Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns

by Paolo Emilio Mistrulli
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ASSESSING FINANCIAL CONTAGION IN THE INTERBANK MARKET:
MAXIMUM ENTROPY VERSUS OBSERVED INTERBANK LENDING PATTERNS

Paolo Emilio Mistrulli*

Abstract

Interbank markets allow banks to cope with specific liquidity shocks. At the same time, they may be a channel allowing a bank default to spread to other banks. This paper analyzes how contagion propagates within the Italian interbank market using a unique data set including actual bilateral exposures. Since information on bilateral exposures was not available in most previous studies, they assumed that banks spread their lending as evenly as possible among all the other banks by maximizing the entropy of interbank linkages. Based on the data available on actual bilateral exposures for all Italian banks, the results obtained by assuming the maximum entropy are compared with those reflecting the observed structure of interbank claims. The comparison indicates that, in line with the thesis prevailing in the literature, the maximum entropy method tends to underestimate the extent of contagion. However, this does not hold in general. Under certain circumstances, depending on the structure of the interbank linkages, the recovery rates of interbank exposures and banks’ capitalization, the maximum entropy approach overestimates the scope for contagion.

Keywords: interbank market, financial contagion, systemic risk, maximum entropy.

JEL classification: G21, G28.

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Systemic risk has always played a prominent role in the economic policy debate. A widely shared view is that the banking system is an important chain in the propagation of shocks to the entire economy. Early theoretical works concerning systemic risk in the banking sector have focused on depositor runs triggered by self-fulfilling expectations (Diamond and Dybvig, 1983) or by signals regarding bank solvency (Chari and Jagannathan, 1988 and Jacklin and Bhattacharya, 1988). More recently, attention has been paid to financial contagion in the interbank market (Rochet and Tirole, 1996). While the interbank market may allow banks to cope with specific liquidity shocks (Bhattacharya and Gale, 1987), interbank relationships may represent a channel for contagion: the economic distress of an insolvent bank may propagate to other banks through interbank linkages. Recent theoretical works (Allen and Gale, 2000; Freixas, Parigi and Rochet, 2000) have also highlighted that the propagation of shocks within the interbank market is dependent on the exact pattern of banks’ financial linkages.

Many contributions have sought to measure the vulnerability of the interbank market to financial contagion. However, these works suffer from two main data limitations. First, detailed data on banks’ bilateral exposures are generally not available. In order to circumvent this problem, two approaches have been adopted: a) the severity of financial contagion in the interbank market has been measured by focusing on a specific segment of

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1. I am grateful to Ugo Albertazzi, Franklin Allen, Guglielmo Barone, Marcello Bofondi, Francesco Columba, Alessio D'Ignazio, Xavier Freixas, Eugenio Gaiotti, Leonardo Gambacorta, Andrea Generale, Giorgio Gobbi, Francesca Lotti, Christian Upper and Andreas Worms and two anonymous referees for helpful suggestions. I am greatly indebted to Paolo Angelini for having provided me with very useful comments at different stages of this work. I would also like to thank participants in the seminars at the Bank of Italy, at the Universitat Pompeu Fabra and at CSEF-University of Salerno, and participants in the workshop on “Systemic Risk in the Financial Sector” hosted by Collegium Budapest (Budapest, 2005) and in the 2006 European Meeting of the Econometric Society. The opinions expressed in this paper are mine and in no way involve the responsibility of the Bank of Italy.


3. A different channel for contagion through the interbank market could be related to signals or self-fulfilling expectations triggering interbank deposits withdrawals.

4. See Upper (2007) and Appendix A for details of the literature on financial contagion.
the market for which actual bilateral exposures were available; in particular, Furfine (2003) has investigated contagion in the Federal Reserve’s large-value transfer system (Fedwire) which, however, represents a small fraction of U.S. total interbank exposures (according to Furfine himself, approximately 14 per cent); b) other contributions have taken into account the whole interbank market. However, since detailed information on banks’ bilateral exposures is not available, these works have had to assume a specific pattern for them. In particular, it is possible to distinguish between two different cases: i) interbank bilateral exposures are obtained on the basis of aggregated interbank assets and liabilities by maximising the entropy\(^5\) of the matrix of bilateral claims (e.g. Upper and Worms, 2004) or ii) when information about large bilateral exposures is available, the entropy maximisation method is applied only to those elements of the matrix of bilateral exposures that are not known (Blavarg and Nimander, 2002; Wells, 2004; Degryse and Nguyen, 2007; van Lelyveld and Liedorp, 2006). The main limitation of this approach is that it assumes a market structure which might be quite different from the actual one. Indeed, the maximisation of the entropy rules out lending relationships in the interbank market\(^6\) since it assumes that each bank lends to all the other banks in the market.

A second data limitation concerns the structure of the banking system. The existing literature has typically overlooked the fact that banks are often affiliated with a conglomerate, even if this may, quite obviously, affect the resilience of the banking system to systemic shocks.

In this paper, financial contagion in the interbank market is investigated using a unique data set including detailed information on each actual bilateral exposure for all the Italian banks. Thus, the analysis deals with all kinds of interbank financial linkages, besides derivatives and shares, for every Italian bank and for each bank-to-bank relationship. According to the previous literature, the paper uses simulation techniques to measure the severity of financial contagion.

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\(^5\) See Appendix B for details.

\(^6\) Cocco et al. (2003) find evidence supporting the existence of lending relationships in the interbank market for overnight funds.
The paper contributes to the literature in two main respects. Firstly, it provides a measure of the vulnerability to financial contagion of the Italian interbank market, thus enriching the evidence so far available for most of the industrialised countries. However, the analysis differs from those made for other countries as it is based on actual bilateral exposures instead of maximum entropy exposures. Furthermore, by taking into account the affiliation of banks with a conglomerate, it provides evidence on the link between financial conglomeration and banking system stability. The paper shows that even if the Italian interbank market is conducive to financial contagion it hardly triggers a systemic crisis. Banks conglomeration tends to improve the resilience of the banking system to shocks. However, in some cases, since conglomeration induces a strong interdependency among affiliated banks, the extent of contagion may be even wider if banks’ affiliation is taken into account than when it is ignored.

Secondly, it shows that the measure of financial contagion depends on the pattern of interbank linkages. To this end, the paper applies the maximum entropy method, it re-runs the simulation exercises and then compares them with the results based on actual bilateral exposures. The comparison indicates that, in line with the thesis prevailing in the literature, the maximum entropy method leads to an undervaluation of the extent of contagion. However, this does not hold in general. Under certain circumstances, depending on the structure of the interbank linkages, the recovery rates of interbank exposures and banks’ capitalisation, the maximum entropy approach implies an overvaluation of the scope for contagion.

These results have two consequences. On the empirical level, it indicates that in some circumstances, depending on the structure of the interbank market, the maximum entropy approach may not be very reliable in assessing the severity of financial contagion. Furthermore, since it does not provide a lower bound to contagion vulnerability, it may not be reliable either for assessing whether the interbank market is conducive to contagion or not. On the theoretical level, following Allen and Gale’s (2000) taxonomy, the comparison between maximum entropy and observed exposures may be interpreted as a comparison

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7 The paper by Angelini et al. (1996) has also analysed the risk of contagion in the Italian interbank market, although the work takes into account only the netting system segment.
between complete and incomplete markets. In this respect, the results obtained in this paper show that complete markets are not always less conducive to contagion than other markets, as Allen and Gale (2000) stated.

The remainder of the paper is organised as follows. The next section describes the methodology and data. The third section deals with the vulnerability to financial contagion of the Italian interbank market. Section 4 investigates how the maximum entropy method affects the severity of financial contagion by comparing, for the Italian case, the results obtained on the basis of the maximum entropy matrix of bilateral exposures with those obtained using the observed matrix. The final section concludes and summarises the main findings.

2. Data and methodology

2.1 Data

Data on gross bilateral interbank exposures are obtained from the Bank of Italy supervisory reports database. Since January 1989 all Italian banks, including their foreign branches, submit to the Bank Italy their end-of-month bilateral exposures to all other banks, foreign intermediaries included. Data refer to all kinds of interbank assets except shares. The database allows for a distinction between different claims (CDs, current accounts, repos, other loans, subordinated and unsubordinated bonds) classified according to their maturity (up to 18 months, over 18 months), currency of denomination and counterpart nationality (Italy vs. rest of the world).

At the end of 2003, interbank exposures accounted for a substantial share of the total assets of Italian banks. The total amount of loans to banks and of securities issued by banks held by the Italian banking system was equal to 24 per cent of total assets at the end of 2003; it was 37 per cent for the banks in the euro area as a whole. More importantly, from the perspective of financial contagion, interbank exposures were over 3 times the total amount of capital and reserves in Italy, 6 times that in the euro area. If one considers only the domestic segment of the market, interbank assets were 18 per cent of total assets in Italy and 21 per cent in the euro area. The interbank asset-to-capital ratio was respectively 2.4 and 3.5 for the Italian banking system and the euro area banks.
2.2 The contagion mechanism

All banks raising funds in the interbank market are allowed to fail one at a time. The losses suffered by banks lending to the failed bank are then computed. If the amount of the losses is greater than lenders’ tier-1 capital (i.e. capital and reserves) then lenders default. The simulation is then iterated by verifying if banks that fail after the first iteration make other banks fail as well. At each iteration banks that failed in the previous one are dropped from the set of banks which may be affected by contagion. The simulation continues until at least one bank default occurs.

More formally, let $B$ be the set of banks and $x_{ij}$ denote the funds that bank $j \in B$ borrows from bank $i \in B$, where $x_{ij} \geq 0 \ \forall (i, j) \in B \times B$ and $x_{ii} = 0 \ \forall i \in B$, by $\epsilon_i > 0$ the initial capital endowment of bank $i$, and by $\alpha \in [0,1]$ the rate of loss (i.e. the incidence of losses due to contagion out of the interbank exposure). Finally, let $z \in B$ denote the first bank that defaults because of some idiosyncratic shock, and define $D^*_z \subseteq B$ and $S^*_z \subseteq B$ as the set of banks, respectively, defaulted and surviving at the $n$-th step of the contagion path initiated by bank $z$, as follows:

(1) \[ D^*_z = \{ k \in B : \epsilon_{k,z}^n \leq 0 \mid \epsilon_{k,z}^{n-1} > 0 \} \] \[ S^*_z = \{ k \in B : \epsilon_{k,z}^n > 0 \} , \quad \forall n \geq 1 \]

where $\epsilon_{k,z}^n$, which is the capital of bank $k$ at the $n$-th step of contagion initiated by bank $z$, is equal to:

(2) \[ \epsilon_{k,z}^n = \epsilon_{k,z}^{n-1} - \alpha \sum_{j \in D_{k,z}^{n-1}} x_{kj} , \quad \forall n \geq 1 \text{ and } \forall k \neq z \]

The contagion path is then the following:
Failed banks

\[ D_\xi^0 = \{ \zeta \} \]

\[ D_\xi^1 = \left\{ k \in S_\xi^0 : \epsilon_{k,\zeta}^1 = (\epsilon_k - \alpha x_{k,\zeta}) \leq 0 \right\} \]

\[ D_\xi^2 = \left\{ k \in S_\xi^1 : \epsilon_{k,\zeta}^2 = (\epsilon_{k,\zeta}^1 - \alpha \sum_{j \in D_\xi^1} x_{kj}) \leq 0 \right\} \]

\[ \vdots \]

\[ D_\xi^n = \left\{ k \in S_\xi^{n-1} : \epsilon_{k,\zeta}^n = (\epsilon_{k,\zeta}^{n-1} - \alpha \sum_{j \in D_\xi^{n-1}} x_{kj}) \leq 0 \right\} \]

\[ D_\xi^N = \{ \emptyset \} \]

Surviving banks

\[ S_\xi^0 = \{ k \in B \setminus \{ \zeta \} \} \]

\[ S_\xi^1 = \left\{ k \in S_\xi^0 : \epsilon_{k,\zeta}^1 = (\epsilon_k - \alpha x_{k,\zeta}) > 0 \right\} \]

\[ S_\xi^2 = \left\{ k \in S_\xi^1 : \epsilon_{k,\zeta}^2 = (\epsilon_{k,\zeta}^1 - \alpha \sum_{j \in D_\xi^1} x_{kj}) > 0 \right\} \]

\[ \vdots \]

\[ S_\xi^N = \left\{ k \in S_\xi^{N-1} : \epsilon_{k,\zeta}^N = (\epsilon_{k,\zeta}^{N-1} - \alpha \sum_{j \in D_\xi^{N-1}} x_{kj}) > 0 \right\} \]

and the process stops after \( N \) iterations when no default occurs.

Before commenting on some shortcomings of the contagion mechanism it is useful to emphasise two characteristics of the process. First, default condition (1) refers to interbank gross exposures instead of net ones. The main reason for this is that bilateral netting is not feasible because when banks fail, their liabilities are pooled and satisfied according to seniority. Moreover, this kind of procedure usually lasts many years. Second, simulations are run on unconsolidated interbank bilateral exposures. An alternative contagion mechanism could be based on consolidated exposures. However, consolidating interbank exposures would amount to assuming that, in the case in which a bank affiliated with a group fails, internal exposures (i.e., interbank exposures among banks affiliated to the same parent company) are senior to the others. On the other hand, one can define an alternative contagion mechanism such that banks affiliated to a group may be bailed out by their parent company if the consolidated tier-1 capital is larger than the losses suffered from contagion (see Section 3.2 below).

The contagion mechanism described above suffers from some drawbacks. In particular:
loss rates are constant across banks and through different stages of financial contagion propagation and they are also not dependent on loan contract covenants;

the analysis focuses on contagion only among banks operating in Italy;

the contagion mechanism concentrates on a specific channel for contagion ignoring any other source which could interact with the propagation of contagion within the interbank market;\(^8\)

banks are not allowed to issue shares in order to compensate for the losses they suffer from the failure of some interbank market counterpart;

creditor runs are ruled out.

As far as loss rates are concerned (i), bank failures are rarely observed and therefore it is difficult to estimate a loss rate for interbank exposures.\(^9\) For this reason, this paper follows the approach adopted by previous works (e.g. Furfine, 2003; Upper and Worms, 2004) of measuring the risk of contagion at different loss rates between 0 and 1. It is also assumed that the loss rate does not depend on the banks involved. While this assumption is not quite realistic, it is not easy to endogenise loss rates. The risk could be that of introducing even increasingly unrealistic parameters in simulations.

The analysis focuses on contagion among banks operating in Italy (ii) because otherwise it would be not possible to analyze all “second-round” effects. Actually, as far as external interbank exposures are concerned, interbank linkages among foreign banks are not known. This implies that is not possible to verify whether, say, the failure of a foreign bank (bank \(i\)) triggers indirectly the failure of a bank operating in Italy (bank \(k\)) by causing first the failure of another foreign bank (bank \(j\)) that has borrowed money from the first (bank \(i\)).

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\(^8\) For example, exposures from the payment and settlement system may provide alternative channels for contagion. Furthermore, loans to non-banks may amplify the effect of a bank failure by causing liquidity shortages and the insolvency for some borrowers which in turn may make the banking system suffer other loan losses.

\(^9\) Some authors have tried to estimate the loss rates for the U.S. banking system. However, their estimates are very different: James (1991) has found a loss rate of 40 per cent while Kaufman’s (1994) estimate is 5 per cent.
The analysis of a more comprehensive contagion mechanism (iii) would certainly add to the realism of the analysis. On the other hand, by making the contagion process more complex it might be more difficult to isolate the contribution of each channel for contagion.

Some of the shortcomings mentioned above would be solved if more detailed information were available (i-iii). On the other hand, others (iv-v) seem to be more difficult to tackle with as they would require a theoretical foundation that, to the best of my knowledge, is not yet available.

All the drawbacks of the analysis mentioned above may affect the measure of the severity of contagion in a specific interbank market. However, they do not affect the comparison between the results based on the maximum entropy matrix of bilateral exposures and those based on actual bilateral exposures as they are obtained by the same contagion mechanism.

3. Is the Italian interbank market vulnerable to financial contagion?

The aim of this section is to assess whether the Italian interbank market is vulnerable to financial contagion. The analysis is divided into two parts. The first one (subsection 3.1), following the literature on financial contagion in the interbank market, addresses the issue of financial contagion by assuming that banks cannot react to a shock by, for example, raising capital to compensate for the losses suffered from the failure of their interbank counterparts. In the second part (subsection 3.2), parent banks are allowed to recapitalise their affiliates by redistributing capital resources within the conglomerate.

3.1 The benchmark analysis

Table 1 reports simulation results referring to December 2003. Different measures of financial contagion propagation have been computed: i) the number of banks whose default causes at least one bank failure by contagion (contagious banks), ii) the number of banks that fail by contagion and their total assets as a percentage of the Italian banking system’s

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10 It is worth noting that results are not affected by an end-of-year effect. Simulations for other months of 2003 (March, June and September) have produced similar results.
total assets (banks failing by contagion), and iii) the number and the total assets, as a percentage of the Italian banking system’s total assets, of banks that never fail independently of the bank failing by some idiosyncratic shock (contagion-proof banks).

Overall, the evidence indicates that financial contagion may occur in the Italian banking system. This result does not hold in general but depends on the bank that fails at the initial stage. In fact, Table 1 shows that, even for high loss rates, only a limited number of banks are contagious: the number of contagious banks ranges from 16 to 67, depending on the loss rate, out of 789 Italian banks operating at the end of 2003. It is also worth noting that contagious banks are not necessarily large ones. Some small bank may be contagious, particularly for high loss rates. On the other hand, even if the loss rate is equal to one, not all large banks (22 at the end of 2003) are able to make other banks fail.

Since many banks are not contagious, the propagation of contagion is fairly limited on average, both in terms of the number of banks failed and their share of the banking system’s total assets.

Furthermore, as far as contagion-proof banks are concerned (i.e. banks which, for a given loss rate and independently of the bank that initiates the contagion process, never fail) the results indicate that a large part of the banking system is immune to contagion. Even in the worst case (i.e. when the loss rate is equal to 1) banks immune to contagion represent a considerable share of the banking system (529 out of 789 banks; about 60 per cent of the banking system’s total assets).

All in all, these results suggest that, even if contagion may occur in the Italian interbank market, there is little scope for financial contagion in Italy. However, one may be interested in other characteristics of the distribution of contagion other than mean values. Following previous works on financial contagion, the maximum impact of financial contagion (worst scenario) has also been reported in Table 1. This is the result of the failure of the bank that produces the maximum contagion impact. While such an event is unlikely it makes sense to consider it in an analysis whose aim is to quantify risks and their consequences. In this case, the number of banks failing by contagion ranges from 4 to 116 and is greater than 50 for a loss rate greater or equal to 0.7; the share of total assets of banks failing by contagion ranges from 0.5 to 15.9 per cent and is greater than 10 per cent for a loss rate greater than or equal to 0.7.
### Severity of financial contagion

(December 2003)

<table>
<thead>
<tr>
<th>Loss rate</th>
<th>Banks failing by contagion</th>
<th>Contagion-proof banks (2)</th>
<th>Contagious banks (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worsen assets</td>
<td>Mean assets</td>
<td>Std.dev.</td>
</tr>
<tr>
<td></td>
<td>All banks</td>
<td>Large banks</td>
<td>All banks</td>
</tr>
<tr>
<td>0.1</td>
<td>0.466</td>
<td>0.203</td>
<td>0.004</td>
</tr>
<tr>
<td>0.2</td>
<td>3.467</td>
<td>0.015</td>
<td>0.007</td>
</tr>
<tr>
<td>0.3</td>
<td>7.422</td>
<td>0.028</td>
<td>0.018</td>
</tr>
<tr>
<td>0.4</td>
<td>7.870</td>
<td>0.037</td>
<td>0.019</td>
</tr>
<tr>
<td>0.5</td>
<td>7.906</td>
<td>0.054</td>
<td>0.031</td>
</tr>
<tr>
<td>0.6</td>
<td>7.986</td>
<td>0.072</td>
<td>0.039</td>
</tr>
<tr>
<td>0.7</td>
<td>8.280</td>
<td>0.092</td>
<td>0.052</td>
</tr>
<tr>
<td>0.8</td>
<td>15.207</td>
<td>11.136</td>
<td>0.115</td>
</tr>
<tr>
<td>0.9</td>
<td>15.637</td>
<td>11.136</td>
<td>0.138</td>
</tr>
<tr>
<td>1.0</td>
<td>15.878</td>
<td>11.136</td>
<td>0.154</td>
</tr>
</tbody>
</table>

(1) Large banks are defined as banks whose total assets are at least equal to 20 billion euro. (2) Banks which are never affected by contagion. (3) Banks which make at least one bank fail.
The effects of contagion are highly concentrated. Up to a loss rate equal to 0.7, financial contagion affects mainly large banks. Only if loss rates exceed that threshold do small banks account for a significant share of the overall extent of contagion. Large banks generally play a substantial role in the contagion mechanism, not only as banks initiating the process (i.e. contagious banks) but also as banks affected by contagion. Among large banks (22 at the end of 2003), most of them are contagious for a loss rate greater than 0.1.

Large banks are relatively less important in the propagation of contagion: for a loss rate lower than 0.9, most of them are immune to contagion. Only in the worst scenario do they account for a large share of the impact of contagion when it is measured in terms of total assets.

Overall these results suggest that for loss rates up to 0.7 the spreading of contagion is very limited. For loss rates greater than 0.7, financial contagion becomes significant, although it seems unable to trigger a systemic crisis.

3.2 The effect of bail-outs within conglomerates

The results obtained in Section 3.1 are based on the hypothesis that banks cannot react to any shock by, for example, raising capital to compensate for the losses suffered from the failure of their interbank counterparts. While this assumption may be quite suitable for stand-alone banks, it seems less appropriate for banks affiliated with a conglomerate. Indeed, it seems quite reasonable to assume that parent companies may recapitalise affiliated banks that would otherwise fail by, at least, redistributing capital resources among subsidiaries. On one hand, this would improve the resilience of the banking system to financial contagion since contagion losses are shared among all banks belonging to a group. On the other, banks that are not financially linked (directly or indirectly) to the bank that fails in the first step may suffer from losses due to financial contagion if other affiliates are financially linked to the bank which triggers the contagion process.

In this section we analyse how results change if we assume that banks can be bailed out by other banks belonging to the same conglomerate. Simulations are still based on unconsolidated interbank exposures and run in the same way as before except for assuming a
slightly different default condition. In particular, that condition now refers to the consolidated capital as it is assumed that losses are now shared among banks affiliated with a certain conglomerate.  

Let $C_i$ denote the set of banks affiliated with conglomerate $s$ and $k_s$ the capital of conglomerate $s$. Let bank $z \in B$ default. The capital of conglomerate $s$ at the $n$-th contagion path initiated by bank $z$ is equal to

$$k^{n}_s = k^{n-1}_s - \alpha \sum_{i \in C_s, j \in D^{n-1}_s} x_{ij}, \quad \forall n \geq 1$$

and

$$D^{n}_s = \left\{ i \in C_s : k^{n}_i \leq 0 \left| k^{n-1}_i > 0 \right. \right\}, \quad \forall n \geq 1$$

It is worth highlighting some important differences with respect to previous simulations: the default of bank $z$ affects the solvency of bank $i \in C_s$ not only because of the losses bank $i$ faces in lending money to bank $z$, but also because of the losses all other banks affiliated with conglomerate $s$ suffer in lending to bank $z$. In fact, all the losses suffered by the banks affiliated with a conglomerate $s$ affect the consolidated capital $k_s$. This bears out the hypothesis that, at each stage of the contagion process, banks can be bailed out by other banks affiliated with the same conglomerate if the consolidated capital is greater than the overall amount of the losses.

When conglomerates are considered another issue arises. As there may be close interdependencies among banks affiliated with a conglomerate, apart from those related to financial interlinkages, it could be that a shock that hits one bank will hit all banks of the same conglomerate. Thus, in this section, the impact of financial contagion is computed under two different hypotheses that $i)$ the initial shock that makes banks fail at the first stage is bank-specific (one bank fails), or that $ii)$ it is conglomerate-specific (i.e. one conglomerate as a whole fails).

11 For stand-alone banks previous conditions hold.
Table 2 shows the results of simulations obtained for bank-specific shocks. The comparison with the results based on individual capital (Table 1) indicates that the effect of contagion is on average smaller if bail-outs within conglomerates are allowed. This reflects the fact that some relatively small banks can be bailed out by their parent company. Thus, one might conclude that banking conglomerates may lower the impact of financial contagion.

However, this result does not hold in general. In some cases, the severity of contagion may be even greater. The results for the worst scenario indicate that, under the hypothesis of bail-outs within conglomerates, financial contagion, for loss rates ranging from 0.4 to 0.7, is significantly more dangerous in terms of total assets affected than in the benchmark case. Similarly, the total assets of contagion-proof banks account for a smaller share of the banking system’s total assets if loss rates are above 0.3 than in the benchmark case. This means that in the latter case, the impact of contagion tends to be more concentrated than in the former.

On the contrary, the maximum impact measured in terms of the number of banks failed by contagion is almost always less than the impact computed when bail-outs are not allowed. The same holds for contagious banks.

This evidence reflects the fact that conglomerate bail-outs have two opposing effects on the severity of contagion: on one hand, they imply a strengthening of subsidiaries’ capital position, on the other, they add another channel for contagion as banks in a conglomerate suffer not only from the losses due to their interbank linkages but also from the losses due to all the other conglomerate affiliates’ interbank exposures. The first effect is relatively greater for small banks, which may be more easily bailed-out by the conglomerate they are affiliated with. This explains why the number of failed banks tends to be lower, even in the worst scenario than in the case in which bail-outs are not allowed. On the contrary, this effect is limited in terms of the total assets affected by contagion as banks that may be bailed out are typically small.

All in all, the severity of contagion when bail-outs are allowed may be even greater compared with the benchmark case.
<table>
<thead>
<tr>
<th>Loss rate</th>
<th>Total assets (as a percentage of banking system total assets)</th>
<th>Number of banks</th>
<th>Total assets (as a percentage of banking system total assets)</th>
<th>Number of Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worst scenario</td>
<td>Mean values</td>
<td>Std.dev. (all banks)</td>
<td>Worst scenario</td>
</tr>
<tr>
<td></td>
<td>All banks</td>
<td>Large banks</td>
<td>All banks</td>
<td>Large banks</td>
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<td>0.466</td>
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<td>0.003</td>
<td>0.000</td>
</tr>
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<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>0.3</td>
<td>0.475</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>0.4</td>
<td>12.767</td>
<td>11.830</td>
<td>0.049</td>
<td>0.037</td>
</tr>
<tr>
<td>0.5</td>
<td>12.767</td>
<td>11.830</td>
<td>0.063</td>
<td>0.046</td>
</tr>
<tr>
<td>0.6</td>
<td>12.786</td>
<td>11.830</td>
<td>0.075</td>
<td>0.055</td>
</tr>
<tr>
<td>0.7</td>
<td>12.786</td>
<td>11.830</td>
<td>0.079</td>
<td>0.055</td>
</tr>
<tr>
<td>0.8</td>
<td>12.786</td>
<td>11.830</td>
<td>0.099</td>
<td>0.069</td>
</tr>
<tr>
<td>0.9</td>
<td>12.786</td>
<td>11.830</td>
<td>0.106</td>
<td>0.069</td>
</tr>
<tr>
<td>1.0</td>
<td>12.827</td>
<td>11.830</td>
<td>0.110</td>
<td>0.069</td>
</tr>
</tbody>
</table>

(1) Large banks are defined as banks whose total assets are at least equal to 20 billion euro. (2) Banks which are never affected by contagion. (3) Banks which make at least one bank fail.
On the other hand, given that the number of banks involved in the contagion process, both contagious and contagion-affected banks, is smaller it could be easier to manage a systemic crisis in this case.

Apart from bail-outs, bank conglomerates may damage the resilience of the interbank market to financial contagion as they also raise the interdependence among banks. Table 3 shows that, if at the first stage of simulations a conglomerate as a whole fails instead of a single bank, the impact of contagion is greater.

4. Maximum entropy versus observed interbank lending patterns

This section compares the results obtained in the previous section with those that would be obtained if the matrix of bilateral exposures were not known and, by following previous contributions, the maximum entropy method (ME) were adopted. This comparison is important in two respects. Firstly, it sheds some light on the reliability of the ME approach for assessing whether the interbank market is vulnerable to financial contagion. Secondly, as the ME matrix represents, according to Allen and Gale’s (2000) taxonomy, a complete market, the comparison between the results obtained using the actual matrix of bilateral exposures and those based on the ME matrix may provide some evidence in support of the hypothesis that the pattern of interbank linkages affects the severity of financial contagion.

Section 4.1, after briefly recalling the main results of Allen and Gale’s (2000) model, shows, in contrast, that complete markets are not necessarily less conducive to contagion than incomplete markets. To do this, some simple numerical examples are considered. Section 4.2 applies the same approach to the Italian interbank market. The results support the analysis of Section 4.1 indicating that in the Italian case the ME method may in certain overrate the vulnerability to contagion.

---

12 See Appendix B for details.
# Conglomerates and the severity of financial contagion: conglomerate-specific shocks

(December 2003)

<table>
<thead>
<tr>
<th>Loss rate</th>
<th>Number of banks</th>
<th>Total assets</th>
<th>Number of banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worst scenario</td>
<td>Mean values</td>
<td>Std.dev.</td>
</tr>
<tr>
<td></td>
<td>Large banks</td>
<td>All banks</td>
<td>Large banks</td>
</tr>
<tr>
<td>All banks</td>
<td></td>
<td></td>
<td>(all banks)</td>
</tr>
<tr>
<td>Large banks</td>
<td></td>
<td></td>
<td>(all banks)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.720</td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>0.2</td>
<td>12.703</td>
<td>11.830</td>
<td>0.042</td>
</tr>
<tr>
<td>0.3</td>
<td>12.751</td>
<td>11.830</td>
<td>0.048</td>
</tr>
<tr>
<td>0.4</td>
<td>12.767</td>
<td>11.830</td>
<td>0.085</td>
</tr>
<tr>
<td>0.5</td>
<td>12.767</td>
<td>11.830</td>
<td>0.092</td>
</tr>
<tr>
<td>0.6</td>
<td>12.786</td>
<td>11.830</td>
<td>0.096</td>
</tr>
<tr>
<td>0.7</td>
<td>12.786</td>
<td>11.830</td>
<td>0.099</td>
</tr>
<tr>
<td>0.8</td>
<td>12.786</td>
<td>11.830</td>
<td>0.122</td>
</tr>
<tr>
<td>0.9</td>
<td>12.786</td>
<td>11.830</td>
<td>0.126</td>
</tr>
<tr>
<td>1.0</td>
<td>12.827</td>
<td>11.830</td>
<td>0.127</td>
</tr>
</tbody>
</table>

(1) Large banks are defined as banks whose total assets are at least equal to 20 billion euro. (2) Banks which are never affected by contagion. (3) Banks which make at least one bank fail. (4) Conglomerates including at least one large bank.
4.1 Interbank lending patterns and the theory of financial contagion

Recent theoretical contributions (Allen and Gale, 2000; Freixas, Parigi and Rochet, 2000) have highlighted that the propagation of shocks within the interbank market is dependent on the exact pattern of banks’ financial linkages. Allen and Gale (2000) identify two key structural characteristics of the market which affect financial contagion: market completeness and market interconnectedness. A market is complete if each bank lends to all the others. On the other hand, a market is perfectly interconnected if each bank is financially linked to all the others, regardless of the kind of linkage, which may be both direct (i.e. each bank lends to all the others) and indirect (the link that a bank could indirectly establish with another if a third one is linked to both of them).

Those authors point out that the more complete the market the less severe is the financial contagion and the more interconnected the market the more severe is the contagion. Thus, there is a trade-off in terms of the propagation of contagion between completeness and interconnectedness. These two concepts are strictly linked. A complete market is a specific case of a perfectly interconnected market where banks are financially linked to all the others only by direct exposures. Otherwise, incomplete markets may be differently interconnected: they may be both perfectly or partially interconnected depending on the pattern of financial linkages. In general, markets that are incomplete and not perfectly interconnected may differ both in terms of completeness and interconnectedness.

The comparison between ME and observed interbank patterns may be interpreted as the comparison between complete and incomplete markets with the notable qualification that Allen and Gale’s (2000) model does not allow for bank heterogeneity. On the contrary, the ME method assumes that banks spread their lending as evenly as possible, consistently with their aggregate interbank assets and liabilities which may, in turn, differ considerable among banks.

Let us ignore for a while the issue of bank heterogeneity, which will be dealt with later. According to Allen and Gale’s (2000) taxonomy, the comparison between actual and ME bilateral exposures may lead to the following cases,

(a) the (actual) market is complete,
(b) the (actual) market is incomplete but perfectly interconnected, and
(c) the (actual) market is incomplete but not perfectly connected.
In the first case, the ME matrix of bilateral exposures and the observed ones coincide. Thus, the measure of the impact of financial contagion is not affected by the ME hypothesis. On the other hand, if the market is incomplete but perfectly interconnected (case b), ME leads to an undervaluation of financial contagion. To show this consider Figure1, which represents the ME matrix of bilateral exposures under the assumption that, for each bank, total interbank liabilities and total interbank assets are equal.

This assumption is needed only to simplify the maximisation of the entropy. It does not imply any loss of generality as contagion is based on gross exposures. Let us assume also that bank capital is equal to \( k \) and that a bank fails if losses exceed its capital and reserves. Thus, if bank \( A \) fails due to a specific shock\(^{13}\) it is easy to verify that there is no contagion unless the loss rate is greater than 0.5 (i.e. losses are greater than \( k \)). In that case, all banks fail by contagion (Table 4).

We now compare these results with those obtained for case b) (Figure 2).\(^{14}\) It is easy to verify that all banks in the market fail if the market is incomplete and perfectly connected unless the loss rate is lower than 0.2 (1/6 to be precise). This confirms that, if the actual market is incomplete but perfectly connected, the maximum entropy method tends to underrate the extent of contagion (in this specific case when the loss rate ranges from 0.2 to 0.5).

This is due to a typical effect of risk diversification. In fact, while complete markets and incomplete but perfectly connected markets do not differ in terms of the number of banks potentially affected by contagion (as a shock may propagate from each bank to all the others in both cases), complete markets allow banks to better diversify credit risk than incomplete ones, given that each bank lends directly to all the other banks instead of holding the financial assets issued by a limited number of counterparts.

\(^{13}\) In the absence of bank heterogeneity the identity of the bank that fails due to some idiosyncratic shock is not relevant.

\(^{14}\) This corresponds to the incomplete market structure depicted in Allen and Gale’s (2000) Figure 1.
Figure 1

A complete market
(maximum entropy)

<table>
<thead>
<tr>
<th>Bank A</th>
<th>Bank B</th>
<th>Bank C</th>
<th>Bank D</th>
<th>Total assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>2k</td>
<td>0</td>
<td>2k</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>2k</td>
<td>2k</td>
<td>0</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Total liabilities</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
</tr>
</tbody>
</table>

Table 4

Financial contagion: complete versus incomplete markets
(number of banks failing by contagion)

<table>
<thead>
<tr>
<th>Loss Rate</th>
<th>Symmetric case</th>
<th>Asymmetric case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Complete markets (fig.1)</td>
<td>Incomplete but perfectly connected market (fig. 2)</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0.7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0.8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0.9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Consider now case c) as depicted in Figure3.\textsuperscript{15} In this case it is straightforward to verify that the maximum entropy method does not affect the measure of contagion if loss rates are lower than 0.2 (1/6); it underrates the scope for contagion if loss rates range from 0.2 to 0.5 and it overrates it for loss rates greater than 0.5.

The reason why, in spite of lower market concentration, complete markets may, \textit{ceteris paribus}, be even less resilient to financial contagion than incomplete ones is that, if the size of the interbank market is sufficiently large with respect to bank capital, there could be limited scope for risk diversification, especially for large loss rates, from spreading interbank exposures over a great number of counterparts. It is easy to verify that if the total amount of interbank assets held by each bank were less than $3k$, complete markets would be totally immune to contagion, thus representing, in line with Allen and Gale (2000), the most resilient market structure. For larger interbank markets, the sole effect of diversification is that market interconnectedness rises, favouring the propagation of contagion.

In general, for $N$ homogenous banks the interbank assets-to-capital ratio has to be at least equal to $(N-1)$ to make the failure of a bank propagate within the market. However, actual interbank markets are not so large. On the other hand, according to the evidence provided by those authors who have resorted to the maximum entropy method, even complete markets are vulnerable to financial contagion. The reason for this result is that,

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textit{Bank A} & \textit{Bank B} & \textit{Bank C} & \textit{Bank D} & \textbf{Total assets} \\
\hline
0 & 6k & 0 & 0 & 6k \\
\hline
0 & 0 & 6k & 0 & 6k \\
\hline
0 & 0 & 0 & 6k & 6k \\
\hline
6k & 0 & 0 & 0 & 6k \\
\hline
\textbf{Total liabilities} & 6k & 6k & 6k & 6k & 24k \\
\hline
\end{tabular}
\caption{An incomplete but perfectly connected market}
\end{table}

\footnote{This corresponds to the disconnected incomplete market structure depicted in Allen and Gale’s (2000) Figure 3. This market is made of two totally disconnected segments: to one of them belong bank A and B, to the other bank C and D.}
although markets are assumed to be complete, the ME does not rule out bank heterogeneity and, as shown below, the condition for contagion propagation then becomes less stringent.

Figure 3

<table>
<thead>
<tr>
<th>Bank</th>
<th>Bank A</th>
<th>Bank B</th>
<th>Bank C</th>
<th>Bank D</th>
<th>Total assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank A</td>
<td>0</td>
<td>6k</td>
<td>0</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Bank B</td>
<td>6k</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Bank C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6k</td>
<td>6k</td>
</tr>
<tr>
<td>Bank D</td>
<td>0</td>
<td>0</td>
<td>6k</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Total liabilities</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>24k</td>
</tr>
</tbody>
</table>

To show how the resilience of the market may deteriorate when bank heterogeneity is allowed, let us consider the interbank market depicted in Figure 4, which is equal to the one represented in Figure 3 apart from the fact that banks differ in terms of interbank asset-to-capital ratio. In this case, the identity of the bank that fails due to some idiosyncratic shock is not irrelevant any more. The maximum impact of financial contagion is obtained when bank A defaults (i.e. the bank that is better capitalised than the others). For a loss rate greater than 0.1 the failure of bank A makes only bank B fail by contagion according to the observed matrix (panel a).

On the other hand, bank A makes all other banks fail unless the loss rate is lower than 0.3. Thus, the two matrices produce the same result up to a loss rate equal to 0.2, the maximum entropy method underrates the severity of contagion for loss rates ranging from 0.1 to 0.3 and overrates the extent of contagion if the loss rate is greater than 0.3. This means that, by allowing for some heterogeneity in banks’ capitalisation, the maximum impact of financial contagion tends to be overstated for lower loss rates than in the case in which banks are homogenous (loss rates greater than 0.5). It may be also verified that in this case, even if each bank held an amount of interbank assets lower than 3k, complete markets would not be immune to contagion as they were in the symmetric case (Figure 1).
Bank Heterogeneity

a) observed matrix of bilateral exposures

An incomplete and disconnected market

<table>
<thead>
<tr>
<th></th>
<th>Bank A</th>
<th>Bank B</th>
<th>Bank C</th>
<th>Bank D</th>
<th>Total assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank A</td>
<td>0</td>
<td>6k</td>
<td>0</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Bank B</td>
<td>6k</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Bank C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6k</td>
<td>6k</td>
</tr>
<tr>
<td>Bank D</td>
<td>0</td>
<td>0</td>
<td>6k</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Total liabilities</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>24k</td>
</tr>
<tr>
<td>Capital and reserves</td>
<td>2k</td>
<td>2/3k</td>
<td>2/3k</td>
<td>2/3k</td>
<td>4k</td>
</tr>
</tbody>
</table>

b) ME matrix of bilateral exposures

A complete market
(maximum entropy)

<table>
<thead>
<tr>
<th></th>
<th>Bank A</th>
<th>Bank B</th>
<th>Bank C</th>
<th>Bank D</th>
<th>Total assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank A</td>
<td>0</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>Bank B</td>
<td>2k</td>
<td>0</td>
<td>2k</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>Bank C</td>
<td>2k</td>
<td>2k</td>
<td>0</td>
<td>2k</td>
<td>6k</td>
</tr>
<tr>
<td>Bank D</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>0</td>
<td>6k</td>
</tr>
<tr>
<td>Total liabilities</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>6k</td>
<td>24k</td>
</tr>
<tr>
<td>Capital and reserves</td>
<td>2k</td>
<td>2/3k</td>
<td>2/3k</td>
<td>2/3k</td>
<td>4k</td>
</tr>
</tbody>
</table>

The main conclusion of this section is that, apart from the cases in which the actual market is complete or is incomplete and perfectly connected, ME may involve, for high loss rates, an overstatement of the severity of financial contagion. This may also happen more easily when banks are heterogeneous.¹⁶

This possible effect seems to be quite realistic as there are at least three reasons why actual interbank markets are, in general, incomplete and to some extent disconnected. First, the existence of informational asymmetries or, in general, transaction costs may prevent

¹⁶ This result is in line with Iori, Jafarey and Padilla (2006).
banks from fully exploiting their capacity to diversify interbank counterparts. Second, for
banks that are affiliated with conglomerates the multiple money centre structure seems to be
suitable to exploit economies of scale in liquidity management. Finally, in many countries
banks differ in size considerably and therefore the assumption that the interbank market is
made of homogenous banks seems unrealistic.

4.2 The Italian case

Some characteristics of the Italian interbank market suggest that it is far from being a
complete market and that the adoption of the ME method may therefore significantly affect
the assessment of financial contagion. Indeed, the Italian interbank market fits quite well in
the multiple money centre structure described in Freixas, Parigi and Rochet (2000), in which
some banks trade with a bank (the money centre) while they do not trade with each other: at
the end of 2003, more than two thirds of interbank claims were traded among banks
belonging to the same group. As far as interconnectedness is concerned, banks were on
average financially linked, both directly and indirectly through other banks, to one half of
the total number of counterparts.\textsuperscript{17}

The comparison of the severity of financial contagion in the Italian interbank market by
using, alternatively, the maximum entropy (ME) matrix and the observed matrix of bilateral
exposures (Figures 5-9) confirms that ME may provide a quite different measure of financial
contagion.

Figure 5 supports the hypothesis that complete markets are more resilient to contagion
than incomplete ones, at least in terms of contagious banks. In this case, one can argue that a
smaller number of banks have to be monitored in order to avoid the propagation of contagion
within the banking system if the market were complete.

Figures 6 and 7 indicate that the ME method leads, on average, to an undervaluation of
the severity of financial contagion, both in terms of the number of banks and of the total
assets affected by contagion, if the loss rate is not above 0.8 and 0.9, respectively. Similarly,
according to Figures 8 and 9, ME underrates the maximum impact of financial contagion,
measured by the maximum number of banks failed and the maximum share of the banking system’s total assets affected by contagion, for loss rates not greater than 0.7.

**Figure 5**

![Number of contagious banks](December 2003)

To sum up, the evidence suggests that in most cases ME tends to underrate the impact of financial contagion. On the contrary, for high loss rates ME may imply an overvaluation of the severity of contagion.

A further complication is that the lowest loss rate at which ME overvalues the severity of financial contagion depends on the characteristics of the market. As shown in Section 4.1, that critical value may also be quite low. From the previous discussion it may also be argued that the lowest loss rate at which ME underrates the extent of contagion is negatively related to the asymmetry among market participants in terms of the amount of bilateral exposures and to the size and the disconnectedness of the interbank market. In other words, when the interbank market is made up of a few big players raising funds from many relatively small counterparts, interbank exposures account for a large share of banks’ capital and the market

---

17 The computation of a measure of interconnectedness for the Italian interbank market refers only to those financial linkages such that contagion is at least possible (i.e. such that for a loss rate equal to 1, interbank exposures exceed bank capital).
Figure 6

Banks failing by contagion
Number of banks affected by contagion - mean values
(December 2003)

Figure 7

Banks failing by contagion
Percentage of banking system total assets affected by contagion - mean values
(December 2003)
Figure 8

Banks failing by contagion
Number of banks affected by contagion - worst scenario
(December 2003)

Figure 9

Banks failing by contagion
Percentage of banking system total assets affected by contagion - worst scenario
(December 2003)
is highly segmented, ME may then be of little help in assessing the severity of financial contagion as it would overstate the impact of financial contagion even for low loss rates. As a consequence, in that situation the ME approach might not be very reliable even for assessing whether a danger of financial contagion exists or not.

The comparison between the ME and actual measures of financial contagion has been done by the Wilcoxon signed-rank test (Table 5). In particular, this test compares, for each bank $i$ and a given loss rate, the severity of contagion triggered by its default ($\Omega_i$), computed on the base of the actual matrix of bilateral exposures, with the severity of contagion triggered by the same bank $i$, calculated on the base of the ME matrix ($\Omega_i^{ME}$). The null hypothesis is that the median of the differences ($\Omega_i - \Omega_i^{ME}$) is zero. Table 5 reports the results for both the distribution of the number of banks failing by contagion and the distribution of the total assets affected by contagion. The results indicate that the ME method tends to provide a measure of the severity of contagion that is statistically different from the one obtained on the base of actual bilateral exposures.

### Table 5

A test for the equality of ME and observed financial contagion distributions (1)

(December 2003)

<table>
<thead>
<tr>
<th>Loss rate</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total assets affected by contagion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All banks</td>
<td>Wilcoxon signed-rank test</td>
<td>1.517</td>
<td>2.516</td>
<td>3.061</td>
<td>2.932</td>
<td>3.250</td>
<td>4.638</td>
<td>4.150</td>
<td>2.946</td>
<td>2.514</td>
</tr>
<tr>
<td></td>
<td>$p$-value</td>
<td>0.1292</td>
<td>0.0119</td>
<td>0.0022</td>
<td>0.0034</td>
<td>0.0012</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0032</td>
<td>0.0119</td>
</tr>
<tr>
<td></td>
<td>$p$-value</td>
<td>0.0112</td>
<td>0.0005</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Number of banks affected by contagion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All banks</td>
<td>Wilcoxon signed-rank test</td>
<td>1.775</td>
<td>1.678</td>
<td>2.123</td>
<td>2.263</td>
<td>2.912</td>
<td>3.343</td>
<td>3.346</td>
<td>2.251</td>
<td>2.311</td>
</tr>
<tr>
<td></td>
<td>$p$-value</td>
<td>0.0760</td>
<td>0.0934</td>
<td>0.0337</td>
<td>0.0236</td>
<td>0.0036</td>
<td>0.0008</td>
<td>0.0008</td>
<td>0.0244</td>
<td>0.0209</td>
</tr>
<tr>
<td></td>
<td>$p$-value</td>
<td>0.0043</td>
<td>0.0300</td>
<td>0.0061</td>
<td>0.0012</td>
<td>0.0002</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(1) Distributions refers to a measure of the propagation of financial contagion in the interbank market computed for each bank failing by an idiosyncratic shock.
5. Conclusion

This paper has investigated whether interbank linkages may be conducive to financial contagion. The contribution to the existing literature is twofold. Firstly, it enriches the evidence so far available on financial contagion by providing the first comprehensive analysis for the Italian interbank market. To this end, unlike previous contributions, which are based on ME bilateral exposures, the paper is based on a unique data set of actual bilateral exposures. It has also taken into account the effect of banking conglomerates in terms of banking system stability. This aspect has been ignored by previous works. Secondly, the paper compares the results obtained by using ME bilateral exposures with those obtained on the base of actual bilateral exposures. This allows us to show how the ME method widely used in the literature may affect the analysis of financial contagion.

The main result is that: the Italian interbank market is conducive to financial contagion. However, even for high loss rates, the default of banks raising funds in the interbank market hardly triggers a systemic crisis. Only in some extreme cases does the severity of financial contagion seem considerable. Simulations also indicate that, by allowing conglomerates to recapitalise their affiliates which otherwise would fail, the resilience to financial contagion of the banking system tends to improve. However, in some cases the fact that losses are shared among banks affiliated to a conglomerate, adding another channel for contagion means that banking stability may even worsen.

The paper also indicates that the maximum entropy procedure, widely adopted in previous contributions, may overvalue the severity of contagion. This contrasts with the common view that complete markets are more resilient to financial contagion. In some circumstances, depending on the size of the interbank market, the presence of large players, and the loss rate, complete markets may be even more conducive to contagion than incomplete ones. In that case, the benefits from diversifying the interbank exposures are more than counterbalanced by the costs due the fact that each bank establishes a large number of financial linkages, thus making a domino effect viable.
References


## Appendix A

The empirical investigation on financial contagion

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Institutions</th>
<th>Type of Data</th>
<th>Period</th>
<th>Type of Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angelini, Maresca and Russo (1996)</td>
<td>Italy</td>
<td>288 banks</td>
<td>Bilateral end-of-day net balances</td>
<td>January 1992</td>
<td>idiosyncratic shock</td>
</tr>
<tr>
<td>Degryse and Nguyen (2007)</td>
<td>Belgium</td>
<td>Belgian banks</td>
<td>Estimated bilateral interbank exposures (RAS algorithm), large bilateral exposures (December 2003), quarterly data</td>
<td>1993-2002</td>
<td>idiosyncratic shock</td>
</tr>
<tr>
<td>Sheldon and Maurer (1998)</td>
<td>Switzerland</td>
<td>Swiss banks</td>
<td>Estimated bilateral interbank exposures (RAS algorithm), overnight interbank loans</td>
<td>1987-95</td>
<td>idiosyncratic shock</td>
</tr>
<tr>
<td>Upper and Worms (2004)</td>
<td>Germany</td>
<td>German banks and foreign bank branches operating in Germany</td>
<td>Estimated bilateral interbank exposures (RAS algorithm)</td>
<td>December 1998</td>
<td>idiosyncratic shock</td>
</tr>
<tr>
<td>Van Lelyveld and Liedorp (2006)</td>
<td>Netherlands</td>
<td>Dutch banks and foreign subsidiaries and branches</td>
<td>Estimated bilateral interbank exposures (RAS algorithm), large bilateral exposures, ad hoc survey</td>
<td>December 2002</td>
<td>idiosyncratic shock</td>
</tr>
</tbody>
</table>
Appendix B

The maximum entropy methodology for obtaining interbank bilateral exposures

The interbank linkages may be represented by the following $N \times N$ matrix

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix}$$

where $a_i = \sum_{j=1}^{N} x_{ij}$, $l_j = \sum_{i=1}^{N} x_{ij}$ are, respectively, the total amount of money bank $i$ lends to other banks and bank $j$ raises from other banks. In the absence of any assumption about the distribution of bilateral exposures, the matrix $X$ cannot be identified as $N^2 - 2N$ unknowns have to be estimated.

The common approach is to assume that banks maximise the dispersion of their interbank exposures. Following an appropriate normalisation this implies that bilateral exposures are given by a simple solution: $x^*_y = a_i/l_j$. However, this solution would imply that for banks that are both lender and borrower in the market they lend to themselves. In order to rule out this outcome it is necessary to assume that $x^*_y = 0, \forall i = j$.

The problem is then to estimate bilateral exposures such that the matrix $\hat{X}$ obtained by the maximisation becomes as close as possible to matrix $X^*$. This is generally obtained by minimising the cross-entropy between the two matrices:

$$\min_{\hat{X}} \sum_{i=1}^{N} \sum_{j=1}^{N} \ln \left( \frac{x^*_{ij}}{\hat{x}_{ij}} \right)$$

s.t.

$$\sum_{j=1}^{N} \hat{x}_{ij} = a_i \quad \sum_{i=1}^{N} \hat{x}_{ij} = l_j \quad \hat{x}_{ij} \geq 0 \quad \forall i \neq j \quad \hat{x}_{ij} = 0 \quad \forall i = j$$

This problem has been solved numerically using the RAS algorithm.\(^\text{18}\)

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\(^{18}\) See Censor and Zenos (1997) for details.
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F. SPADAFORA, Il pilastro privato del sistema previdenziale: il caso del Regno Unito, Economia Pubblica, 34, 5, pp. 75-114, TD No. 503 (June 2004).


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G. Ferrero, Monetary policy, learning and the speed of convergence, Journal of Economic Dynamics and Control, TD No. 499 (June 2004).

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R. Bronzini, FDI Inflows, Agglomeration and host country firms’ size: Evidence from Italy, Regional Studies, TD No. 526 (December 2004).


R. Felici and M. Pagnini, Distance, bank heterogeneity and entry in local banking markets, The Journal of Industrial Economics, TD No. 557 (June 2005).


M. Bugamelli and A. Rosolia, Produttività e concorrenza estera, Rivista di politica economica, TD No. 578 (February 2006).


S. Federico and G. A. Minerwa, Outward FDI and local employment growth in Italy, Review of World Economics, TD No. 613 (February 2007).


B. Roffia and A. Zaghini, Excess money growth and inflation dynamics, International Finance, TD No. 629 (June 2007).