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FORECASTING AGGREGATE DEMAND: ANALYTICAL COMPARISON OF TOP-DOWN AND BOTTOM-UP APPROACHES IN A MULTIVARIATE EXPONENTIAL SMOOTHING FRAMEWORK*

by Giacomo Sbrana[§] and Andrea Silvestrini[§§]

Abstract

Forecasting aggregate demand is a crucial matter in all industrial sectors. In this paper, we provide the analytical prediction properties of top-down (TD) and bottom-up (BU) approaches when forecasting aggregate demand, using multivariate exponential smoothing as demand planning framework. We extend and generalize the results obtained by Widiarta, Viswanathan and Piplani (2009) by employing an unrestricted multivariate framework allowing for interdependency between the variables. Moreover, we establish the necessary and sufficient condition for the equality of mean squared errors (MSEs) of the two approaches. We show that the condition for the equality of MSEs also holds even when the moving average parameters of the individual components are not identical. In addition, we show that the relative forecasting accuracy of TD and BU depends on the parametric structure of the underlying framework. Simulation results confirm our theoretical findings. Indeed, the ranking of TD and BU forecasts is led by the parametric structure of the underlying data generation process, regardless of possible misspecification issues.

JEL Classification: C32, C43.
Keywords: top-down and bottom-up forecasting, multivariate exponential smoothing.

Contents

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1 Introduction[1]

Forecasting aggregate demand is a crucial matter in all industrial sectors (see Zotteri and Kalchschmidt, 2007, and Kalchschmidt et al., 2006). Consider, for instance, a retail company which offers a broad range of items to its customers. In order to reduce inventory costs and to manage efficiently the supply chain planning process, the company has to rely on accurate predictions for each demand segment and for the whole aggregate demand (see Kerkkänen et al., 2009, for a discussion on the impacts that sales forecast errors have on the supply chain). In the field of industrial maintenance, a related issue is faced when forecasting future spare parts demand, which is needed in order to keep equipment operating properly. This problem is very relevant, for instance, in military logistics, which represents one of the largest outlays of military budgets (see Moon et al., 2012a, 2012b, for an analysis of demand for spare parts in the South Korean Navy). A similar problem occurs in the automobile industry (Fliedner and Lawrence, 1995).

In the context of aggregate demand forecasting, one of the most important issues faced by both theoretical and empirical literature can be summarized as in the abstract of Dunn et al. (1976, p. 68): "*Should statistical forecasts be constructed by aggregating data to each level for which forecasts are required or aggregating the forecasts from the lower levels? The relevant literature suggests no general answer*". Despite that this paper dates back to the seventies, the question raised by the authors remains an open issue.

In general, the aggregate demand can be forecasted using different procedures. In this paper, we compare the forecasting performance of top-down (TD) and bottom-up (BU) approaches whose definition can be found, for example, in Zotteri et al. (2005, p. 480): ''*In the bottom-up approach, individual forecasts for each demand segment (e.g., single stock-keeping unit, single day, or single store) are combined to produce a forecast of aggregate demand (e.g., group of products, week or group of stores). This is referred to as the cumulative forecast since it is the sum of individual lower level forecasts. In the top-down process, aggregate demand data are used to forecast aggregate demand, etc.*".

The goal of this article is to provide explicit analytical expressions for the TD and BU approaches when forecasting the aggregate demand. We assume as production planning framework the multivariate version of the simple exponential smoothing. The simple exponential smoothing, also known as exponentially weighted moving average (EWMA), has a long tradition in forecasting economic time series (Muth, 1960).

Technically, an EWMA smoothing recursion leads to the same forecasts produced by an IMA(1,1) model, which is the reduced form of a random walk plus noise structural time series model (Harvey, 1989). Regarding the IMA(1,1), it is worth quoting the Nobel prize winner Clive Granger: "*This model provides a very good representation of a wide range of economic*

[1] While assuming the scientific responsibility for any errors in the paper, the authors wish to thank Luc Bauwens, Christian M. Hafner, Marco Lippi, Helmut Lütkepohl and David Veredas for useful suggestions and discussion. The paper is the responsibility of its authors and the opinions expressed here do not necessarily reflect those of the Bank of Italy or the Eurosystem.

time series [...] we do not advocate the adoption of this model in all occasions. However, if some simple specific model is to be assumed on a priori grounds, we feel that the first-order integrated moving average process is a serious candidate for economic time series in general." (the quotation is taken from Granger and Newbold, 1977, p. 203). Indeed, despite its simplicity, exponential smoothing represents a strong candidate compared to other complex models as also discussed by several authors (see for example Dekker et al., 2004, Fliedner and Lawrence, 1995, Fliedner, 1999, Moon et al., 2012a, 2012b).

As a distinctive feature, we adopt a multivariate framework in order to allow for interdependencies in the demand environment, while most previous studies featured univariate demand models (with few exceptions such as Chen and Blue, 2010, and Kremer et al. 2012). Therefore, our results generalize the results achieved by Widiarta et al. (2009) by avoiding coefficient restrictions and allowing for interdependency among the individual components. This is relevant since in empirical applications the concept of interdependency is usually neglected in order to avoid complications.

Furthermore, we derive the necessary and sufficient condition for the equality of mean squared errors (MSEs) of the TD and BU approaches. In particular, our results shed light on the analytical properties of TD and BU when assuming a first order vector integrated moving average model, which corresponds to the multivariate exponential smoothing with no restrictions on parameters. To our knowledge, such a framework has never been used to compare TD and BU approaches.

Recently, other papers have compared the forecasting properties of alternative demand planning approaches based on MSEs. For instance, Chen and Blue (2010) consider a bivariate first order vector autoregressive framework. Moreover, Widiarta et al. (2009) assume simple exponential smoothing as the forecasting technique for both TD and BU. They prove that TD and BU are equally efficient in terms of MSE if the individual components follow univariate MA(1) processes with identical MA coefficients and if the smoothing constants used for forecasting the individual components are equal (p. 91). Whereas these authors give a condition for equal efficiency, relying on rather strong restrictions, our results are valid in general, regardless of any restrictions. Indeed, contrary to Widiarta et al. (2009), we relax the assumption of identical parameters of the single subaggregate components. This is clearly more realistic since, in a standard production planning context, the parametric structure of the components is hardly ever identical.

The remainder of the paper is structured as follows. After a brief literature review in Section 2, in Section 3 we present the methodological framework, which is based on the multivariate exponential weighted moving average. In Section 4 we derive the parameters and the MSE of the TD approach, while in Section 5 we focus on the BU approach. In Section 6, we give the necessary and sufficient condition for the equality of MSEs. Then, in Section 7, using a simple bivariate model, we show that the mentioned condition can be achieved even when the single components differ in dynamics. To this purpose, we provide conditions under which the equality of MSEs holds while the equality of predictors does not. In Section 8 we present results of a simulation study to compare the out-of sample MSEs and mean absolute errors (MAEs) of TD and BU. Section 9 concludes. All proofs are relegated to the Appendix.

2 Literature review

The literature on TD and BU approaches is wide and considers different frameworks. For this reason, we do not attempt to survey all the contributions, but rather to give the main references related to our work.

The first reference literature is the time series literature where TD and BU are often referred to as aggregate and disaggregate specifications. When the data generation process is assumed to be a vector ARMA model, the consequences of contemporaneous (cross-sectional) aggregation have been discussed since the original contributions of Granger and Morris (1976) and Box and Jenkins (1976). Most of the theoretical results on the aggregation of ARMA are collected in Chapter 4 of Lütkepohl (1987), in Section 2.4 of Lütkepohl (2006) and in Lütkepohl (2009). Other prominent contributions are those of Rose (1977), Tiao and Guttman (1980), Wei and Abraham (1981), Kohn (1982), Lütkepohl (1984a, 1984b, 1987).

The second reference literature encompasses the demand planning studies focusing on the effectiveness of TD and BU approaches. Fliedner (1999) argues that forecast performance is dependent upon the statistical nature of the disaggregate items comprising the aggregate series: In particular, higher positive/negative correlation leads to improved forecast performance at the aggregate level. Weatherford et al. (2001) show that a fully disaggregated forecasting method outperforms aggregated forecasts. More recently, Dekker et al. (2004) focus on seasonal demand forecasts, while Moon et al. (2012a, 2012b) consider in detail alternative forecasting methods for predicting the demand for spare parts by the South Korean Navy. In general, the choice between competitive specifications seems to be based on the specific framework employed and case study analyzed. Indeed, as argued by Zotteri et al. (2005) and Zotteri and Kalchschmidt (2007), the choice of the appropriate aggregation level depends on the underlying data generation process.

Our contribution is related to these two types of literature focusing on the issue of forecasting aggregated variables. One of the major gap left by the previous literature is the lack of analytical results establishing conditions under which the TD approach outperforms the BU and vice versa. This paper fills this gap by shedding light on the algebraic conditions determining the forecasting performance of TD and BU assuming the multivariate exponential smoothing as the framework. Clearly, this is only one of the possible production planning framework and therefore it can be considered as the starting point to be further developed as future research.

3 The demand planning framework

In this section we present our assumptions on the demand planning framework, the forecasting problem and the methodology adopted.

We consider a system describing the demand estimates for $i = 1, 2, \ldots, N$ products at time $t = 1, 2, \ldots, T$. We assume that demand estimates follow an unrestricted multivariate exponential weighted moving average (EWMA) process where the vector x_t is N-variate and $\hat{x}_{it}$ denotes the one-step ahead forecast of x_{it}:

$$\begin{bmatrix} \hat{x}_{1t} \\ \hat{x}_{2t} \\ \vdots \\ \hat{x}_{Nt} \end{bmatrix} = \Phi \begin{bmatrix} \hat{x}_{1,t-1} \\ \hat{x}_{2,t-1} \\ \vdots \\ \hat{x}_{N,t-1} \end{bmatrix} + (I_N - \Phi) \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ \vdots \\ x_{N,t-1} \end{bmatrix} \tag{1}$$

with

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \ldots & \phi_{1N} \\ \phi_{21} & \phi_{22} & \ldots & \phi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N1} & \phi_{N2} & \ldots & \phi_{NN} \end{bmatrix}$$

and I_N the identity matrix of size N. It is well known that the process in Eq. (1) can be reparameterized as an integrated vector moving average of order one (a vector IMA(1,1)) representing the reduced form of the multivariate EWMA (see Harvey, 1989, p. 432).[2]

Therefore, our focus is on the following system of equations:

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{Nt} \end{bmatrix} - \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ \vdots \\ x_{N,t-1} \end{bmatrix} = \begin{bmatrix} (1-\phi_{11}L) & -\phi_{12}L & \ldots & -\phi_{1N}L \\ -\phi_{21}L & (1-\phi_{22}L) & \ldots & -\phi_{2N}L \\ \vdots & \vdots & \ddots & \vdots \\ -\phi_{N1}L & -\phi_{N2}L & \ldots & (1-\phi_{NN}L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \tag{2}$$

where L is the back shift operator such that $Lx_t = x_{t-1}$ and $\varepsilon_t = x_t - \hat{x}_t$. In addition, $\varepsilon_t' = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt})$ is a vector of white noise innovations such that $E(\varepsilon_t) = 0$ and

$$E(\varepsilon_t \varepsilon_t') = \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \ldots & \sigma_{1N} \\ \sigma_{12} & \sigma_{22} & \ldots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \ldots & \sigma_{NN} \end{bmatrix}$$

It follows that $E(\varepsilon_t \varepsilon_s') = 0$ for any $s \neq t$.

[2]In the univariate framework, it is well known that the EWMA is equivalent to an IMA(1,1) process. Yet, it should be noted that EWMA captures an additional case: quoting from Hyndman et al. (2008), p. 169: "The EWMA parameter space $\alpha \in (0, 2)$ corresponds exactly to the ARIMA parameter space $|\theta_1| < 1$. However, we observe that the finite start-up assumption enables the EWMA scheme to handle $\alpha = 0$, corresponding to a constant mean; the ARIMA model does not include this case."

It is interesting to note that the specifications as in equations (1) and (2) represent the most general framework in the context of multivariate exponential smoothing. In general it is assumed that all eigenvalues of the matrix Φ are in modulus smaller than one. However, our results are valid regardless of the invertibility condition of the system in (2). Therefore, the invertibility assumption can be relaxed and the matrix Φ need not be restricted.

The main objective of our analysis consists of comparing top-down and bottom-up approaches in forecasting the aggregated process $z_t = Fy_{it}$, where F is a $(1 \times N)$ vector of weights (i.e., $F = [\omega_1 \quad \omega_2 \quad \ldots \quad \omega_N]$) and $y_{it} = (1 - L)x_{it}$. It is relevant to note that no restriction is imposed on the values of the vector F. This means that several aggregation schemes can be considered. Indeed, one can compare the forecasts of TD and BU for specific sub-aggregates. Consider, for example, a firm producing three different items. In addition, assume that the management aims at forecasting the aggregate demand for two items only. Then we have a trivariate system for equations (1) and (2) and setting $F = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ allows focusing on the simple sum of the first two components. At the same time, implicitly, our framework restricts the analysis to flat hierarchies. Yet, a recent literature applicable to demand planning focuses on hierarchical structures with two or more levels (Athanasopoulos et al., 2009; Hyndman et al., 2011). Extending our analysis to generic K levels hierarchical structures is an interesting development which we leave for further research.

As already pointed out by the previous literature (Wei and Abraham, 1981, and Lütkepohl, 1984b, 1987), when the data generation process is known and no estimation uncertainty is faced, aggregating the forecasts of the multivariate process in (2) represents the "optimal" procedure, since it makes use of the largest information set and delivers the smallest forecast MSE ($MSE_{opt} = E(z_t - F\hat{y}_{it})^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \sigma_{ij} \omega_j)$). On the other hand, TD and BU may represent two sub-optimal procedures being $MSE_{TD} \geq MSE_{opt}$ and $MSE_{BU} \geq MSE_{opt}$. More specifically, TD and BU may produce the optimal forecasts under specific conditions that we provide in the next two sections.

Since, in general, the data generation process is unknown, most of the previous literature has compared the forecasting performances of TD and BU. However, little indication has been provided about the conditions allowing MSE_{TD} to be smaller, equal or larger than MSE_{BU}.[3] Only recently, Widiarta et al. (2009) present a sufficient condition for the equality of MSEs, which is valid under strict homogeneous conditions.

In the next sections, the forecasting properties of TD and BU are fully investigated and compared. A necessary and sufficient condition for the equality of MSEs is also shown, assuming that the system in Eq. (2) represents the data generation process.

[3]See, for instance, the last section in Wei and Abraham (1981). These authors show examples when the aggregate (TD) approach outperforms the disaggregate (BU) one and vice versa. Yet, they do not provide any general conditions for the equality of MSEs.

4 Parameters of the aggregated process: Top-Down approach

This section focuses on the TD approach, which refers to the process of forecasting the demand for the aggregate of items.

In the vector IMA(1,1) model in (2), each component, as well as the aggregated data, follow an IMA(1,1).[4] Therefore, the aggregated process $Fy_{it} = F(1-L)x_{it}$ is an MA(1), that is:

$$z_t = Fy_{it} = \psi(L)a_t = (1-\psi L)a_t \tag{3}$$

In addition, from the well known properties of the MA(1) process, we have that

$$E(z_t^2) = (1+\psi^2)\sigma_a^2 \qquad E(z_t z_{t-1}) = -\psi\sigma_a^2 \tag{4}$$

Defining $\alpha_j = \sum_{i=1}^N \phi_{ij}$, one can see that $a_t = \sum_{i=1}^N \varepsilon_{it}\omega_i - \sum_{i=1}^N \alpha_i\varepsilon_{i,t-1}\omega_i + \psi a_{t-1}$ is a white noise process since: $E(a_t a_{t-1}) = \frac{\sum_{i=1}^N \sum_{j=1}^N -\alpha_i\sigma_{ij}\omega_i\omega_j + \psi E(a_{t-1}^2)}{1-\psi^2} = 0$.

The α_j $(j = 1, 2, \ldots, N)$ parameters have a precise interpretation. In general, the contribution of the j-th innovation at time $t-1$ to the i-th variable at time t corresponds to the element in row i and column j of the Φ matrix in Eq. (1). Thus, when considering the TD approach the generic α_j measures the impact of the j-th innovation at time $t-1$ on the aggregate variable at time t. The smaller the distance between the α_j's the more similar the impact of the innovations in the TD specification. On the other hand, the wider the distance between the α_j's the more different the impact of the innovations on the aggregate variables is.

We now derive the "macro" parameters (ψ and σ_a^2) as functions of the "micro" parameters of the system in (2), namely α_j and σ_{ij}. Consider the following equation:

$$\delta = \frac{E(z_t^2)}{2E(z_t z_{t-1})} = \frac{(1+\psi^2)}{-2\psi} = \frac{\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j + \sum_{i=1}^N \sum_{j=1}^N \alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)}$$

We can express ψ as function of δ as follows: $\psi = \delta \pm \sqrt{\delta^2 - 1}$. Note that, given that δ is negative, the positive sign in front of the squared root guarantees that this solution is invertible (i.e., $0 \leq \psi < -1$). Hence, the MA parameter in (4) is:

$$\psi = \frac{\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j + \sum_{i=1}^N \sum_{j=1}^N \alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)} + \sqrt{\left(\frac{\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j + \sum_{i=1}^N \sum_{j=1}^N \alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)}\right)^2 - 1} \tag{5}$$

In addition, the innovations variance can be expressed as:

$$\sigma_a^2 = \sigma_{TD}^2 = \frac{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)}{\frac{\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j + \sum_{i=1}^N \sum_{j=1}^N \alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)} + \sqrt{\left(\frac{\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j + \sum_{i=1}^N \sum_{j=1}^N \alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^N \sum_{j=1}^N \alpha_i\sigma_{ij}\omega_i\omega_j\right)}\right)^2 - 1}} \tag{6}$$

[4] A proof that the aggregate is also an MA(1) can be found in Lütkepohl (2007), p. 436. In general, summing up across i moving average process of order q_i lead to an MA(q*) where $q^* \leq max(q_i)$.

Therefore, equations (5) and (6) represent two direct computational tools to recover the MA parameter of the aggregated process and its variance, which corresponds to the variance of TD (σ^2_{TD}). In addition, it can easily be seen that the variance of the aggregated process achieves its minimum value whenever $\alpha_1 = \alpha_2 = \ldots = \alpha_N$. That is, when $a_t = \omega_1\varepsilon_{1t} + \omega_2\varepsilon_{2t} + \ldots + \omega_N\varepsilon_{Nt}$, TD has the same variance of the optimal forecasting procedure (i.e., $\sigma^2_{TD} = \sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j$).

To see that, note that when the equality of α_i ($i = 1, 2, \ldots, N$) holds we can rewrite δ as follows:

$$\delta = \frac{(1+\alpha^2)(\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j)}{-2\alpha(\sum_{i=1}^N \sum_{j=1}^N \omega_i\sigma_{ij}\omega_j)}$$

such that: $\psi = \alpha$. This case represents the condition of equality of predictors provided by Lütkepohl (1987, p. 107). As a consequence, the variance of the TD approach depends on the distance among the α_i. More specifically, the more $\alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_N$, the higher the TD variance.

Consider a bivariate version of the model in (2), in which for simplicity we let $E(\varepsilon_t\varepsilon_t') = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$. Figure 1 displays the contour and three-dimensional plots of σ^2_{TD} in (6) as a function of the parameters of a bivariate vector IMA(1,1): $\alpha_1 = (\phi_{11} + \phi_{21})$ and $\alpha_2 = (\phi_{12} + \phi_{22})$. In both panels, the extra-diagonal element of the covariance matrix of the innovations ρ is set equal to 0.3.

The three-dimensional plot of σ^2_{TD} is clearly symmetric across the 45 degree line on the (α_1,α_2) cartesian plane, and has its minima where $\alpha_1 = \alpha_2$, on the same plane. As we move away from the 45 degree line, σ^2_{TD} increases.

5 Parameters of the disaggregated process: Bottom-Up approach

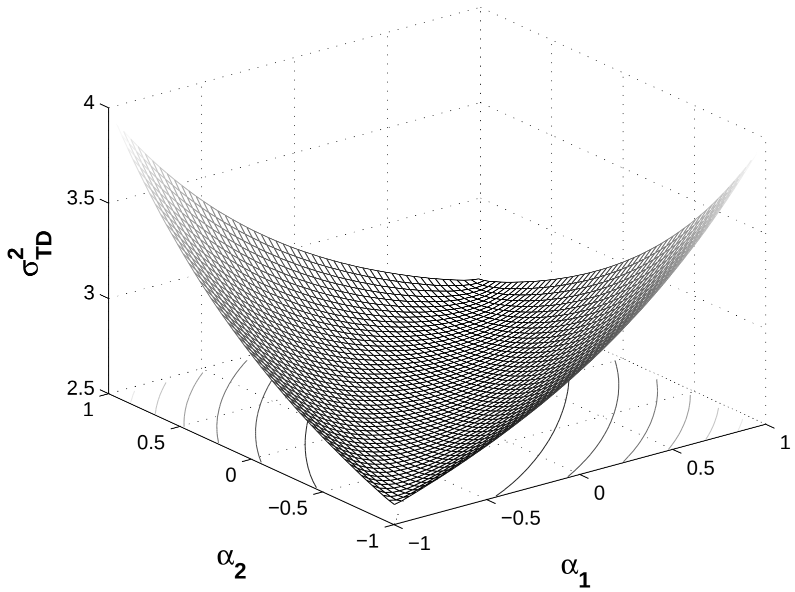
This section describes the properties of BU approach. Here, the focus is on modeling and forecasting equation by equation the system of multiple equations. In particular, this procedure aims at forecasting ex-ante each of the equations contained in (2) and to aggregate ex-post the forecasts. That is, considering the data generation process as in Eq. (2), we can rewrite the system as follows:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Nt} \end{bmatrix} = \begin{bmatrix} (1-\theta_1 L) & 0 & \ldots & 0 \\ 0 & (1-\theta_2 L) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & (1-\theta_N L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \vdots \\ \eta_{Nt} \end{bmatrix} \tag{7}$$

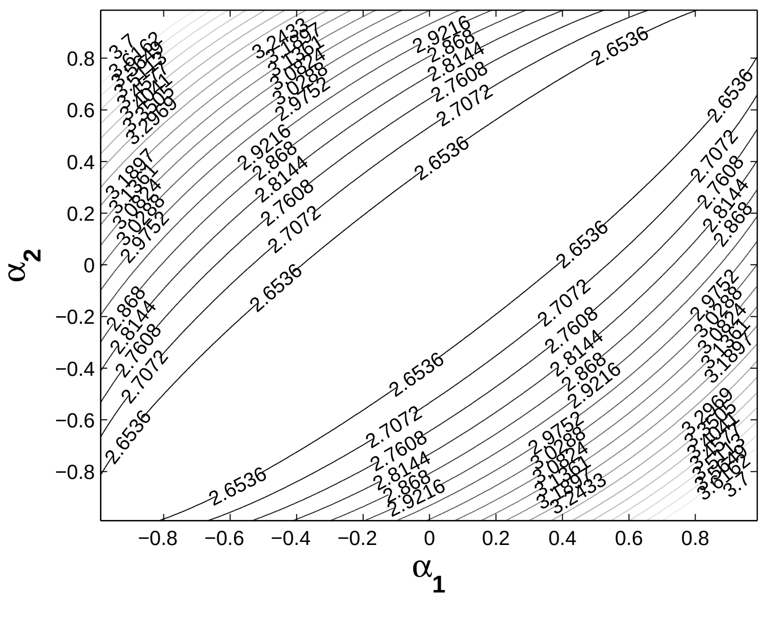
Let

$$\delta_i = \frac{E(y_{it}^2)}{2E(y_{it}y_{i,t-1})} = \frac{\sigma_{ii} + \sum_{j=1}^N \sum_{k=1}^N \phi_{ij}\phi_{ik}\sigma_{jk}}{-2\sum_{j=1}^N \phi_{ij}\sigma_{ij}} \qquad (i = 1, 2, \ldots, N)$$

Figure 1: TD approach: contour and three-dimensional plots of σ^2_{TD}



(a) three-dimensional plot of $\sigma^2_{TD} = f(\alpha_1; \alpha_2)$



(b) contour plot of $\sigma^2_{TD} = f(\alpha_1; \alpha_2)$

From top to down: three-dimensional and contour plots of σ^2_{TD}, i.e., the variance of the TD approach, as a function of the parameters of a vector IMA(1,1): $\alpha_1 = (\phi_{11} + \phi_{21})$ and $\alpha_2 = (\phi_{12} + \phi_{22})$. In both figures, the extra-diagonal element of the covariance matrix of the innovations ρ is set equal to 0.3.

Then, it follows that each of the θ_i ($i = 1, 2, \ldots, N$) in (7) can be expressed as $\theta_i = \delta_i + \sqrt{\delta_i^2 - 1}$:

$$\theta_i = \frac{\sigma_{ii} + \sum_{j=1}^{N} \sum_{k=1}^{N} \phi_{ij}\phi_{ik}\sigma_{jk}}{-2\sum_{j=1}^{N} \phi_{ij}\sigma_{ij}} + \sqrt{\left(\frac{\sigma_{ii} + \sum_{j=1}^{N} \sum_{k=1}^{N} \phi_{ij}\phi_{ik}\sigma_{jk}}{-2\sum_{j=1}^{N} \phi_{ij}\sigma_{ij}}\right)^2 - 1} \tag{8}$$

Bearing in mind that $\eta_{it} = \varepsilon_{it} - \phi_{i1}\varepsilon_{1,t-1} - \ldots - \phi_{iN}\varepsilon_{N,t-1} + \theta_i\eta_{i,t-1}$, we have the following expression for the variance of the disaggregate error:

$$E(\eta_{it}^2) = \frac{\sigma_{ii} + \sum_{k=1}^{N} \sum_{u=1}^{N} \phi_{ik}\phi_{iu}\sigma_{ku} - 2\theta_i \sum_{k=1}^{N} \phi_{ik}\sigma_{ik}}{(1 - \theta_i^2)}$$

Moreover, the covariance between the innovations is:

$$E(\eta_{it}\eta_{jt}) = \frac{\sigma_{ij} + \sum_{k=1}^{N} \sum_{u=1}^{N} \phi_{ik}\phi_{ju}\sigma_{ku} - \theta_j \sum_{k=1}^{N} \phi_{ik}\sigma_{jk} - \theta_i \sum_{k=1}^{N} \phi_{jk}\sigma_{ik}}{(1 - \theta_i\theta_j)}$$

Therefore, the MSE of the BU approach can be computed as:

$$\sigma_{BU}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\frac{\sigma_{ij} + \sum_{k=1}^{N} \sum_{u=1}^{N} \phi_{ik}\phi_{ju}\sigma_{ku} - \theta_j \sum_{k=1}^{N} \phi_{ik}\sigma_{jk} - \theta_i \sum_{k=1}^{N} \phi_{jk}\sigma_{ik}}{(1 - \theta_i\theta_j)}\right) \omega_i\omega_j \tag{9}$$

One can see that the variance of the BU approach process depends on the magnitude of the extra-diagonal parameters ϕ_{ij} (with $i \neq j$). First of all, it achieves the minimum whenever $\phi_{ij} = 0$ (that is, when the micro units are independent processes). In other words, the BU approach forecasts as well as the data generation process whenever $\varepsilon_{it} = \eta_{it}$, that is $\theta_i = \phi_{ii}$ (this is the condition provided by Widiarta et al. 2009, p. 91). On the other hand, the variance of BU increases with the magnitude of the extra diagonal parameters ϕ_{ij}. In fact, it can be seen that $\sigma_{BU}^2 > \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i\sigma_{ij}\omega_j$ whenever $\phi_{ij} \neq 0$ with $i \neq j$.

6 A comparison of the TD and BU predictors via forecast mean squared errors

Notice that so far we have expressed the corresponding MSEs as a function of the parameters of the data generation process as in Eq. (2). The structure and the parameter values of the latter determine the accuracy of the forecasting procedures built on individual components and on the contemporaneously aggregated process.

We are now able to rank the MSEs of the TD and BU approaches. In what follows, we provide a condition that guarantees equal predictive efficiency. This is the main contribution of the paper.

Theorem 1 *Given the vector process in Eq. (2), Top-Down and Bottom-Up approaches have identical MSE if and only if the following condition holds:*

$$\frac{-2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\sigma_{ij}\omega_i\omega_j\right)}{\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\sigma_{ij}\omega_j+\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\sigma_{ij}\omega_i\omega_j\right)}+\sqrt{\left(\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\sigma_{ij}\omega_j+\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j}{-2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\sigma_{ij}\omega_i\omega_j\right)}\right)^2-1}}$$
$$=$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\sigma_{ij}+\sum_{k=1}^{N}\sum_{u=1}^{N}\phi_{ik}\phi_{ju}\sigma_{ku}-\theta_j\sum_{k=1}^{N}\phi_{ik}\sigma_{jk}-\theta_i\sum_{k=1}^{N}\phi_{jk}\sigma_{ik}}{(1-\theta_i\theta_j)}\right)\omega_i\omega_j$$

It is interesting to note that this condition is particularly flexible since the equality holds regardless of the values of θ_i and ψ. Indeed, below we show that the equality holds even when the values of θ_i are not identical.

To interpret Theorem 1, which is a highly non-linear function of several parameters, it can be useful to fix some of them and set $N = 2$, moving the others and drawing the equality condition. This latter can be viewed as an implicit function of the form: $\sigma^2_{TD} - \sigma^2_{BU} = 0$. As before, we let $E(\varepsilon_t\varepsilon_t') = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$.

Figures 2 and 3 display three-dimensional and contour plots of $\sigma^2_{TD} - \sigma^2_{BU}$, as a function of the parameters of a bivariate vector IMA(1,1), i.e., $\phi_{11}, \phi_{21}, \phi_{12}, \phi_{22}$ and ρ. In the plots, two of the parameters (i.e., ϕ_{11}, ϕ_{22}) vary while the other three parameters (i.e., ρ, ϕ_{12}, ϕ_{21}) are kept fixed and set equal to the values reported below each panel.

[FIGURES 2 AND 3 ABOUT HERE]

Some interesting conclusions may be easily drawn from Figures 2 and 3. First, from panels (a) in Figure 2 and (a) in Figure 3, we remark that as ϕ_{12} and ϕ_{21} move toward the point (0, 0) in absolute value, the function values increase, i.e., the performance of the BU approach is improved. This is also evident by looking at panels (b) in Figure 2 and (b) in Figure 3, where the level curves are depicted. Second, from a careful look at the contours sketched in panel (b) in Figure 2, we note that for the chosen combination $\rho = 0.3$, $\phi_{12} = 0$, $\phi_{21} = 0$, the BU outperforms the TD approach across all the displayed region. In particular, the difference between the MSEs is almost zero close to the $\alpha_1 = \alpha_2$ line, and increases steadily as we move away from the 45 degree line. In panel (b) in Figure 3, for the chosen combination $\rho = 0.3$, $\phi_{12} = 0.8$, $\phi_{21} = 0.8$, TD outperforms BU in a rather wide region, close to the $\alpha_1 = \alpha_2$ line and on the top left of the graph, where the level curves are negative. As we move away from the 45 degree line toward the bottom right of the graph, the contours become positive, and the ranking changes, i.e., BU outperforms TD.

7 On the conditions for equal forecasting efficiency

In the context of VARMA models, Lütkepohl (1987) gives the necessary and sufficient condition for the equality of h-step ahead predictors based on the individual components and on the aggregated process. That is, Corollary 4.1.1, case ii, p. 107 in Lütkepohl (1987) states that:

$$z_t^{BU}(h) = z_t^{TD}(h) \iff F\Theta(L) = \psi(L)F \tag{10}$$

where, in our framework, $\Theta(L)$ is the disaggregate polynomial matrix on the RHS of (7) and $\psi(L)$ as in (3).

Condition (10) states that we get identical predictions for Fy_{it} using TD and BU if and only if the MA parameters of the individual components (θ_i) are equal to the MA parameter of the aggregated process (ψ):[5]

$$\theta_1 = \theta_2 = \ldots = \theta_N = \psi \tag{11}$$

Here it is crucial to note that although (11) is necessary and sufficient for the equality of the predictors, *it is sufficient but not necessary for the equality of the corresponding MSEs.* To show this, for simplicity, we consider a bivariate vector IMA(1,1) model. Note that the bivariate framework has been widely used in the theoretical literature (see for instance Chen and Blue, 2010, who use a bivariate Vector AR(1)).

Assuming a bivariate vector IMA(1,1) model with $\sigma_{12} = 0$ and hence focusing on a diagonal covariance matrix of the innovations, we introduce a condition for equal forecasting efficiency that does not necessarily satisfy (11). That is, we are able to provide a condition that allows $MSE_{TD} = MSE_{BU}$ whereas $\theta_1 \neq \theta_2$.

Proposition 1 *Consider the following bivariate system:*

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} (1+\phi_{11}L) & \phi_{12}L \\ \phi_{21}L & (1+\phi_{22}L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{12}$$

with $\sigma_{11} = \sigma_{22} = 1$ and $\sigma_{12} = E(\varepsilon_{1t}\varepsilon_{2t}) = 0$. In addition, assume that the vector of weights is $F = [1 \quad 1]$. If we impose $\phi_{12} = \phi_{21}$, then

$$\phi_{12} = \pm\frac{\phi_{22} - \phi_{11}}{2} \tag{13}$$

is a sufficient condition for the equality of MSEs of the TD and BU approaches.

The linear combination in (13) guarantees the equality of forecasting performance of the competitive predictors despite that $\theta_1 \neq \theta_2$. The proof of Proposition 1 is given in the Appendix. Note that, given the above results, the conditions given in the Theorem in Widiarta et al. (2009)

[5] We briefly summarize the steps of the proof in Lütkepohl (1987) to show necessity of (10). Let us focus on the bivariate framework of (2). To show that (10) is a necessary condition for the equality of one-step ahead predictors, assume that $z_t^{BU}(1) = z_t^{TD}(1)$ holds. Remind that $z_{t+1} - z_t^{TD}(1) = a_{t+1}$ and $z_{t+1} - z_t^{BU}(1) = Fy_{t+1}^{BU}$ by construction. Hence $z_t^{BU}(1) = z_t^{TD}(1) \Rightarrow Fy_{t+1}^{BU} = a_{t+1}$.

$$F\Theta(L)\eta_t := y_t := \psi(L)a_t = \psi(L)F\eta_t$$

and thereby (10).

are necessary and sufficient for the equality of predictors. However, they are sufficient but not necessary for the equality of MSE.

It is interesting to note that all the illustrations and numerical examples proposed by the aggregation literature focus on σ_{12} equal to zero (e.g., Wei and Abraham, 1981, Lütkepohl, 1984c, 1987, 2007). In other words, it is always assumed a diagonal covariance matrix of the innovations. To our knowledge, nowhere in the literature is the case $\sigma_{ij} \neq 0$ discussed and analyzed. Yet, this latter case deserves particular attention due to its great practical importance in empirical analysis since, very often, the individual components series are correlated. Here we present another sufficient condition for equal forecasting performance in the bivariate case, which holds when the innovations covariance matrix is full (not diagonal).

Proposition 2 *Consider the system as in Eq. (12) with $\sigma_{11} = \sigma_{22} = 1$ and $\sigma_{12} = \rho$. In addition, assume that the vector of weights is $F = [1 \quad 1]$. For any ϕ_{11}, ϕ_{22} and assuming $\sigma_{12} = \rho$, the following*

$$\begin{aligned} \phi_{21} &= (\phi_{11} - \phi_{22}) \left(\frac{1}{2} + \rho \right) \\ \phi_{12} &= \frac{\phi_{11} - \phi_{22}}{2} \end{aligned} \qquad (14)$$

are sufficient conditions for the equality of MSEs of TD and BU approaches.

We defer the proof of Proposition 2 to the Appendix. The reader can check that when (14) holds condition (11) is not met, since $\theta_1 \neq \theta_2$.

It is worth focusing on the relevance of these results for the applied research. It is well known that empirical forecasting accuracy is mainly based on the comparison of mean squared errors of competitive models and not on the equality of predictors (being a particularly strong condition). Therefore, our analysis has some direct consequences on the empirical debate on the use of TD versus BU forecasts. Indeed, it is not possible to establish *a priori* which is the best forecasting model, since both the TD and the BU predictors are sub-optimal procedures if compared with the optimal procedure, i.e., aggregating the forecasts based on the original data generation process in Eq. (2). In addition, Propositions 1 and 2 provide conditions for the equality of MSEs when the IMA(1,1) parameters (i.e. θ_i) are not identical. They both reinforce the fact that (11) is sufficient but not necessary for equal forecasting efficiency.

A relevant issue is to establish conditions under which the TD approach outperforms the BU approach (i.e., $\sigma^2_{TD} < \sigma^2_{BU}$) or the other way around (i.e., $\sigma^2_{TD} > \sigma^2_{BU}$). This is given by comparing equations (6) and (9). These equations are highly nonlinear with respect to the demand planning framework parameters. Nevertheless, we can still identify two conditions affecting the magnitude of the MSEs of the approaches. As of (6), one can see that the wider (the narrower) the distance between the α_i's, the higher (the smaller) the MSE of the TD approach. As of (9), the farther to zero (the closer to zero) the extra-diagonal parameters of Φ, the higher (the smaller) the MSE of the BU approach. The next section will further clarify these considerations.

8 A simulation study

This section focuses on the main results from a Monte Carlo simulation. We adopt the framework already suggested by Lütkepohl (1984c, 1987), taking into account the potential problems of model misspecification and estimation uncertainty linked to small sample size.

More specifically, we consider a bivariate vector MA(1) process as the data generation process (DGP):

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1+\phi_{11}L & \phi_{12}L \\ \phi_{21}L & 1+\phi_{22}L \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \quad t = 1, 2, \ldots, T \tag{15}$$

with

$$\boldsymbol{\varepsilon}_t \sim i.i.d.\ N\left(\mathbf{0}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right).$$

Note that, in (15), we introduce a positive contemporaneous covariance between the innovations, that is, $\rho \neq 0$. This is a novelty with respect to the Monte Carlo simulations presented in Lütkepohl (1984c, 1987).

The structure of the parameters is the only feature that makes our analysis differ from the previously mentioned Monte Carlo simulations. That is, we assume three different Data Generation Processes (DGP):

- DGP 1 ($\sigma_{TD} > \sigma_{BU}$): $\phi_{11} = 0.7$; $\phi_{12} = 0$; $\phi_{21} = 0$; $\phi_{22} = -0.4$; $\rho = 0.3$;
- DGP 2 ($\sigma_{TD} = \sigma_{BU}$): $\phi_{11} = 0.7$; $\phi_{12} = 0.2$; $\phi_{21} = 0.32$; $\phi_{22} = 0.3$; $\rho = 0.3$;
- DGP 3 ($\sigma_{TD} < \sigma_{BU}$): $\phi_{11} = 0.1$; $\phi_{12} = 0.8$; $\phi_{21} = 0.8$; $\phi_{22} = 0.1$; $\rho = 0.3$;

The implied parameters and forecast MSEs of BU and TD predictors are shown in Table 1 for DGP 1, DGP 2 and DGP 3.

All the processes are invertible. The DGP 1 represents the theoretical case when the BU outperforms the TD approach. On the other hand, the DGP 3 is the case when the TD performs better than the BU approach. DGP 2 satisfies condition (14). In fact, in this case, we have shown that TD and BU have exactly the same one-step ahead forecasting performance in terms of MSE. Note that for five-steps ahead the three DGPs are equivalent in terms of MSEs.

The aim of the experiment is to compare the accuracy of TD and BU approaches in forecasting the aggregate variable $\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = y_{1t} + y_{2t}$.

For each DGP, the number of replications is 10,000. The number of observations used to estimate the model in-sample is $T = 30, 50, 100, 200, 500$. Five observations are kept for out-of-sample evaluation. Moreover, we assume that the data generation process is unknown and we take into account possible model misspecification. That is, only autoregressive processes are used to fit and forecast the simulated data. More specifically, as in Lütkepohl (1987), we

Table 1: Implied parameters of BU and TD predictors using DGPs 1, 2, 3

	DGP 1 $\sigma_{TD} > \sigma_{BU}$	DGP 2 $\sigma_{TD} = \sigma_{BU}$	DGP 3 $\sigma_{TD} < \sigma_{BU}$
BU PREDICTOR			
θ_1	0.70	0.70	0.21
θ_2	-0.40	0.36	0.21
σ^2_{BU}	2.60	2.80	3.90
TD PREDICTOR			
ψ	0.13	0.70	0.90
σ^2_{TD}	3.03	2.80	2.60
FORECAST ACCURACY			
$\ln\left(\frac{MSE_{TD}}{MSE_{BU}}\right)$ $(h = 1)$	0.16	0.00	-0.41
$\ln\left(\frac{MSE_{TD}}{MSE_{BU}}\right)$ $(h = 5)$	0.00	0.00	0.00

employ AR(p) processes with $p = 1, 2, ..., 6$.[6] This makes sense as the invertible MA(1) can be closely approximated with an AR with finite lags. The standard information criteria are applied for model selection (in particular, the Akaike Information Criterion, AIC, and the Schwartz Information Criterion, BIC).

One-step ahead and five-steps ahead forecasts are considered. It should be noted that, given the vector MA(1) as the framework, the MSEs of TD and BU correspond for h-steps ahead forecasts with $h \geq 2$. In other words, for $h \geq 2$, the MSEs of TD and BU are equal to the MSE resulting from the aggregation of the system in Eq. (2).[7] That is the reason why in the last line in Table 1 we have zeros for all three DGPs. On the other hand for one-step ahead Theorem 1 holds.

The out-of-sample mean squared error (MSE) and the mean absolute error (MAE) are used for comparing the forecasting accuracy of TD and BU. The MSE is employed since all theoretical results in this paper are based on the comparison of MSEs. However, since the use of the MSE has been criticized by the previous literature (see for example Hyndman et al., 2008; Gardner, 2006; Tashman, 2000), we decide to employ also the ratio of MAEs.[8] The use of different measures of accuracy helps shedding light on comparing the forecasting performance of TD and

[6] According to Lütkepohl (1984c), six is not a severe restriction for the maximum AR order.

[7] Indeed, one can easily check that for any matrix Φ in Eq. (2), any positive definite variance covariance matrix Σ in Eq. (2) and vector of weights F, when $h \geq 2$ we have that $MSE_{TD} = MSE_{BU} = \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\sigma_{ij}\omega_j + \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j$.

[8] An interesting discussion on the importance of selecting the most appropriate error measure for evaluating the forecasting accuracy can be found in Davydenko and Fildes (2013).

BU approaches.

Table 2: $\ln\left(\frac{MSE_{TD}}{MSE_{BU}}\right)$ using DGPs 1, 2, 3

		DGP 1		DGP 2		DGP 3	
		$\sigma_{TD} > \sigma_{BU}$		$\sigma_{TD} = \sigma_{BU}$		$\sigma_{TD} < \sigma_{BU}$	
Sample	Steps ahead	AIC	BIC	AIC	BIC	AIC	BIC
	h=1	0.16	0.12	0.05	0.03	-0.14	-0.11
T=30	h=5	0.00	0.00	0.04	0.03	-0.01	-0.02
	h=1	0.12	0.11	0.01	0.00	-0.13	-0.11
T=50	h=5	0.00	0.00	0.01	0.01	0.00	0.00
	h=1	0.09	0.08	0.00	-0.01	-0.11	-0.08
T=100	h=5	0.02	0.00	0.01	0.01	0.00	-0.01
	h=1	0.08	0.08	0.00	-0.01	-0.12	-0.09
T=200	h=5	0.00	0.00	0.01	0.01	0.00	0.00
	h=1	0.10	0.10	0.00	0.00	-0.13	-0.12
T=500	h=5	0.01	0.00	0.01	0.00	0.01	0.00

Table 2 reports the Monte Carlo results using the three different DGPs: each cell contains the natural log of the $\frac{MSE_{TD}}{MSE_{BU}}$ ratio. Table 3 shows the natural log of the $\frac{MAE_{TD}}{MAE_{BU}}$ ratio for the different DGPs. Values greater than zero indicate that $MSE_{TD} > MSE_{BU}$ in Table 2 and $MAE_{TD} > MAE_{BU}$ in Table 3. Values smaller than zero indicate the reverse. The use of the logarithm is recommended by Dangerfield and Morris (1992) in order to reduce the bias of summary statistics computed as simple ratios (which, since cannot be less than zero by construction, are positively skewed – see Alexander and Francis, 1986).

Results relative to DGP 1 are clearly in favour of the BU predictor for one-step ahead forecasts. This is true comparing both MSE and MAE when using both the Akaike and Schwartz information criteria. On the contrary, for five-steps ahead there seems to be no evidence that one approach outperforms the other. This result is expected given the last line of Table 1.

Focusing on DGP 3, on the other hand, we face the opposite situation in which the TD predictor outperforms the BU (this is evident in both Tables). More specifically, for one-step ahead we observe negative outcomes for all T (indicating the better performance of TD predictor), whereas for five-steps ahead the results are closed to zero (this is especially evident in Table 3). Indeed no specific approach outperforms its competitor for five-steps ahead forecasts. Again, this is in line with the results as in Table 1.

Looking at DGP 2, for which the condition of equality of predictors (14) holds, the MSE

Table 3: $\ln\left(\frac{MAE_{TD}}{MAE_{BU}}\right)$ using DGPs 1, 2, 3

		DGP 1 $\sigma_{TD} > \sigma_{BU}$		DGP 2 $\sigma_{TD} = \sigma_{BU}$		DGP 3 $\sigma_{TD} < \sigma_{BU}$	
Sample	Steps ahead	AIC	BIC	AIC	BIC	AIC	BIC
	h=1	0.08	0.07	0.02	0.01	-0.08	-0.06
T=30	h=5	0.00	-0.01	0.00	0.00	0.01	0.01
	h=1	0.09	0.08	0.01	0.00	-0.13	-0.09
T=50	h=5	0.00	0.00	0.00	0.00	0.01	0.01
	h=1	0.07	0.06	0.00	0.00	-0.12	-0.10
T=100	h=5	0.00	0.00	0.00	0.00	0.01	-0.01
	h=1	0.09	0.08	0.01	0.00	-0.15	-0.15
T=200	h=5	0.00	0.00	0.00	0.00	-0.01	-0.01
	h=1	0.08	0.07	0.00	0.00	-0.15	-0.14
T=500	h=5	0.00	0.00	0.00	0.00	0.00	0.00

results in Table 2 show that BU tends to perform slightly better than the TD in very small samples, when $T = 30$, but not as much as observed for DGP 1. On the other hand, when $T \geq 50$, the forecasting performances are clearly the same. In general, when the number of observations increases, the log of the ratio of MSEs is equal to zero using both AIC and BIC. This is true for one-step ahead and for five-steps ahead forecasts. In summary, the differences between MSEs are very small, especially in large samples, where estimation uncertainty is reduced. Results comparing the MAE of TB and BU in Table 3 are similar to those for the MSE. Here, however, even for $T = 30$, the differences between the MAE of the two competitors are very small. In other words, the comparison based on the MAEs yields results that reinforce the equality of forecasting performance of TB and BU.

These results deserve some interpretation. The necessary and sufficient condition in Theorem 1 has been derived assuming an IMA(1,1) model as the framework. In the experimental design, only finite order AR(p) models – with p selected on the basis of standard information criteria – are used to fit and forecast the simulated MA(1) data: In doing so, we take into account possible model misspecification and estimation uncertainty. Indeed, it is well known that an invertible MA(1) process corresponds to an infinite-order AR. For example, the process $y_t = e_t - \alpha e_{t-1}$ with $|\alpha| < 1$ can be expressed as $y_t = e_t + \sum_{i=1}^{\infty} \alpha^i y_{t-i}$. Therefore, the finite order AR(p) represents a close approximation to the infinite-order AR process and hence to the MA(1).

Thus, the results suggest that, as long as the equality condition holds for the underlying vector IMA(1,1) as in Eq. (2), BU and TD perform almost identically when employing models that closely approximate the data generation process. Similarly, when assuming a framework in which BU outperforms TD (or vice versa), these differences stand out in the simulation results even assuming a certain degree of misspecification. Clearly, these considerations might not hold when using misspecified models that do not approximate the data generation process.

Although not reported, simulations have also been carried out assuming several values of ρ, in order to check whether the correlation coefficient might influence the performances of BU and TD approaches. The results, available upon request, show that all the previous results are confirmed. For instance, when $\rho = 0$ and DGP 1 is considered, BU outperforms TD and the differences between the two competitors are even more pronounced.

Overall, from this Monte Carlo experiment, we can conclude that the simulation results confirm our theoretical findings and shed further light on the ρ parameter's influence on the accuracy of the competing predictors. In particular, DGP 1 and DGP 3 represent two opposite frameworks in which one forecasting method clearly outperforms its competitor. Moreover, Table 2 shows that when DGP 2 is considered the condition of equal forecasting performance in terms of MSE in (14) is validated by simulations. This is true for both one and five-steps ahead forecasts, regardless of misspecification issues.

9 Conclusions

Assuming an unrestricted multivariate exponential smoothing as the data generation process, we show that the parameters of both top-down and bottom-up forecasting approaches can be expressed as analytical functions of the parameters of the reduced form vector IMA(1,1) model. In addition, after having analytically derived the forecasting properties of both top-down and bottom-up, we provide the necessary and sufficient condition for the equality of their MSEs. Our results are valid in general, regardless of restrictions on the parameters. We also show that the condition for equality of predictors is sufficient but not necessary for the equality of MSEs. Monte Carlo simulations seem to confirm all our theoretical achievements.

It is worth noting that our results are valid only when assuming the simple exponential smoothing as the demand planning framework. This is clearly a limitation since we are aware that this framework represents only one of the possible candidates for demand planning. Depending on the context, several other frameworks might be more appropriate. For example, for conventional fast-moving consumer goods, models featuring seasonality should be considered. The double exponential smoothing and trend versions are also very relevant: In particular, the damped trend model has been shown to be a very strong contender. See Gardner (2006) and McKenzie and Gardner (2010).

Moreover, the paper focuses on a sub-case of hierarchical forecasting, restricting the analysis to flat hierarchies with only two levels. This represents another restriction since the recent demand planning literature is extending the analysis to hierarchical structures with more levels

(Athanasopoulos et al., 2009; Hyndman et al., 2011).

Finally, this paper contains useful results assuming full knowledge of the parameters of the multivariate exponential smoothing. We are aware that this represents an ideal situation since, in empirical analysis, practitioners do not have such information and misspecification issues do usually arise.

However, we believe that our theoretical findings might have important practical implications. First of all, one general implication is that neither TD approach nor BU approach should be preferred *a priori* in any empirical analysis. This is not only due to possible misspecification issues but also because of the peculiar structure of the matrix of parameters that plays a crucial role in determining the forecasting performance of the competitive approaches.

Secondly, we provide the conditions when the TD outperforms the BU and vice versa. This is relevant information for empirical analysis when the system in (2) is estimated and TD and BU are implied and compared. In other words, once the system is estimated, any industrial managers can now easily evaluate which approach dominates in terms of MSE. We note that, in empirical analysis, if the system in (2) is estimated and the forecasts of the aggregate demand are implied, it is also worth estimating TD and BU specifications and comparing their forecasting performance with those of the estimated multivariate model. In general, we expect our results to hold also in empirical cases as suggested by the simulations. However, there might be empirical cases where our results may not hold for specific reasons such as potential misspecification of the demand planning framework or small sample estimation issues.

The simulation results represent a relevant finding. That is, if the demand planning framework is the multivariate exponential smoothing, then the ranking of TD and BU forecasts seems to be led by the underlying framework regardless of possible misspecification issues. Needless to say that if the demand planning framework is not the one adopted in this paper, the previous statement is not valid. In fact, our paper represents a first step in the analytical evaluation of the forecasting properties of TD and BU. Future research might investigate the forecasting properties of TD and BU when the demand planning framework differs from (2). For example, our results might be extended to more general trend exponential smoothing as well as damped trend exponential smoothing models. Finally, extending our analysis to generic K levels hierarchical structures would be an interesting development for further research.

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APPENDIX

Proof of Proposition 1

Let $\phi_{12} = \phi_{21} = \frac{\phi_{22}-\phi_{11}}{2}$ and $\rho = 0$. Similar results, *mutatis mutandis*, hold for $\phi_{12} = \phi_{21} = \frac{\phi_{11}-\phi_{22}}{2}$ and $\rho = 0$.

Bearing in mind equation (5), it is easy to see that θ_1 and θ_2 simplify to

$$\theta_1 = \frac{4+5\phi_{11}^2 - 2\phi_{11}\phi_{22} + \phi_{22}^2}{8\phi_{11}} - \sqrt{\frac{(4+5\phi_{11}^2 - 2\phi_{11}\phi_{22} + \phi_{22}^2)^2}{64\phi_{11}^2} - 1}$$

and

$$\theta_2 = \frac{4+\phi_{11}^2 - 2\phi_{11}\phi_{22} + 5\phi_{22}^2}{8\phi_{22}} - \sqrt{\frac{(4+\phi_{11}^2 - 2\phi_{11}\phi_{22} + 5\phi_{22}^2)^2}{64\phi_{22}^2} - 1},$$

hence, $\theta_1 \neq \theta_2$. The ψ parameter in (5) is equal to

$$\psi = \frac{1}{8\phi_{22}}\left(\left(4+\phi_{11}^2 - 2\phi_{11}\phi_{22} + 5\phi_{22}^2\right) - \sqrt{\left(4+\phi_{11}^2 - 2\phi_{11}\phi_{22} + 5\phi_{22}^2 - 8\phi_{22}\right)\left(4+\phi_{11}^2 - 2\phi_{11}\phi_{22} + 5\phi_{22}^2 + 8\phi_{22}\right)}\right).$$

It differs from θ_1. Moreover $\psi = \theta_2$.

Furthermore, when $\rho = 0$, the variance of the aggregated process is $\sigma_{TD}^2 = \frac{2\phi_{22}}{\psi}$, that is,

$$\sigma_{TD}^2 = \frac{2\phi_{22}}{\theta_2}.$$

The MSE of the optimal one-step ahead predictor of y_t based on the univariate components of $\mathbf{x}_t$ is given in (9), which for $\phi_{12} = \phi_{21} = \frac{\phi_{22}-\phi_{11}}{2}$ and $\rho = 0$ becomes

$$\sigma_{BU}^2 = \frac{\phi_{11}}{\theta_1} + \frac{\phi_{22}}{\theta_2} + \frac{\phi_{22}-\phi_{11}}{1-\theta_1\theta_2}\left(\phi_{11} + \phi_{22} - \theta_1 - \theta_2\right).$$

To have equal forecasting performance, it has to be $\sigma_{BU}^2 = \sigma_{TD}^2$. For this condition to be verified, it must hold

$$\frac{\phi_{22}}{\theta_2} - \frac{\phi_{11}}{\theta_1} = \frac{\phi_{22}-\phi_{11}}{1-\theta_1\theta_2}\left(\phi_{11} + \phi_{22} - \theta_1 - \theta_2\right).$$

Hence, to be $\sigma_{BU}^2 = \sigma_{TD}^2$, we need to show that

$$\frac{\phi_{22}\theta_1}{\phi_{11}\theta_2} = \frac{\theta_1^2 - \phi_{11}\theta_1 + 1}{\theta_2^2 - \phi_{22}\theta_2 + 1}. \tag{16}$$

Let us focus on the right-hand side (RHS) of (16). After some tedious calculations, we notice that the numerator $\theta_1^2 - \phi_{11}\theta_1 + 1$ can be factorized as

$$\frac{1}{32\phi_{11}^2}\left(-2\phi_{22}\phi_{11} + \phi_{22}^2 + 4 + \phi_{11}^2\right)$$
$$\times\left(4 + 5\phi_{11}^2 + \phi_{22}^2 - 2\phi_{22}\phi_{11} - \sqrt{\frac{(\phi_{22}^2 + 4 - 8\phi_{11} - 2\phi_{22}\phi_{11} + 5\phi_{11}^2)(\phi_{22}^2 + 4 + 8\phi_{11} - 2\phi_{22}\phi_{11} + 5\phi_{11}^2)}{\phi_{11}^2}}\phi_{11}\right).$$

Similarly, the denominator $\theta_2^2 - \phi_{22}\theta_2 + 1$ may be factorized as

$$\frac{1}{32\phi_{22}^2}\left(-2\phi_{22}\phi_{11} + \phi_{22}^2 + 4 + \phi_{11}^2\right)$$
$$\times\left(4 + 5\phi_{22}^2 + \phi_{11}^2 - 2\phi_{22}\phi_{11} - \sqrt{\frac{(\phi_{11}^2 + 4 - 8\phi_{22} - 2\phi_{22}\phi_{11} + 5\phi_{22}^2)(\phi_{11}^2 + 4 + 8\phi_{22} - 2\phi_{22}\phi_{11} + 5\phi_{22}^2)}{\phi_{22}^2}}\phi_{22}\right).$$

Consequently we can express the ratio $\frac{\theta_1^2 - \phi_{11}\theta_1 + 1}{\theta_2^2 - \phi_{22}\theta_2 + 1}$ as

$$\frac{\phi_{22}^2\left(4 + 5\phi_{11}^2 + \phi_{22}^2 - 2\phi_{22}\phi_{11} - \sqrt{\frac{(\phi_{22}^2+4-8\phi_{11}-2\phi_{22}\phi_{11}+5\phi_{11}^2)(\phi_{22}^2+4+8\phi_{11}-2\phi_{22}\phi_{11}+5\phi_{11}^2)}{\phi_{11}^2}}\phi_{11}\right)}{\phi_{11}^2\left(4 + 5\phi_{22}^2 + \phi_{11}^2 - 2\phi_{22}\phi_{11} - \sqrt{\frac{(\phi_{11}^2+4-8\phi_{22}-2\phi_{22}\phi_{11}+5\phi_{22}^2)(\phi_{11}^2+4+8\phi_{22}-2\phi_{22}\phi_{11}+5\phi_{22}^2)}{\phi_{22}^2}}\phi_{22}\right)}.$$

In addition, focus on the left-hand side (LHS) of (16). We can express θ_2 as

$$\theta_2 = \frac{1}{8\phi_{22}}\left(\left(4+\phi_{11}^2-2\phi_{11}\phi_{22}+5\phi_{22}^2\right)-\sqrt{\left(4+\phi_{11}^2-2\phi_{11}\phi_{22}+5\phi_{22}^2-8\phi_{22}\right)\left(4+\phi_{11}^2-2\phi_{11}\phi_{22}+5\phi_{22}^2+8\phi_{22}\right)}\right). \tag{17}$$

Similarly to θ_2, we can express θ_1 as

$$\theta_1 = \frac{1}{8\phi_{11}}\left(\left(4+5\phi_{11}^2-2\phi_{11}\phi_{22}+\phi_{22}^2\right)-\sqrt{\left(4+5\phi_{11}^2-2\phi_{11}\phi_{22}+\phi_{22}^2-8\phi_{11}\right)\left(4+5\phi_{11}^2-2\phi_{11}\phi_{22}+\phi_{22}^2+8\phi_{11}\right)}\right). \tag{18}$$

From (18) and (17), it is immediately evident that the LHS and the RHS of (16) are equal. Therefore, the result follows.

□

Proof of Proposition 2

We assume $\phi_{21} = (\phi_{11}-\phi_{22})(\frac{1}{2}+\rho)$, $\phi_{12} = \frac{\phi_{11}-\phi_{22}}{2}$ and $\rho \neq 0$. As stated in Proposition 2, in what follows we are going to show that this linear combination of the DGPs parameters guarantees the equality of forecasting performance of the competitive processes, no matter the values of ϕ_{11}, ϕ_{22} and ρ.

Bearing in mind equation (5), it is easy to see that θ_1 and θ_2 simplify to

$$\begin{aligned}\theta_1 = {} & \tfrac{4+(5+4\rho)\phi_{11}^2+\phi_{22}^2-2\phi_{11}\phi_{22}(1+2\rho)}{4((2+\rho)\phi_{11}-\rho\phi_{22})} \\ & -\tfrac{\sqrt{(4+8\phi_{11}+4\rho\phi_{11}+5\phi_{11}^2+4\rho\phi_{11}^2-4\rho\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+\phi_{22}^2)(4-8\phi_{11}-4\rho\phi_{11}+5\phi_{11}^2+4\rho\phi_{11}^2+4\rho\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+\phi_{22}^2)}}{4((2+\rho)\phi_{11}-\rho\phi_{22})},\end{aligned} \tag{19}$$

and

$$\begin{aligned}\theta_2 = {} & \tfrac{4+(\phi_{11}+2\rho\phi_{11})^2+(5-4\rho^2)\phi_{22}^2-2\phi_{11}\phi_{22}(1+2\rho)}{(4\rho(1+2\rho)(\phi_{11}-\phi_{22})+8\phi_{22})} \\ & -\tfrac{\sqrt{(4+4\rho\phi_{11}+8\rho^2\phi_{11}+\phi_{11}^2+4\rho\phi_{11}^2+4\rho^2\phi_{11}^2+8\phi_{22}-4\rho\phi_{22}-8\rho^2\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+5\phi_{22}^2-4\rho^2\phi_{22}^2)}}{\sqrt{(4\rho(1+2\rho)(\phi_{11}-\phi_{22})+8\phi_{22})}} \\ & \times\tfrac{\sqrt{(4-4\rho\phi_{11}-8\rho^2\phi_{11}+\phi_{11}^2+4\rho\phi_{11}^2+4\rho^2\phi_{11}^2-8\phi_{22}+4\rho\phi_{22}+8\rho^2\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+5\phi_{22}^2-4\rho^2\phi_{22}^2)}}{\sqrt{(4\rho(1+2\rho)(\phi_{11}-\phi_{22})+8\phi_{22})}}.\end{aligned} \tag{20}$$

As a consequence $\theta_1 \neq \theta_2$. Some straightforward calculations show that the ψ parameter in (5) is equal to θ_1.

Furthermore, the variance of the aggregated process is provided in (6), which for $\phi_{12} = \frac{\phi_{11}-\phi_{22}}{2}$ and $\phi_{21} = (\phi_{11}-\phi_{22})(1/2+\rho)$ is

$$\sigma_{TD}^2 = \frac{(1+\rho)((2+\rho)\phi_{11}-\rho\phi_{22})}{\theta_1}. \tag{21}$$

The MSE of the optimal one-step ahead predictor of y_t based on the univariate components of $\mathbf{x}_t$ is given in (9), which for $\phi_{12} = \frac{\phi_{11}-\phi_{22}}{2}$ and $\phi_{21} = (\phi_{11}-\phi_{22})(1/2+\rho)$ becomes

$$\begin{aligned}\sigma_{BU}^2 = {} & \tfrac{\phi_{11}+\frac{1}{2}\rho(\phi_{11}-\phi_{22})}{\theta_1}+\tfrac{\rho(\frac{1}{2}+\rho)(\phi_{11}-\phi_{22})+\phi_{22}}{\theta_2} \\ & +\tfrac{\frac{1}{2}\left(2\rho^2(\phi_{11}-\phi_{22})^2+2(\phi_{11}-\phi_{22})(\phi_{11}+\phi_{22}-\theta_1-\theta_2)+\rho(4+5\phi_{11}^2+\phi_{22}^2-2\phi_{11}(\phi_{22}+2(\theta_1+\theta_2)))\right)}{1-\theta_1\theta_2}\end{aligned} \tag{22}$$

To have equal forecasting performance, it has to be $\sigma_{BU}^2 = \sigma_{TD}^2$. For this condition to be verified, on the basis of (21) and (22), it must hold

$$\begin{gathered}\tfrac{\frac{1}{2}(1+2\rho)((2+\rho)\phi_{11}-\rho\phi_{22})}{\theta_1}-\tfrac{\rho(\frac{1}{2}+\rho)(\phi_{11}-\phi_{22})+\phi_{22}}{\theta_2} \\ = \\ \tfrac{\frac{1}{2}\left(2\rho^2(\phi_{11}-\phi_{22})^2+\rho(4+5\phi_{11}^2+\phi_{22}^2-2\phi_{11}(\phi_{22}+2(\theta_1+\theta_2)))+2(\phi_{11}-\phi_{22})(\phi_{11}+\phi_{22}-\theta_1-\theta_2)\right)}{1-\theta_1\theta_2},\end{gathered}$$

which yields

$$
\frac{\theta_1\theta_2}{1-\theta_1\theta_2} = \frac{-2\rho^2(\phi_{11}-\phi_{22})(\theta_1-\theta_2)-\rho(\phi_{11}\theta_1-\phi_{22}\theta_1-5\phi_{11}\theta_2+\phi_{22}\theta_2)+2\phi_{11}\theta_2-2\phi_{22}\theta_1}{2\rho^2(\phi_{11}-\phi_{22})^2+\rho(4+5\phi_{11}^2+\phi_{22}^2-2\phi_{11}(\phi_{22}+2(\theta_1+\theta_2)))+2(\phi_{11}-\phi_{22})(\phi_{11}+\phi_{22}-\theta_1-\theta_2)}.
$$

Let us focus on the first ratio $\frac{\theta_1\theta_2}{1-\theta_1\theta_2}$. Substituting for (19) and (20), we get

$$
\begin{aligned}
\frac{\theta_1\theta_2}{1-\theta_1\theta_2} = & -\frac{1}{2} + \frac{(\rho(1+2\rho)(\phi_{11}-\phi_{22})+2\phi_{22})\sqrt{(4+8\phi_{11}+4\rho\phi_{11}+5\phi_{11}^2+4\rho\phi_{11}^2-4\rho\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+\phi_{22}^2)}}{\sqrt{(4(\rho-1)(\rho+1)(\phi_{11}-\phi_{22})(4+(1+2\rho)\phi_{11}^2-2(3+2\rho)\phi_{11}\phi_{22}+(1+2\rho)\phi_{22}^2))}} \\
& \times \frac{\sqrt{(4-8\phi_{11}-4\rho\phi_{11}+5\phi_{11}^2+4\rho\phi_{11}^2+4\rho\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+\phi_{22}^2)}}{\sqrt{(4(\rho-1)(\rho+1)(\phi_{11}-\phi_{22})(4+(1+2\rho)\phi_{11}^2-2(3+2\rho)\phi_{11}\phi_{22}+(1+2\rho)\phi_{22}^2))}} \\
& + \frac{(-(2+\rho)\phi_{11}+\rho\phi_{22})\sqrt{(4+4\rho\phi_{11}+8\rho^2\phi_{11}+\phi_{11}^2+4\rho\phi_{11}^2+4\rho^2\phi_{11}^2+8\phi_{22}-4\rho\phi_{22}-8\rho^2\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+5\phi_{22}^2-4\rho^2\phi_{22}^2)}}{\sqrt{(4(\rho-1)(\rho+1)(\phi_{11}-\phi_{22})(4+(1+2\rho)\phi_{11}^2-2(3+2\rho)\phi_{11}\phi_{22}+(1+2\rho)\phi_{22}^2))}} \\
& \times \frac{\sqrt{(4-4\rho\phi_{11}-8\rho^2\phi_{11}+\phi_{11}^2+4\rho\phi_{11}^2+4\rho^2\phi_{11}^2-8\phi_{22}+4\rho\phi_{22}+8\rho^2\phi_{22}-2\phi_{11}\phi_{22}-4\rho\phi_{11}\phi_{22}+5\phi_{22}^2-4\rho^2\phi_{22}^2)}}{\sqrt{(4(\rho-1)(\rho+1)(\phi_{11}-\phi_{22})(4+(1+2\rho)\phi_{11}^2-2(3+2\rho)\phi_{11}\phi_{22}+(1+2\rho)\phi_{22}^2))}}.
\end{aligned}
$$

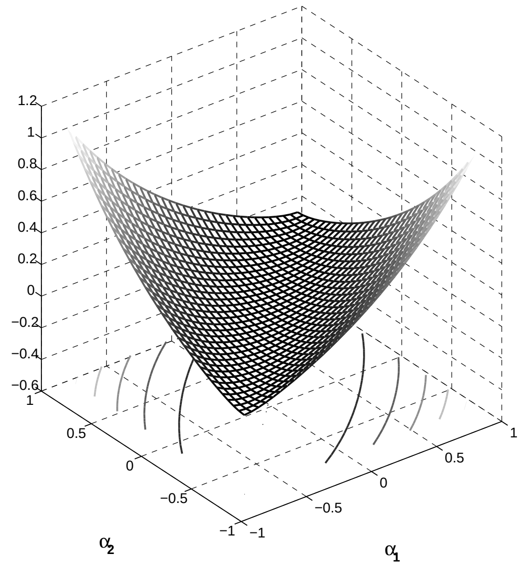
After tedious calculations it is possible to see that we get exactly the same expression as above after plugging (19) and (20) in the second ratio

$$
\frac{-2\rho^2(\phi_{11}-\phi_{22})(\theta_1-\theta_2)-\rho(\phi_{11}\theta_1-\phi_{22}\theta_1-5\phi_{11}\theta_2+\phi_{22}\theta_2)+2\phi_{11}\theta_2-2\phi_{22}\theta_1}{2\rho^2(\phi_{11}-\phi_{22})^2+\rho(4+5\phi_{11}^2+\phi_{22}^2-2\phi_{11}(\phi_{22}+2(\theta_1+\theta_2)))+2(\phi_{11}-\phi_{22})(\phi_{11}+\phi_{22}-\theta_1-\theta_2)}.
$$

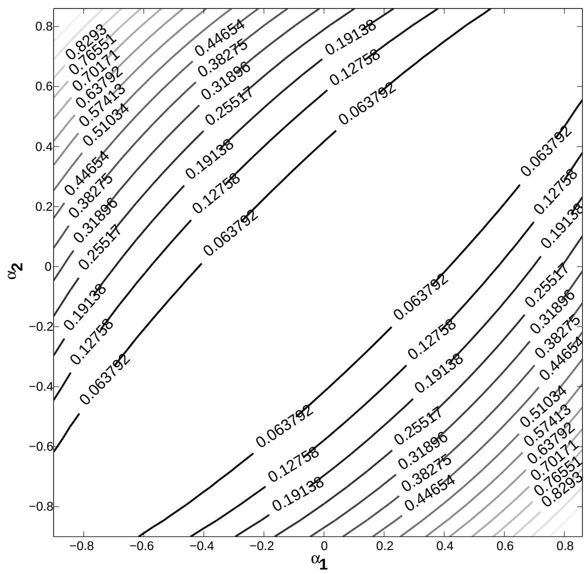
This completes the proof.

$\square$

Figure 2: Three-dimensional and contour plots of $\sigma_{TD}^2 - \sigma_{BU}^2$ when $\rho = 0.3$, $\phi_{12} = 0$, $\phi_{21} = 0$



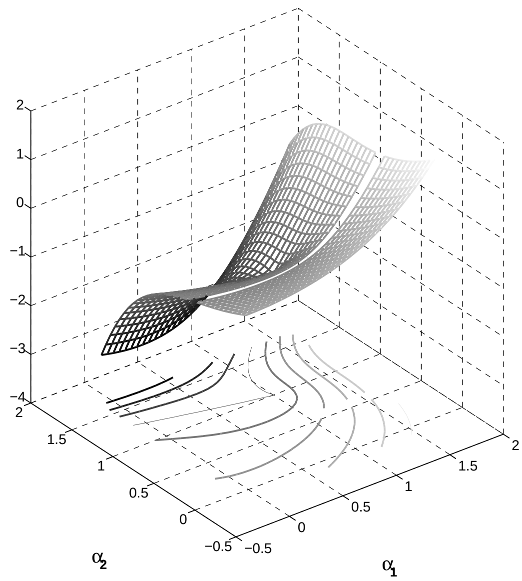
(a) $\rho = 0.3$, $\phi_{12} = 0$, $\phi_{21} = 0$;



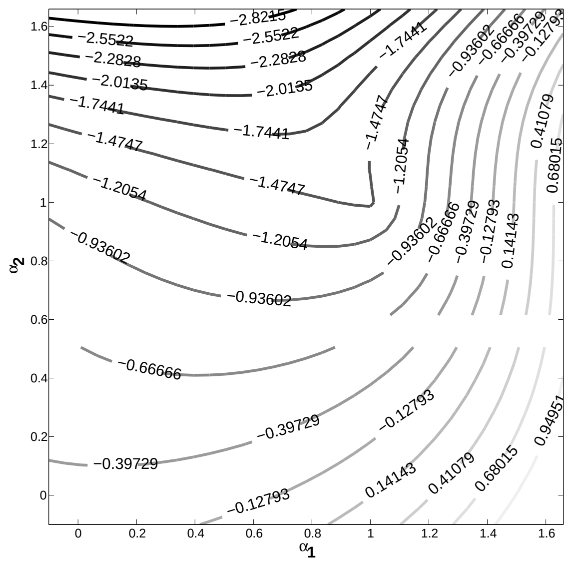
(b) $\rho = 0.3$, $\phi_{12} = 0$, $\phi_{21} = 0$;

Three-dimensional and contour plots of $\sigma_{TD}^2 - \sigma_{BU}^2$, i.e., the variance of the TD approach minus the variance of the BU approach, as a function of the parameters of a bivariate vector IMA(1,1): $\alpha_1 = (\phi_{11} + \phi_{21})$ and $\alpha_2 = (\phi_{12} + \phi_{22})$. In the figures, two of the parameters (i.e., ϕ_{11}, ϕ_{22}) vary while the other three parameters (i.e., ρ, ϕ_{12}, ϕ_{21}) are set equal to the values below each panel.

Figure 3: Three-dimensional and contour plots of $\sigma^2_{TD} - \sigma^2_{BU}$ when $\rho = 0.3$, $\phi_{12} = 0.8$, $\phi_{21} = 0.8$



(a) $\rho = 0.3$, $\phi_{12} = 0.8$, $\phi_{21} = 0.8$;



(b) $\rho = 0.3$, $\phi_{12} = 0.8$, $\phi_{21} = 0.8$

Three-dimensional and contour plots of $\sigma^2_{TD} - \sigma^2_{BU}$, i.e., the variance of the TD approach minus the variance of the BU approach, as a function of the parameters of a bivariate vector IMA(1,1): $\alpha_1 = (\phi_{11} + \phi_{21})$ and $\alpha_2 = (\phi_{12} + \phi_{22})$. In the figures, two of the parameters (i.e., ϕ_{11}, ϕ_{22}) vary while the other three parameters (i.e., ρ, ϕ_{12}, ϕ_{21}) are set equal to the values below each panel.

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