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Macroeconomics of international price discrimination

by Giancarlo Corsetti and Luca Dedola

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MACROECONOMICS OF INTERNATIONAL PRICE DISCRIMINATION

by Giancarlo Corsetti* and Luca Dedola**

Abstract

This paper builds a baseline two-country model of real and monetary transmission in the presence of optimal international price discrimination by firms. Distributing traded goods to consumers requires nontradables, intensive in local labor. Because of distributive trade the price elasticity of demand depends on country-specific shocks to productivity and the exchange rate. Hence, within limits dictated by the possibility of arbitrage, profit-maximizing monopolistic firms drive a wedge between prices across countries at both wholesale and retail level. Optimal pricing thus results in possibly large deviations from the law of one price and incomplete pass-through on import prices. Consistent with the received wisdom on international transmission, nominal and real depreciations worsens the terms of trade. In general, the nominal and real exchange rate are more volatile than fundamentals, and large movements in the international prices translate into small changes in consumption, employment and the price level. Finally, we provide an example showing that international policy cooperation may be redundant even when asset trading is ruled out, despite incomplete pass-through and less than optimal risk sharing.


Keywords: exchange rate pass-through, wholesale and retail prices, nominal rigidities, international cooperation.

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1. Introduction

One of the most striking stylized facts of the international economy is the magnitude of cross-border price differentials. First, even in regions with the longest track record of free trade, the empirical evidence shows that the law of one price fails to hold for most types of goods and services. Although this law also fails to hold within national boundaries, the deviations are much more dramatic at the international level — which has led some researchers to posit a specific ‘border effect’ (i.e. the effect of switching currencies across jurisdictions) on the prices of tradables. Second, prices seem to respond only mildly, if at all, to changes in the nominal exchange rate. To the extent that incomplete exchange rate pass-through is due to destination-specific markup adjustment by firms with market power, this is evidence of market segmentation. As firms ‘price-to-market’ (henceforth PTM), buyers across national markets face systematically different prices for otherwise identical goods.

While a deeper understanding of these facts is an exciting challenge to microeconomic research, the evidence of large international price discrepancies raises important macroeconomic issues. Namely, cross-border relative price adjustments are at the core of the conventional wisdom on international transmission, exemplified by the enduring contributions

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2 See Rogoff [1996] for an excellent survey on the evidence on the failure of the law of one price. In his analysis of US exchange rate movements, using both consumers and producers price indices, Engel [1999] finds that a great deal of the amount of deviations from purchasing power parity are due to a failure of the law of one price for internationally traded goods.

3 Engel and Rogers [1996] find that nominal currency movements appear to be important determinants of international price discrepancies.

4 Exchange rate pass-through, quite low for consumer prices, is far from complete also for international prices. According to the evidence surveyed by Goldberg and Knetter [1997], 1/2 is the median fraction by which exporters to the US offset a dollar appreciation by lowering their export prices.

of Friedman [1953] and Mundell [1960]. If firms optimally insulate local prices from exchange rate movements, the exchange rate may have a lesser impact on the relative prices of domestic and foreign goods, in relation to what is implied by the received view. For instance, Krugman [1989] notes that, in the presence of deviations from the law of one price and incomplete pass-through, large swings in the exchange rate may bring about only negligible changes in the equilibrium allocation. But what are the overall implications of endogenous PTM for the international transmission? Does international market segmentation justify pessimism about the allocative role of exchange rate movements? Does it create policy spillovers that call for international cooperation in policy design?

To address these issues, we build a general-equilibrium two-country model with nominal wage rigidities and monopolistic competition in production, in which upstream firms with monopoly power optimally charge different prices to competitive retailers situated in different locations. What makes the elasticity of demand differ across markets is the need for local-input-intensive distribution services. We show that this way of modelling vertical relationships among firms located in different markets is effective in bringing the model more closely into line with key stylized facts of the international economy and leads to a number of novel results.

First, deviations from the law of one price at both wholesale and retail levels in our model derive endogenously from optimal pricing by monopolistic firms. Different from most contributions in the literature, we solve the firms’ problem under the constraint that prices should not provide opportunities for arbitrage across market locations. Secondly, since the price elasticity of export demand is increasing in the wholesale price and non-linear in the exchange rate, optimal international price discrimination leads to incomplete exchange rate pass-through. Third, despite low pass-through, a nominal depreciation can worsen the terms of trade — consistent with the received wisdom about international transmission and the possibility of expenditure-switching effects. Furthermore, nominal and real exchange rates are positively correlated in equilibrium. Fourth, the exchange rate tends to be more volatile than

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6 To enhance comparison with the literature our open economy model with endogenous price discrimination builds on the analytical framework of Corsetti and Pesenti [2001a,b] and Obstfeld and Rogoff [1995, 2000]. The specification of consumption preferences in these models is such that terms-of-trade movements in response to worldwide shocks can be sufficient to generate optimal risk-sharing: introducing Arrow-Debreu securities would not change the equilibrium allocation. This is no longer the case when we allow for distributive trade: no equilibrium with trade in international bonds can lead to optimal risk sharing — not even when nominal rigidities and monopoly power distortions are removed.
fundamentals. However, because of low equilibrium pass-through, large movements in the nominal and real exchange rates translate into small changes in consumption, employment and price levels. Finally, we provide an example showing that, despite segmentation of both the assets and the goods markets, monetary rules targeting the domestic output gap are (constrained-) optimal, and there are no gains from international policy cooperation.

Distribution services are traditionally invoked as a key reason for the failure of Purchasing Power Parity (henceforth PPP). Dornbusch [1989], for instance, suggests that these services may account for his finding that the price of an identical consumption basket is higher in high-income economies than in low-income ones. Overall, distributive trade accounts for an important share of the retail price of consumption goods: for the US, including wholesale and retail services, marketing, advertisement and local transportation, the average distribution margin is as high as 50 percent (see Burstein, Neves and Rebelo [2000]).

Strong evidence of the importance of distribution services in accounting for international price discrimination is presented by Goldberg and Verboven [2001], based on comprehensive and detailed data for automobile prices in five European countries. According to their estimates, a 1 per cent change in the nominal exchange rate induces a 0.46 per cent adjustment in the export prices in exporter currency (i.e., equivalent to a 0.54 pass-through coefficient). Of this, between 0.37 and 0.39 per cent can be attributed to a change in local wages (i.e., nominal wages in the destination country).

In recent years, a number of contributions have included distributive trade in open macro models in order to account for the very low exchange rate pass-through at the consumer price level. Yet, this literature has not analyzed market segmentation resulting from the vertical interaction among monopolistic producers and retailers, nor the implications of such interaction for international transmission and policy design. This is precisely the goal of this paper.

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7 Recent literature has explored the role of barriers to trade and transportation costs, but without linking them explicitly to international price discrimination. See Obstfeld and Rogoff [2001] on the role of transportation costs in explaining major puzzles in international finance, and the evidence in Parsley and Wei [2000] on transportation costs and the border effect.

8 Goldberg and Verboven [2001] estimate that local costs account for up to 35 per cent of the price of a car, mainly due to distribution services provided by local dealers.

9 Erceg and Levin [1995], McCallum and Nelson [1999], and Burstein, Neves and Rebelo [2000] assume that distribution requires local inputs, focusing on the case of perfect competition in the goods market.
By modelling vertical relationships between firms located in different national markets, we take a different approach from the recent contributions that view market segmentation as a direct implication of price rigidities. In this literature, foreign exporters preset consumer prices in local currency. One problem with this class of models is that, although they do account for price differences across markets and the border effect, they imply that an exchange rate depreciation should improve a country’s terms of trade. Obstfeld and Rogoff [2000a] however find evidence at odds with such prediction: in the data, exchange rate depreciations tend to be associated with a worsening of the terms of trade, as in the conventional wisdom.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses optimal pricing by monopolistic firms facing country-specific demand elasticities. Section 4 analyzes the novel features of the equilibrium with endogenously segmented markets, highlighting the link between exchange rate determination and relative prices. Section 5 derives a set of predictions that are broadly consistent with the main stylized facts on international prices. Section 6 looks at the limiting case of no monopoly power in the goods market and characterizes the optimal monetary policy, showing that in equilibrium there are no international policy spillovers that would call for international policy cooperation. Section 7 concludes.

2. The model

The world economy consists of two countries of equal size, $H$ and $F$. Each country is specialized in one type of tradable good, produced in a number of varieties or brands defined over a continuum of unit mass. Brands of tradable goods are indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. In addition, each country produces an array of differentiated nontradable goods, indexed by $n \in [0, 1]$. Nontraded goods are either consumed or used to make intermediate tradable goods $h$ and $f$ available to domestic consumers.

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10 An incomplete list of papers assuming local currency prices includes Betts and Devereux [2000], Chari, Kehoe and McGrattan [2000], Devereux and Engel [2000], and Kollman [1997], among others (see Lane [2001] and Engel [2002] for a survey of the literature, and Corsetti and Pesenti [2001b] for a generalization of this approach). Early contributions simply assumed that foreign exporters quote prices in local currency. Recent works by Bacchetta and van Wincoop [2000], Corsetti and Pesenti [2002] and Devereux and Engel [2001] analyze the problem of producers who can choose whether to preset prices in domestic currency only or in both domestic and foreign currencies.

11 Another approach to modelling price discrimination (clearly complementary to ours) consists of introducing non-constant elasticity preferences — see the recent work by Bergin and Feenstra [2001] on the persistence of real exchange rates following monetary shocks.
Firms producing tradable and nontradable goods are monopolistic suppliers of one brand of goods only. These firms employ differentiated domestic labor inputs in a continuum of unit mass. Each worker occupies a point in this continuum, and acts as a monopolistic supplier of a differentiated type of labor input to all firms in the domestic economy. Households/workers are indexed by \( j \in [0,1] \) in the Home country and \( j^* \in [0,1] \) in the Foreign country. Firms operating in the distribution sector, by contrast, are assumed to operate under perfect competition. They buy tradable goods and distribute them to consumers using nontraded goods as the only input in production.

In our baseline model, we allow for nominal rigidities by assuming that workers and firms agree on the nominal wage rate one period in advance. In what follows, we describe our set up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy — whereas variables referred to Foreign firms and households are marked with an asterisk.

**Technology**

Let \( Y(h) \) denote total output of a differentiated tradable good \( h \), and \( L(h, j) \) the demand for labor input of type \( j \) by the producer of good \( h \). By the same token, \( Y(n) \) denotes total production of a differentiated nontradable good \( n \), and \( L(n, j) \) the corresponding demand for labor input \( j \). The production function of the Home traded and nontraded goods are, respectively:

\[
Y_t(h) = Z_{H,t} \left[ \int_0^1 L_t(h, j) \frac{\phi-1}{\phi} dj \right]^{\frac{\phi}{\phi-1}}, \quad Y_t(n) = Z_{N,t} \left[ \int_0^1 L_t(n, j) \frac{\phi-1}{\phi} dj \right]^{\frac{\phi}{\phi-1}}; \quad (1)
\]

where \( \phi \) is the elasticity of substitution among labor inputs, which is the same across sectors, and \( Z \) denotes stochastic productivity parameters, which are sector-specific. Similar expressions hold for firms in the Foreign country, whereas the elasticity of substitution is also \( \phi \), but the productivity shocks are not necessarily symmetric.

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12 Due to this assumption, we note from the start that the equilibrium allocation studied below would be identical in a vertically integrated economy, where exporters with monopoly power own local retailers.

13 Christiano, Eichenbaum and Evans [2001] and Smets and Wouters [2001] are recent structural models providing convincing evidence that price stickiness induced by wage stickiness is an important determinant of macroeconomic fluctuations. Here we abstract from issues in inflation dynamics that could be analyzed, for instance, by assuming Calvo-style adjustment of prices or wages. See Kollman [1997] and Chari et al. [2000] among others.
Our specification of the distribution sector is in the spirit of the matter-of-fact remark by Tirole ([1995], page 175) that “production and retailing are complements, and consumers often consume them in fixed proportions”. As in Erceg and Levin [1995] and Burstein, Neves and Rebelo [2000], we thus assume that bringing one unit of traded goods to consumers requires \( \eta \) units of a basket of differentiated nontraded goods

\[
\eta = \left[ \int_0^1 \eta(n)^{\frac{\theta-1}{\theta}} dn \right]^{\frac{1}{\theta-1}}.
\] (2)

We note here that the Dixit-Stiglitz index above also applies to the consumption of differentiated nontraded goods, specified in the next subsection. In equilibrium, then, the basket of nontraded goods required to distribute tradable goods to consumers will have the same composition as the basket of nontradable goods consumed by the representative domestic household.\(^{14}\)

Preferences

Home agent \( j \)'s lifetime expected utility \( U \) is defined as:

\[
U_t(j) \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U \left[ \ln C_\tau(j) + \frac{\chi_\tau}{1-\varepsilon} \left( \frac{M_\tau(j)}{P_\tau} \right)^{1-\varepsilon} - \kappa_\tau \ell_\tau(j) \right],
\] (3)

where \( \beta < 1 \) is the discount rate and the instantaneous utility is a function of a consumption index \( C(j) \), to be defined below, real balances \( M(j) / P \), and labor effort \( \ell(j) \). Instantaneous utility is state-dependent, as we potentially allow for velocity shocks in the form of a stochastically varying utility of real balances, and shocks to the disutility of labor.

Households consume all types of (domestically-produced) nontraded goods, and both types of traded goods. So \( C_t(n, j) \) is consumption of brand \( n \) of Home nontraded good by agent \( j \) at time \( t \); \( C_t(h, j) \) and \( C_t(f, j) \) are the same agent’s consumption of Home brand \( h \) and Foreign brand \( f \). For each type of good, we assume that one brand is an imperfect substitute for all other brands, with constant elasticity of substitution \( \theta > 1 \). Consumption of Home and

\(^{14}\) For simplicity, we do not distinguish between nontradable consumption goods, which directly enter the agents’ utility, and nontraded distribution services, which are jointly consumed with traded goods. This distinction may however be important in more empirically oriented studies (e.g., see MacDonald and Ricci [2001]). By the same token, we ignore distribution costs incurred in the non-traded good market, as these can be accounted for by varying the level of productivity in the nontradable sector.
Foreign goods by Home agent $j$ is defined as:

$$ C_{H,t}(j) \equiv \left[ \int_0^1 C_t(h, j)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t}(j) \equiv \left[ \int_0^1 C_t(f, j)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \quad C_{N,t}(j) \equiv \left[ \int_0^1 C_t(n, j)^{\frac{\theta-1}{\theta}} dn \right]^{\frac{\theta}{\theta-1}}. \tag{4} $$

The consumption aggregator of tradable goods and the full consumption basket of individuals $j$ are, respectively:

$$ C_{T,t}(j) \equiv 2C_{H,t}(j)^{1/2}C_{F,t}(j)^{1/2} \tag{5} $$

$$ C_t(j) \equiv \frac{C_{T,t}(j)^\gamma C_{N,t}(j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}. \tag{6} $$

As in Corsetti and Pesenti [2001a], the parameters describing consumption preferences are the same in the Home and Foreign country.$^{15}$

**Price indexes**

Let $p_t(h)$ and $p_t^*(h)$ denote the *retail* price of brand $h$ expressed in the Home and Foreign currency, respectively. The utility-based price indexes of Home-produced tradables are:

$$ P_{H,t} = \left[ \int_0^1 [p_t(h)]^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P_{H,t}^* = \left[ \int_0^1 [p_t^*(h)]^{1-\theta} dh \right]^{\frac{1}{1-\theta}}. \tag{7} $$

The price indexes $P_{N,t}, P_{N,t}^*, P_{F,t}$ and $P_{F,t}^*$, are analogously defined. The utility-based price indexes of tradable and the utility-based CPI are:

$$ P_t = P_{H,t}^{1/2} P_{F,t}^{1/2}, \quad P_{T,t} = (P_{H,t}^*)^{1/2} (P_{F,t}^*)^{1/2}, \quad P_t = P_{T,t}^{1-\gamma} P_{N,t}^{1-\gamma}. \tag{8} $$

**Household budget constraints and asset markets**

Home agents hold Home currency $M$, two international bonds, $B_H$ and $B_F$, respectively denominated in Home and Foreign currency, and a well-diversified portfolio of domestic equities. They earn labor income $W^\ell$ and pay non-distortionary (lump-sum) net taxes $T$, $^{15}$ Consistent with the assumption that each country specializes in the production of a single type of traded good, the elasticity of substitution between goods produced in different countries (set equal to one) is below the elasticity of substitution among goods produced in one country ($\theta > 1$).
denominated in Home currency. The individual flow budget constraint for agent $j$ in the Home country is:

$$
M_t(j) + B_{H,t+1}(j) + \mathcal{E}_t B_{F,t+1}(j) \leq M_{t-1}(j) + (1 + i_t) B_{H,t}(j) + (1 + i_t^*) \mathcal{E}_t B_{F,t}(j) + \int_0^1 \Pi(h,j) dh + \int_0^1 \Pi(n,j) dn + W_t(j) e_t(j) - T_t(j) - P_{H,t} C_{H,t}(j) - P_{F,t} C_{F,t}(j) - P_{N,t} C_{N,t}(j)
$$

(9)

where $\mathcal{E}_t$ is the nominal exchange rate, expressed as Home currency per unit of Foreign currency; $i_t$ and $i_t^*$ are the nominal yields in Home and Foreign currency, paid at the beginning of period $t$ but known at time $t-1$; and $\int \Pi(h,j) dh + \int \Pi(n,j) dn$ is the agent’s share of profits from all firms $h$ and $n$ in the economy.

The international bonds are assumed to be in zero net-supply, so that in the aggregate $B_{H,t} = -B_{H,t}^*$ and $B_{F,t} = -B_{F,t}^*$.

**Government budget constraint and policy instruments**

The government budget constraint in the Home country is:

$$
\int_0^1 [M_t(j) - M_{t-1}(j)] dj + \int_0^1 T_t(j) dj = 0.
$$

(10)

We abstract from government spending, and seigniorage revenue is rebated to households in a lump-sum fashion. To characterize monetary policy, it is convenient to define a variable $\mu_t$ such that

$$
\frac{1}{\mu_t} = \beta (1 + i_{t+1}) E_t \left( \frac{1}{\mu_{t+1}} \right).
$$

(11)

Given the time path of $\mu$, there is a corresponding sequence of Home nominal interest rates. Without loss of generality, we assume that the government affects the stock of Home monetary assets by controlling the short-term rate $i_{t+1}$: Home monetary easing at time $t$ leading to a lower $i_{t+1}$ is associated with a higher $\mu_t$.\(^{17}\)

3. **Pricing-to-market and incomplete pass-through**

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\(^{16}\) The notation conventions follow Obstfeld and Rogoff [1996, ch.10]. Specifically, $M_t(j)$ denotes agent $j$’s nominal balances accumulated during period $t$ and carried over into period $t+1$, while $B_{H,t}(j)$ and $B_{F,t}(j)$ denote agent $j$’s bonds accumulated during period $t-1$ and carried over into period $t$.

\(^{17}\) With logarithmic utility, in equilibrium $\mu_t$ is equal to nominal spending $P_t C_t$. 
3.1 Firms’ optimization and optimal price discrimination

International price discrimination is a key feature of the international economy captured by our model. In what follows we show that, even if Home and Foreign consumers have identical constant-elasticity preferences for consumption, the need for distribution services intensive in local nontraded goods implies that the elasticity of demand for the \( h \) (\( f \)) brand at wholesale level is not generally the same across markets. Firms will thus want to charge different prices at Home and in the Foreign country. We will focus our analysis on Home firms — optimal pricing by Foreign firms can be easily derived from it.

Consider first the optimal pricing problem faced by firms producing nontradables for the Home market. The demand for their product is

\[
    C(n) + \eta(n) = [p_t(n)]^{-\theta} P_{N,t}^{\theta} \left[ C_{N,t} + \eta \left( \int_0^1 C_t(h) dh + \int_0^1 C_t(f) df \right) \right].
\]  

(12)

It is easy to see that their optimal price will result from charging a constant markup over the unit labor costs:

\[
    p_t(n) = P_{N,t} = \frac{\theta}{\theta - 1} \frac{W_t}{Z_{N,t}}.
\]  

(13)

Note that nominal wage rigidities do not translate into price rigidities: the price of the nontraded good \( p_t(n) \) in fact moves inversely with productivity in the sector.

Now, let \( \bar{p}_t(h) \) denote the price of brand \( h \) expressed in the Home currency, at producer level. With a competitive distribution sector, the consumer price of good \( h \) is simply

\[
    p_t(h) = \bar{p}_t(h) + \eta P_{N,t}.
\]  

(14)

In the case of firms producing tradables, “pricing to market” derives endogenously from the solution to the problem of the Home representative firm in the sector:

\[
    \begin{align*}
    \max_{\bar{p}_t(h)} & \quad \left[ \bar{p}_t(h) C_t(h) + E_t \bar{p}_t^* (h) C_t^* (h) \right] - \frac{W_t}{Z_{H,t}} \left[ C_t(h) + C_t^* (h) \right]
    \end{align*}
\]  

(15)

where

\[
    C_t(h) = \left( \frac{P_{H,t}}{\bar{p}_t(h) + \eta P_{N,t}} \right)^{-\theta} C_{H,t}, \quad C_t^* (h) = \left( \frac{P_{H,t}^*}{\bar{p}_t^* (h) + \eta P_{N,t}^*} \right)^{-\theta} C_{H,t}^*.
\]  

(16)
Making use of (13), the optimal wholesale prices $\bar{p}(h)$ and $\bar{p}^*(h)$ are:

$$
\bar{p}_t(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\eta}{\theta - 1} Z_{H,t} \right) \frac{W_t}{Z_{H,t}},
$$

(17)

$$
\mathcal{E}_t \bar{p}^*(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\eta}{\theta - 1} \mathcal{E}_t W^*_t \frac{Z^*_H}{Z^*_N} \right) \frac{W_t}{Z_{H,t}}.
$$

(18)

Unlike the case of nontraded goods (13), in this case the markups charged by the Home firms include a state-contingent component (in brackets in the above expressions) that varies as a function of productivity shocks. In the export market, the markup also responds to monetary innovations (affecting the exchange rate) and relative wages.

In general, the optimal wholesale price of tradable goods will not obey the law of one price ($\bar{p}_t(h) \neq \mathcal{E}_t \bar{p}_t(h)$). With $\eta > 0$, the elasticity of the demand for the Home goods as perceived by the upstream monopolist will be different at Home and abroad, reflecting any asymmetry in relative productivity and/or relative wages. Home monopolistic firms take account of the implications of distributive trade on the demand elasticity for their product, and find it optimal to charge different prices to firms distributing in the Home and in the Foreign market.

In the Home market the price elasticity of the demand for the good $h$ depends on relative productivity across domestic sectors:

$$
-\frac{\partial C_t(h)}{\partial \bar{p}_t(h)} \frac{\bar{p}_t(h)}{C_t(h)} = \frac{\bar{p}_t(h)}{\bar{p}_t(h) + \eta P_{N,t}} = \frac{\theta}{1 + \frac{\eta}{\theta - 1} Z_{H,t}}.
$$

(19)

In the export market the price elasticity of the demand for the good $h$ depends on productivity shocks at Home and abroad, and the exchange rate:

$$
-\frac{\partial C^*_t(h)}{\partial \bar{p}^*(h)} \frac{\bar{p}^*(h)}{C^*_t(h)} = \frac{\bar{p}^*(h)}{\bar{p}^*(h) + \eta P^*_{N,t}} = \frac{\theta}{1 + \frac{\eta}{\theta - 1} \mathcal{E}_t W^*_t \frac{Z^*_H}{Z^*_N}}.
$$

(20)

Observe that the price elasticity of export demand is non linear in the exchange rate: a relatively appreciated Home currency (a low $\mathcal{E}_t$) corresponds to a relatively large price...
elasticity. Furthermore, it is increasing in the wholesale price — as shown by the literature on international trade, this is a sufficient condition for incomplete exchange-rate pass-through.\(^{18}\)

Note also that the above expressions are monotonic functions of the distribution margin, defined as the share of distributive trade in the consumer price of the good \(h\). This margin is \(\eta P_{N,t}^*/p_t^*(h)\) in the export market and \(\eta P_{N,t}/p_t(h)\) in the domestic market. In either market, the higher the distribution margin, the lower the price elasticity.

### 3.2 Optimal pricing in the presence of arbitrage between retail and wholesale markets

While charging (17) and (18) would maximize firms’ profits, arbitrage in the goods market may prevent optimal price discrimination between domestic and foreign dealers.\(^{19}\) We now analyze firms’ optimal pricing decisions allowing for the possibility of arbitrage across wholesale and retail markets.

Consider the consumer price of the good \(h\) in both markets, calculated adding the distribution costs (\(\eta P_N\) and \(\eta^* P_N^*\)) to the optimal producer prices above:

\[
p_t(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\theta}{\theta - 1} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{W,t}}, \tag{21}
\]

\[
\mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} \left( 1 + \frac{\theta}{\theta - 1} \frac{\mathcal{E}_t W_t^*}{W_t} \frac{Z_{H,t}}{Z_{N,t}} \right) \frac{W_t}{Z_{W,t}}. \tag{22}
\]

If the representative Home firm set the wholesale price in the Foreign country above the consumer price of its own good in the Home country, firms distributing good \(h\) in the Foreign country would find it profitable to buy it from Home retailers rather than in the wholesale market. This implies that optimal price discrimination is possible only as long as the following no-arbitrage conditions are verified:

\[
\mathcal{E}_t p_t^*(h) = \mathcal{E}_t (\tilde{p}_t^*(h) + \eta P_{N,t}^*) \geq \tilde{p}_t(h) \tag{23}
\]

\[
p_t(h) = \tilde{p}_t(h) + \eta P_{N,t} \geq \mathcal{E}_t \tilde{p}_t^*(h).
\]

\(^{18}\) As far as the elasticity of substitution between the tradable good and the nontradable bundle in the retailer distribution technology is less than 1, the producer’s price elasticity will be increasing in \(p^*(h)\).

\(^{19}\) We are assuming that markets can be segmented along national lines. In our model this could be easily justified with a system of selective and exclusive distribution, in which the manufacturer can choose dealers and restrain them from reselling to anyone but end-users. We observe here that regulation 123/85 of the European Commission has allowed these practices in the European Union to some extent (see Goldberg and Verboven [2001] for the implications of the Regulation in the European car market).
Using optimal prices, these conditions can be synthetically written as:

\[
\frac{1}{\theta} \leq \frac{E_t W_t^* Z_{N,t}}{W_t Z_{N,t}^*} \leq \theta.
\] (24)

According to this expression, for given relative nominal marginal costs in the nontraded goods sector, a large depreciation of the nominal exchange rate could reduce the Home consumer price of \(h\) in Foreign currency below the optimal export price \(p_t^* (h)\) — violating the second inequality above. In that case, arbitrage in the goods market would force the domestic price and the foreign wholesale price to be set equal: \(p_t (h) = E_t \tilde{p}_t^* (h)\). By the same token, a large appreciation of the exchange rate could reduce the foreign retail price of \(h\) in the Home currency below the wholesale price at Home. In this case, ruling out arbitrage requires firms to set \(\tilde{p}_t (h) = E_t p_t^* (h)\).

Leaving the characterization of prices when shocks are such that the no-arbitrage condition is binding to the Appendix, we provide an intuitive account of our main results. Suppose that to rule out arbitrage Home firms must set: \(p_t (h) = E_t \tilde{p}_t^* (h)\). Relative to the optimal prices (17) and (18), Home firms will now raise \(\tilde{p}_t (h)\) above (17) while lowering \(\tilde{p}_t^* (h)\) below (18). As the two prices cannot be set independently, the drop in the markup in the foreign market is partly offset by a higher markup at home. Even when the no-arbitrage condition is binding, wholesale prices will still be different in the Home and Foreign markets: when distribution services are required, the law of one price never holds.

It is easy to verify that, holding (23), it can never be profitable to buy good \(h\) at the Home (Foreign) retail price and sell it at the Foreign (Home) retail price after paying local distribution costs. We note here that, to the extent that domestic households need local distribution services even if they buy tradables abroad (i.e. technical assistance), our model endogenously rules out consumers’ arbitrage across retail markets.

In concluding this section, it is appropriate to discuss briefly our assumption of perfect competition in the distribution sector. Allowing for monopoly power in the retail sector would imply double marginalization at consumer prices. In our setup, however, monopolistic retailers face a constant-elasticity demand, so that their markup over marginal cost would be constant (the right hand side of (14) would be multiplied by the constant \(\frac{\theta}{\theta - 1}\)). Thus, double marginalization would influence the level of consumer prices (both (21) and (22) would be multiplied by \(\frac{\theta}{\theta - 1}\)), but not their wholesale counterpart (17) and (18). For this reason, it would not induce substantial differences in our results on the equilibrium response of prices
to nominal and real shocks. Assuming monopolistic wholesale suppliers and competitive distributors is a natural benchmark specification for our paper.

3.3 Exchange rate pass-through

In addition to explaining deviations from the law of one price, distribution costs also have key implications for the optimal degree of exchange rate pass-through. Assume at first that the no-arbitrage condition (24) is satisfied as a strict inequality. Other things equal, the change of producer prices in response to exchange rate movements is:

\[
\frac{\partial p_t^*(h)}{\partial \bar{e}_t} \frac{\partial \bar{e}_t(h)}{\partial \bar{E}_t} = 1 + \frac{\eta}{\theta - 1} \frac{Z_{h,t} \bar{E}_t W_t}{Z_{n,t}^* N_t} \frac{1}{<1}.
\]

(25)

Given wages and productivity, the degree of exchange rate pass-through on import prices is less than 100 per cent. Pass-through is even lower on consumer prices, as the denominator of the above expression is augmented by \( \eta P_N^* \) (which is equivalent to multiplying the second term in the denominator by \( \theta > 1 \)).

Note that the right-hand side of the above expression is just the inverse of the state-contingent component of the markup charged by Home firms in the export market. Clearly, anything that increases the Home-export markup will dampen the effect of the exchange rate on Home-export prices. So a relatively appreciated currency (a low \( E_t \)), low monopoly power (a high \( \theta \)), and a small need for distribution services (a low \( \eta \)) all imply a relatively small markup in the export market, and a correspondingly high degree of exchange rate pass-through.

Allowing for arbitrage between the retail and the wholesale markets cannot but reinforce our results. Above we have shown that, when the exchange rate depreciation is large enough, Home firms will set the foreign wholesale price equal to the domestic consumer price. To the extent that shocks cause Home firms to raise the Home price of their goods, the degree of pass-through will clearly be even lower than implicit in (18).

As will be clear from the rest of the paper, our model provides a framework to contrast partial and general equilibrium results. Using our analysis below, it can be easily verified that the expression (25) will hold in general equilibrium in the presence of shocks to Home and Foreign money, and/or to productivity in the Home nontradables and Foreign tradables sectors. In the general case, (i.e. including all shocks), exchange rate pass-through cannot be captured in such a simple way — although it will still be incomplete.
It will also be clear that nominal shocks have real effects only because of nominal rigidities. Without wage contracts, nominal shocks would not alter relative wages (the exchange rate and the wage rate would move in the same direction), and pass-through would be 100 per cent. But even if all prices and wages were fully flexible, price differentials would still reflect productivity differentials across sectors, as well as wage differentials arising from country-specific shocks to labor supply. In other words, in our model, firms’ willingness to engage in price discrimination across markets is independent of nominal rigidities.

4. Exchange rates and relative prices in general equilibrium

We are now ready to analyze how the exchange rate and the terms of trade respond to shocks in general equilibrium. We focus on symmetric equilibria within a country (though not necessarily symmetric across countries): we hereafter drop the indexes $j$ and $j^*$ and interpret all variables in per-capita (or aggregate) terms.

The world equilibrium is characterized as follows. Given the stochastic processes driving monetary stances ($\mu_t$ and $\mu_t^*$) and the shocks to productivity and preferences (all the $Z$’s, $\chi_t$, $\kappa_t$, and $\kappa_t^*$), and given the initial holdings of bonds ($B_{H,0}$ and $B_{F,0}$) and money ($M_0$ and $M_0^*$), for $t \geq 0$ the equilibrium (symmetric across firms) is a set of processes for the nominal exchange rate $E_t$, the Home allocations and prices ($l_t$, $C_{H,t}$, $C_{N,t}$, $M_{t+1}$, $B_{H,t+1}$, $B_{F,t+1}$, $P_{H,t}$, $P_{N,t}$ and $W_t$) and their Foreign counterparts ($l_t^*$, $C_{H,t}^*$, $C_{N,t}^*$, $M_{t+1}^*$, $B_{H,t+1}^*$, $B_{F,t+1}^*$, $P_{H,t}$, $P_{N,t}$ and $W_t^*$) that (a) satisfy the Home and Foreign consumers’ optimality conditions, (b) maximize firms’ profits, (c) satisfy the market clearing conditions for each asset and each good, in all the markets where it is traded, and (d) satisfy the resource constraints.

Table 1 presents a subset of equilibrium conditions that completely characterize the model. In the table, $\bar{A}_{t+1}$ denotes Home non-monetary wealth at the beginning of time $t+1$, i.e.

$$\bar{A}_{t+1} = (1 + i_{t+1}) B_{H,t+1} + (1 + i_{t+1}^*) E_{t+1} B_{F,t+1}; \quad (26)$$

$\bar{T}$ is similarly defined. Equations (29) state the risk-adjusted uncovered interest parity condition, where we have used the fact that in equilibrium $\mu_t = P_t C_t$ and $\mu_t^* = P_t^* C_t^*$. Equation (30) is the bond market-clearing equation, while (31) is the current account. These three equations simultaneously determine the nominal exchange rate and external borrowing. Employment and consumption in both countries (corresponding to the expressions (32), (34),
(33) and (35)) can then be derived as a function of exogenous shocks, wages and the nominal exchange rate. In each country, the nominal wage rate (27) (or (28)) is preset given the joint distribution of employment, $\kappa$ (or $\kappa^*$) and domestic monetary shocks. In the Appendix, we show that this economy can have multiple non-stochastic steady states, for (perhaps too) large values of the distribution margin.

Two observations help understanding the role of distribution costs in our specification. First, by using the definition of the real exchange rate, we can write:

$$
\frac{\mathcal{E}_t P_t}{P_t} = \left( \frac{\mathcal{E}_t P_{t,t}^*}{\bar{P}_{t,t}} \right) \left( \frac{P_{N,t}}{P_{T,t'}} \right)^{1-\gamma} \left[ \left( \frac{\mathcal{E}_t P_{N,t}^*}{\bar{P}_{N,t}} \right) \left( \frac{\mathcal{E}_t P_{T,t}^*}{\bar{P}_{T,t}} \right) \right] \frac{1}{\bar{P}_{N,t}^*} \left( \frac{P_{N,t}}{P_{T,t}} \right)^{1-\gamma}.
$$

In our model, movements in the real exchange rate are due both to differences in prices (in the same currency) of traded goods across countries and to movements of the relative price of tradables in terms of nontradables. This is in sharp contrast with models adopting a similar specification, but not allowing for distributive trade. By setting $\eta = 0$, in fact, there would be no deviation from the law of one price. The first term on the right-hand side of the above definition would be constant, and the variability of the real exchange rate would only depend on the variability of the relative price of nontradables within each country — a prediction that is inconsistent with the findings for the US real exchange rate in Engel [1999].

Second, as the Home demand for imports can be written:

$$
C_{F,t} = \gamma \frac{\mu_t}{2 \bar{P}_{F,t}} = \gamma \frac{\mu_t}{2 \bar{P}_{F,t} + \eta P_{N,t}},
$$

the elasticity of the imports demand with respect to the wholesale price $\bar{P}_{F,t}$ is:

$$
\frac{\partial C_{F,t}}{\partial \bar{P}_{F,t} C_{F,t}} = \frac{\bar{P}_{F,t}}{\bar{P}_{F,t} + \eta P_{N,t}} = \frac{\mathcal{E}_t + \frac{\eta}{\theta - 1} W_t Z_{F,t}}{\mathcal{E}_t + \frac{\eta}{\theta - 1} W_t^* Z_{N,t}},
$$

an expression that is clearly decreasing in $\eta$, and equal to 1 for $\eta = 0$. Distribution services reduce the price elasticity of imports demand, below what is implied by Cobb-Douglas preferences. But by lowering the price elasticity of imports, distributive trade induces larger price movements for any given quantity change.

20 Since tradables have equal shares in Home and Foreign consumption baskets, terms of trade movements do not impinge on the real exchange rate.
Table 1: Equilibrium characterization

\[ W_t = \frac{\phi}{\phi - 1} \frac{E_{t-1} \kappa_t \ell_t}{E_{t-1}} \tag{27} \]

\[ W^*_t = \frac{\phi}{\phi - 1} \frac{E_{t-1} \kappa^*_t \ell^*_t}{E_{t-1}} \tag{28} \]

\[ \mathcal{E}_t = \frac{\mu_t E_t (\mathcal{E}_{t+1}^* \mu_{t+1})}{\mu_t E_t (\mu_{t+1})} = \frac{\mu_t E_t (\mathcal{E}_{t+1}^* \mu_{t+1})}{\mu_t E_t (\mu_{t+1})} \tag{29} \]

\[ \overline{\mathcal{A}}_t = -\mathcal{E}_t \overline{\mathcal{A}}^*_t \tag{30} \]

\[ E_t \left\{ \beta \mu_t \overline{\mathcal{A}}_{t+1} \right\} = \overline{\mathcal{A}}_t - \frac{\gamma}{2} \left[ \frac{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}}{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}} - 1 + \frac{\eta}{\theta} \mathcal{E}_t W_t \mathcal{Z}_{N,t} \right] \tag{31} \]

\[ \ell_t = \frac{\eta}{\theta} \mu_t \left[ 1 + \frac{2 \mathcal{E}_t W_t}{\eta \theta} + \frac{2 \gamma}{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}} + \frac{2 \gamma}{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}} \right] \tag{32} \]

\[ \ell^*_t = \frac{\eta}{\theta} \mu^*_t \left[ 1 + \frac{2 \mathcal{E}_t W_t}{\eta \theta} + \frac{2 \gamma}{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}} + \frac{2 \gamma}{1 + \frac{\eta}{\theta} \frac{E_t W_t}{W_t} \mathcal{Z}_{N,t}} \right] \tag{33} \]

\[ C_t = \frac{\theta - 1}{\theta} \frac{\mu_t}{W_t} (Z_{N,t}) \left( \frac{Z_{N,t}}{Z_{H,t}} + \frac{\eta}{\theta - 1} \right)^{-\frac{1}{2}} \left( \frac{Z_{N,t}}{Z_{H,t}} \bar{E}_t W_t + \frac{\eta}{\theta - 1} \right)^{-\frac{1}{2}} \tag{34} \]

\[ C^*_t = \frac{\theta - 1}{\theta} \frac{\mu^*_t}{W_t^*} (Z_{N,t}^*) \left( \frac{Z_{N,t}^*}{Z_{H,t}^*} + \frac{\eta}{\theta - 1} \right)^{-\frac{1}{2}} \left( \frac{Z_{N,t}^*}{Z_{H,t}^*} \bar{E}_t W_t + \frac{\eta}{\theta - 1} \right)^{-\frac{1}{2}} \tag{35} \]
To analyze further the properties of our model, it is convenient to proceed at first under the assumption of financial autarky. Under this assumption, the exchange rate is implicitly determined by the balanced trade condition \( (E_t \bar{P}_{H,t}^* C_{H,t}^* = \bar{P}_{F,t} C_{F,t}) \) as a non-linear function of relative monetary policy stance \( \mu/\mu^* \) and relative productivity:

\[
\frac{1 + \frac{\eta}{\theta - 1} \frac{E_t W_t^* Z_{H,t}^*}{W_t Z_{N,t}^*}}{1 + \frac{\eta \theta}{\theta - 1} \frac{E_t W_t^* Z_{H,t}^*}{W_t Z_{N,t}^*}} = \frac{\mu_t}{E_t + \frac{\eta}{\theta - 1} \frac{W_t Z_{F,t}}{W_t Z_{N,t}}}. 
\]

As we will show in the next section, most results under financial autarky are qualitatively identical in our economy with international trade in bonds.\(^21\)

In what follows, we linearize the model around a symmetric equilibrium with \( Z_H = Z_F = Z_T, Z_N^* = Z_N \) and equal wage levels \( \kappa/\mu \) across countries. First, we derive the exchange rate response to nominal and real shocks, focusing on the relative volatility of this variable: the exchange rate tends to move more than the underlying economic fundamentals. Next, we show that notwithstanding the low pass-through nominal depreciations tend to worsen the terms of trade. Finally, we show that nominal and real exchange rates are positively correlated in equilibrium.

### 4.1 Exchange rate volatility

Consider first the response of the exchange rate to monetary policy shocks, in the form of an unexpected changes in the ratio of \( \mu \) to \( \mu^* \). In the long run, the nominal exchange rate moves one-to-one with relative monetary stances. As all prices are ex-ante flexible, money is neutral: relative wages do not respond to anticipated monetary innovations. The short-run response of the nominal exchange rate (given nominal wages) can instead be written as

\[
\tilde{E}_t = \frac{\left(1 + \frac{\eta}{\theta - 1} Z_T^* \right) \left(1 + \frac{\eta \theta}{\theta - 1} Z_T^* \right)}{\left(1 + \frac{\eta}{\theta - 1} Z_N^* \right) \left(1 + \frac{\eta \theta}{\theta - 1} Z_N \right) - 2 \eta Z_T^* \left(\tilde{\mu}_t - \tilde{\mu}_t^*\right)}. 
\]

When the conditions for the existence of multiple steady states derived in the appendix do not hold, the coefficient multiplying the relative monetary shock in the above equation is always

\[^{21}\text{In addition, the static version of our model below is directly comparable to many recent contributions in open macro that, by assumption or endogenously, rule out current account dynamics (see Corsetti and Pesenti [2001a], Heathcote and Perri [2000] and Obstfeld and Rogoff [2002]).}\]
positive and larger than one. Interestingly, a permanent monetary expansion depreciates on
impact the nominal exchange rate by more than its long-run value, generating expectations of
appreciation in the future.

In response to real productivity shocks, the exchange rate jumps to its new equilibrium
value on impact:

\[
\hat{E}_t = \left( \frac{\eta}{\eta - 1} \frac{Z_T}{Z_N} \right) \left( 1 + \frac{\eta\theta Z_T}{Z_N} \right) - 2\eta \frac{Z_T}{Z_N} \left[ \hat{Z}_{N,t} + \hat{Z}_{W,t} - \hat{Z}^*_N - \hat{Z}^*_F \right].
\]

(39)

The response to shocks to the traded and the nontraded sector is symmetric: the nominal
exchange rate depreciates with any domestic productivity shock and appreciates with any
Foreign shock. Intuitively, a positive shock to productivity in the Home tradable sector reduces
the wholesale price of the Home goods in the Foreign market. Although the retail price also
falls, increasing Foreign demand, the value of exports drops: for a given value of Home
imports, balanced trade in equilibrium requires a depreciation of the currency (the less elastic
the demand for imports, the larger the rate of depreciation).

By the same token, a positive productivity shock to the Home nontradable sector reduces
unit distribution costs in the Home market, increasing the price elasticity of Home demand for
imports: import prices tend to fall. However, because of falling distribution costs, retail prices
fall by more, boosting Home import demand. Thus, the exchange rate must depreciate to
ensure a zero trade balance. Note that the size of exchange rate movements in response to
productivity shocks is amplified when \( \eta \) and \( \theta \) are relatively high.

---

22 The above coefficient depends inversely on the exchange rate elasticity of import and export demand. We
checked numerically that the steady state is unique for a range of values of \( \eta \frac{Z_T}{Z_N} \) such that the conditions for
miform multiplicity in the Appendix do not hold.

23 Note that the above result is not comparable to Dornbusch’s overshooting. Without trade in financial assets
uncovered interest parity does not hold. Moreover, by definition of \( \mu \), permanent shocks to this variable affect
neither Home nor Foreign interest rates.

24 When the distribution margin is so high that the economy may exhibit multiple equilibria, the response
of the exchange rate to nominal and real shocks may change sign even for equilibria that are in a neighborhood
of the symmetric steady state. Clearly, for values of the parameters close to those that make the sign switch, the
exchange rate becomes extremely volatile.
4.2 Terms of trade

Consider now the link between nominal exchange rate movements and the terms of trade. Linearizing the latter around a symmetric steady state, we obtain

\[
\frac{\hat{P}_{F,t}}{\hat{P}^{*}_{F,t}} = \left(1 - \frac{\eta}{\theta - 1}\right) \left(\hat{E}_t - \hat{W}^{*}_t + \hat{W}_t\right) + \left(\hat{Z}_{H,t} - \hat{Z}^{*}_{F,t}\right) - \frac{\eta}{\theta - 1} \left(\hat{Z}_{N,t} - \hat{Z}^{*}_{N,t}\right)\left(1 + \frac{\eta}{\theta - 1}\right),
\]

(40)

where \(\hat{E}_t\) is given by either (38) or (39), depending on the nature of the shock. Under mild conditions on the degree of monopoly power and distribution margins, such that \(\eta < \theta - 1\), the coefficient of \(\hat{E}_t\) in the above expression is positive. Hence, monetary shocks induce a positive correlation between the terms of trade and the exchange rate.

The correlation between the terms of trade and the nominal exchange rate is also positive in the presence of real shocks to productivity in the Home tradable sector. As these shocks unambiguously depreciate the Home currency, they worsen the terms of trade both directly (second term on the right-hand side of the expression above) and through their effect on \(\hat{E}\). For this reason, it is possible that shocks to the tradable sector cause the terms of trade to be more volatile than \(\hat{E}\).

Conversely, the correlation between \(\hat{E}\) and the terms of trade induced by shocks to productivity to the Home nontradable sector is not necessarily positive. While unambiguously depreciating the Home currency, these shocks also have a positive effect on the terms of trade. This is because, by reducing the cost of distributive trade in the Home market, they raise the price elasticity of the Home demand for Foreign products. A higher price elasticity tends to lower the optimal price charged by Foreign wholesalers. While this effect is small for reasonable values of \(\eta/ (\theta - 1)\), its presence implies that the volatility of the terms of trade in response to shocks to nontradables tends to be lower than the volatility of \(\hat{E}\).

4.3 The real exchange rate

Linearizing the real exchange rate around a symmetric steady state, and focusing on nominal shocks only, we obtain:

\[
\frac{\left(\hat{E}_t P^{*}_{F,t}\right)}{\hat{P}^{*}_{F,t}} = \left[\frac{(1 - \gamma) + \frac{\rho^q Z_t}{Z^{*}}}{1 + \frac{\rho^q Z_t}{Z^{*}}}\right] \hat{E}_t,
\]

(41)
where $\hat{E}_t$ is given by (38). Since $0 < \gamma < 1$, this expression shows that monetary shocks always move nominal and real exchange rates in the same direction. Thus, an unexpected monetary expansion at home will bring about both a nominal and a real depreciation. However, since the coefficient of $\hat{E}_t$ in the above expression is less than one, the real exchange rate will move by less.

With shocks to traded and nontraded productivity, we have, respectively:

$$
\left( \frac{\xi_t P_t^*}{P_t} \right) = \left[ 1 - \frac{\gamma}{(1+\eta/\theta)Z_N} \right] \hat{E}_t
$$

$$
\left( \frac{\xi_t P_t^*}{P_t} \right) = \left[ 1 - \frac{\gamma}{(1+\eta/\theta)Z_N} \right] \left( 1 + \hat{E}_t \right)
$$

(42)

where $\hat{E}_t$ is given by (39). We have seen above that, regardless of the sector in which they occur, domestic productivity shocks always depreciate the domestic currency in nominal terms. It is then apparent from the above expressions that they also depreciate it in real terms. Observe that the real depreciation will be attenuated in the case of shocks to $Z_H,t$, relative to the case of shocks to $Z_N,t$.

5. Equilibrium under alternative specifications of the asset market

In this section, we analyze our model under alternative specifications of the assets market by means of numerical exercises assuming financial autarky, trade in nominal bonds, and trade in nominal claims contingent on real and monetary shocks — i.e., claims to receiving one unit of currency in each state of nature. We study the response of our economy to unexpected nominal and real shocks, defined as a 1 per cent deviation from their initial steady state values, allowing for both permanent shocks and temporary shocks (lasting only one period). When trade in assets is limited to bonds, it is well known that the effects of shocks on the wealth distribution across countries will generate endogenous dynamics (see Obstfeld and Rogoff [1996], Chapter 10). Thus, we solve for the equilibrium path assuming that after the shock the economy evolves under perfect foresight to its new steady state, characterized by a different world distribution of wealth.

In our exercises, we assume that labor is 4 times as productive in the tradable sector relative to the nontradable sector — namely, we set $Z_T/Z_N = 4$; the values of $\eta$ and $\theta$ are set equal to 0.28 and 10, such that markups vary between 11 and 25 percent across sectors, and the distribution margin is 50 percent — a number that is not far from available estimates for
the US and other OECD countries (see Burstein, Neves and Rebelo [2000]). Tradables and nontradables are given the same weight in consumption, i.e. $\gamma = .5$. Using the conditions derived in the appendix, it can be verified that these parameters values ensure a unique steady state.

The results of our exercises are shown in Table 2, which reports the percentage changes in the nominal exchange rate (NER), the real exchange rate (RER), the terms of trade (TOT), and real GDP (at constant steady-state prices) for the Home country under different asset trading arrangements. Observe that, qualitatively, the equilibrium response to shocks in the economies with trade in bonds are in line with our analysis in the previous section.

Relative to the economy with no assets trade, however, there are apparent quantitative differences. Notably, there is a large drop in the exchange rate volatility between financial autarky and the bond economy. What explains such differences? In general, a shock increasing the supply of Home tradables requires a worsening of the terms of trade to absorb the greater relative abundance of the Home traded good. Not only is this due to a larger supply of Home goods: it also reflects a shift of labor inputs in the Foreign country, out of the tradable sector into the nontradable sector. Such a shift is necessary to take advantage of lower import prices — more nontradables being demanded by retailers in the Foreign economy.

When a complete set of nominal contingent assets is available, perfect risk sharing calls for the Home country to run a trade deficit, as it is efficient for the more productive country to consume relatively more. In turn, this entails an increase in the Foreign labor supply and in the production of Foreign tradables that in equilibrium greatly limit the relative abundance of Home tradables and the movements in goods prices required to clear markets.

In the bond-only economy, the two countries can to some extent replicate the efficient risk sharing arrangement by having the Home country run a current account deficit and borrow from abroad — thus limiting the relative supply of Home goods and containing in equilibrium the decline in its terms of trade. In financial autarky, instead, all trade has to be quid pro quo. Relative to the bond economy, the Home country must thus export more and import less. But higher net exports in equilibrium require larger movements in the terms of trade and in nominal and real exchange rates. With this intuitive explanation in mind, we now turn to discussing our exercises in detail.
5.1 Nominal and real exchange rate

Consider first the response of the nominal exchange rate to monetary shocks (first two columns in Table 2). With incomplete markets, the NER is more volatile than the underlying shock. A 1 percent temporary increase in $\mu$ — equivalent to a 1 percentage point drop in the short-term nominal interest rate — depreciates $E$ by 1.3 per cent in the bond economy, and by a striking 8.8 per cent under financial autarky. The same pattern of volatility emerges in response to permanent monetary shocks.\(^{25}\) Note that the result shown by (38) for the case of financial autarky carries over in our bond-only economy — although its size is relatively contained in the presence of international borrowing and lending.

It is easy to show that with contingent nominal bonds the exchange rate and $\mu$ move in proportion as

$$E_t = \frac{\mu_t}{\mu^*_t}. \quad (43)$$

We should note here that the above solution also characterizes the exchange rate in our incomplete-market economy in the limiting case $\eta \to 0$ — when our specification becomes similar to Corsetti and Pesenti [2001a], Obstfeld and Rogoff [2000] and Devereux and Engel [2000]. With constant demand elasticities (no distributive trade), nominal shocks cannot produce any excess volatility. Furthermore, the exchange rate responds to real shocks in the economy only through endogenous changes in the relative monetary policy stance $\mu_t/\mu^*_t$.

Without trade in contingent nominal bonds and $\eta > 0$, instead, real shocks to productivity depreciate NER independently of monetary policy (third through sixth column in the table). The magnitude of the effects of innovations to $Z_H$ and $Z_N$ on NER is comparable (slightly lower in the case of shocks to $Z_N$). In the bond economy, temporary productivity shocks have a much smaller impact than permanent shocks (.2 percent vs. 3.9 percent), against which bonds provide no insurance mechanism.

Finally, the real exchange rate is positively correlated with the nominal exchange rate (so that it always weakens in response to positive nominal and real shocks), although relative volatility depends on the nature of the underlying shock. Note that RER is less volatile than

\(^{25}\) In the bond economy, a temporary increase in $\mu$ decreases the domestic interest rate for one period. By the uncovered interest parity, the domestic currency is expected to appreciate in the future, as the nominal exchange rate returns to the initial steady state level. Following a permanent shock to $\mu$, instead, the exchange rate displays no endogenous dynamics. It immediately jumps to the new depreciated level and stays there. Recall that a permanent shock to $\mu$ implies constant domestic and foreign interest rates. The price of Home goods will however increase proportionally between the period in which the shock occurs, and the next.
NER in response to nominal shocks and productivity shocks in the tradable goods sector; it is as volatile or more volatile in response to shocks to the nontradable goods sector.

5.2 Terms of trade

In all our exercises, a nominal depreciation of the Home currency worsens the Home terms of trade, regardless of the nature of the shock, raising the possibility of expenditure-switching effects from exchange rate movements.\textsuperscript{26} The terms of trade can be more volatile than the nominal (and real) exchange rate or less volatile, again depending on the nature and relative size of the shocks hitting the economy. For instance, whether or not agents can trade international bonds, monetary shocks cause TOT to worsen by 30 percent less than NER.

Shocks in the nontradable sector have a small effect on the terms of trade. Conversely, shocks to the tradable sector raise the volatility of the terms of trade above that of the nominal (and real) exchange rate. Combining both shocks — i.e. allowing for an aggregate macro shock to domestic productivity — relative volatility depends on how persistent the shock is (see the last two columns of Table 2). Namely, permanent innovations cause the nominal exchange rate to move more than the real exchange rate and the terms of trade. The opposite pattern emerges in the case of temporary shocks.

5.3 Real GDP and price levels

In most exercises, exchange rate movements are several times larger than the average changes in output. For instance, in response to permanent shocks to productivity, the nominal and real exchange rates are between 5 and 20 times more volatile than real output, which, in turn, is in general more volatile than consumption. So the model may be consistent with a high volatility of the real exchange rate relative to real variables.

In general, domestic price movements tend to be quite limited with respect to those in international prices, due to the low degree of pass-through that characterizes the model. Changes in the domestic price level are in general a great deal smaller than changes in the nominal exchange rate, particularly so in the face of nominal shocks: in this case, the CPI response is ten times smaller than that of the nominal exchange rate.\textsuperscript{27}

\textsuperscript{26} In light of our discussion in the previous section, this is so because the ratio $\eta/(\theta - 1)$ is small in our parameterization.

\textsuperscript{27} Corsetti, Dedola and Leduc [2002] carry out extensive quantitative analysis of the model, using more general specifications of preferences and technology. Specifically, this paper is focused on the lack of international risk sharing as exemplified by the negative correlation of relative consumption and the real exchange rate — the so-called Backus and Smith [1993] anomaly.
Table 2: Impact Responses of Selected Variables to Nominal and Real Shocks under Alternative Asset Markets Structures
(Percentage deviations from steady-state values)

<table>
<thead>
<tr>
<th></th>
<th>Monetary Shock</th>
<th>Shock to Tradable</th>
<th>Shock to Nontradable</th>
<th>Economy-wide Shock</th>
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<tr>
<td></td>
<td>Temporary</td>
<td>Permanent</td>
<td>Temporary</td>
<td>Permanent</td>
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<tr>
<td>NER</td>
<td>8.8%</td>
<td>8.8%</td>
<td>3.9%</td>
<td>3.9%</td>
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<tr>
<td>RER</td>
<td>6.8%</td>
<td>6.8%</td>
<td>3.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>TOT</td>
<td>6.8%</td>
<td>6.8%</td>
<td>3.9%</td>
<td>3.9%</td>
</tr>
<tr>
<td>GDP</td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Financial Autarky</td>
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<tr>
<td>Bond economy</td>
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<tr>
<td>Complete Markets</td>
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</tbody>
</table>
6. Monetary policy, international spillovers and gains from cooperation

Despite nominal wage rigidities, it is easy to show that in our economy there exists a simple monetary rule that can sustain a flex-price, flex-wage allocation — obtained from Table 1 by evaluating the optimal wage rates (27) and (28) without the expectation operator. Let $\mu$ and $\mu^*$ be determined as follows

$$
\mu_t = \frac{\Gamma_t}{\kappa_t}, \quad \mu^*_t = \frac{\Gamma^*_t}{\kappa_t^*} \quad (44)
$$

where $\Gamma_t$ and $\Gamma^*_t$ are deterministic functions of time (therefore known at time $t - 1$), scaling the level of wages in the economy. Following the above rules monetary authorities tightens in those states of the world when the disutility of working is higher. As monetary policy in either country completely stabilizes the marginal disutility of working, according to (27) and (28) workers will find it optimal to set a nominal wage equal to $\Gamma_t \phi / (\phi - 1)$ and $\Gamma^*_t \phi / (\phi - 1)$. It is easy to check that this is exactly the flexible wage that would result when (44) is implemented. It follows that the above monetary rules support the flexible-wage allocation.28

In a recent paper, Obstfeld and Rogoff [2002] argue that gains from international monetary cooperation are small, once monetary authorities can credibly commit to self-oriented policies of domestic stabilization, such as (44). In their model, there are no deviations from the law of one price for tradable goods; moreover, in their benchmark economy, terms of trade movements guarantees perfect risk sharing. Using our framework, we can show that a similar result holds in an environment with deviations from the law of one price and low pass-through to consumer prices, and less than optimal risk sharing. As in Obstfeld and Rogoff [2002], we measure national welfare in terms of the expected utility of the national representative agents, but abstracting from liquidity services (i.e. assuming $\gamma \to 0$).

Consider the limiting case in which different brands of tradable goods are perfect substitutes $(\theta \to \infty)$, so that, with competitive goods markets, firms are no longer able to segment markets across national borders.29 The law of one price holds at the producer price level (this is apparent when evaluating expressions (17) and (18) as $\frac{\theta}{\theta - 1}$ approaches unity).

---

28 This result shows that market segmentation due to distributive trade does not prevent the possibility that monetary policy rules may sustain the allocation without nominal rigidities. As shown by Corsetti and Pesenti [2001b], however, this is impossible when market segmentation and incomplete pass-through can be attributed to local currency pricing (even partial local currency pricing) by firms.

29 The case of perfect competition in a small open economy without nominal rigidities is analyzed by Burstein, Neves and Rebelo [2000].
Table 3: Equilibrium with competitive good markets in financial autarky

\[ W_t = \frac{\phi}{\phi - 1} E_{t-1} \kappa_t \mu_t \]  
\[ W_t^* = \frac{\phi}{\phi - 1} E_{t-1} \kappa^*_t \mu^*_t \]  
\[ E_t = \frac{\mu_t - \eta \mu_t^* E_{t-1} \kappa_t \mu_t Z_{F,t}}{\mu_t^* - \eta \mu_t E_{t-1} \kappa_t \mu_t Z_{N,t}} \]  
\[ \ell_t = \frac{\phi - 1}{\phi} \frac{\mu_t}{E_{t-1} \kappa_t \mu_t} \]  
\[ C_t = \frac{\phi - 1}{\phi} \frac{\mu_t}{E_{t-1} \kappa_t \mu_t} \left( \frac{Z_{N,t}}{Z_{H,t}} + \eta \right)^{-\frac{\gamma}{\gamma - 1}} \left[ \frac{E_{t-1} \kappa_t \mu_t}{\mu_t} \left( \frac{Z_{N,t}}{Z_{F,t}} \right) \right]^{-\frac{\gamma}{\gamma - 1}} \]  
\[ C_t^* = \frac{\phi - 1}{\phi} \frac{\mu_t^*}{E_{t-1} \kappa^*_t \mu^*_t} \left( \frac{Z_{N,t}^*}{Z_{F,t}^*} + \eta \right)^{-\frac{\gamma}{\gamma - 1}} \left[ \frac{E_{t-1} \kappa_t \mu_t}{\mu_t} \left( \frac{Z_{N,t}^*}{Z_{F,t}^*} \right) \right]^{-\frac{\gamma}{\gamma - 1}} \]

When prices are measured net of distribution costs, it also holds at the consumer price level. With perfectly competitive goods markets, the solution of the model is considerably simplified. For the case of financial autarky, we are actually able to solve the model in closed form. The allocation with perfect competition in the goods market and without trade in assets is shown in Table 3.

The following proposition states that, with competitive goods markets, monetary authorities can do no better than implementing (44) — even though under financial autarky the resulting allocation does not coincide with the command optimum.30 Most crucially, international cooperation is completely superfluous.

Proposition 1 Without trade in assets, if \( \frac{\theta}{\theta - 1} = 1 \), the monetary policy rule \( \mu_t = \Gamma_t / \kappa_t \) — as specified in (44) — is optimal in both the world Nash equilibrium and under cooperation (independently of the welfare weights attributed to the Home and Foreign country).

---

30 See the working paper version of this paper for a detail characterization of the command optimum allocation (Corsetti and Dedola [2002]).
To prove this proposition, we note that the first order conditions of the policy problem in a world Nash equilibrium include the following equation:

$$\frac{1}{\mu_t} - \frac{\kappa_t}{E_{t-1}(\kappa_t\mu_t)} + \frac{\kappa_t\mu_t - E_{t-1}(\kappa_t\mu_t)}{2\mu_t} \left[ E_{t-1}(\kappa_t\mu_t) - \eta \frac{E_{t-1}(\kappa_t^*\mu_t^*)}{\mu_t^*} \frac{Z_{H,t}}{Z_{N,t}} \right] = 0$$

(52)

It is easy to check that the monetary rule \(\mu_t = \Gamma_t / \kappa_t\) is a solution to the above equation for any Foreign monetary policy stance \(\mu_t^*\). To prove the second part of the proposition, consider the problem of a benevolent centralized planner, setting \(\mu_t\) and \(\mu_t^*\) to maximize a weighted average of the expected utility of the Home and Foreign agents, according to arbitrary weights \(\Omega\) and \(1 - \Omega\):

$$\max_{\mu_t,\mu_t^*} E_{t-1} \left[ \Omega (\log C_t - \kappa_t l_t) + (1 - \Omega) (\log C_t^* - \kappa_t^* l_t^*) \right]$$

(53)

subject to (48)-(51). The first order condition for \(\mu_t\) is

$$\Omega \left[ \frac{1}{\mu_t} - \frac{\kappa_t}{E_{t-1}(\kappa_t\mu_t)} + \frac{\kappa_t\mu_t - E_{t-1}(\kappa_t\mu_t)}{2\mu_t} \left[ E_{t-1}(\kappa_t\mu_t) - \eta \frac{E_{t-1}(\kappa_t^*\mu_t^*)}{\mu_t^*} \frac{Z_{H,t}}{Z_{N,t}} \right] \right] + (1 - \Omega) \frac{\gamma}{2} \left[ \frac{1}{\mu_t} - \frac{\kappa_t}{E_{t-1}(\kappa_t\mu_t)} - \eta \frac{Z_{V,t}\kappa_t\mu_t - E_{t-1}(\kappa_t\mu_t)}{\mu_t^2} \right] = 0$$

(54)

It is apparent that setting \(\mu_t\) according to (44) solves the above equation regardless of the value of \(\mu_t^*\) and \(\Omega\).

Intuitively, in the absence of firms’ market power, the monetary authority in each country can achieve a constrained Pareto-efficient allocation by redressing the distortion from wage stickiness. It cannot, however, address the distortion due to monopoly power in the labor market, nor can it engineer enough risk sharing, as is required to achieve the command optimum allocation.

Despite financial and asset market frictions, a very high degree of competition in the goods market implies that there are no monetary policy spillovers in equilibrium. However, international spillovers are not zero, and the monetary rules (44) may not be optimal, when we move away from this limiting case. In analogy to Obstfeld and Rogoff [2002], the question is whether the magnitude of spillovers in the general version of our model is large enough to motivate international policy coordination — an issue that we leave to future research.
7. Conclusions

In the last few years, a vast and growing literature in international macroeconomics has been devoted to the study of interdependence using general-equilibrium models with nominal rigidities in either prices or wages. This literature has broken new ground in the analysis of stabilization policies in an open-economy, improving our understanding of the international transmission of both policy and real shocks.

Relative to the existing literature, this paper opens a new perspective to model the macroeconomy and raises new issues. Several recent contributions stress the importance of placing international price discrimination centerstage in the analysis of open economies. In this paper we have shown that, among the alternative ways to do this, modelling vertical relationships among firms located in different markets is a realistic and promising strategy, bringing open macro theory more closely into line with key stylized facts of the international economy related to large discrepancies in international prices.

In our model, due to the presence of downstream retailers, upstream firms with monopoly power may face different demand elasticities in national markets even under symmetric, constant elasticity preferences across countries. Thus, they will optimally charge different prices to domestic and foreign dealers — within the limits dictated by the possibility of international arbitrage between wholesale and retail markets. As a consequence, the law of one price fails to hold at both producer and consumer levels, independently of nominal rigidities. Secondly, as firms optimally adjust markups in the face of demand fluctuations, the response of prices to exchange rate movements is muted at both consumer and producer levels: exchange rate pass-through is not complete. Third, our model yields the result that a depreciation of the currency generally worsens the terms of trade. Thus, despite low pass-through the international transmission of monetary shocks can have expenditure-switching effects. Fourth, our specification accounts for the high volatility of the exchange rate relative to consumption and output, whereas large changes in the nominal and real exchange rate are associated with small changes in the real allocation and the consumer price level.

Key to our approach is that distributive trade requires local inputs; that is, that there are vertical interactions among firms across national boundaries. It is worth stressing that vertical interactions are not exclusively due to distributive trade. Realistically, local inputs can be employed in some final stage of manufacturing of the final product at local level, combined with traded intermediate goods. Encompassing both distributive trade and manufacturing at
local level, the share of the consumer prices that can be attributed to the cost of local inputs may actually become quite high, potentially reinforcing many of the novel results of our analysis.

The model in this paper has been purposely kept simple by means of convenient assumptions. For instance, the elasticity of substitution among individual goods (brands) is the same in all sectors, the elasticity of substitution among types of good is set equal to one, and there is no difference between nontraded goods and distribution services. Relaxing these assumptions is a key step to confront the model more directly with the data.

Most crucially, we have assumed the most basic vertical structure: an upstream monopolist sells its product to a perfectly competitive downstream firm, the retailer. In this case, without distortionary taxation at national level, vertical integration would be completely neutral as regards the allocation. As we argued in section 3, allowing for monopoly power in the retail sector would imply double marginalization at consumer prices. However, it would not induce substantial differences relative to our results on the equilibrium response of prices to nominal and real shocks — there are no significant losses in adopting our simpler benchmark specification. Nevertheless, an important task for future research is to generalize our setup to richer strategic interactions between upstream and downstream firms (e.g., allowing for non-linear pricing, or nominal rigidities in the goods market). bringing more realism – and more insights from trade and industrial organization theory – into the construction of open macro models.
Appendix I

Firms’ pricing and the no-arbitrage conditions
This section of the Appendix studies the representative firm’s problem under the constraint that prices should not provide opportunities for arbitrage. The (arbitrage-constrained) optimal prices by the representative Home firm is the solution to the following profit maximization problem

$$\max_{\pi(h), \pi^*(h)} \left[ \pi_t(h) C_t(h) + \mathcal{E}_t \pi_t(h) C^*_t(h) \right] - \frac{W_t}{Z_{hi,t}} \left[ C_t(h) + C^*_t(h) \right]$$

s.t.

$$C_t(h) + C^*_t(h) = \left[ \pi_t(h) + \eta P_{N,t} \right]^{\theta} \left[ P_{H,t}^0 C_{H,t} + \pi_t(h) + \eta P_{N,t} \right]^{-\theta} \left( P_{H,t}^0 \right)^{\theta} C_{H,t}^*$$

$$\pi_t(h) + \eta P_{N,t} - \mathcal{E}_t \pi_t(h) \geq 0$$

$$\mathcal{E}_t \left( \pi_t(h) + \eta P_{N,t} \right) - \pi_t(h) \geq 0.$$ 

The relevant FOC’s for $\pi(h)$ and $\pi^*(h)$ (including the complementary slackness conditions and dropping time subscripts) are, respectively:

$$\left[ \pi(h) + \eta P_N \right]^{-\theta} P_{H}^0 C_{H} \left( 1 - \theta \frac{\pi(h)}{\pi(h) + \eta P_N} + \theta \frac{W}{Z_{H} \pi(h) + \eta P_N} \right) + \xi_1 = \xi_2$$

$$\left[ \pi^*(h) + \eta P^*_N \right]^{-\theta} P_{H}^0 C_{H} \left( 1 - \theta \frac{\pi^*(h)}{\pi^*(h) + \eta P^*_N} + \theta \frac{W}{Z_{H} \mathcal{E}(\bar{\pi}^*(h) + \eta P^*_N)} \right) + \xi_2 = \xi_1$$

$$\xi_1 \left[ \pi(h) + \eta P_N - \mathcal{E}\pi^*(h) \right] = 0, \quad \pi(h) + \eta P_N - \mathcal{E}\pi^*(h) \geq 0, \quad \xi_1 \geq 0$$

$$\xi_2 \left[ \mathcal{E}(\pi^*(h) + \eta P^*_N) - \pi(h) \right] = 0, \quad \mathcal{E}(\pi^*(h) + \eta P^*_N) - \pi(h) \geq 0, \quad \xi_2 \geq 0.$$ 

The optimal prices discussed in the main text are derived for the case in which $\xi_1 = \xi_2 = 0$. An important implication of these prices is that, if the Home monopolist can freely discriminate across national markets, the same is true of the Foreign one. Before proceeding further, note that if $\xi_1 \geq 0$, and $\xi_2 \geq 0$, then it must be that $\eta \left( P_N + \mathcal{E} P^*_N \right) = 0$, which can be true only for $\eta = 0$. Obviously, as long as distribution costs are strictly positive, the two constraints cannot be binding at the same time.

Thus, we can characterize optimal price-setting when either condition is binding. Without loss of generality set $\xi_1 \geq 0$ and $\xi_2 = 0$, i.e. $\pi(h) + \eta P_N = \mathcal{E}\pi^*(h)$. The relevant
FOC’s for \( \bar{p}(h) \) and complementary slackness conditions are:

\[
P^\theta_H C_H \left( 1 - \frac{\theta \bar{p}(h) + \eta P_N}{\bar{p}(h) + \eta P_N} + \frac{1}{Z_H \bar{p}(h) + \eta P_N} \right) = -\xi_1 \leq 0,
\]

\[
(\mathcal{E} P^*_H)^\theta C_H \left( 1 - \frac{\theta \bar{p}(h) + \eta P_N}{\bar{p}(h) + \eta (P_N + \mathcal{E} P^*_N)} + \frac{1}{Z_H \bar{p}(h) + \eta (P_N + \mathcal{E} P^*_N)} \right) = \xi_1 \geq 0.
\]

It follows that, under symmetry, the optimal price \( \bar{p}_H \) should satisfy

\[
\left( 1 + \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} < \bar{p}_H < \left( 1 - \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} + \frac{\eta}{\theta - 1} \frac{\mathcal{E} W^*}{Z_N},
\]

while solving the following quadratic equation:

\[
C_H \frac{\theta \left( 1 + \frac{\eta}{\theta - 1} \frac{Z_H}{Z_N} \right) \frac{W}{Z_H} - (\theta - 1) \bar{p}_H}{\bar{p}_H + \eta \frac{\theta}{\theta - 1} \frac{W}{Z_N}} + C_{H}^* \frac{\theta \left( \frac{1}{Z_H} - \frac{\eta}{\theta - 1} \frac{1}{Z_N} \right) W + \frac{\eta}{\theta - 1} \frac{E W^*}{Z_N} - (\theta - 1) \bar{p}_H}{\bar{p}_H + \eta \frac{\theta}{\theta - 1} \left( \frac{W}{Z_N} + \frac{E W^*}{Z_N} \right)} = 0.
\]

The optimality condition for \( \bar{P}_F^* \) mirrors the above expression.

**Steady state characterization and multiplicity**

In this section of the appendix, we show that our economy can have either a unique or three steady states, depending on whether the exchange rate elasticity of Home export expenditure is larger or smaller than the exchange rate elasticity of Home import expenditure, both evaluated at the symmetric equilibrium with \( \mathcal{E} = 1 \) — a condition reminiscent of the Marshall-Lerner condition.

Consider a deterministic steady state in which external wealth is zero, i.e. \( B_H = B_F = 0 \), so that the trade balance is zero, and all exogenous variables are set equal to constant values, symmetric across countries. The steady state exchange rate is given by (37).

Proposition 2  If \( \frac{\mu^*}{\mu} = 1 \), \( \frac{Z_F}{Z_N} = \frac{Z_H}{Z_N} = \frac{Z_T}{Z_N} \), and \( \frac{\kappa^*}{\kappa} = 1 \), there always exists a steady state in which \( \frac{W}{W^*} = \frac{\kappa \mu}{\kappa^* \mu^*} = 1 \), the nominal exchange rate \( \mathcal{E} \) is equal to 1, and the consumption and labor allocation as determined by equations (32) and (34) are equal across countries. Moreover, when the exchange-rate elasticity of import expenditure is larger than
the exchange-rate elasticity of export expenditure in Home currency, both evaluated at the symmetric equilibrium with $\mathcal{E} = 1$

$$
\left[ \frac{\partial \left( \bar{P}_t C_F \right)}{\partial \mathcal{E}} \frac{\mathcal{E}}{\bar{P}_t C_F} \right]_{\mathcal{E}=1} \geq \left[ \frac{\partial \left( \bar{E}_t^* C_H^* \right)}{\partial \mathcal{E}} \frac{\mathcal{E}}{\bar{E}_t^* C_H^*} \right]_{\mathcal{E}=1}, \tag{55}
$$

there will be two more steady-state equilibria in which $\frac{W}{W^*} = 1$, but the nominal exchange rate is equal to $\mathcal{E}_h$ and $\mathcal{E}_l$, respectively, where $\mathcal{E}_h > 1$, and $0 < \mathcal{E}_l < 1$. The consumption and labor allocations, as determined by equations (32) and (34), will differ across countries.

As the proof of this proposition is the same as the proof of multiple temporary equilibria in a version of our economy with balanced trade and flexible wages, in what follows we write variables with time subscripts. Since the trade balance is identically equal to zero in equilibrium, the value of Home exports is equal to the value of Home imports:

$$
\mathcal{E}_t \bar{P}_H^* C_{H,t} = \bar{P}_F^* C_{F,t},
$$

that is

$$
\mathcal{E}_t \mu_t^* \frac{\bar{P}_H^*}{\bar{P}_F^*} = \frac{\bar{P}_F^*}{\bar{P}_F^*} \mu_t,
$$

or, using the expressions for $P_{F,t}$, $\bar{P}_F^*$, $P_{H,t}^*$ and $\bar{P}_H^*$:

$$
\mathcal{E}_t \mu_t^* \frac{1 + \frac{\eta}{\theta - 1} \mathcal{E}_t W_t^* Z_{H,t}^*}{1 + \frac{\eta}{\theta - 1} \mathcal{E}_t W_t^* Z_{N,t}^*} = \mathcal{E}_t + \frac{\eta}{\theta - 1} \mathcal{E}_t W_t^* Z_{F,t}^*.
$$

It is straightforward to verify that, for $\frac{\mu_t}{\mu_t^*} = \frac{W_t}{W_t^*} = 1$ and $\frac{Z_{F,t}}{Z_{N,t}} = \frac{Z_{H,t}}{Z_{N,t}}$, the equality above is always satisfied for $\mathcal{E}_t = 1$. This establishes the first part of the proposition.

In order to prove the second part, note that both the left-hand side and the right-hand side of the above expression are nonlinear functions of $\mathcal{E}_t$. For $\mathcal{E}_t = 0$, the left-hand side term of the above expression is equal to zero, but the right-hand side term is positive and equal to $\mu_t \eta \mathcal{E}_t W_t^* Z_{F,t}^* Z_{N,t}^* > 0$ (i.e., the value of the Home country imports is above the value of its exports). Second, for $\mathcal{E}_t \to \infty$, the left-hand side diverges, while the right hand side converges to $\mu_t$ (i.e., Home country net exports are surely positive). Therefore, if the function of $\mathcal{E}_t$
defined by the left-hand side expression cuts the function defined by the right-hand side one at $E_t = 1$ from above — i.e. the derivative of the first function with respect to $E_t$ is smaller in absolute value than the derivative of the second function both evaluated at $E_t = 1$, then the equilibrium condition is satisfied by two more values of the nominal exchange rate, $E_t$ and $E_h$, such that $E_h > 1 > E_t > 0$. This proves the second part of our proposition.

Finally, the two derivatives of interest are given by the following expressions:

$$
\frac{\partial}{\partial E_t} \left[ \frac{1}{\eta} \left( \frac{\partial}{\partial t} E_t W_t^* Z_{H,t} \right) \right] = 
\frac{\mu^*}{E_t W_t Z_{N,t}} + \frac{\eta}{\theta - 1} \left( \frac{E_t W_t^* Z_{H,t}}{Z_{N,t}} \right)^2
$$

$$
\frac{\partial}{\partial E_t} \left[ \frac{1}{\eta} \left( \frac{\partial}{\partial t} E_t W_t^* Z_{H,t} \right) \right] = 
\frac{\mu^*}{E_t W_t Z_{N,t}} + \frac{\eta}{\theta - 1} \left( \frac{E_t W_t^* Z_{H,t}}{Z_{N,t}} \right)^2
$$

For $\frac{\mu^*}{\mu_t} = 1$ and $\frac{Z_{F,t}}{Z_{N,t}} = \frac{Z_{H,t}}{Z_{N,t}} = \frac{Z_{T,t}}{Z_{N,t}}$ a sufficient condition for the existence of multiple equilibria is that

$$
\eta \frac{Z_{T,t}}{Z_{N,t}} \geq 1 + 2 \frac{\eta Z_{T,t}}{\theta - 1 Z_{N,t}} + \theta \left( \frac{\eta Z_{T,t}}{\theta - 1 Z_{N,t}} \right)^2,
$$

$$
0 \geq 1 + \eta \left( \frac{2}{\theta - 1} - 1 \right) \frac{Z_{T,t}}{Z_{N,t}} + \theta \left( \frac{\eta Z_{T,t}}{\theta - 1 Z_{N,t}} \right)^2 \iff
$$

$$
(\theta - 1) \frac{\eta}{2} \leq \frac{2}{\theta} \leq \frac{\eta}{2}.
$$

By way of example, consider the case in which $\theta = 10$ and the productivity level is identical in the two sectors. In this case, the above condition is satisfied by $\frac{9}{5} \leq \eta \leq \frac{9}{2}$. 
Appendix II: Solving the model

The problem of the Home representative consumer

The Home agent \( j \) chooses a consumption plan, a portfolio plan and a wage rate, such as to maximize utility (3) subject to the budget constraint (9) and total labor demand (76).

The first order conditions of the Home consumer’s problem with respect to \( C_{H,t}(j) \), \( C_{F,t}(j) \), \( C_{N,t}(j) \), \( B_{H,t+1}(j) \), \( B_{F,t+1} \) and \( M_t(j) \) are, respectively:

\[
\frac{\gamma}{C_{H,t}(j)} = 2\lambda_t(j) P_{H,t} \quad (56)
\]

\[
\frac{\gamma}{C_{F,t}(j)} = 2\lambda_t(j) P_{F,t} \quad (57)
\]

\[
\frac{1 - \gamma}{C_{N,t}(j)} = \lambda_t(j) P_{N,t} \quad (58)
\]

\[
\lambda_t(j) = \beta E_t \lambda_{t+1}(j) (1 + i_{t+1}) \quad (59)
\]

\[
\lambda_t(j) = \beta E_t \lambda_{t+1}(j) (1 + i^*_{t+1}) \quad (60)
\]

\[
\lambda_t(j) = \beta E_t \lambda_{t+1}(j) + \chi_t \left( \frac{M_t(j)}{P_t} \right)^{-\epsilon} P_t \quad (61)
\]

From these conditions, it is easy to see that, at the optimum, the individual demand for Home, Foreign and non-traded consumption goods is a constant share of total consumption expenditure:

\[
P_t C_t(j) = 2 \frac{P_{H,t} C_{H,t}(j)}{\gamma} = 2 \frac{P_{F,t} C_{F,t}(j)}{\gamma} = \frac{1}{1 - \gamma} P_{N,t} C_{N,t}(j). \quad (62)
\]

Using these expressions, it is easy to verify that

\[
P_t C_t(j) = P_{H,t} C_{H,t}(j) + P_{F,t} C_{F,t}(j) + P_{N,t} C_{N,t}(j). \quad (63)
\]
The intertemporal allocation of consumption is determined according to the Euler equation:

\[
\frac{1}{C_t(j)} = \beta (1 + i_{t+1}) E_t \left( \frac{P_t}{P_{t+1} C_{t+1}(j)} \right) \tag{64}
\]

Finally, condition (61) can be written as the money demand function:

\[
\left( \frac{M_t(j)}{P_t} \right) \varepsilon = \chi_t \frac{1 + i_{t+1}}{i_{t+1}} C_t(j) \tag{65}
\]

Define the variable \(Q_{t,t+1}(j)\) as:

\[
Q_{t,t+1}(j) \equiv \beta \frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)} \tag{66}
\]

which is agent \(j\)'s stochastic discount rate. Comparing (66) with (59) and (60), we obtain:

\[
E_t Q_{t,t+1}(j) = \frac{1}{1 + i_{t+1}}, \quad E_t \left[ Q_{t,t+1}(j) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] = \frac{1}{1 + i_{t+1}} \tag{67}
\]

It follows that the risk-adjusted uncovered interest parity linking domestic and foreign nominal interest rates is:

\[
\frac{1 + i_{t+1}}{1 + i^*_t} = E_t \left( \frac{\mathcal{E}_{t+1}}{P_{t+1} C_{t+1}(j)} \right) \left[ E_t \left( \frac{\mathcal{E}_t}{P_{t+1} C_{t+1}(j)} \right) \right]^{-1} \tag{68}
\]

Note that, in the absence of uncertainty the previous condition collapses to the familiar expression \(1 + i_{t+1} = (1 + i^*_t) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\).

Using (67) we can write:

\[
M_t(j) + B_{H,t+1}(j) = \frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \{ Q_{t,t+1}(j) [M_t(j) + (1 + i_{t+1}) B_{H,t+1}(j)] \} \tag{69}
\]

and:

\[
\mathcal{E}_t B_{F,t+1}(j) = E_t \{ Q_{t,t+1}(j)(1 + i^*_t) \mathcal{E}_{t+1} B_{F,t+1}(j) \} \tag{70}
\]

It follows that the flow budget constraint (9) is:

\[
\frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \{ Q_{t,t+1}(j) A_{t+1}(j) \} \leq A_t(j) + R_t(j) - T_t(j) - P_t C_t(j) \tag{71}
\]
where $A_{t+1}$ is wealth (net assets) at the beginning of period $t + 1$, defined as:

$$A_{t+1}(j) \equiv M_t(j) + (1 + i_{t+1}) B_{H,t+1}(j) + (1 + i^*_t) E_{t+1} B_{F,t+1}(j).$$ (72)

Optimization implies that households exhaust their intertemporal budget constraint: the flow budget constraint hold as equality and the transversality condition below is satisfied:

$$\lim_{N \to \infty} E_t [Q_{t,N}(j) A_N(j)] = 0$$ (73)

where $Q_{t,N} \equiv \prod_{s=t+1}^N Q_{s-1,s}$. If an interior solution exists (as is the case given our parameterization), the resource constraint holds as equality as well. Similar results characterize the optimization problem of Foreign agent $j^*$.

**Wage setting**

Consider now the problem of choosing an optimal nominal wage rate one period in advance. Let $W(j)$ denote the nominal wage of worker $j$, and define the Home country wage index $W$ as

$$W_t = \left[ \int_0^1 W_t(j)^{1-\phi} dj \right]^{\frac{1}{1-\phi}}$$ (74)

The (constant-elasticity) demand for labor input $j$ by the firm $n$ can be expressed as follows

$$L_t(n, j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} \frac{Y_t(n)}{Z_{N,t}}$$ (75)

Using the fact that $\phi$ is the same across sectors, the total demand for the labor input supplied by $j$ to all domestic firms is

$$L_t(i, j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} \left[ \int_{h=0}^1 \frac{Y_t(h)}{Z_{n,t}} dh + \int_{n=0}^1 \frac{Y_t(n)}{Z_{N,t}} dn \right]$$ (76)

Workers are assumed to be monopolistic suppliers of a particular type of labor; thus, they take into account the above demand schedule when fixing the nominal wage rate. We posit that the number of workers is large enough so that they ignore the impact of their own pricing decision on the aggregate wage index. As in Obstfeld and Rogoff [2000a], the first order condition for
this problem is the expression

$$ W_t(j) = \frac{\phi}{\phi - 1} \frac{E_{t-1} (\kappa_t \ell_t(j))}{E_{t-1} \left( \frac{\ell_t(j)}{P_t C_t(j)} \right)} \quad (77) $$

With a competitive labor market, the nominal wage rate equates the disutility of labor to the marginal utility of consumption of an additional unit of nominal revenue. Because of workers’ monopoly power, the wage rate is set with a markup $\phi / (\phi - 1)$ over the expected utility cost of labor effort, expressed in units of domestic currency. Having set the wage rate optimally, workers stand ready to provide any amount of labor to firms at the going rate, as long as the real wage is above the marginal disutility of labor. We restrict the size of shocks in such a way that this will always be the case.

**The current account**

We focus on an equilibrium in which domestic agents are symmetric within a country (although there could be asymmetries across countries). Aggregating the individual budget constraints and using the government budget constraint we obtain an expression for the Home current account:

$$ E_t \{ Q_{t+1} \overline{A}_{t+1} \} = \overline{A}_t + R_t - P_t C_t \quad (78) $$

where $\overline{A}$ is defined as wealth net of money balances, or

$$ \overline{A}_{t+1} \equiv A_{t+1} - M_t. \quad (79) $$

Now, $R_t$ is defined as:

$$ R_t \equiv \left( P_{H,t} + \eta P_{N,t} \right) C_{H,t} + P_{N,t} C_{N,t} + \mathcal{E}_t \bar{P}^*_{H,t} C^*_{H,t} + \eta P_{N,t} C_{F,t} $$

$$ = P_t C_t - P_{F,t} C_{F,t} + \mathcal{E}_t \bar{P}^*_{H,t} C^*_{H,t} + \eta P_{N,t} \frac{P_{F,t}}{P_{H,t}} C_{F,t} - \eta \mathcal{E}_t \bar{P}^*_{N,t} \frac{P^*_{F,t}}{P^*_{H,t}} C^*_{H,t} $$

$$ = P_t C_t - \frac{\gamma}{2} P_t C_t + \frac{\gamma}{2} \mathcal{E}_t \bar{P}^*_{H,t} C^*_{H,t} + \eta \frac{\gamma}{2} \frac{P_{N,t}}{P_{H,t}} P_t C_t - \eta \frac{\gamma}{2} \frac{P^*_{N,t}}{P^*_{H,t}} \mathcal{E}_t \bar{P}^*_{H,t} C^*_{H,t} $$

$$ = P_t C_t \left( 1 - \frac{\gamma}{2} + \frac{\gamma}{2} \frac{P_{N,t}}{P_{H,t}} \right) + \frac{\gamma}{2} \mathcal{E}_t \bar{P}^*_{H,t} C^*_{H,t} \left( 1 - \eta \frac{P^*_{N,t}}{P^*_{H,t}} \right) $$
Thus the Home current account becomes

\[ E_t \{ Q_{t+1} \hat{A}_{t+1} \} = \hat{A}_t - \frac{\gamma}{2} \mu_t \left( 1 - \eta \frac{P_{N,t}}{P_{F,t}} \right) + \frac{\gamma}{2} \epsilon_t \mu_t^* \left( 1 - \eta \frac{P_{N,t}}{P_{H,t}} \right) \]

Clearly, in equilibrium \( \hat{A}_t = -\epsilon_t \hat{A}_t^* \), for all \( t \).
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