A non-parametric model-based approach to uncertainty and risk analysis of macroeconomic forecasts

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A NON-PARAMETRIC MODEL-BASED APPROACH
TO UNCERTAINTY AND RISK ANALYSIS OF MACROECONOMIC FORECASTS

by Claudia Miani* and Stefano Siviero*

Abstract

It has increasingly become standard practice to supplement point macroeconomic forecasts with an appraisal of the degree of uncertainty and the prevailing direction of risks. Several alternative approaches have been proposed in the literature to compute the probability distribution of macroeconomic forecasts; all of them rely on combining the predictive density of model-based forecasts with subjective judgment about the direction and intensity of prevailing risks. We propose a non-parametric, model-based simulation approach, which does not require specific assumptions to be made regarding the probability distribution of the sources of risk. The probability distribution of macroeconomic forecasts is computed as the result of model-based stochastic simulations which rely on re-sampling from the historical distribution of risk factors and are designed to deliver the desired degree of skewness. By contrast, other approaches typically make a specific, parametric assumption about the distribution of risk factors. The approach is illustrated using the Bank of Italy’s Quarterly Macroeconometric Model. The results suggest that the distribution of macroeconomic forecasts quickly tends to become symmetric, even if all risk factors are assumed to be asymmetrically distributed.

JEL Classification: C14, C53, E37.
Keywords: macroeconomic forecasts, stochastic simulations, balance of risks, uncertainty, fan-charts.

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1 Introduction

Conveying an assessment of the risks inherent to macroeconomic forecasts is being increasingly recognized not simply as an embellishment to the projections but, rather, as an all-essential component of the projections themselves, particularly when they are meant to provide support to the policy-making process. Indeed, policymakers are not solely interested in knowing the most likely future evolution of the economy. Instead, they are very keen to know what are likely to be the main sources of risks\(^2\) to a given forecast; what impact those risks may have on the variables of interest, should they materialise; what probability may be assigned to the event that each of them actually takes place; whether and how those risks are inter-related. Indeed, if the goal of the forecasting process is not simply to “produce a take-it-or-leave-it figure,” but, rather, to track the “story behind the figures,”\(^3\) such goal would not be fully reached if the story was not supplemented by an appraisal of the uncertainty surrounding it and of the likely impact of the factors of risks on the outlook for the variables of interest.

The financial crisis that originated in 2007 and the widespread recession that followed have even more compellingly shown — one may argue — that forecasts to be used as an input in the policy decision-making process should be systematically accompanied by an assessment of the risks surrounding them, and that such risks should be carefully and properly pondered. The point is explicitly and forcefully made by Visco (2009), who argues that “[p]erhaps the lesson to be learnt from recent experience is that a forecast should always be given and taken in its entirety: so, not only the central figure, but also the evaluations of the main risks, their size and the likelihood of their realization.” This conclusion rests on the observation that

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\(^1\)This paper benefitted from very insightful and useful comments by: participants in the National Bank of Poland Workshop “Experiences and Challenges of Forecasting at Central Banks,” Warsaw, 4 November 2009, and particularly the discussant, Malte Knüppel; Filippo Altissimo; Fabio Busetti; Alberto Locarno. The usual disclaimer applies. The opinions expressed here are the authors’ own and do not necessarily reflect those of the Bank of Italy.

\(^2\)By “source of risk” we mean any factor which may result in a discrepancy between macroeconomic projections and actual realizations observed \textit{ex post}.

\(^3\)Siviero and Terlizzese (2007).
“[v]irtually all forecasts of recent years have systematically indicated a risk that economic performance might not be as good as outlined in the central projection and that the feared downward adjustments to demand and output might be sudden and dramatic.” And yet, those warnings were by and large ignored, since, unfortunately, “end users (policy-makers, professional forecasters and the public at large) focus on the numbers summarizing the central or ‘basic’ scenario (i.e. the one judged most likely), [even though] the products of forecasting are often more complex and include an evaluation of the size and direction of the risks surrounding the specific estimate.” Overcoming this state of affairs requires, on the one hand, that users of forecasts become more attentive to the analyses of risks which almost always accompany central projections; and, on the other, that “forecasters [themselves] learn to communicate their message with more determination and force, adapting their alarm signals to suit” (Visco (2009)). This, in turn, calls for the design of ever more effective and accurate ways to convey the appraisal of the risks surrounding the projections — their direction, intensity and likely implications.

In the last decade or so, many central banks have taken to disseminate their views as to the balance of risks around their published macroeconomic projections, adopting (variants of) the approach originally developed at the Bank of England and used in its Inflation Reports, where uncertainty is summarised by means of so-called “fan-charts”. The main appeal of that approach rests in its combining explicit judgmental assessments as to the source, direction and intensity of risks, on the one hand, with the structure and stochastic properties of the model underlying the projections themselves, on the other. A recipe which were to rely only on the latter ingredient would not deliver particularly interesting insight: it would simply reproduce the average uncertainty observed in history, leaving unanswered the question of how risks are perceived to be here and now. By contrast, a wholly judgmental appraisal, lacking any rigor, would inevitably look arbitrary and hence largely unreliable. By combining the two ingredients, instead, risk analysis à la Bank of England blends together both (i) one’s judgment about the source, size and skewness of risks — as they are perceived
to be in the present circumstances — and (ii) the uncertainty associated with the forecasting tool one is using — where and how it originates and how it propagates. Because of this fortunate combination of ingredients, the original idea developed at the Bank of England (see Britton, Cunningham and Whitley (1997) and Britton, Fisher and Whitley (1998) for a description) has been taken up by several other central banks, with various refinements and variants (see, e.g., Blix and Sellin (1998) and (2000) for the Sveriges Riksbank; Novo and Pinheiro (2003) and Pinheiro and Esteves (2008) for the Banco de Portugal).

The original Bank of England’s proposal, as well as those presented in the other contributions listed above, rely on a parametric approach; i.e., they require an explicit assumption to be made about: (i) the family of probability distributions generating the various risk factors (inputs), as well as (ii) the family of the resulting probability distributions for the variables of interest (outputs) — typically posited to be the same as the one assumed for the inputs. In the original Bank of England’s approach, all risk factors are assumed to be generated by a two-piece normal probability distribution; the resulting probability distribution for the variables of interest (consumer price inflation and GDP growth) is also assumed to belong to the same family. However, as shown by Novo and Pinheiro (2003), the two-piece normal distribution is not closed with respect to linearity: i.e., even if all risk factors are generated by a two-piece normal distribution, the resulting distribution for the outputs is unknown, and in general does not belong to the two-piece normal family — even in the simplest possible case where outputs are a linear function of inputs. As a solution to this issue, Novo and Pinheiro (2003) (with a successive refinement introduced by Pinheiro and Esteves (2008)) propose to posit, instead, that all risk factors are generated by a skewed generalized normal distribution. While this does represent a step forward, there still may be situations in which the distribution of the outputs is not skewed generalized normal. Also, if the model is nonlinear (as it often is), the distribution of the variables of interest is in general unknown, whatever distribution is assumed for the risk factors.

In this paper, an approach to risk analysis is proposed that imposes no constraint to
the distribution of inputs and is likewise wholly agnostic as to the distribution of outputs. The distribution for the variables of interest is obtained by means of stochastic (bootstrap) simulations of the forecasting model, re-sampling from the various sources of risks in the forecasting model (residuals of selected stochastic equations and historical projection errors of the main exogenous variables),\(^4\) and imposing that the drawings satisfy the desired degree of skewness. The stochastic simulation results are then used to compute the various quantiles of the implied distribution for the outputs or other features of interest. The approach may thus be characterised as \textit{non-parametric model-simulation-based}.

The remainder of the paper is organized as follows: Section 2 describes the general set-up of risk analysis based on combining the predictive density of model-based forecasts with subjective estimates of risks and uncertainty and reviews the existing literature on the parametric approaches; Section 3 presents the core of our approach; Section 4 reports the results of an empirical application; we typically find that the distribution of the output variables quickly tends to converge to a normal distribution as the number of risk factors and/or the number of simulation periods increase; Section 6 concludes.

\section{Parametric approach to risk analysis: a short survey of the literature}

Appraising forecast uncertainty by means of judgment-informed probability distributions for the main macroeconomic variables may be traced back to second half of the 1990’s, when the Bank of England began publishing, in its Inflation Reports, a graphical representation of the distribution of risks around the central projection (so-called “fan-charts”). The core idea behind the Bank of England’s approach —retained in all subsequent refinements (including

\(^4\)It is worthwhile noting that, while a blanket-assumption for the distribution of risk factors is usually made in other approaches (the distribution being the same for all factors), the approach proposed here does not need uniformity of distributions to be posited.
the one proposed here)—consists of combining a subjective appraisal of selected risk factors\(^5\) with objective measures of the uncertainty associated to the forecasting tool(s).

To describe the main feature of the Bank of England’s approach, let us consider a linear dynamic macroeconometric model:

\[
\Gamma y_t = By_{t-1} + Ax_t + e_t
\]  

(1)

where \(y_t\) is an \((n \times 1)\) vector of endogenous variables, \(x\) is an \((m \times 1)\) vector of exogenous variables, \(e\) is an \((n \times 1)\) vector of structural disturbances, and \(\Gamma, B\) and \(A\) are matrices of parameters, of appropriate sizes.

If the sole sources of risk are the exogenous variables and the residual of stochastic equations\(^6\), then the (overall) forecast error in period \(t > \tau\) (\(\tau\) being the last period for which observations are available) may be written as follows:

\[
y_t - \hat{y}_t = \Pi_1(x_t - x^*_t) + \Pi_0 \Pi_1(x_{t-1} - x^*_{t-1}) + \cdots + \Pi_0^k \Pi_1(x_{\tau+1} - x^*_{\tau+1}) + e_t + \Pi_0 e_{t-1} + \cdots + \Pi_0^k \Pi_1 e_{\tau+1}
\]

(2)

where \(x^*\) are the assumptions formulated for the exogenous variables at times \(t > \tau\), \(\Pi_0 = \Gamma^{-1} B\), \(\Pi_1 = \Gamma^{-1} A\) and \(e_t = \Gamma^{-1} e_t\).

Hence, the forecast error is a linear function of four components:

- the contemporaneous impact of the error made in projecting the exogenous variables at time \(t\);
- the lagged impact of the error made in projecting the exogenous variables for all pre-

\(^5\)In the case of the Bank of England, the responsibility for appraising risk factors rests with the Monetary Policy Committee.
\(^6\)Others potential sources of risk (e.g., model mis-specification; mis-measurement of initial conditions; parameter uncertainty) are thus neglected here. See Clements and Hendry (1998) for a full taxonomy of sources of forecast error.
vious times \( s, \tau < s < t \);

- the contemporaneous impact of the “pure” forecast error in time \( t \), itself a combination of the (structural) shocks to all stochastic equations of the model;

- the lagged impact of all previous “pure” forecast errors.

Given the (joint) probability distribution of the projection errors of the exogenous variables and of the shocks to all stochastic equations of the model, the expression above may be used to compute the probability distribution of \( y_t \). In particular, if all \((x_t - x^*_t)’s\) and \(e_t’\)s are normally distributed, the distribution of \((y_t - \widehat{y}_t)\) will be normal too (and hence symmetric).

However, it may be the case that, on the basis of available information, the forecaster subjectively judges the risks surrounding the \(x_t’\)s and the \(e_t’\)s not to be symmetrically distributed around the baseline assumptions \(x_t = x^*_t\) and \(e_t = 0\). Once assumptions are formulated regarding the (joint) asymmetric distribution of \((x_t - x^*_t)’s\) \(e_t’\)s, the resulting (most likely asymmetric) distribution of all \((y_t - \widehat{y}_t)’s\) may in principle be computed.

Note that the number \(m\) of exogenous variables and \(n\) of structural disturbances may easily be rather large; indeed, in some of the models in use in central banks and other institutions, there are as many as hundreds of both \(x_t’\)s and \(e_t’\)s. Assessing the perceived degree of asymmetry for each of those factors is clearly not feasible. Therefore, usually a subset of (the most relevant) risk factors is pre-selected and retained for risk analysis; around 15-20 risk factors seems to be the standard choice (the choice aiming of course at selecting the most relevant sources of risk). This in turn implies that the distribution of the outputs resulting from risk analysis may differ from the distribution observed in history: specifically, while the latter reflects the uncertainty deriving from hundreds of sources of risk, the former limits itself to a much smaller number. In other words, even in the symmetric case, \(\sigma^2_{\hat{y}_{it}}\) (the variance of \((y_{it} - \widehat{y}_{it})\) computed on the basis of historical observations) will in general differ from \(\sigma^2_{y_{it}}\) (the corresponding variance computed in history using the formula above only for a subset of the risk factors). To tackle this issue, the variance of the \(i\)-th endogenous variable
resulting from risk analysis is multiplied by the adjustment factor \( \frac{\sigma^2_p}{\sigma^2_{yit}} \). This guarantees that, even though risk analysis necessarily relies only on a subset of all conceivable risk factors, the resulting size of risk will by default be in line with historical experience.

To complete the description of the risk analysis approach à la the Bank of England, one only needs to specify the assumption adopted for the probability distribution of risk factors. Britton, Cunningham and Whitley (1997) and Britton, Fisher and Whitley (1998) adopt the “two-piece normal” (tpn) distribution (see Johnson,Kots and Balakrishnan (1994)) as a suitably flexible representation of asymmetries in the distribution of risks.

The tpn distribution is formed by combining the left half of a normal distribution with parameters \((\mu, \sigma^2_1)\) with the right half of a normal distribution with parameters \((\mu, \sigma^2_2)\); the two distributions are re-scaled so as to take the same value at the mode. If \(\sigma_1 > \sigma_2\), the distribution is positively skewed, with mean > median > mode; vice versa if \(\sigma_1 < \sigma_2\). The tpn distribution is fully described by three parameters: mean, mode (or, equivalently, mean-mode difference) and variance; alternatively and equivalently, one of the three parameters may be the probability mass lying above (or below) the mode.

In the Bank of England’s approach, it is first posited that the baseline projection (also called central or point projection below) corresponds to the situation in which all inputs (exogenous variables, stochastic disturbances) and all outputs (endogenous variables of interest) take the modal value of their respective distributions. For each risk factor, a tpn distribution is then calibrated which — given the constraint that the mode coincides with the baseline assumptions for all \(x_i\)’s and \(e_i\)’s— reflects the perceptions about the prevailing direction of risks. In the case of the tpn distribution, this may be directly formulated in terms of mean-mode difference, or, alternatively and equivalently, in terms of the probability mass perceived to be above (or below) the mode.\(^7\)

One now faces the problem of computing the resulting distribution for the variables of interest, given the tpn distributions assumed for the risk factors.

\(^7\)This description ignores the possibility of expressing a judgment about the intensity of risks; a more general set-up of the problem will be presented below.
The Bank of England’s approach relies on a (possibly linearized) representation of the model of the economy underlying the projections, positing that (i) the marginal distributions of the outputs (endogenous variables of interest) are all tpn, and (ii) the mean-mode difference characterizing the distribution of each output is a linear function of the mean-mode differences assumed when calibrating the tpn’s for the inputs. In other words, considering for simplicity a static version of the linear model above, the mean-mode difference posited to hold for the tpn distribution of the $y_t$’s is:

$$\text{mean} - \text{mode}|_{y_t} = \Pi_1 \text{mean} - \text{mode}|_{x_t} + \text{mean} - \text{mode}|_{x_t}$$  \quad (3)

Finally, absent for the time being any judgment on the intensity of risks, the variance of the tpn distributions of the outputs is assumed to coincide with the historical variance of the respective forecast errors. Given that the mode of the tpn distribution is assumed to coincide, for all outputs, with the central projection, three parameters of the tpn distributions are now known (the mode, the mean-mode difference and the variance) and hence the distribution is fully characterised.

The approach adopted at the Sveriges Riksbank (see Blix and Sellin (1998) and (2000)) is similar to the one outlined above.

The approach developed at the Banco de Portugal hinges on a different functional assumption for the asymmetric distribution of risk factors. The tpn distribution is abandoned because, as shown by Novo and Pinheiro (2003), a linear combination of tpn distributions is not, in general, a tpn distribution; even less justified is the assumption that the mean-mode difference is given, for all inputs, by a linear combination of the mean-mode differences for the inputs (i.e., eq. (3) above does not hold).

To overcome those inconsistencies, Novo and Pinheiro (2003) propose adopting, instead, a different asymmetric distribution for the inputs, the “skewed generalized normal” (sgn), resulting from the linear combination of a normal distribution and an exponential one. As the tpn, the sgn is identified by three parameters, and collapses to the normal distribution
for a particular case.

It is worthwhile noting that the $sgn$ distribution can only accommodate limited degrees of skewness; furthermore, a specific condition must be fulfilled in order for the resulting distribution of the outputs to belong to the $sgn$ family too.

As a further refinement, Pinheiro and Esteves (2008) assume a joint probability distribution for the risk factors (“multivariate normal skewed”), with the aim of overcoming some of the limitations imposed by the adoption of the $sgn$ distribution; the marginal distribution of the variables of interest is obtained by numerical simulation.

The short survey above suggests that parametric approaches to risk analysis present a major weakness: if an asymmetric distribution is assumed for input variables, the resulting distribution for output variables is, in the most general case, unknown; this is a fortiori the case if the model is nonlinear. The non-parametric approach presented in the next section aims at overcoming that limitation.

3 Non-parametric bootstrap approach to risk analysis

As an alternative to formulating a specific assumption regarding the functional form of (asymmetric) distributions for input and output variables, Altissimo and Siviero (1998), at the Banca d’Italia, and Knuppel and Todter (2007), at the Deutsche Bundesbank, have developed non-parametric approaches, similar, in their general features, to the one outlined below. In both cases, the forecast distribution is generated by means of stochastic simulations based on asymmetric bootstrap applied to the risk factors.

All non-parametric approaches rest on obtaining the distribution of the variables of interest by means of stochastic (bootstrap) simulations of the forecasting model; they differ in the way subjective judgement and risk factors are combined to obtain skewed distributions, and in the treatment of correlated risk factors. In our empirical application below, the correlation among risk factors is virtually nil; we thus may safely ignore the complications associated
with dealing with correlated risk factors.\textsuperscript{8}

The core issue may now be tackled of how to extract replications from the empirical distribution of risk factors in such a way that the replications conform to the desired degree of asymmetry in accordance with the formulated assumptions. To do that, we build on a mechanism proposed by Ferreira and Steel (2006) to introduce skewness into any continuous, unimodal and symmetric distribution.

Let $f(\nu)$ be the empirical distribution of the risk factors $\nu$’s. Ferreira and Steel (2006) propose using a probability density function $p(u), u \in (0, 1)$, as a weight function to define a skewed probability density function corresponding to the original symmetric one, $f(\nu)$:

\[ s(\nu) = f(\nu)p(F(\nu)), \]  \hspace{1cm} (4)

where $s$ is the probability density function of the skewed distribution and $F$ is the cumulative density function corresponding to $f$.

As shown by Ferreira and Steel (2006), if $p(u)$ is unimodal with mode at $\frac{1}{2}$, then the skewed distribution $s(\nu)$ is also unimodal, with the same mode as the original distribution $f(\nu)$.

To make this approach operational, all that one needs to do is to choose a functional form for $p(u)$. Let us first consider the simplest possible choice, i.e., the case in which $p(u)$ is a function as similar as possible to a uniform distribution, defined between 0 and 1. Specifically, let $p(u)$ be a scale function with, say, $p^*$ of the distribution below $\frac{1}{2}$, and the remaining $(1 - p^*)$ above. For this condition to be fulfilled, the following step-wise $p(u)$

\hspace{1cm} \textsuperscript{8} In theory, one may easily deal with the case of correlated risk factors, by simply computing the Cholesky decomposition of the risk factors’ variance-covariance matrix. However, this poses both a practical and a theoretical problem. As to the former, in case orthogonal risk factors are computed, eliciting subjective judgment is arguably a much more challenging task, as economic intuition may scarcely provide an intuitive guidance when it comes to appraising unobservable variables. Turning to the theoretical problem, prior to computing the Cholesky decomposition one needs to choose a suitable ordering of the original risk factors. As already noted in the literature, different orderings imply different marginal distributions of the (actual) risk factors. Specifically, the skewness of the latter is not invariant with respect to the choice of the ordering—a highly undesirable feature. The solutions proposed in the literature (see Pinheiro and Esteves (2008)) are extremely costly from a computational viewpoint and at any rate inconclusive, at least in the most general case.
function could be used:

\[ p(u) = \begin{cases} 2p^*, & u \in (0, 0.5) \\ 2(1 - p^*), & u \in (0.5, 1) \end{cases} \]  

(5)

where \( p^* = \Pr(u < \frac{1}{2}) \).

To illustrate how such a distribution would shape the resulting \( s(\nu) \), given the original distribution \( f(\nu) \), let us consider the case in which \( f(\nu) \) is the normal distribution. The resulting distribution \( s(\nu) \) will be asymmetric, with \( p^* \) of the distribution below zero (in the numerical example below, \( p^* = 0.70 \)). Note that, given the choice of a discontinuous \( p(u) \), \( s(\nu) \) is discontinuous as well.

The original (normal) \( f(\nu) \), the \( p(u) \) distribution defined as above and the resulting \( s(\nu) \) are reported in Figure 1. Selected statistics of the original and modified distributions are reported in Table 1.

**Figure 1: Original distribution, discrete skewing function and resulting skewed distribution**

![Figure 1](image)

(a) Normal distribution  
(b) \( p \) distribution  
(c) Skewed distribution

**Table 1: Selected statistics of the original and skewed distributions, with discrete skewing function**

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>mode</th>
<th>median</th>
<th>variance</th>
<th>kurtosis</th>
<th>( p(&lt;0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric distribution</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Asymmetric distribution</td>
<td>-0.32</td>
<td>0</td>
<td>-0.36</td>
<td>0.9</td>
<td>3.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

15
While such approach seems promising, discontinuity at the modal value of the resulting asymmetric distribution may be viewed as undesirable. To avoid discontinuity, the following smooth function of the hyperbolic family may be used instead:

\[
p(u) = \begin{cases} 
A - s_1 + s_1 \cdot \text{hyps}\left((\gamma/10)(u - 0.5)\right), & u \in (0, 0.5) \\
A - s_2 + s_2 \cdot \text{hyps}\left((\gamma/10)(u - 0.5)\right), & u \in (0.5, 1)
\end{cases}
\]

where \(\text{hyps}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}\) is the hyperbolic secant. The parameter \(\gamma\) provide a measure of the curvature around \(u = \frac{1}{2}\) (a low value for \(\gamma\) corresponds to high value for the curvature).

The parameter \(A\) (which equals the value of the pdf at \(u = \frac{1}{2}\)) is the maximum value which the \(p(u)\) distribution can take, and increases the density at the mode of the original distribution.

Constraints must be imposed on the parameters \(\gamma\) and \(A\) in order for: (i) \(p(u)\) to be a unimodal distribution with mode at \(\frac{1}{2}\); (ii) the integral of \(p(u)\) between 0 and \(\frac{1}{2}\) be equal to a given chosen probability (say \(p^*\)).

The original (normal) \(f(\nu)\), the continuous \(p(u)\) distribution defined as above and the resulting \(s(\nu)\) are shown in Figure 2. Selected statistics of the original and modified distributions are reported in Table 2.

This is the skewing mechanism we adopt below for all risk factors.

Finally, it may be the case that, in one’s judgment, the current degree of uncertainty differs from the one observed on average in history. This case may be easily accomodated by multiplying the variance of the skewed distribution of each risk factor, prior to proceding to extracting realizations of risk factors, by an uncertainty adjustment coefficient: a coefficient larger than 1 corresponds to higher-than-usual uncertainty; viceversa if the coefficient is lower than 1.

---

\(^9\)In particular, the following constraints must hold: \(\gamma > \pi(1 - p^*)/p^*\), and \(2p^* < A < 2(1 - p^*)\gamma/\pi\); also, it must be that \(s_1 = \frac{2p^*-A}{\pi/\gamma - 1}\) and \(s_2 = \frac{2(1-p^*)-A}{\pi/\gamma - 1}\). We set \(A = 1.45\) and \(\gamma = 2.5\). This choice keeps the variance and kurtosis of the resulting asymmetric distribution close to those of the original one.
Operationally, we proceed as follows. Firstly, a judgmental assessment is formulated on the probability $p^*$ that the outcome of each (structural) source of risk turns out to be lower than what assumed in the baseline simulation. Secondly, we generate (pseudo-) observed asymmetric risk factors by applying the skewing mechanism outlined above. Thirdly, we extract a high number of random draws from the resulting skewed (empirical) distribution, using rejection sampling techniques. Fourthly, for each draw, the (possibly nonlinear) macroeconomic model underlying the baseline simulation is simulated. This delivers a high number of realisations for all variables of interest. Finally, these realisation may be used to compute the (empirical) distribution of the variables of interest and its key features.

Compared with standard parametric approaches, the approach proposed here is very flexible as to the choice of the skewing mechanism (for instance, different $p(u)$ distributions may be straightforwardly assumed for different risk factors) and imposes no restriction on
the resulting distribution of the variables of interest. Moreover, it can handle the case of nonlinear models. Finally, while most other approaches are static (i.e., the risk assessment for time $t$ does not impact on uncertainty at time $t + 1$), this approach is dynamic: any judgment on risks at time $t$ affects uncertainty in all subsequent periods, according to the dynamic structure of the model in use.$^{10}$

4 An application to the Banca d’Italia’s Quarterly Macro-
Econometric Model

In this section we present an application of the approach outlined above to the Banca d’Italia’s Quarterly Macro-Econometric Model (BIQM). The BIQM is a large scale, dynamic, non-linear, structural macro-econometric model of the Italian economy, comprising about 700 endogenous variables (around 70 behavioural equations).$^{11}$ The BIQM serves several purposes, including providing short-to-medium-term projections several times a year. Since July 2008, macroeconomic projections for Italy produced with the BIQM are published, twice a year, in the Banca d’Italia’s (quarterly) Economic Bulletin (January and July issues).

To illustrate the approach, we apply it to the projections published in the Economic Bulletins No. 55. The horizon of those projections ranged from the last quarter of 2008 (unknown at the time when the forecasts were finalised) to the last quarter of 2010.

The point forecasts for the two macroeconomic variables of interest (the annual rate of change of GDP; the annual rate of change of the consumption deflator) are presented in Table 3.

$^{10}$As shown in the next section, this has relevant consequences on the shape of the distribution of the variables of interest as the forecast horizon lengthens.

$^{11}$For a short description of the model structure and main characteristics see Busetti, Locarno and Monteforte (2005)).
The projection for Italian GDP (-2.0 per cent) was, at back then, among the lowest in the range of available forecasts; however, as is well known, later developments were dramatically worse than anticipated at the beginning of last year: in July 2009, the GDP growth projection for 2009 was revised downwards by about 3 percentage points.

The exercise we conduct is the following: for each risk factor, we set the probability that the outcome be lower than assumed in January 2009, in such a way that the mean of the resulting skewed distribution turns out to be roughly in line with the assumptions underlying the July 2009 projections (i.e., for each risk factor, we require that the mean and the mode of the skewed distribution coincide with the July and January 2009 assumptions, respectively). In some cases, when the July assumptions differ very markedly from those formulated in January, we also amplify the degree of uncertainty.

Table 4 presents the resulting appraisal of risk for each of the 16 risk factors we consider.\footnote{The ordering of the risk factors in Table 2 is the same as the ordering assumed for the Cholesky decomposition described in the previous section; however, as mentioned earlier, the choice of the ordering impacts little, if at all, on the results.}

Due to the exceptional, unprecedented and unexpected fall of world trade between the end of 2008 and the beginning of 2009 (with quarterly rates of change of the order of up to -8 per cent, hugely worse than assumed in January), only an extreme assumption on the degree of skewness, combined with an extraordinarily large (though temporary) widening of the uncertainty adjustment factor, can deliver a mean value of the distribution not far from the assumption that looked realistic in July.

\begin{table}[h]
\centering
\caption{Macroeconomic projections for Italy - Annual rates of change, (source: Banca d’Italia, Economic Bulletin No. 55, Jan. 2009)}
\begin{tabular}{lccc}
\hline
          & 2008 & 2009 & 2010 \\
\hline
GDP       & -0.6 & -2   & 0.5   \\
Consumption deflator & 3.5  & 1.1  & 1.4   \\
\hline
\end{tabular}
\end{table}
### Table 4: Risk factors assumptions

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Uncertainty adjustment coefficient</td>
<td>Pr(shock&lt;0)</td>
<td>Uncertainty adjustment coefficient</td>
<td>Pr(shock&lt;0)</td>
<td>Uncertainty adjustment coefficient</td>
<td>Pr(shock&lt;0)</td>
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<tr>
<td>Exogenous variables</td>
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<tr>
<td>World demand</td>
<td>70.0</td>
<td>0.99</td>
<td>70.0</td>
<td>0.50</td>
<td>70.0</td>
<td>0.55</td>
</tr>
<tr>
<td>Price of foreign manufactures (export weights)</td>
<td>1.0</td>
<td>0.90</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>Price of foreign manufactures (import weights)</td>
<td>1.0</td>
<td>0.90</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>Dollar/euro exchange rate</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>Commodity prices (excluding agricultural goods)</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
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<tr>
<td>Commodity prices (agricultural goods)</td>
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<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
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<tr>
<td>Stochastic equations</td>
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<td></td>
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<td>Economic consumption</td>
<td>1.0</td>
<td>0.99</td>
<td>1.0</td>
<td>0.20</td>
<td>1.0</td>
<td>0.45</td>
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<tr>
<td>Consumption of durable goods</td>
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<td>0.99</td>
<td>1.0</td>
<td>0.20</td>
<td>1.0</td>
<td>0.55</td>
</tr>
<tr>
<td>Investments in machinery</td>
<td>2.5</td>
<td>0.99</td>
<td>2.5</td>
<td>0.20</td>
<td>2.5</td>
<td>0.45</td>
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<tr>
<td>Imports</td>
<td>30.0</td>
<td>0.99</td>
<td>30.0</td>
<td>0.20</td>
<td>30.0</td>
<td>0.45</td>
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<tr>
<td>Exports</td>
<td>40.0</td>
<td>0.99</td>
<td>40.0</td>
<td>0.20</td>
<td>40.0</td>
<td>0.45</td>
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<tr>
<td>Private sector employment</td>
<td>2.0</td>
<td>0.99</td>
<td>2.0</td>
<td>0.20</td>
<td>2.0</td>
<td>0.55</td>
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<td>Private sector value added deflator</td>
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<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
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<tr>
<td>Deflator of consumption of non-durable goods</td>
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<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
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<tr>
<td>Inflation expectations</td>
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<td>0.50</td>
<td>1.0</td>
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<tr>
<td>Private sector wages</td>
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<td>0.50</td>
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<tr>
<td>Pass-through</td>
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<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
<td>1.0</td>
<td>0.50</td>
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Historical observations for the empirical distribution of the risk factors are available for the period 1984Q1 to 2007Q4; the variance-covariance matrix of the risk factors was estimated using those observations.

Stochastic simulations were then computed for the period comprised between 2008Q4 to 2010Q4, drawing from the empirical distribution of the (orthogonalised) risk factors, 1984Q1 to 2007Q4, modified so as to match the judgmental assessments presented in Table 4 above; the example below is based on 10,000 drawings.

Figures 3 and 4 show the estimated marginal distributions of the variables of interest (year-on-year consumption deflator inflation and quarter-on-quarter GDP growth), for the percentiles comprised between the 10th and the 90th; colour bands are centered on the median and jointly cover 80 per cent of the estimated distribution; each pair of the same colour nuance correspond to a probability mass of 20 per cent. The central line indicates the median; the other line indicates the projection formulated in January 2009. Figures 5 and 6 present the same information, in tri-dimensional graphs. Figures 7 and 8 present the joint distribution of GDP and consumer prices, for the years 2009 and 2010, respectively.
Figure 3: Fan-chart, January 2009 Italian GDP projection (qoq rate of change)

Notes: (1) Black line: baseline projection (2) Dotted line: median of the probability distribution of the projection (3) Each pair of color bands correspond to a probability mass of 20 per cent.

Figure 4: Fan-chart, January 2009 Italian consumption deflator projection (yoy rate of change)

Notes: see Fig.3
Figure 5: Probability distribution of the January 2009 Italian GDP projection (qoq rate of change)

Figure 6: Probability distribution of the January 2009 Italian consumption deflator projection (yoy rate of change)
Figure 7: Joint probability distribution of the January 2009 Italian GDP (qoq rate of change) and consumption deflator (yoy rate of change) projections for 2009

Figure 8: Joint probability distribution of the January 2009 Italian GDP (qoq rate of change) and consumption deflator (yoy rate of change) projections for 2010
A few remarks are in place: first, the width of the bands opens up as the forecast horizon lengthens; such widening of the bands is much more pronounced for inflation than for GDP growth; second, the degree of asymmetry of the resulting distributions for the variables of interest is limited in the initial periods of the projection horizon and tends to vanish completely as the horizon lengthens, *despite the considerable degree of asymmetry embedded in the assumptions for almost all risk factors*. Thus, not only the mean of the distribution shifts, but also the mode; this contrasts with the (imposed) constancy of the mode in the approaches surveyed in Section 2. Such features may be attributed to the dynamic nature of our approach: as time goes by and shocks pile up, the resulting distributions of the endogenous variables tend to converge to the normal distribution as a consequence of the CLT. This feature, one may speculate, is a useful reminder of one fundamental truth: as one looks farther into the future, her/his opinions matter less and less; uncertainty becomes wider, and more evenly distributed, no matter what one may think.

## 5 Concluding remarks

Conveying an assessment of the risks inherent to macroeconomic forecasts is increasingly becoming an all-essential component of the forecasts themselves. The need to design ever more effective and accurate ways to convey the appraisal of the risks surrounding the projections —their direction, intensity and likely implications— has been magnified, it may be argued, by the evidence of difficulties in communicating the results of risk analysis in the run-up to the current crisis. In this paper we proposed an approach to risk analysis that imposes no parametric constraint on the distribution of risk factors and is generally more flexible than previous approaches (e.g., it does not require the model to be linear; also, it may easily accommodate extreme degrees of asymmetry in the sources of risk). An empirical application shows that the results one obtains with this approach differ markedly from those associated with other approaches.
References


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2008

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2009


M. BUGAMELLI and F. PATERNÒ, Do workers’ remittances reduce the probability of current account reversals?, World Development, v. 37, 12, pp. 1821-1838, TD No. 573 (January 2006).

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