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REAL TERM STRUCTURE
AND INFLATION COMPENSATION IN THE EURO AREA

by Marcello Pericoli*

Abstract

Estimates of the real term structure for the euro area implied by French index-linked bonds are obtained by means of a smoothing spline methodology. The real term structure allows computation of the constant-maturity inflation compensation, which is compared with the surveyed inflation expectations in order to obtain a rough measure of the inflation risk premium. The comparison between the inflation compensation and the inflation swap shows that the two variables are closely interlinked but differently affected by illiquidity during periods of stress. The methodology used in this paper is quite effective at capturing the general shape of the real term structure while smoothing through idiosyncratic variations in the yields of index-linked bonds. Real interest rates tend to be quite stable at longer horizons and the average 10-year real rate from 2002 to 2009 is close to 2 per cent. Furthermore, evidence is found that inflation compensation was held down in the period 2008-09 by an increase in the liquidity premium of index-linked bonds.

JEL Classification: C02, G10, G12.
Keywords: index-linked bond, real term structure, inflation compensation, inflation risk premium, smoothing spline.

Contents

* Bank of Italy, Economics, Research and International Relations.

1 Introduction[1]

In the last decade government-issued inflation-indexed bonds have become available in a number of euro-area countries and have provided a fundamentally new instrument popular sought after by institutional investors and households, especially for retirement saving. A bond linked to an inflation index allows the computation of a real yield to maturity, which is not directly comparable with the corresponding nominal yield to maturity since they differ as to maturities, coupon rates and cash-flow structures. Thus, it is worthwhile estimating the real term structure implied by the index-linked bonds, first, to obtain an estimate of the zero-coupon real interest rate across the maturity spectrum and, second, to compare it with the nominal term structure and derive the inflation compensation requested by market participants to hold index-linked bonds, a proxy of their expectations of inflation.

The paper presents an estimate of the real term structure for the euro area derived from the index-linked (IL) bonds issued by the French Treasury, *Obligations Assimilables au Trésor* (OAT).[2] The French Treasury has been issuing OATi bonds indexed to the domestic Consumer Price Index (CPI) since July 1998 and OAT€i bonds indexed to the euro-area Harmonized Index of Consumer Prices excluding tobacco (HICP excluding tobacco, henceforth HICP) since July 2001. The progressive introduction of IL bonds denominated in euros and with an indexation to the euro-area HICP has made it possible to extract the inflation compensation, also known as breakeven inflation rate (BEIR), requested by investors to hold nominal bonds, as the difference between the yield on a nominal bond and the corresponding yield on a real bond. This compensation consists, for the most part, of expected inflation over the corresponding period, but there is also an inflation risk premium component linked to the inflation uncertainty. Since the expected inflation rate is a key variable for investment decisions and for determining the stance of monetary policy, the timeliness and the variety of horizons, characteristics of the expectations based on quoted bonds, are extremely desirable features for investors and policy-makers; by contrast, surveyed data of expected inflation rates are released quarterly or semi-annually and for very few maturities.

The enrichment of the market through successive issues of IL bonds across the maturity spectrum has made it possible i) to estimate the term structure of real interest rates denominated in euros, and ii) to derive a constant-maturity BEIR for the euro area and its term structure.

The first part of the paper presents the term structure of the real interest rates for the euro area implied in the IL bonds indexed to the HICP. The real term structure is estimated with a smoothing spline with a penalty factor, a methodology initially proposed

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[2] See the documentation available at the French Treasury website (http://www.aft.gouv.fr/article_1715.html) for further information.

by Fisher et al. (1995) and enriched by Anderson and Sleath (2001) in the estimate of the nominal and real term structure for the UK government bond market. A spline methodology with a penalty factor is preferred to other popular methodologies, such as the seminal Nelson-Siegel model, first because it is more stable when the number of bonds is small and second because it does not impose an asymptote on long-term forward rates, which are the key ingredients to obtain long-term market expectations for interest rates. Other interesting features of this method are presented in the Appendix, in which the results of Fisher et al. (1995) are compared with those obtained by other methods.

An important criterion for choosing a term structure model is the purpose which the model itself serves. Clearly, there is no best model for the term structure as it depends on the application. If the aim is to price off-the-run bonds, a general criterion should be the minimization of the pricing error. Conversely, when attempting to extract interest-rate expectations for monetary policy purposes, a smooth term structure is desirable. However, term structure has manifold uses in a central bank and more than one model should be welcome. A parsimonious model, such as the Nelson-Siegel, seems appropriate for monetary policy and macroeconomic analysis as it shapes the term structure on the basis of few identifiable parameters that have a clear interpretation. A more flexible and stable approach, such as that implied by methodologies backed by pure interest-rate models, can be useful for pricing purposes, even if no-arbitrage considerations are clearly not taken into account. This paper uses a smoothing spline which is extremely stable even when there are very few coupon bonds available; it gives results similar to the Nelson-Siegel model, the benchmark of many central banks, and it outperforms the other methodologies in terms of pricing errors.

The second part of the paper presents estimates of the constant-maturity inflation compensation (or BEIR) by subtracting the zero-coupon real rate from the corresponding zero-coupon nominal rate. The use of the constant-maturity BEIR presents two advantages with respect to the BEIR computed as the difference between the nominal and the real yield to maturity. First, on a long time horizon, the difference between a specific nominal yield and a specific real yield changes maturity as time passes and is not easily comparable with previous figures; the practice of substituting old bonds with the new issue is a palliative. Second, the BEIR computed as difference in yield to maturity depends heavily on the different duration of the bonds and their different cash-flow structure, while that computed as the difference between zero-coupon rates is insulated from the cash flows.

Real interest rates combined with the rate implied in the nominal government bond yield provide a measure of inflation expectations since in real terms the payoff of a nominal bond should be close to that of an IL bond over its entire life. Such breakeven inflation rates are usually taken as proxies for inflation expectations and provide a measure of central bank credibility about targeting a specific inflation rate. The primary objective of the European Central Bank (ECB) is to maintain price stability within the euro area, defined as a rate of inflation below, but close to, 2 per cent over the medium run. One forward-looking way to evaluate the success of monetary policy is to look at expectations of inflation; in fact, if monetary policy is successful at keeping expectations well-anchored, then financial market participants will tend to "look through" the cycles of inflation and not change expectations about the rate of inflation over the longer run. The low level of inflation and the unorthodox monetary policy recorded over the past years has raised concerns about the possibility that market participants were still seeing ECB policy as consistent with longer-run price stability.

However, the comparison between nominal and real rates is biased by the presence of risk premia due to liquidity and inflation risks. Moreover, the comparison is further biased by the presence of seasonality in the daily price reference index used to index the coupon and the principal of the IL bond.

Results show that the spline methodology used in this paper is quite effective at capturing the general shape of the real term structure while smoothing through idiosyncratic variations in the yields of IL bonds. Real interest rates tend to be quite stable at longer horizons and the average 10-year real rate from 2002 to 2009 is close to 2 per cent even after correcting estimates for the seasonality of the euro-area reference price index. Furthermore, euro-area IL bonds are characterized by low liquidity, especially in comparison with the corresponding nominal bonds, which may be due to the fact that index-linked investors tend to hold these bonds until maturity. In addition, evidence is found that inflation compensation was held down in the period 2008-09 by a premium associated with the illiquidity of OAT€i, by analysing the indication from the asset-swap spread. Finally, an approximation of the inflation risk premium is introduced by comparing the inflation compensation implied by the nominal and real term structures and the inflation expectations surveyed by Consensus Economics and the ECB's Survey of Professional Forecasters. The burden of developing a model for the term structure of inflation risk premia is left to future work.

The paper is organized as follows. Section 2 reviews the literature; Section 3 presents the data. Estimates of the real term structure are presented in Section 4, those of the inflation compensation in Section 5. Section 6 documents the different information deriving from the zero-coupon inflation swap market; Section 7 concludes. The Appendix presents the methodology for correcting seasonality, introduces a primer on real term structure estimation methods, and compares the estimates of the real term structure derived from several methodologies.

2 A review of the literature

The literature on real term structure originated in the United States and in the United Kingdom, countries characterized by liquid and deep markets for IL bonds since the beginning of the 1990s in the UK and from 1997 in the US. Only recently has a similar stream of literature grown up in the eurozone, thanks to the issuance of this type of bond in the major euro-area countries. Before the introduction of IL bonds the real term structure was derived by a no-arbitrage restriction in a nominal term structure model constrained by inflation expectations (for example, Campbell and Shiller, 1996, and Hördal and Tristani, 2007, for the period before 2002). Only the introduction of IL bonds has allowed researchers to estimate the real term structure from quoted bonds.[3]

There is no clear consensus on the best methodology to adopt in the estimation, in analogy with the debate on the nominal term structure. Some researchers tend to prefer a spline methodology, while others use the seminal method first introduced by Nelson and Siegel (1987) and its refinements – the Appendix presents a primer on the methodologies for real term structure estimates. In addition, the real term structure estimate is complicated by the scarcity of IL bonds, by the absence of securities with maturity shorter than one year, and by the presence of outliers, which matter most when the number of securities is small.

In a seminal paper, Evans (1998) introduces a simple parametric approach to estimate the UK real term structure, which reduces the parameters of the four-parameter Nelson-Siegel approach, originally used by the Bank of England until the late 1990s .

The Fisher et al. (1995) method, used by the Federal Reserve Board to estimate the nominal term structure for the US government bond market, is also used by Sack (2000) in the estimate of the real term structure derived from nominal and index-linked STRIPs. Yields to maturity on coupon and principal STRIPs are evenly-spaced zero-coupon rates and so the construction of the term structure is extremely simplified.

Anderson and Sleath (2001) present the new methodology used by the Bank of England; they introduce the Variable Roughness-Penalty (VRP) spline, which extends the original smoothing spline method with a penalty factor introduced by Fisher et al. (1995) and Waggoner (1997) for the US nominal bond market. In the VRP spline, a different penalty is assigned in the pricing error depending on the maturity of the bond; in the original work, Waggoner (1997) uses three penalties depending on the type of bonds used to estimated the term structure, namely T-bills, T-notes and T-bonds.

For the euro-area IL bond market, Hördal and Tristani (2007) use the spline methodology introduced by McCulloch and Kochin (2000). The methodology is specifically designed to work even when bond data are only available for few maturities. For the US market, McCulloch posts on his website estimates of monthly real zero-coupon rates derived from U.S. TIPS, obtained by means of the McCulloch and Kochin (2000) methodology.

Gürkaynak et al. (2010) and D'Amico et al. (2008) estimate the term structure implied by US IL bonds (Treasury Inflation Protected Securities, TIPS) with the seminal approach first introduced by Nelson and Siegel (1987) and also use the Nelson-Siegel methodology

[3] McCulloch posts on his website ("The US real term structure of interest rates", http://economics.sbs.ohio-state.edu/jhm/ts/ts.html) the end-of-month real and nominal term structures. The Bank of England publishes the estimates of the UK real and nominal term structures obtained by means of the Variable-Roughness-Penalty spline methology by Anderson and Sleath (2001) (http://www.bankofengland.co.uk/statistics/yieldcurve/index.htm).

for the US TIPSs. Similarly, Ejsing et al. (2007) estimate the real term structure for the euro area with the approach of Nelson and Siegel (1987) from French OAT€i.

Finally, Jarrow and Yildirim (2003) estimate a piecewise constant function with non-linear least squares to obtain four real zero-coupon rates from the US TIPSs. This method, although simple, is too restrictive in terms of the number of zero-coupon rates.

This paper uses the smoothing spline methodology introduced by Fisher et al. (1995) and compares the results derived with those alternative methodologies described in the Appendix. The Fisher et al. (1995) methodology is preferred to other methods for several reasons. First, it is very stable across the sample period with respect to other approaches, in particular the Nelson-Siegel; in fact, the small number of issues available makes convergence very hard with the Nelson-Siegel setup in many estimates. Second, with respect to the other spline methodologies used in the literature, the Fisher et al. (1995) spline does not impose a limiting forward rate like the McCulloch-Kochin (2000) model and it does not require a fine-tuning of the short-term end of the term structure like the Anderson and Sleath (2001) model; moreover, the Anderson and Sleath (2001) model performs very poorly when the number of bonds is small. Finally, the Fisher et al. (1995) model outperforms many other methodologies in in-sample pricing and is outperformed in out-of-sample pricing only by the Anderson and Sleath (2001) model for bonds with maturity longer than 10 years.

3 The data

The euro-area IL bond market started in 1998 with the issue of French government bonds, OATi, indexed to the domestic French Consumer Price Index (CPI); in the following years the French Treasury continued issuing IL bonds of the same class in order to enrich the maturity spectrum of the French IL bond market. In 2002 there was the first issue of French government bonds, OAT€i, indexed to the euro-area HICP excluding tobacco, the reference price index of the eurozone. Similarly, in 2003 the Italian and in 2006 the German Treasuries started issuing IL bonds indexed to the euro-area HICP excluding tobacco. This work considers only French IL bonds for two reasons; first, in the sample they are given an AAA rating, against the lower rating given to the Italian government securities and, second, the time series start from 1998, considering indexation to the French CPI, and from 2002, considering indexation to the euro-area HICP excluding tobacco, thus allowing a long-term comparison with the corresponding nominal bonds. From 1998 to date there have been seven issues of OATi and from 2001 to date five issues of OAT€i. Coupons are paid once a year on 25th July (see Tables 6 and 7 in the Appendix).

Since 2004 the French IL bond market has been further enriched by the possibility of stripping the principal and the coupons of OATis and OAT€is; namely, STRIPs (Separated Trading of Registered Interest and Principal) are OATs whose interest and principal portions of the security have been separated, or "stripped", and may then be sold separately in the secondary market.[4] Given that STRIPs are quoted as discount bonds and are available along the entire time-to-expiration of the bond, they increase the number of bonds and allow a substantial improvement in the estimation of the real term structure. However, since quotes for STRIPs derived from OATis and OAT€is are available only from August 2009, they have been used only to cross-validate this paper's model on some specific dates.

Similarly, the nominal term structure is estimated by using the quotes of the euro repo rates with maturity 1 week, 2 weeks, 3 weeks, 1 month, 2 months, 3 months, 6 months, 9 months, 12 months for the short term, of the BTANs (*Bon à Taux Annuel Normalisé*) with time to maturity greater than 1 year and below 5 years, and of standard OATs with maturity greater than 1 year.

Daily mid-quotes are obtained from Bloomberg and Thomson Financial Reuters. The daily consumer price index reference is obtained from the website of the European Central Bank (www.ecb.int) and from the website of the French Treasury (www.aft.gouv.fr).

OAT€is (OATis) are government bonds indexed to the euro-area HICP excluding tobacco (domestic French CPI); their principal is protected from inflation thanks to the indexation to a daily price index reference, even if it is paid out by the issuer at the moment of the bond's redemption. The daily price index reference P_t^{lag} on day t of month m is computed as a linear interpolation of the monthly index, namely

$$P_t^{lag} = I_{m-3} + \frac{(\#\ of\ days\ since\ the\ start\ of\ month\ m) - 1}{(\#\ of\ days\ in\ month\ m)}(I_{m-2} - I_{m-3})$$

where I_m is the monthly HICP excluding tobacco for month m. The euro-area IL bond market has a 3-month indexation lag to account for the delays in the publication of the price index by the statistical agency. In the euro area, Eurostat releases the HICP for month m around the 15th of month $m+1$.

The redemption value of the bond is given by 100$\cdot$*(daily price reference at maturity)/(base index)*=$100 \cdot P_T^{lag}/P_0^{lag}$ where T is the reimbursement date and 0 is the issue date.

OAT€i are guaranteed by a redemption at par. This implies that in case of deflation throughout the life of the bond the redemption value is equal to 100. In our model I do not consider this case likely, although it is possible, as this option will have value only if one assumes an average deflation over the entire life of the bond to be likely; thus I rule out, a priori, the case of redemption at par.

The annual coupon (the real coupon) is paid once a year as a fixed percentage of the index-linked principal and is determined at the time of issue; so the *paid coupon* is given by

[4] The name derives from the days before computerization, when paper bonds were physically traded: traders would literally tear the interest coupons off paper securities for separate resale.

$$\begin{aligned} & 100 \cdot (real\ \ coupon) \cdot \frac{(daily\ \ inflation\ \ reference\ \ at\ \ payment\ \ date)}{(base\ \ index)} \\ = \ & 100 \cdot (real\ \ coupon) \cdot \frac{P^{lag}_{h_i}}{P^{lag}_0} \end{aligned}$$

where h_i is the payment date of the i-th coupon, namely the coupon date. The ratio *(daily inflation reference at payment date)/(base index)*=$P^{lag}_{h_i}/P^{lag}_0$ is defined as the indexation coefficient (IC).

4 The estimate of the real term structure

The real term structure is estimated with the smoothing spline proposed by Fisher et al. (1995). The spline methodology is superior to parametric methodologies, such as the Nelson-Siegel families, when the numbers of bonds is extremely small (McCulloch and Kochin, 2000, Hördahl and Tristani, 2007). In addition, according to McCulloch and Kochin (2000) the smoothing spline, which is estimated on forward rates, does not impose an asymptote for longer maturities rates and is, therefore, capable of capturing the expectations implied in long-term rates (see the Appendix for the technical description)

The spline methodology satisfies the three main properties which are supposed to be sought-after in term structure estimates. First, this technique gives smooth forward curves rather than attempting to fit every data point, as the aim is to supply a measure of market expectations for monetary policy purposes instead of a precise pricing of all bonds in the market. Second, the technique is sufficiently flexible to capture movements in the underlying term structure. Third, estimates of the term structure at any particular maturity are stable, in the sense that small changes in data at one maturity, especially at the extremes of the maturity spectrum, do not have a disproportionate effect on forward rates at other maturities.[5]

The sample of the IL bonds is split into two subperiods. The first runs from November 2001 to December 2003; in this sample the real term structure is obtained from the OATis and OAT€is. The second runs from January 2004 to December 2009 and considers only OAT€is. The use of OATis in the first subsample is necessary given the very few issues of OAT€is before 2004. However, when one compares the estimates obtained from OAT€is with those obtained from OAT€is and OATis for the second period, results do not differ substantially. For consistency, the same spline methodology is used to compute the nominal term structure.

Table 1 shows the sample statistics for the IL and the nominal bonds while Figure 1 plots the daily time series for the 3-year, 5-year, 7-year, 10-year and 20-year zero-coupon

[5] The class of models introduced by Waggoner (1997) and by Anderson and Sleath (2001) is also relatively less flexible at the long end than at shorter maturities, where expectations are likely to be better defined.

interest rates. Daily data show that real rate averages are increasing in maturity, ranging from 1.48 per cent for the 3-year rate to 2.29 per cent for the 25-year rate. Conversely, the standard deviation decreases with the maturity. Asymmetry, measured by the skewness coefficient, has a V-shape with peaks at the shortest and highest maturities, while fat-tails, approximated by the kurtosis coefficient, are evenly distributed across maturities. The real rates show a strong autocorrelation from the 1-day lag to the 20-day lag. More importantly, real rates show very strong similarities with the corresponding nominal rates in terms of standard deviation, skewness, kurtosis and autocorrelation.

By inspecting the time series of the real rates it can be seen that the term structure of real rates shows an inverted shape in 2002, computed as the difference between the 10-year and the 3-year interest rates, while it has a standard natural positive slope from January 2003 onwards. Moreover, from January 2003 until the middle of 2006, the steepness of the term structure is strictly positive even with decreasing real interest rates; it flattens from mid-2006 until the end of 2007. From the beginning of 2009 it starts steepening, influenced by the sharp decrease in interest rates at the shortest maturity, which – as in 2005 – hit negative territory. Sample statistics show a positive slope of the real term structure, with and average of $1.96 - 1.50 = 0.46$ percentage points between the 10- and 3-year maturities and $2.25 - 1.50 = 0.75$ percentage points between the 20- and 3-year maturities.

4.1 The correction for seasonality[6]

As shown by Ejsing et al. (2007) the construction of a constant maturity inflation expectation measure has to encompass the seasonality of the euro-area HICP excluding tobacco. The dynamics of the seasonality factor – computed with an X12-ARIMA methodology and shown in Figure 2 – widen progressively from January 2002. This implies that the gross price of IL bonds, computed as the clean price plus the accrued interest and the inflation accrual, depends on the time of year. However, the order of magnitude of the adjustment required to compare IL bond quotes in different days of the year is small (the average of the daily correction factor is around 1.003, with a range of 0.012) and the correction mostly impacts bonds with the shortest maturities (Figure 3); see the Appendix for a formal introduction to the correction of seasonality in IL bonds.

The difference between corrected and uncorrected real interest rates is over 12 basis points for the shortest maturities, but decreases below 2 basis points for real interest rates with maturity greater than 15 years (Figure 4).

4.2 Robustness

A set of robustness tests confirms the main findings of the paper. Real term structure estimates are compared with those obtained with the Nelson-Siegel model, with a simplified Nelson-Siegel model, i.e. monotonic, with the McCulloch and Kochin (2000) methodology,

[6] I am indebted to Alessandro Secchi and Marco Taboga who showed me the algorithm for the correction due to seasonality.

and with the Variable-Roughness Penalty model by Anderson and Sleath (2001). Some partial comparisons are presented in the Appendix. The real term structure has been estimated from August 2009 to December 2009 with STRIP quotes. Moreover, estimates have been made including German IL bonds from January 2006 to December 2009. Results, available on request, do not differ substantially.

5 The inflation compensation

The real term structure can also used be to extract the inflation compensation requested by investors to hold IL bonds. This compensation, known as the breakeven inflation rate (BEIR), is equal to the difference between the nominal and the real interest rates, namely

$$BEIR_t^n = y_t^n - r_t^n \tag{1}$$

where y_t^n is the nominal interest rate at time t with maturity n, and r_t^n is the corresponding real interest rate. Time series of BEIRs are shown in Figure 5 and their statistics in Table 2. The statistics are described jointly with the inflation swap rates in a following section.

Note that, since the OATi€ is indexed to the euro-area HICP, the real term structure is compared with the corresponding nominal term structure extracted from nominal OATs issued by the French Treasury; differently, Hördal and Tristani (2007) compare the real term structure extracted from OATi€ with the German nominal term structure. The nominal term structure for French government bonds is also estimated with the methodology of Fisher et al. (1995) used for IL bonds.

BEIRs are very volatile at short-term maturities and tend to stabilize as maturity increases. The dynamics of the BEIRs suggests two main conclusions. First, the dispersion of inflation forecasts across the maturity spectrum is very large at the beginning of the sample, which coincides with the introduction of the single monetary policy; the dispersion can also be explained by possible pricing errors due to the scarcity of IL bonds. Second, the BEIR tends to be highly stable for longer maturities; 10-year and 20-year BEIRs fluctuate in the range $2.0 - 2.5$ per cent from the beginning of 2002 to the end of 2008, with an abrupt drop in the last quarter of 2008 on the back of deteriorating conditions in the interbank market. The difference in volatility between short- and long-term BEIRs can be explained by the anchoring of inflation expectations in the long term, by the volatility of the inflation risk premia or by a combination of the two.

Estimates of the inflation compensation around the end of 2008 and the beginning of 2009 show very low figures which are difficult to interpret as expectations of deflation in the euro area, but can be ascribed to the disfunctioning of the IL bond market. The financial literature for the US markets (see Campbell et al., 2009) has documented the impact of market-specific factors on inflation-indexed bond yields; the increase in volatility of Treasury Inflation-Protected Security (TIPS) yields in the autumn of 2008 appears to have resulted in part from the unwinding of large institutional positions after the failure of Lehman Brothers. These institutional influences on yields can alternatively be described as liquidity, market segmentation, or demand and supply effects. No research has been conducted in a similar vein for the euro-area IL bond market. I believe that the functioning of this market in the euro area can be of fundamental importance in assessing the reliability of readily available inflation expectations and thus the necessary monetary policy intervention. A sharper analysis of the IL bond market in the euro area should be a priority in the research agenda of financial economists.

However, the BEIR is not a pure expectation of the inflation rate since, as shown by Evans (1998), it can be thought of as the sum of the expected inflation rate at time t during the n periods to maturity, $\pi_t^{e,n}$, and the inflation risk premium at period t, IRP_t^n, namely $BEIR_t^n = \pi_t^{e,n} + IRP_t^n$. It can be shown that if variables are jointly lognormal, the risk premium is given by $IRP_t^n = Cov(m_t^n, \pi_t^n) - \frac{1}{2}Var(\pi_t^n)$, where m is the stochastic discount factor and π the inflation rate (see Appendix); in other words, the premium requested by investors to hold IL bonds and to hedge against unexpected changes in inflation depends on the negative covariance between the marginal rate of substitution (the stochastic discount factor) and the inflation rate; the second term is a Jensen inequality. Sometimes, the first term, $Cov(m_t^n, \pi_t^n)$, of the inflation risk premium is referred to as the 'pure inflation risk premium'.

The inflation risk premium, i.e. the compensation for risk due to uncertainty of future inflation, can be evaluated by mean of ad-hoc models and it is not the aim of this paper. However, it is worthwhile spending some words on this variable since it affects the computation of the inflation compensation. This premium is required by investors to hold assets whose real payoff is affected by unanticipated changes in inflation. Thus, investors require a premium as compensation for changes in inflation they are not able to forecast. This premium, in a standard representative-agent power-utility model, is positive when the covariance between the stochastic discount factor and inflation is negative (in other words when expected consumption growth is low and inflation is high).

A first evidence of the risk premium embedded in the BEIR can be obtained by comparing it with the corresponding long-term inflation expectations surveyed by professional forecasters (Figure 6); quarterly expectations for the 5-year-ahead annual inflation rate are collected by the ECB's Survey of Professional Forecasters (SPF) while semi-annual expectations of the annual inflation rate between five and ten years ahead are collected by Consensus Economics. As a first approximation, the inflation risk premium, IRP, is the difference between the BEIR and the expected inflation rate at the corresponding maturity; Evans (1998) uses a similar approach to approximate the UK inflation risk premium. As can be seen, the IRP is constantly positive with the exception of the 2002-03 period for the 5-year horizon and for the 2008-2009 for the 5-year and the 10-year horizon. Even if the main driver of the IRP is the covariance between the discount factor and the inflation rate, which can partly explain the drop in risk premium around the end of 2008, other factors may be at play. In the following part of this section I consider the other factors that can explain the dynamics of the BEIR.

5.1 Bond liquidity and inflation compensation

As explained by Sack (2000) and Gürkaynak et al. (2010) the comparison between nominal and real interest rates is made difficult by the different degree of liquidity of nominal bonds with respect to IL bonds. Accordingly, equation (1) becomes

$$\begin{aligned} BEIR_t^n &= y_t^n - r_t^n \\ &= \pi_t^{e,n} + IRP_t^n + \eta_t^n \end{aligned} \tag{2}$$

where η_t^n captures a different degree of liquidity of the two types of bonds.

The liquidity premium requested by investors to hold IL bonds can be proxied by the premium requested by investors to hold less liquid bonds with the same credit risk. Part of the literature focuses on nominal bonds with low liquidity and compares them with IL bonds. Alternatively, the liquidity premium can be computed directly from two nominal bonds with different liquidity and then added to the BEIR.

Along the first line of research, Gürkaynak et al. (2010) consider the difference between off-the-run nominal bonds and IL bonds, under the assumption that the former are less liquid than the benchmark nominal bonds which are used to build a standard term structure. In a similar vein, Sack (2000) compares the nominal and the real term structure both implied in the corresponding STRIPs. Unfortunately, this method cannot be applied to the French government bonds as index-linked STRIPs have been available only since August 2009.

Alternatively, Ejsing et al. (2007) and the ECB (2009) correct the difference between the nominal and the real term structure by the spread between the term structure obtained from IL bonds issued by the French government agency CADES and the term structure obtained from the nominal OATs. In fact, IL bonds issued by the CADES have the same credit risk as government bonds – the French Treasury is the guarantor – but a much lower degree of liquidity.[7] In the same vein, the ECB (2009) computes a liquidity correction for German government bonds using bonds issued by the state-owned KfW Bankengruppe, which have the same characteristics as the French CADES bonds.

Following Ejsing et al. (2007), the liquidity premium is computed as the difference between the zero-coupon rate extracted from CADES bonds and the corresponding rate extracted from the nominal French OATs (Figure 7). The CADES rates for the 10-year maturity are not available from end-2002 to end-2004 and thus the corresponding liquidity premium cannot be computed. The 5-year and 10-year liquidity premia have an average of 70-80 basis points for the available sample, but show large variations. In particular, the

[7] CADES – Caisse d'Amortissement de la Dette Sociale – is a French administrative public agency supervised by the French government. Its mission is to pay off the social security debt transferred to it, to contribute to the general budget of the French government, and to make payments to various social security funds and organizations. The company only operates in France. Like most companies in its industry (small companies that only issue bonds), CADES publishes very little information regarding sustainability. Still, in the field of sustainability, CADES belongs to the 50% best performing companies in the industry.

premia are quite low in the period 2006-07 and increase at the end of 2008 on the back of market uncertainty in the international financial markets.

Even if it were too simplistic to add the liquidity premia to the BEIRs, it would be intuitive to see that the large drop in BEIRs recorded in the last quarter of 2008 and first quarter of 2009 can be entirely explained by the large liquidity premium investors demanded for holding less liquid bonds, such as French IL bonds. It is worth noticing that the liquidity of 5-year bonds remained low after the autumn of 2008.

Similar conclusions in terms of liquidity are obtained by the BEIR implied by the nominal STRIPs and by the index-linked STRIPs quoted from August 2009. Results, available on request, do not differ substantially from that obtained by comparing the standard nominal and IL bond term structures, even if the sample period is too short to be statistically significant.

6 Comparison with the inflation swap rates

Important indications about the role of liquidity premia since the autumn of 2008 are provided by comparing the BEIR implied by the OAT€i cash market and BEIRs implied by zero-coupon inflation swaps. Zero-coupon inflation swaps are derivatives contracts where one of the parties pays the other cumulative HICP inflation over the term of the contract at maturity in exchange for a predetermined fixed rate.[8] This rate is known as the 'synthetic' BEIR because, if inflation grew at this fixed rate over the life of the contract, the net payment on the contract at maturity would be equal to zero. As with the 'cash' BEIR implied by OAT€i and nominal OAT, this rate reflects both expected inflation over the relevant period and an inflation risk premium.

Statistics for the difference between inflation swap rates and for the BEIR are summarized in Table 2. Results show that the average 'synthetic' BEIR is higher than the 'cash' BEIR; the variability of the inflation swap rate is smaller than that of the cash BEIR; there are similar indications in terms of asymmetry, skewness and serial autocorrelation.

However, a formal test of difference between the averages of the two measures rejects the null hypothesis that the two time series are on average different over the sample period. Table 3 reports the arithmetic mean of the difference between the inflation swap rates and the 'cash' BEIR and the p-values of the test of significance of the arithmetic mean. For all the maturity spectrum, the test rejects the null of arithmetic mean different from zero.

[8] In a zero-coupon inflation swap one party – the inflation seller – pays a fixed rate on a given notional amount, compounded annually and paid in a single payment at the maturity of the swap. The counterparty — or inflation payer — pays the aggregate percentage rise in non-seasonally adjusted CPI. So, for example, in a EUR 10 million 5-year zero-coupon swap the fixed payer might pay 2.5 per cent versus receiving inflation. The pay-off of the payer would be $[(1.025)^5 - 1] * 10mm$, the pay-off of the receiver $[CPI(in\ 5\ yrs)/CPI(today) - 1] * 10mm$. The fixed-rate payer wins if compounded inflation is above 2.5 per cent and loses if it is less than 2.5 per cent. This fixed rate is the rate quoted in the market.

Table 3 – Test for difference between inflation swap rates (ISR) and BEIRs

	daily (1)		weekly (2)		monthly (3)	
	μ	$p-value$	μ	$p-value$	μ	$p-value$
ISR-BEIR 1y	0.0112	0.4949	0.0016	0.4992	0.0073	0.4942
ISR-BEIR 2y	0.0767	0.4561	0.0745	0.4502	0.0841	0.4131
ISR-BEIR 3y	0.0876	0.4403	0.0856	0.4311	0.0914	0.3909
ISR-BEIR 4y	0.0925	0.4278	0.0922	0.4132	0.0946	0.3679
ISR-BEIR 5y	0.0999	0.4137	0.1004	0.3963	0.0971	0.3505
ISR-BEIR 6y	0.1072	0.4023	0.1069	0.3803	0.1109	0.3377
ISR-BEIR 7y	0.1124	0.3949	0.1138	0.3741	0.1126	0.3263
ISR-BEIR 8y	0.1124	0.3903	0.1143	0.3683	0.1133	0.3271
ISR-BEIR 9y	0.1090	0.3872	0.1109	0.3639	0.1109	0.3265
ISR-BEIR 10y	0.1006	0.3873	0.1024	0.3622	0.0986	0.3249

(1) 1,451 daily data from 21 June 2004 to 30 December 2009; (2) 291 weekly data from Wednesday 23 June 2004 to Wednesday 30 December 2009; (3) 67 end-of-month data from 30 June 2004 to 30 December 2009. The Table reports the average difference in percentage points between the inflation swap rate (ISR) and the corresponding BEIR for ten maturities (column μ) and the p-value for the test that the average difference is different from 0 (column p-value) A p-value greater than 0.05 implies that the null hypothesis that the difference is equal to 0 is not rejected at the 5% significance level; the p-value is computed for a Student-t distribution with T-1 degrees of freedom, where T is the length of the time series. The standard errors are computed with the Newey-West estimator with a Bartlett window.

A visual inspection documents that the BEIRs implied by the zero-coupon inflation swaps (the 'synthetic' BEIR) recorded an abrupt drop in the autumn of 2008, albeit a smaller one than the 'cash' BEIR; the 10-year cash BEIR reaches a minimum of 1.21 percentage points against a 1.61 of the 'synthetic' BEIR (similarly the 5-year 'cash' BEIR drops to 0.54 against 0.89 of the 'synthetic' BEIR). The problem with these measures is that they are not immune to the counterparty risk which affected the interbank market in that period and so a liquidity premium – in the IL bond market – could have been substituted by a counterparty risk premium – in the private inflation swap market.

6.1 The information content of liquidity from the asset swap spread

Campbell et al. (2009) and Haubrich et al. (2011) show that the Treasury asset swap spread gives an indication of the illiquidity of the TIPS market during market turmoil, and in particular in the autumn of 2008. This section investigates whether this result holds for the euro-area market as well.

In normal times, inflation swap rates are linked to BEIRs by no-arbitrage trading strategies which replicate long positions in OAT€i and short positions in nominal OAT in the asset swap market. In fact, the supplier of inflation protection, i.e. the seller of a zero-coupon inflation swap, usually hedges his/her position by simultaneously taking long positions in OAT€i and short positions in nominal OAT. This strategy is equivalent to two levered positions in the asset swap market;[9] the first position implies the investor receives the OAT€i cash-flow in exchange for the LIBOR plus a fixed spread, and the

[9] In a bond asset swap, the buyer of the bond swaps the fixed-rate coupon with the LIBOR plus a spread, known as the asset-swap spread.

second position implies the investor receives the LIBOR plus a fixed spread in return for the OAT cashflow. By a no-arbitrage argument the difference in this asset swap spread should be equal to the 'synthetic' BEIR.

Typically, the asset swap spread is negative for government bonds, since the credit rating of the bond issuer is higher than that of the counterparty, and its absolute magnitude is larger for IL bonds (OAT€i) than nominal bonds (OAT). Thus, the inflation-hedging strategy of an inflation swap faces a positive financing cost derived from the 'long OAT€i – short OAT' position.

Figure 9 shows that starting in mid-September 2008 the OAT€i asset-swap spread increased from normal levels of about -200 basis points to about -75 basis points, while the nominal OAT asset-swap spread increased from -25 basis points to +25 basis points. That is, financing the strategy 'long OAT€i – short OAT' became extremely expensive relative to historical levels just as their cash prices fell abruptly. This evidence points to an episode of intense selling in the cash OAT€i market with insufficient demand to absorb those sales and simultaneously another shortage of capital to finance levered positions in markets other than nominal OAT, that is, a 'flight-to-liquidity' episode. Under this interpretation, in the autumn of 2008 the 'synthetic' BEIR was a better proxy for inflation expectations than the 'cash' BEIR.

7 Conclusion

This paper presents an estimate of the euro-area term structure which is quite effective at capturing the general shape of the term structure while smoothing through idiosyncratic variations in the yields of IL bonds. The methodology is also extremely stable and able to give a good fit in-sample and out-of-sample. The estimated yield curve can be expressed in a variety of ways, including zero-coupon yields, par yields, and forward rates. Moreover, it can be compared with the corresponding nominal term structure to obtain estimates of inflation compensation (or breakeven inflation rates).

It is shown that real interest rates tend to be quite stable at longer horizons and that the average 10-year real rate from 2002 to 2009 is close to 2 per cent. The correction for the seasonality of the euro-area reference price index does not change results greatly. Furthermore, the analysis documents that euro-area IL bonds are characterized by low liquidity, especially in comparison with the corresponding nominal bonds; this can be due, according to market intelligence, to the fact that index-linked investors tend to hold these bonds until maturity. In addition, evidence is found that inflation compensation was held down in the period 2008-09 by a premium associated with the illiquidity of OAT€i, by analysing the indication from the asset-swap spread.

Finally, an approximation of the inflation risk premium is introduced by comparing the inflation compensation implied by the nominal and real term structures and the inflation expectations surveyed by Consensus Economics and by the ECB's Survey of Professional Forecasters.

Having the real term structure should greatly benefit our efforts to better understand the behaviour of nominal yields. It allows us to parse nominal yields and forward rates into their real rate component and their inflation compensation component. These two components may behave quite differently, in which case simply looking at a nominal yield might mask important information.

8 Appendix

8.1 Relation between yields and the inflation premium

The price R of an IL bond with maturity $n+1$ is

$$R_t^{n+1} = E_t\left(R_{t+1}^n \cdot M_{t+1}\right) . \tag{3}$$

where M is the real pricing kernel.[10] The price Y of a nominal bond with maturity $n+1$ is

$$Y_t^{n+1} = E_t\left(Y_{t+1}^n \cdot \frac{M_{t+1}}{\Pi_{t+1}}\right), \tag{4}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the ratio between the price index at time $t+1$ and t and, thus, M_{t+1}/Π_{t+1} is the nominal pricing kernel. First, consider one-period bonds, so that (3) and (4) become $R_t^1 = E_t\left(1 \cdot M_{t+1}\right)$ and $Y_t^1 = E_t\left(1 \cdot \frac{M_{t+1}}{\Pi_{t+1}}\right)$. Assume that variables are jointly lognormal, namely

$$[R, Y, M, \Pi]' \sim \exp(N([\mu_r, \mu_y, \mu_m, \mu_\pi]', \Sigma)),$$

and define $m = \ln(M)$, $\pi = \ln(\Pi)$, $y^n = -\frac{1}{n}\ln(Y^n)$ and $r^n = -\frac{1}{n}\ln(R^n)$. By substituting prices with yields and subtracting (3) from (4) I obtain

$$\begin{aligned}
y_t^1 - r_t^1 &= -\ln E_t\left(\exp(m_{t+1} - \pi_{t+1})\right) + \ln E_t\left(\exp(m_{t+1})\right) \\
&= -\left(\mu_m - \mu_\pi + \frac{1}{2}V_t(m_{t+1} - \pi_{t+1})\right) + \left(\mu_m + \frac{1}{2}V_t(m_{t+1})\right) \\
&= -\left(\mu_m - \mu_\pi + \frac{1}{2}V_t(m_{t+1}) + \frac{1}{2}V_t(\pi_{t+1}) - CV_t(m_{t+1}, \pi_{t+1})\right) \\
&\quad + \left(\mu_m + \frac{1}{2}V_t(m_{t+1})\right) \\
&= \mu_\pi + CV_t(m_{t+1}, \pi_{t+1}) - \frac{1}{2}V_t(\pi_{t+1}).
\end{aligned} \tag{5}$$

When bonds are multi-period, take logarithm of (3) and (4) and apply the law of iterated expectations $E_t(E_{t+1}(E_{t+h}(\cdot)))) = E_t(\cdot)$, $V_t(V_{t+1}(V_{t+h}(\cdot)))) = V_t(\cdot)$ and furthermore assume $E_t \sum_{i=1}^n x_{t+i} = n \cdot \mu_x$, $V_t\left(\sum_{i=1}^n x_{t+i}\right) = n \cdot V_t(x_{t+1})$, $CV_t\left(\sum_{i=1}^n x_{t+i}y_{t+i}\right) = n \cdot CV_t(x_{t+1}, y_{t+1})$; it follows

$$\begin{aligned}
r_t^{n+1} &= -\frac{1}{n}\left(n \cdot \mu_m + \frac{n}{2}V_t\left(m_{t+1}\right)\right) \\
y_t^{n+1} &= -\frac{1}{n}\left(n \cdot \mu_m - n \cdot \mu_\pi + \frac{n}{2}V_t(m_{t+1}) + \frac{n}{2}V_t(\pi_{t+1}) - n \cdot CV_t(m_{t+1}, \pi_{t+1})\right),
\end{aligned}$$

which gives the result of (5). Note that (5) holds as an approximation when variables are not lognormal. (5) shows that the difference between the zero-coupon nominal and real rates is equal to the expected inflation rate over the maturity of the interest rates plus the inflation risk premium, defined as the sum of the covariance between the pricing kernel and the inflation rate, usually negative, and a correction due to the variability of inflation.

[10] E_t is the conditional expectation operator, V_t the conditional variance operator and CV_t the conditional covariance operator.

8.2 The correction for seasonality

The price an investor has to pay at time t to purchase an IL bond is $(B_t + AI_t) \cdot P_t^{lag}/P_0$, where B is the clean price, AI the accrued interest and P_t^{lag}/P_0 the accrued inflation in period t, the indexation coefficient (see the main text for the definition). Therefore, quoted prices are clean not only in the sense that they do not include interest accrual, but also clean in the sense that they are free of inflation compensation.

I define $\delta(h)$ the nominal price of a zero-coupon bond indexed to the consumer price index which pays off P_{t+h}^{lag}/P_t^{lag} in h periods, i.e. at time $t+h$. Therefore, the nominal price of a coupon-bearing bond indexed to the same consumer price index, which has face value equal to 1 and expires at period H, is

$$(B_t + AI_t)\,\frac{P_t^{lag}}{P_0^{lag}} = C \cdot \sum_{h=1}^{H} \delta_t(h)\frac{P_t^{lag}}{P_0^{lag}} + \delta_t(H)\frac{P_t^{lag}}{P_0^{lag}}, \tag{6}$$

where C is the coupon and $\delta(\tau)$ is the discount factor τ periods ahead. The term P_t^{lag}/P_0 appears because it is the inflation accrual the bond holder gets, while it differs from the definition of the discount function. (6) can be reduced to the standard bond formula $(B_t + AI_t) = C \cdot \sum_{h=1}^{H} \delta_t(h) + \delta_t(H)$ where $\delta_t(\cdot)$ can be interpreted as the real discount function.

Similarly, define $\delta^{SA}(h)$ the price of a seasonally-adjusted zero-coupon bond indexed to the consumer price index, which pays off $P_{t+h}^{lag,SA}/P_t^{lag,SA}$ in h periods, i.e. at time $t+h$. Assuming a multiplicative seasonality (namely $P_t = SF_t \cdot P_t^{SA}$, where SF_t is the seasonality factor for period t) it follows

$$\frac{P_{t+h}^{lag}}{P_t^{lag}} = \frac{P_{t+h}^{lag,SA}}{P_t^{lag,SA}}\frac{SF_{t+h}^{lag}}{SF_t^{lag}}, \tag{7}$$

which implies

$$\delta_t(h) = \frac{SF_{t+h}^{lag}}{SF_t^{lag}}\delta_t^{SA}(h). \tag{8}$$

By substituting equation (8) into (6) it follows

$$(B_t + AI_t) = C \cdot \sum_{h=1}^{H} \delta_t^{SA}(h)\frac{SF_{t+h}^{lag}}{SF_t^{lag}} + \delta_t^{SA}(H)\frac{SF_{t+H}^{lag}}{SF_t^{lag}}.$$

Furthermore, assuming that the seasonality factors are constant during the residual time to expiration, namely $SF_{t+h} = SF_{t+H}$, it follows

$$(B_t + AI_t)\,\frac{SF_t^{lag}}{SF_{t+H}^{lag}} = C \cdot \sum_{h=1}^{H} \delta_t^{SA}(h) + \delta_t^{SA}(H). \tag{9}$$

Equation (9) defines the seasonally-adjusted real discount factor $\delta^{SA}(h)$, which is obtained by simply multiplying the gross price of the bond, $B+AI$, by the ratio between two seasonal factors.

8.3 The yield to maturity of an index-linked bond

The yield to maturity of an IL bond is the yield y which equates the current bond price accrued by the interest, AI, and by the price index changes, measured by the indexation coefficient γ, and the discounted sum of the cash flow adjusted by the indexation coefficient γ. Namely, y solves

$$(B_t + AI_t) \cdot \gamma_t = \sum_{h=1}^{H} e^{-yh} \cdot C \cdot \gamma_t + e^{-yH} \cdot 1 \cdot \gamma_t$$

which, dividing both terms by γ_t, reduces to

$$(B_t + AI_t) = \sum_{h=1}^{H} e^{-yh} \cdot C + e^{-yH} \cdot 1$$

The computation of the yield to maturity of an IL bond can then be made without the knowledge of the indexation coefficient.

8.4 A primer on non-parametric term structure models

Given a set of current gross IL bond prices, $B + AI = B^+$, and coupon/principal payments C, the term structure is defined by the discount function $\delta(h;\theta)$, where θ is a vector of parameters and h is the maturity of the discount factor (hereafter I use $\delta(h;\theta)$ as a symbol for the discount function for the seasonally-adjusted real discount factor). This function prices the IL bonds such that

$$B^+ = \sum_{h=1}^{H} \delta(h;\theta) \cdot C_h + \varepsilon \tag{10}$$

where ε is the pricing error and H is the maturity of the bond. The optimal set of parameters θ^* solves

$$\theta^* = \arg\min \left\{ \varepsilon'\varepsilon | \varepsilon = B^+ - \sum_{h=1}^{H} \delta(h;\theta) \cdot C_h \right\}$$

Given the objective function, there are different methods of estimating the term structure – see James and Webber (2001). First, parametric models, which can be linear and non-linear, assume the term structure is derived from an interest rate model – such as that of Vasicek, Longstaff and Schwartz as well as the affine interest-rate models. Also, non-parametric models can be split into linear and non-linear classes.[11] The linear non-parametric models assume the term structure can be expressed as a linear combination of basis functions, which span the vector space; an example of this class is the spline. In

[11] Note the non-parametric models also depend upon parameters. My taxonomy for parametric and non-parametric follows James and Webber (2000). Anderson et al. (1996) use a different taxonomy and include Nelson-Siegel types in the parametric family.

the non-linear models the term structure cannot be expressed as a function of basis and, conversely, can be described by a smaller number of parameters; the Nelson-Siegel and Svensson methods belong to this category. In what follows I briefly sketch the best known non-parametric term structure models applied to the estimation of index-linked bond term structures.

Given the discount function $\delta(h;\theta)$, one can obtain the zero-coupon rate

$$r(h;\theta) = -\frac{\log\left[\delta(h;\theta)\right]}{h} ,$$

the par yield

$$p(h;\theta) = \frac{1-\delta(h;\theta)}{\int_0^h \delta(m;\theta)\cdot dm} ,$$

and the forward rate

$$f(h;\theta) = -\frac{\partial\left[\delta(h;\theta)\right]}{\partial h}\cdot\frac{1}{\delta(h;\theta)} .$$

8.5 Non-parametric non-linear families: parsimonious models

8.5.1 The Nelson-Siegel model

In the Nelson-Siegel model the discount function has four parameters, i.e. $\theta_{NS} = [\tau, \beta_0, \beta_1, \beta_2]'$, and is specified by

$$\delta(h;\theta_{NS}) = e^{-\beta_0 h - \tau\cdot(\beta_1+\beta_2)}\cdot\left(1-e^{-\frac{h}{\tau}}\right) + \beta_2 h\cdot e^{-\frac{h}{\tau}}.$$

Given that $\delta(h;\theta) = e^{-y(h)\cdot h}$ the zero-coupon interest rate for the maturity h is given by

$$y(h;\theta_{NS}) = \beta_0 + (\beta_1+\beta_2)\frac{\tau}{h}(1-e^{-\frac{h}{\tau}}) - \beta_2 e^{-\frac{h}{\tau}},$$

where $y(0) = \beta_0 + \beta_1$ is the short-term rate and $\lim_{h\to\infty} y(h) = \beta_0$ is the long-term rate; τ and β_2 control for location, height and hump of the curve. Although a variety of objective functions is available, a standard choice is

$$\theta^*_{NS} = \arg\min\left\{\varepsilon'\varepsilon | \varepsilon = \frac{B^+_n - \widehat{B}^+_n(y|\theta_{NS})}{D_n}\right\}, \tag{11}$$

where $B^+_n - \widehat{B}^+_n(y)$ is the pricing error as a function of the zero coupon and D is the modified duration of each bond. The Nelson-Siegel model has only four parameters and is, therefore, very simple and flexible; conversely, its simplicity does not allow double humps to be shaped in the term structure. Moreover, it is not suitable for no-arbitrage modelling. Further, this method models the forward-rate curve as well as the spot-rate curve but it is not suited to modelling the discount-rate curve.[12]

The monotonicity of the real term structure has motivated some researchers to use a reduced Nelson-Siegel model, defined the monotonic model (Bliss, 1997). Given the vector of parameters $\theta_{MO} = [\tau, \beta_0, \beta_1]'$, the zero coupon is given by

$$y(h; \theta_{MO}) = \beta_0 + (\beta_1)\frac{\tau}{h}(1 - e^{-\frac{h}{\tau}}). \tag{12}$$

8.6 Non-parametric non-linear families: smoothing splines

8.6.1 The smoothing spline of Fisher-Nychka-Zervos

A spline is a special function defined piecewise by polynomials, which, in interpolating problems, is often preferred to polynomial interpolation because it yields similar results even when using low-degree polynomials. Spline have constraints imposed to ensure that the overall term structure is continuous and smooth. This contrasts with the parametric approach which specifies a single functional form to describe the entire term structure. The ability of the individual segments of the spline curve to move to some degree independently of one another (subject to the continuity and smoothness constraints) gives rise to the superior performance of the spline with respect to the parametric methods.

Specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline; thus, the minimization problem can be stated as

$$\min_{f(h;\theta)} \left[\sum_{n=1}^{N} \left(\frac{B_n^+ - \widehat{B}_n^+(f(h;\theta))}{D_n} \right)^2 + \int_0^T \lambda(h) \cdot \left[f''(h;\theta) \right]^2 dh \right]. \tag{13}$$

The first term is the difference between the observed price B_n^+ of the n-th bond and the estimated price, $\widehat{B}_n^+(f(h;\theta)) = \sum_{h=1}^{H} \delta(h;\theta) \cdot C_h$, weighted by its duration, D, summed over all bonds in our data set; the second term is the penalty term, with $f(h;\theta)$ being the cubic spline of the forward-rate function, whose argument is the maturity h and parameters the vector β, and $\lambda(h)$ is the penalty function.

[12] The Svensson model augments the Nelson-Siegel model by introducing two parameters which allow more flexibility in the shape of the curve, in particular by allowing the existence of double humps. Given the vector of parameters $\theta_{SV} = [\tau_1, \tau_2, \beta_0, \beta_1, \beta_2, \beta_3]'$, the zero-coupon interest rate is defined by

$$y(h; \theta_{SV}) = \beta_0 + \beta_1(1 - e^{\frac{-h}{\tau_1}})(-\frac{\tau_1}{h}) + \beta_2((1 - e^{\frac{-h}{\tau_1}})\frac{\tau_1}{h} - e^{\frac{h}{\tau_1}}) + \beta_3((1 - e^{\frac{-h}{\tau_2}})\frac{\tau_2}{h} - e^{\frac{h}{\tau_2}}).$$

However, this method performs very poorly given the low number of bonds in each period. Moreover, the double-hump case is very rare in the real term structure, which is in general monotonic.

Fisher et al. (1995) use $\lambda(h)$ constant across maturity but time-varying — in the sense that its value is computed on a daily basis — and the penalty function is defined by

$$\lambda \int_0^T \left(\frac{\partial^2 f(h;\theta)}{\partial h^2}\right)^2 dh = \lambda\theta^\intercal \left(\int_0^T f''(h;\theta)^\intercal f''(h;\theta) dh\right)\theta = \lambda \cdot \theta^\intercal \cdot \mathcal{H} \cdot \theta,$$

where β is the vector of parameters of the spline, $\mathcal{H}$ is a diagonal matrix defined by the structure of the spline basis. Since any θ that makes $f(h;\theta)$ linear in h is not penalized, $\mathcal{H}$ has two zero eigenvalues. By defining $\Pi(\theta(\lambda)) = \widehat{B}_n^+(f(h;\theta))$, the vector of parameters which minimizes (13) is given by

$$\theta^*_{FNZ}(\lambda) = \arg\min \left[\left(\frac{B^+ - \Pi(\theta(\lambda))}{D}\right)^\intercal \left(\frac{B^+ - \Pi(\theta(\lambda))}{D}\right) + \lambda \cdot \theta(\lambda)^\intercal \cdot H \cdot \theta(\lambda)\right],$$

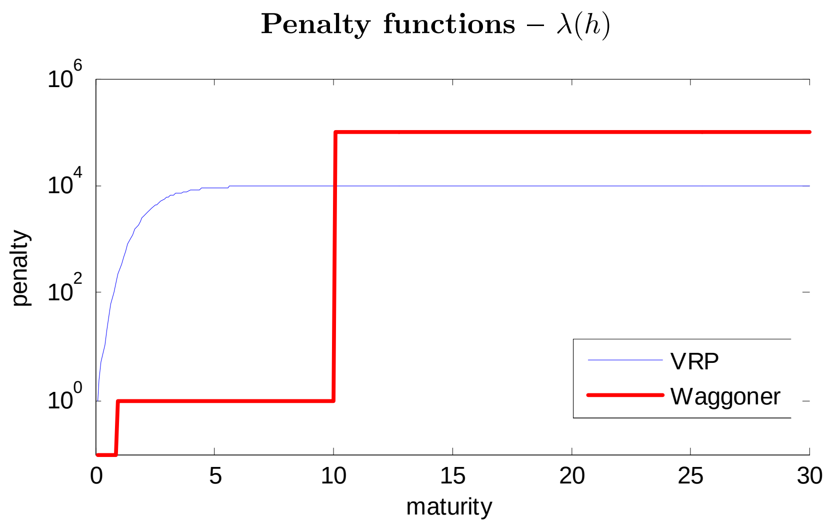
and, in general, the minimizer $\theta^*_{FNZ}(\lambda)$ is found by non-linear least squares.

8.6.2 The Variable-Roughness Penalty (VRP) smoothing spline

The Variable-Roughness Penalty (VRP), introduced by Waggoner (1997) and refined by Anderson and Sleath (2001), is a smoothing spline approach where the pricing error is weighted by the reciprocal of the bond duration, D, and the penalty function $\lambda(h)$ is increasing in the bond maturity. The minimization problem is identical to equation (13) but with the penalty function $\lambda(h)$ given by

$$\ln(\lambda(h)) = L - (L - S)e^{-\frac{h}{\mu}}, \tag{14}$$

where L, S, μ are three parameters typically estimated from historical data. The penalty function works in a way that curvature at any maturity is not penalized equally. Since the yield curve tends to have much more curvature at the short than at the long end, the penalty function is decreasing in h and assigns smaller weights to shorter maturities. A similar way to choose a penalty function which varies with the maturity is adopted by Waggoner (1997); however, Waggoner (1997) chooses only three weights according to the segmentation of the US market into bills, notes and treasuries. The penalty function of the VRP approach is shown below.



According to Anderson and Sleath (2001), the VRP smoothing spline methodology outperforms the other non-parametric methodologies for several reasons. The VRP shares with the other smoothing spline methods the possibility of i) obtaining smooth forward curves which supply a measure of market expectations for monetary policy purposes; ii) a large flexibility which allows movements in the underlying term structure to be captured and iii) stability in the sense that small changes in data at one maturity (such as at the very long end) do not have a disproportionate effect on forward rates at other maturities. With respect to other smoothing spline, the VRP has the advantage of being relatively less flexible at the long end than at shorter maturities, where expectations are likely to be better defined.

8.6.3 The quadratic-natural spline of McCulloch-Kochin

McCulloch and Kochin (2000) introduce the quadratic-natural spline to construct zero-coupon equivalents for index-linked yields, which should outperform other methods as it is designed to work with yield data that are only available for few maturities.[13] The McCulloch-Kochin spline is based on a discount function linear in the unknown parameters a of the form

$$\delta(h; a) = \exp\left[-\sum_{j=1}^{n} a_i \psi_i(h)\right], \tag{15}$$

where h is the time to maturity and n is the number of maturities available from the data, while $\psi_i(h)$-s are splines defined by

$$\psi_i(h) = \vartheta_j(h) - \frac{\vartheta_j''(h_n)}{\vartheta_{n+1}''(h_n)}\vartheta_{n+1}(h), \quad j = 1, ..., n,$$

[13] The author is indebted to Professor McCulloch for providing the GAUSS programs to implement this estimation method. Estimation has been done with translated Matlab codes. See http://economics.sbs.ohio-state.edu/jhm/ts/ts.html for estimates of the US real term structure.

and the functions $\vartheta_j(h)$ are given by

$$\begin{aligned}\vartheta_1(h) &= h \\ \vartheta_2(h) &= h^2 \\ \vartheta_j(h) &= \max(0, h - h_{j-2})^3,\ j = 3, ..., n+1.\end{aligned}$$

The log discount function defined by (15) is a quadratic-natural spline function linear in the parameters a; when you plug (15) into equation (10), the latter can be easily solved numerically by iteratively evaluating coupons using last iteration, subtracting from bond prices and fitting residual principal values until yields converge.

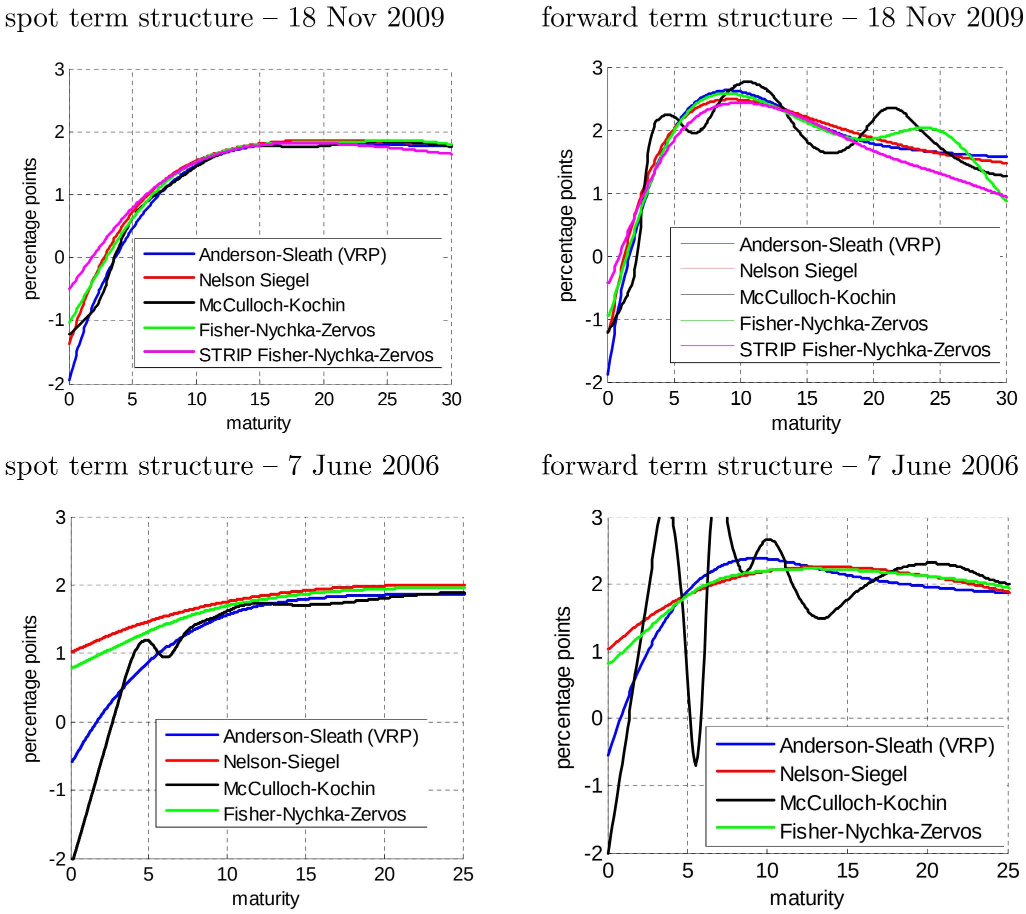
8.7 Comparison of methodologies

Basically, there are three forces that shape the term structure: (1) expectations, (2) risk premia, and (3) convexity. Roughly speaking, risk premia are linear in maturity and tend to raise yields, while convexity is quadratic in maturity and tends to lower yields. Both effects tend to be larger with greater uncertainty. The kind of curvature found in the spline forward-rate estimates, and in particular in the Fisher-Nychka-Zervos forward rates, is capturing those two effects; in fact, the convexity component only becomes significant after the 15- or 20-year maturity. Alternatively, one can directly observe the convexity implied in the yields on STRIPs, which are zero-coupon rates; convexity cannot be seen in coupon yields because they are averages of zero rates.

In addition to this consideration, it must be said that, by and large, when the number of bonds is small a parsimonious model can sometimes converge with great difficulty. However, under normal circumstances, all the methodologies presented tend to give similar results for the term structure of spot rates while giving different results for the term structure of the forward rates. The main differences are due to the fact that the non-linear models (such as the Nelson-Siegel and Svensson models) impose an asymptote on the spot curve and the McCulloch-Kochin model imposes an asymptote on the curve shape. Conversely, the Fisher-Nychka-Zervos and the VRP smoothing splines are more flexible and, thus, can give better information on long-term interest-rate expectations.

Figure 10 reports the term structure for real spot and real forward rates on 18 November 2009 and on 7 June 2006 computed with different methods. All in all, the real spot term structure is very different in levels between maturities zero and 10 years, while it converges to a single figure thereafter. The Fisher-Nychka-Zervos term structure obtained from STRIPs – available only after August 2009 – is much more bent at the long end thanks to the convexity effect stemming from the separate trading of zero-coupon bonds. Conversely, the real forward term structures differ substantially. In particular, the McCulloch-Kochin term structure shows large humps over the entire maturity spectrum, while the Fisher-Nychka-Zervos term structure is very stable until the 20-year maturity and converges around the figures recorded by the Anderson-Sleath, the McCulloch-Kochin and the Nelson-Siegel estimates; incidentally, on 18 November 2009, the forward Fisher-Nychka-Zervos term structure converges to the long-term forward rate obtained from the same methodology applied to STRIPs. The Anderson-Sleath and the Nelson-Siegel forward term structures behave quite similarly at the long end. The forward Fisher-Nychka-Zervos term structure implied by STRIPs is the most bent at the long end thanks to the convexity effect which is averaged out in the other estimates. By comparing the spot and forward term structure it appears that the Fisher-Nychka-Zervos and the Nelson-Siegel have very similar features over the two dates; on the other hand, the Anderson-Sleath and the McCulloch methods bend the term structure excessively at the short end, both for the spot and the forward rates.

Figure 10 – Spot and forward real term structures



8.7.1 Pricing errors

A standard way to compare term structure models is the computation of in-sample and out-of-sample performance measures across estimation methods for various subsets and subperiods. The in-sample performance is evaluated by examining the ability of five estimation methods to fit bond prices and is measured by the mean absolute fitted-price errors (MAEs) and by the mean absolute fitted-price errors weighted by the bond duration (WMAEs); Table 4 reports the MAEs and the WMAEs over different time samples and over different maturity samples for several methodologies, namely the Fisher-Nychka-Zervos (FNZ), the

McCulloch-Kochin (MC), the Variable-Roughness-Penalty spline (VRP), the Nelson-Siegel (NS) and the Monotonic model (MO).

The out-of-sample performance (also defined cross-validation by the literature) of the term structure models is evaluated as the MAEs and as the WMAEs over the issues excluded from the subsample used to estimate the underlying term structure; the out-of-sample MAEs and WMAEs are the averages of the pricing errors computed for each traded bond which is left out of the estimation of the term structure;[14] analogously to Table 4, Table 5 reports the out-of-sample MAEs and the WMAEs for several models.[15]

As far as the in-sample pricing errors are concerned (Table 4), the Fisher-Nychka-Zervos model outperforms the other models. When you consider the entire maturity-sample, i.e. 6-month – 30-year, the MAE is equal to 0.287 (average error in basis points) over the full sample 2001-09; the same ranking in MAEs holds for the other maturity samples and for the other time samples. The same ranking among models holds for the WMAEs.

As far as the out-of-sample MAEs are concerned (Table 5), the Fisher-Nychka-Zervos is consistently outperformed by the Variable-Roughness-Penalty spline for bonds with maturity over 10 years but outperforms the latter for shorter maturities over the entire sample and in the period 2004-09. When you consider the WMAEs, the Fisher-Nychka-Zervos is always outperformed in the first subsample, 2001-03, over the entire maturity samples, while it is outperformed over the second subsample, 2004-09, and for the entire period only for bonds with maturity between 10 and 30 years.

In particular, the Variable-Roughness-Penalty spline model performs very poorly at the short-term end of the real term structure, 6-month – 5-year, both in-sample and out-of-sample, while it has a better out-of-sample performance in the bracket 10-year – 30-year, where the number of outstanding issues is scarce. All in all, the Fisher-Nychka-Zervos model gives a better out-of-sample performance. Moreover, it can be shown that, among the best performers in terms of pricing errors, the Fisher-Nychka-Zervos model is much faster than the other methodologies.

[14] In the out-of-sample performance test, pricing erros are not computed for the bonds with the shortest and with the longest maturity.

[15] A simple bootstrap estimation of the real term structure reveals good in-sample MAEs and poor out-of-sample MAEs and WMAEs.

Table 4 – In-sample performance of term structure models

	Nov-2001 / Dec-2009				Nov-2001 / Dec-2003				Jan-2004 / Dec-2009			
	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]
FNZ (*MAE*)	0.258	0.356	0.249	0.287	0.128	0.339	0.132	0.231	0.263	0.364	0.286	0.304
MC (*MAE*)	0.631	0.556	0.665	0.621	0.835	0.702	0.763	0.736	0.623	0.485	0.632	0.585
VRP (*MAE*)	0.421	0.439	0.734	0.573	0.326	0.416	0.752	0.578	0.424	0.450	0.728	0.571
NS (*MAE*)	0.516	0.758	1.074	0.857	1.853	1.004	1.485	1.266	0.468	0.642	0.945	0.738
MO (*MAE*)	0.703	1.462	1.641	1.396	2.759	2.360	2.581	2.480	0.629	1.037	1.346	1.078
FNZ (*WMAE*)	0.088	0.052	0.020	0.044	0.028	0.047	0.010	0.028	0.091	0.055	0.023	0.048
MC (*WMAE*)	0.187	0.079	0.039	0.080	0.165	0.094	0.042	0.071	0.188	0.072	0.038	0.083
VRP (*WMAE*)	0.142	0.067	0.047	0.073	0.073	0.058	0.048	0.053	0.145	0.072	0.047	0.078
NS (*WMAE*)	0.202	0.110	0.068	0.109	0.414	0.142	0.091	0.125	0.194	0.095	0.061	0.104
MO (*WMAE*)	0.249	0.223	0.100	0.171	0.616	0.355	0.154	0.264	0.236	0.160	0.083	0.144

Note: The Table reports the pricing errors for different term structure estimates. The pricing error is defined as (i) the mean absolute pricing error (*MAE*) and as (ii) the duration-weighted mean absolute pricing error (*WMAE*). The pricing errors are computed for bonds with maturity longer than 6 months and shorter than 5 years (6m-5y], longer than 5 years and shorter than 10 years (5y-10], larger than 10 years and shorter than 30 years (10y-30) and for the entire sample (6m-30). Pricing errors are computed for the entire period and for two subperiods. The FNZ refers to the Fisher-Nychka-Zervos model (13), MC to the McCulloch-Kochin model (15), VRP to Variable-Roughness Spline model (14), NS to the Nelson-Siegel model (11), MO to the monotonic model (12). For example, 0.258 in the first cell means that the FNZ model has on average an absolute in-sample pricing error of 0.258 ticks for bonds with maturity between 6 months and 5 years for the entire sample period Nov-2001/Dec-2009.

Table 5 – Out-of-sample performance of term structure models

	Nov-2001 / Dec-2009				Nov-2001 / Dec-2003				Jan-2004 / Dec-2009			
	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]	(6m-5y]	(5y-10y]	(10y-30y)	(6m-30y]
FNZ (*MAE*)	0.497	0.692	1.599	1.025	NaN	1.104	2.603	1.694	0.497	0.575	1.411	0.897
MC (*MAE*)	0.725	1.096	1.695	1.263	NaN	1.421	2.956	1.957	0.725	0.984	1.456	1.115
VRP (*MAE*)	0.891	0.719	1.341	0.992	NaN	1.084	2.200	1.525	0.891	0.617	1.178	0.893
NS (*MAE*)	1.885	1.636	1.885	1.777	NaN	2.954	2.890	2.929	1.885	1.269	1.694	1.562
MO (*MAE*)	1.824	2.333	1.733	2.010	NaN	3.917	3.431	3.725	1.824	1.888	1.411	1.688
FNZ (*WMAE*)	0.132	0.093	0.108	0.105	NaN	0.138	0.158	0.146	0.132	0.080	0.099	0.097
MC (*WMAE*)	0.196	0.148	0.117	0.144	NaN	0.178	0.184	0.180	0.196	0.137	0.105	0.137
VRP (*WMAE*)	0.259	0.101	0.092	0.124	NaN	0.137	0.132	0.135	0.259	0.090	0.084	0.122
NS (*WMAE*)	0.527	0.233	0.127	0.243	NaN	0.385	0.170	0.299	0.527	0.190	0.119	0.232
MO (*WMAE*)	0.519	0.334	0.115	0.280	NaN	0.506	0.201	0.385	0.519	0.286	0.099	0.260

Note: The out-of-sample performance of the term structure models is evaluated as the MAEs and as the WMAEs over the issues excluded from the subsample used to estimate the underlying term structure. Pricing errors are computed when the number of bonds is larger than 4. See note to Table 4.

9 Figures

Figure 1 – Term structure of real zero-coupon rates (daily data)

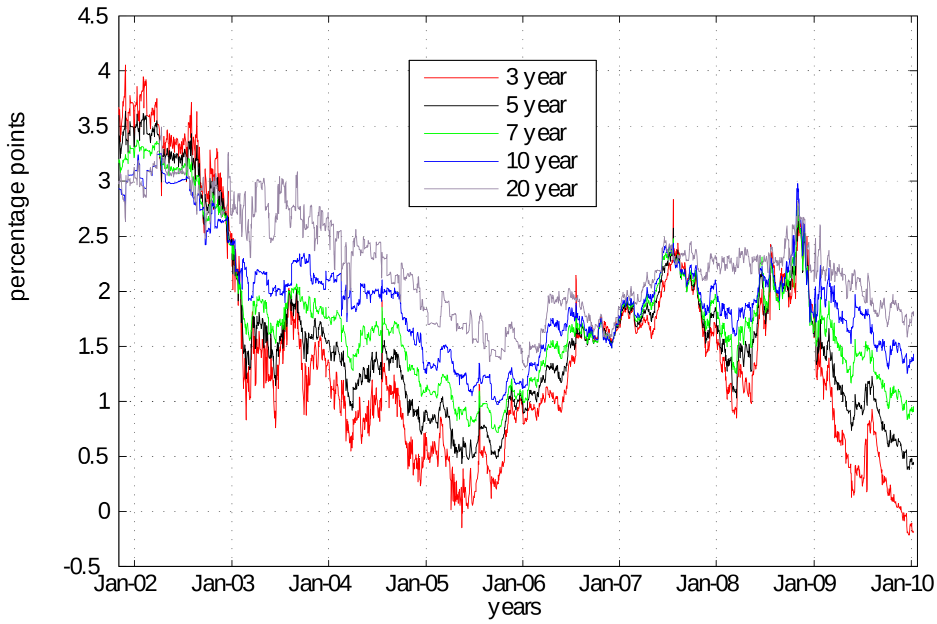
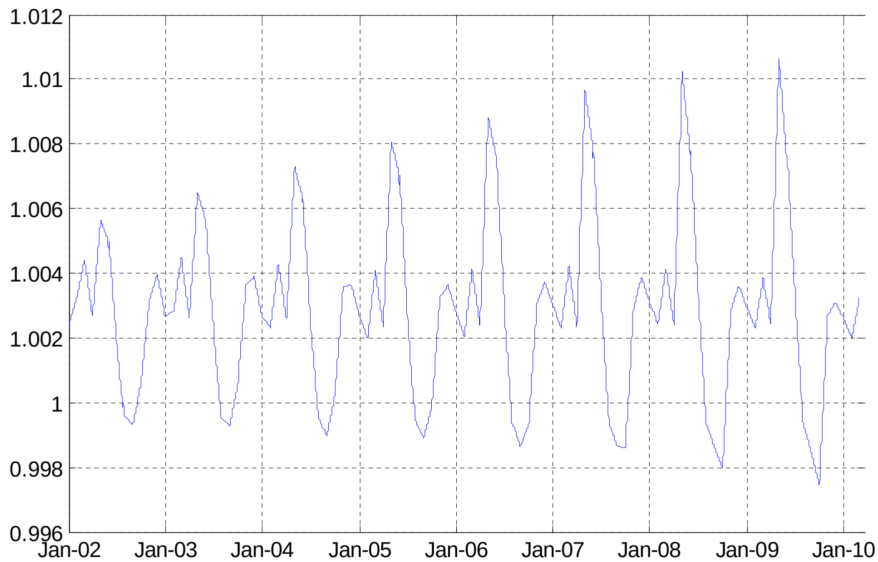
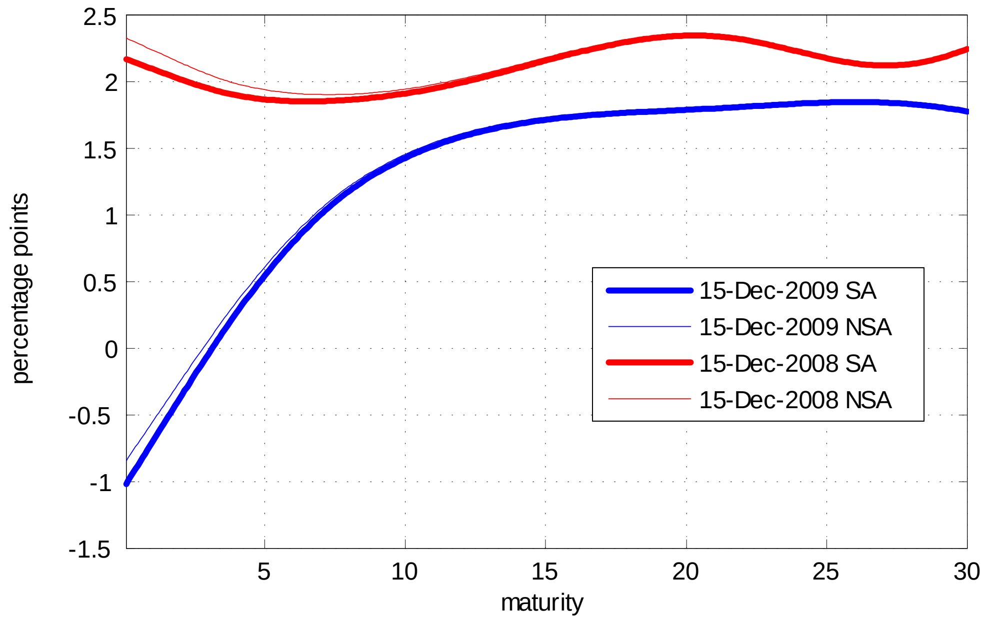


Figure 2 – Daily correction factor for the seasonality of the euro-area HICP



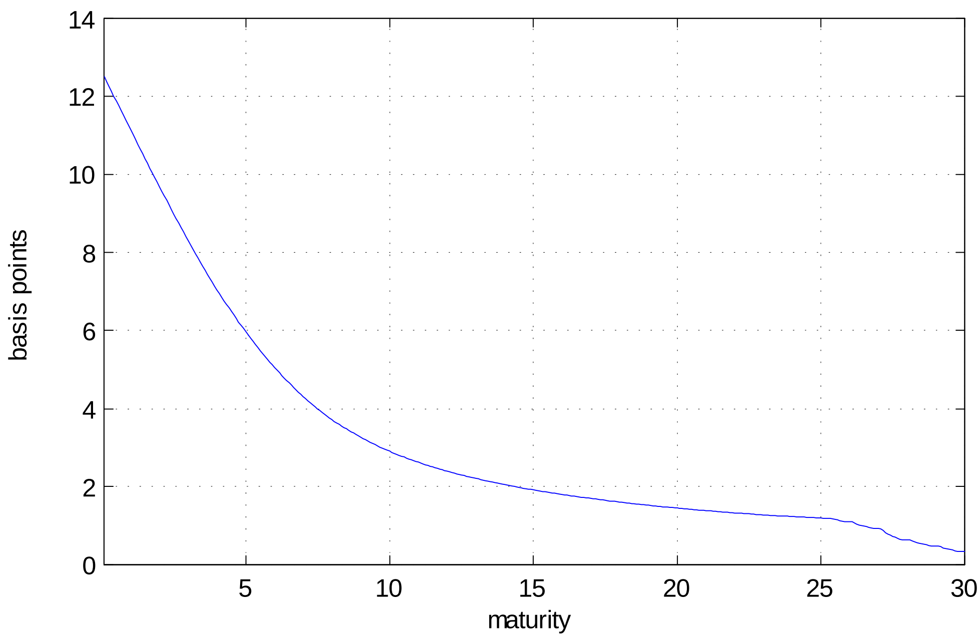
Derived from the multiplicative seasonal component of the euro-area HICP (ex. tobacco) estimated with the X12-ARIMA methodology.

Figure 3 – Seasonally adjusted and not-adjusted term structure



The bold lines show the not-seasonally-adjusted (NSA) real term structure, the thin line the seasonally-adjusted (SA) term structure.

Figure 4 – Average differences due to correction for HICP seasonality



Average difference in basis points between the daily not-seasonally-adjusted term structure and the seasonally-adjusted term structure. Sample: 1 November 2001 – 31 December 2009.

Figure 5 – Term structure of breakeven inflation rates (daily data)

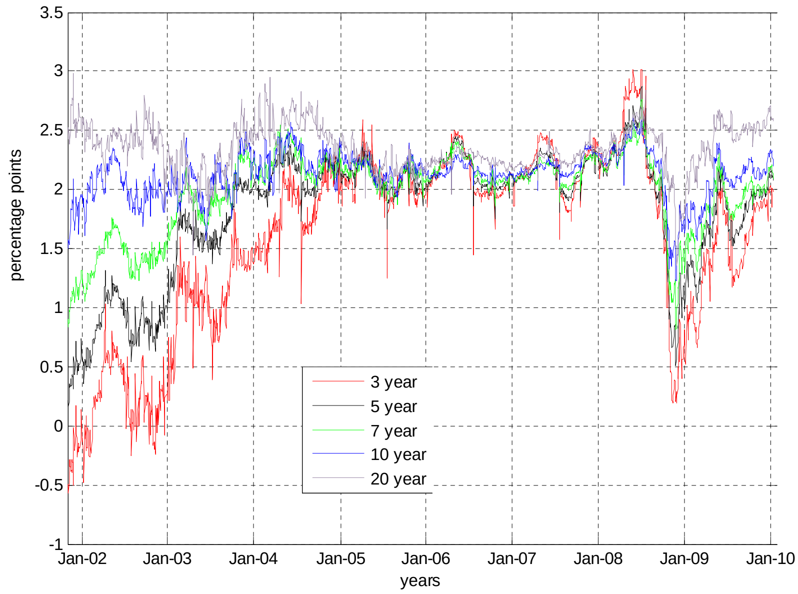
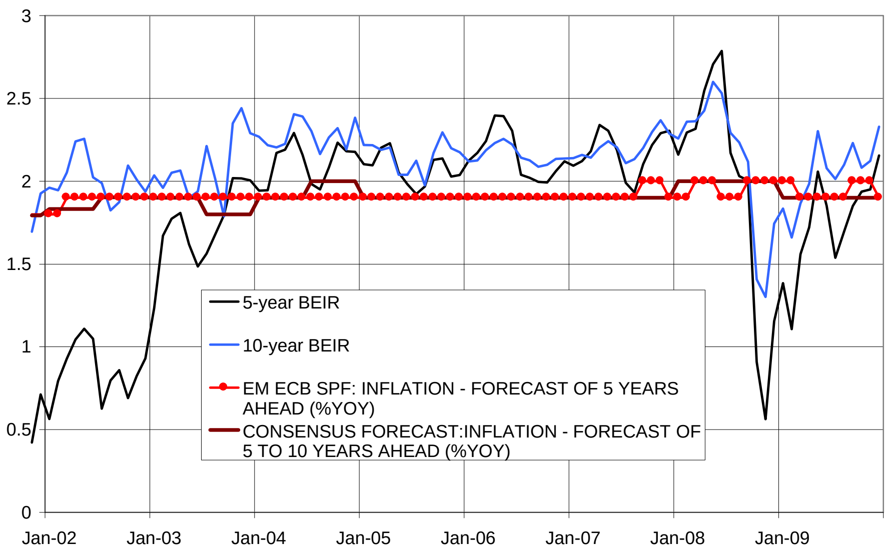
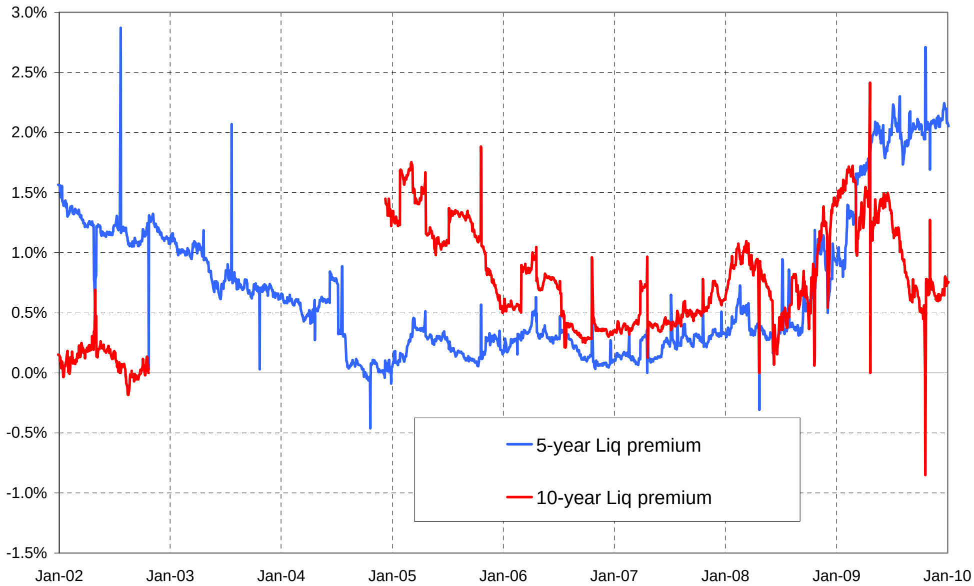


Figure 6 – BEIR and inflation forecast 5 and 10 years ahead (monthly and quarterly data)



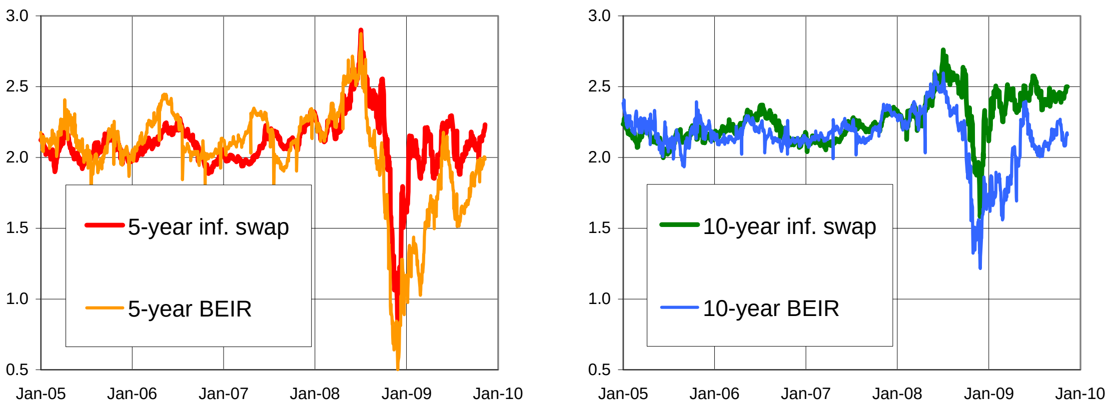
Note: Data from the Survey of Professional Forecasters are quarterly; data from Consensus Economics are half-yearly; data for the BEIRs are end-of-month

Figure 7 – Liquidity premia in the French government market



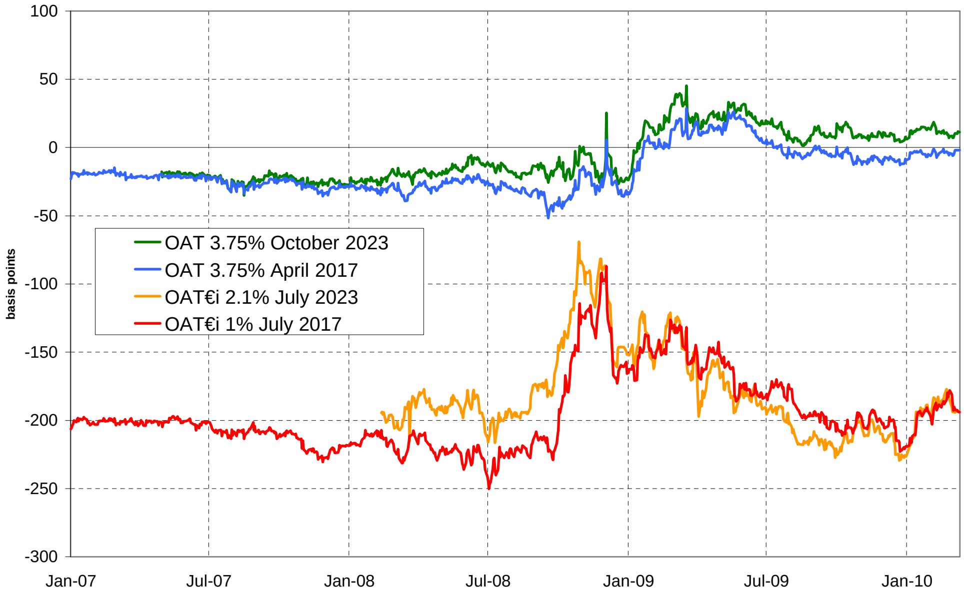
The risk premium for liquidity is computed as the difference between the zero-coupon rate on the CADES bonds and the corresponding rate on the French OATs. The 10-year zero-coupon rate on the CADES is missing from end-2002 to end-2004 due to the scarcity of bonds with longer maturity.

Figure 8 – BEIR implied in OAT€i and in zero-coupon inflation swaps



Source: Bloomberg and Thomson Financial Reuters.

Figure 9 – Asset-swap spreads of selected OAT and OAT€i



In an asset swap, one party pays the cash flows on a specific bond and receives in exchange the LIBOR plus a spread known as the asset-swap spread. Typically, this spread is negative and its absolute magnitude is larger for OAT€i than for OAT.

10 Tables

Table 1a - Statistics for daily zero-coupon rates (1)

	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ_1^a	ρ_5^a	ρ_{20}^a	ρ_{60}^a	ρ_{120}^a	ρ_{250}^a
ILB-3y	1.48	1.63	0.55	14.38	0.99	0.97	0.88	0.66	0.38	-0.08
ILB-5y	1.65	0.99	0.67	12.03	0.99	0.97	0.89	0.69	0.40	-0.04
ILB-7y	1.79	0.71	0.54	11.24	0.99	0.98	0.90	0.71	0.43	0.02
ILB-10y	1.96	0.68	-0.32	14.04	0.99	0.97	0.91	0.74	0.48	0.14
ILB-15y	2.14	0.89	-0.08	23.43	0.99	0.97	0.90	0.78	0.56	0.28
ILB-20y	2.25	0.97	-0.13	17.46	0.99	0.96	0.90	0.80	0.62	0.34
ILB-25y	2.29	0.86	0.42	23.09	0.99	0.97	0.92	0.80	0.63	0.38
nom-3y	3.04	1.13	1.28	32.07	0.99	0.98	0.94	0.77	0.47	0.00
nom-5y	3.45	0.82	0.54	9.93	0.99	0.98	0.92	0.70	0.34	-0.13
nom-7y	3.75	0.79	0.34	6.97	0.99	0.97	0.90	0.67	0.31	-0.13
nom-10y	4.08	0.73	0.33	12.31	0.99	0.98	0.92	0.72	0.42	0.03
nom-15y	4.41	0.67	-0.04	28.06	1.00	0.98	0.94	0.81	0.59	0.25
nom-20y	4.59	0.73	-0.32	24.43	0.99	0.98	0.93	0.81	0.62	0.33
nom-25y	4.60	0.76	0.20	7.53	0.99	0.97	0.92	0.79	0.58	0.31

Table 1b - Statistics for weekly zero-coupon rates (2)

	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ_1^a	ρ_2^a	ρ_4^a	ρ_{13}^a	ρ_{26}^a	ρ_{52}^a
ILB-3y	1.48	1.27	0.26	7.90	0.96	0.94	0.88	0.63	0.33	-0.10
ILB-5y	1.65	0.83	0.09	6.11	0.97	0.95	0.90	0.66	0.36	-0.06
ILB-7y	1.79	0.65	-0.24	6.79	0.97	0.95	0.90	0.68	0.38	-0.00
ILB-10y	1.95	0.65	-0.47	7.02	0.97	0.95	0.90	0.72	0.42	0.12
ILB-15y	2.13	0.78	-0.09	8.48	0.96	0.94	0.89	0.77	0.51	0.27
ILB-20y	2.24	0.83	0.05	6.78	0.96	0.94	0.90	0.78	0.58	0.33
ILB-25y	2.30	0.67	-0.21	7.61	0.97	0.96	0.92	0.79	0.60	0.36
nom-3y	3.03	0.84	-0.21	4.65	0.98	0.97	0.94	0.75	0.42	-0.02
nom-5y	3.45	0.75	0.07	3.71	0.98	0.96	0.92	0.67	0.28	-0.15
nom-7y	3.75	0.72	0.20	3.57	0.97	0.95	0.90	0.64	0.25	-0.16
nom-10y	4.08	0.64	0.26	3.73	0.98	0.96	0.91	0.70	0.37	0.00
nom-15y	4.41	0.58	0.46	7.20	0.98	0.97	0.94	0.79	0.55	0.23
nom-20y	4.58	0.67	0.48	11.35	0.98	0.96	0.93	0.79	0.59	0.31
nom-25y	4.60	0.76	0.18	8.48	0.97	0.95	0.92	0.77	0.54	0.28

Table 1c - Statistics for monthly zero-coupon rates (3)

	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ_1^a	ρ_2^a	ρ_3^a	ρ_6^a	ρ_9^a	ρ_{12}^a
ILB-3y	1.47	1.01	-0.22	3.01	0.87	0.74	0.61	0.31	0.01	-0.13
ILB-5y	1.64	0.77	-0.41	4.64	0.88	0.76	0.65	0.33	0.06	-0.08
ILB-7y	1.78	0.67	-0.34	6.58	0.88	0.77	0.68	0.36	0.12	-0.01
ILB-10y	1.95	0.63	-0.07	7.31	0.88	0.77	0.71	0.40	0.21	0.11
ILB-15y	2.14	0.63	0.25	5.80	0.88	0.79	0.76	0.50	0.33	0.26
ILB-20y	2.24	0.62	0.24	4.50	0.88	0.81	0.77	0.56	0.40	0.30
ILB-25y	2.28	0.53	0.23	3.19	0.91	0.84	0.78	0.58	0.44	0.33
nom-3y	3.04	0.71	-0.45	3.72	0.93	0.84	0.74	0.41	0.17	-0.02
nom-5y	3.44	0.66	-0.15	2.81	0.91	0.79	0.67	0.27	0.02	-0.16
nom-7y	3.75	0.65	-0.12	2.89	0.89	0.75	0.64	0.24	0.00	-0.18
nom-10y	4.07	0.61	-0.01	3.08	0.90	0.78	0.70	0.36	0.15	-0.01
nom-15y	4.41	0.54	0.04	3.77	0.93	0.84	0.79	0.54	0.36	0.20
nom-20y	4.57	0.54	-0.55	6.77	0.92	0.84	0.80	0.59	0.44	0.28
nom-25y	4.58	0.58	-0.54	5.67	0.92	0.83	0.78	0.55	0.41	0.26

(1) $2,131$ daily data from 1 November 2001 to 31 December 2009; (2) 426 weekly data from Wednesday 5 November 2001 to Wednesday 28 December 2009; (3) 110 end-of-month data from 30 November 2001 to 28 December 2009; a) statistics for the level of zero-coupon interest rates; b) in annual terms; c) statistics for the first differences of zero-coupon interest rates; μ is the arithmetic mean; σ is the standard deviation, σ_3/μ_3 the skewness, σ_4/μ_4 the uncentred kurtosis, ρ_i is the autocorrelation coefficient at lag i.

Table 2a - Statistics for daily inflation swap rates (ILS) and BEIR (1)										
	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ^a_1	ρ^a_5	ρ^a_{20}	ρ^a_{60}	ρ^a_{120}	ρ^a_{250}
ILS-1y	1.86	1.67	-0.22	15.66	0.98	0.96	0.88	0.59	0.22	-0.04
ILS-2y	1.98	0.89	-0.86	21.14	0.99	0.97	0.86	0.53	0.12	-0.12
ILS-3y	2.04	0.69	-0.06	11.39	0.99	0.97	0.84	0.48	0.07	-0.16
ILS-4y	2.09	0.61	0.01	14.06	0.99	0.96	0.81	0.44	0.02	-0.21
ILS-5y	2.13	0.50	0.10	18.53	0.99	0.96	0.78	0.40	-0.03	-0.25
ILS-6y	2.16	0.53	-0.23	17.46	0.99	0.95	0.76	0.36	-0.07	-0.27
ILS-7y	2.19	0.52	-1.20	20.46	0.98	0.94	0.76	0.34	-0.10	-0.27
ILS-8y	2.22	0.48	-1.14	21.98	0.98	0.93	0.77	0.33	-0.09	-0.25
ILS-9y	2.24	0.44	-0.85	16.24	0.98	0.94	0.78	0.36	-0.05	-0.24
ILS-10y	2.26	0.39	-0.35	9.16	0.99	0.94	0.79	0.39	-0.01	-0.23
BEIR-1y	1.85	2.65	-0.27	21.08	0.97	0.94	0.86	0.55	0.20	0.02
BEIR-2y	1.90	2.03	-0.30	21.93	0.98	0.95	0.86	0.52	0.15	0.01
BEIR-3y	1.96	1.52	-0.39	21.99	0.98	0.95	0.85	0.49	0.10	-0.02
BEIR-4y	2.00	1.15	-0.51	20.34	0.98	0.96	0.84	0.47	0.06	-0.05
BEIR-5y	2.03	0.92	-0.56	17.10	0.98	0.96	0.83	0.46	0.02	-0.09
BEIR-6y	2.06	0.81	-0.51	14.69	0.98	0.95	0.82	0.45	-0.00	-0.12
BEIR-7y	2.08	0.77	-0.47	14.85	0.98	0.94	0.81	0.42	-0.03	-0.16
BEIR-8y	2.11	0.75	-0.41	15.65	0.98	0.93	0.79	0.40	-0.06	-0.20
BEIR-9y	2.13	0.72	-0.27	16.21	0.97	0.92	0.77	0.37	-0.10	-0.23
BEIR-10y	2.16	0.71	-0.06	17.15	0.97	0.91	0.75	0.35	-0.13	-0.25

Table 2b - Statistics for weekly inflation swap rate (ILS) and BEIR (2)										
	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ^a_1	ρ^a_2	ρ^a_4	ρ^a_6	ρ^a_{13}	ρ^a_{52}
ILS-1y	1.85	0.53	-0.22	6.07	0.96	0.94	0.87	0.56	0.21	0.23
ILS-2y	1.98	0.40	-0.43	7.88	0.96	0.93	0.86	0.49	0.10	0.12
ILS-3y	2.04	0.34	-0.76	11.12	0.96	0.93	0.83	0.44	0.05	0.08
ILS-4y	2.09	0.30	-1.26	13.85	0.96	0.92	0.81	0.40	-0.01	0.03
ILS-5y	2.13	0.26	-1.29	12.85	0.95	0.91	0.78	0.37	-0.05	-0.02
ILS-6y	2.16	0.28	-1.31	15.38	0.93	0.88	0.75	0.32	-0.10	-0.07
ILS-7y	2.19	0.23	-1.18	12.91	0.93	0.88	0.74	0.30	-0.13	-0.10
ILS-8y	2.22	0.21	-1.46	13.92	0.93	0.88	0.76	0.30	-0.13	-0.10
ILS-9y	2.24	0.20	-1.40	12.69	0.93	0.88	0.77	0.32	-0.09	-0.07
ILS-10y	2.26	0.19	-1.40	15.70	0.93	0.88	0.78	0.34	-0.06	-0.03
BEIR-1y	1.85	0.85	-0.74	11.96	0.94	0.92	0.85	0.51	0.16	0.21
BEIR-2y	1.90	0.68	-0.92	12.48	0.94	0.92	0.85	0.49	0.12	0.17
BEIR-3y	1.96	0.54	-1.01	11.41	0.95	0.92	0.84	0.46	0.07	0.12
BEIR-4y	2.00	0.44	-0.92	8.94	0.95	0.92	0.83	0.44	0.02	0.07
BEIR-5y	2.03	0.37	-0.74	6.69	0.95	0.91	0.82	0.43	-0.01	0.04
BEIR-6y	2.05	0.33	-0.58	5.87	0.94	0.91	0.82	0.41	-0.04	0.01
BEIR-7y	2.08	0.31	-0.46	5.93	0.94	0.90	0.81	0.39	-0.07	-0.02
BEIR-8y	2.10	0.29	-0.37	5.94	0.93	0.89	0.79	0.37	-0.11	-0.05
BEIR-9y	2.13	0.28	-0.25	5.75	0.92	0.87	0.78	0.34	-0.15	-0.09
BEIR-10y	2.15	0.27	-0.08	5.46	0.91	0.85	0.76	0.32	-0.19	-0.13

Table 2c - Statistics for monthly inflation swap rate (ILS) and BEIR (3)

	μ^a	$\sigma^{b,c}$	$\sigma_3/\mu_3{}^c$	$\sigma_4/\mu_4{}^c$	ρ_1^a	ρ_2^a	ρ_3^a	ρ_6^a	ρ_{12}^a	ρ_{24}^a
ILS-1y	1.86	1.11	-1.54	9.42	0.86	0.69	0.55	0.18	-0.05	-0.09
ILS-2y	1.99	0.97	-1.63	10.23	0.82	0.62	0.47	0.07	-0.12	-0.03
ILS-3y	2.05	0.88	-1.78	11.03	0.79	0.55	0.41	-0.00	-0.14	0.00
ILS-4y	2.09	0.78	-2.26	14.79	0.77	0.51	0.39	-0.04	-0.20	0.06
ILS-5y	2.13	0.72	-1.77	13.58	0.75	0.48	0.37	-0.08	-0.24	0.11
ILS-6y	2.17	0.65	-1.42	13.82	0.70	0.38	0.30	-0.11	-0.23	0.16
ILS-7y	2.19	0.58	-1.75	17.38	0.70	0.37	0.27	-0.14	-0.23	0.18
ILS-8y	2.22	0.52	-1.55	15.60	0.69	0.34	0.25	-0.12	-0.23	0.18
ILS-9y	2.24	0.46	-1.82	15.32	0.72	0.39	0.28	-0.10	-0.25	0.18
ILS-10y	2.25	0.43	-2.19	18.14	0.72	0.42	0.32	-0.08	-0.26	0.18
BEIR-1y	1.86	1.29	-1.26	6.04	0.85	0.67	0.50	0.11	0.06	-0.14
BEIR-2y	1.91	1.13	-1.52	7.45	0.84	0.64	0.47	0.07	0.04	-0.10
BEIR-3y	1.96	1.00	-1.76	9.17	0.83	0.61	0.44	0.02	0.01	-0.05
BEIR-4y	2.00	0.88	-1.85	10.49	0.81	0.58	0.43	-0.01	-0.04	0.00
BEIR-5y	2.03	0.79	-1.70	10.75	0.80	0.56	0.42	-0.04	-0.08	0.05
BEIR-6y	2.06	0.71	-1.39	10.11	0.78	0.53	0.40	-0.06	-0.12	0.08
BEIR-7y	2.08	0.65	-1.10	9.35	0.77	0.51	0.39	-0.08	-0.16	0.11
BEIR-8y	2.11	0.60	-0.94	8.99	0.76	0.49	0.37	-0.11	-0.19	0.14
BEIR-9y	2.13	0.56	-0.95	9.14	0.75	0.48	0.36	-0.14	-0.22	0.16
BEIR-10y	2.16	0.52	-1.09	9.65	0.74	0.47	0.35	-0.17	-0.25	0.17

(1) $1,451$ daily data from 21 June 2004 to 31 December 2009; (2) 289 weekly data from Wednesday 23 June 2004 to Wednesday 30 December 2009; (3) 67 end-of-month data from 30 June 2004 to 28 December 2009; a) statistics for the level of zero-coupon interest rates; b) in annual terms; c) statistics for the first differences of zero-coupon interest rates; μ is the arithmetic mean; σ is the standard deviation, σ_3/μ_3 the skewness, σ_4/μ_4 the uncentred kurtosis, ρ_i is the autocorrelation coefficient at lag i.

Table 6.a – French government bonds (OAT) linked to
the French index (France CPI) and euro-area HICP

ISIN	Bond denomination	coupon rate	issue date	red. date	indexation
FR0000571424	OAT-I FRANCE	3.00	25/07/1998	25/07/2009	France CPI
FR0000186413	OAT-I FRANCE	3.40	25/07/1999	25/07/2029	France CPI
FR0000188013	OAT-EI FRANCE	3.00	25/07/2001	25/07/2012	euro-area HICP
FR0000188799	OAT-EI FRANCE	3.15	25/07/2002	25/07/2032	euro-area HICP
FR0000188955	OAT-I FRANCE	2.00	25/07/2002	25/07/2013	France CPI
FR0000188955	OAT-I FRANCE	2.50	25/07/2002	25/07/2013	France CPI
FR0010094375	OAT-I FRANCE	1.60	25/07/2003	25/07/2011	France CPI
FR0010050559	OAT-EI FRANCE	2.25	25/07/2003	25/07/2020	euro-area HICP
FR0010135525	OAT-EI FRANCE	1.60	25/07/2004	25/07/2015	euro-area HICP
FR0010235176	OAT-I FRANCE	1.00	25/07/2005	25/07/2017	France CPI
FR0010447367	OAT-EI FRANCE	1.80	25/07/2006	25/07/2040	euro-area HICP
FR0010585901	OAT-I FRANCE	2.10	25/07/2007	25/07/2023	France CPI

Table 6.b – German government bonds (Bund) linked to euro-area HICP

ISIN	Bond denomination	coupon rate	issue date	red. date	indexation
DE0001030518	BUNDESREPUB.DTL.	2.25	15/04/2007	15/04/2013	euro-area HICP
DE0001030500	BUNDESREPUB.DTL.	1.50	15/03/2006	15/04/2016	euro-area HICP
DE0001030526	BUNDESREPUB.DTL.	1.75	15/04/2009	15/04/2020	euro-area HICP

Table 7 – French OAT coupon and principal STRIPs linked to euro-area HICP

ISIN	Bond denomination	issue date	red. date
FR0010482331	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2007
FR0010482349	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2008
FR0010482364	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2009
FR0010482372	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2010
FR0010482380	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2011
FR0010482398	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2012
FR0010482406	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2013
FR0010482414	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2014
FR0010482422	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2015
FR0010482430	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2016
FR0010482448	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2017
FR0010482455	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2018
FR0010482463	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2019
FR0010482471	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2020
FR0010482497	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2021
FR0010482505	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2022
FR0010482513	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2023
FR0010482521	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2024
FR0010482539	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2025
FR0010482547	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2026
FR0010482554	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2027
FR0010482562	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2028
FR0010482570	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2029
FR0010482588	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2030
FR0010482596	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2031
FR0010482604	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2032
FR0010482612	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2033
FR0010482620	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2034
FR0010482638	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2035
FR0010482646	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2036
FR0010482653	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2037
FR0010482679	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2038
FR0010482695	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2039
FR0010482703	OAT FRANCE COUPON STRIP	07/09/2006	25/07/2040
FR0010482257	OAT FRANCE PRINCIPAL STRIP	25/07/2001	25/07/2012
FR0010482315	OAT FRANCE PRINCIPAL STRIP	25/07/2002	25/07/2032
FR0010482299	OAT FRANCE PRINCIPAL STRIP	25/07/2003	25/07/2020
FR0010482273	OAT FRANCE PRINCIPAL STRIP	25/07/2004	25/07/2015
FR0010482232	OAT FRANCE PRINCIPAL STRIP	25/07/2005	25/07/2010
FR0010482323	OAT FRANCE PRINCIPAL STRIP	25/07/2006	25/07/2040

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