External trade and monetary policy in a currency area

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EXTERNAL TRADE AND MONETARY POLICY IN A CURRENCY AREA

by Martina Cecioni*

Abstract

For historical and geographical reasons, the member countries of the European Monetary Union (EMU) display different degrees of external trade openness. The paper lays out a model for a currency area composed of two regions. One region is more open to trade with a third country outside the area than the other. Using the utility-based loss function for the currency area, the optimal monetary policy is compared to the one for a homogeneous area. In the model with heterogeneity, the relative competitiveness across regions influences the extent to which shocks are transmitted to the area-wide inflation and output gap. Under a plausible calibration for the EMU, the optimal policy plan exhibits a stronger tendency towards currency area exchange rate stabilization than the one in the homogeneity case. Moreover, it is welfare-improving to forgo some area-wide inflation stabilization to dampen inflation differentials.

JEL Classification: E52, F41.
Keywords: Monetary union, optimal monetary policy, loss function.

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* Bank of Italy, Economics, Research and International Relations
1 Introduction*

With the creation of the European Monetary Union (EMU) the member countries agreed to relinquish individual control on both the nominal interest rate and the nominal exchange rate. Some empirical studies confirm that the establishment of the currency area, by irrevocably fixing the exchange rates across member countries and thereby eliminating the related risk in foreign transactions, stimulated intra euro area trade. However, there is no evidence of diversion of trade away from non-member countries (Micco et al., 2003). We observe instead that some of the EMU countries have strong trade linkages with countries that do not belong to the euro area for mainly historical or geographical reasons, while others show a pattern of trade that is more oriented toward intra euro area goods. About ten years after the establishment of the EMU and despite ever increasing trade integration, member countries continue to display heterogeneity in their degree of openness toward countries outside the currency area.2

The heterogeneity along this dimension is relevant for setting the monetary policy stance in a currency area as the transmission mechanism of external shocks may differ across regions and monetary policy may become less effective in controlling inflation and stabilizing the output gap. Honohan and Lane (2003 and 2004) show that due to differences in the degree of external openness, since the adoption of the common currency, exchange rate movements have had an impact on inflation differentials within the EMU.

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1 One of the most recent estimates by Frankel (2008) found that the euro boosted intra-Eurozone trade by around 10 to 20% on average, in line with what was shown in previous studies (e.g. Baldwin, 2006).

2 Ireland, the Netherlands and Belgium stand out in this respect. Between 1997 and 2006, both the average external trade volume over GDP and the average external imports over GDP ratios of Belgium, Ireland and the Netherlands were about three times higher than the corresponding ratios of the remaining euro-area countries. Furthermore, this is a rather stable fact over the sample considered.
The paper sets up a model for an open currency area composed of countries that have different trade linkages with the rest of the world. After deriving the microfounded welfare measure, it then solves for the optimal monetary policy from a currency area point of view. Under a plausible calibration for the EMU it analyzes how the degree of heterogeneity in the external dimension affects the optimal responses of the macroeconomic variables to area-wide and asymmetric shocks.

The adoption of a common currency by European countries has spawned a large body of work on monetary policy in currency areas. The common driver of these studies is the fact that the economies of EMU member countries display some structural differences. One strand of literature has been devoted to studying and documenting these differences. For instance, since the euro-area member countries display large and persistent inflation differentials relative to other currency areas (in particular the US), considerable effort has gone into understanding the sources of this dispersion.

Building on these facts, a second strand of literature has focused on the optimal design of monetary and fiscal policy in currency areas. Benigno (2004) and Benigno and Lopez-Salido (2006) study the implications of, respectively, heterogeneous degrees of stickiness and different degrees of inflation persistence among regions for the optimal target of inflation in a currency area. Galí and Monacelli (2008) and Ferrero (2009) consider the role of independent fiscal policies in a currency area in which regions share the same structural features but are hit by asymmetric technology shocks.

The main contribution of this paper is to build external trade linkages with the rest of the world into a standard model for a currency area and to study the effect of cross-country dissimilarities on the degree of openness of the optimal monetary policy design. The main issue is not whether the EMU is an optimal currency area, but whether, given the institutional setting, the monetary policy prescriptions that are valid for a homogeneous economy can be applied to a heterogeneous economy.

The model shares most of its features with the New Open Economy macroeconomics literature. The parameter for the degree of external openness coincides with the preferences of households in each region for goods produced by the rest of the world. Symmetrically, the same parameter indi-

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3See ECB (2003) for a careful analysis of different measures of inflation dispersion and a comparison with the US.

cates the preferences of the rest of the world for the goods produced in each of the two regions.

When there is heterogeneity as to the degree of external openness, the dynamics of the area-wide inflation and output gap are affected by the relative competitiveness between regions. Therefore to describe fully the behavior of the area-wide economy the fluctuations of price differentials must be taken into account. The intuition for this result is the following. In an open currency area, shifts in the terms of trade between the monetary union and the rest of the world (hereafter external terms of trade) affect the demand for the goods produced in the area and thus its output gap and inflation. As to changes in the relative price between the goods produced in the two regions (hereafter internal terms of trade), their effects depend on the rest of the world’s preferences for the goods produced in the two regions (i.e. the heterogeneity as defined above). If the rest of the world is neutral about consuming goods from one or the other region of the monetary union, price differentials among the regions induce the rest of the world to substitute the more expensive good with the cheaper one and the aggregate demand for currency area goods is not affected by them. If, instead, the rest of the world has a stronger preference for the goods produced by one of the two regions, this substitution is not perfect and the fluctuations of the internal terms of trade affect the aggregate demand of currency area goods coming from the rest of the world.

After deriving a quadratic utility-based loss function for the currency area, I solve for the full commitment optimal monetary policy from a timeless perspective. Several studies (e.g. De Paoli, 2009 and Faia and Monacelli, 2008) on the optimal monetary policy in an open economy have shown that, once one departs from a special parametrization\textsuperscript{5}, the strict domestic inflation stabilization is no longer the first best policy and a partial stabilization of the exchange rate is desirable. This result holds in the case of a homogeneous currency area open to external trade. The paper shows that in the presence of heterogeneity the extent to which the exchange rate should be stabilized is reinforced and higher volatility of area-wide inflation is prescribed.

In response to an area-wide technology shock, for instance, the optimal response in a homogeneous economy is to partially accommodate the shock\textsuperscript{5}\textsuperscript{Namely, when the elasticity of intertemporal substitution of consumption or the intratemporal elasticity of substitution between domestic and foreign goods are different from one (Corsetti and Pesenti, 2001).}
by reducing the nominal interest rate to stabilize area-wide inflation and the output gap. This policy generates a depreciation of the monetary union currency that boosts the external demand of currency area goods. When the regions of the currency area are heterogeneous, the shock generates higher inflation in the more open country. A depreciation of the nominal exchange rate would amplify these inflation differentials, as the more open region encounters stronger inflationary pressures. Since the inflation differentials cause deadweight losses according to the utility-based welfare of the currency area (see Benigno, 2004), it is welfare improving to reduce the nominal interest rate by less in order to attenuate the depreciation of the monetary union currency. This implies that higher fluctuations of the area-wide inflation are allowed in order to partially dampen those of the inflation differentials caused by the shock.

The result sheds light on an important point. It is commonly believed that in a currency area the centralized monetary authority has the ability to react only to aggregate disturbances while region-specific shocks can be stabilized only through other policy instruments, such as national fiscal policies (see Galí and Monacelli, 2008 and Ferrero, 2009). This paper shows that in a heterogeneous currency area the central bank, by internalizing that the transmission mechanism of monetary policy impulses differs across regions, is able to respond to idiosyncratic shocks. Furthermore, the optimal policy plan prescribes to make active use of this channel to influence inflation differentials since the aggregate welfare measure for the currency area also depends on them (see Benigno, 2004).

The rest of the paper is organized as follows. Section 2 illustrates some stylized facts about the external openness of EMU countries and describes the way in which this is incorporated in the model. Section 3 sets up the model for the currency area. Section 4 analyzes the effects of heterogeneity on the dynamics of the model. Section 5 derives and illustrates the optimal monetary policy plan. Section 6 concludes. The appendix illustrates the details of the solution of the model, the derivation of the approximated welfare measure and the equations that characterize the optimal plan.
2 The external openness of EMU countries

Figure 1 plots the degree of total openness against the degree of external openness of 11 EMU member countries. Both indicators are measured as the ratio of imports plus exports to GDP. The degree of total openness refers to trade flows with all countries whereas the degree of external openness refers to trade flows with countries that are not in the euro area.

Figure 1: Degrees of total and external openness

According to figure 1, the countries that are more open to external trade are also those that are more open overall. Furthermore, one can identify three groups of countries. The first includes countries with a relatively low degree of external and total openness, namely France, Greece, Italy, Portugal and Luxembourg, Slovenia, Cyprus, Malta and Slovakia are excluded.

Luxembourg, Slovenia, Cyprus, Malta and Slovakia are excluded.

The average degree of total openness across EMU members from 1997 to 2006 is 0.86 and the average degree of external openness over the same period is 0.34. The cross-country dispersions of the total and external degree of openness, as measured by the relative standard deviation, are 47% and 54% respectively.
Spain. Another group includes countries with a relatively high degree of external and total openness, namely Belgium, Ireland and the Netherlands. Finally, there is a third group of countries (Austria, Finland and Germany) that are in an intermediate position between the two above. This pattern has been quite stable over the last decade and a similar one can be observed if the degrees of openness are measured as (total and external) imports over GDP (see figure 2, panel (a)). In the composition of trade there is some dispersion across countries as well (see figure 2, panel (b)).

I introduce these features of the data in the model by assuming that the inhabitants of the two regions of the currency area differ in their preferences for goods produced by the rest of the world (r.o.w.) outside the area. The consumption bundles of citizens from regions 1 and 2 are given respectively by

\[
C_{1,t} = C_{1,t}^{1} - \alpha - \omega + \varepsilon C_{1,t}^{\alpha} C_{row,t}^\omega - \varepsilon \\
C_{2,t} = C_{2,t}^{1} - \alpha - \omega - \varepsilon C_{2,t}^{\alpha} C_{row,t}^\omega + \varepsilon
\]

where \(C_{i,t}\) is the consumption good produced in region \(i = 1, 2\) and r.o.w.. This specification of preferences implies that the consumption basket of each region in the currency area includes, in addition to domestically produced goods, those produced in the other region of the currency area and in the rest of the world.\(^8\)

The parameters \(\alpha\) and \((\omega \pm \varepsilon)\) can be thought of as degrees of openness toward each trading partner. In fact, the steady state solution of the model described in the next section implies that these are the shares of imported consumption goods in total consumption. Thus, \(\alpha + \omega - \varepsilon\) and \(\alpha + \omega + \varepsilon\) are the degrees of total openness of regions 1 and 2 respectively.

The proposed assumption on the composition of the consumption bundles in the two regions captures the fact that inside the EMU more open countries are also more open to external trade. The region that is more open to external trade (region 2) is also the one that has a lower degree of home bias (i.e. a higher degree of overall openness) and vice versa. The parameter \(\alpha\) that indicates trade inside the currency area is constant across countries. This assumption is motivated by the need to keep the model simple by shut-

\(^8\)In this way the degrees of external and total openness are modeled as a structural feature of the regions. This is what we observe since the pattern of figure 1 is quite stable over the period from 1997 to 2006.
Figure 2: Other measures of the external openness of EMU countries

(a) Total and external imports over the GDP

(b) External trade over total trade

Notes: Average of quarterly data between 1997 and 2006. Source: Eurostat.

...ting down one possible source of asymmetry. Indeed the paper focuses on the

9 Andrés et al. (2008) allows for differences in the degree of openness toward the other member countries in a model for a closed currency area.
effects of asymmetry in the degree of exposure to shocks originating outside the monetary union.

In a trade-balanced steady state, the chosen specification implies that the parameters \((\omega - \varepsilon)\) and \((\omega + \varepsilon)\) also indicate the share of the goods produced respectively in region 1 and 2 requested by the rest of the world. Under the chosen specification, region 2 is more open to external trade than region 1, not only because its consumption basket is more oriented towards foreign-produced goods, but also because the rest of the world demands more of its good. This is one way to capture the fact that some countries in the EMU have, for historical or geographical reasons, closer linkages with partners outside the currency area.

The parameter \(\varepsilon\) is an indicator of the heterogeneity in the preference toward goods from the r.o.w.. When \(\varepsilon = 0\), the two regions have identical preferences for goods produced outside the currency area and symmetric preferences for their own produced goods. By setting \(\varepsilon \neq 0\), the two regions diverge and the model is able to capture the heterogeneity described in figure 1.

To introduce heterogeneity in the external dimension of EMU countries, an alternative modeling strategy would have been the following

\[
C_t^1 = C_{1,t}^{(1-\alpha)} C_{2,t}^{\alpha(\omega+\varepsilon)} C_{\text{row},t}^{\alpha(1-\omega-\varepsilon)}
\]
\[
C_t^2 = C_{2,t}^{(1-\alpha)} C_{1,t}^{\alpha(\omega-\varepsilon)} C_{\text{row},t}^{\alpha(1-\omega+\varepsilon)}.
\]

where now \(\alpha\) indicates the degree of overall openness and \(\alpha(1 - \omega \pm \varepsilon)\) the degree of external openness. In this case it is assumed that the two regions differ in their patterns of trade but not in the degree of total openness. This assumption, while consistent with the data shown in figure 2, is at odds with the data about the degree of total openness shown in figure 1. The assumption of preferences in (1) and (2) is consistent not only with the facts depicted in figure 1 but also with the existence of heterogeneity in the composition of the trade structure.\(^{10}\)

\(^{10}\)According to preferences (1) and (2), the share of trade with the rest of the world as a proportion of total trade is given by \(\frac{\omega \pm \varepsilon}{\alpha \pm \omega \pm \varepsilon}\) and is therefore influenced by the degree of heterogeneity in trade, \(\varepsilon\).
3 The model

The model presented in this section is a slight modification of the New Keynesian framework for open economies.\textsuperscript{11} The world is composed of a currency area (U) and the rest of the world (ROW). The currency area is formed by two regions (hereinafter region 1 and 2). Both regions are inhabited by a continuum of identical households of mass one and are the same size. The two regions of the monetary union are explicitly modeled while the rest of the world is described by an exogenous stochastic process on the vector of the variables of interest.

3.1 Households

The representative household of each region $i = 1, 2$ has the following lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_i^t)^{1-\sigma}}{1-\sigma} - \frac{(N_i^t)^{1+\varphi}}{1+\varphi} \right]$$

(3)

where $N_i^t$ are the hours of labor supplied by the agent living in region $i$ and $C_i^t$ is a bundle of consumption goods coming from the two regions of the currency area and from the rest of the world. The bundles for the two regions are defined in equations (1) and (2). The Cobb-Douglas specification of preferences implies that the elasticity of substitution among goods is one.

The goods $C_{1,t}$, $C_{2,t}$ and $C_{row,t}$ are CES aggregates of differentiated varieties of each good and, for $i = 1, 2$ and r.o.w., they are defined as follows\textsuperscript{12}

$$C_{i,t} \equiv \left[ \int_0^1 c_{i,t}(h)^{(\theta-1)/\theta} dh \right]^\theta/(\theta-1)$$

where $h$ is a generic variety of the differentiated good $i$ and $\theta > 1$ is the elasticity of substitution among varieties.

The representative household living in country $i$ maximizes (3) subject to a sequence of budget constraints of the form

$$P_{c,t}C_{i,t} + E_tQ_{t,t+1}D_{i,t+1}^i \leq D_i^i + W_i^iN_i^t + T_i^t$$

(4)

where $P_{c,t}$ is the price index of the consumption bundle relevant for welfare in region $i$ expressed in the currency of the monetary union; $D_{i,t+1}^i$ is the nominal

\textsuperscript{11}See Lane (2001) for a survey on the so-called New Open Economy macroeconomics.

\textsuperscript{12}Hereinafter the subscript indicates the provenance of the good.
payoff in $t + 1$ of the portfolio of state-contingent claims held at the end of period $t$; $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoff relevant to the households living in country $i$; $W_i^t$ is the nominal wage and $T_i^t$ are lump-sum transfers. I assume that markets are complete both at the domestic and international level.

As commonly found in the literature, the household problem can be solved in two steps. The first step implies solving for the optimal allocation of expenditure across the bundle of goods produced in different countries and across the varieties of each bundle. The solution yields to the following demand functions for $i, j = 1, 2$

$$c_{j,t}^i(h) = \left(\frac{p_{j,t}(h)}{P_{j,t}}\right)^{-\theta} C_{j,t}^i$$

$$C_{j,t}^i = \alpha \left(\frac{P_i^t}{P_{j,t}}\right) C_{t}^i \quad \text{for } j \neq i$$

$$C_{1,t}^1 = (1 - \alpha - \omega + \varepsilon) \left(\frac{P_{1,t}^c}{P_{1,t}}\right) C_{t}^1$$

$$C_{2,t}^2 = (1 - \alpha - \omega - \varepsilon) \left(\frac{P_{2,t}^c}{P_{2,t}}\right) C_{t}^2.$$  

Similarly, the demand for goods imported from the rest of the world by households in the two regions are

$$C_{row,t}^1 = (\omega - \varepsilon) \left(\frac{P_{1,t}^c}{P_{row,t}^* S_t}\right) C_{t}^1$$

$$C_{row,t}^2 = (\omega + \varepsilon) \left(\frac{P_{2,t}^c}{P_{row,t}^* S_t}\right) C_{t}^2$$

where $S_t$ is the nominal exchange rate between the currency of the monetary union and that of the rest of the world and the asterisk on a variable indicates that it is expressed in the currency of the rest of the world. Given these demand functions one can define the following producer price (PPI) and consumer price (CPI) indices for the region $i = 1, 2$ of the monetary union

$$P_{i,t} = \left(\int_0^1 p_{i,t}(h)(1-\theta) \, dh\right)^{1/(1-\theta)}$$

$$P_{1,t}^c = P_{1,t}^{(1-\alpha-\omega+\varepsilon)} P_{2,t}^\alpha \left(\frac{P_{row,t}^* S_t}{P_{row,t}^* S_t}\right)^{(\omega-\varepsilon)}$$

$$P_{2,t}^c = P_{2,t}^{(1-\alpha-\omega-\varepsilon)} P_{1,t}^\alpha \left(\frac{P_{row,t}^* S_t}{P_{row,t}^* S_t}\right)^{(\omega+\varepsilon)}.$$
The second step involves the household optimal choice of the hours worked and of the intertemporal allocation of consumption. The first order conditions for the representative agent of country \(i = 1, 2\) are

\[
(C^i_t)^\sigma (N^i_t)^{\phi} = \frac{W^i_t}{P^c_{i,t}} \tag{8}
\]

\[
\beta \left( \frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left( \frac{P^c_{i,t}}{P^c_{i,t+1}} \right) = Q_{t,t+1}. \tag{9}
\]

By taking expectations conditional on the information available at time \(t\) on both sides of (9) I obtain the following Euler equation

\[
\beta R_t E_t \left\{ \left( \frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left( \frac{P^c_{i,t}}{P^c_{i,t+1}} \right) \right\} = 1 \tag{10}
\]

where \(R_t = 1/E_t Q_{t,t+1}\) is the return on a riskless bond in the currency of the monetary union. Under the assumption of complete markets, combining the Euler equations of each region and assuming that the initial debt holdings are equal across regions, one obtains the following risk sharing condition

\[
C^1_t = \left( \frac{P^c_{2,t}}{P^c_{1,t}} \right)^{1/\sigma} C^2_t ; \quad C^i_t = \left( \frac{P^c_{row,t} S_t}{P^c_{i,t}} \right)^{1/\sigma} C^{row}_t \text{ for } i = 1, 2. \tag{11}
\]

If \(\sigma = 1\), i.e. the utility is logarithmic in consumption, the risk sharing is such that the consumption expenditure in the three countries is the same at each time \(t\) and the model does not exhibit a foreign asset dynamics. Even in this special parametric case, however, risk sharing is not perfect. In fact, the existence of home bias in consumption implies that purchasing power parity (PPP) does not hold.

Moreover, under the assumption of complete financial markets the equilibrium price of the foreign riskless bond in terms of the monetary union’s currency is equal to \((R^*_t)^{-1} S_t = E_t Q_{t,t+1} S_{t+1}\). Combining it with the equilibrium price of the riskless bond in the currency area (that is \(R_t = 1/E_t Q_{t,t+1}\)), one obtains the uncovered interest parity (UIP) condition

\[
E_t \left\{ Q_{t,t+1} \left[ R_t - R^*_t \left( \frac{S_{t+1}}{S_t} \right) \right] \right\} = 0. \tag{12}
\]
3.2 Firms

In each region there is a continuum of mass one of firms. Each firm produces a variety of the differentiated good in a regime of monopolistic competition. If a firm is located in region 1, it can produce only a variety of the consumption good produced in that region \((C_{1,t})\). The technology available to produce the variety \(h\) is linear in the labor input, i.e.

\[
Y_{i,t}(h) = A_{i,t}N_t^i(h)
\]

where \(A_{i,t}\) is a technology shock that is region-specific.

Each firm chooses the price that maximizes its profit taking as given demand for its variety. The demand of variety \(h\) of the good produced in region \(i = 1, 2\) and \(j \neq i\) of the currency area is given by

\[
y_{i,t}(h) = \left(\frac{p_{i,t}(h)}{P_{i,t}}\right)^{-\theta} \left[ (1 - \alpha + \omega \pm \varepsilon) \left(\frac{P_{c,i}^i}{P_{i,t}}\right) C_t^i + \alpha \left(\frac{P_{c,j}^j}{P_{i,t}}\right) C_t^j + \right.
\]

\[+ (\omega \pm \varepsilon) \left(\frac{P_{c,\text{row}}^iS_t}{P_{i,t}}\right) \right] C_{\text{row},t}.
\]

(13)

However, price stickiness à la Calvo implies that firms are not able to change their price whenever they want. Each period there is a probability \(\delta\), independent of time, that the price cannot be changed. This implies that the optimal price is decided in a forward-looking manner, as a markup charged over the expected value of future discounted marginal costs. The log-linear expression for the optimal new price is\(^{13}\)

\[
p_{i,t}^{\text{new}} = \mu + (1 - \beta \delta) \sum_{k=0}^{\infty} (\beta \delta)^k E_t \{ \text{rmc}_{i,t+k} + p_{i,t} \}.
\]

where \(\mu\) is the steady state level of the markup and depends on the substitutability among varieties (i.e. \(\mu \equiv \frac{\theta}{\theta - 1}\)) and \(\text{rmc}_{i,t+k}\) are the (log deviation) of the real marginal costs, defined in the next section.

\(^{13}\)Variables written in lower case letters are log deviations from the deterministic steady state of that variable.
3.3 Equilibrium

The labor market is competitive and its clearing condition implies that firms’ real marginal costs for \( i = 1, 2 \) are

\[
RM C_{i,t} = (1 - \tau)(C_i^i)^\sigma(N_i^i)^\rho \frac{P_{c,i}}{P_{i,t} A_{i,t}}. \tag{14}
\]

where \( \tau \) is a subsidy to the labor supply and it is assumed to be the same across regions of the currency area. Before deriving the clearing condition of the market for goods in each region, one must first aggregate across all varieties both the demand and the supply side. The index for aggregate output in region \( i \) is defined as \( Y_{i,t} \equiv \left[ \int_0^1 y_{i,t}(h)(\theta - 1)/\theta \, dh \right]^{\theta/(\theta - 1)} \). Aggregating over all varieties \( h \) in equation (13), I obtain the following aggregate demand function for the goods 1 and 2

\[
Y_{i,t} = (1 - \alpha - \omega \pm \varepsilon) \left( \frac{P_{c,i}}{P_{i,t}} \right) C_i^i + \alpha \left( \frac{P_{c,j}}{P_{i,t}} \right) C_j^j +
+(\omega \pm \varepsilon) \left( \frac{P_{row,i}}{P_{i,t}} \right) C_{row,i} \text{ for } i, j = 1, 2 \text{ and } i \neq j \tag{15}
\]

To obtain the aggregate production function I make use of the index of aggregate employment defined by Gali and Monacelli (2005): \( N_i^i \equiv \int_0^1 N_i^i(h)dh = \frac{Y_{i,t} Z_{i,t}}{A_{i,t}} \), where \( Z_{i,t} \equiv \int_0^1 \frac{y_{i,t}(h)}{Y_{i,t}} \, dh \). They show that equilibrium variations of \( z_{i,t} \equiv \log Z_{i,t} \) around the perfect foresight steady state are of second order and can thus be ignored in a first order approximation. The log linear expression for the aggregate production function is then

\[
y_{i,t} = a_{i,t} + n_{i,t}.
\]

3.4 Solution of the model

In this section I propose a solution of the model described above that has three features. First, it is a log linear solution. The optimality conditions of the firms and households in each region together with the market clearing conditions are log linearized around the trade-balanced deterministic steady state. In this steady state the three countries share the same level of consumption, output and hours. Whenever \( \varepsilon \) is different from zero, the only
difference among the steady state equilibrium of each country is the composition of their consumption bundles.\textsuperscript{14}

Second, instead of obtaining the dynamics of the macroeconomic variables of each of the two regions of the currency area, I rewrite the model so as to have a solution for the dynamics of area-wide variables and a solution for the differentials between the two regions. I obtain the area-wide variables by aggregating those of the two regions weighted by their relative size and the differentials by comparing the variables in the more open region with those in the more closed one. In other terms, considering the generic variable $x$, I define the area-wide corresponding variable as $x_u \equiv mx_1 + (1-m)x_2$, where $m$ is the size of the region 1, and the differentials as $x_r \equiv x_2 - x_1$. The model has therefore two blocks: the area-wide and the differentials block. Solving the model in this way allows me to shed light on what is the main contribution of introducing heterogeneity into an otherwise standard framework of a homogeneous currency area. In the benchmark case I assume that the regions are of equal size.

Third, I solve the model in the case of fully flexible prices and sticky prices. As is standard in the literature I then rewrite it in terms of the gaps between these two equilibria to highlight the effects of nominal rigidities.

Henceforth I use the following definitions. The \textit{internal terms of trade} is the relative price of the good produced in the more open region of the currency area and the one produced in the less open one, i.e. $T^{in} \equiv \frac{P_2}{P_1}$. The \textit{external terms of trade} is the relative price between the good produced in the rest of the world and the aggregate of the goods produced in the currency area, i.e. $T^{ex} \equiv \frac{SP_{row}}{P_u}$. Similarly, the \textit{internal real exchange rate} is the ratio of the CPIs of the two regions, i.e. $Q^{in} \equiv \frac{P_{c2}}{P_{c1}}$. The \textit{external real exchange rate} is the ratio of the CPIs of the rest of the world and the aggregate of the currency area, i.e. $Q^{ex} \equiv \frac{SP_{c2}}{P_{cu}}$. The internal terms of trade and real exchange rate can be thought of as measures of the producer and consumer price level differentials, respectively.

As mentioned above, the rest of the world is not explicitly modeled and fluctuations in its macroeconomic variables are specified as exogenous stochastic processes. In this section I report the equations that describe the dynamics of the small-scaled version of the model. The reader should refer

\textsuperscript{14}It can be shown that both terms of trade, the one between the two regions of the currency area and the one between the currency area and the rest of the world, are uniquely pinned down in the perfect foresight steady state.
to the appendix for the algebraic derivations behind them.

### 3.4.1 The equations of the area-wide block

By combining the aggregate demand for the goods produced in the two regions of the currency area (equation (15) for $i = 1, 2$) and using the risk sharing conditions (11), I obtain the following aggregate demand for the goods produced in the currency area

$$y_{u,t} = \frac{1}{\sigma_\omega} t^{ex}_t + y_{row,t} - \sigma_\epsilon t^{im}_t$$

where $y_{row,t}$ is the output of the rest of the world. The parameter $\sigma_\epsilon$ and $\sigma_\omega$ are combinations of the structural parameters that affect the openness dimension of the economy ($\alpha, \omega, \varepsilon$), the inverse of the intertemporal elasticity of substitution ($\sigma$) and the size of the regions, which is assumed to be equal to 0.5 throughout the paper (see appendix A).

Equation (16) is similar to the aggregate demand equation of the open economy of Galí and Monacelli (2005). Since the currency area economy is open to the rest of the world and given the assumption of market completeness, a change in the rest of the world output ($y_{row,t}$) affects the demand for currency area goods. Similarly, an increase of the external terms of trade enhances the competitiveness of the monetary union goods, thus boosting the aggregate demand for them. When there is no heterogeneity in the preferences, that is $\varepsilon = 0$, the parameter $\sigma_\epsilon$ is equal to zero. In this case the rest of the world is indifferent between the goods produced in the two regions and the average level of competitiveness of the currency area goods ($t^{ex}_t$) is sufficient to determine the aggregate demand for the area goods. When there is heterogeneity in preferences, that is $\varepsilon \neq 0$, the last term in (16) is different from zero and the price dispersion across regions matters. If, *ceteris paribus*, the goods produced by the more open region become more competitive (i.e. $t^{im}_t$ decreases), the demand of currency area goods coming from the rest of the world increases.

As regards the supply side of the area-wide economy, log linearizing equation (14) for $i = 1, 2$, substituting for the risk sharing conditions (11) and aggregating across regions of the currency area, I obtain the following

---

15In this case, it also turns out that $\sigma_\omega = \frac{\omega \sigma + (\sigma - 1)(1 - \omega)}{(1 - \omega)^2 + (1 - \omega)}$ and the currency area economy behaves exactly like the small open economy of Galí and Monacelli (2005).
expression for the area-wide real marginal costs

\[ rmc_{u,t} = -\nu + (\sigma_{\omega} + \varphi)y_{u,t} + (\sigma - \sigma_{\omega})y_{row,t} + \sigma_{\omega}\sigma_{\epsilon}t_{\epsilon}^m - (1 + \varphi)a_{u,t}. \]  

(17)

where \( \nu = -\log(1 - \tau) \). Since the currency area is an open economy, the real marginal costs are affected not only by the demand for domestic goods \( (y_{u,t}) \) and by the domestic productivity level \( (a_{u,t}) \), but also by the world aggregate demand \( (y_{row,t}) \). For instance, an increase of the domestic and international demand calls for a higher labor supply that can be provided by households at higher real wages, driving up the real marginal cost.

In addition, when \( \epsilon \neq 0 \), the internal terms of trade matter as well. In fact, in an open economy the real marginal costs depend also on the ratio between the consumer price level and the producer price level. This is because the real wage that matters for firms is deflated with the producer price index, while the one relevant for households is deflated with the consumer price index. If the two economies have the same degree of home bias the area-wide real marginal cost, aggregated from the two regions real marginal costs, is affected by the gap between the union CPI and the union PPI which can be rewritten in terms of the external terms of trade.\(^{16}\) When countries are heterogeneous in the degree of home bias (or external trade openness) the gap between the currency area CPI and the PPI does not depend only on the external terms of trade but also on the internal terms of trade.

Given the expression for the real marginal costs, the natural level of output, that is the level of output when prices are fully flexible, is given by\(^{17}\)

\[ \bar{y}_{u,t} = \frac{\nu - \mu}{(\sigma_{\omega} + \varphi)} - \frac{\sigma - \sigma_{\omega}}{\sigma_{\omega}}y_{row,t} - \frac{\sigma_{\omega}\sigma_{\epsilon}}{(\sigma_{\omega} + \varphi)}t_{\epsilon}^m + \frac{1 + \varphi}{(\sigma_{\omega} + \varphi)}a_{u,t}. \]  

(18)

Linearizing and aggregating across regions the firms’ price optimality conditions imply that the inflation dynamics is described by the following New Keynesian Phillips curve\(^{18}\)

\[ \pi_{u,t} = \beta E_{t+1}\pi_{u,t+1} + \lambda rmc_{u,t} \]

where \( \lambda = \frac{(1-\beta)(1-\delta)}{\delta} \). Substituting for the real marginal costs I obtain

\(^{16}\)Using equation (16) the external terms of trade can then be rewritten in terms of domestic and foreign demand, as in equation (17).

\(^{17}\)The variables at their natural level are indicated by a tilde \( \tilde{\text{...}} \).

\(^{18}\)For a complete derivation see Appendix B of Galí and Monacelli (2005).
\[ \pi_{u,t} = \beta E_t \pi_{u,t+1} + \kappa_x u x_{u,t} + \kappa_T u (t^i_t - \tilde{t}^i_t) \]  

(19)

where \( x_{u,t} \) is the output gap (i.e. \( x_{u,t} \equiv y_{u,t} - \tilde{y}_{u,t} \)) and the parameters are given by

\[
\begin{align*}
\kappa_x^u & \equiv \lambda (\sigma_\omega + \varphi) > 0 \\
\kappa_T^u & \equiv \lambda \sigma_\omega \sigma_\varepsilon > 0
\end{align*}
\]

In the absence of heterogeneity, the last term of (19) drops and the dynamics of the currency area inflation is described as the standard one of an open economy. When preferences are heterogeneous, inflationary or deflationary pressures arise also in the case in which the relative price of the currency area goods is not at its natural level.\(^\text{19}\) A positive internal terms of trade gap indicates that the price of good 1, the one preferred by most of the currency area citizens, is lower than it should be in the flexible price equilibrium. This brings about an inefficient reallocation of resources from region 2 toward region 1, generating an increase in the aggregate demand for currency area goods (beyond the one captured by the area-wide output gap term) that drives up inflation. The opening of an internal terms of trade gap implies that there is a trade-off between the output gap and the inflation stabilization in the currency area and that the complete stabilization of inflation is not equivalent to the stabilization of the gap between the actual and the natural output level. This trade-off arises endogenously after any misalignments of the price differentials from their natural level \( \tilde{r}_t \).

From the household intertemporal first order conditions I obtain the following IS curve

\[
x_{u,t} = E_t x_{u,t+1} + \sigma_\varepsilon E_t \Delta (t^i_t - \tilde{t}^i_t) - \frac{1}{\sigma_\omega} (r_t - E_t \pi_{u,t+1} - \tilde{r}_{u,t})
\]

(20)

where \( \tilde{r}_{u,t} \) is the natural rate of interest, which is a function of the primitive shocks in the currency area economy

\[
\tilde{r}_{u,t} \equiv \phi_{yrow} E_t \Delta y_{row,t+1} + \phi_{t,in} E_t \Delta \tilde{t}^i_t + \phi_{a_u} E_t \Delta a_{u,t+1}
\]

\(^{19}\)Note that in a first order approximation the only relevant relative price in the currency area is the internal terms of trade.
where $\phi_{y_{row}}$, $\phi_{in}$ and $\phi_{au}$ are defined in the appendix. Interestingly, the internal terms of trade gap also plays a role in the aggregate demand side of the currency area economy.\footnote{The determinants of natural internal terms of trade are defined in the next section.}

### 3.4.2 The equations of the differentials block

The dynamics of the inflation differentials in the currency area are given by

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \lambda rmc_{r,t}. $$

After solving for the difference of the real marginal costs in the more and less open region, the inflation differentials are given by the following equation

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \kappa_r(t^\text{in}_t - \tilde{t}^\text{in}_t) + \kappa_x x_{u,t}. $$

(21)

where

\begin{align*}
\kappa_r &\equiv \lambda \xi_{ex} \sigma_u > 0 \\
\kappa_T &\equiv \lambda (\xi_{in} + \xi_{ex} \sigma_u \sigma_e) < 0
\end{align*}

As specified in the appendix, the parameter $\xi_{ex}$ is equal to zero when $\varepsilon = 0$. Hence, heterogeneity implies that the area-wide variables, namely the output gap, affect the dynamics of the differentials.

Benigno (2004) points out the disconnection of the terms of trade dynamics from the area-wide block of the model in a currency area with homogenous regions hit by asymmetric shocks and the inability for the authority that controls the area-wide nominal interest rate to affect the differentials. This is a feature of the proposed model when $\varepsilon = 0$. Instead, when $\varepsilon \neq 0$, the area-wide and difference blocks of the model cannot be solved separately.

When prices are fully flexible, the internal terms of trade is affected by asymmetric shocks (i.e. $a_{r,t}$) and, whenever there is heterogeneity across regions, by the area-wide shocks (i.e. $a_{u,t}$ and $y_{row,t}$). The natural internal terms of trade is thus

$$\tilde{t}^\text{in}_t = \gamma_{y_{row}} y_{row,t} + \gamma_{au} a_{u,t} - \gamma_{ar} a_{r,t}. $$

(22)

where $\gamma_{y_{row}}$, $\gamma_{au}$ and $\gamma_{ar}$ are defined in the appendix.

The block of the model that describes the behavior of the differentials within the currency area is closed by specifying the dynamics of the internal
terms of trade gap. From the definition of the internal terms of trade (i.e. $t_{\text{in}}^{t} \equiv p_{2,t} - p_{1,t}$) this is given by

$$t_{\text{in}}^{t} - \tilde{t}_{\text{in}}^{t} = t_{\text{in}}^{t-1} - \tilde{t}_{\text{in}}^{t-1} + \pi_{r,t} - (\tilde{t}_{t}^{m} - \tilde{t}_{t-1}^{m}).$$

(23)

4 The effects of heterogeneity

This section analyzes quantitatively the effect of heterogeneity on the dynamics of the area-wide inflation and output gap, the inflation differentials and the terms of trade in the model set up above. I first describe the calibration for the EMU and then I study the transmission mechanism of a symmetric technology shock under an ad hoc monetary policy rule.

4.1 Calibration

The time period is the quarter. The value for the time-discount parameter $\beta$ is set equal to 0.99 so that the steady state real interest rate is 4% in annual terms. The inverse of the intertemporal elasticity of substitution $\sigma$ is set equal to 2. There is no clear evidence on what should be the value of this parameter. However, in studying the effects of external developments on the domestic economy in the case of complete markets, one wants to depart from the case of logarithmic utility (i.e. $\sigma = 1$) because it implies that the external influences on the domestic economy are shut down and the open economy is isomorphic to the closed one.\(^{21}\) The parameter $\varphi$ represents the inverse of the labor supply elasticity and, as is common in the real business cycle literature, it is set at 3. The elasticity of substitution between varieties of each differentiated good ($\theta$) is equal across countries and it is set so that the steady state mark up is 1.2. Therefore, $\theta$ equals 6. I calibrate the Calvo parameter $\delta$, which indicates the degree of price stickiness using the standard value of 0.75, as it is frequent in the literature. This implies an average duration of the price contract of four quarters.

The parameters for the degrees of openness that need to be calibrated are $\alpha$, $\omega$ and $\varepsilon$. The first one is the degree of internal openness and it is calibrated as the average of the GDP share of imports from other members of the union across 11 European countries.\(^ {22}\) In fact, there is heterogeneity

\(^{21}\)See Clarida et al. (2001).

\(^{22}\)Luxembourg, Slovenia, Cyprus, Malta and Slovakia are excluded.
across member countries in this parameter too. However, for the reasons explained in section 2, I restrict it to be the same across countries so as to insulate the effect of heterogeneity in the degree of external openness. Hence, I set $\alpha = 0.25$. The parameter for external openness $\omega$ is set equal to the average of the GDP share of the external imports across 11 member countries of the euro area, that is $\omega = 0.17$. The parameter that measures heterogeneity $\varepsilon$ takes the values of 0 and 0.14. The former represents the theoretical case of homogeneous regions whereas the latter is set so that $\omega + \varepsilon$ is the average degree of external openness across the more open countries (the Netherlands, Belgium and Ireland).

There are three sources of exogenous fluctuations in the model: the aggregate technology shock $a_{u,t}$, the asymmetric technology shock $a_{r,t}$ and the external shock $y_{row,t}$. I assume that their stochastic processes are all AR(1). To calibrate the parameters of the external shock process I use the log of the world demand variable of the Area Wide Model dataset (Fagan et al., 2001) and I fit the following univariate trend-stationary process.

$$
y_{row,t} = 0.60 + 0.0003t + 0.94y_{row,t-1} + \epsilon_{row,t}.
$$

The standard deviation of the innovations is $\sigma_{y_{row}} = 0.005859$. In order to calibrate the shocks for the area-wide productivity and differentials, I consider the primitive technology shocks in the two regions and I specify them as

$$
a_{1,t} = \rho a_{1,t-1} + \epsilon_{1,t} \\
a_{2,t} = \rho a_{2,t-1} + \epsilon_{2,t}.
$$

The persistence parameter $\rho$ and the standard deviation of the innovation $\sigma_{\epsilon}$ are the same across regions and equal respectively to 0.94 and 0.0061, based on the evidence of Smets and Wouters (2005) for the entire euro area. Moreover the shocks are uncorrelated across regions (i.e. $corr(\epsilon_{1,t}, \epsilon_{2,t}) = 0$). Hence, the processes for $a_{u,t} \equiv ma_{1,t} + (1 - m)a_{2,t}$ and $a_{r,t} \equiv a_{2,t} - a_{1,t}$ have the following specification

$$
a_{u,t} = \rho a_{u,t-1} + \epsilon_{u,t} \\
a_{r,t} = \rho a_{r,t-1} + \epsilon_{r,t}.
$$

where the variance of the innovations are $\sigma_u^2 = [m^2 + (1 - m)^2] \sigma_\epsilon^2$ and $\sigma_r^2 = 2\sigma_\epsilon^2$.

---

4.2 The transmission mechanism of area-wide technology shocks

To highlight the effect of heterogeneity in the transmission mechanism of an exogenous disturbance and its interaction with nominal rigidities, I consider the responses to an i.i.d. area-wide technology shock under a monetary policy regime of strict inflation targeting. The monetary policy is represented by the following targeting rule

\[ \pi_{u,t} = 0. \]

Figure 3 displays the responses of the area-wide and differentials variables to a positive one percent symmetric shock in technology \((a_u)\) in the case of homogeneous (solid line) and heterogeneous regions (starred line).

A positive technology shock induces a decline in the natural interest rate of the currency area. In order to keep inflation stable the monetary authority fully accommodates this decline by decreasing the nominal interest rate. If the rest of the world does not respond to the shock, the monetary policy response generates a depreciation of the nominal exchange rate on impact and an increase of the external terms of trade.

When the regions of the currency area are homogeneous (i.e. \(\varepsilon = 0\)) the dynamics of the model are standard. Under the policy of strict inflation stabilization, the central bank adjusts the nominal interest rate so that the area-wide inflation remains stable. This policy results in closing the gap between the actual and natural output level. The currency area goods are more competitive following the exogenous rise in productivity and the depreciation of the nominal exchange rate induced by the monetary policy response. Moreover, the internal terms of trade is not affected by the area-wide technology shock and no inflation differentials are generated.

When the regions of the currency area have different degrees of exposure to the rest of the world (i.e. \(\varepsilon \neq 0\)), the nominal depreciation generated by the monetary policy response has a different impact on the two regions. The demand for the goods produced by the more open region (good 2) increases more than the one for the other goods produced in the area. The different demand pressures induce an adjustment of the relative price of the goods in the two regions, that is an increase in the internal terms of trade.

The dynamics of the inflation differentials are driven by the gap between the actual internal terms of trade and its flexible price level and by the area-wide output gap. As pointed out by Aoki (2001) and Benigno (2004), the
Figure 3: IRFs to a positive area-wide technology shock under strict inflation targeting

Notes: The shock is equal to 1% st dev of the area-wide technology. The solid line is the homogeneity case ($\varepsilon = 0$); the starred line the heterogeneity one ($\varepsilon = 0.14$).

complete price stabilization brought about by the policy rule considered here has the effect of inducing a sluggish response of the actual internal terms of trade. The internal terms of trade gap is thus negative on impact and positive in subsequent periods. A negative internal terms of trade gap indicates that, due to nominal rigidities, the relative price of the goods produced in

\[ Aoki (2001) \text{ and Benigno (2004) focus on changes in the natural level of the relative price (or internal terms of trade) caused by asymmetric shocks across countries (or sectors). Here these changes are brought about by the heterogeneous transmission mechanism of a symmetric shock.} \]
the more open region is lower than in the flexible price equilibrium. This
generates inflationary pressures on that region and thus an inflation differ-
ential. The increase of the area-wide output gap in the quarter of the shock
exacerbates the effect on the inflation differentials. Inheriting the sluggi-
ghess of the internal terms of trade, the inflation differentials are slightly more
persistent than the shock that generates them.  

5 Optimal monetary policy

I compute and analyze the solution of the model presented above under a
policy that maximizes the welfare of the currency area households. I obtain a
microfounded measure of welfare by taking a second order approximation of
the utilities of the currency area citizens. The optimal monetary policy prob-
take the form of a standard linear-quadratic optimization problem as is
common in the literature. In this section I describe the distortions of the
market allocation with respect to the Pareto efficient one, the welfare mea-
sure adopted and some moments that characterize the optimal policy plan,
namely the impulse response functions after an area-wide and an asymmet-
ric technology shock. Throughout the analysis it is assumed that the two
regions are of equal size. In the last subsection, I consider the case in which
the more open region is smaller in order to check how size affects the results.

5.1 Flexible price equilibrium and the Pareto efficient
allocation

In the open currency area described above three distortions move the market
allocation away from the Pareto efficient one. Firstly, firm’s market power
lowers the output relative to the one in the efficient equilibrium. Secondly,
the presence of nominal rigidities alter the relative prices, thereby causing
a misallocation of resources across the varieties of the differentiated goods.

25Whenever there is heterogeneity across regions, the sluggishness of the internal terms
of trade is transmitted to the area-wide variables. The effect is, however, quantitatively
small in a two-region economy. Carlstrom et al. (2006) show that differences in the degree
of price stickiness or asymmetric sectoral monetary policy responses generate movements
in relative prices that affect the inflation and output gap in aggregate. In this case,
asymmetric responses to area-wide shocks, due to heterogeneity in preferences, is what
generates a terms of trade dynamics.

26See Benigno and Woodford (2006).
Furthermore, the market equilibrium allocation does not take into account the incentive to distort the terms of trade with the rest of the world in a way that is beneficial to the currency area citizens.\textsuperscript{27} The social planner of the currency area, in fact, would tend to increase domestic consumers’ purchasing power internationally. An improvement in the external terms of trade induces the currency area citizens to consume more of the imported good, reducing the hours worked without diminishing their overall level of consumption. Heterogeneity \textit{per se} does not introduce a distortion in the economy. It is the stickiness of prices coupled with the fact that the nominal exchange rate across regions is irrevocably fixed that generates an additional distortion in the economy: the sluggish adjustment of the relative price of the goods produced in the two regions after a shock implies a misallocation of the resources.\textsuperscript{28}

Galí and Monacelli (2005) show the optimality of strict domestic inflation stabilization for a special parametrization under which both the intertemporal and the intratemporal elasticities of substitution are equal to unity. In this case a constant subsidy to monopolistic competitive firms in steady state is sufficient to guarantee the coincidence between the flexible price allocation and the efficient one. When the subsidy is implemented, the only distortion left in the economy is price stickiness; setting inflation to zero delivers the Pareto efficient allocation. However, in this special case the economy behaves as if it is closed. Faia and Monacelli (2008) and De Paoli (2009) relax these assumptions and show that, in the more realistic case in which the three distortions mentioned above are in place, the markup stabilization implemented by the flexible price allocation is not optimal and some inflation volatility, together with a partial stabilization of the exchange rate, is instead advisable.

Since this paper studies how the degree of openness affects monetary policy design in a currency area it is important to consider a calibration so that the currency area economy is not isomorphic to a closed one. For this reason in the benchmark calibration the intertemporal elasticity of substitution is different from one. Thus, in the open currency area, even when the labor subsidy is in place, the flexible price equilibrium allocation is not optimal, independently to the degree of heterogeneity across regions.

\textsuperscript{27}See, among others, Corsetti and Pesenti (2001) and De Paoli (2009).
\textsuperscript{28}Aoki (2001) and Benigno (2004) highlight this distortion.
5.2 The currency area welfare function

This subsection describes the welfare measure for the currency area economy, which is based on the discounted sum of the future utility flows of the households of both regions of the monetary union. For region $i = 1, 2$ the utility flows are given by

$$w^i_t = U(C^i_t, N^i_t) \equiv \frac{(C^i_t)^{1-\sigma} - (N^i_t)^{1+\varphi}}{1 - \sigma}. \frac{1+\varphi}{1+\varphi}.$$

Depending on the region in which they live, there are two types of agents in the currency area. As monetary policy is decided at an area-wide level, the relevant welfare function must be only one and a criterion to aggregate the utility functions of each type of agent is needed. I consider a utilitarian social welfare function. Among the aggregation functions, this specification places less emphasis on the heterogeneity across regions and implies that the currency area welfare function is a weighted average of the welfare of each region, where the weights are equal to the size of the regions, that is

$$W^u = E_0 \sum_{t=0}^{\infty} \beta^t \left[ mw_1^i + (1 - m) w_2^i \right].$$

In order to have the optimal monetary policy problem in the standard linear-quadratic form I take a second order Taylor expansion of the utility flows of each region around the optimal deterministic steady state. Then I aggregate across regions and, after substituting for the linear terms using the second order approximation of the model’s equilibrium condition, I obtain a purely quadratic approximation of the currency area welfare function. The objective of the currency area benevolent monetary authority can be written as follows

$$-E_0 \sum_{t=0}^{\infty} \beta^t L^u_t$$

where $L^u_t$ is a measure of the deadweight losses:

$$L^u_t = \left\lbrace \Phi_{\pi_u \pi_{u,t}}^2 + \Phi_{x_u} \left( y_{u,t} - \bar{y}_{u,t} \right)^2 + \Phi_{\pi_{t^h}} \left( t_{t^h} - \bar{t}_{t^h} \right)^2 + \Phi_{\pi_r \pi_{r,t}} + \Phi_{x_u \pi_{t^h}} \left( y_{u,t} - \bar{y}_{u,t} \right) \left( t_{t^h} - \bar{t}_{t^h} \right) + t.i.p. + o(\|\xi\|^3) \right\rbrace.$$

Details on the derivation of this function are in the Appendix. A double tilde $\tilde{\tilde{\cdot}}$ on a variable indicates that it is at its efficient level.
where a double tilde on a variable indicates its efficient level, \textit{t.i.p.} stands for terms independent from policy and \(o(\|\xi\|^3)\) contains all terms that are of order higher than two in the given bound for the vector of shocks \(\xi \equiv [y_{row} \ a_u \ a_r]'\). The deadweight losses are generated by any deviation of the actual variables from their efficient level. The parameters \(\Phi\)s are functions of the structural parameters of the model and their values under the baseline calibration described in section 4.1 are reported in table 1.

**Table 1:** The coefficients of the utility-based loss function under the benchmark calibration

<table>
<thead>
<tr>
<th>(\varepsilon)</th>
<th>0</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Phi_{\pi_u})</td>
<td>69.9</td>
<td>69.9</td>
</tr>
<tr>
<td>(\Phi_{x_u})</td>
<td>3.54</td>
<td>3.52</td>
</tr>
<tr>
<td>(\Phi_{\pi_r})</td>
<td>17.5</td>
<td>17.5</td>
</tr>
<tr>
<td>(\Phi_{t_{in}})</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>(\Phi_{x_u,t_{in}})</td>
<td>0</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

When \(\varepsilon = 0\), the loss function in (24) is similar to the one obtained in Benigno (2004). In particular, compared with the loss function of a closed economy, there are two additional quadratic terms beyond the ones on inflation and the output gap: the internal terms of trade gap and the inflation differentials. The deviation of the internal terms of trade from its efficient level generates losses as resources are inefficiently moved from one region to the other. As explained above, the internal terms of trade does not adjust immediately to region-specific fluctuations because there is price stickiness and the nominal exchange rate is irrevocably fixed.

Moreover, equation (24) features not only a term that penalizes the fluctuations in area-wide inflation, but also one that penalizes those in the inflation differentials. As shown by Woodford (2003, ch. 6) the presence of nominal rigidities in the form of Calvo price stickiness generates a motive for inflation stabilization. Since we assumed that this distortion in the market equilibrium is present in the production of goods in both regions, implementing the efficient allocation requires that the inflation of the domestic goods produced in each region is stabilized. A loss function that penalizes only the aggregate currency area inflation does not guarantee that the volatilities of the national inflations will be minimized. What is required is that both the
area-wide inflation and the differentials across regions are stabilized.\textsuperscript{30}

When \( \varepsilon \neq 0 \), the negative comovement between the area-wide output gap and the internal terms of trade gap gives rise to a deadweight loss (\( \Phi_{u,t}^{in} \) is zero when \( \varepsilon = 0 \) and negative when \( \varepsilon > 0 \)). A positive internal terms of trade gap indicates a competitive advantage for the country that is less oriented towards producing goods for the rest of the world market, inducing a decline in the demand for currency area goods coming from the rest of the world. When this is coupled with a negative area-wide output gap the efficiency cost of the shift of the work effort toward the more competitive country is higher.

5.3 The IRFs to an area-wide technology shock under the optimal plan

Figure 4 describes the impulse response functions (IRFs) to an area-wide technology shock under the optimal policy plan, comparing the case in which regions are homogeneous (\( \varepsilon = 0 \)) with the one in which regions are heterogeneous (\( \varepsilon = 0.14 \)).\textsuperscript{31}

An area-wide technology shock increases the relative competitiveness of the currency area goods with respect to those produced in the rest of the world. The optimal policy accommodates this increase in competitiveness through a nominal exchange rate depreciation and an increase in the external terms of trade \( t_{t}^{ex} \). Due to the external terms of trade externality, however, the social planner wants to partially dampen the response of the exchange rate, therefore the area-wide inflation is not perfectly stabilized after the shock. When regions are homogeneous, an area-wide shock affects all member countries of the area equally; thus the internal terms of trade gap and the inflation differential do not move.

When regions are open to different degrees toward the rest of the world, the more open region (region 2) benefits more from the gain in competitiveness as the rest of the world prefers its goods to the ones produced in the other region. Hence, in the flexible price equilibrium, the higher demand of good 2 leads to an increase in its relative price. In the sticky price equilibrium, the internal terms of trade gap cannot adjust immediately and a negative terms

\textsuperscript{30}Notice that in equation (24) \( \Phi_{\pi_{a}} = m(1 - m)\Phi_{\pi_{a}} \). Thus, I could have written \( \Phi_{\pi_{a}} [m\pi_{1,t}^{2} + (1 - m)\pi_{2,t}^{2}] \) instead of \( \Phi_{\pi_{a}}\pi_{u,t}^{2} + \Phi_{\pi_{a}}\pi_{r,t}^{2} \).

\textsuperscript{31}The equations that fully describe the optimal policy plan are specified in the appendix.
**Figure 4:** IRFs to a positive area-wide technology shock under the optimal policy

Notes: The shock is equal to 1% st dev of the area-wide technology. The starred line is the homogeneity case ($\epsilon = 0$); the solid line the heterogeneity one ($\epsilon = 0.14$).

of trade gap opens. This induces higher inflationary pressures in the more open region and therefore an increase in inflation differentials.

The optimal policy must balance the objective of stabilizing the area-wide output gap and inflation with the one of stabilizing the inflation dif-
ferentials and the internal terms of trade. It thus prefers to attenuate the
depreciation of the nominal exchange rate relative to the homogeneous case
since this depreciation would amplify the inflation differentials across regions.
This implies that the area-wide inflation displays a larger volatility when re-
gions are heterogeneous. The presence of heterogeneity, therefore, induces a
stronger motive for exchange rate stabilization and goes against the complete
stabilization of inflation.

Figure 5 displays the responses of the nominal interest rate, following
each of the shocks of the model, which is implied by the optimal policy
plan. After a positive technology shock the nominal interest rate reacts
more on impact when regions are heterogeneous than in the homogeneous
case; afterward it decreases more sharply in the homogeneous case in order
to allow for a stronger depreciation in the monetary union currency. The
opposite happens when a shock coming from the rest of the world hits the
currency area economy.

Figure 6 shows the responses of the main macroeconomic variables to
an area-wide technology shock under a policy of strict inflation stabilization
and under the optimal policy when regions are heterogeneous.\(^\text{32}\) In this case
the complete stabilization of the area-wide inflation does not imply the sta-
bilization of the welfare-relevant output gap. The heterogenous transmission
mechanism across regions induces fluctuations of the distance of the internal
terms of trade from its efficient level, which in turn affect the relationship
between area-wide inflation and the output gap as described by the New
Keynesian Phillips curve. Compared to the optimal policy, a strict inflation
targeting rule induces a stronger response of the nominal exchange rate of
the currency area and generates higher inflation differentials. Both these
variables are instead partially stabilized under the optimal policy.

Under the benchmark calibration, the relatively considerable weight on
the inflation differentials in the objective of the currency area central bank
induces the social planner to care about inflation differentials stabilization.
However, it is the interaction between the area-wide and the difference block
of the model introduced by the heterogeneity in the external openness that
allows the central bank to affect them even if it controls only the area-wide
interest rate. The optimal policy plan internalizes that with heterogeneity the
transmission mechanism of monetary policy impulses differs across regions
and makes active use of it in order to influence the differentials.

\(^{32}\)The strict inflation stabilization policy is described by the targeting rule \(\pi_{u,t} = 0\).
5.4 The IRFs to an asymmetric technology shock under the optimal plan

Figure 7 displays the impulse response functions to an asymmetric technology shock ($a_{r,t}$). It can be interpreted as a shock in the efficiency level of the production of one good of the currency area relative to the other. In particular, a positive variation of $a_{r,t}$ implies a gain in competitiveness in the more open region (region 2) relative to the more closed one (region 1).

Following the shock, a positive internal terms of trade gap opens because the presence of price stickiness implies that the relative price across regions
Figure 6: The IRFs to a positive area-wide technology shock under different policy regimes

Notes: The IRFs are obtained for a calibration of $\varepsilon = 0.14$ under the optimal policy (solid line) and the strict inflation targeting (IT) regime (starred line)

does not adjust immediately. This gap affects the inflation differentials: inflation is higher in region 1 because the distortion in the relative price caused by the stickiness inefficiently boosts the demand of good 1.
Figure 7: The IRFs to an asymmetric technology shock under the optimal policy plan

Notes: The asymmetric shock is an exogenous increase in the relative productivity of the more open region; the starred line is the homogeneity case ($\varepsilon = 0$); the solid line the heterogeneity one ($\varepsilon = 0.14$).

When the regions are homogenous, the area-wide variables are not affected by the shock and only the relative prices and quantities adjust. The differential block of the model is disconnected from the area-wide one and
monetary policy, having as the only instrument the currency union nominal interest rate, cannot undo the distortion in the internal terms of trade generated by nominal rigidities. When, instead, $\varepsilon \neq 0$ the area-wide inflation and output gap are affected. Since sizable inflation differentials generate welfare losses, the central bank raises the output gap under the optimal policy in order to partially mitigate the inflation differentials (see equation (21)). This in turn affects the area-wide inflation that it is slightly increasing after the shock.

### 5.5 The effect of the size of the regions

One of the common features of the more open block of countries of the EMU is that they are smaller in size compared to the less open block. It is thus of interest to analyze how the size of the two regions affects the results.

**Figure 8:** The effect of region size on the inflation response to a technology shock in the optimal policy

Figure 8 displays the optimal response of the area-wide inflation to a
positive 1% standard deviation of the area-wide technology shock when the regions have the same size (left panel) and when the size of region 2, the one that is more open to the rest of the world, is one third of the total size of the currency area (right panel).

When the regions are the same size, the response of inflation in an economy with heterogeneous degrees of external openness (i.e. $\varepsilon = 0.14$) is almost twice as strong as that in the region with homogeneous degrees of openness. When instead region 2 is smaller than region 1, the decrease of inflation on impact is about 40% stronger when $\varepsilon = 0.14$ than in the homogeneity case.

6 Conclusions

The degree of external openness is an important element that shapes the transmission mechanism of global shocks in the EMU countries. Evidence shows that the member countries differ in this respect.

The paper highlights the effects of heterogeneity on external trade openness among the regions of a currency area in the optimal design of monetary policy. It builds a framework for an open currency area economy composed of two regions that have different preferences toward the goods produced by the rest of the world outside the currency area. After obtaining a welfare measure that is consistent with the model and microfounded, the optimal monetary policy from the currency area viewpoint is derived.

When the currency area regions have different degrees of external openness, both the transmission mechanism of the shocks and the microfounded welfare function differ from the baseline case of a homogeneous currency area. Heterogeneity in external trade generates a stronger motive for exchange rate stabilization in the optimal policy plan. Accordingly, more inflation volatility is advisable. For example, after a positive area-wide technology shock the central bank should not accommodate the shock perfectly and should decrease the nominal interest rate by less than what is predicted by the optimal policy for a homogeneous currency area. In this way the union nominal exchange rate depreciation is lower and the inflation differentials, generated by the different exposure of the two regions toward the rest of the world, are partially alleviated.

The results of the paper suggest that some degree of heterogeneity across regions is sufficient for the cross-region dispersion to influence the area-wide
variables dynamics and to be of importance for welfare. The optimal monetary policy plan takes into account the structural differences across regions and balances the welfare losses coming from area-wide inflation and the output gap with those coming from the inflation differentials.

References


Appendix

A The (Sum and Difference) Solution of the Model

In this appendix I derive in detail the solution of the model in terms of area-wide and differentials variables (that is the equations described in Section 3.4). I consider the case in which the two regions of the currency area have equal size (i.e. \( m = 0.5 \)). I loglinearized the model around the trade-balanced deterministic steady state. For a generic variable \( x \) I define the area-wide corresponding variable as \( x_u \equiv mx_1 + (1 - m)x_2 \) and the differentials as \( x_r \equiv x_2 - x_1 \). As specified above, I assume that the rest of the world is a large country relative to the currency area (specifically, the amount of goods produced in the currency area and consumed by the rest of the world is negligible in its basket of consumption, thus one can say \( C^\text{row} = Y^\text{row} \)).

A.1 Derivation of equation (16)

Consider first the aggregate demand of goods produced in region 1 (equation (15))

\[
Y_{1,t} = (1 - \alpha - \omega + \varepsilon) \left( \frac{P_{1,t}}{P_{1,t}} \right)^{-1} C^1_t + \alpha \left( \frac{P_{1,t}}{P_{2,t}} \right)^{-1} C^2_t + (\omega - \varepsilon) \left( \frac{P_{1,t}}{P^e_{\text{row},t} S_t} \right)^{-1} C^\text{row}_t.
\]

Using the risk sharing conditions (11) to substitute for the consumption of region 2 and of the rest of the world, I obtain

\[
Y_{1,t} = \left( \frac{P_{1,t}}{P^e_{\text{row},t} S_t} \right)^{-1} C^1_t \left[ (1 - \alpha - \omega + \varepsilon) + \alpha \left( \frac{P^e_{1,t}}{P^e_{2,t} S_t} \right)^{\frac{1}{\sigma}} + (\omega - \varepsilon) \right].
\]

Loglinearizing around the steady state I obtain

\[
y_{1,t} = \left[ \alpha + \frac{1}{2} (\omega - \varepsilon) \right] t^\text{in} + (\omega - \varepsilon) t^\text{ex} + c^1_t + \frac{\sigma - 1}{\sigma} \left[ \alpha q^\text{in} + (\omega - \varepsilon) q^\text{ex} \right] (25)
\]

where I used the following definitions for the internal and external terms of trade and real exchange rates

\[
t^\text{in} = p_2 - p_1
\]

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\begin{align*}
    t_{it}^e &= p_{it}^e + s - p_{it}^u \\
    q_{it}^{in} &= p_{it}^2 - p_{it}^1 \\
    q_{it}^{ex} &= p_{it}^f + s - p_{it}^u.
\end{align*}

Using (6) and (7) I derived the (log linearized) relationships between the (internal and external) terms of trade and the real exchange rates

\begin{align*}
    q_{it}^{in} &= (1 - 2\alpha - \omega)t_{it}^{in} + 2\varepsilon t_{it}^{ex} \quad (26) \\
    q_{it}^{ex} &= \frac{\varepsilon}{2}t_{it}^{in} + (1 - \omega)t_{it}^{ex}. \quad (27)
\end{align*}

Substituting (26) and (27) into the aggregate demand of goods produced in region 1 I have

\begin{align*}
    y_{1,t} = \left[\alpha\Phi + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right]t_{it}^{in} + (\omega - \varepsilon)\Gamma_1 t_{it}^{ex} + c_{it}^1. \quad (28)
\end{align*}

Analogously, the aggregate demand for goods produced in region 2 is

\begin{align*}
    y_{2,t} = -\left[\alpha\Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2\right]t_{it}^{in} + (\omega + \varepsilon)\Gamma_2 t_{it}^{ex} + c_{it}^2. \quad (29)
\end{align*}

where

\begin{align*}
    \Phi &\equiv 1 + \frac{\sigma - 1}{\sigma} (1 - 2\alpha - 2\omega) \\
    \Gamma_1 &\equiv 1 + \frac{\sigma - 1}{\sigma} \left(\frac{2\alpha\varepsilon}{\omega - \varepsilon} + 1 - \omega + \varepsilon\right) \\
    \Gamma_2 &\equiv 1 + \frac{\sigma - 1}{\sigma} \left(1 - \omega - \varepsilon - \frac{2\alpha\varepsilon}{\omega + \varepsilon}\right).
\end{align*}

Notice that when \(\varepsilon = 0\), \(\Gamma_1 = \Gamma_2 = 1 + \frac{\sigma - 1}{\sigma}(1 - \omega)\). When the utility is logarithmic (i.e. \(\sigma = 1\)), \(\Gamma_1 = \Gamma_2 = \Phi = 1\). Aggregating over the regions of the currency area, the aggregate demand of the currency area goods is

\begin{align*}
    y_{u,t} = \frac{1}{4}[(\omega - \varepsilon)\Gamma_1 - (\omega + \varepsilon)\Gamma_2]t_{it}^{in} + \frac{1}{2}[(\omega - \varepsilon)\Gamma_1 + (\omega + \varepsilon)\Gamma_2]t_{it}^{ex} + c_{it}^u \quad (30)
\end{align*}

Using (11), the following risk sharing condition between the currency area and the rest of the world holds

\(C_t^{\text{ex}} = (Q_t^{\text{ex}})^{1/\sigma} C_t^{\text{row}}\).
Loglinearizing and using (26) and (27), one obtains
\[
c_t^u = \frac{1}{\sigma} \left[ (1 - \omega) t_t^{ex} + \frac{\varepsilon}{2} t_t^{in} \right] + c_t^{row} \tag{31}
\]
which, substituted into (30), yields to equation (16) of the text
\[
y_{u,t} = \frac{1}{\sigma_t} t_t^{ex} + y_{row,t} - \sigma_t t_t^{in}
\]
where
\[
\sigma_{\omega}^{-1} = \frac{1}{2} \left[ (\omega - \varepsilon) \Gamma_1 + (\omega + \varepsilon) \Gamma_2 + \frac{2(1 - \omega)}{\sigma} \right]
\]
\[
\sigma_{\varepsilon} = \frac{1}{4} \left[ (\omega + \varepsilon) \Gamma_2 - (\omega - \varepsilon) \Gamma_1 + \frac{2\varepsilon}{\sigma} \right].
\]

When \( \sigma = 1 \) (i.e., log utility in consumption) the parameter \( \sigma_\omega \) is equal to one and \( \sigma_\varepsilon \) is equal to zero independently of \( \varepsilon^{33} \).

### A.2 Derivation of equation (17)

From the labor market clearing condition (14) for both regions of the currency area using the production function and the definition of the internal and external terms of trade, I obtain the following log linear expressions for the real marginal costs in region 1 and 2
\[
rmc_{1,t} = -\nu + \sigma c_t^1 + \varphi y_{1,t} + \left[ \alpha + \frac{1}{2} (\omega - \varepsilon) \right] t_t^{in} + (\omega - \varepsilon) t_t^{ex} - (1 + \varphi) a_{1,t}, \tag{32}
\]
\[
rmc_{2,t} = -\nu + \sigma c_t^2 + \varphi y_{2,t} - \left[ \alpha + \frac{1}{2} (\omega + \varepsilon) \right] t_t^{in} + (\omega + \varepsilon) t_t^{ex} - (1 + \varphi) a_{2,t}. \tag{33}
\]

Aggregating over the regions of the currency area the area-wide real marginal costs are
\[
rmc_{u,t} = -\nu + \sigma c_t^u + \varphi y_{u,t} - \frac{\varepsilon}{2} t_t^{in} + \omega t_t^{ex} - (1 + \varphi) a_{u,t}.
\]

Using the risk sharing condition (31) and substituting \( t_t^{ex} \) using (16), I obtain equation (17) of the text, that is
\[
rmc_{u,t} = -\nu + (\sigma_\omega + \varphi) y_{u,t} + (\sigma - \sigma_\omega) y_{row,t} + \sigma_\omega \sigma_\varepsilon t_t^{in} - (1 + \varphi) a_{u,t}
\]

\(^{33}\)This is the special case in which an open economy is isomorphic to a closed economy. See Clarida et al. (2001).
A.3 Derivation of equation (20)

From the intertemporal first order conditions of the households living in the currency area I have the following (area-wide) Euler equation

\[ c^u_t = E_t c^u_{t+1} - \frac{1}{\sigma} \left( r_t - E_t \pi^c_{u,t+1} \right). \]

Using (6) and (7) and the definition of the terms of trade, the following is the relationship between the CPI and the PPI inflation in the currency area

\[ \pi^c_{u,t} = \pi_{u,t} + \omega \Delta t^e_t - \frac{\varepsilon}{2} \Delta t^i_t. \]

Substituting the last expression and (30) into the area-wide Euler equation above, I have

\[ y_{u,t} = E_t y_{u,t+1} + \sigma \varepsilon E_t \Delta t^i_{t+1} - \Theta E_t \Delta t^e_t - \frac{1}{\sigma} (r_t - E_t \pi_{u,t+1}), \]

where \( \Theta \equiv \frac{1}{2} \left[ (\omega - \varepsilon) \Gamma_1 + (\omega + \varepsilon) \Gamma_2 - \frac{2\omega}{\sigma} \right]. \) Using equation (16) to substitute for \( t^e_t \) and rearranging the terms, I obtain

\[ y_{u,t} = E_t y_{u,t+1} + \sigma \varepsilon E_t \Delta t^i_{t+1} - \frac{1}{\sigma \omega} (r_t - E_t \pi_{u,t+1} + \Theta \sigma E_t \Delta y^\text{row}_{u,t+1}) \]

Adding and subtracting the natural rate of the currency area output (18) and rearranging terms, I can rewrite the IS equation in terms of the output gap and the real interest rate gap as in equation (20), that is

\[ x_{u,t} = E_t x_{u,t+1} + \sigma \varepsilon E_t \Delta (\tilde{t}^i_{t+1} - \tilde{r}^i_{t+1}) - \frac{1}{\sigma \omega} (r_t - E_t \pi_{u,t+1} - \tilde{r}^r_{u,t}). \]

The natural rate of the real interest rate is defined as

\[ \tilde{r}^r_{u,t} \equiv \phi^r E_t \Delta y^*_t + \phi^i E_t \Delta \tilde{t}^i_{t+1} + \phi^u E_t \Delta a_{u,t+1} \]

where the coefficients are defined as

\[
\begin{align*}
\phi^r_y &\equiv \sigma \omega \left( \Theta \sigma - \sigma - \sigma\omega \right) \\
\phi^i_t &\equiv +\frac{\sigma \omega \varepsilon \varphi}{\sigma \omega + \varphi} \\
\phi^u a &\equiv \frac{\sigma \omega (1 + \varphi)}{\sigma \omega + \varphi}
\end{align*}
\]
A.4 Derivation of equation (21) and (22)

In order to obtain the equation that describes the dynamics of the inflation differentials between the two regions of the currency area, I derive the real marginal costs differentials. Subtracting (32) from (33) I have the differentials of real marginal costs

$r_{mc{r,t}} = \sigma c_t^r + \varphi y_{r,t} - (2\alpha + \omega) t_{t}^{in} + 2\varepsilon t_{t}^{ex} - (1 + \varphi)a_{r,t}$.  \hspace{1cm} (34)

Subtracting (28) from (29), I have the following expression for the differentials of output in the two regions

$y_{r,t} = -\left[2\alpha \Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2 + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right] t_{t}^{in} + [(\omega + \varepsilon)\Gamma_2 - (\omega - \varepsilon)\Gamma_1] t_{t}^{ex} + c_{r,t}$.

The risk sharing condition (11) together with (26) and (27) implies that

$c_t^r = -\frac{1}{\sigma}(1 - 2\alpha - \omega) t_{t}^{in} - \frac{2\varepsilon}{\sigma} t_{t}^{ex}$.

Combining the last two equations with (34), I obtain

$r_{mc{r,t}} = \xi_{in} t_{t}^{in} + \xi_{ex} t_{t}^{ex} - (1 + \varphi)a_{r,t}$,

where

$\xi_{in} \equiv -\frac{\sigma + \varphi}{\sigma}(1 - 2\alpha - \omega) - \varphi \left[2\alpha \Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2 + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right] - (2\alpha + \omega)$

$\xi_{ex} \equiv -\frac{\sigma + \varphi}{\sigma} 2\varepsilon + \varphi \left[(\omega + \varepsilon)\Gamma_2 - (\omega - \varepsilon)\Gamma_1\right] + 2\varepsilon$.

Notice that when $\varepsilon = 0$, $\xi_{ex} = 0$ implying that the external competitiveness of the currency area toward the rest of the world is not affecting the real marginal cost differentials inside the union. Substituting $t_{t}^{ex}$ using equation (16), I have

$r_{mc{r,t}} = (\xi_{in} + \xi_{ex}\sigma_\omega \sigma_\varepsilon) t_{t}^{in} + \xi_{ex}\sigma_\omega y_{u,t} - \xi_{ex}\sigma_\omega y_{row,t} - (1 + \varphi)a_{r,t}$  \hspace{1cm} (35)

In the flexible price equilibrium the real marginal costs are equal to the opposite of the mark up in both regions. Since the monopolistic distortion is assumed to be the same across regions of the currency area, the differential of the real marginal costs is equal to zero when prices are fully flexible. Making
use of this fact, by setting equation (35) equal to zero I obtain the expression for the internal terms of trade that is in (22), that is

$$\tilde{t}^i_t = \gamma_{y_{row}}y_{row,t} + \gamma_{a_u}a_{u,t} - \gamma_{a_r}a_{r,t}$$

where

$$\gamma_{y_{row}} \equiv \frac{\xi_{ex}\sigma_{\omega} (\sigma + \varphi)}{\xi_{in}(\sigma_{\omega} + \varphi) + \varphi \xi_{ex}\sigma_{\omega}\sigma_{\varepsilon}}$$

$$\gamma_{a_u} \equiv -\frac{(1 + \varphi)\xi_{ex}\sigma_{\omega}}{\xi_{in}(\sigma_{\omega} + \varphi) + \varphi \xi_{ex}\sigma_{\omega}\sigma_{\varepsilon}}$$

$$\gamma_{a_r} \equiv \frac{(1 + \varphi)(\sigma_{\omega} + \varphi)}{\xi_{in}(\sigma_{\omega} + \varphi) + \varphi \xi_{ex}\sigma_{\omega}\sigma_{\varepsilon}}.$$ 

The New Keynesian Phillips curve for the inflation differentials is given by

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \lambda rmc_{r,t}.$$ 

Thus, using (35) and (22), I obtain equation (21) of the text.

**B Derivation of the approximated welfare loss function**

The utility of the representative agent in each region $i$ of the currency area is the following

$$U(C_i^t, N_i^t) \equiv \frac{(C_i^t)^{1-\sigma}}{1-\sigma} - \frac{(N_i^t)^{1+\varphi}}{1+\varphi}$$

Taking a second order Taylor expansion around the optimal steady state I have

$$W_{i,t} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \bar{C}_i^{1-\sigma} \left[ \bar{c}_{i,t} + \frac{(1-\sigma)}{2} \bar{c}_{i,t}^2 \right] - \bar{N}_i^{1+\varphi} \left[ y_{i,t} + z_{i,t} + \frac{(1+\varphi)}{2} y_{i,t}^2 \right] \right\} + t.i.p. + o(||\xi||^3)$$

where in $t.i.p.$ are collected all terms that are independent from the monetary policy and in $o(||\xi||^3)$ the terms that are of order higher than two. To substitute for the hours worked $n_{i,t}$ I used the production function aggregated
for all varieties produced in the same country. That is, in log linear terms,

\[ y_{i,t} = n_{i,t} + a_{i,t} - z_{i,t}. \]

I define the welfare function for the whole currency area as the weighted average of the welfare measures in the two regions. The weights are given by the size of each region. Among the possible aggregation function, the specification of an utilitarian social welfare function is the one that gives less weight to the heterogeneity across regions. The welfare in the two regions are aggregated using, according to which the utility flows of the two regions are aggregated using a linear function. Thus

\[ W_{u,t} = mW_{1,t} + (1 - m)W_{2,t} \]

For now I am assuming that the two regions have equal size \((m = 0.5)\).

Galì and Monacelli (2005) showed that \(z_{i,t} = \frac{\theta}{2} \text{var}_j (\pi_{i,t}) = \frac{\theta}{2} \pi_{i,t}^2\). Using this result, the currency area approximated welfare function can be rewritten as follows

\[
W_{u,t} = C^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left[ w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_{\pi} \xi_t - \frac{1}{2} \pi'_t W_{\pi} \pi_t \right] + t.i.p. + o(\|\xi\|^3) \tag{36}
\]

where

\[
y'_t = [ y_{u,t} \ c_{u,t} \ t^n_t \ t^e_t \ q^n_t \ q^e_t \ y_{r,t} \ c_{r,t} ]
\]

\[
\xi'_t = [ y_{row,t} \ a_{u,t} \ a_{r,t} ]
\]

\[
\pi'_t = [ \pi_{u,t} \ \pi_{r,t} ]
\]

\[
w'_y = \left[ -\frac{1}{\Phi} \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \right]
\]

\[
W_y = \begin{bmatrix}
\frac{(1+\varphi)}{\delta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (1-\sigma) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[ 34 \text{A bar on a variable indicates that it is at the steady state level. I define } \Phi \equiv \frac{\delta^{1+\varphi}}{\delta^{\varphi}}\text{ such that } (1-\Phi)\bar{Y} \text{ is the output distorted steady state.} \]
\[
W_{\xi} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
W_{\pi} = \begin{bmatrix}
\frac{\theta}{\lambda \sigma} & 0 \\
0 & m(1 - m) \frac{\theta}{\lambda \sigma}
\end{bmatrix}
\]

In order to get the appropriate welfare approximation I need to eliminate the linear terms \( w'y_t \) in the second order approximation (36). Following Benigno and Woodford (2006), I can do it using the second order Taylor expansion of some of the equilibrium conditions of the model. I consider the following equilibrium conditions:

a) The aggregate demand of goods produced in the currency area (i.e. the weighted average of equations (15) for \( i, j = 1, 2 \) and \( i \neq j \));

b) The relative demand of goods produced in the two regions of the currency area (i.e. the difference between (15) when \( i = 2 \) and \( i = 1 \));

c) The risk sharing condition between the currency area and the rest of the world (i.e. \( C_{u,t} \));

d) The risk sharing condition between the regions inside the currency area (i.e. \( C_{r,t} \));

e) The definition of the real exchange rate of the currency area with the rest of the world (i.e. \( Q_{ex}^t = (Q_{ex}^t)^{1 - \omega}(T_{ex}^t)\xi \));

f) The definition of the real exchange rate between the regions of the currency area (i.e. \( Q_{in}^t = (T_{in}^t)^{1 - \omega}(T_{ex}^t)^{2\xi} \));

g) The labor market clearing condition aggregated for all regions in the currency area (i.e. the weighted average of (14) for \( i = 1, 2 \));

h) The labor market clearing condition in differences between regions in the currency area (i.e. the difference between (14) when \( i = 2 \) and when \( i = 1 \)).

a) Demand of area-wide goods:

\[
\sum_{t=0}^{\infty} \beta^t [a_y y_t + \frac{1}{2} y_t' A_y y_t - y_t' A_\xi \xi_t] + o(\|\xi\|^3) = 0 \quad (37)
\]

\[
a_y' = \begin{bmatrix}
-1 & 1 & -\frac{\varepsilon}{2} & \omega - \frac{\varepsilon}{2} \frac{(\sigma^2 - 1)}{\sigma} & \omega \frac{(\sigma - 1)}{\sigma} & 0 & 0
\end{bmatrix}
\]

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where

\[
x_{5,5} = -\frac{1}{4} (\frac{\sigma - 1}{\sigma^2})^2 \left( 4\alpha^2 + \omega^2 + \varepsilon^2 + 4\alpha\omega - \omega - \alpha \right) \frac{\varepsilon}{2}(\sigma - 1)
\]

\[
x_{5,6} = x_{6,5} = \frac{1}{2} (\frac{\sigma - 1}{\sigma^2}) \varepsilon (-1 + 2\alpha + 2\omega)
\]

\[
x_{6,6} = -\frac{1}{2} \frac{(\sigma - 1)^2}{\sigma^2} (\omega^2 + \varepsilon^2) - \frac{(\sigma - 1)}{\sigma^2} \omega
\]

\[
A_{\xi} = 0
\]

b) Demand differentials:

\[
\sum_{t=0}^{\infty} \beta^t [b'_yt_t + \frac{1}{2} y'_t B_y y_t - y'_t B_{\xi} \xi_t] + o(||\xi||^3) = 0 \quad (38)
\]

\[
b'_y = \begin{bmatrix} 0 & 0 & -2\alpha - \omega & 2 \varepsilon & \frac{(\sigma - 1)}{\sigma} (2\alpha + \omega) & 2 \frac{(\sigma - 1)}{\sigma} \varepsilon & -1 & 1 \end{bmatrix}
\]

\[
B_y = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} \frac{(\sigma - 1)^2}{\sigma^2} \varepsilon (4\alpha + 2\omega - 1) & \frac{(\sigma - 1)^2}{\sigma^2} (-\omega + 2\alpha\omega + \omega^2 + \varepsilon^2) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} \frac{(\sigma - 1)^2}{\sigma^2} (-\omega + 2\alpha\omega + \omega^2 + \varepsilon^2) & -\frac{(\sigma - 1)^2}{\sigma^2} 4\omega - \frac{2(\sigma - 1)}{\sigma^2} \varepsilon & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

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\[ \sum_{t=0}^{\infty} \beta^t[y_t + \frac{1}{2}y'_t C_{yt} - y'_t C_{\xi_t}] + o(||\xi||^3) = 0 \] (39)

\[ c'_y = \begin{bmatrix} 0 & -1 & 0 & 0 & \frac{1}{\sigma} & 0 & 0 \end{bmatrix} \]

\[ C_y = 0 \]

\[ C_{\xi} = 0 \]

d) Risk sharing - region 1 vs region 2 of the Currency area

\[ \sum_{t=0}^{\infty} \beta^t[d'_t y_t + \frac{1}{2}y'_t D_{yt} - y'_t D_{\xi_t}] + o(||\xi||^3) = 0 \] (40)

\[ d'_y = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{\sigma} & 0 & 0 & -1 \end{bmatrix} \]

\[ D_y = 0 \]

\[ D_{\xi} = 0 \]

e) Real exchange rate definition - Extra currency area

\[ \sum_{t=0}^{\infty} \beta^t[e'_t y_t + \frac{1}{2}y'_t E_{yt} - y'_t E_{\xi_t}] + o(||\xi||^3) = 0 \] (41)

\[ e'_y = \begin{bmatrix} 0 & 0 & \frac{\xi}{2} & (1-\omega) & 0 & -1 & 0 & 0 \end{bmatrix} \]

\[ E_y = 0 \]
\[ E_\xi = 0 \]

f) Real exchange rate definition - Intra currency area

\[
\sum_{t=0}^{\infty} \beta^t [f'_y y_t + \frac{1}{2} y_t F_y y_t - y'_t F_\xi \xi_t] + o(||\xi||^3) = 0
\]  

(42)

\[ f'_y = \begin{bmatrix} 0 & 0 & (1 - 2\alpha - \omega) & 2\varepsilon & -1 & 0 & 0 & 0 \end{bmatrix} \]

\[ F_y = 0 \]

\[ F_\xi = 0 \]

g) Labor market clearing - aggregate currency area

\[
\sum_{t=0}^{\infty} \beta^t [g'_y y_t + \frac{1}{2} y_t G_y y_t - y'_t G_\xi \xi_t] + o(||\xi||^3) = 0
\]  

(43)

\[ g'_y = \begin{bmatrix} \varphi & \sigma & -\frac{\varepsilon}{2} & \omega & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ G_y = 0 \]

\[ G_\xi = \begin{bmatrix} 0 & -(1 + \varphi) & \varphi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

h) Labor market clearing - differences between regions

\[
\sum_{t=0}^{\infty} \beta^t [h'_y y_t + \frac{1}{2} y_t H_y y_t - y'_t H_\xi \xi_t] + o(||\xi||^3) = 0
\]  

(44)
\[
\begin{bmatrix}
\frac{0}{0} & -2\alpha - \omega & 2\varepsilon & 0 & 0 & \varphi & \sigma
\end{bmatrix}
\]

\[\mathbf{H}_y = 0\]

\[
\mathbf{H}_\xi =
\begin{bmatrix}
0 & 0 & 0 \\
0 & -(1 + \varphi)\varphi & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Eliminating the linear term in (36) implies finding the vector \(\mathbf{L}_x\) such that

\[\mathbf{w}_y = \left[ a_y \ b_y \ c_y \ d_y \ e_y \ f_y \ g_y \ h_y \right] \mathbf{L}_x\]

where \(\mathbf{L}_x\) has dimension 8 by 1.

I can therefore rewrite equation (36) as

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} y'_t L_y y_t + y'_t L_\xi \xi_t + \frac{1}{2} L_\pi \pi_t^2 \right] + \text{t.i.p.} + o(\|\xi\|^3) \tag{45}
\]

where

\[
\begin{align*}
L_y &= W_y + A_y L_1 + B_y L_2 \\
L_\xi &= W_\xi + G_\xi L_7 + H_\xi L_8 \\
L_\pi &= W_\pi
\end{align*}
\]

In order to write the welfare in terms of the currency area output, the output differentials in the two regions, the internal and external terms of trade, the currency area inflation and the regions' inflation differentials, I construct the mapping \(N\) such that

\[
y'_t = N[y_{u,t} \ y_{r,t} \ t_{i,t}^{in} \ t_{i,t}^{ex} \ \xi_t] + N_\xi \xi_t
\]

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where
\[
N = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & \frac{\varepsilon}{2\sigma} & \frac{(1-\omega)}{\sigma} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & (1-2\alpha-\omega) & 2\varepsilon \\
0 & 0 & \frac{\varepsilon}{2} & (1-\omega) \\
1 & 0 & 0 & 0 \\
0 & 0 & -(1-2\alpha-\omega) & \frac{2\varepsilon}{\sigma}
\end{bmatrix}
\]

and
\[
N_\xi = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

I can substitute for \(y_{R,t}\) using the following mapping
\[
\begin{bmatrix}
y_{u,t} \\
y_{r,t} \\
t_{t}^{in} \\
t_{t}^{ex}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & \frac{1}{\varphi} \left[ (1 - \frac{1}{\sigma}) (2\alpha + \omega) \sigma^{-1} \right] & 0 \\
0 & 1 & -\frac{1}{\varphi} \left( 1 - \frac{1}{\sigma} \right) 2\varepsilon \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
y_{u,t} \\
t_{t}^{in} \\
t_{t}^{ex}
\end{bmatrix} + M_\xi \xi_{t}'
\]

where
\[
M_\xi = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1+\omega}{\sigma} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

In order to reduce further the number of variables, one can write
\[
\begin{bmatrix}
y_{u,t} \\
y_{r,t} \\
t_{t}^{in} \\
t_{t}^{ex}
\end{bmatrix} = Q \begin{bmatrix}
y_{u,t} \\
t_{t}^{in} \\
t_{t}^{ex}
\end{bmatrix} + Q_\xi \xi_{t}'
\]

where
\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\sigma_{\omega} & -\sigma_{\omega} \sigma_{\varepsilon} & -\sigma_{\omega} \sigma_{\varepsilon}
\end{bmatrix}
\]
and

\[
Q_\xi = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
-\sigma & 0 & 0
\end{bmatrix}
\]

The new coefficients are in the matrix \( H = Q'M'N'L_yNMQ \) and \( H_\xi = Q'M'N'L_y (NMQ_\xi + NM_\xi + N_\xi) + Q'M'N'L_\xi \). Thus, I can rewrite the loss function (45) as

\[
L_{t_0}^u = E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left\{ \frac{1}{2} [y_{u,t} t_{t}^{in}] H [y_{u,t} t_{t}^{in}]' + [y_{u,t} t_{t}^{in}] H_\xi [y_{row,t} a_{u,t} a_{r,t}]' + \frac{1}{2} \pi_\xi' L_\pi \pi_t \right\} + t.i.p. + o(\|\xi\|^3)
\]

The period loss function, written in terms of gap of the variables from their efficient level, is given by

\[
L_t^u = \left\{ \Phi_{xu} \pi_{u,t}^2 + \Phi_{xt} (y_{u,t} - \tilde{y}_{u,t})^2 + \Phi_{tin} (t_{t}^{in} - \tilde{t}_{t}^{in})^2 + \Phi_{fr} \pi_{r,t}^2 \right. \\
+ \Phi_{xu,t} (y_{u,t} - \tilde{y}_{u,t}) (t_{t}^{in} - \tilde{t}_{t}^{in}) \left. \right\} + t.i.p. + o(\|\xi\|^3)
\]

where

\[
\Phi_{xu} = H(1,1) \\
\Phi_{tin} = H(2,2) \\
\Phi_{xu,t} = [H(1,2) + H(2,1)] \\
\Phi_{\pi u} = W_\pi(1,1) \\
\Phi_{\pi r} = W_\pi(2,2)
\]

The efficient equilibrium level of the area-wide output and the internal terms of trade is given by the following equations

\[
\tilde{y}_{u,t} = \frac{1}{H(1,1)} \left\{ \frac{1}{2} [H(1,2) + H(2,1)] t_{t}^{in} - H_\xi(1,1)y_{row,t} - H_\xi(1,2)a_{u,t} - H_\xi(1,3)a_{r,t} \right\}
\]

\[
\tilde{t}_{t} = \frac{1}{H(2,2)} \left\{ \frac{1}{2} [H(1,2) + H(2,1)] y_{u,t} - H_\xi(2,1)y_{row,t} - H_\xi(2,2)a_{u,t} - H_\xi(2,3)a_{r,t} \right\}
\]
In the special case of $\sigma = 1$ and $\varepsilon = 0$ the coefficients are the following

$$\Phi_{xu} = \frac{(1 + \varphi)}{\Phi}$$
$$\Phi_{t^m} = m(1 - m)\frac{(1 + \varphi)}{\Phi}$$
$$\Phi_{xu,t^m} = 0$$

These coefficients are equal to those of Galí and Monacelli (2005) with the exception of the internal terms of trade that is a result of aggregating the welfare functions of the two regions.

C The Optimal Policy Plan

The benevolent monetary authority of the currency area solves the following linear quadratic problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t L_t^u$$

subject to

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \kappa_{u}^u (y_{u,t} - \tilde{y}_{u,t}) + \kappa_{t}^u \left(t_{t}^{\in} - \tilde{t}_{t}^{\in}\right) - \kappa_{u}^u (\tilde{y}_{u,t} - \tilde{\tilde{y}}_{u,t}) - \kappa_{t}^u (\tilde{t}_{t}^{\in} - \tilde{\tilde{t}}_{t}^{\in})$$

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \kappa_{r}^r (y_{u,t} - \tilde{\tilde{y}}_{u,t}) + \kappa_{r}^r \left(t_{t}^{\in} - \tilde{t}_{t}^{\in}\right) - \kappa_{r}^r (\tilde{y}_{u,t} - \tilde{\tilde{y}}_{u,t}) - \kappa_{r}^r (\tilde{t}_{t}^{\in} - \tilde{\tilde{t}}_{t}^{\in})$$

$$t_{t}^{\in} - \tilde{t}_{t}^{\in} = t_{t-1}^{\in} - \tilde{t}_{t-1}^{\in} + \pi_{r,t} - (\tilde{t}_{t} - \tilde{\tilde{t}}_{t-1})$$

Calling $\lambda_{1,t}$, $\lambda_{3,t}$ and $\lambda_{3,t}$ the Lagrange multipliers associated with the three constraints above, the following are the first order conditions with respect to $\pi_{u,t}$, $\pi_{r,t}$, $x_{u,t}$ and $(t_{t}^{\in} - \tilde{t}_{t}^{\in})$ of the full commitment optimal policy problem.

$$\Phi_{xu,\pi_{u,t}} + \lambda_{1,t} - \lambda_{1,t-1} = 0 \quad (46)$$
$$\Phi_{xu} \left(y_{u,t} - \tilde{y}_{u,t}\right) + \Phi_{xu,t^m} \left(t_{t}^{\in} - \tilde{t}_{t}^{\in}\right) - \lambda_{1,t} \kappa_{u}^u - \lambda_{2,t} \kappa_{r}^u = 0 \quad (47)$$
$$\Phi_{xu,\pi_{r,t}} + \lambda_{2,t} - \lambda_{2,t-1} - \lambda_{3,t} = 0 \quad (48)$$
$$\Phi_{xu,t^m} \left(t_{t}^{\in} - \tilde{t}_{t}^{\in}\right) + \Phi_{xu,t^m} \left(y_{u,t} - \tilde{y}_{u,t}\right) - \lambda_{1,t} \kappa_{r}^u - \lambda_{2,t} \kappa_{r}^u + \lambda_{3,t} - \lambda_{3,t+1} = 0 \quad (49)$$
Equations (46)-(49), the structural equations of the model (equations (19), (21) and (23)), i.e. the first order conditions with respect to the Lagrange multipliers, together with the stochastic processes specified for the three exogenous variables, $a_{u,t}$, $a_{r,t}$ and $y_{row,t}$, fully describe the dynamics of the currency area economy under the optimal policy plan.
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