left[\frac{1}{\sigma-1+\lambda} \right]^{\frac{1}{\sigma-1}+1}$. As I shown in the proof of Lemma 2.1, this is always positive, implying $\frac{\partial U}{\partial \lambda} > 0$. When $\hat{\lambda} \leq \lambda < 1$, then also $\frac{\partial \varphi^*}{\partial \lambda} > 0$ and, given $\frac{\partial U}{\partial \varphi^*} > 0$, this implies again $\frac{\partial U}{\partial \lambda} > 0$. The welfare depends on variable trade costs τ only through φ^* . According to Proposition 7, $\partial \varphi^* / \partial \tau < 0$ when a is sufficiently small and $\partial \varphi^* / \partial \tau > 0$ otherwise. It follows that $\partial U / \partial \tau < 0$ when a is sufficiently small and $\partial U / \partial \tau > 0$ otherwise.

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