# BANCA D'ITALIA

# Temi di discussione

del Servizio Studi

# Canonical term-structure models with observable factors and the dynamics of bond risk premiums

by M. Pericoli and M. Taboga



Number 580 - February 2006

The purpose of the Temi di discussione series is to promote the circulation of working papers prepared within the Bank of Italy or presented in Bank seminars by outside economists with the aim of stimulating comments and suggestions.

The views expressed in the articles are those of the authors and do not involve the responsibility of the Bank.

*Editorial Board:* Giorgio Gobbi, Marcello Bofondi, Michele Caivano, Stefano Iezzi, Andrea Lamorgese, Marcello Pericoli, Massimo Sbracia, Alessandro Secchi, Pietro Tommasino, Fabrizio Venditti.

Editorial Assistants: ROBERTO MARANO, ALESSANDRA PICCININI.

## CANONICAL TERM-STRUCTURE MODELS WITH OBSERVABLE FACTORS AND THE DYNAMICS OF BOND RISK PREMIUMS

by Marcello Pericoli and Marco Taboga\*

#### Abstract

We study the dynamics of risk premiums on the German bond market, employing no-arbitrage term-structure models with both observable and unobservable state variables, recently popularized by Ang and Piazzesi (2003). We conduct a specification analysis based on a new canonical representation for this class of models. We find that risk premiums display a considerable variability over time, are strongly counter-cyclical and bear no significant relation to inflation.

JEL classification: C5, G1.

Keywords: term structure models, yield curve, risk premium.

#### Contents

1. Introduction	7
2. The model	10
3. The data	15
4. Empirical evidence	16
5. Conclusions	20
6. Appendix	22
References	27
Tables	29
Figures	36

<sup>\*</sup>Bank of Italy. Economic Research Department.

E-mail: marcello.pericoli@bancaditalia.it, marco.taboga@bancaditalia.it .

### **1.** Introduction<sup>1</sup>

We study the dynamics of risk premiums on the German bond market and their relation to macroeconomic variables. We employ no-arbitrage affine multifactor term-structure models following the approach, recently popularized by Ang and Piazzesi (2003), of including both observable and unobservable factors in the set of state variables. Conforming to the existing literature, the observable state variables we include are inflation and a measure of the output gap.

We derive a canonical representation for the class of affine models with both observable and unobservable variables including as special cases the models of Ang and Piazzesi (2003), Ang, Dong and Piazzesi (2004), Ang, Piazzesi and Wei (2005), Hördal, Tristani and Vestin (2005) and Rudebusch and Wu (2005). The new set of identifying restrictions implied by this representation is less restrictive than the set of restrictions first proposed by Ang and Piazzesi (2003). Ang and Piazzesi correctly acknowledge that identification schemes provided by Dai and Singleton (2000) for affine term-structure models cannot be applied to models with observable variables, since equivalent representations of the models can be obtained only by rotations and translations of the variables which leave the observable variables unchanged. Some of the over-identifying restrictions in Ang and Piazzesi (2003) are rejected by formal statistical tests in our sample. However, they have substantial consequences on estimated risk premiums only when their two-stage estimation procedure is employed instead of a joint estimation procedure. We use our canonical representation to perform a specification analysis and find that three unobservable state variables must be added to the two observables to obtain an accurate description of yieldcurve dynamics. Hence, the classical finding that multifactor models with three unobservable factors provide the best balance between parsimony and statistical fit (e.g. Litterman and Scheinkman - 1991 and Knez, Litterman and Scheinkman -

<sup>&</sup>lt;sup>1</sup>Any views expressed in this article are the authors' and do not necessarily represent those of the Bank of Italy. We thank Paolo Angelini, Giuseppe Grande, Marco Protopapa, Glenn Rudebusch, Oreste Tristani, Paolo Zaffaroni, two anonymous referees and seminar participants at the Bank of Italy and at the FFM 2005 Conference for helpful discussion.

1994) is not altered by the inclusion of observable state variables. The inclusion of macroeconomic variables is nevertheless worthwhile: a variance decomposition analysis reveals that shocks to output and inflation explain a significant portion of the variability of risk premiums, hence they play a key role in determining the dynamics of bond yields. Estimating both the physical and the risk-neutral dynamics of the factors driving interest rates, we are able to separate risk premiums from the other components of bond yields, namely expectations and Jensen's inequality adjustments: this is achieved by deriving the no-arbitrage bond yields that would be observed if the market were populated by risk-neutral investors and then subtracting them from the yields implicit in actually observed market prices. We find that risk premiums display a considerable variability over time and that both the level and the variability of the premiums are increasing with the maturity of bonds. High negative correlation with the output gap provides evidence that the premiums are countercyclical, but there seems to be no systematic link between inflation and the premiums. When unobservable variables are included in the model, observable variables make only a minimal contribution to explaining the overall variability of yields. However, concentrating only on risk premiums we find that macroeconomic variables explain a significant portion of their variability. These pieces of evidence are only apparently conflicting: since the short-term interest rate is procyclical and risk premiums are countercyclical, long-term bond yields, being a sum of the two, do not react much to output shocks because the separate effects of these shocks on premiums and the short rate offset each other. This hypothesis is confirmed when we perform a variance decomposition on yields of different maturities: the proportion of variance explained by output shocks is higher for shorter maturities and lower for longer maturities; this result is due to the fact that risk premiums on shorter maturities are smaller and hence the compensating effect explained above is only partial.

Our study belongs to a recent strand of the literature which uses no-arbitrage pricing models to analyze the relation between the yield curve and macroeconomic fundamentals: some examples are Ang and Piazzesi (2003), Ang, Dong and Piazzesi (2004), Ang, Piazzesi and Wei (2005), Hördal, Tristani and Vestin (2005) and Rudebusch and Wu (2005). For a survey, we refer the reader to Diebold, Piazzesi and Rudebusch (2005). Earlier studies investigating the relation between the yield curve and macroeconomic variables, such as Fama (1990), Mishkin (1990), Estrella and Mishkin (1995) and Evans and Marshall (1998) do not consider no-arbitrage relations among yields and do not model bond pricing. As a consequence, they are able to make predictions only about the yields explicitly analyzed (typically no more than three), they do not rule out theoretical inconsistencies due to the presence of arbitrage opportunities along the yield curve and they make no predictions about risk premiums and their evolution over time. For these reasons, the more recent studies we mentioned above have proposed to enrich macro-finance models with rigorous asset pricing relations, imposing no-arbitrage constraints on bond prices. All these studies employ Gaussian affine term-structure models where risk premiums are allowed to vary over time. Their primary focus, however, is on the relation among economic growth, inflation and interest rates, while they analyze the time-variation of risk premiums only incidentally. They implicitly characterize risk premiums and their dependence on macroeconomic variables, by specifying and estimating a pricing kernel, but they do not provide explicit measures of risk premiums across maturities and over time. The aim of our paper, instead, is to provide measures of risk premiums which have a straightforward economic interpretation; for each bond and at each point in time, we measure the extra-return per period required by bond market investors to bear interest rate risk. The bond pricing model we use allows for a rigorous separation of the risk premiums from the other components of the term spreads, namely expectations of future interest rates and Jensen's inequality adjustments. Furthermore, estimating a no-arbitrage pricing functional defined also on observable variables, we are able to assess separately the impact of changes in macroeconomic fundamentals on risk premiums and to understand to what extent the variability of risk premiums is generated by macroeconomic uncertainty or by other factors.

The paper is organized as follows: Section 2 presents the class of affine models we estimate and gives the minimal identifying conditions; Section 3 describes our dataset; Section 4 discusses the empirical evidence, as well as some important details regarding the numerical procedures adopted to estimate the model; Section 5 concludes. The Appendix contains all the technical details.

### 2. The model

Our model of the term structure is a standard Gaussian affine model, set in discrete time, as in the majority of the recent literature about macro term structure models. The model consists of three equations. The first equation describes the dynamics of the vector of state variables  $X_t$  (a k-dimensional vector,  $k \in \mathbb{N}$ ):

$$X_t = \mu + \rho X_{t-1} + \Sigma \varepsilon_t \tag{1}$$

where  $\varepsilon_t \sim N(0, I_k)$ ,  $\mu$  is a  $k \times 1$  vector and  $\rho$  and  $\Sigma$  are  $k \times k$  matrices. Without loss of generality, it can be assumed that  $\Sigma$  is lower triangular. Furthermore, to ensure stationarity of the process, we assume that all the eigenvalues of  $\rho$  strictly lie inside the unit circle. The probability measure associated with the above specification of  $X_t$  will be denoted by P.

The second equation relates the one-period interest rate  $r_t$  to the state variables (positing that it be an affine function of the state variables):

$$r_t = a + b^{\mathsf{T}} X_t \tag{2}$$

where a is a scalar and b is a  $k \times 1$  vector.

The third equation is related to bond pricing in an arbitrage-free market. A sufficient condition for the absence of arbitrage on the bond market is that there exists a risk-neutral measure Q, equivalent to P, under which the process  $X_t$  follows the dynamics:

$$X_t = \overline{\mu} + \overline{\rho} X_{t-1} + \Sigma \eta_t \tag{3}$$

where  $\eta_t \sim N(0, I_k)$  under Q and such that the price at time t of a bond paying a unitary amount of cash at time t + n (denoted by  $p_t^n$ ) equals:

$$p_t^n = \mathcal{E}_t^Q \left[ \exp(-r_t) \, p_{t+1}^{n-1} \right] \tag{4}$$

where  $\mathbf{E}_t^Q$  denotes expectation under the probability measure Q, conditional upon the information available at time t.

The vector  $\overline{\mu}$  and the matrix  $\overline{\rho}$  are in general different from  $\mu$  and  $\rho$ , while equivalence of P and Q guarantees that  $\Sigma$  is left unchanged. The link between the risk-neutral distribution Q and the physical distribution P is given by the (timevarying) price of risk  $\lambda_t$ :

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{5}$$

where  $\lambda_0 = \Sigma^{-1} (\mu - \overline{\mu})$  and  $\lambda_1 = \Sigma^{-1} (\rho - \overline{\rho})$ . According to Cameron, Martin and Girsanov's theorem (e.g. Kallenberg - 1997)

$$\mathbf{E}_{t}^{P}\left[\frac{dQ}{dP}\right] = \prod_{j=1}^{\infty} \exp\left[-\frac{1}{2}\lambda_{t+j-1}^{\top}\lambda_{t+j-1} - \lambda_{t+j-1}^{\top}\varepsilon_{t+j}\right]$$
(6)

so that the pricing kernel

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^{\mathsf{T}}\lambda_t - \lambda_t^{\mathsf{T}}\varepsilon_{t+1}\right)$$
(7)

can be used to recursively price bonds:

$$p_t^n = \mathcal{E}_t^P \left[ m_{t+1} p_{t+1}^{n-1} \right]$$
(8)

Multifactor affine models of the term structure, such as the one just described, are very popular in the finance literature and their properties have long been studied by many researchers. Thorough specification analyses of these models have been conducted (e.g. Dai and Singleton, 2000) and their properties are now well-known. A distinguishing feature of these models is that they are able to describe the dynamics of yields in terms of a small set of unobservable state variables: typically three variables are deemed a sufficient number to describe the whole yield curve and this is also supported by empirical studies, such as the seminal paper by Litterman and Scheinkman (1991). Although such models are capable of describing accurately and parsimoniously the evolution of interest rates over time, the factors they identify as the driving forces of interest rates often lack economic intuition and are difficult to relate to relevant economic variables. This is one of the reasons why recent studies have proposed to augment the usual set of unobservable state variables with some observable variables. Typically, inflation and a measure of the output gap are the two observable variables, while a small number of unobservable factors, ranging from one to three, are included in the models: recent examples are Ang and Piazzesi (2003), Rudebusch and Wu (2005), Hördal, Tristani and Vestin (2005) and Ang, Piazzesi and Wei (2005). All these works impose some set of restrictions on the system of equations (1-3) and, after estimating the coefficients, derive bond prices using equation (4).

We take the same approach, adding inflation and output gap to the unobservable factors. However, rather than imposing ad hoc set of restrictions on the parameters of the model and arbitrarily defining the number of unobservable variables, we derive a set of minimal identifying restrictions and, placing only these restrictions on the model, we perform a specification analysis to select the number of unobservable factors.

Our minimal set of identifying restrictions is not the standard set of restrictions usually imposed for identification of affine term-structure models (e.g.: Dai and Singleton - 2000). Standard models of the term structure include only unobservable factors and equivalent representations of the factor dynamics can be obtained by performing any rotation and translation of the factors. On the contrary, our set of identifying restrictions takes into account the fact that in a model with both observable and unobservable factors equivalent representations can be obtained only with rotations and translations which leave the observable factors unchanged.

Suppose that the first  $k^o$  variables included in the model are observable and the remaining  $k^u = k - k^o$  are unobservable. Collect their values at time t into the  $k^o \times 1$  vector  $X_t^o$  and the  $k^u \times 1$  vector  $X_t^u$  respectively. Equations (1-3) can be written as follows:

Short-rate  
process
$$\begin{cases}
r_t = a + b^{o^{\top}} X_t^o + b^{u^{\top}} X_t^u \\
Law of motion under P
\end{cases}
\begin{cases}
X_t^o = \mu^o + \rho^{oo} X_{t-1}^o + \rho^{ou} X_{t-1}^u + \Sigma^{oo} \varepsilon_t^o \\
X_t^u = \mu^u + \rho^{uo} X_{t-1}^o + \rho^{uu} X_{t-1}^u + \Sigma^{uo} \varepsilon_t^o + \Sigma^{uu} \varepsilon_t^u
\end{cases}$$
(9)
Law of motion 
$$\begin{cases}
X_t^o = \overline{\mu}^o + \overline{\rho}^{oo} X_{t-1}^o + \overline{\rho}^{ou} X_{t-1}^u + \Sigma^{oo} \eta_t^o \\
X_t^u = \overline{\mu}^u + \overline{\rho}^{uo} X_{t-1}^o + \overline{\rho}^{uu} X_{t-1}^u + \Sigma^{uo} \eta_t^o + \Sigma^{uu} \eta_t^u
\end{cases}$$

where all the matrices are obtained by separating into blocks the matrices in equations (1-3).

The following proposition, proved in the Appendix, gives the minimal set of restrictions to be imposed in order to identify the model:

**Proposition 1** Let  $\overline{\rho}$  have distinct and real eigenvalues. Then model (9) always admits an equivalent representation (eventually after renaming the unobservable factors and the error terms) with the following restrictions:

- $b^u = \overline{1}$  (a vector of 1s)
- $\overline{\mu}^u = 0$
- $\overline{\rho}^{uo} = 0$
- $\overline{\rho}^{uu}$  is diagonal
- $\Sigma^{oo}$  and  $\Sigma^{uu}$  are lower triangular

We impose the above set of minimal restrictions on the models we estimate. Proposition (1) allows us to understand restrictions imposed by models previously proposed in the literature. For example, Ang and Piazzesi's (2003) model, which can be re-parametrized as a special case of the general model in (9), imposes a set of over-identifying restrictions equivalent to the following:  $\rho^{uo} = 0$ ,  $\rho^{ou} = 0$ ,  $\overline{\rho}^{ou} = 0$ and  $\Sigma^{uo} = 0$ . Hördahl, Tristani and Vestin (2005) also build a model which is a special case of (9): they impose on the *P*-dynamics a set of restrictions which are derived from a structural model of the economy using Söderlind's (1999) procedure and they specify the dynamics under *Q* with a restricted parametrization of the prices of risk  $\lambda_0$  and  $\lambda_1$ . Another structural model encompassed as a special case by (9) is derived in Rudebusch and Wu (2005).

Note that within this Gaussian framework bond yields are affine functions of the state variables:

$$y_t^n = -\frac{1}{n}\ln(p_t^n) = A_n + B_n^{\mathsf{T}}X_t$$
 (10)

where  $y_t^n$  is the yield at time t of a bond maturing in n periods and  $A_n$  and  $B_n$  are coefficients obeying the following simple system of Riccati equations, derived from (4): <sup>2</sup>

$$A_1 = a \tag{11}$$

$$B_1 = b \tag{12}$$

$$A_{n} = \frac{1}{n} \left[ a + (n-1) \left( A_{n-1} + B_{n-1}^{\mathsf{T}} \overline{\mu} - \frac{n-1}{2} B_{n-1}^{\mathsf{T}} \Sigma \Sigma^{\mathsf{T}} B_{n-1} \right) \right]$$
(14)

$$B_{n} = \frac{1}{n} \left[ b + (n-1) \,\overline{\rho}^{\mathsf{T}} B_{n-1} \right] \tag{15}$$

The yields  $\widetilde{y}_t^n$  and the bond prices  $\widetilde{p}_t^n$  that would obtain in an arbitrage-free market populated by risk neutral investors are instead obtained setting the prices of risk to zero ( $\lambda_t = 0$ ) in (7) and (8):

$$\widetilde{p}_t^n = \mathcal{E}_t^P \left[ \exp\left(-r_t\right) \widetilde{p}_{t+1}^{n-1} \right]$$
(16)

They obey the same system of recursive equations (11), where  $\overline{\mu}$  and  $\overline{\rho}$  are substituted by  $\mu$  and  $\rho$ . Subtracting the risk-neutral yields  $\tilde{y}_t^n$  thus calculated from the actual yields  $y_t^n$  one obtains the risk premiums  $\pi_t^n$ :

 $<sup>^{2}</sup>$ A proof by induction for a more general case can be found, for example, in Dai, Singleton and Yang (2003).

$$\pi_t^n = y_t^n - \widetilde{y}_t^n \tag{17}$$

 $\pi_t^n$  is the additional interest per unit of time required by investors to bear the risk associated with the fluctuations in the price of a bond expiring in n periods. Such premiums are in general time varying and they are constant only when  $\rho = \overline{\rho}$ .

### 3. The data

For our empirical analysis of the term structure we rely on a dataset of zero coupon rates extracted from German government bond yields and recorded at a monthly frequency, provided by the Deutsche Bundesbank : the yield curve consists of ten maturities, from 1 to 10 years. Since we estimate the model at a monthly frequency, we also include a one-month interest rate taken from the money market, as a control. The sample goes from January 1973 to September 2004 and the yields are registered on the last trading day of each month. We utilize all the eleven maturities to carry out estimation of the models. In this respect our paper differs from most existing studies, which select only small subsets of the available maturities and typically do not employ yields of maturities longer than five years. We prefer not to exclude a priori any maturity from our sample, because we are also interested in understanding the capability of the models to fit the entire yield curve.

We include two macroeconomic variables in our model: an inflation rate and a measure of the output gap. The inflation rate is the twelve-month growth rate of the German consumer price index. The output gap is derived from industrial production, applying band-pass filters with different frequency ranges (2-4, 3-5 and 2-8 years), as in Baxter and King (1995). We rely on industrial production to construct a measure of the output gap, because it is available at monthly frequency and it is widely considered a coincident indicator of the business cycle.

#### 4. Empirical evidence

The first step in our estimation strategy is to select the number of unobservable variables to include in the model. We estimate three models, all having inflation and the output gap as observable variables. The three models have one, two and three unobservable variables respectively and are estimated imposing only the minimal set of identifying restrictions given in Proposition 1.

The models are estimated by maximum likelihood using Chen and Scott's (1993) methodology: given a set of parameters, observed bond prices are used to infer the values of the unobservable factors (see the Appendix for details). In order to do so, one has to assume that a number of bonds equal to the number of unobservable factors are exactly priced and their prices are measured without error: we choose the 3-year bond for the model with one unobservable factor and we add first the 5-year and then the 10-year when we increase the number of unobservable variables to two and three. Different choices of the set of exactly priced bonds do not seem to change parameter estimates significantly, as long as shorter maturities (up to 2 years) are excluded from the set. The estimated standard deviations of the pricing errors on longer maturities are always very small (usually less than 5 basis points), indicating that the assumption of exact pricing is not overly restrictive for these maturities. On the contrary, when shorter maturities are not assumed to be exactly priced, the estimated standard deviations of their errors are quite high (in some cases more than 50 basis points), suggesting that one should exclude these maturities from the set of exactly priced bonds.

Due to the highly non-linear dependence of the likelihood function on the parameters of the risk-neutral distribution, numerical maximization is computationally quite burdensome. We find that a considerable increase in speed is achieved using the simulated annealing algorithm (this should also avoid local maxima) and using a Schur decomposition (see Meyer - 2001) to parametrize the matrix  $\rho$  and the block  $\overline{\rho}^{oo}$  of the matrix  $\overline{\rho}$ . We use the following Schur decomposition of an *n*-dimensional square matrix A (Khuri - 2002):

$$A = UTU^{\top} \tag{18}$$

$$U = (I - Q) (I + Q)^{-1}$$
(19)

where Q is skew-symmetric (n(n-1)/2 parameters), T is is upper triangular (n(n+1)/2 parameters) and U is orthogonal by construction. By constraining the elements on the principal diagonal of T (as well as the elements on the diagonal of  $\overline{\rho}^{uu}$ ) to be strictly less than 1 in absolute value, we ensure that  $\rho$  and  $\overline{\rho}$  have all their eigenvalues inside the unit circle, so that the process  $X_t$  is stationary both under the physical and the risk neutral measure. Although at an optimum we never find the latter constraints to be binding, the constraints considerably restrict the parameter space where numerical search is performed, hence increasing speed.<sup>3</sup>

Standard information criteria (SBC and AIC) suggest that the model with three unobservable variables is the most appropriate to describe the joint dynamics of interest rates and macro variables, hence we comment the results obtained with this model. Note that, although no lags of inflation and output gap are explicitly included in our model, the unobservable factors provide a flexible device to eventually capture lagged effects of the variables in the system (both linear and non-linear).

All the models were estimated three times, one for each of the three frequency ranges used to filter the output gap. The correlations between the measures of risk premiums obtained in each estimation were always higher than 0.995, indicating that the results are robust to different choices of the measure of output gap. We report the results obtained with the widest frequency range (2-8 years).

Table 5 displays the coefficients of the estimated model. The standard errors are obtained numerically, using two-sided approximated first and second order derivatives. Since the log-likelihood of the sample is the sum of non-independent conditional log-likelihoods, we account for serial correlation in the scores by using a

<sup>&</sup>lt;sup>3</sup>The constraints are implicitly imposed parametrizing each constrained parameter p as  $p = 0.9999 \cdot \cos(\theta)$ , so that an unconstrained maximization algorithm can still be used.

Newey-West estimator to compute the long-run covariance matrix. The bandwidth is set equal to 12 months, in view of the fact that annual inflation can artificially induce autocorrelations up to the eleventh lag. Further enlarging the bandwidth does not seem to produce relevant changes in estimated standard deviations. Standard deviations are generally quite small, making most parameters significantly different from zero. An exception are some off-diagonal elements of  $\rho$ , which are close to zero and have high standard deviations. The inferences to be drawn from the model are not altered when we apply the two-stage procedure adopted, for example, by Dai and Singleton (2000), which consists in re-estimating the model imposing zero constraints on the parameters not significantly different from zero. We find that the restrictions  $\overline{\rho}^{ou} = 0$  and  $\Sigma^{uo} = 0$  imposed by Ang and Piazzesi (2003) are rejected at all conventional confidence levels in our sample. However, their restrictions do not have substantial consequences on estimated risk premiums (Table 2 and Figure 7). Instead, we find a dramatic change when we adopt their two-stage consistent estimation strategy, which consists in estimating the parameters  $a, b^{o}, \mu^{o}, \rho^{oo}$  and  $\Sigma^{oo}$ in a first step and the remaining parameters in a second step. With their two-stage procedure, estimated risk premiums are on average lower and more variable (Table 2 and Figure 7) and more than 25 per cent of the times estimated risk premiums are negative at all maturities.

Figure 1 displays the time series of risk premiums for the 3, 5 and 10-year bonds, calculated as the difference between the yield that the market required on those bonds at any point in time and the yield that a risk-neutral investor would have required to hold the same bonds. It is evident that bond risk premiums display a considerable variability across time: for example, the premium on the 10-year bond, which averages 186 basis points throughout our sample, has a standard deviation of 72 points and reaches a peak of 388 basis points in March 1975 and a trough of 15 in March 1992. Table 1 reports more details about the sample distribution of risk premiums for all the maturities.

The average risk premium in our sample is increasing with maturity (see Figure 2): it is quite small for shorter maturity bonds (3 and 35 basis points for the 1 and 2-

year bonds respectively), it averages about one hundred points for the intermediate maturities (4 to 6 years) and reaches a maximum of 186 points for the 10-year maturity. Dividing the sample into two sub-samples (before and after 1990), we find that risk premiums have been lower during the last fifteen years. Also the variability of risk premiums is increasing with the maturity: the standard deviation is about 27 points for the 1-year bond and increases to about 72 points for the 10-year bond.

The variation of risk premiums over time seems to be strongly related to macroeconomic variables. In particular, high negative (partial) correlation with the output gap at all maturities (see Table  $5^4$ ) suggests that the risk premiums are countercyclical. If one assumes positive correlation between consumption and production, this is consistent with the hypothesis that the price of future consumption is low (interest rates are high) when current consumption is low (hence current marginal utility is high). Moreover, an impulse response analysis carried out on some yields (see Figure 5) suggests that an increase in output causes a decrease in risk premiums. The link of risk premiums to inflation seems to be less evident: there is a small negative partial correlation between inflation and risk premiums on shorter maturity bonds, while the correlation is positive, but not statistically significant, for longer maturities. Note that the pricing equation (4) is defined in nominal terms, but it is fully equivalent to one defined in real terms, once a proper change of numeraire has been performed: hence, a correlation of risk premiums with inflation cannot be attributed to the fact that we are estimating the model with nominal quantities. The impulse-response analysis (Figure 6) shows that risk premiums tend to decrease slightly when inflation increases, but they eventually revert and then remain above the equilibrium level for some time.

Tables 3 and 4 show the variance decomposition of risk premiums and interest

<sup>&</sup>lt;sup>4</sup>The estimates of the regressions reported in Table 5 are to be interpreted as estimates of the coefficients of an orthogonal projection of the risk premiums on the two-macroeconomic variables. The regressions have therefore no structural interpretation, also in view of the fact that the regressors are endogenous, but they provide a joint evaluation of the predictability of risk premiums, which takes into account the correlation between predictors.

rates. Within our sample, the proportion of the variance of interest rates explained by macroeconomic fundamentals is never greater than 20 per cent. As a general rule, the proportion of variance explained by macroeconomic variables increases with the forecasting horizon and with the maturity of the bonds. When we look only at risk premiums, the proportion of variance explained by macro-factors dramatically increases, up to almost 50 per cent: shocks to output play a prominent role in determining unexpected changes in the risk premiums; inflation plays a relevant role only for shorter maturities. The fact that the proportion of variance explained by macro-factors is much higher for risk premiums than for yields might seem puzzling at first. However this is explained by the fact that the short-term interest rate is pro-cyclical and risk premiums are counter-cyclical: long-term bond yields, being a sum of the two, do not react much to output shocks because the separate effects of these shocks on premiums and the short rate compensate each other.

#### 6. Conclusions

We have analyzed the dynamics of risk premiums on the German bond market, employing no-arbitrage multifactor affine term-structure models. We have followed the approach, recently popularized by Ang and Piazzesi (2003), of including both unobservable and observable variables in the set of state variables, in order to assess the link between macroeconomic fundamentals and risk premiums. We carried out a specification analysis, based on a new set of identifying conditions, in order to select the best model. We found that, even after including inflation and output gap in the set of state variables, three unobservable variables are still needed to describe accurately yield curve dynamics, confirming what is already well-established for models with latent variables only. We have proposed a methodology to quantify risk premiums, which gives easily interpretable measures of the additional interest per unit of time required by investors for bearing the risk associated with bond price fluctuations. Our sample provides evidence that such premiums are strongly time-varying and a considerable portion of this variability is due to output and inflation shocks. There is a systematic relation between output and premiums, the latter being countercyclical, but we find no systematic relation between inflation and

premiums. Both findings are consistent with the predictions of economic theory.

## Appendix

## 0.1 Proposition 1

**Proof.** The law of motion of the process under Q is:

$$X_t = \overline{\mu} + \overline{\rho} X_{t-1} + \Sigma \eta_t \tag{20}$$

Define a matrix C as follows:

$$C = \begin{bmatrix} e_1 & \dots & e_{k^o} & v_1^\top & \dots & v_{k^u}^\top \end{bmatrix}^\top$$
(21)

where:

$$e_1 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}_{\top}^{\top}$$
(22)

$$e_2 = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \end{bmatrix}^{+}$$
 (23)

$$\dots$$
 (24)

are the first  $k^o$  vectors of the Euclidean basis of  $\mathbb{R}^{k^o+k^u}$  and  $v_1, \ldots, v_{k^u}$  are  $k^u$  independent left eigenvectors of  $\overline{\rho}$ . Since  $\overline{\rho}$  has got distinct eigenvalues, it is always possible to choose  $v_1, \ldots, v_{k^u}$  in such a way that C is invertible. Denote by  $\Lambda$ the diagonal matrix whose diagonal elements are the eigenvalues associated with  $v_1, \ldots, v_{k^u}$  and define:

$$C^{u} = \left[ \begin{array}{cc} v_{1}^{\top} & \dots & v_{k^{u}}^{\top} \end{array} \right]^{\top}$$

$$(25)$$

Pre-multiplying (20) by C, one obtains:

$$X_t^o = \overline{\mu}^o + \left[ \overline{\rho}^{oo} \ \overline{\rho}^{ou} \right] X_{t-1} + \Sigma^{oo} \eta_t^o$$
(26)

$$C^{u}X_{t} = C^{u}\overline{\mu}^{u} + \Lambda C^{u}X_{t-1} + C^{u}\Sigma^{uo}\eta_{t}^{o} + C^{u}\Sigma^{uu}\eta_{t}^{u}$$
(27)

Transform the first equation in (26) as follows:

$$X_t^o = \overline{\mu}^o + \left[ \overline{\rho}^{oo} \quad \overline{\rho}^{ou} \right] X_{t-1} + \Sigma^{oo} \eta_t^o$$
(28)

$$= \overline{\mu}^{o} + \left[ \overline{\rho}^{oo} \quad \overline{\rho}^{ou} \right] C^{-1} C X_{t-1} + \Sigma^{oo} \eta_t^o$$
<sup>(29)</sup>

$$= \overline{\mu}^{o} + \left[ \overline{\rho}^{oo} \quad \overline{\rho}^{ou} \right] C^{-1} \left[ X_{t-1}^{o\top} \quad \left( C^{u} X_{t-1} \right)^{\top} \right]^{\top} + \Sigma^{oo} \eta_{t}^{o}$$
(30)

Redefining  $X_t^u := C^u X_t$  and setting  $F = \begin{bmatrix} \overline{\rho}^{oo} & \overline{\rho}^{ou} \end{bmatrix} C^{-1}$  one gets:

$$X_t^o = \overline{\mu}^o + F X_{t-1} + \Sigma^{oo} \eta_t^o \tag{31}$$

$$X_t^u = C^u \overline{\mu}^u + \Lambda X_{t-1}^u + C^u \Sigma^{uo} \eta_t^o + C^u \Sigma^{uu} \eta_t^u$$
(32)

Since the eigenvectors of  $\overline{\rho}$  strictly lie inside the unit circle, it is possible to redefine  $X_t^u$  again as  $X_t^u := X_t^u - (I - \Lambda)^{-1} C^u \overline{\mu}^u$  so that it has zero mean. Multiplying each unobservable factor by its corresponding coefficient in  $b^u$ , one obtains the representation in Proposition 1 by appropriately matching the coefficients in Proposition 1 with those in (31) (note that redefining the unobservable factors also affects the law of  $X_t$  under P, so that in general no restriction can be imposed on the P-dynamics).

## 0.2 Inversion of yields

Suppose that at each time period bond yields of m (with  $m > k^u$ ) different maturities  $(n_1, n_2, \ldots, n_m)$  are observable. Performing an "inversion" of  $k^u$  observable yields (in the spirit of Duffie and Kan - 1996 and Pang and Hodges - 1995), it is possible to express the unobservable factors as linear combinations of observable yields and observable factors. This procedure allows to recover a set of equations to be estimated where the unobservable factors do not appear:

$$\begin{cases} X_t^o = \alpha^o + \beta^{oo} X_{t-1}^o + \beta^{oe} y_{t-1}^e + T^{oo} \varepsilon_t^o \\ y_t^e = \alpha^e + \beta^{eo} X_{t-1}^o + \beta^{ee} y_{t-1}^e + T^{eo} \varepsilon_t^o + T^{eu} \varepsilon_t^u \\ y_t^f = \alpha^f + \beta^{fo} X_{t-1}^o + \beta^{fe} y_{t-1}^e + T^{fo} \varepsilon_t^o + T^{fu} \varepsilon_t^u \end{cases}$$
(33)

In the above system of equations  $y_t^e$  is the vector of  $k^u$  observable yields used to invert the unobservable factors,  $y_t^f$  is the vector containing the remaining  $m - k^u$ yields and the matrices  $\alpha^i$ ,  $\beta^{ik}$  and  $T^{ik}$  (of appropriate dimensions) are non-linear functions of the parameters of the model (the exact functional forms are reported below). Any choice of the  $k^u$  yields to be included in the vector  $y_t^e$  gives rise to an equivalent representation of the system. (33) is a VAR, where the observable factors and  $k^u$  yields are regressed on their own lags, to which a system of regression equations explaining the remaining  $m - k^u$  yields has been adjoined. As it stands, the system can not be subjected to statistical estimation, because there are only  $k^u + k^o$  sources of error for a total of  $m + k^o > k^u + k^o$  equations to be estimated and the covariance matrix of the error terms is singular. The hypothesis usually made in order to estimate the system is that observed yields are subject to measurement or pricing errors, that is the econometrician does not observe  $y_t^e$  and  $y_t^f$ , but  $\tilde{y}_t^e$  and  $\tilde{y}_t^f$ , where:

$$\widetilde{y}_t^e = y_t^e + D^e z_t^e \tag{34}$$

$$\widetilde{y}_t^f = y_t^f + D^f z_t^f, aga{35}$$

 $z_t^e$  and  $z_t^f$  are  $k^u \times 1$  and  $(m - k^u) \times 1$  multivariate standard normal random vectors respectively and  $D^e$  and  $D^f$  are conformable matrices. It is often assumed (e.g. Chen and Scott - 1993 and Ang and Piazzesi - 2003) that the  $k^u$  yields in  $y_t^e$  are measured without error ( $\tilde{y}_t^e = y_t^e$ ): although theoretically restrictive, this assumption allows us to identify all the error terms, because it makes the number of errors equal to the number of equations; furthermore, both  $y_t^e$  and its lag  $y_{t-1}^e$  appear in the second equation of (33), hence if  $D^e \neq 0$  error terms are serially correlated and statistical estimation of (33) becomes much more involved.

Assuming exact pricing of  $y_t^e$ , the system of equations to be estimated is:

$$\begin{cases} X_t^o = \alpha^o + \beta^{oo} X_{t-1}^o + \beta^{oe} y_{t-1}^e + T^{oo} \varepsilon_t^o \\ y_t^e = \alpha^e + \beta^{eo} X_{t-1}^o + \beta^{ee} y_{t-1}^e + T^{eo} \varepsilon_t^o + T^{eu} \varepsilon_t^u \\ \widetilde{y}_t^f = \alpha^f + \beta^{fo} X_{t-1}^o + \beta^{fe} y_{t-1}^e + T^{fo} \varepsilon_t^o + T^{fu} \varepsilon_t^u + D^f z_t^f \end{cases}$$
(36)

The above equations are simply regressions of the observable yields and the observable variables on one-period lags of the observable variables and the exactly priced yields. Although the same regressors appear on the right-hand side of all equations, OLS estimation is not feasible, because the regression coefficients and the covariance matrix are functions of the same parameters and cannot be estimated independently. Following the majority of the literature on term-structure models, we propose maximum likelihood estimation of the system.

### 0.2.1 Functional form of the regression coefficients

The yields are affine in the observable and unobservable factors:

$$\begin{cases} y_t^e = A^e + B^{eo} X_t^o + B^{eu} X_t^u \\ y_t^f = A^f + B^{fo} X_t^o + B^{fu} X_t^u \end{cases}$$
(37)

Note that the coefficients  $A^i$  and  $B^{ij}$  are functions of the parameters of the process  $X_t$  under the risk-neutral measure Q. Lag the first equation by one period and invert, to obtain:

$$X_{t-1}^{u} = (B^{eu})^{-1} \left( y_{t-1}^{e} - A^{e} - B^{eo} X_{t-1}^{o} \right)$$
(38)

The VAR (under P) is:

$$\begin{cases} X_t^o = \mu^o + \rho^{oo} X_{t-1}^o + \rho^{ou} X_{t-1}^u + \Sigma^{oo} \varepsilon_t^o \\ X_t^u = \mu^u + \rho^{uo} X_{t-1}^o + \rho^{uu} X_{t-1}^u + \Sigma^{uo} \varepsilon_t^o + \Sigma^{uu} \varepsilon_t^u \end{cases}$$
(39)

Substituting (38) into (39), we get:

$$\begin{cases} X_{t}^{o} = \mu^{o} + \rho^{oo} X_{t-1}^{o} + \rho^{ou} \left( B^{eu} \right)^{-1} \left( y_{t-1}^{e} - A^{e} - B^{eo} X_{t-1}^{o} \right) + \Sigma^{oo} \varepsilon_{t}^{o} \\ X_{t}^{u} = \mu^{u} + \rho^{uo} X_{t-1}^{o} + \rho^{uu} \left( B^{eu} \right)^{-1} \left( y_{t-1}^{e} - A^{e} - B^{eo} X_{t-1}^{o} \right) + \Sigma^{uo} \varepsilon_{t}^{o} + \Sigma^{uu} \varepsilon_{t}^{u} \end{cases}$$

$$\tag{40}$$

Now, use the two equations in (40) to eliminate  $X_t^o$  and  $X_t^u$  from the two equations in (37) and adjoin the first equation in (40) to obtain the following system of regression equations, involving only observable variables (factors and yields):

$$y_t^e = \alpha^e + \beta^{eo} X_{t-1}^o + \beta^{ee} y_{t-1}^e + T^{eo} \varepsilon_t^o + T^{eu} \varepsilon_t^u$$
  

$$y_t^f = \alpha^f + \beta^{fo} X_{t-1}^o + \beta^{fe} y_{t-1}^e + T^{fo} \varepsilon_t^o + T^{fu} \varepsilon_t^u$$
  

$$X_t^o = \alpha^o + \beta^{oo} X_{t-1}^o + \beta^{oe} y_{t-1}^e + T^{oo} \varepsilon_t^o$$
(41)

$$T^{oo} = \Sigma^{oo} \tag{55}$$

$$\beta^{oe} = \rho^{ou} (B^{eu})^{-1} \tag{54}$$

$$\beta^{oo} = \rho^{oo} - \rho^{ou} \left(B^{eu}\right)^{-1} B^{eo}$$
(53)

$$\alpha^{o} = \mu^{o} - \rho^{ou} \left(B^{eu}\right)^{-1} A^{e}$$
(52)

$$T^{fu} = B^{fu} \Sigma^{uu} \tag{51}$$

$$T^{fo} = B^{fo}\Sigma^{oo} + B^{fu}\Sigma^{uo}$$
<sup>(50)</sup>

$$\beta^{fe} = B^{fo} \rho^{ou} (B^{eu})^{-1} + B^{fu} \rho^{uu} (B^{eu})^{-1}$$
(49)

$$\beta^{fo} = B^{fo} \left( \rho^{oo} - \rho^{ou} \left( B^{eu} \right)^{-1} B^{eo} \right) + B^{fu} \left( \rho^{uo} - \rho^{uu} \left( B^{eu} \right)^{-1} B^{eo} \right)$$
(48)

$$\alpha^{f} = A^{f} + B^{fo} \left( \mu^{o} - \rho^{ou} \left( B^{eu} \right)^{-1} A^{e} \right) + B^{fu} \left( \mu^{u} - \rho^{uu} \left( B^{eu} \right)^{-1} A^{e} \right)$$
(47)

$$T^{eo} = B^{eo} \Sigma^{oo} + B^{eu} \Sigma^{uo}$$
(45)  
$$T^{eu} = B^{eu} \Sigma^{uu}$$
(46)

$$T^{eo} = B^{eo} \Sigma^{oo} + B^{eu} \Sigma^{uo} \tag{45}$$

$$\beta^{ee} = B^{eo} \rho^{ou} (B^{eu})^{-1} + B^{eu} \rho^{uu} (B^{eu})^{-1}$$
(44)

$$\beta^{eo} = B^{eo} \left( \rho^{oo} - \rho^{ou} \left( B^{eu} \right)^{-1} B^{eo} \right) + B^{eu} \left( \rho^{uo} - \rho^{uu} \left( B^{eu} \right)^{-1} B^{eo} \right)$$
(43)

$$\alpha^{e} = A^{e} + B^{eo} \left( \mu^{o} - \rho^{ou} \left( B^{eu} \right)^{-1} A^{e} \right) + B^{eu} \left( \mu^{u} - \rho^{uu} \left( B^{eu} \right)^{-1} A^{e} \right)$$
(42)

where:

#### References

- Ang, A., S. Dong and M. Piazzesi (2004), "No-Arbitrage Taylor Rules", mimeo, Columbia University.
- Ang, A. and M. Piazzesi (2003), "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables", Journal of Monetary Economics, 50, pp. 745-787.
- Ang, A., M. Piazzesi and M. Wei (2005), "What Does the Yield Curve Tell us about GDP Growth?", forthcoming in *Journal of Econometrics*.
- Baxter, M. and R. G. King (1995), "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series", NBER Working Paper No. 5022.
- Chen, R. R. and L. Scott (1993), "Maximum Likelihood Estimation for a Multifactor Equilibrium Model of the Term Structure of Interest Rates", Journal of Fixed Income, 3, pp. 14-31.
- Dai, Q. and K. J. Singleton (2000), "Specification Analysis of Affine Term Structure Models", Journal of Finance, 55, pp. 1943-78.
- Dai, Q., K. J. Singleton and W. Yang (2003), "Regime Shifts in a Dynamic Term Structure Model of U.S. Treasury Bond Yields", mimeo.
- Diebold, X. F., M. Piazzesi and G. D. Rudebusch (2005), "Modelling Bond Yields in Finance and Macroeconomics", forthcoming in American Economic Review, Papers and Proceedings.
- Duffie, D. and R. Kan (1996), "A Yield-Factor Model of Interest Rates", Mathematical Finance, 6, pp. 379-406.
- Estrella, A. and F. S. Mishkin (1995), "The Term Structure of Interest Rates and its Role in Monetary Policy for the European Central Bank", NBER Working Paper No. 5279, September.

- Evans, C. L. and D. A. Marshall (1998), "Monetary Policy and the Term Structure of Nominal Interest Rates: Evidence and Theory", Carnegie Rochester Series on Public Policy, 49, pp. 53-111.
- Fama, E. (1990), "Term-structure Forecasts of Interest Rates, Inflation and Real Returns", Journal of Monetary Economics, 25, pp. 59-76.
- Hördal, P., O. Tristani and D. Vestin (2005), "A Joint Econometric Model of Macroeconomic and Term Structure Dynamics", forthcoming in *Journal of Econometrics*.
- Knez, P. K., R. Litterman and J.A. Scheinkman (1994), "Explorations into factors explaining money market returns", *Journal of Finance*, 49, pp. 1861-1882.
- Litterman, R. and J. A. Scheinkman (1991), "Common factors affecting bond returns", *Journal of Fixed Income*, 1, pp. 54-61.
- Kallenberg, O. (1997), Foundations of Modern Probability, Springer.
- Khuri, A. I. (2002), Advanced Calculus with Applications in Statistics, Wiley.
- Meyer, C. D. (2001), Matrix Analysis and Applied Linear Algebra, Society for Industrial and Applied Math.
- Mishkin, F. S. (1990), "What Does the Term-structure Tell Us About Future Inflation?", Journal of Monetary Economics, 25, pp. 76-95.
- Pang, K. and Hodges, S. (1995), "Non-Negative Affine Models of the Term-Structure", mimeo, University of Warwick.
- Rudebusch, G. D. and T. Wu (2005), "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy", forthcoming in *Journal of Money, Credit* and Banking.
- Söderlind, P. (1999), "Solution and Estimation of RE Macromodels with Optimal Policy", *European Economic Review*, 43, pp. 813-823.

# 1 Tables

Sample Period: Jan 1973 to Sept 2004											
Maturity	1	2	3	4	5	6	7	8	9	10	
Mean	0.03	0.35	0.65	0.91	1.13	1.32	1.48	1.62	1.75	1.86	
Std deviation	0.27	0.39	0.48	0.55	0.60	0.64	0.67	0.69	0.70	0.72	
Min	-0.80	-0.83	-0.75	-0.64	-0.50	-0.36	-0.21	-0.07	0.05	0.15	
First quartile	-0.16	0.12	0.35	0.55	0.73	0.90	1.04	1.16	1.28	1.37	
Median	0.06	0.36	0.66	0.92	1.11	1.30	1.45	1.60	1.71	1.81	
Third quartile	0.22	0.61	0.94	1.25	1.52	1.73	1.88	2.02	2.16	2.27	
Max	0.57	1.25	1.88	2.37	2.75	3.07	3.32	3.54	3.72	3.88	
Sample Period: Jan 1973 to Dec 1989											
		Sam	ple Peri	od: Jan	1973 to	Dec 198	39				
Maturity	1	Sam 2	ple Perie 3	od: Jan 4	1973 to 5	Dec 198 6	89 7	8	9	10	
Maturity Mean	1 0.05	Sam 2 0.42	ple Perio 3 0.76	od: Jan 4 1.05	1973 to 5 1.29	Dec 198 6 1.50	$\frac{39}{1.67}$	8 1.82	9 1.96	10 2.08	
Maturity Mean Std deviation	$1 \\ 0.05 \\ 0.27$	Sam 2 0.42 0.36	ple Perio 3 0.76 0.45	od: Jan 4 1.05 0.51	1973 to 5 1.29 0.56	Dec 198 6 1.50 0.59	$\frac{7}{1.67}$	8 1.82 0.63	9 1.96 0.64	10 2.08 0.65	
Maturity Mean Std deviation Min	1 0.05 0.27 -0.52	Sam 2 0.42 0.36 -0.39	ple Perio 3 0.76 0.45 -0.27	od: Jan 4 1.05 0.51 -0.11	1973 to 5 1.29 0.56 0.06	Dec 198 6 1.50 0.59 0.23	$     \frac{39}{1.67} \\     0.61 \\     0.39   $	8 1.82 0.63 0.42	9 1.96 0.64 0.44	10 2.08 0.65 0.46	
Maturity Mean Std deviation Min First quartile	1 0.05 0.27 -0.52 -0.16	Sam 2 0.42 0.36 -0.39 0.19	ple Perio 3 0.76 0.45 -0.27 0.47	bd: Jan     4     1.05     0.51     -0.11     0.70	1973 to 5 1.29 0.56 0.06 0.92	Dec 198 6 1.50 0.59 0.23 1.11	39 7 1.67 0.61 0.39 1.30	8 1.82 0.63 0.42 1.46	$9 \\ 1.96 \\ 0.64 \\ 0.44 \\ 1.59$	$     \begin{array}{r}       10 \\       2.08 \\       0.65 \\       0.46 \\       1.72     \end{array} $	
Maturity Mean Std deviation Min First quartile Median	1 0.05 0.27 -0.52 -0.16 0.07	Sam 2 0.42 0.36 -0.39 0.19 0.42	ple Perio 3 0.76 0.45 -0.27 0.47 0.78	0d: Jan 4 1.05 0.51 -0.11 0.70 1.06	$   \begin{array}{r}     1973 to \\     5 \\     1.29 \\     0.56 \\     0.06 \\     0.92 \\     1.29   \end{array} $	Dec 198 6 1.50 0.59 0.23 1.11 1.48	$     \frac{7}{1.67} \\     0.61 \\     0.39 \\     1.30 \\     1.64   $	8 1.82 0.63 0.42 1.46 1.80	$9 \\ 1.96 \\ 0.64 \\ 0.44 \\ 1.59 \\ 1.94$	$     \begin{array}{r}       10 \\       2.08 \\       0.65 \\       0.46 \\       1.72 \\       2.04     \end{array} $	
Maturity Mean Std deviation Min First quartile Median Third quartile	$ \begin{array}{r} 1\\ 0.05\\ 0.27\\ -0.52\\ -0.16\\ 0.07\\ 0.24 \end{array} $	Sam 2 0.42 0.36 -0.39 0.19 0.42 0.67	ple Perio 3 0.76 0.45 -0.27 0.47 0.78 1.07	$\begin{array}{c} \text{od: Jan} \\ \hline 4 \\ \hline 1.05 \\ 0.51 \\ -0.11 \\ 0.70 \\ 1.06 \\ 1.42 \end{array}$	$     \begin{array}{r}       1973 \text{ to} \\       5 \\       1.29 \\       0.56 \\       0.06 \\       0.92 \\       1.29 \\       1.70 \\     \end{array} $	Dec 198 6 1.50 0.59 0.23 1.11 1.48 1.90	$     \frac{39}{7}     1.67     0.61     0.39     1.30     1.64     2.04     $	8 1.82 0.63 0.42 1.46 1.80 2.18	$9 \\ 1.96 \\ 0.64 \\ 0.44 \\ 1.59 \\ 1.94 \\ 2.31$	$     \begin{array}{r}       10 \\       2.08 \\       0.65 \\       0.46 \\       1.72 \\       2.04 \\       2.43 \\     \end{array} $	
Maturity Mean Std deviation Min First quartile Median Third quartile Max	$\begin{array}{c} 1 \\ 0.05 \\ 0.27 \\ -0.52 \\ -0.16 \\ 0.07 \\ 0.24 \\ 0.57 \end{array}$	Sam 2 0.42 0.36 -0.39 0.19 0.42 0.67 1.25	ple Perio 3 0.76 0.45 -0.27 0.47 0.78 1.07 1.88	$\begin{array}{r} \text{ad: Jan} \\ \hline 4 \\ \hline 1.05 \\ 0.51 \\ -0.11 \\ 0.70 \\ 1.06 \\ 1.42 \\ 2.37 \end{array}$	$   \begin{array}{r}     1973 \text{ to} \\     5 \\     1.29 \\     0.56 \\     0.06 \\     0.92 \\     1.29 \\     1.70 \\     2.75 \\   \end{array} $	Dec 198 6 1.50 0.59 0.23 1.11 1.48 1.90 3.07	$     \frac{7}{1.67} \\     0.61 \\     0.39 \\     1.30 \\     1.64 \\     2.04 \\     3.32     $	$     \begin{array}{r}       8 \\       1.82 \\       0.63 \\       0.42 \\       1.46 \\       1.80 \\       2.18 \\       3.54 \\     \end{array} $	$\begin{array}{r} 9\\ 1.96\\ 0.64\\ 0.44\\ 1.59\\ 1.94\\ 2.31\\ 3.72 \end{array}$	$     \begin{array}{r}       10 \\       2.08 \\       0.65 \\       0.46 \\       1.72 \\       2.04 \\       2.43 \\       3.88 \\     \end{array} $	

 ${\bf Table \ 1} \ {\rm .} \ {\rm \ \ ~ \ .} \ \ \ \ \ \ \ \ .} \ \ \ \ .} \ {\rm .} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$ 

Sample Period: Jan 1990 to Sept 2004												
Maturity	1	2	3	4	5	6	7	8	9	10		
Mean	0.00	0.26	0.52	0.75	0.94	1.11	1.25	1.38	1.50	1.60		
Std deviation	0.26	0.39	0.48	0.55	0.60	0.63	0.66	0.68	0.69	0.71		
Min	-0.80	-0.83	-0.75	-0.64	-0.50	-0.36	-0.21	-0.07	0.05	0.15		
First quartile	-0.17	0.01	0.23	0.45	0.64	0.76	0.89	0.99	1.10	1.19		
Median	0.03	0.31	0.56	0.76	0.93	1.08	1.21	1.32	1.44	1.53		
Third quartile	0.19	0.49	0.79	1.05	1.24	1.42	1.57	1.68	1.81	1.92		
Max	0.43	0.98	1.49	1.90	2.23	2.49	2.71	2.90	3.06	3.20		

Moments and quartiles of the time series of estimated risk premiums (in percentage points).

Unrestricted model											
Maturity	1	2	3	4	5	6	7	8	9	10	
Mean	0.03	0.35	0.65	0.91	1.13	1.32	1.48	1.62	1.75	1.86	
Std deviation	0.27	0.39	0.48	0.55	0.60	0.64	0.67	0.69	0.70	0.72	
Min	-0.80	-0.83	-0.75	-0.64	-0.50	-0.36	-0.21	-0.07	0.05	0.15	
First quartile	-0.16	0.12	0.35	0.55	0.73	0.90	1.04	1.16	1.28	1.37	
Median	0.06	0.36	0.66	0.92	1.11	1.30	1.45	1.60	1.71	1.81	
Third quartile	0.22	0.61	0.94	1.25	1.52	1.73	1.88	2.02	2.16	2.27	
Max	0.57	1.25	1.88	2.37	2.75	3.07	3.32	3.54	3.72	3.88	
Ang and Piazzesi's (2003) restrictions - one-stage estimation											
Maturity	1	2	3	4	5	6	7	8	9	10	
Mean	0.01	0.34	0.64	0.90	1.12	1.31	1.47	1.60	1.72	1.83	
Std deviation	0.22	0.34	0.44	0.51	0.57	0.60	0.63	0.65	0.67	0.68	
Min	-0.67	-0.54	-0.44	-0.31	-0.17	-0.02	0.13	0.27	0.41	0.52	
First quartile	-0.15	0.09	0.32	0.53	0.71	0.87	1.01	1.12	1.23	1.31	
Median	0.00	0.31	0.60	0.84	1.05	1.24	1.41	1.54	1.68	1.78	
Third quartile	0.14	0.58	0.95	1.27	1.51	1.71	1.85	2.02	2.15	2.25	
Max	0.53	1.00	1.62	2.16	2.54	2.78	3.05	3.28	3.48	3.66	
	Ang an	d Piazze	esi's $(20)$	03) resti	rictions ·	- two-sta	age estii	nation			
Maturity	1	2	3	4	5	6	7	8	9	10	
Mean	-0.20	0.03	0.20	0.32	0.41	0.48	0.52	0.55	0.57	0.59	
Std deviation	0.71	0.70	0.73	0.76	0.80	0.82	0.84	0.86	0.88	0.89	
Min	-2.28	-2.08	-1.91	-1.75	-1.68	-1.66	-1.65	-1.63	-1.61	-1.59	
First quartile	-0.64	-0.33	-0.23	-0.19	-0.15	-0.10	-0.05	-0.05	-0.08	-0.06	
Median	-0.09	0.09	0.24	0.41	0.53	0.54	0.61	0.64	0.66	0.68	
Third quartile	0.31	0.53	0.72	0.84	1.01	1.09	1.16	1.20	1.22	1.23	
Max	1.16	1.42	1.80	2.11	2.35	2.53	2.67	2.78	2.87	2.93	

**Table 2** - The empirical distribution of risk premiums over timeComparison with restricted models (sample period: Jan 1973 to Sept 2004)

Moments and quartiles of the time series of estimated risk premiums (in percentage points).

	10 year forecasting horizon													
Maturity	1	2	3	4	5	6	7	8	9	10				
Inflation	23.2	13.2	7.9	5.3	3.8	3.0	2.5	2.2	2.1	2.1				
Output	22.4	28.9	28.4	26.3	23.9	21.7	19.6	17.6	15.7	14.1				
Other factors	54.4	58.0	63.7	68.5	72.3	75.4	78.0	80.2	82.2	83.9				
3 year forecasting horizon														
Maturity	1	2	3	4	5	6	7	8	9	10				
Inflation	24.4	14.1	8.4	5.4	3.6	2.6	1.9	1.4	1.2	1.0				
Output	21.3	27.9	27.6	25.4	23.1	20.8	18.7	16.7	14.9	13.2				
Other factors	54.3	58.0	64.1	69.2	73.3	76.6	79.5	81.9	84.0	85.8				
			1 year	forecas	ting ho	rizon								
Maturity	1	2	3	4	5	6	7	8	9	10				
Inflation	25.4	18.6	12.4	8.4	5.8	4.1	3.0	2.1	1.5	1.1				
Output	18.8	25.0	24.2	21.4	18.5	15.9	13.7	11.7	10.0	8.6				
Other factors	55.8	56.4	63.4	70.2	75.7	80.0	83.4	86.2	88.4	90.3				

Table 3 - Risk premiums - variance decomposition

Variance decomposition of the errors in forecasting risk premiums. Contribution (in percentage points) of the orthogonalized disturbances relative to each factor to the mean-squared forecast error, for different forecasting horizons (1, 3 and 10 years).

	10 year forecasting horizon												
Maturity	1	2	3	4	5	6	7	8	9	10			
Inflation	6.1	6.4	6.9	7.4	7.9	8.4	8.8	9.2	9.5	9.8			
Output	13.8	11.6	9.7	8.3	7.3	6.5	6.0	5.6	5.3	5.0			
Other factors	80.1	82.1	83.4	84.3	84.8	85.1	85.2	85.2	85.2	85.2			
3 year forecasting horizon													
Maturity	1	2	3	4	5	6	7	8	9	10			
Inflation	4.8	4.6	4.9	5.2	5.7	6.1	6.6	7.0	7.4	7.7			
Output	13.3	10.7	8.4	6.6	5.3	4.4	3.7	3.2	2.9	2.6			
Other factors	81.9	84.7	86.7	88.2	89.0	89.5	89.7	89.7	89.7	89.6			
			1 year	forecas	ting ho	rizon							
Maturity	1	2	3	4	5	6	7	8	9	10			
Inflation	3.9	3.1	2.9	2.9	3.0	3.2	3.4	3.7	3.9	4.0			
Output	9.6	8.4	7.0	5.9	4.9	4.2	3.7	3.3	3.0	2.7			
Other factors	86.5	88.6	90.1	91.3	92.1	92.6	92.9	93.1	93.2	93.3			

 Table 4 - Yields - variance decomposition

Variance decomposition of the errors in forecasting yields. Contribution (in percentage points) of the orthogonalized disturbances relative to each factor to the mean-squared forecast error, for different forecasting horizons (1, 3 and 10 years).

Maturity	1	2	3	4	5	6	7	8	9	10
Constant	0.19	0.51	0.79	1.03	1.22	1.38	1.52	1.64	1.74	1.82
	(0.02)	(0.02)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
Inflation	-0.05	-0.05	-0.05	-0.04	-0.03	-0.02	-0.01	0.00	0.01	0.01
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Output Gap	-0.28	-0.44	-0.52	-0.56	-0.58	-0.58	-0.58	-0.57	-0.55	-0.54
	(0.02)	(0.02)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)

 ${\bf Table} \ {\bf 5} \ {\rm - Regressions} \ {\rm of} \ {\rm risk} \ {\rm premiums} \ {\rm on} \ {\rm inflation} \ {\rm and} \ {\rm output} \ {\rm gap}$ 

	$a_0$				
	4.5923				
	(0.1748)				
	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
	1.4876	0.22231	1	1	1
	(0.0114)	(0.0035)	-	-	-
	· · · ·	~ /			
	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$
	0.10721	-0.04044	0.30042	-0.3377	-0.2132
	(0.0354)	(0.0902)	(0.1589)	(0.3524)	(0.2703)
	$\overline{\mu}_1$	$\overline{\mu}_2$	$\overline{\mu}_3$	$\overline{\mu}_4$	$\overline{\mu}_5$
	-0.38181	2.5069	0	0	0
	(0, 0007)	(0.9947)		_	-
	(0.0227)	(0.2247)	-	_	
	(0.0227)	(0.2247)	-	_	
	(0.0227)	(0.2247)	-	_	
	$(0.0227)$ $\rho_{i1}$	$(0.2247)$ $\rho_{i2}$	$\rho_{i3}$	$ ho_{i4}$	$ ho_{i5}$
$\rho_{1i}$	$\frac{\rho_{i1}}{1.0014}$	$(0.2247)$ $\rho_{i2}$ $-0.0141$	$\frac{\rho_{i3}}{0.0424}$	$\frac{\rho_{i4}}{0.0429}$	$\frac{\rho_{i5}}{0.0373}$
$\rho_{1j}$	$\frac{\rho_{i1}}{1.0014}$ (0.0614)	$\frac{\rho_{i2}}{-0.0141}$ (0.0539)	$-\frac{\rho_{i3}}{0.0424}$ (0.1393)	$\frac{\rho_{i4}}{0.0429}$ (0.1720)	$\frac{ ho_{i5}}{0.0373}$ (0.0478)
$\rho_{1j}$ $\rho_{2j}$	$(0.0227)$ $\frac{\rho_{i1}}{1.0014}$ $(0.0614)$ $0.0081$	(0.2247) $-0.0141$ $(0.0539)$ $0.9837$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662	$\rho_{i4}$ 0.0429 (0.1720) -0.0816	$\rho_{i5}$ 0.0373 (0.0478) -0.0016
$ ho_{1j} ho_{2j}$	$(0.0227)$ $\frac{\rho_{i1}}{1.0014}$ $(0.0614)$ $0.0081$ $(0.0122)$	(0.2247) $-0.0141$ $(0.0539)$ $0.9837$ $(0.0207)$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662 (0.0443)	$\rho_{i4}$ 0.0429 (0.1720) -0.0816 (0.0504)	$\rho_{i5}$ 0.0373 (0.0478) -0.0016 (0.0147)
$\begin{array}{c} \rho_{1j} \\ \rho_{2j} \\ \rho_{3j} \end{array}$	$(0.0227)$ $\frac{\rho_{i1}}{1.0014}$ $(0.0614)$ $0.0081$ $(0.0122)$ $-0.0413$	$\begin{array}{c} \rho_{i2} \\ \hline -0.0141 \\ (0.0539) \\ 0.9837 \\ (0.0207) \\ -0.3142 \end{array}$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662 (0.0443) 1.5011	$\rho_{i4}$ 0.0429 (0.1720) -0.0816 (0.0504) 0.6488	$\begin{array}{r} \rho_{i5} \\ \hline 0.0373 \\ (0.0478) \\ -0.0016 \\ (0.0147) \\ 0.0000 \end{array}$
$\begin{array}{c} \rho_{1j} \\ \rho_{2j} \\ \rho_{3j} \end{array}$	$\begin{array}{c} \rho_{i1} \\ \hline 1.0014 \\ (0.0614) \\ 0.0081 \\ (0.0122) \\ -0.0413 \\ (0.1866) \end{array}$	(0.2247) $-0.0141$ $(0.0539)$ $0.9837$ $(0.0207)$ $-0.3142$ $(0.2221)$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662 (0.0443) 1.5011 (0.0881)	$\frac{\rho_{i4}}{0.0429}$ (0.1720) -0.0816 (0.0504) 0.6488 (0.0721)	$\begin{array}{r} \rho_{i5} \\ 0.0373 \\ (0.0478) \\ -0.0016 \\ (0.0147) \\ 0.0000 \\ (0.0450) \end{array}$
$\begin{array}{c} \rho_{1j} \\ \rho_{2j} \\ \rho_{3j} \\ \rho_{4j} \end{array}$	$\begin{array}{c} \rho_{i1} \\ \hline 1.0014 \\ (0.0614) \\ 0.0081 \\ (0.0122) \\ -0.0413 \\ (0.1866) \\ 0.0579 \end{array}$	$\begin{array}{c} \rho_{i2} \\ \hline -0.0141 \\ (0.0539) \\ 0.9837 \\ (0.0207) \\ -0.3142 \\ (0.2221) \\ 0.3276 \end{array}$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662 (0.0443) 1.5011 (0.0881) -0.5879	$\begin{array}{c} \rho_{i4} \\ \hline 0.0429 \\ (0.1720) \\ -0.0816 \\ (0.0504) \\ 0.6488 \\ (0.0721) \\ 0.2527 \end{array}$	$\begin{array}{r} \rho_{i5} \\ 0.0373 \\ (0.0478) \\ -0.0016 \\ (0.0147) \\ 0.0000 \\ (0.0450) \\ 0.0000 \end{array}$
$\rho_{1j}$ $\rho_{2j}$ $\rho_{3j}$ $\rho_{4j}$	$\begin{array}{c} \rho_{i1} \\ \hline 1.0014 \\ (0.0614) \\ 0.0081 \\ (0.0122) \\ -0.0413 \\ (0.1866) \\ 0.0579 \\ (0.2230) \end{array}$	$(0.2247)$ $-\rho_{i2}$ $-0.0141$ $(0.0539)$ $0.9837$ $(0.0207)$ $-0.3142$ $(0.2221)$ $0.3276$ $(0.1698)$	$\rho_{i3}$ 0.0424 (0.1393) -0.0662 (0.0443) 1.5011 (0.0881) -0.5879 (0.2295)	$\begin{array}{c} \rho_{i4} \\ 0.0429 \\ (0.1720) \\ -0.0816 \\ (0.0504) \\ 0.6488 \\ (0.0721) \\ 0.2527 \\ (0.1171) \end{array}$	$\begin{array}{c} \rho_{i5} \\ 0.0373 \\ (0.0478) \\ -0.0016 \\ (0.0147) \\ 0.0000 \\ (0.0450) \\ 0.0000 \\ (0.0510) \end{array}$
$\begin{array}{c} \rho_{1j} \\ \rho_{2j} \\ \rho_{3j} \\ \rho_{4j} \\ \rho_{5j} \end{array}$	$\begin{array}{r} \rho_{i1} \\ \hline 1.0014 \\ (0.0614) \\ 0.0081 \\ (0.0122) \\ -0.0413 \\ (0.1866) \\ 0.0579 \\ (0.2230) \\ -0.0212 \end{array}$	$\begin{array}{c} \rho_{i2} \\ \hline -0.0141 \\ (0.0539) \\ 0.9837 \\ (0.0207) \\ -0.3142 \\ (0.2221) \\ 0.3276 \\ (0.1698) \\ 0.1052 \end{array}$	$\begin{array}{c} \rho_{i3} \\ \hline 0.0424 \\ (0.1393) \\ -0.0662 \\ (0.0443) \\ 1.5011 \\ (0.0881) \\ -0.5879 \\ (0.2295) \\ -0.1168 \end{array}$	$\begin{array}{c} \rho_{i4} \\ 0.0429 \\ (0.1720) \\ -0.0816 \\ (0.0504) \\ 0.6488 \\ (0.0721) \\ 0.2527 \\ (0.1171) \\ -0.1381 \end{array}$	$\begin{array}{c} \rho_{i5} \\ 0.0373 \\ (0.0478) \\ -0.0016 \\ (0.0147) \\ 0.0000 \\ (0.0450) \\ 0.0000 \\ (0.0510) \\ 0.9287 \end{array}$

The subscripts refer to: 1) Inflation 2) Output gap 3-5) Unobservable factors. Standard errors in parentheses.

	$\overline{ ho}_{i1}$	$\overline{ ho}_{i2}$	$\overline{ ho}_{i3}$	$\overline{ ho}_{i4}$	$\overline{ ho}_{i5}$
$\overline{\rho}_{1j}$	1.0261	0.0062	-0.5014	-0.7066	0.9965
	(0.0063)	(0.0017)	(0.0086)	(0.0155)	(0.0167)
$\overline{ ho}_{2j}$	-0.2956	0.9419	3.2777	4.5729	-5.4499
-	(0.0147)	(0.0053)	(0.0253)	(0.0336)	(0.0200)
$\overline{\rho}_{3i}$	0	0	0.9928	0	0
0	-	-	(0.0020)	-	-
$\overline{\rho}_{4i}$	0	0	0	0.9905	0
0	-	-	-	(0.0024)	-
$\overline{\rho}_{5i}$	0	0	0	0	0.7421
5	-	-	-	-	(0.0175)

	$\Sigma_{i1}$	$\Sigma_{i2}$	$\Sigma_{i3}$	$\Sigma_{i4}$	$\Sigma_{i5}$
$\Sigma_{1j}$	0.3030	0	0	0	0
	(0.0230)	-	-	-	-
$\Sigma_{2j}$	0.0076	0.1270	0	0	0
	(0.0073)	(0.0100)	-	-	-
$\Sigma_{3j}$	-0.2212	0.0812	1.0621	0	0
	(0.0120)	(0.0234)	0.0108	-	-
$\Sigma_{4j}$	0.2041	-0.0756	-1.0442	0.1742	0
	(0.0246)	(0.0144)	(0.0159)	(0.0195)	-
$\Sigma_{5j}$	-0.3294	0.0397	-0.2054	0.0000	0.3317
Ŭ	(0.0195)	(0.0296)	(0.0253)	(0.0234)	(0.0314)

Standard deviations of pricing errors

		1	0	
$1\mathrm{m}$	1y	2y	4y	6y
0.7668	0.3381	0.0778	0.0101	0.0116
(0.0827)	(0.0340)	(0.0081)	(0.0010)	(0.0012)
7y	8y	9y		
0.0180	0.0180	0.0116		
(0.0019)	(0.0019)	(0.0012)		



 $\mathbf{Figure}~\mathbf{1} \text{ - Risk premiums (in percentage points per annum) on bonds of different maturities.}$ 



Figure 2 - The term structure of risk premiums. The quartiles of the empirical distribution of estimated risk premiums are plotted against bond maturities.



Figure  ${\bf 3}$  - The output gap and the risk premium on the 10-year bond.



Figure 4 - Inflation and the risk premium on the 10-year bond.



Figure 5 - Impulse-response analysis. Response of risk premiums to a one standard deviation positive shock to output.



Figure  ${\bf 6}$  - Impulse-response analysis. Response of risk premiums to a one standard deviation positive shock to inflation.



Figure 7 - Risk premium on the 10-year bond. Comparison with restricted models.

- N. 555 Do capital gains affect consumption? Estimates of wealth effects from italian households' behavior, by L. Guiso, M. PAIELLA and I. VISCO (June 2005).
- N. 556 *Consumer price setting in Italy*, by S. FABIANI, A. GATTULLI, R. SABBATINI and G. VERONESE (June 2005).
- N. 557 *Distance, bank heterogeneity and entry in local banking markets*, by R. FELICI and M. PAGNINI (June 2005).
- N. 558 International specialization models in Latin America: the case of Argentina, by P. CASELLI and A. ZAGHINI (June 2005).
- N. 559 Caratteristiche e mutamenti della specializzazione delle esportazioni italiane, by P. MONTI (June 2005).
- N. 560 Regulation, formal and informal enforcement and the development of the household loan market. Lessons from Italy, by L. CASOLARO, L. GAMBACORTA and L. GUISO (September 2005).
- N. 561 Testing the "Home market effect" in a multi-country world: a theory-based approach, by K. Behrens, A. R. LAMORGESE, G. I. P. OTTAVIANO and T. TABUCHI (September 2005).
- N. 562 Banks' participation in the eurosystem auctions and money market integration, by G. BRUNO, M. ORDINE and A. SCALIA (September 2005).
- N. 563 *Le strategie di prezzo delle imprese esportatrici italiane*, by M. BUGAMELLI and R. TEDESCHI (November 2005).
- N. 564 Technology transfer and economic growth in developing countries: an economic analysis, by V. CRISPOLTI and D. MARCONI (November 2005).
- N. 565 La ricchezza finanziaria nei conti finanziari e nell'indagine sui bilanci delle famiglie italiane, by R. BONCI, G. MARCHESE and A. NERI (November 2005).
- N. 566 Are there asymmetries in the response of bank interest rates to monetary shocks?, by L. GAMBACORTA and S. IANNOTTI (November 2005).
- N. 567 Un'analisi quantitativa dei meccanismi di riequilibrio del disavanzo esterno degli Stati Uniti, by F. PATERNÒ (November 2005).
- N. 568 *Evolution of trade patterns in the new EU member States*, by A. ZAGHINI (November 2005).
- N. 569 *The private and social return to schooling in Italy*, by A. CICCONE, F. CINGANO and P. CIPOLLONE (January 2006).
- N. 570 *Is there an urban wage premium in Italy?*, by S. DI Addario and E. PATACCHINI (January 2006).
- N. 571 *Production or consumption? Disentangling the skill-agglomeration connection,* by GUIDO DE BLASIO (January 2006).
- N. 572 Incentives in universal banks, by Ugo Albertazzi (January 2006).
- N. 573 Le rimesse dei lavoratori emigrati e le crisi di conto corrente, by M. BUGAMELLI and F. PATERNÒ (January 2006).
- N. 574 Debt maturity of Italian firms, by SILVIA MAGRI (January 2006).
- N. 575 *Convergence of prices and rates of inflation*, by F. BUSETTI, S. FABIANI and A. HARVEY (February 2006).
- N. 576 Stock market fluctuations and money demand in Italy, 1913-2003, by MASSIMO CARUSO (February 2006).
- N. 577 Skill dispersion and firm productivity: an analysis with employer-employee matched data, by S. IRANZO, F. SCHIVARDI and E. TOSETTI (February 2006).
- N. 578 Produttività e concorrenza estera, by M. BUGAMELLI and A. ROSOLIA (February 2006).
- N. 579 Is foreign exchange intervention effective? Some micro-analytical evidence from the Czech Republic, by ANTONIO SCALIA (February 2006).

<sup>(\*)</sup> Requests for copies should be sent to:

Banca d'Italia – Servizio Studi – Divisione Biblioteca e pubblicazioni – Via Nazionale, 91 – 00184 Rome (fax 0039 06 47922059). They are available on the Internet www.bancaditalia.it.

- L. GUISO and G. PARIGI, *Investment and demand uncertainty*, Quarterly Journal of Economics, Vol. 114 (1), pp. 185-228, **TD No. 289** (November 1996).
- A. F. POZZOLO, *Gli effetti della liberalizzazione valutaria sulle transazioni finanziarie dell'Italia con l'estero*, Rivista di Politica Economica, Vol. 89 (3), pp. 45-76, **TD No. 296 (February 1997)**.
- A. CUKIERMAN and F. LIPPI, Central bank independence, centralization of wage bargaining, inflation and unemployment: theory and evidence, European Economic Review, Vol. 43 (7), pp. 1395-1434, TD No. 332 (April 1998).
- P. CASELLI and R. RINALDI, *La politica fiscale nei paesi dell'Unione europea negli anni novanta*, Studi e note di economia, (1), pp. 71-109, **TD No. 334 (July 1998)**.
- A. BRANDOLINI, The distribution of personal income in post-war Italy: Source description, data quality, and the time pattern of income inequality, Giornale degli economisti e Annali di economia, Vol. 58 (2), pp. 183-239, TD No. 350 (April 1999).
- L. GUISO, A. K. KASHYAP, F. PANETTA and D. TERLIZZESE, Will a common European monetary policy have asymmetric effects?, Economic Perspectives, Federal Reserve Bank of Chicago, Vol. 23 (4), pp. 56-75, TD No. 384 (October 2000).

- P. ANGELINI, Are banks risk-averse? Timing of the operations in the interbank market, Journal of Money, Credit and Banking, Vol. 32 (1), pp. 54-73, **TD No. 266 (April 1996).**
- F. DRUDI and R. GIORDANO, *Default Risk and optimal debt management,* Journal of Banking and Finance, Vol. 24 (6), pp. 861-892, **TD No. 278 (September 1996)**.
- F. DRUDI and R. GIORDANO, *Wage indexation, employment and inflation,* Scandinavian Journal of Economics, Vol. 102 (4), pp. 645-668, **TD No. 292 (December 1996)**.
- F. DRUDI and A. PRATI, Signaling fiscal regime sustainability, European Economic Review, Vol. 44 (10), pp. 1897-1930, TD No. 335 (September 1998).
- F. FORNARI and R. VIOLI, *The probability density function of interest rates implied in the price of options*, in: R. Violi, (ed.), Mercati dei derivati, controllo monetario e stabilità finanziaria, Il Mulino, Bologna, **TD No. 339 (October 1998)**.
- D. J. MARCHETTI and G. PARIGI, Energy consumption, survey data and the prediction of industrial production in Italy, Journal of Forecasting, Vol. 19 (5), pp. 419-440, TD No. 342 (December 1998).
- A. BAFFIGI, M. PAGNINI and F. QUINTILIANI, Localismo bancario e distretti industriali: assetto dei mercati del credito e finanziamento degli investimenti, in: L.F. Signorini (ed.), Lo sviluppo locale: un'indagine della Banca d'Italia sui distretti industriali, Donzelli, TD No. 347 (March 1999).
- A. SCALIA and V. VACCA, *Does market transparency matter? A case study*, in: Market Liquidity: Research Findings and Selected Policy Implications, Basel, Bank for International Settlements, **TD No. 359** (October 1999).
- F. SCHIVARDI, *Rigidità nel mercato del lavoro, disoccupazione e crescita*, Giornale degli economisti e Annali di economia, Vol. 59 (1), pp. 117-143, **TD No. 364 (December 1999)**.
- G. BODO, R. GOLINELLI and G. PARIGI, *Forecasting industrial production in the euro area*, Empirical Economics, Vol. 25 (4), pp. 541-561, **TD No. 370** (March 2000).
- F. ALTISSIMO, D. J. MARCHETTI and G. P. ONETO, *The Italian business cycle: Coincident and leading indicators and some stylized facts*, Giornale degli economisti e Annali di economia, Vol. 60 (2), pp. 147-220, **TD No. 377 (October 2000)**.
- C. MICHELACCI and P. ZAFFARONI, (*Fractional*) *Beta convergence*, Journal of Monetary Economics, Vol. 45, pp. 129-153, **TD No. 383 (October 2000)**.
- R. DE BONIS and A. FERRANDO, *The Italian banking structure in the nineties: testing the multimarket contact hypothesis*, Economic Notes, Vol. 29 (2), pp. 215-241, **TD No. 387 (October 2000)**.

- M. CARUSO, Stock prices and money velocity: A multi-country analysis, Empirical Economics, Vol. 26 (4), pp. 651-72, TD No. 264 (February 1996).
- P. CIPOLLONE and D. J. MARCHETTI, *Bottlenecks and limits to growth: A multisectoral analysis of Italian industry*, Journal of Policy Modeling, Vol. 23 (6), pp. 601-620, **TD No. 314 (August 1997)**.
- P. CASELLI, *Fiscal consolidations under fixed exchange rates*, European Economic Review, Vol. 45 (3), pp. 425-450, **TD No. 336 (October 1998)**.
- F. ALTISSIMO and G. L. VIOLANTE, Nonlinear VAR: Some theory and an application to US GNP and unemployment, Journal of Applied Econometrics, Vol. 16 (4), pp. 461-486, TD No. 338 (October 1998).
- F. NUCCI and A. F. POZZOLO, *Investment and the exchange rate*, European Economic Review, Vol. 45 (2), pp. 259-283, **TD No. 344 (December 1998)**.
- L. GAMBACORTA, On the institutional design of the European monetary union: Conservatism, stability pact and economic shocks, Economic Notes, Vol. 30 (1), pp. 109-143, **TD No. 356 (June 1999)**.
- P. FINALDI RUSSO and P. ROSSI, Credit costraints in italian industrial districts, Applied Economics, Vol. 33 (11), pp. 1469-1477, TD No. 360 (December 1999).
- A. CUKIERMAN and F. LIPPI, *Labor markets and monetary union: A strategic analysis,* Economic Journal, Vol. 111 (473), pp. 541-565, **TD No. 365 (February 2000)**.
- G. PARIGI and S. SIVIERO, An investment-function-based measure of capacity utilisation, potential output and utilised capacity in the Bank of Italy's quarterly model, Economic Modelling, Vol. 18 (4), pp. 525-550, TD No. 367 (February 2000).
- F. BALASSONE and D. MONACELLI, *Emu fiscal rules: Is there a gap?*, in: M. Bordignon and D. Da Empoli (eds.), Politica fiscale, flessibilità dei mercati e crescita, Milano, Franco Angeli, **TD No. 375** (July 2000).
- A. B. ATKINSON and A. BRANDOLINI, Promise and pitfalls in the use of "secondary" data-sets: Income inequality in OECD countries, Journal of Economic Literature, Vol. 39 (3), pp. 771-799, TD No. 379 (October 2000).
- D. FOCARELLI and A. F. POZZOLO, *The determinants of cross-border bank shareholdings: An analysis with bank-level data from OECD countries*, Journal of Banking and Finance, Vol. 25 (12), pp. 2305-2337, **TD No. 381 (October 2000)**.
- M. SBRACIA and A. ZAGHINI, *Expectations and information in second generation currency crises models*, Economic Modelling, Vol. 18 (2), pp. 203-222, **TD No. 391 (December 2000)**.
- F. FORNARI and A. MELE, Recovering the probability density function of asset prices using GARCH as diffusion approximations, Journal of Empirical Finance, Vol. 8 (1), pp. 83-110, TD No. 396 (February 2001).
- P. CIPOLLONE, La convergenza dei salari manifatturieri in Europa, Politica economica, Vol. 17 (1), pp. 97-125, TD No. 398 (February 2001).
- E. BONACCORSI DI PATTI and G. GOBBI, The changing structure of local credit markets: Are small businesses special?, Journal of Banking and Finance, Vol. 25 (12), pp. 2209-2237, TD No. 404 (June 2001).
- CORSETTI G., PERICOLI M., SBRACIA M., Some contagion, some interdependence: more pitfalls in tests of financial contagion, Journal of International Money and Finance, 24, 1177-1199, TD No. 408 (June 2001).
- G. MESSINA, Decentramento fiscale e perequazione regionale. Efficienza e redistribuzione nel nuovo sistema di finanziamento delle regioni a statuto ordinario, Studi economici, Vol. 56 (73), pp. 131-148, TD No. 416 (August 2001).

- R. CESARI and F. PANETTA, *Style, fees and performance of Italian equity funds*, Journal of Banking and Finance, Vol. 26 (1), **TD No. 325 (January 1998)**.
- L. GAMBACORTA, Asymmetric bank lending channels and ECB monetary policy, Economic Modelling, Vol. 20 (1), pp. 25-46, **TD No. 340 (October 1998)**.

- C. GIANNINI, "Enemy of none but a common friend of all"? An international perspective on the lender-oflast-resort function, Essay in International Finance, Vol. 214, Princeton, N. J., Princeton University Press, TD No. 341 (December 1998).
- A. ZAGHINI, Fiscal adjustments and economic performing: A comparative study, Applied Economics, Vol. 33 (5), pp. 613-624, TD No. 355 (June 1999).
- F. ALTISSIMO, S. SIVIERO and D. TERLIZZESE, *How deep are the deep parameters?*, Annales d'Economie et de Statistique,.(67/68), pp. 207-226, **TD No. 354 (June 1999)**.
- F. FORNARI, C. MONTICELLI, M. PERICOLI and M. TIVEGNA, *The impact of news on the exchange rate of the lira and long-term interest rates*, Economic Modelling, Vol. 19 (4), pp. 611-639, **TD No. 358** (October 1999).
- D. FOCARELLI, F. PANETTA and C. SALLEO, *Why do banks merge?*, Journal of Money, Credit and Banking, Vol. 34 (4), pp. 1047-1066, **TD No. 361 (December 1999)**.
- D. J. MARCHETTI, *Markup and the business cycle: Evidence from Italian manufacturing branches*, Open Economies Review, Vol. 13 (1), pp. 87-103, **TD No. 362 (December 1999)**.
- F. BUSETTI, Testing for stochastic trends in series with structural breaks, Journal of Forecasting, Vol. 21 (2), pp. 81-105, TD No. 385 (October 2000).
- F. LIPPI, *Revisiting the Case for a Populist Central Banker*, European Economic Review, Vol. 46 (3), pp. 601-612, **TD No. 386 (October 2000)**.
- F. PANETTA, The stability of the relation between the stock market and macroeconomic forces, Economic Notes, Vol. 31 (3), TD No. 393 (February 2001).
- G. GRANDE and L. VENTURA, Labor income and risky assets under market incompleteness: Evidence from Italian data, Journal of Banking and Finance, Vol. 26 (2-3), pp. 597-620, TD No. 399 (March 2001).
- A. BRANDOLINI, P. CIPOLLONE and P. SESTITO, *Earnings dispersion, low pay and household poverty in Italy, 1977-1998*, in D. Cohen, T. Piketty and G. Saint-Paul (eds.), The Economics of Rising Inequalities, pp. 225-264, Oxford, Oxford University Press, **TD No. 427** (November 2001).
- L. CANNARI and G. D'ALESSIO, La distribuzione del reddito e della ricchezza nelle regioni italiane, Rivista Economica del Mezzogiorno (Trimestrale della SVIMEZ), Vol. XVI (4), pp. 809-847, Il Mulino, TD No. 482 (June 2003).

- F. SCHIVARDI, *Reallocation and learning over the business cycle*, European Economic Review, , Vol. 47 (1), pp. 95-111, **TD No. 345 (December 1998)**.
- P. CASELLI, P. PAGANO and F. SCHIVARDI, Uncertainty and slowdown of capital accumulation in Europe, Applied Economics, Vol. 35 (1), pp. 79-89, **TD No. 372 (March 2000).**
- P. ANGELINI and N. CETORELLI, *The effect of regulatory reform on competition in the banking industry*, Federal Reserve Bank of Chicago, Journal of Money, Credit and Banking, Vol. 35, pp. 663-684, **TD No. 380 (October 2000)**.
- P. PAGANO and G. FERRAGUTO, Endogenous growth with intertemporally dependent preferences, Contribution to Macroeconomics, Vol. 3 (1), pp. 1-38, **TD No. 382 (October 2000).**
- P. PAGANO and F. SCHIVARDI, *Firm size distribution and growth,* Scandinavian Journal of Economics, Vol. 105 (2), pp. 255-274, **TD No. 394 (February 2001)**.
- M. PERICOLI and M. SBRACIA, A Primer on Financial Contagion, Journal of Economic Surveys, Vol. 17 (4), pp. 571-608, TD No. 407 (June 2001).
- M. SBRACIA and A. ZAGHINI, *The role of the banking system in the international transmission of shocks*, World Economy, Vol. 26 (5), pp. 727-754, **TD No. 409 (June 2001)**.
- E. GAIOTTI and A. GENERALE, Does monetary policy have asymmetric effects? A look at the investment decisions of Italian firms, Giornale degli Economisti e Annali di Economia, Vol. 61 (1), pp. 29-59, TD No. 429 (December 2001).
- L. GAMBACORTA, *The Italian banking system and monetary policy transmission: evidence from bank level data*, in: I. Angeloni, A. Kashyap and B. Mojon (eds.), Monetary Policy Transmission in the Euro Area, Cambridge, Cambridge University Press, **TD No. 430 (December 2001)**.

- M. EHRMANN, L. GAMBACORTA, J. MARTÍNEZ PAGÉS, P. SEVESTRE and A. WORMS, *Financial systems and the role of banks in monetary policy transmission in the euro area*, in: I. Angeloni, A. Kashyap and B. Mojon (eds.), Monetary Policy Transmission in the Euro Area, Cambridge, Cambridge University Press, **TD No. 432 (December 2001)**.
- F. SPADAFORA, Financial crises, moral hazard and the speciality of the international market: further evidence from the pricing of syndicated bank loans to emerging markets, Emerging Markets Review, Vol. 4 (2), pp. 167-198, TD No. 438 (March 2002).
- D. FOCARELLI and F. PANETTA, Are mergers beneficial to consumers? Evidence from the market for bank deposits, American Economic Review, Vol. 93 (4), pp. 1152-1172, **TD No. 448 (July 2002)**.
- E.VIVIANO, Un'analisi critica delle definizioni di disoccupazione e partecipazione in Italia, Politica Economica, Vol. 19 (1), pp. 161-190, **TD No. 450 (July 2002)**.
- M. PAGNINI, *Misura e Determinanti dell'Agglomerazione Spaziale nei Comparti Industriali in Italia*, Rivista di Politica Economica, Vol. 3 (4), pp. 149-196, **TD No. 452** (October 2002).
- F. BUSETTI and A. M. ROBERT TAYLOR, *Testing against stochastic trend and seasonality in the presence of unattended breaks and unit roots*, Journal of Econometrics, Vol. 117 (1), pp. 21-53, **TD No. 470** (February 2003).

- F. LIPPI, *Strategic monetary policy with non-atomistic wage-setters*, Review of Economic Studies, Vol. 70 (4), pp. 909-919, **TD No. 374 (June 2000)**.
- P. CHIADES and L. GAMBACORTA, *The Bernanke and Blinder model in an open economy: The Italian case*, German Economic Review, Vol. 5 (1), pp. 1-34, **TD No. 388 (December 2000)**.
- M. BUGAMELLI and P. PAGANO, *Barriers to Investment in ICT*, Applied Economics, Vol. 36 (20), pp. 2275-2286, **TD No. 420 (October 2001)**.
- A. BAFFIGI, R. GOLINELLI and G. PARIGI, *Bridge models to forecast the euro area GDP*, International Journal of Forecasting, Vol. 20 (3), pp. 447-460, **TD No. 456 (December 2002)**.
- D. AMEL, C. BARNES, F. PANETTA and C. SALLEO, Consolidation and Efficiency in the Financial Sector: A Review of the International Evidence, Journal of Banking and Finance, Vol. 28 (10), pp. 2493-2519, TD No. 464 (December 2002).
- M. PAIELLA, *Heterogeneity in financial market participation: appraising its implications for the C-CAPM*, Review of Finance, Vol. 8, pp. 1-36, **TD No. 473 (June 2003)**.
- E. BARUCCI, C. IMPENNA and R. RENÒ, *Monetary integration, markets and regulation*, Research in Banking and Finance, (4), pp. 319-360, **TD No. 475 (June 2003)**.
- E. BONACCORSI DI PATTI and G. DELL'ARICCIA, *Bank competition and firm creation*, Journal of Money Credit and Banking, Vol. 36 (2), pp. 225-251, **TD No. 481 (June 2003)**.
- R. GOLINELLI and G. PARIGI, Consumer sentiment and economic activity: a cross country comparison, Journal of Business Cycle Measurement and Analysis, Vol. 1 (2), pp. 147-172, TD No. 484 (September 2003).
- L. GAMBACORTA and P. E. MISTRULLI, *Does bank capital affect lending behavior?*, Journal of Financial Intermediation, Vol. 13 (4), pp. 436-457, **TD No. 486 (September 2003)**.
- F. SPADAFORA, *Il pilastro privato del sistema previdenziale: il caso del Regno Unito*, Rivista Economia Pubblica, (5), pp. 75-114, **TD No. 503** (June 2004).
- G. GOBBI and F. LOTTI, Entry decisions and adverse selection: an empirical analysis of local credit markets, Journal of Financial services Research, Vol. 26 (3), pp. 225-244, TD No. 535 (December 2004).
- F. CINGANO and F. SCHIVARDI, *Identifying the sources of local productivity growth*, Journal of the European Economic Association, Vol. 2 (4), pp. 720-742, **TD No. 474 (June 2003)**.
- C. BENTIVOGLI and F. QUINTILIANI, *Tecnologia e dinamica dei vantaggi comparati: un confronto fra quattro regioni italiane*, in C. Conigliani (a cura di), *Tra sviluppo e stagnazione: l'economia dell'Emilia-Romagna*, Bologna, Il Mulino, **TD No. 522 (October 2004)**.
- E. GAIOTTI and F. LIPPI, Pricing behavior and the introduction of the euro:evidence from a panel of restaurants, Giornale degli Economisti e Annali di Economia, 2004, Vol. 63(3/4):491-526, TD No. 541 (February 2005).

- L. DEDOLA and F. LIPPI, *The monetary transmission mechanism: evidence from the industries of 5 OECD countries*, European Economic Review, 2005, Vol. 49(6): 1543-69, **TD No. 389 (Decembre 2000)**.
- G. DE BLASIO and S. DI ADDARIO, *Do workers benefit from industrial agglomeration?* Journal of regional Science, Vol. 45 n.4, pp. 797-827, **TD No. 453 (October 2002).**
- M. OMICCIOLI, Il credito commerciale: problemi e teorie, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 494 (June 2004).
- L. CANNARI, S. CHIRI and M. OMICCIOLI, Condizioni del credito commerciale e differenzizione della clientela, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 495 (June 2004).
- P. FINALDI RUSSO and L. LEVA, Il debito commerciale in Italia: quanto contano le motivazioni finanziarie?, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 496 (June 2004).
- A. CARMIGNANI, Funzionamento della giustizia civile e struttura finanziaria delle imprese: il ruolo del credito commerciale, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 497 (June 2004).
- G. DE BLASIO, Credito commerciale e politica monetaria: una verifica basata sull'investimento in scorte, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 498 (June 2004).
- G. DE BLASIO, *Does trade credit substitute bank credit? Evidence from firm-level data.* Economic notes, Vol. 34 n.1, pp. 85-112, **TD No. 498 (June 2004).**
- A. DI CESARE, *Estimating Expectations of Shocks Using Option Prices*, The ICFAI Journal of Derivatives Markets, Vol. II (1), pp. 42-53, **TD No. 506 (July 2004).**
- M. BENVENUTI and M. GALLO, Perché le imprese ricorrono al factoring? Il caso dell'Italia, in L. Cannari, S. Chiri e M. Omiccioli (a cura di), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, Il Mulino, TD No. 518 (October 2004).
- P. DEL GIOVANE and R. SABBATINI, *L'euro e l'inflazione. Percezioni, fatti e analisi*, Bologna, Il Mulino, **TD No. 532 (December 2004).**