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An empirical comparison of alternative credit default swap pricing models

by Michele Leonardo Bianchi

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AN EMPIRICAL COMPARISON OF ALTERNATIVE CREDIT DEFAULT SWAP PRICING MODELS

by Michele Leonardo Bianchi*

Abstract

Most of the important models in finance rest on the assumption that randomness is explained through a normal random variable because, in general, the use of alternative models is obstructed by the difficulty of calibrating and simulating them. In this paper, we empirically study models for pricing credit default swaps under a reduced-form framework, assuming different dynamics for the default intensity process. After reviewing the most recent results on this subject, we explore both pricing performance and parameter stability during the highly volatile period from 30 June 2008 to 31 December 2010 for different classes of processes: one driven by the Brownian motion, three driven by non-Gaussian Lévy processes, and the last one driven by a Sato process. The models are analysed from both a static and dynamic perspective.

JEL Classification: C46, C58, C61.

Keywords: credit default swap, Cox-Ingersoll-Ross, non-Gaussian Ornstein-Uhlenbeck processes, Lévy processes, Sato processes, filtering methods, unscented Kalman filter, particle filter.

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1 Introduction¹

Credit derivatives are relatively new financial instruments compared with stock options or interest rate derivatives. Even if each product has distinctive features, the mathematical setting needed for the pricing of credit derivatives can be viewed as a slight modification of models employed to price those derivatives with a longer history. Most of the approaches studied to improve market standard models in the area of stock options or interest rate derivatives have also been extended to obtain more reliable credit derivative pricing engines. In particular, certain arguments widely discussed in the literature, such as the analysis of risk premia, of the change of measure problem, of affine processes, of jumps in the underlying, or of a suitable dependence structure for multivariate modelling, have been treated and extended in the context of credit derivatives. The extraordinary growth of the market for credit-linked products has led to increasing interest in the sector.²

The debate between academia, financial industry and regulators has examined whether the mathematical and statistical tools employed in risk management and valuation of complex financial instruments played a role in the recent crisis. In particular, models to measure the default probabilities of baskets of loans have been placed at the centre of the discussion and used to criticize both the mathematics and the quantitative models extensively employed in the industry (readers are referred to Donnelly and Embrechts (2010) for a discussion on this topic).

A correct understanding of different pricing models has implications not only from the methodological standpoint but also for proper financial reporting. Both International Financial Reporting Standards (IFRS) and US Generally Accepted Accounting Principles (US GAAP) allow the so-called *mark-to-model* valuation for financial instruments not having an active market, that is, a particular valuation technique (i.e. a pricing model) can be used to compute the *fair value* of these instruments (see FASB (2006)).

From a practical perspective, the *fair value* of complex financial derivatives, such as exotic options or baskets of credit default swaps, can be computed in two steps: (1) calibration of the parameters to the prices of liquid instruments traded in an active market; and (2) use of the parameters coming out of the previous calibration exercise to price more complex derivatives. This procedure implies that the *fair value* depends on the model's assumptions in both the calibration exercise and the evaluation part. Therefore, the use of different models can lead to different *fair valuations*. For this reason, the first property a model must have is the ability to explain the prices of liquid financial instruments traded on active and regulated markets, first, to ensure a proper risk assessment, then, to limit possible arbitrage opportunities in more complex derivatives. Additionally, a model must be flexible enough to explain certain *stylized facts* observed in financial markets but, at the

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² To have an idea of the phenomenon readers are referred to the website www.defaultrisk.com.

same time, it must possess a satisfactory degree of computational tractability (see Cont (2001)).

Even though there exists a vast amount of research on the pricing of complex credit derivatives, such as first-to-default swaps or synthetic collateralized debt obligations (CDOs), there are only a few empirical studies of the behaviour of credit default swap (CDSs) pricing models during the last financial downturn. Although CDSs are the building blocks of more complex credit derivatives, most of the quantitative models proposed by the industry to price synthetic CDOs do not consider a framework consistent with the CDS prices quoted in the market. In fact, standard factor models for the valuation of CDOs usually assume that all firms in the portfolio have the same default probability and that the dependence between firms can be explained through a single parameter (see Eberlein et al. (2008)). This lack of consistency is mostly due to the absence of proper data and models to calibrate a dependence structure between default events for a large number of different entities, the inherent difficulty in dealing with these complex products, the computational intractability caused by the dimensionality of the problem, and the too fast growth of the market for these products in the last decade without a correct understanding of their associated risks (for a further discussion see Brigo et al. (2010)).

It is within this context that we discuss in this paper the performance of some pricing models to evaluate the spreads of CDSs observed in the market. More precisely, risk-neutral parameters are extracted from the term structure of CDS spreads for more than one hundred companies included in the Markit iTraxx Europe Index by considering five different models (one driven by the Brownian motion, three driven by non-Gaussian Lévy processes, and the last one driven by a Sato process) and we compare their pricing errors in the period from 30 June 2008 to 31 December 2010. In other terms, starting from three different classes of processes we analyse *reduced-form* or *intensity-based* models by assuming five different distributional hypotheses.

The underlying assumption made in most models is that the uncertainty in the financial markets can be explained through a normal distribution. However, there is considerable empirical evidence that the normal distribution is not flexible enough to explain the dynamics of complex financial products. The drawbacks of the normal model are by no means recent. The first fundamental criticism came in the 1960s from Mandelbrot (1963). He strongly rejected normality as a distributional model for asset returns based on his study of commodity returns and interest rates. For this reason, particularly in the last decade, both academia and the industry have started applying more complex mathematical tools to finance, in order to deal with possible mispricing caused by the use of normal-based models. The introduction of jumps and heavy-tails in the dynamics of stock returns (see Rachev and Mittnik (2000), Schoutens (2003), Cont and Tankov (2004), Rachev et al. (2011)) has been followed by the introduction of jumps in default modelling. Jumps in credit risk models can be introduced in two ways: (1) they can be included in the dynamics of the firm value process or (2) they can be included in the dynamics of the intensity process (Schoutens and Cariboni (2009)).³ We focus

³ There is also the possibility of assuming a stochastic interest rate and adding jumps in both

on this second approach.

In the empirical analysis we investigate the classical Cox, Ingersoll and Ross (CIR) process applied to the modelling of default probabilities as proposed by Duffie and Singleton (1999); we then study Ornstein-Uhlenbeck (OU) default intensity processes completely driven by jumps. Finally, we analyse a different framework where the default probability is explained via a Sato process, as proposed by Kokholm and Nicolato (2010). In practice, we compare three different families of stochastic processes in our empirical study: (1) the Brownian motion driving the CIR process; (2) Ornstein-Uhlenbeck processes based on Lévy processes,⁴ that is, non-Gaussian processes with independent and homogeneous increments (gamma, inverse Gaussian and variance gamma); and (3) additive processes (Sato), that is processes with independent but non-necessary homogeneous increments (see Sato (1999)). Jumps are introduced in the default intensity dynamics because CDS spreads exhibit large fluctuations and therefore it is necessary to account for this heavy-tailed nature of the risk (see Cont (2010)) characterized by rare events and irregular jumps.

We consider two methodologies to estimate the proposed models: (1) we fit the models to the daily spreads observed in the market and check both the models' capabilities and the parameter stability (we refer to it as *static estimation*); and (2) we extract the unobservable default intensity process using filtering methods (we refer to it as *dynamic estimation*). Regarding the stability of the parameters, the static analysis shows that the Sato based model outperforms its competitors and the parameter stability can be improved by incorporating regularization techniques into the optimization procedure. For the CDS spreads that we analyse and for the time period that we investigate, under the static perspective the variance gamma based model seems to be satisfactory in terms of calibration error compared with all other competitor models, and from the dynamics perspective it explains CDS behaviour better than the CIR model. Thus, we empirically assess that the skewness and fat-tail properties of the default intensity process are also important for the pricing of CDSs.

The remainder of this paper is organized as follows. In Section 2 we discuss the more recent approaches proposed in the literature to model single-name credit default swaps. A brief review showing the mathematical framework of intensity-based models is reported in Section 3 and the stochastic processes employed in the empirical analysis are described in Section 4. In Section 5 we describe the data considered in the empirical study and in Section 6 we discuss two different estimation methodologies, the first based on calibration on a daily basis, the second based on filtering methods that make use of the entire sample over the observed period. We then report and discuss the results. Section 7 concludes.

the instantaneous interest rate process and the default intensity process.

⁴ The Brownian motion is a special case of Lévy process. It can be proved that it is the only Lévy process with continuous path.

2 Actual vs risk-neutral default probability

In this section we discuss the change of measure problem in the context of default risk valuation to provide an explanation of default risk premia. Before doing this, we briefly recall the history of credit risk models.

It is well known that models for the valuation of single-name credit derivatives are divided in two categories: *firm value* or *structural* models pioneered by Merton (1974) and *reduced-form* or *intensity-based* models originating with Jarrow and Turnbull (1992). The former approach is built on the idea that a default occurs when the value of the firm reaches a certain threshold, the latter directly models the default time as the first jump of a Poisson process with a given intensity λ_t . In practice, the firm value model may consider both balance-sheet and historical default frequencies, while the intensity-based model generally deals only with financial instrument market information (see also Jarrow and Protter (2004)). Each approach has its drawbacks and advantages. For a more detailed comparison between the two approaches see the recent work of Jarrow (2011).

Firm value models use a larger set of information, in particular balance-sheet data, historical default frequencies, and market information, and they can be used to compute the *real world* (*actual* or *actuarial based*) default probability. For this reason these models are often used by rating agencies like Moody's and Fitch (see Dwyer and Qu (2007), and Liu et al. (2007)). However, the firm value process is unobservable, as recently pointed out also by Duan and Fulop (2009). It can be extracted only by considering the observable equity value (or stock price) process, and by taking into consideration that the equity value can be viewed as a contingent claim written on the firm value. Then, there is no general consensus on the data to be considered as inputs for the pricing engine, and on the way to transform the actual probability into a risk-neutral probability to compute the value of default-based derivatives. This last point is of extreme importance for those models that allow for jumps in the dynamics of the firm value process. Similar arguments about the change of measure problem in the context of option pricing can be extended to credit risk models, as discussed by Le Courtois and Quittard-Pinon (2006).

Reduced-form models allow for a relatively simple parameter estimation based on financial instruments traded in the market, such as defaultable bonds or credit default swaps. However, this procedure extracts a probability measure which is by construction risk-neutral. By doing so, the risk premia embedded in the prices of traded instruments cannot be evaluated without a suitable change of measure, that is without a function that combines both information coming from historical data (i.e. historical default frequencies) and information provided by an active market of derivative securities.

As has already been observed for equity options, risk premia are inherently hard to measure (see Jarrow (2012)). A possible approach to evaluating default risk premia has been proposed by Berndt et al. (2008).⁵ Starting from the *real world* default probability provided by Moody's, the authors estimate the parameters of a Black-Karasinski default intensity process under the actual probability measure and the risk-neutral intensity process is extracted from the spreads of CDS

⁵ An interesting study in a structural setting has recently been proposed by Berg (2010).

traded in the market. By exploring the rich structure of all possible changes of measure (see Jarrow et al. (2005)), the authors show that the ratio of risk-neutral to actual default intensities can be viewed as a proxy for the default risk premium. As observed above, default risk premia can only be analysed by combining different sources of information, that is, historical default probabilities, market implied probabilities, and the relations between them.

In this paper we investigate the pricing performance of five different distributional assumptions directly under the risk-neutral measure, but we do not analyse the behaviour of default risk premia, principally because their estimation is very difficult (see Jarrow (2012)). Pricing a derivative following this route is therefore a case of *relative pricing* as defined by Rebonato (2004), and nothing can be said about the real world probability. The purpose of these models is the pricing and hedging of derivatives without allowing arbitrage opportunities.

3 Evaluate CDS spreads

We consider a *reduced-form* approach to model the default probability, and we assume that the default intensity is a stochastic process (see Duffie et al. (2003)). There is a general consensus in assuming a stochastic intensity instead of a deterministic intensity to model uncertainty about the future dynamics of the credit risk of a given reference entity. A similar framework has been used to price interest rate derivatives. However, in the credit derivatives case one models the default intensity process while in the interest rate derivatives case one models the spot rate or the factors explaining the term structure. More in detail, in a reduced-form model one assumes that the time of default is determined by the first jump time of a Cox process N_t starting from zero and with a stochastic intensity rate λ_t , where $0 \leq t \leq T$. Under this setting the default time τ is defined as

$$\tau = \inf \{t > 0 | N_t > 0\} = \inf \{t > 0 | A_t > E_1\},$$

where the process A_t is usually defined as an *integrated process*, i.e.

$$A_t = \int_0^t \lambda_s ds$$

where λ_t is often a stationary affine process and E_1 is an exponential random variable with mean equal to 1. It follows that the survival probability up to time t is equal to

$$P_{Surv}(\tau > t) = \mathbf{E}[\exp(-A_t)] = \phi_{A_t}(i),^6 \tag{1}$$

where $\phi_{A_t}(u)$, with $u \in \mathbb{R}$, is the characteristic function relative to A_t and $i = \sqrt{-1}$. By assuming a model for the intensity process λ_t we can compute the corresponding survival probability. If one knows the characteristic function of the process A_t , it is straightforward to compute the expectation in equation (1).

Recently, Kokholm and Nicolato (2010) analysed a reduced-form approach where the default time is defined by the direct modelling of the process A_t . In

⁶ The characteristic function of a random variable X is defined as $\phi_X(u) = \mathbf{E}[\exp(iuX)]$, $u \in \mathbb{R}$.

their paper the default is driven by an increasing and additive process, that is a process A_t with independent but not necessarily stationary increments.

By following the computations reported in O’Kane and Turnbull (2003), given a model for survival probability, one can easily derive the fair spread of a CDS expiring at time T ,

$$c_T^{CDS} = \frac{(1 - R) \sum_i^n D(0, t_i)(P_{Surv}(t_{i-1}) - P_{Surv}(t_i))}{\sum_i^n D(0, t_i)P_{Surv}(t_i)\Delta t_i + \frac{1}{2} \sum_i^n D(0, t_i)(P_{Surv}(t_{i-1}) - P_{Surv}(t_i))\Delta t_i}, \quad (2)$$

where R is the recovery rate (usually assumed equal to 40 per cent), $D(0, t)$ is the discount factor, $t_1, \dots, t_n = T$ represent the dates of payment at the end of each period t_i until maturity T .⁷ Fee and loss payments are assumed to be made at the end of each period. Furthermore, it is taken into account that if default occurs between some payment dates, the fee has to be paid only for the portion between the last payment date and the time of default as the insurance buyer is protected only for that period.

The extension of equation (1) to the sum of two or more default intensity factors $\lambda_t^1, \dots, \lambda_t^n$ is straightforward when pairwise independence is assumed between factors.⁸ The formula in equation (1) becomes

$$P_{Surv}(\tau > t) = \phi_{A_t^1}(i) \cdots \phi_{A_t^n}(i). \quad (3)$$

However, if one considers dependent factors, the decomposition in equation (3) no longer holds. Normal-based models may still have a closed-form solution, but in general, if one assumes a richer dependence structure (for example, a multivariate model or a copula allowing for tail dependence), Monte Carlo simulations or numerical methods are needed to evaluate the survival probability.⁹

4 Cox-Ingersoll-Ross, Ornstein-Uhlenbeck and Sato reduced-form models

In this section we describe different models based on the short-rate process approach first defined in the interest rate context. However, there are two main differences between interest rate and credit derivatives. First, as observed by Cont (2010), unlike the interest rate swap, the payoff of a CDS has a binary nature, that is, while the mark-to-market value of a CDS position prior to default may be small, the actual exposure upon default may represent a large fraction of the

⁷ Here we assume a deterministic discount factor. Chen et al. (2008, forthcoming) considered a stochastic interest rate, and Dunbar (2008) proposed a framework with stochastic factors to model interest rate and liquidity.

⁸ Feldhütter and Lando (2008) proposed a model with six independent factors to calibrate Treasury bonds, corporate bonds, and swap rates using both cross-sectional and time-series properties of the observed yields. This independence assumption may be restrictive, although the advantage is that pricing formulas have explicit solutions, and the model is more parsimonious with fewer parameters to estimate.

⁹ A multi-factor Lévy based short-rate model has been proposed by Zhang (2006). He considered three dependent factors driven by non-normal processes and calibrate the model to fixed income derivatives.

notional. Second, as outlined by Brigo and El-Bachir (2010), while the interest rate derivatives market is one of the most active financial markets with a large number of caps/floors, swaptions and other derivatives, the single-name default swap market presents a very small number of traded derivatives and the calibration of any model with a large number of parameters (for example, Rebonato et al. (2010)) becomes unfeasible. In view of these differences, we start with the classical CIR model and extend the short-rate approach by modifying the distributional assumption without significantly increasing the number of parameters.

4.1 The CIR process

A well-known way to model the default intensity process is to assume the CIR dynamics by considering the following mean-reverting process

$$d\lambda_t = \kappa(\eta - \lambda_t)dt + \vartheta\sqrt{\lambda_t}dW_t, \quad (4)$$

with $\lambda_0 > 0$, κ , η and ϑ positive parameters, and $2\kappa\eta > \vartheta^2$ in order to ensure that the origin is inaccessible, that is, the intensity process does not reach zero. Under the CIR assumption there exists a closed-form expression for the characteristic function of the integrated CIR process and the survival probability in equation (1) can be computed as follows

$$P_{Surv}^{CIR}(\tau > t) = \frac{\exp(\kappa^2\eta t/\vartheta^2) \exp(-2\lambda_0/(\kappa + \gamma \coth(\gamma t/2)))}{(\coth(\gamma t/2) + \kappa \sinh(\gamma t/2)/\gamma)^{2\kappa\eta/\vartheta^2}}, \quad (5)$$

where $\gamma = \sqrt{\kappa^2 + 2\vartheta^2}$.¹⁰

4.2 Ornstein-Uhlenbeck processes

The CIR model can be enhanced by adding jumps and by considering the so-called jump diffusion CIR (JCIR) model as described by Brigo and Mercurio (2006) and by Lando (2004). Alternatively, one can consider pure jumps mean-reverting processes of the Ornstein-Uhlenbeck family. Over the past decade non-Gaussian Ornstein-Uhlenbeck (OU) processes introduced by Barndorff-Nielsen and Shephard (2001) have been widely studied by practitioners and academics from both empirical and theoretical points of view and used in finance, economics, engineering and other applied sciences. This family of processes can capture important distributional properties observed in real data and offers a more flexible structure with respect to Gaussian-based models. This flexibility, the possibility to explain certain *stylized facts* of financial time series, and a suitable degree of computational tractability have increased the number of applications to finance, in particular, to stochastic volatility and interest rate models, together with a vast amount of theoretical research papers.

As defined by Barndorff-Nielsen and Shephard (2001), an OU process λ_t is a solution of a stochastic differential equation of the form¹¹

$$d\lambda_t = -\theta\lambda_t dt + dz_{\theta t}. \quad (6)$$

¹⁰ See Schoutens and Cariboni (2009) and references therein for the derivation of the formula.

¹¹ The unusual timing θt is deliberately chosen so that the marginal distribution of λ_t is independent of the choice of θ (see Barndorff-Nielsen and Shephard (2001)).

If z_t is an increasing Lévy process with finite variation starting from 0 and if $\lambda_0 > 0$, it can be proved that the process λ_t is strictly positive and bounded from below by $\lambda_0 \exp(-\theta t)$. If λ_t is an OU process with marginal law D ,¹² then it is named a D-OU process. Under certain assumptions¹³ and given a marginal law for D , one can compute the characteristic function of the process z_t (the so-called *background driving Lévy process* - BDLP).¹⁴ The definition can be extended to non-increasing processes, as introduced by Barndorff-Nielsen (1997) and widely studied in academia (see the recent work of Rosiński and Sinclair (2010)).

In the rest of this section we describe OU processes with only positive jumps based on the gamma and on the inverse Gaussian (IG) distributional assumption and an enhancement allowing for both positive and negative jumps based on the variance-gamma distributional assumption.

4.2.1 Mean-reverting processes with only positive jumps

In the following we consider two examples of OU processes: the Gamma-OU and the IG-OU. Both hypotheses are particularly convenient as the characteristic functions of both the integrated Gamma-OU and the integrated IG-OU process are known in closed-form. Furthermore, there exist efficient algorithms to draw random paths by assuming these dynamics (see Barndorff-Nielsen and Shephard (2001) and Zhang and Zhang (2008)).

Without further analysis of the properties of these processes, we provide the necessary formulas to evaluate the survival probability under these distributional assumptions.¹⁵ In the Gamma-OU, case by assuming a $\Gamma(a, b)$ process it can be proved that

$$P_{Surv}^{Gamma-OU}(\tau > t) = \exp \left(-\frac{\lambda_0}{\theta}(1 - \exp(-\theta t)) - \frac{\theta a}{1 + \theta b} \left(b \log \left(\frac{b}{b + \theta^{-1}(1 - \exp(-\theta t))} \right) + t \right) \right), \quad (7)$$

In the IG-OU case by assuming a IG(a,b) process it can be proved that

$$P_{Surv}^{IG-OU}(\tau > t) = \exp \left(-\frac{\lambda_0}{\theta}(1 - \exp(-\theta t)) - \frac{2a}{b\theta} A(t) \right), \quad (8)$$

where

$$A(t) = \frac{1 + \sqrt{1 + \kappa(1 - \exp(-\theta t))}}{\kappa} + \frac{1}{\sqrt{1 + \kappa}} \left(\operatorname{arctanh} \left(\frac{\sqrt{1 + \kappa(1 - \exp(-\theta t))}}{\sqrt{1 + \kappa}} \right) - \operatorname{arctanh} \left(\frac{1}{\sqrt{1 + \kappa}} \right) \right),$$

with $\kappa = \frac{2}{b^2\theta}$.

¹² This means that if one starts the process with an initial value sampled from the D distribution, at each future time t , λ_t is distributed as D .

¹³ One has to assume the law D *self-similar*. For the definition of self-similarity readers are referred to Sato (1999).

¹⁴ See Section A.1 in the Appendix.

¹⁵ See Schoutens and Cariboni (2009) for the derivation of the formula.

4.2.2 Mean-reverting processes with two-sided jumps

In this section we describe the variance-gamma Ornstein-Uhlenbeck process in the context of default modelling. As discussed in Section 6.2, under a state-space framework the use of a OU mean-reverting process with only positive jumps is not sufficient to obtain a satisfactory calibration error in the pricing of observed CDS spreads. To obtain greater flexibility in the calibration exercise we introduce a two-sided jumps Lévy-based OU model. This mean-reverting process belongs to the class of tempered stable as well as of hyperbolic Ornstein-Uhlenbeck processes (see Bibby and Sørensen (2003)). This process can be viewed as the two-sided extension of the Gamma-OU process and also in this case equation (1) can be written in closed-form. Let C , λ_+ , λ_- be positive constants, the law X is said to have a variance-gamma (VG) distribution if the characteristic function of X is given by

$$\phi_X(u) = E[\exp(iuX)] = \left(\frac{\lambda_+ \lambda_-}{\lambda_+ \lambda_- + (\lambda_+ - \lambda_-)iu + u^2} \right)^C. \quad (9)$$

The VG distribution can be viewed as the difference between two independent gamma distributions $\Gamma(C, \lambda_+)$ and $\Gamma(C, \lambda_-)$. Given a VG distribution, one can define the corresponding VG-OU process by assuming $\text{VG}(C, \lambda_+, \lambda_-)$ and it can be proved that¹⁶

$$\begin{aligned} P_{Surv}^{VG-OU}(\tau > t) = \exp & \left(-\frac{\lambda_0}{\theta}(1 - \exp(-\theta t)) \right. \\ & - \frac{\theta C}{1 + \theta \lambda_+} \left(\lambda_+ \log \left(\frac{\lambda_+}{|\lambda_+ + \theta^{-1}(1 - \exp(-\theta t))|} \right) + t \right) \\ & \left. + \frac{\theta C}{1 - \theta \lambda_-} \left(\lambda_- \log \left(\frac{\lambda_-}{|\lambda_- - \theta^{-1}(1 - \exp(-\theta t))|} \right) - t \right) \right). \end{aligned} \quad (10)$$

Even if the process may become negative, the asymmetric shape of the VG distribution and the additional assumption $\lambda_+ < \lambda_-$ make the VG-OU process proper in many empirical applications. Furthermore, the inequalities $\lambda_+ < \lambda_-$ and $\lambda_0 > 0$ ensure that the survivor probability in equation (10) is well-defined.

4.3 The Sato family

Sato processes can be viewed as an enhancement of Lévy processes as they have still independent but not necessarily stationary increments. This class of processes was introduced by Sato (1991), but it has been applied to finance only recently by Carr et al. (2007). As observed in Section 3, Kokholm and Nicolato (2010) analysed a reduced-form approach where the default time is defined by the direct modelling of the process A_t . In the previous Sections 4.1 and 4.2, the increasing process A_t has been modelled as the integral of a OU process. In this section, the process A_t is assumed increasing and additive, that is, a process with independent

¹⁶ See Appendix A.1 for the derivation of the formula.

but not necessarily stationary increments. Here we review the construction of Sato processes, starting from the definition of self-similarity. A process A_t is *self-similar* if

$$A_{\alpha t} = \alpha^\gamma A_t, \quad (11)$$

where the equality is in distribution and γ is named the self-similarity coefficient. Given a *self-decomposable* law¹⁷ and a parameter γ , one can construct a self-similar process. By equation (11) the following equalities

$$\mathbf{E} [e^{iuA_t}] = \mathbf{E} [e^{iut^\gamma A_1}] = \phi_{A_1}(ut^\gamma) \quad (12)$$

hold. If one knows the characteristic function of A_1 , it is possible to have a closed-form expression for (12) and thus for (1). Furthermore, the proposed model has a deterministic intensity process given by the equality

$$\lambda_t = \frac{d}{dt} \log \mathbf{E} [e^{-A_t}]. \quad (13)$$

Both the gamma and the IG random variable are self-decomposable, and the expectation in (12) has a closed-formula. If A_1 is a $\Gamma(a, b)$ random variable, A_t is named a *Sato-Gamma* process, and if A_1 is a IG(a,b) random variable, A_t is named a *Sato-IG* process. As these processes are similar we consider only the Sato-Gamma assumption in the empirical analysis. In the Sato-Gamma model it can be proved that¹⁸

$$P_{Surv}^{Sato-Gamma}(\tau > t) = \frac{1}{(1 + t^\gamma b^{-1})^a}. \quad (14)$$

5 The data

This section provides a description of the data used in the empirical analysis. We consider CDS spread data for 117 companies included in the Markit iTraxx Europe Index (Series 12)¹⁹ from 30 June 2008 to 31 December 2010. The dataset includes the highly volatile period after Lehman Brothers filed for Chapter 11 bankruptcy protection (15 September 2008).

Mid, bid and ask spreads for maturities 1, 3, 5, 7, and 10 years are obtained from Bloomberg. We are aware of the fact that (1) Bloomberg data does not necessarily refer to real transactions because it picks up market quotes (and not transaction prices) provided by its contributors, (2) the collected bid-ask difference could not reflect the real liquidity of the market, (3) the data can be different among data providers, as recently observed by Mayordomo et al. (2010), and, furthermore, (4) the transactions generally involve the 5-year maturity contracts as described in Amadei et al. (2011). Eight companies have been discarded as the spreads are not available for the entire period; therefore we analyse the default term structure for a total of 117 companies. For each company and for each maturity we have 655 observations, for a total of more than 380,000 CDS spreads. Table 1 provides summary statistics on the distribution of the companies analysed in the sample

¹⁷ Readers are referred to Sato (1999) for further details.

¹⁸ See the work of Kokholm and Nicolato (2010).

¹⁹ The index includes a total of 125 companies.

period. In general, for all companies and for all maturities, spreads increased sharply after Lehman's failure and bid-ask spreads widened, particularly for the shortest maturity. Recall that on 17 September 2008, Bloomberg reported that

U.S. Treasury three-month bill rates dropped to the lowest since World War II as a loss of confidence in credit markets worldwide prompted investors to abandon higher-yielding assets for the safety of the shortest-term government securities.

and in the same days the EURIBOR-OIS spread²⁰ reached unexpected levels. Figure 1 shows the behaviour of the mean and the median of CDS and bid-ask spreads in the period analysed.

Note that the use of a one-factor model is justified by the fact that the principal component analysis, conducted using the covariance matrix of the spread levels for 5-year maturity, shows that the first principal component accounts for at least 81 per cent of the variation in all spreads. Pan and Singleton (2008) observed that the use of a stochastic interest rate model does not greatly modify the estimates; for this reason, we assume the risk-free term structure to be known. Risk-free rates are extracted from the LIBOR swap rate for short-term maturities up to 9 months and the EU swap curve for maturities from 1 year to 30 years.²¹ At each given day and for each maturity the discount factor is computed by a linear interpolation of the risk-free term structure.

6 The empirical study: parameter estimation

There are two possible methodologies to estimate a reduced-form model: (1) one can fit the model to the daily spreads observed in the market and check both the model capabilities and the parameter stability; or (2) one can extract the unobservable default intensity process (or processes) by using a *filter*²² as described by Lando (2004) and empirically tested by Jarrow et al. (2009). The first approach can be viewed as a short-term or static estimation for the purpose of pricing and hedging on a daily basis, and for this reason we name it the *market maker approach*. Under this perspective, at each given point in time the model prices have to be as close as possible to the prices traded in the market and even if the market is quoting unreasonable prices the market makers have to find the parameters that replicate those prices.²³ The market maker needs to achieve static consistency in order to provide at each given point in time two-sided quotes. The second approach is a long-term or dynamic estimation where the default intensity process is estimated by *filtering* the new information, for the purpose of assessing the long-term default probability of the reference entity. Estimated model parameters can be used to

²⁰ The difference between the 3-month Euro interbank offered rate (EURIBOR) and the overnight indexed swap (OIS) is a measure of both credit and liquidity risk.

²¹ For a discussion on the choice of the *risk-free* rate see Hull et al. (2004).

²² The Kalman filter and its extensions can be taken into consideration when the intensity model is Gaussian, otherwise a *particle filter* framework has to be applied (see van der Merwe et al. (2001)).

²³ The goal is usually achieved by considering models with a large number of parameters.

make projections on price movements and trade the difference between the model prediction and the market quotes. These investment strategies (known as *statistical arbitrage strategies*) are commonly used by hedge funds and banks' proprietary trading desks and, for this reason, we name this second estimation methodology the *long-term convergence trader approach*. Under this second perspective, one assumes that the model can beat the market or, more clearly, one trusts the model prices and tries to find possible arbitrage opportunities by looking at the differences between model and market prices. Dynamic consistency is important for long-term convergence trading, and pricing errors represent trading opportunities.

In this Section we investigate both approaches. In Section 6.1 we analyse one-factor models based on different distributional assumptions, namely CIR, Gamma-OU, IG-OU, Sato-Gamma and VG-OU based model. Then, in Section 6.2 we cast the model into a state-space framework and calibrate with filtering methods one-factor models based on the CIR and VG-OU processes.

6.1 A static perspective: the market maker approach

First, we consider in the empirical study the market maker approach in order to study the pricing performance on a daily basis and the parameter stability during a market downturn. From a practical perspective, on each trading day we minimize the root mean square error (RMSE) given by

$$RMSE(\Theta) = \sqrt{\sum_{T_i} \frac{(c_{T_i}^{CDS\ market} - c_{T_i}^{CDS\ model}(\Theta))^2}{\text{number of observations}}} \quad (15)$$

where T_i are the different maturities and Θ is the parameter vector according to a given model. Since the minimization of equation (15) with respect to the parameter vector Θ has neither a closed-form solution nor a global minimum, a numerical optimization routine is needed to find a relative minimum.²⁴ As already observed by Fang et al. (2010), the minimization of equation (15) is a well-known ill-posed problem, mainly because the solution is not necessarily unique and there is no guarantee that a solution exists. For this reason we consider a regularization term of the form

$$f(\Theta) = \rho \|\omega \cdot (\Theta - \Theta_0)\|^2 \quad (16)$$

where ρ is a given constant parameter, Θ_0 is a given set of model parameters, ω is a vector selected to give the same weight to each parameter (see Section A.3 in the Appendix), and “ \cdot ” denotes the inner product of vectors. The optimization problem becomes

$$\hat{\Theta} = \min_{\Theta} (RMSE(\Theta)^2 + \rho \|\omega \cdot (\Theta - \Theta_0)\|^2). \quad (17)$$

Furthermore, this approach leads to more parameter stability over time. The choice of the parameter ρ influences the model calibration; however, ρ cannot be fixed in advance but depends on the data at hand and the level of error present in it

²⁴ We use the Matlab r2008b function *fmincon*. The procedure run on a Pentium D 3.00GHz with 2GB of Ram.

(see Cont and Tankov (2004) and Fang et al. (2010)). In the calibration exercise, we consider two different values for ρ : (1) first, we solve the optimization problem (17) without regularization techniques, that is $\rho = 0$, and then (2) we solve it with $\rho = 100$. This last value for ρ shows a good balance between pricing performance and parameter stability (see Section A.3 in the Appendix).

Then, we add some constraints to the problem in order to obtain more stable parameters over time and avoid the CIR and the VG-OU process hitting the zero bound. In the CIR case we force the parameters $(\kappa, \eta, \vartheta, \lambda_0)$ in the region between $(0.1, 0.005, 0.05, 1e-5)$ and $(0.8, 0.05, 0.25, 2.5)$. In the one-sided jumps OU cases we choose the following constraints for $(\theta, a, b, \lambda_0)$: for the Gamma-OU model $(0.1, 0.1, 10, 1e-5)$ are the lower bounds and $(4, 150, 40000, 2.5)$ the upper ones, for the IG-OU model $(0.25, 0.2, 10, 1e-5)$ are the lower bounds and $(3, 2, 100, 2.5)$ the upper ones. In the Sato case we fix $a = 0.5$ and (γ, b) can vary between $(0.5, 5)$ and $(5, 1500)$. Finally, in the VG-OU case $(0.1, 0.1, 10, 10, 1e-5)$ are the lower bounds and $(4, 150, 10000, 10000, 2.5)$ the upper ones of the parameters $(\theta, C, \lambda_+, \lambda_-, \lambda_0)$ and we add the inequality constraint $\lambda_+ < \lambda_-$, which in practice means that the positive tail is “fatter” than the negative one. Furthermore, in the non-regularized problem, the starting point of the optimization procedure is kept fixed and in the regularized problem the starting point of a given day is the optimal solution computed for the previous day.

We observe that in the one-sided jumps OU models, the calibration can be problematic as spread values are mainly governed by the ratio a/b , but the parameters a and b are difficult to calibrate separately. By fixing as starting point a large value for a (for example, $a = 100$) one obtains in the calibration procedure a large value for b as well ($b \gg a$), therefore the variance of the intensity OU process becomes very small (in this case, the ratio $a/b = k$). By fixing as starting point a smaller value for a' (for example, $a' = 2$) one obtains in the calibration procedure a smaller value for b' ($b' < b$). At the same time the ratio $a'/b' = a/b = k$, the pricing error is quite close to the previous and the estimated variance is greater. In order to obtain a greater variance for the calibrated OU processes comparable with that of the CIR based model, we select as starting points in the Gamma-OU and in the IG-OU cases $a = 2$ and $a = 0.5$, respectively (see also Figure 11 in Appendix A.2). Under this assumption we obtain much smaller estimated parameters than in Cariboni and Schoutens (2009). A careful selection of the initial parameters is needed in the VG-OU case as well.

After some preliminary attempts, as starting point in the optimization procedure we consider $(0.3, 0.025, 0.065, 0.005)$ for the CIR model, $(0.75, 2, 100, 0.005)$ and $(0.5, 0.5, 25, 0.005)$ for the Gamma-OU and the IG-OU model, $(1, 0.5, 100)$ for the Sato-Gamma model, and $(0.75, 20, 250, 400, 0.005)$ in the VG-OU model.

Figure 2 shows the behaviour of the RMSE of the calibrated one-factor models across companies (median, 95th and 15th percentile are reported) with and without regularization. The pricing error is high between the end of 2008 and the first half of 2009. At first sight, it might seem that the two optimization approaches provide similar pricing errors and for this reason we report only the results of the regularized problem. In the following we show only median, mean, 90th and 10th percentile values and we do not report minimum and maximum values of the variables of interest. We do this to exclude possible outliers that may affect the

explanatory power of the results shown in Figure 2. These outliers are principally due to companies with the highest spreads.

Furthermore, we define a market distress measure²⁵ (MDM) given by the distance between bid and ask spreads, that is

$$MDM = \sqrt{\sum_{T_i} \frac{(c_{T_i}^{CDS \text{ bid}} - c_{T_i}^{CDS \text{ ask}})^2}{4 \cdot \text{number of observations}}}. \quad (18)$$

For each given model and for each company we count the number of days in which the RMSE is greater than the measure MDM, that is

$$\text{Exceedances} = \frac{\sum_t I_{\{RMSE_t > MDM_t\}}}{\text{numbers of days}}$$

where t indicates the time component. The number 4 in the denominator of equation (18) implies that we have a satisfactory pricing performance if the difference between the market price and the model price is not greater than an half of the bid-ask spread, or equivalently, the model price is larger than the ask and smaller than the bid price. This is a possible way to include a market distress component into the pricing exercise. The true price lies somewhere between the bid and ask price, therefore we have a bad calibration when the model price is not, in mean, between these prices. In practice, when the RMSE exceeds the MDM, we have a bad calibration if not we have a good performance of the model. As the market becomes less stable, the measure MDM starts to increase, therefore a larger pricing error is allowed. Figure 3 reports the behaviour of this measure, where median, mean and percentiles are computed across all 117 companies. Furthermore, by considering all the companies analysed, Figure 4 shows for all calibrated models the boxplot on the number of days on which an exceedance occurs. First, the figure shows that the regularization enhances the calibration exercise in the IG-OU and the VG-OU case. Under the IG distributional assumption the choice of the starting point and the regularization play a major role. Then, in the regularized optimization approach on average the VG-OU model is the best performing and the Sato-Gamma is the worst. In the CIR and in the Gamma-OU case, the pricing error of the model with $\rho = 100$ is slightly greater than the model with $\rho = 0$. Under the Sato assumption the regularization does not affect the finding of the optimal parameters. In both non-regularized and regularized approaches, the VG-OU model produces slightly smaller errors, at least for the data considered in this study. This is not surprising since the VG-OU model has the largest number of parameters.

Regarding parameter stability, as outlined by Kokholm and Nicolato (2010), the Sato process, even if with only 2 degrees of freedom, demonstrates more stable parameters as shown in Figures 5 and 6. As outlined in similar empirical studies, we estimate the autocorrelation of the parameters, in order to assess parameter stability over time. Figure 7 shows the average autocorrelation for each model

²⁵ The number 4 ($=2^2$) in the denominator comes from the fact that we are considering the semi-difference between bid and ask spreads.

parameter across companies.²⁶ As expected, the regularized optimization technique improves parameter stability over time in all the cases considered (see also Section A.3 in the Appendix). As far as parameter stability is concerned, the Sato-Gamma model outperforms the four competitor models. Furthermore, the Sato based model does not show big differences between the regularized and the non-regularized optimization approaches.

6.2 A dynamic perspective: the long-term convergence trader approach

In the market maker case the model parameters are calibrated on CDS spreads at each given point in time and these parameters can vary over time. In the long-term convergence trader case the dynamics of CDS spreads are explained by a unique set of parameters kept fixed over time. For each given company, under the static approach we perform a point-in-time estimation and under the dynamic approach we extract both the unobservable intensity process and the related parameters by the whole term structure of CDS spreads (the so-called *state and parameter estimation*). It is clear that a dynamic consistency is more difficult to achieve than a static consistency, and a more sophisticated calibration method is needed.

Filtering methods are standard tools to explore the behaviour of the interest rate term structure (see, for example, the works of Brigo and Hanson (1998) and Duan and Simonato (1999)) and they have been successfully applied in a state-space framework to extract the unobservable factors from observed rates. However, there are only few research papers that cast the default term structure into a state-space form and calibrate it with a filter (see Chen et al. (2008), Jarrow et al. (2009), Carr and Wu (2010), and Chen et al. (forthcoming)).²⁷

In all the cases we are interested in, the model can be written in the following form

$$\begin{aligned}\lambda_t &= f(\lambda_{t-1}, \Theta, v_{t-1}) \\ z_t &= h(\lambda_t, \Theta, \varepsilon_t)\end{aligned}\tag{19}$$

where t is the day counter, λ_t is the state variable (also referred to as the latent or unobservable factor) and it can be also a multidimensional variable, v_{t-1} is the randomness from the state variable. The state variable follows the dynamics described by f . The variable z_t represents the set of observations, in our case the CDS spreads observed in the market. Then, the function h is the so-called measurement function, which in our case is given by the CDS pricing formula, and it depends on the state variable, model parameters and measurement noise ε_t . A standard hypothesis assumes that this measurement noise is normally distributed: since we consider five CDS spread observations each day, we have a five dimensional normally distributed error. Even though the measurement error covariance matrix R can be set as a non-diagonal matrix, it is chosen to be diagonal in this study and therefore the covariance structure of default intensity is represented only by the

²⁶ A value around 0 indicates a very low stability; a value around 1 indicates high stability.

²⁷ Recently, Li (2011) analysed and explained the use of filtering methods and applied them to option pricing models.

model itself and not by the measurement error covariance matrix. Furthermore, in order to provide a better estimation error, the measurement error covariance is adjusted for the daily observed bid-ask spreads, that is, the diagonal covariance matrix is multiplied by a diagonal matrix with diagonal entries equal to observed bid-ask spreads.

The state variable in equation (19) can be *filtered* in order to obtain a likelihood function for the error.²⁸ As described in Chen et al. (forthcoming), given the definition of the state-propagation equation and measurement equations, at each time step t , we filter out the mean and covariance matrix of the state variables conditional on the observed series and construct the predictive mean (z_t^{model}) and covariance matrix (P_t) of the observed series based on the filtered state variables. Then, model parameters are estimated by maximizing the sum of the likelihood functions. Under the assumption of normally distributed errors,²⁹ it is possible to obtain the following quasi-maximum likelihood estimates, that is, one can compute (up to a constant term) the log-likelihood function at time t

$$LL_t = -\frac{1}{2} \log(\det(P_t)) - \frac{1}{2} ((z_t^{model} - z_t^{market})' P_t^{-1} (z_t^{model} - z_t^{market})) \quad (20)$$

and the joint log-likelihood for the entire observed sample is

$$LL(\Theta) = \sum_{t=1}^T LL_t. \quad (21)$$

Then, the maximum likelihood estimator (MLE) can be computed by solving the following optimization problem

$$\hat{\Theta} = \max_{\Theta} LL(\Theta). \quad (22)$$

As discussed above, the algorithm provides the simultaneous state and parameter estimation. To resume, the estimation algorithm can be implemented as follows.

Algorithm

1. at time 0, take an initial guess for the set of parameters Θ_0 , the state variable λ_0 and for the diagonal matrix R ;
2. estimate the state variable with a filtering method and generate model prices;
3. compare the model prices with the market ones and evaluate the log-likelihood function in equation (21);
4. insert the log-likelihood evaluation procedure (steps 1 to 3) into an optimization procedure in order to find the solution of the problem (22).

²⁸ A more detailed description of the algorithm can be found in the Appendix A.4.

²⁹ We are assuming that the error component is normal, but the state variable can be any Markov process. For each step, we assume that the measurement errors on each series are independent and identically distributed.

Since the model proposed in equation (19) is non-Gaussian with respect to the state variable and not linear with respect to the measurement function, the classical Kalman filter cannot be used. The filtering methods considered here are based on the extensions of the Kalman filter, that is the unscented Kalman filter (UKF) and the particle filter (PF), as described by van der Merwe et al. (2001). Inference with filtering methods has been widely studied, see for example the works of Lopes and Tsay (2010), and applied in engineering and finance. Ad-hoc statistical Matlab libraries are available on the web.³⁰ In this paper we do not go into detailed explanations of computational issues related to this field: they have little to do with financial modelling and more to do with statistical and programming problems.

In the rest of this section we analyse under a filtering perspective the same one-factor models already studied in Section 6.1. However, we focus only on the CIR and the VG-OU model. By considering that the VG distribution in the case in which $\lambda_+ < \lambda_-$ is asymmetric, the proposed VG-OU process presents positive and negative jumps of different sizes, and at least in theory, it allows a more flexible calibration. However, this greater flexibility comes at the expense of the positivity of simulated trajectories. In order to overcome this theoretical drawback, we use a practical trick (see O’Sullivan (2008)). Indeed, in both the CIR and the VG-OU case, to ensure the factor does not become negative we replace possible negative values of the factor with 10^{-5} (though this occurs rarely). In this part of the paper we do not deal with Sato based models. By construction, Sato based models do not present a stochastic default intensity (as stated in equation (13) the intensity is deterministic), as they are defined by directly modelling the process A_t in equation (1). Then, we exclude also the Gamma and IG based models because, under a dynamic perspective, they do not seem to provide an acceptable level of calibration error in the fitting of observed CDS spreads. The main drawback is the lack of negative jumps in the trajectory that makes slow return to stability levels after large positive jumps.³¹

6.2.1 Assessing the filters’ capabilities: a simulation study

In this section we propose a simulation study of two different filtering methods, that is the UKF and the PF, in order to assess the estimation methodology. First, intensity processes are simulated and then CDS prices are computed with a certain normal measurement random noise with given covariance matrix $\sigma_\varepsilon I$, where σ_ε is a constant and I is the unit matrix.

The simulation study is performed by assuming two different dynamics for the mean-reverting process driving the default intensity (CIR and VG-OU). As already discussed in Section 6.2, the Gamma-OU and the IG-OU are not considered since they show a poor performance, mostly due to the fact that they allow only for positive jumps. We extract the risk-free rate from the LIBOR swap rate for short-term maturities up to 9 months and the EU swap curve for maturities from 1 year

³⁰ A slight modification of the code proposed in <http://www.cs.ubc.ca/~nando/> has been used in the empirical study, in both the UKF and the PF case.

³¹ Multi-factor OU models have been already applied to finance to explain the behaviour of electricity spot prices (see Benth et al. (2007)).

to 30 years from 30 June 2008 to 31 December 2010, for a total of 655 observation days, as described in Section 5. We simulate 655 CDS spreads for each model and for each maturity (1, 3, 5, 7, and 10 years). More precisely, we use the following procedure:

1. by considering a one-factor model, we simulate an intensity process with a given parameters set Θ_0 ;
2. given the intensity process (the state variable), we compute the corresponding CDS spreads;
3. we simulate the normal measurement noise with zero mean and covariance matrix $\sigma_\varepsilon I$ ($\sigma_\varepsilon = 10$) and then add the noise to the CDS spreads;
4. for each day, we calibrate the one-factor model by following the method proposed in Section 6.1 (for a total of 655 sets of parameters);
5. we compute the median values Θ_{median} of the parameters computed in Step 4 (across all 655 sets of parameters) and only λ_0 is set to be equal to the parameter estimated for the first observation day;
6. by taking as starting point Θ_{median} , we estimate the default intensity process and the parameters with a filtering method;
7. finally, we compare the estimated intensity process with the simulated one.

In the CIR case we consider the set of parameters $(\kappa, \eta, \vartheta, \lambda_0, \sigma_\varepsilon)$ equal to $(0.35, 0.02, 0.1, 0.0025, 10)$. Figure 8 shows simulated CDS spread data, the simulated CIR process, the estimated unobservable CIR process extracted through the unscented Kalman filter from the simulated CDS spread data, and the pricing errors across different maturities. As shown in Figure 8, the estimated (and unobservable) default intensity process is close to the simulated one, showing a suitable performance of the estimation methodology.

The CIR model does not allow for jumps in the default intensity, and this implies that it may be not able to capture sharp movements in the market. Conversely, the VG-OU model allows for jumps in the default intensity and, at least in theory, can capture sharp movements in the market and for this reason we also study it. In the VG-OU case we consider the set of parameters $(\theta, C, \lambda_+, \lambda_-, \lambda_0, \sigma_\varepsilon)$ equal to $(0.75, 20, 500, 1000, 0.0025, 10)$. Figure 9 shows simulated CDS spread data, the simulated VG-OU process, the estimated unobservable VG-OU process extracted through the UKF and the PF from the simulated CDS spread data, and the pricing error across different maturities for both filtering methods. As shown in Figure 9, the unobservable default intensity process estimated with the PF method has a smaller pricing error, showing a suitable performance of the estimation methodology. The intensity process estimated with the UKF method presents a worse performance than the process estimated with the PF method, as it is not able to capture large jumps.

However, as expected, in the UKF case, the optimization procedure converges faster towards a local minimum. As observed by Kantas et al. (2009), in the PF case

the maximum likelihood parameter estimation method is more challenging, because one has to deal with a likelihood function estimate that is not continuous with respect to Θ and, consequently, the optimization routine shows poor convergence properties. Furthermore, to avoid the degeneration of the particle weights, which leads to a few particles containing most of the probability mass, and to make the sequential simulation-based techniques viable (see Chapter 7 in Candy (2009)), we consider a stratified resampling scheme (see Douc and Cappé (2005), and references therein).

We point out that under the VG-OU assumption the likelihood evaluation procedure in the unscented Kalman filter case is more than 50 times faster than in a particle filter case with 1,500 particle simulations. The computing time increases if one considers more particles or a more complex Lévy based model. The computational cost of the resampling step is of minor concern with respect to the computational cost of the simulation step and of the evaluation of the measurement function h . The evaluation of the measurement function h accounts for more than 70 per cent of the computational time and the simulation step for 25 per cent. We stress that we face the same problem under the CIR assumption and have a similar computational cost.³² In the particle filter case we simulate 1,500 particles at each time step. Furthermore, in order to reduce the variance and to improve the stability of the algorithm, random variates are kept fixed into the optimization routine. In Lévy based models, one may have to deal also with memory allocation problems as the number of particle increases. This is of minor concern if one can use the 64-bit version of Matlab on a 64-bit operating system or if the random numbers generator algorithm allows the seed to be set. We run the particle filter estimate on an Opteron platform with Matlab R2011a and, in order to reduce the algorithm variance, we fix six matrices of uniform variates of which the four largest have the dimension $1500 \times 100 \times 655$ or, alternatively, we set the seed of the generator .

6.2.2 Estimation on market data

In this section we investigate the filtering based estimation on real CDS spread data described in Section 5. In Figure 10 we report the results of the maximum likelihood estimates based on the UKF and the PF method. First, we consider the UKF approach. Under this approach both the RMSE and average relative percentage error (ARPE)³³ are smaller in the VG model. The median ARPE is 10.02 per cent in the CIR case and 9.39 in the VG case (in mean, it is 10.13 and 9.82 respectively). The computational time, evaluated as the number of function evaluations into the optimization procedure, is larger in the VG case in comparison with the CIR case (in median, more than 950 against around 500 function evaluations). As already

³² If one considers an unscented particle filter algorithm (see van der Merwe et al. (2001)), even with a small number of random variates (200 particles), the computational time increases, because most of the time is spent in the computation of the proposal distribution (UKF step).

³³ The ARPE is defined as

$$ARPE = \frac{1}{\text{number of observations}} \sum_{\text{observations}} \frac{|c^{CDS \text{ market}} - c^{CDS \text{ model}}|}{c^{CDS \text{ market}}}.$$

observed in Section 6.2, for each single company the dynamics of CDS spreads over time is explained by a unique set of parameters. Therefore, the boxplots in Figure 10 show the variations of these parameters across all 117 companies analysed. By taking into consideration the results in Section 6.1, in the optimization procedure the parameters $(\kappa, \eta, \vartheta, \lambda_0)$ move in the region between $(0.1, 0.005, 0.05, 1e-5)$ and $(0.8, 0.05, 0.25, 2.5)$ in the CIR based model and $(\theta, C, \lambda_+, \lambda_-)$ move in the region between $(0.1, 1, 50, 50)$ and $(4, 100, 750, 1000)$ in the VG-OU based model. In both cases the diagonal elements of the diagonal matrix R range between 1 and 100. We underline that the parameter C hits the boundary in most cases and the median value is 100 (the mean value, 65.40). By construction, positive jumps are always greater than negative ones, as the marginal distribution VG is assumed to be asymmetric with $\lambda_+ < \lambda_-$. The median difference between λ_- and λ_+ is slightly more than 40 (the mean difference is slightly more than 120). This difference between the positive and the negative tail in the VG based model seems to be the factor that decreases the calibration error with respect to the normal-based model. However, as already proved in Section 6.2.1, the UKF approach is not able properly to assess the benefits arising from the presence of jumps in the dynamics of the default intensity and for this reason we calibrate the model by applying a particle filter approach.

As already observed, in Figure 10 we also report the results of the maximum likelihood estimates based on the PF method. As starting point of the optimization problem we consider the UKF estimates. In the PF case too, the estimates do not show remarkable differences. The computational time of the two models, evaluated as the number of function evaluations into the optimization procedure, is similar. The median ARPE is 9.19 per cent in the CIR case and 8.92 in the VG case (in mean, it is 9.67 and 9.30 respectively). We underline that the parameter C hits the boundary in most cases, and the median value across all 117 companies analysed is 99.98 (in mean, 79.39). The median difference between λ_- and λ_+ is nearly 43 (the mean difference is nearly 120).

The estimates related to the matrix R show that the calibration error depends on the maturity. In particular, Figure 10 shows large values for the elements r_{11} and r_{55} of the diagonal matrix R , which correspond to the 1-year and 10-year maturity, respectively. Conversely the element r_{22} corresponding to the 3-year maturity is the smallest in the CIR case (r_{33} in the VG-OU case, respectively). This means that the calibration error is large for the shortest and the longest maturity and, conversely it is small for the 3-year maturity (5-year, respectively). The use of a one-factor model allows us to calibrate only partially the inverted spreads observed during the recent market downturns. The calibration error is large in mean (nearly 10 per cent). The preliminary principal component analysis described in Section 5 and the present calibration exercise confirm that the proposed one-factor models are not able exactly to calibrate the dynamics of CDS spreads over time. Even if the VG-OU model outperforms the CIR model under the static setting, it does not show remarkable differences with respect to the Gaussian competitor in the dynamic setting analysed.

Besides estimating the parameters in both models, we apply the Akaike information criterion (AIC) to identify the superior model. The AIC is evaluated

as

$$AIC = 2np - 2LL$$

where np is the number of parameters and LL is the model's log-likelihood. According to the Akaike information criterion, the VG-OU model (PF estimate) is better because in 110 cases out of 117 its AIC value is smaller than the AIC value of the CIR model (UKF estimate).³⁴ The Akaike information criterion shows that the proposed VG-OU model can be useful in explaining sharp movements in the CDS market.

7 Conclusions

In this paper we present an empirical study of CDS *no-arbitrage* pricing models by following two different estimation methods: (1) we study the models' performance under a static perspective and (2) we analyse the dynamics of the default process over time by considering filtering methods.

In the first empirical analysis, pricing performance, parameter stability and model performance adjusted for general market behaviour are measured across different models and across a wide range of companies. It is shown that, although all selected models have a large pricing error during the period of market distress, the RMSE relative to CIR and OU processes is small, with median values across all companies less than 4bp over the entire time window. The Sato assumption does not allow for an effective calibration of observed spreads, even if parameters are quite stable over time. As far as parameter stability is concerned, the CIR parameters have autocorrelations similar to the OU model parameters, and the Sato-Gamma parameters are more stable over time. The parameter stability can be improved by considering regularization techniques into the optimization procedure. Furthermore, we note that in some cases the CIR parameters hit the boundary included in the optimization procedure to avoid the intensity process reaching zero. Regarding the models' performance adjusted for general market behaviour, and measured as the number of times the model price is between bid and ask prices, both CIR and OU processes show a satisfactory performance for at least 50 per cent of the companies included in the sample, with less than 15 per cent exceedances (10 per cent in the VG-OU case). For the Sato process the number of exceedances is greater.

The dynamic analysis shows that only the CIR and the VG-OU models can effectively be used to calibrate market CDS spreads under a state-space approach. The use of Lévy based OU processes that allow only for positive jumps (i.e. Gamma-OU and IG-OU) is obstructed by the path properties of these processes, which do not seem suitable to replicate the behaviour of observed CDS spreads quoted in the market. The VG-OU process allows for two-sided jumps and it may be used in practical applications. However, this process is driven by a non-Gaussian random variable and therefore the use of more complex filtering method is needed to obtain reliable estimates and to assess the benefits arising from the

³⁴ In 89 cases out of 117 the AIC of the VG-OU model (PF estimate) is smaller than the AIC value of the CIR model (PF estimate).

presence of jumps in the dynamics of the default intensity. Finally, because the calibration error is in mean nearly 10 per cent, we can conclude that the proposed one-factor models are not always able exactly to calibrate the dynamics of CDS spreads over time. The VG-OU model does not show remarkable differences in term of calibration error with respect to the Gaussian competitor under the dynamic setting analysed. However, it can be useful to calibrate more volatile CDS spreads to explain sharp variations in CDS spreads. This is confirmed by the fact that, applying the Akaike information criterion, we find empirical evidence that the VG-OU model outperforms the common CIR model.

A Appendix

A.1 Ornstein-Uhlenbeck processes

Consider the stochastic process λ_t defined as

$$d\lambda_t = -\theta\lambda_t dt + dZ_{\theta t}, \quad (\text{A.1})$$

where $\theta > 0$ and Z_t is a *background driving Lévy process* (BDLP), or equivalently

$$\lambda_t = \exp(-\theta t)\lambda_0 + \exp(-\theta t) \int_0^t \exp(\theta s) dZ_{\theta s}. \quad (\text{A.2})$$

We refer to processes of this family as Ornstein-Uhlenbeck processes. This family of stochastic processes has been widely investigated in the literature (see Barndorff-Nielsen and Shephard (2001)). In financial applications we are interested in the integrated process A_t defined as

$$A_t = \int_0^t \lambda_s ds, \quad (\text{A.3})$$

that can be rewritten as

$$A_t = \theta^{-1}(1 - \exp(-\theta t))\lambda_0 + \theta^{-1} \int_0^t (1 - \exp(-\theta(t-s))) dZ_{\theta s},$$

and in defining λ_t such that the characteristic function of A_t

$$\phi_{A_t}(u) = E[\exp(iuA_t)] \quad (\text{A.4})$$

has a closed-form solution. Stochastic volatility models to price stock options and intensity-based models to price credit default swaps have been studied in the related literature as described by Nicolato and Venardos (2003) and Cariboni and Schoutens (2009).

Given a one-dimensional distribution D (not necessarily restricted to the positive half-line), there exists a (stationary) OU process whose marginal law is D (i.e. a D-OU process) if and only if D is self-decomposable (Schoutens (2003)). The cumulant function of Z_1 , that is

$$k_Z(u) = \log E[\exp(-uZ_1)] \quad (\text{A.5})$$

can be derived by the cumulant function of the law D , indeed

$$k_Z(u) = uk'_D(u). \quad (\text{A.6})$$

Given the log of the moment generating function of a random variable L_1

$$\vartheta(u) = \log E[\exp(uL_1)],$$

and considering the Lemma proven in Eberlein and Raible (1999) and Nicolato and Venardos (2003), the following equality holds

$$E \left[\exp \left(\int_0^t f(s) dL_s \right) \right] = \exp \left(\int_0^t \vartheta(f(s)) ds \right), \quad (\text{A.7})$$

where L is a Lévy process with some finite exponential moments and f is a bounded function $\mathbb{R} \rightarrow \mathbb{C}$, that is $|\Re(f)| < M$. It follows that

$$E[\exp(iuA_t)] = \exp\left(iu\lambda_0\theta^{-1}(1 - \exp(-\theta t)) + \int_0^t \theta \vartheta_Z(iu\theta^{-1}(1 - \exp(-\theta(t-s)))) ds\right). \quad (\text{A.8})$$

Closed-form solutions for equation (A.8) are known in the Gamma-OU and the IG-OU case. The integral (A.8) can be explicitly computed also in the variance-gamma case (VG-OU), as proved in the following.

A law X is said to have a variance-gamma (VG) distribution with mean m ³⁵ if the characteristic function of X is given by

$$\phi_X(u) = E[\exp(iuX)] = \exp(iu(m - C(\lambda_- - \lambda_+)\lambda_+^{-1}\lambda_-^{-1}) - C \log(\lambda_+\lambda_- + (\lambda_+ - \lambda_-)iu + u^2) + C \log(\lambda_+\lambda_-)).$$

Then, we can define a Lévy process X_t and refer to it as a VG process. By simple calculations, we can write the cumulant function of Z_1 , that is given by

$$k_{Z_{VG}}(u) = uk'_{VG}(u) = -u(m - C(\lambda_- - \lambda_+)\lambda_+^{-1}\lambda_-^{-1}) + \frac{Cu}{\lambda_- - u} - \frac{Cu}{\lambda_+ + u} \quad (\text{A.9})$$

then,

$$\vartheta_{Z_{VG}}(u) = u(m - C(\lambda_- - \lambda_+)\lambda_+^{-1}\lambda_-^{-1}) + \frac{Cu}{\lambda_+ - u} - \frac{Cu}{\lambda_- + u} \quad (\text{A.10})$$

Thus, in order to evaluate equation (A.8) we compute the integral

$$\int_0^t \theta \vartheta_{Z_{VG}}(iu\theta^{-1}(1 - \exp(-\theta(t-s)))) ds \quad (\text{A.11})$$

and by changing variable $x = 1 - \exp(-\theta(t-s))$ in the integral above, we write

$$\int_0^{1-\exp(-\theta t)} \frac{\vartheta_{Z_{VG}}(iu\theta^{-1}x)}{(1-x)} dx$$

Therefore, we have the integral

$$\int_0^{1-\exp(-\theta t)} \left(\frac{iu\theta^{-1}x(m - C(\lambda_- - \lambda_+)\lambda_+^{-1}\lambda_-^{-1})}{(1-x)} + \frac{Ciu\theta^{-1}x}{(\lambda_+ - iu\theta^{-1}x)(1-x)} - \frac{Ciu\theta^{-1}x}{(\lambda_- + iu\theta^{-1}x)(1-x)} \right) dx. \quad (\text{A.12})$$

³⁵ We add a shift with respect to equation (9) in order to obtain a zero mean random variable when $m = 0$.

The first part of the integral (A.12) can easily be evaluated

$$\begin{aligned} & \int_0^{1-\exp(-\theta t)} \frac{i u \theta^{-1} x (m - C(\lambda_- - \lambda_+) \lambda_+^{-1} \lambda_-^{-1})}{(1-x)} dx \\ &= -i u \theta^{-1} (m - C(\lambda_- - \lambda_+) \lambda_+^{-1} \lambda_-^{-1}) (1 - \exp(-\theta t) - \theta t). \end{aligned} \quad (\text{A.13})$$

By setting $k = i u \theta^{-1}$, the second part of the integral (A.12) can be rewritten as

$$\begin{aligned} & \int_0^{1-\exp(-\theta t)} \left(\frac{C k x}{(\lambda_+ - k x)(1-x)} - \frac{C k x}{(\lambda_- + k x)(1-x)} \right) dx \\ &= C k \int_0^{1-\exp(-\theta t)} \left(-\frac{\lambda_+}{(\lambda_+ - k)(\lambda_+ - k x)} + \frac{1}{(\lambda_+ - k)(1-x)} \right. \\ & \quad \left. + \frac{\lambda_-}{(\lambda_- + k)(\lambda_- + k x)} - \frac{1}{(\lambda_- + k)(1-x)} \right) dx \\ &= C k \left[\frac{\lambda_+}{k(\lambda_+ - k)} \log(|\lambda_+ - k x|) + \frac{\lambda_-}{k(\lambda_- + k)} \log(|\lambda_- + k x|) \right. \\ & \quad \left. - \left(\frac{1}{\lambda_+ - k} - \frac{1}{\lambda_- + k} \right) \log(|1-x|) \right]_0^{1-\exp(-\theta t)} \\ &= C \left[\frac{\lambda_+}{\lambda_+ - k} \log(|\lambda_+ - k(1 - \exp(-\theta t))|) + \frac{\lambda_-}{\lambda_- + k} \log(|\lambda_- + k(1 - \exp(-\theta t))|) \right. \\ & \quad \left. + \left(\frac{1}{\lambda_+ - k} - \frac{1}{\lambda_- + k} \right) k \theta t - \frac{\lambda_+ \log \lambda_+}{\lambda_+ - k} - \frac{\lambda_- \log \lambda_-}{\lambda_- + k} \right] \end{aligned} \quad (\text{A.14})$$

Thus, the integral has the following solution

$$\begin{aligned} & \int_0^{1-\exp(-\theta t)} \left(\frac{C k x}{(\lambda_+ - k x)(1-x)} - \frac{C k x}{(\lambda_- + k x)(1-x)} \right) dx \\ &= \frac{C}{\lambda_+ - i u \theta^{-1}} \left(\lambda_+ \log \left(\frac{|\lambda_+ - i u \theta^{-1} (1 - \exp(-\theta t))|}{\lambda_+} \right) + i u t \right) \\ & \quad + \frac{C}{\lambda_- + i u \theta^{-1}} \left(\lambda_- \log \left(\frac{|\lambda_- + i u \theta^{-1} (1 - \exp(-\theta t))|}{\lambda_-} \right) - i u t \right) \\ &= \frac{\theta C}{i u - \theta \lambda_+} \left(\lambda_+ \log \left(\frac{\lambda_+}{|\lambda_+ - i u \theta^{-1} (1 - \exp(-\theta t))|} \right) - i u t \right) \\ & \quad - \frac{\theta C}{i u + \theta \lambda_-} \left(\lambda_- \log \left(\frac{\lambda_-}{|\lambda_- + i u \theta^{-1} (1 - \exp(-\theta t))|} \right) + i u t \right) \end{aligned}$$

and in the case

$$m = \frac{C}{\lambda_+} - \frac{C}{\lambda_-},$$

equation (10) follows.

A.2 Conditional moments and simulation algorithms

In the following we briefly recall some results on CIR and OU processes in order to compute conditional moments (mean and variance) and simulate random variates.

Conditional mean and variance are needed to use the unscented Kalman filter, simulation algorithms are necessary to apply the particle filter.

In the CIR case

$$d\lambda_t = \kappa(\eta - \lambda_t)dt + \vartheta\sqrt{\lambda_t}dW_t, \quad (\text{A.15})$$

we can compute the corresponding conditional mean and conditional variance, that is, given λ_t we have

$$\begin{aligned} E[\lambda_{t+\Delta t}|\lambda_t] &= \lambda_t e^{-\kappa\Delta t} + \eta(1 - e^{-\kappa\Delta t}), \\ \text{Var}[\lambda_{t+\Delta t}|\lambda_t] &= \lambda_t \frac{\vartheta^2}{\kappa}(e^{-\kappa\Delta t} - e^{-2\kappa\Delta t}) + \frac{\eta\vartheta^2}{2\kappa}(1 - e^{-\kappa\Delta t})^2. \end{aligned} \quad (\text{A.16})$$

Then, random variates can be drawn by considering a discretization scheme of the form

$$\lambda_{t+\Delta t} = \lambda_t + \kappa(\eta - \lambda_t)\Delta t + \vartheta\sqrt{\lambda_t}\sqrt{\Delta t}N, \quad (\text{A.17})$$

where $\Delta t = 1/250$ and N is a normal random number with zero mean and unit variance.

In the OU case

$$d\lambda_t = -\theta\lambda_t dt + dz_{\theta t}. \quad (\text{A.18})$$

we can write

$$\lambda_t = \exp(-\theta t)\lambda_0 + \exp(-\theta t) \int_0^{\theta t} e^s dz_s.$$

If mean and variance of the BDLP z_t are known, we can compute the conditional mean and variance of the process λ_t , as proved in Theorem 2.1 in Norberg (2004). In particular, in the Gamma-OU case we have

$$\begin{aligned} E[\lambda_{t+\Delta t}|\lambda_t] &= \lambda_t e^{-\theta\Delta t} + \frac{a}{b}(1 - e^{-\theta\Delta t}), \\ \text{Var}[\lambda_{t+\Delta t}|\lambda_t] &= \frac{a}{b^2}(1 - e^{-2\theta\Delta t}), \end{aligned} \quad (\text{A.19})$$

similarly, in the IG-OU case

$$\begin{aligned} E[\lambda_{t+\Delta t}|\lambda_t] &= \lambda_t e^{-\theta\Delta t} + \frac{a}{b}(1 - e^{-\theta\Delta t}), \\ \text{Var}[\lambda_{t+\Delta t}|\lambda_t] &= \frac{a}{b^3}(1 - e^{-2\theta\Delta t}), \end{aligned} \quad (\text{A.20})$$

and in the VG-OU case

$$\begin{aligned} E[\lambda_{t+\Delta t}|\lambda_t] &= \lambda_t e^{-\theta\Delta t} + C\left(\frac{1}{\lambda_+} - \frac{1}{\lambda_-}\right)(1 - e^{-\theta\Delta t}), \\ \text{Var}[\lambda_{t+\Delta t}|\lambda_t] &= C\left(\frac{1}{\lambda_+^2} + \frac{1}{\lambda_-^2}\right)(1 - e^{-2\theta\Delta t}). \end{aligned} \quad (\text{A.21})$$

Methods to draw random numbers from Gamma-OU and IG-OU processes are proposed by Barndorff-Nielsen and Shephard (2001) and by Zhang and Zhang (2008). Their algorithms can easily be implemented and are fast enough to be used in a particle filter estimation method. The Gamma-OU simulation algorithm can easily be extended to simulate VG-OU random variates.

In Figure 11 we show for each distributional assumption the simulated paths of three different intensity processes with initial value equal to 0.005. In order to visually assess the difference between different processes we consider in all cases a mean-reverting level equal to 0.02; consequently, in the one-sided OU cases the ratio a/b is constant and equal to 0.02, and in the two-sided OU case $C(1/\lambda_+ - 1/\lambda_-)$ is constant and equal to 0.02.

A.3 Stability analysis of the regularized optimization problem

The non-linear least square optimization problem defined in equation (17), that is

$$\hat{\Theta} = \min_{\Theta} (RMSE(\Theta))^2 \quad (\text{A.22})$$

has neither a closed-form solution nor a global minimum. A numerical optimization routine is needed to find a relative minimum also because the gradient vector and the Hessian matrix related to the problem are difficult to express in closed-form: even if they can be computed, they have a messy expression. In this paper we follow the practical approach described in Fang et al. (2010); indeed we define the regularized problem

$$\hat{\Theta} = \min_{\Theta} (RMSE(\Theta))^2 + \rho \|\omega \cdot (\Theta - \Theta_0)\|^2. \quad (\text{A.23})$$

where ρ is a constant term and ω is a vector defined to provide a comparable parameter sensitivity as the parameters may differ significantly in magnitude. The vector ω is given by $(1/\Theta_0^1, \dots, 1/\Theta_0^N)$, where N is the length of Θ and with “ \cdot ” we indicate the inner products of vectors. The choice is aimed at achieving a satisfactory calibration error and parameter stability over time.

In this section we study how, by increasing the value of the parameter ρ , the parameters, calibration errors and computational time vary. The selection of a proper ρ is itself an optimization problem which has to be solved to find a solution to the original least squares problem. As already observed in Section 6.1, ρ depends on the data at hand and on the level of error present in it. In Table 2 we report the results of the empirical study conducted over time and across all the 117 companies analysed. More precisely, we show the lag-5 autocorrelation computed by considering the parameter time series of each company. Then we compute median and mean values across all companies. As expected, the parameter stability increases by increasing ρ , even if some parameters are more volatile than others. Additionally, we report median and mean values of the RMSE, of the average relative percentage error (ARPE) and of the number of function evaluations into the optimization routine. The number of function evaluation is a proxy for the computational time. These values are computed both over time and across all the 117 companies analysed. By increasing the value of ρ , we obtain that the calibration error increases in the CIR and in the Gamma-OU case, it remains quite stable in the Sato Gamma case, and in the IG-OU and in the VG-OU cases it reaches the minimum value when $\rho = 10$. The computational time decreases in the CIR case and, conversely, increases in the Sato-Gamma case. The value $\rho = 100$ shows

a good balance between the calibration error and the parameter stability and for this reason we selected this value in the main text of the paper. In the empirical study we solve a large number of problems of the form (A.23): for each model and across the 117 companies we consider 655 daily observations for a total of more than 380,000 daily calibration exercises. For this reason, even if the selected value of ρ may be not the optimal value, it is sufficient for our purposes as it provides us with an acceptable calibration error and parameter stability. As shown in Table 2, when ρ is equal to 100, the median values for the ARPE are just over 2 per cent (less than 1.5 per cent if we do not consider the Sato based model) and the median lag-5 autocorrelations are all above 0.9. Finally, we note that Table 2 confirms that the VG-OU model outperforms its competitor models while having a comparable degree of parameter stability over time and of computational complexity, not only when ρ is equal to 100, but also for all other selected values of ρ .

A.4 The filtering algorithms

We model the CDS time series z_t using a state-space framework with the state λ_t assumed to be markovian.³⁶ This means that the CDS pricing model can be written in the following form

$$\begin{aligned}\lambda_t &= f(\lambda_{t-1}, \Theta, v_{t-1}) \\ z_t &= h(\lambda_t, \Theta, \varepsilon_t)\end{aligned}\tag{A.24}$$

where t is the day counter, λ_t is the state variable modelled as a Markov process with initial distribution $p(\lambda_0)$ and transition law $p(\lambda_t|\lambda_{t-1})$. The state variable follows the dynamics described by the transition function f . The variable z_t represents the set of given observations (in our case the CDS spreads observed in the market). It is assumed to be conditionally independent given the state λ_t and with distribution $p(z_t|\lambda_t)$. Then, v_{t-1} and ε_t are independent state noise and observation noise with mean 0 and variance matrix Q_{t-1} and R_t , respectively (these matrices may be time dependent). The noise ε_t is assumed to be normal with diagonal covariance matrix R_t . Furthermore, the measurement error covariance is adjusted for the daily observed bid-ask spreads, that is, we define a matrix R_t given by the product between a diagonal covariance matrix and a diagonal matrix with diagonal entries equal to the observed bid-ask spreads. The function h is the so-called measurement function, which in our case is given by the CDS pricing formula, and Θ is a set of static parameters.

The filtering problem deals with the estimation of the unobserved stochastic process λ_t based on the past and current measurement of a related process z_t , that is finding the posterior distribution $p(\lambda_t|z_{1:t})$. The basic idea is to define the probability density function corresponding to the state λ_t given all the measurements made up to the time t .

As already observed in Section 6.2, if the measurement function h is linear and the state is Gaussian, the Kalman filter can be used for state and parameter

³⁶ This section is drawn by considering the works of van der Merwe et al. (2001), Bhar (2010) and Malik and Pitt (2011).

estimation. In all the cases we are interested in, we have a non-linear measurement function or a non-Gaussian state and for this reason we apply the unscented Kalman filter in the CIR case and the particle filter in the VG-OU case. These algorithms are briefly described in the following Sections A.4.1 and A.4.2.

A.4.1 Unscented Kalman filter

The algorithm here described is drawn by van der Merwe et al. (2001). Under this approach the state is propagated through a set of deterministic points with given weights generated by the so-called *unscented transformation*. The posterior density $p(\lambda_t|z_{1:t})$ is approximated by assuming a known measurement density $h(z_t|\lambda_t)$ and the ability to compute the conditional mean and covariance of the Markov state λ_t . As pointed out by Christoffersen et al. (2009), when the state vector is not Gaussian, an approximate transition equation can be obtained by exploiting the existence of the two first conditional moments in closed-form and replacing the original state vector with a new Gaussian state vector with identical two first conditional moments (see also Duan and Simonato (1999)). The state-space model can be written as

$$\begin{aligned}\lambda_t &= a + b\lambda_{t-1} + v_{t-1} \\ z_t &= h(\lambda_t, \Theta) + \varepsilon_t\end{aligned}$$

where v_{t-1} and ε_t are independent normal noises with mean 0 and variance matrix Q_{t-1} and R_t , respectively.³⁷ We indicate with $\hat{\lambda}_t$ and P_{λ_t} the mean and covariance of λ_t . The basic idea behind the scaled unscented transformation is to generate a set of points with the first two sample moments equal to $\hat{\lambda}_t$ and P_{λ_t} . We assume that $\hat{\lambda}_0$ and P_{λ_0} are given and define the augmented state

$$\lambda_{t-1}^a = [\lambda'_{t-1}, v'_{t-1}, \varepsilon'_t]'$$

with mean $\hat{\lambda}_{t-1}^a = E[\lambda_{t-1}^a]$ and covariance

$$P_{t-1}^a = \begin{bmatrix} P_{\lambda_{t-1}} & 0 & 0 \\ 0 & Q_{t-1} & 0 \\ 0 & 0 & R_t \end{bmatrix}.$$

The unscented transformation provides a set of $2n_a$ sigma points, where $n_a = n_\lambda + n_v + n_\varepsilon$, with n_λ , n_v , n_ε being dimensions of the state, the state noise and the observation noise, respectively. Chosen the parameters $k \geq 0$, $0 \leq \alpha \leq 1$, $\beta \geq 0$, and set $l = \alpha^2(n_a + k) - n_a$, we define the sigma points Λ_{t-1}^a and their related

³⁷ The parameters a , b and Q_t depend on the model parameter set Θ .

weights W^m and W^c as follows

$$\begin{aligned}
(\Lambda_{t-1}^a)_0 &= \hat{\lambda}_{t-1}^a \\
(\Lambda_{t-1}^a)_i &= \hat{\lambda}_{t-1}^a + \left(\sqrt{(n_a + l)P_{t-1}^a} \right)_i \quad \text{for } i = 1, \dots, n_a \\
(\Lambda_{t-1}^a)_i &= \hat{\lambda}_{t-1}^a - \left(\sqrt{(n_a + l)P_{t-1}^a} \right)_i \quad \text{for } i = n_a + 1, \dots, 2n_a \\
\Lambda_{t-1}^a &= \left[(\Lambda_{t-1}^\lambda)', (\Lambda_{t-1}^v)', (\Lambda_{t-1}^\varepsilon)' \right]' \\
W_0^m &= \frac{l}{n_a + l} \\
W_0^c &= \frac{l}{n_a + l} + (1 - \alpha^2 + \beta) \\
W_i^m &= W_i^c = \frac{1}{2(n_a + l)} \quad \text{for } i = 1, \dots, 2n_a
\end{aligned}$$

where $\left(\sqrt{(n_x + l)P_{t-1}^a} \right)_i$ is the i -th row of the matrix square root of $(n_x + l)P_{t-1}^a$. After computing the sigma points we proceed as follows:

1. we perform the prediction

$$\begin{aligned}
\Lambda_{t|t-1}^\lambda &= a + b\Lambda_{t-1}^\lambda + \Lambda_{t-1}^v \\
\hat{\lambda}_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^m (\Lambda_{t|t-1}^\lambda)_i \\
P_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^c \left[(\Lambda_{t|t-1}^\lambda)_i - \hat{\lambda}_{t|t-1} \right] \left[(\Lambda_{t|t-1}^\lambda)_i - \hat{\lambda}_{t|t-1} \right]' \\
\mathcal{Z}_{t|t-1}^\lambda &= h(\Lambda_{t-1}^\lambda, \Theta) + \Lambda_{t-1}^\varepsilon \\
\hat{z}_{t|t-1} &= \sum_{i=0}^{2n_a} W_i^m (\mathcal{Z}_{t|t-1}^\lambda)_i
\end{aligned}$$

2. we perform the measurement update

$$\begin{aligned}
P_{t|t-1}^{zz} &= \sum_{i=0}^{2n_a} W_i^c \left[(\mathcal{Z}_{t|t-1}^\lambda)_i - \hat{z}_{t|t-1} \right] \left[(\mathcal{Z}_{t|t-1}^\lambda)_i - \hat{z}_{t|t-1} \right]' \\
P_{t|t-1}^{\lambda z} &= \sum_{i=0}^{2n_a} W_i^c \left[(\Lambda_{t|t-1}^\lambda)_i - \hat{\lambda}_{t|t-1} \right] \left[(\mathcal{Z}_{t|t-1}^\lambda)_i - \hat{z}_{t|t-1} \right]' \\
K_t &= P_{t|t-1}^{\lambda z} (P_{t|t-1}^{zz})^{-1} \\
\hat{\lambda}_t &= \hat{\lambda}_{t|t-1} + K_t (z_t - \hat{z}_{t|t-1}) \\
P_{\lambda_t} &= P_{\lambda_{t-1}} - K_t P_{t|t-1}^{zz} K_t'
\end{aligned}$$

3. we compute (up to a constant term) the log-likelihood $L_t = \log p(z_t | \lambda_t)$, indeed

$$L_t = -\log |P_{t|t-1}^{zz}| - (z_t - \hat{z}_{t|t-1})' (P_{t|t-1}^{zz})^{-1} (z_t - \hat{z}_{t|t-1}),$$

where $|\cdot|$ indicates the determinant of a matrix.

Then, to estimate the parameters Θ , we build the joint log-likelihood over the entire observation period, indeed

$$L(\Theta, z_{1:T}) = \sum_{t=1}^T L_t,$$

and, finally, we insert this function into an optimization procedure. By considering Section A.2 and O'Sullivan (2008), we consider the following input for the algorithm. In the CIR case we have

$$\begin{aligned} \hat{\lambda}_0 &= \eta & P_{\lambda_0} &= \eta \frac{\vartheta^2}{2\kappa} \\ a &= \eta(1 - e^{-\kappa\delta t}) & b &= e^{-\kappa\delta t} \\ Q_t &= \hat{\lambda}_t \frac{\vartheta^2}{\kappa} (e^{-\kappa\Delta t} - e^{-2\kappa\Delta t}) + \frac{\eta\vartheta^2}{2\kappa} (1 - e^{-\kappa\Delta t})^2 \end{aligned}$$

and similarly in the VG-OU case we have

$$\begin{aligned} \hat{\lambda}_0 &= C \left(\frac{1}{\lambda_+} - \frac{1}{\lambda_-} \right) & P_{\lambda_0} &= C \left(\frac{1}{\lambda_+^2} + \frac{1}{\lambda_-^2} \right) \\ a &= C \left(\frac{1}{\lambda_+} - \frac{1}{\lambda_-} \right) (1 - e^{-\theta\delta t}) & b &= e^{-\theta\delta t} \\ Q_t &= C \left(\frac{1}{\lambda_+^2} + \frac{1}{\lambda_-^2} \right) (1 - e^{-2\theta\delta t}). \end{aligned}$$

A.4.2 Particle filter

Particle filter is a sequential Monte Carlo method for recursively approximating the posterior density $p(\lambda_t | z_{1:t})$ by assuming a known measurement density $h(z_t | \lambda_t)$ and the ability to simulate from the Markov transition density $f(\lambda_{t+1} | \lambda_t)$. The algorithm estimates the posterior density by considering a set of random samples with associated weights $\{\lambda_t^i, w_t^i\}_{i=1}^N$ where N is the number of samples at each given point in time t . The algorithm includes three main steps: (a) sampling, (b) weights computation, and (c) resampling. In our empirical test we proceed as follows:

1. we sample λ_t^i from the distribution $p(\lambda_t | \lambda_{t-1})$;
2. we compute z_t^i through the measurement function h ;
3. by considering the normal assumption of the measurement error, we evaluate the likelihood $p(z_t | \lambda_t^i)$ and recursively compute the weights

$$w_t^i = w_{t-1}^i p(z_t | \lambda_t^i);$$

4. the current state can be estimated by $\hat{\lambda}_t = \frac{1}{N} \sum_{i=1}^N w_t^i \lambda_t^i$;
5. by considering the weights w_t^i we compute (up to a constant term) the log-likelihood $L_t^i = \log p(z_t | \tilde{\lambda}_t^i)$, indeed

$$L_t^i = -\frac{1}{2} \log |R| - \frac{1}{2} (z_t^{\text{market}} - z_t^i)' R^{-1} (z_t^{\text{market}} - z_t^i),$$

where $|\cdot|$ indicates the determinant of a matrix, and we evaluate the likelihood estimate $\widehat{L}_t = \frac{1}{N} \sum_{i=1}^N w_t^i L_t^i$ and $\widehat{z}_t = \frac{1}{N} \sum_{i=1}^N w_t^i z_t^i$;

6. we resample by taking into consideration the stratified resampling algorithm (see by Douc and Cappé (2005)) and we obtain a new set of samples $\tilde{\lambda}_t^i$ approximately distributed according to $p(\lambda_t|z_{1:t})$.

To estimate the parameters Θ , we build the joint log-likelihood over the entire observation period, indeed

$$L(\Theta, z_{1:T}) = \sum_{t=1}^T \widehat{L}_t,$$

and, finally, we insert this function into an optimization procedure.

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Figure 1: Mean and median CDS and bid-ask spreads for the 117 companies considered in the empirical study.

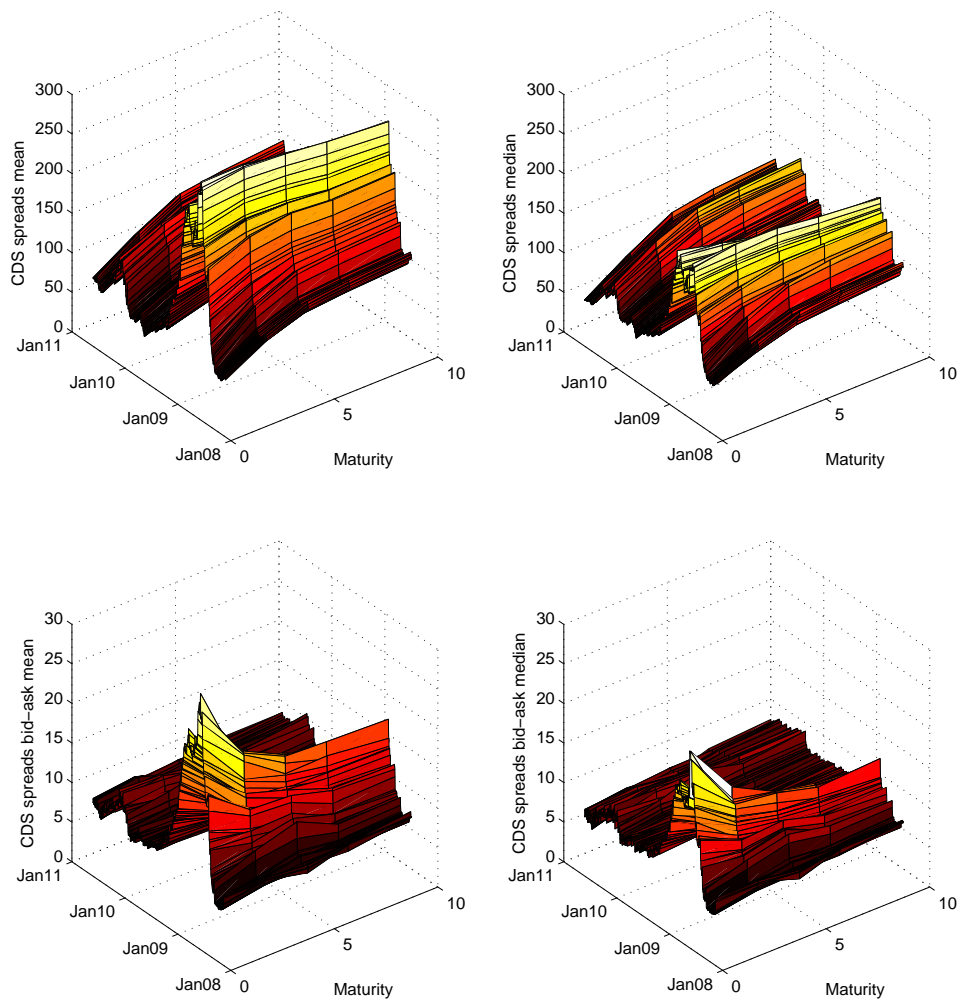


Figure 2: Median, mean, 95th and 5th percentile of the RMSE for the 117 companies considered in the analysis. We report the pricing error obtained with $\rho = 100$.

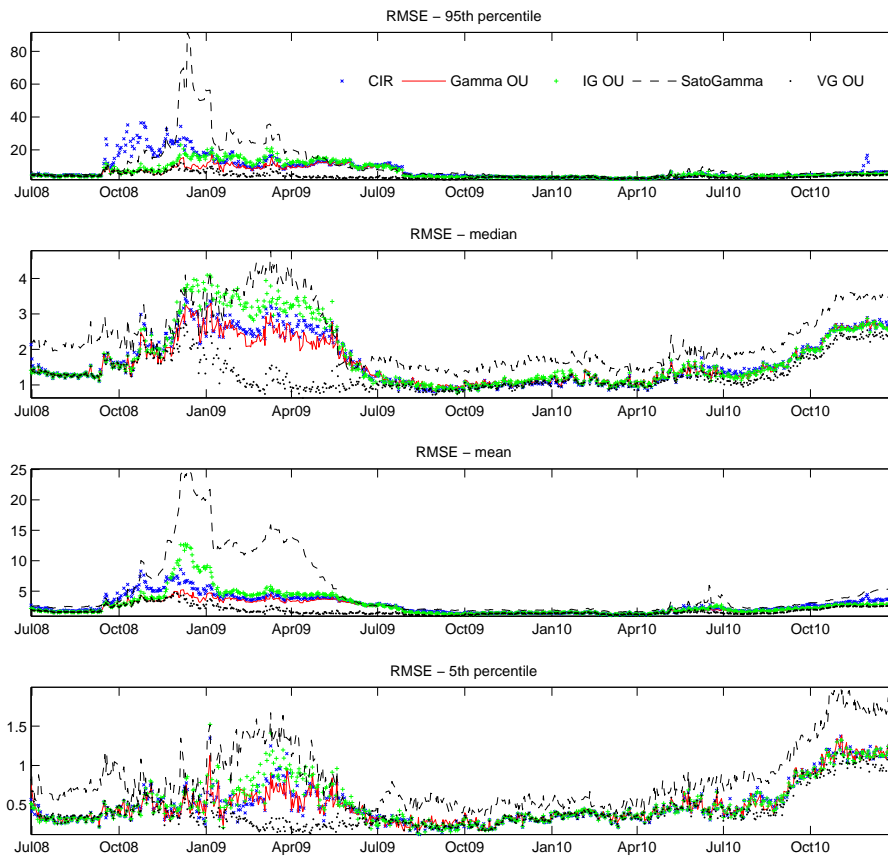


Figure 3: Behavior of the median, mean, 90th and 10th percentile over 117 companies included in the iTraxx Index of the market distress measure (in bp) from 30 June 2008 to 31 December 2010.

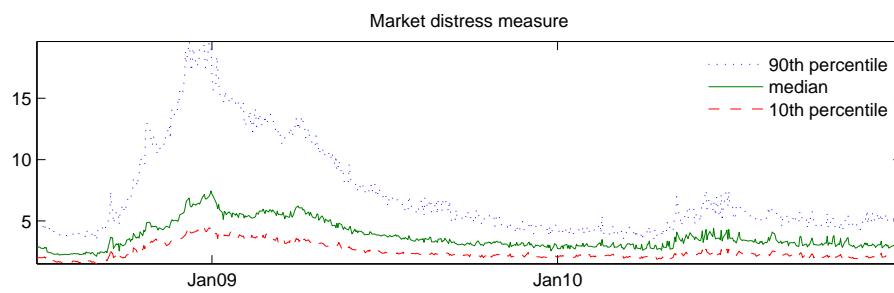


Figure 4: Percentage of exceedances (across companies) for the CIR, the Gamma-OU, the IG-OU, the Sato-Gamma, and the VG-OU models with regularization term $\rho = 0$ (left side) and with regularization term $\rho = 100$ (right side). On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually.

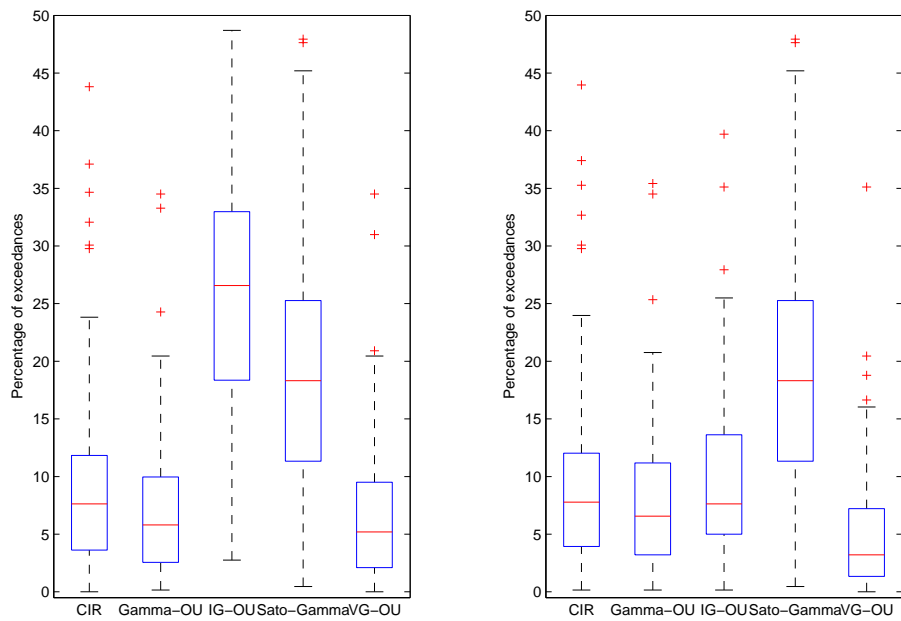


Figure 5: Parameters median, 90th and 10th percentile for 117 companies included in the iTraxx Index estimated by considering the CIR, the Gamma-OU, the IG-OU, the Sato Gamma, and the VG-OU model from 30 June 2008 to 31 December 2010 without regularization, that is $\rho = 0$. The parameter λ_- of the VG-OU model is not reported.

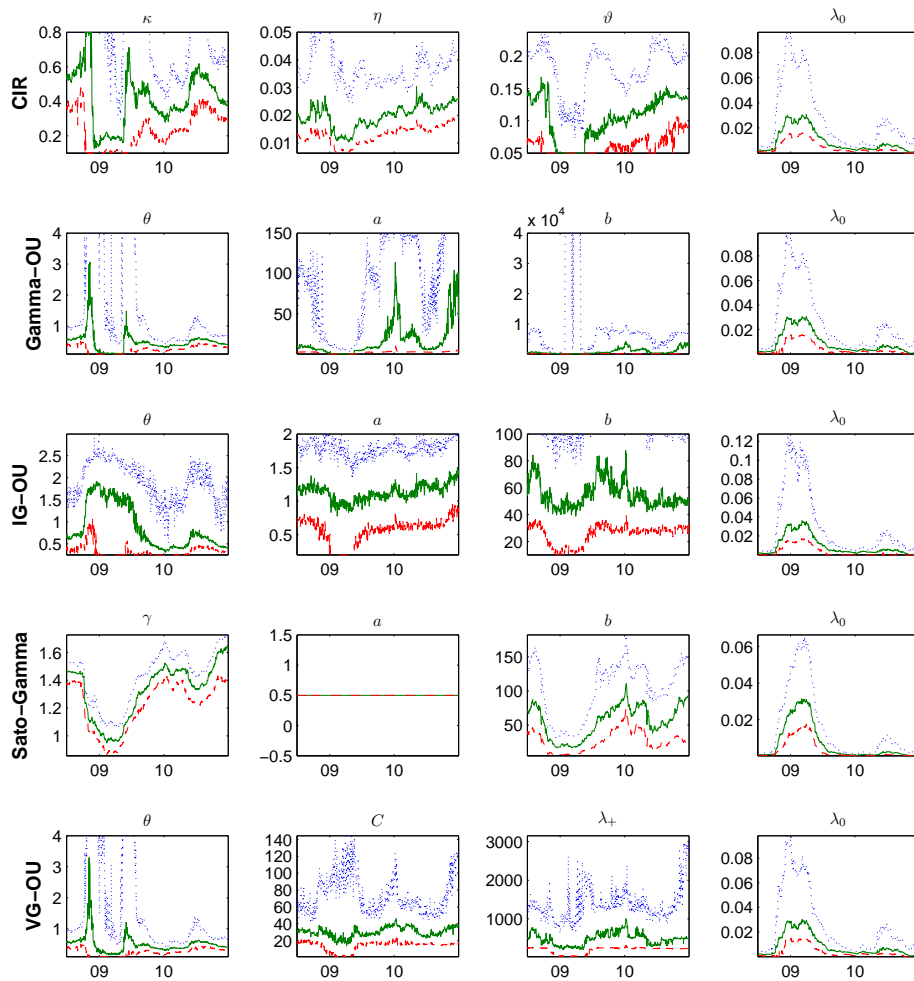


Figure 6: Parameters median, 90th and 10th percentile for 117 companies included in the iTraxx Index estimated by considering the CIR, the Gamma-OU, the IG-OU, the Sato Gamma, and the VG-OU model from 30 June 2008 to 31 December 2010 with regularization term, that is $\rho = 100$. The parameter λ_- of the VG-OU model is not reported.

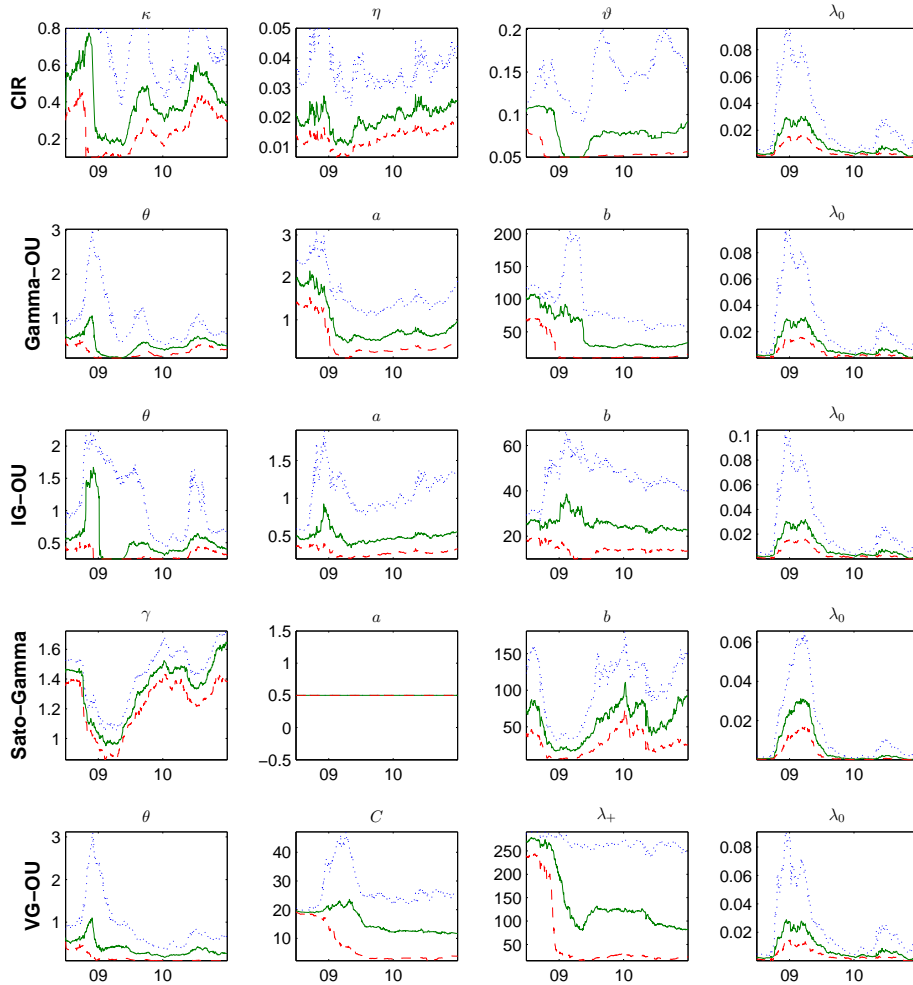


Figure 7: Autocorrelation as a function of the lag for the CIR, the Gamma-OU, the IG-OU, the Sato-Gamma, and the VG-OU estimated parameters. Average values across companies. On the left, we report the autocorrelations without regularization term into the optimization problem, that is $\rho = 0$, on the right, we report the autocorrelations with regularization term $\rho = 100$ into the optimization problem.

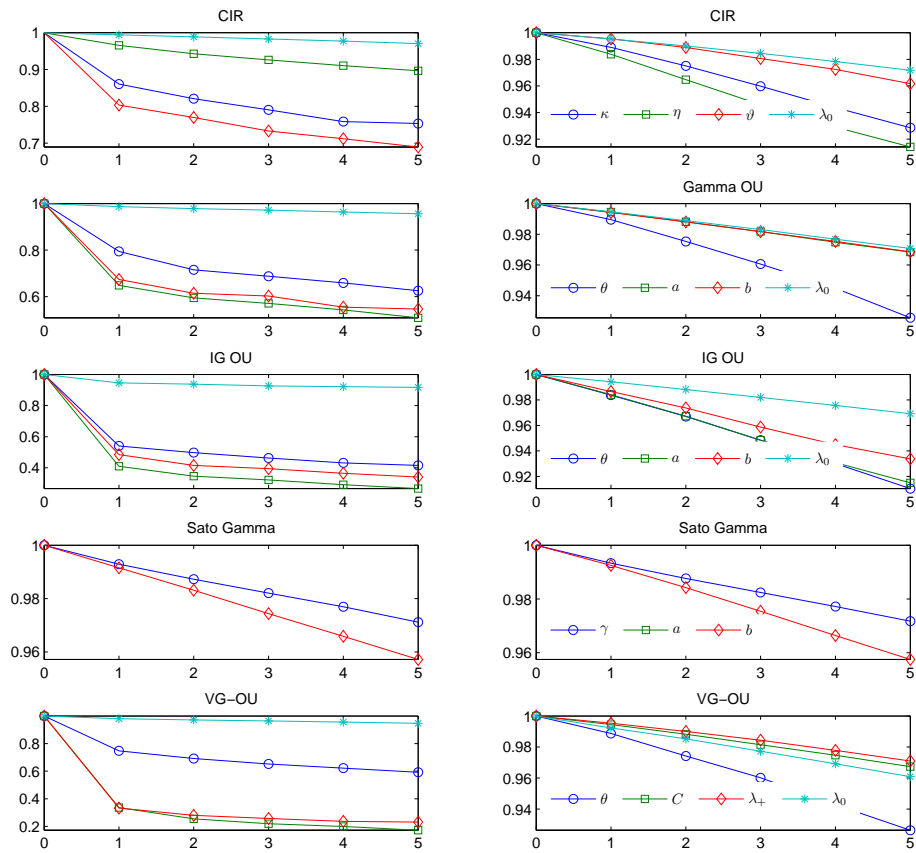


Figure 8: CIR model: (1) Simulated CDS spreads with normal noise, (2) CIR default intensity process and UKF estimate, (3) UKF calibration error. Risk-free rates are extracted from market data between 30 June 2008 to 31 December 2010. The unscented Kalman filter is considered to extract the unobservable default intensity process.

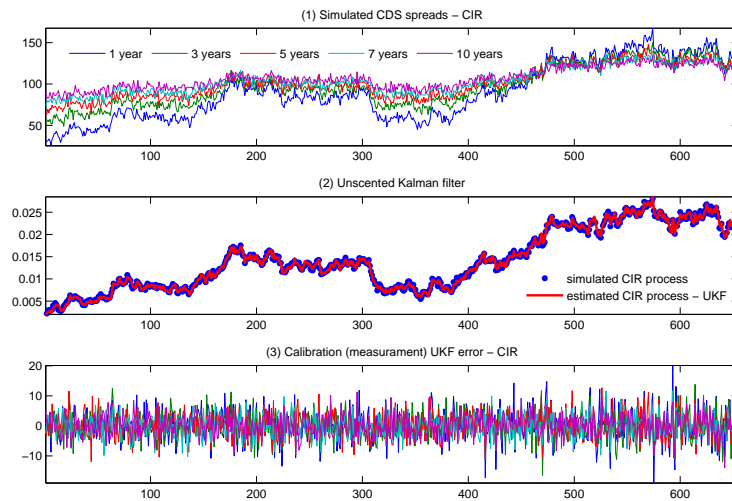


Figure 9: VG-OU model: (1) Simulated CDS spreads with normal noise, (2) VG-OU default intensity process and UKF estimate, (3) UKF calibration error, (4) VG-OU default intensity process and PF estimate, (5) PF calibration error. Risk-free rates are extracted from market data between 30 June 2008 to 31 December 2010. The unscented Kalman filter and the particle filter are considered to extract the unobservable default intensity process.

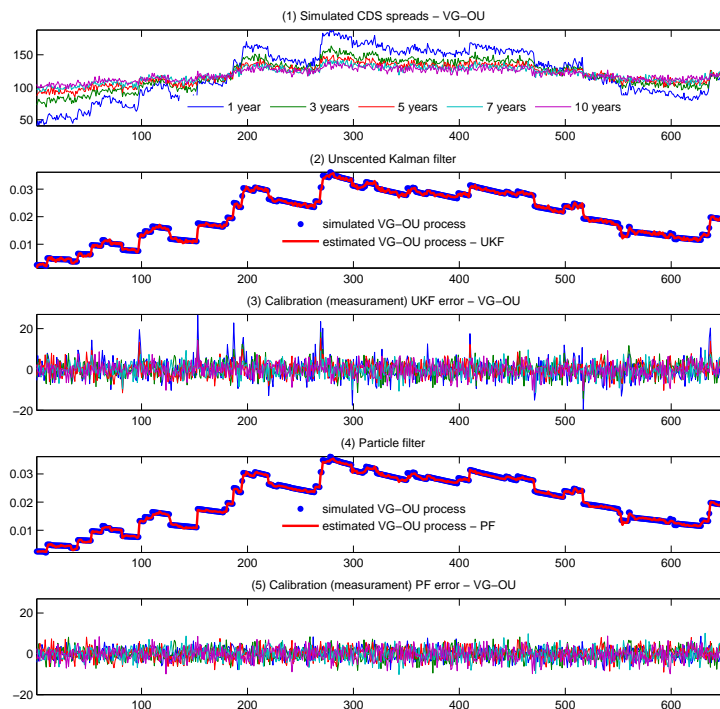


Figure 10: Maximum likelihood estimates based on the UKF and the PF: RMSE, ARPE and parameters boxplot across all 117 companies analysed are reported. The estimates of the CIR and the VG-OU are compared by considering both the UKF and the PF approach. On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually.

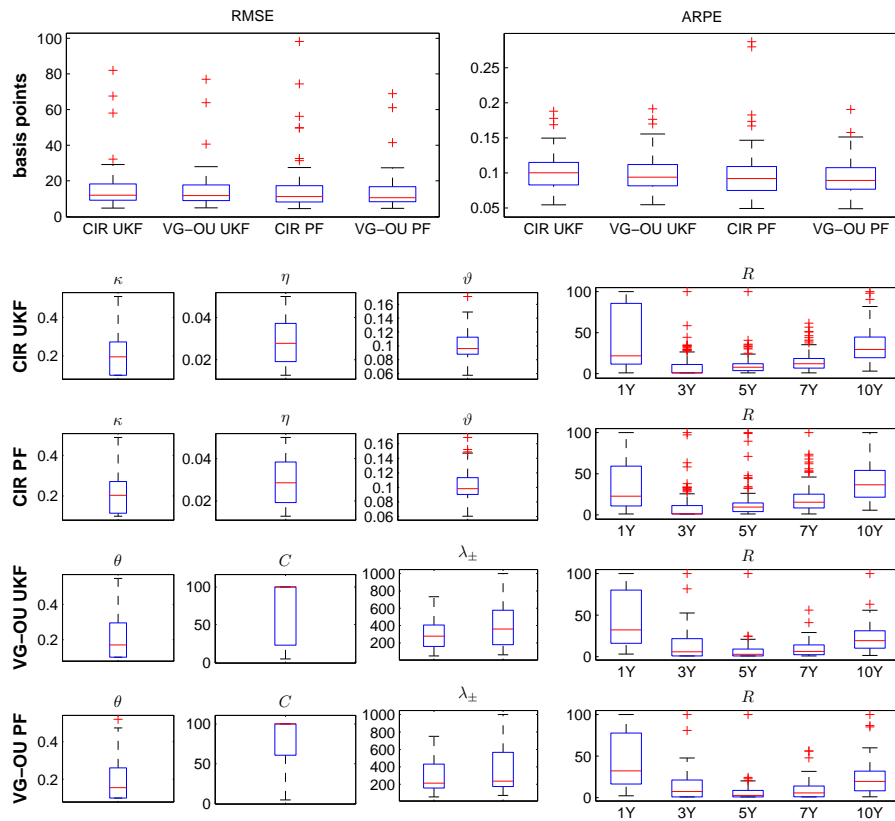
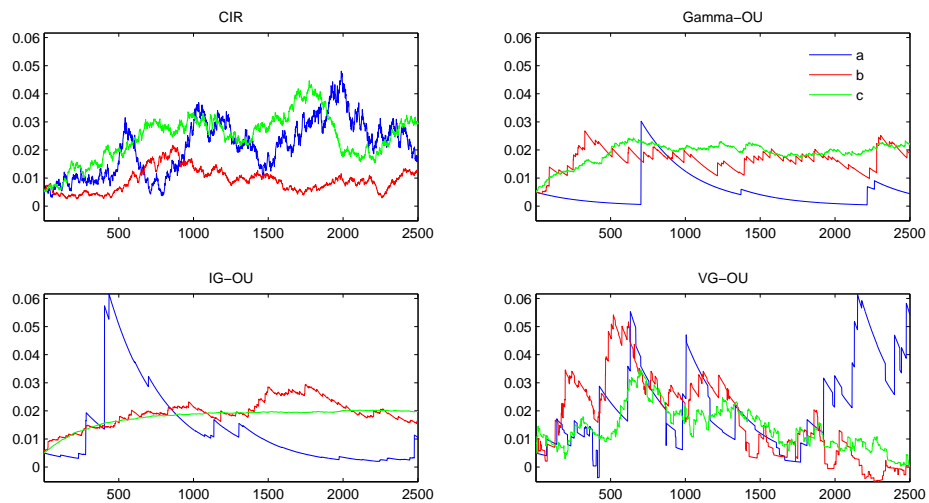


Figure 11: Simulated CIR, Gamma-OU and IG-OU processes with initial value λ_0 equal to 0.005. For the CIR process the parameters $(\kappa, \eta, \vartheta)$ are (a) (0.8, 0.02, 0.1), (b) (0.4, 0.02, 0.05), (c) (0.2, 0.02, 0.05). For both the one sided OU processes we set the parameter $\theta = 0.75$, then for the Gamma-OU (a,b) are (a) (1, 50), (b) (10, 500), (c) (100, 5000) and for the IG-OU (a,b) are (a) (0.1, 5), (b) (1, 50), (c) (10, 500). In the VG-OU case we set the parameter $\theta = 0.75$ and $(C, \lambda_+, \lambda_-)$ (a) (4, 100, 200), (b) (16, 400, 800), (c) (32, 800, 1600).



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	1Y	3Y	5Y	7Y	10Y
minimum	5.49	14.57	22.43	25.54	29.15
5th percentile	20.75	35.14	45.93	50.22	53.55
mean	93.41	110.36	120.69	122.28	124.70
median	56.44	78.89	92.93	96.05	100.13
95th percentile	283.51	290.06	283.40	274.41	267.78
maximum	3257.54	3261.32	3127.44	2912.14	2800.06
standard deviation	132.94	121.92	112.30	105.44	100.70
mean absolute deviation	67.34	63.67	59.74	56.76	54.88
median absolute deviation	27.48	30.47	31.46	30.49	30.26
lag-5 autocorrelation	0.96	0.95	0.93	0.92	0.91

Table 1: Summary statistics of the CDS spreads between 30 June 2008 and 31 December 2010 for the 117 companies considered in the empirical study.

		ρ							
		0	10	50	100	200	500	1000	
CIR	κ	0.7535 <i>0.7354</i>	0.8427 <i>0.8333</i>	0.9063 <i>0.8973</i>	0.9286 <i>0.9217</i>	0.9471 <i>0.9408</i>	0.9701 <i>0.9587</i>	0.9755 <i>0.9676</i>	
	η	0.8969 <i>0.8789</i>	0.9049 <i>0.8902</i>	0.9112 <i>0.8995</i>	0.9141 <i>0.9039</i>	0.9164 <i>0.9068</i>	0.9183 <i>0.9079</i>	0.9187 <i>0.9076</i>	
	ϑ	0.6897 <i>0.6806</i>	0.9219 <i>0.9150</i>	0.9491 <i>0.9392</i>	0.9618 <i>0.9548</i>	0.9728 <i>0.9642</i>	0.9789 <i>0.9690</i>	0.9809 <i>0.9689</i>	
	λ_0	0.9707 <i>0.9631</i>	0.9712 <i>0.9636</i>	0.9717 <i>0.9642</i>	0.9718 <i>0.9646</i>	0.9720 <i>0.9649</i>	0.9725 <i>0.9652</i>	0.9721 <i>0.9653</i>	
	RMSE	1.4025 <i>2.6759</i>	1.4067 <i>2.6950</i>	1.4379 <i>2.7290</i>	1.4985 <i>2.7544</i>	1.5211 <i>2.7902</i>	1.5966 <i>2.8672</i>	1.7498 <i>2.9609</i>	
	ARPE	0.0148 <i>0.0189</i>	0.0149 <i>0.0190</i>	0.0150 <i>0.0194</i>	0.0153 <i>0.0197</i>	0.0154 <i>0.0200</i>	0.0161 <i>0.0209</i>	0.0173 <i>0.0220</i>	
	function evaluation	173 <i>179.3526</i>	111 <i>127.2661</i>	100 <i>110.7324</i>	99 <i>106.5334</i>	99 <i>103.9400</i>	99 <i>101.7892</i>	100 <i>101.1559</i>	
	Gamma-OU	θ	0.6252 <i>0.5969</i>	0.8018 <i>0.7771</i>	0.8866 <i>0.8683</i>	0.9257 <i>0.8999</i>	0.9516 <i>0.9340</i>	0.9679 <i>0.9607</i>	0.9724 <i>0.9696</i>
		a	0.5094 <i>0.5034</i>	0.9549 <i>0.9470</i>	0.9678 <i>0.9526</i>	0.9684 <i>0.9526</i>	0.9667 <i>0.9530</i>	0.9631 <i>0.9500</i>	0.9563 <i>0.9441</i>
b		0.5470 <i>0.5089</i>	0.9561 <i>0.9354</i>	0.9659 <i>0.9543</i>	0.9686 <i>0.9591</i>	0.9664 <i>0.9594</i>	0.9661 <i>0.9615</i>	0.9635 <i>0.9592</i>	
λ_0		0.9560 <i>0.9488</i>	0.9671 <i>0.9581</i>	0.9702 <i>0.9610</i>	0.9708 <i>0.9623</i>	0.9711 <i>0.9635</i>	0.9718 <i>0.9646</i>	0.9719 <i>0.9651</i>	
RMSE		1.2899 <i>2.1841</i>	1.3287 <i>2.2086</i>	1.3857 <i>2.2782</i>	1.4165 <i>2.3190</i>	1.4797 <i>2.3748</i>	1.6557 <i>2.4779</i>	1.7717 <i>2.5811</i>	
ARPE		0.0141 <i>0.0171</i>	0.0143 <i>0.0175</i>	0.0147 <i>0.0182</i>	0.0151 <i>0.0185</i>	0.0157 <i>0.0191</i>	0.0164 <i>0.0202</i>	0.0175 <i>0.0212</i>	
function evaluation		161 <i>183.0975</i>	97 <i>100.7687</i>	93 <i>94.8180</i>	90 <i>92.1618</i>	88 <i>89.7360</i>	84 <i>86.7029</i>	83 <i>84.8406</i>	
IG-OU		θ	0.4160 <i>0.4038</i>	0.8759 <i>0.8645</i>	0.8887 <i>0.8848</i>	0.9105 <i>0.9037</i>	0.9302 <i>0.9246</i>	0.9586 <i>0.9494</i>	0.9720 <i>0.9647</i>
		a	0.2672 <i>0.2796</i>	0.9185 <i>0.9027</i>	0.9145 <i>0.8979</i>	0.9150 <i>0.9035</i>	0.9276 <i>0.9142</i>	0.9389 <i>0.9193</i>	0.9401 <i>0.9288</i>
	b	0.3409 <i>0.3494</i>	0.9267 <i>0.9054</i>	0.9335 <i>0.9143</i>	0.9338 <i>0.9191</i>	0.9351 <i>0.9264</i>	0.9433 <i>0.9300</i>	0.9480 <i>0.9429</i>	
	λ_0	0.9164 <i>0.8924</i>	0.9655 <i>0.9572</i>	0.9682 <i>0.9602</i>	0.9692 <i>0.9616</i>	0.9704 <i>0.9621</i>	0.9715 <i>0.9642</i>	0.9726 <i>0.9653</i>	
	RMSE	1.9670 <i>5.8222</i>	1.4611 <i>2.8714</i>	1.4880 <i>2.6187</i>	1.4787 <i>2.8030</i>	1.5127 <i>2.6454</i>	1.5534 <i>2.6749</i>	1.6158 <i>2.7028</i>	
	ARPE	0.0196 <i>0.0391</i>	0.0156 <i>0.0199</i>	0.0154 <i>0.0196</i>	0.0157 <i>0.0197</i>	0.0158 <i>0.0200</i>	0.0163 <i>0.0205</i>	0.0172 <i>0.0210</i>	
	function evaluation	137 <i>159.1480</i>	96 <i>109.2036</i>	87 <i>103.1158</i>	87 <i>101.0573</i>	87 <i>100.2680</i>	88 <i>99.2605</i>	88 <i>97.1306</i>	
	Sato-Gamma	γ	0.9712 <i>0.9667</i>	0.9712 <i>0.9668</i>	0.9713 <i>0.9670</i>	0.9718 <i>0.9672</i>	0.9719 <i>0.9674</i>	0.9725 <i>0.9679</i>	0.9728 <i>0.9682</i>
		b	0.9572 <i>0.9520</i>	0.9573 <i>0.9522</i>	0.9574 <i>0.9526</i>	0.9575 <i>0.9531</i>	0.9586 <i>0.9538</i>	0.9608 <i>0.9556</i>	0.9631 <i>0.9579</i>
RMSE		2.1090 <i>5.0287</i>	2.1090 <i>5.0287</i>	2.1091 <i>5.0291</i>	2.1091 <i>5.0299</i>	2.1091 <i>5.0323</i>	2.1126 <i>5.0423</i>	2.1352 <i>5.0634</i>	
ARPE		0.0219 <i>0.0270</i>	0.0219 <i>0.0271</i>	0.0220 <i>0.0271</i>	0.0219 <i>0.0271</i>	0.0220 <i>0.0272</i>	0.0222 <i>0.0274</i>	0.0226 <i>0.0278</i>	
function evaluation		56 <i>55.4030</i>	36 <i>37.0428</i>	36 <i>37.0146</i>	36 <i>36.9764</i>	36 <i>36.8857</i>	36 <i>36.6916</i>	36 <i>36.4880</i>	
VG-OU		θ	0.5920 <i>0.5780</i>	0.8311 <i>0.8032</i>	0.8862 <i>0.8744</i>	0.9264 <i>0.9033</i>	0.9423 <i>0.9273</i>	0.9595 <i>0.9517</i>	0.9646 <i>0.9645</i>
	C	0.1724 <i>0.2059</i>	0.9465 <i>0.9330</i>	0.9676 <i>0.9562</i>	0.9674 <i>0.9550</i>	0.9721 <i>0.9645</i>	0.9735 <i>0.9604</i>	0.9778 <i>0.9600</i>	
	λ_-	0.2801 <i>0.2950</i>	0.9607 <i>0.9580</i>	0.9669 <i>0.9544</i>	0.9692 <i>0.9510</i>	0.9734 <i>0.9526</i>	0.9757 <i>0.9538</i>	0.9783 <i>0.9582</i>	
	λ_+	0.2308 <i>0.2581</i>	0.9556 <i>0.9501</i>	0.9658 <i>0.9573</i>	0.9710 <i>0.9628</i>	0.9753 <i>0.9698</i>	0.9799 <i>0.9731</i>	0.9822 <i>0.9727</i>	
	λ_0	0.9468 <i>0.9399</i>	0.9389 <i>0.9188</i>	0.9544 <i>0.9397</i>	0.9609 <i>0.9484</i>	0.9664 <i>0.9547</i>	0.9698 <i>0.9616</i>	0.9707 <i>0.9633</i>	
	RMSE	1.2255 <i>2.0193</i>	1.0152 <i>1.5918</i>	1.0656 <i>1.7065</i>	1.1284 <i>1.7950</i>	1.1969 <i>1.8805</i>	1.3060 <i>2.0379</i>	1.4217 <i>2.1731</i>	
	ARPE	0.0134 <i>0.0163</i>	0.0103 <i>0.0140</i>	0.0114 <i>0.0150</i>	0.0120 <i>0.0157</i>	0.0126 <i>0.0163</i>	0.0141 <i>0.0176</i>	0.0149 <i>0.0187</i>	
	function evaluation	188 <i>258.2815</i>	145 <i>162.6107</i>	129 <i>140.6695</i>	127 <i>133.6337</i>	123 <i>127.1655</i>	116 <i>118.6974</i>	112 <i>113.1985</i>	

Table 2: Lag-5 autocorrelation of the parameters, calibration error and computing time. For each model and for each parameter median and mean values (in italics) of the lag-5 autocorrelation are computed over the 117 companies analysed. Then, median and mean (in italics) values of the root mean square error (RMSE), of the the average relative percentage error (ARPE) and of the number of function evaluations into the optimization routine are computed both over time and over the 117 companies analyzed.

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