BANCA D'ITALIA

Temi di discussione

del Servizio Studi

Expectations and information in second generation currency crises models

by M. Sbracia and A. Zaghini

Number 391 - December 2000

EXPECTATIONS AND INFORMATION IN SECOND GENERATION CURRENCY CRISES MODELS

by Massimo Sbracia* and Andrea Zaghini*

Abstract

We explore the role of expectations in second generation currency crisis models, proving that sudden shifts in speculators' beliefs can trigger currency devaluations, even without any sizable worsening in the fundamentals. In our incomplete information game, mean-preserving changes in speculators' expectations may drive agents to a unique equilibrium with a self-fulfilling attack. In particular, our model supports the thesis that *uncertainty matters*, since a sufficiently large increase in speculators' uncertainty over the fundamentals is likely to trigger a currency crisis. Following a recent line of research, we also compare the results of private and public information models and find the following paradox: if speculators have private information, the fact that the state of fundamentals is publicly revealed turns out to be more advantageous to the government when fundamentals are bad.

JEL classification: F31, F33, D82.

Keywords: currency crises, speculative attack, multiple equilibria.

Contents

1. Introduction ... 7
2. The complete information model ... 10
3. Incomplete information: the role of agents' expectations ... 12
4. Public and private information ... 18
5. Conclusions ... 25
Appendix: Proof of Theorem 2 ... 27
References ... 31

---

*Banca d'Italia, Research Department.

1. Introduction[1]

The financial turmoil that hit East Asian countries in the summer of 1997 has revealed the limits of theoretical models in explaining actual episodes of currency crises. According to many accounts, the event that is considered the most likely cause of crisis (a definite worsening in the fundamentals, possibly implied by an unsustainable economic policy) did not occur, at least in some of the Asian economies struck by speculative attacks.[2] Thus, many economists believe that other factors might have a crucial role in determining the dynamics of crisis.

In first generation currency crisis models (FGMs), originally developed by Krugman (1979) and Flood and Garber (1984), financial crises follow a deterioration of the fundamentals, typically due to inconsistent economic policies. By contrast, in second generation models (SGMs), first developed by Obstfeld (1994), attention turned to the costs and benefits of the fixed exchange rate policy, stressing the importance of the trade-off faced by the government between defending a fixed currency peg and other policy targets.[3] In these models, devaluation is a government's optimal response to the actions of speculators and it can take place as a result of self-fulfilling beliefs, without a previous worsening in the fundamentals. Since speculative attacks raise the cost of defending a fixed exchange rate, SGMs may exhibit self-fulfilling multiple equilibria.[4] In SGMs the space of fundamentals is usually divided into three regions: when fundamentals are "sound", there is a unique equilibrium in which the exchange rate is maintained ("good" equilibrium); when fundamentals are *fragile*, the currency depreciates; when fundamentals fall in an "intermediate" range (the "ripe for attack" zone), both outcomes are feasible.

---

[1] We are very grateful to Matteo Bugamelli, Robert Flood, Olivier Jeanne, Francesco Lippi, Alessandro Prati, Nicolas Sahuguet and Fabiano Schivardi for useful comments and helpful discussions. We also thank an anonymous referee and seminar participants at the Bank of Italy, at Georgetown University (1999 Southeast International Economics Conference), at the University of Rome "Tor Vergata" (1999 Financial Conference), at the University of Bonn (Center for European Integration Studies – ZEI) and at the VIII World Congress of the Econometric Society for insightful suggestions. Massimo Sbracia acknowledges the IMF for the kind hospitality in November 1999, when the paper was under revision. This paper does not necessarily reflect the views of the Bank of Italy or of the IMF. E-mail: m.sbracia@flashnet.it; a.zaghini@poboxes.com

[2] Corsetti et al. (1998) and Radelet and Sachs (1998) express different views of the causes of the Asian crisis.

[3] Economic policy targets that might conflict with the defense of a fixed currency peg include: achieving a low level of unemployment, stimulating economic growth, reducing the fiscal burden, and supporting a sound banking system. For an overview, see Obstfeld (1996).

[4] For a discussion on the self-fulfilling feature of currency crises, see Obstfeld (1986 and 1994), Krugman (1996) and the commentaries therein.

In a recent paper, Morris and Shin (1998) started a promising line of analysis by developing an SGM with incomplete information. They considered speculators having a uniform prior probability distribution over the state of fundamentals that is updated according to the observation of a private signal. Their model, as well as the earlier complete information models, does not allow one to examine the role of the distribution of agents' beliefs about the fundamentals. This issue has been neglected in the literature, presumably because one could think that only the mean of speculators' probability assessment of the fundamentals matters in an incomplete information framework. Hence, one could imagine that the equilibrium (or the equilibria) of the game depends on the region where the mean of the probability distribution falls. In such hypothetical case, the incomplete information framework would not enrich the benchmark analysis and would not modify the structure of the equilibria. Nevertheless, agents' beliefs have often been used to explain actual currency crises. For instance, after the Russian crisis many commentators pointed to an increase in agents' uncertainty as a possible explanation for the transmission of speculative pressures to other countries (especially in Latin America) that had very limited trade linkages with Russia [see BIS (1999) and IMF (1999)]. Yet, typical SGMs do not explain why uncertainty should influence speculative attacks.

In this paper we present two different variants of the incomplete information model studied by Morris and Shin (1998). Our first SGM allows us to study the role played by agents' beliefs on the fundamentals. By explicitly introducing these beliefs, we find that the mean of the distribution of the fundamentals is not the sole relevant parameter. In particular, we show that mean-preserving changes in speculators' expectations can result in a shift from a model with multiple equilibria to a model with a unique "attack" equilibrium. Our model differs from Morris and Shin's because it takes into account a generic prior probability distribution and studies how equilibria change together with it. To focus on the effects of the uncertainty of agents' beliefs, we simplify the model by neglecting private information.

Our study supports the thesis that *uncertainty matters*, since an increase in speculators' uncertainty over the fundamentals is likely to trigger a currency crisis. It also offers some interesting insights on the multiple equilibria zone of the complete information game. We show that when the mean of the distribution of agents' beliefs falls in this region and uncertainty is sufficiently large, the incomplete information model has a unique equilibrium that entails a speculative attack. In other words, when the mean is in the “ripe for attack” zone, the “good” equilibrium *is not robust* to an increase in uncertainty.

In the second SGM presented in this paper, we analyze how public information affects the structure of the equilibria. Along the lines of the global games study of Carlsson and van Damme (1993), Morris and Shin (1998) proved that removing the hypothesis of common knowledge, which is implicit in their private information framework, determines a unique equilibrium. Here, we develop a model in which agents observe the same public signal (so that “some” common knowledge is restored) and we find that multiple equilibria no longer disappear.

Interestingly, by comparing the results of private and public information models, the following paradox emerges: if speculators have private information, the fact that the state of fundamentals is publicly revealed turns out to be more advantageous to the government when fundamentals are bad! Examining the reasons behind this paradox provides some useful insights on the likelihood of the events upon which equilibria rest. When the private information model predicts an equilibrium with a currency attack, the public information model’s equilibrium with no attack envisages an implausible (but still feasible) situation in which speculators forego a big payoff that could have been obtained with a small coordination effort. Analogously, when the private information model predicts an equilibrium with no attack, the public information model’s equilibrium with a currency attack foresees speculators getting a small payoff and requires a very large coordination effort. Hence, the comparison highlights that equilibria discarded by the private information model but that re-emerge in the public information framework seem hardly plausible. Therefore, the paradox warns that policy conclusions drawn from models with multiple equilibria can be misleading, especially when considerations on the likelihood of the outcomes are neglected.

The paper is organized as follows. The next section briefly reviews the benchmark model with complete information. In Section 3, we analyze a basic incomplete information framework where speculators’ expectations are represented by a generic prior probability distribution and we study the consequences of changing its variance. In Section 4, we present the model with public information, and we compare it with the private information model. Section 5 concludes.

2. The complete information model

In this section, we present the simple game-theoretic formulation of an SGM of complete information proposed by Morris and Shin (1998). This model provides the standard framework on which we further develop the analysis of currency crises in the following sections.

Let us suppose that the economy is characterized by a state of fundamentals θ that can take values over the set $[0,1]$, with $\theta = 1$ corresponding to a situation of "strong fundamentals". In the absence of government interventions, the "natural" (or shadow) exchange rate is given by $f(\theta)$, where f is a continuous and strictly increasing function.[5] The actual exchange rate is pegged by the government at a level e^*, with $e^* \geq f(\theta)$ for all θ. A speculator can either *attack* the currency (e.g., by short-selling one unit of it over the exchange rate market) or *not attack*. If she attacks the currency and the government abandons the peg, the speculator obtains the difference between the peg and the "natural" exchange rate, free of the transaction cost: $e^* - f(\theta) - t$; on the other hand, if the government successfully defends the peg, the speculator ends up with a negative payoff (the transaction cost t). If the speculator refrains from attacking, she gets 0. We assume that $e^* - f(1) < t$; namely, in the best state of fundamentals the "natural" exchange rate is sufficiently close to the peg e^* so that the profit resulting from a depreciation is outweighed by the transaction cost t.

The government derives a value $v > 0$ by defending the peg, but he also faces management costs. In particular, the cost of defending the peg is a function of the state of fundamentals and of the share of speculators who attack the currency. We denote this cost function by c. Hence, if the government defends the peg when a proportion α of speculators attacks the currency and the fundamentals are at the level θ, his payoff is $v - c(\alpha, \theta)$; if he abandons the peg, we assume that his payoff is zero. The cost function is supposed to be continuous, differentiable, increasing in α and decreasing in θ. Moreover, two hypotheses are introduced to make the problem economically interesting: $c(0,0) > v$ and $c(1,1) > v$. The first inequality states that in the worst state of fundamentals, the cost of defending the peg exceeds the benefit from maintaining it, even when no speculator attacks;[6] the second states

---

[5] The exchange rate is defined in terms of units of foreign currency per unit of national currency.

[6] Recall that the main insight of SGMs concerns the trade-off faced by government between defending a fixed currency peg and other policy targets. It follows that the government may benefit from abandoning the peg even if nobody attacks, for a given range of fundamentals (see also Footnote 3).

that when all speculators sell the currency, the cost of defending the peg exceeds the value v, even in the best state of fundamentals.

Let us denote with $\underline{\theta}$ the value of the fundamentals that solves $c(0,\underline{\theta}) = v$; i.e., $\underline{\theta}$ is the value of θ at which, in the absence of any speculative selling, the government is indifferent between defending the peg and abandoning it. When $\theta < \underline{\theta}$, the government finds it profitable to abandon the peg even if no speculator attacks the currency. Similarly, we denote by $\bar{\theta}$ the value that solves $e^* - f(\bar{\theta}) - t = 0$. Whenever $\theta > \bar{\theta}$, speculators could force the government to abandon the peg, but they would get a negative payoff from successfully attacking the currency.

Let us assume $\underline{\theta} \leq \bar{\theta}$.[7] The government, who knows the state of fundamentals θ, makes his decision about the peg after speculators' have made their choices. Hence, he will use a decision rule $\psi(\alpha, \theta)$ such that:[8]

$$\psi(\alpha,\theta) = \begin{cases} \textit{abandon}, \text{ if } v - c(\alpha,\theta) \leq 0 \\ \textit{defend}, \text{ otherwise} \end{cases} .$$

In particular, $\psi(1,\theta)$ =*abandon* for any θ in $[0,1]$ and $\psi(0,\theta)$ =*defend* for any θ in $(\underline{\theta},1]$. Given ψ, the solution of the reduced-form game of speculators allows us to classify the soundness of the fixed exchange rate by referring to the states of fundamentals:

• In the interval $[0,\underline{\theta})$ fundamentals are such that the government's cost of defending the peg exceeds the value v, even if no speculator attacks. In the reduced-form game, therefore, the action *attack* is strictly dominant for any agent and the peg is "unstable" since the currency will depreciate.

• In the interval $[\underline{\theta},\bar{\theta}]$, if all speculators attack the currency, the government's cost of defending the peg exceeds the value v and the currency depreciates; on the other hand, if no speculator attacks the currency, the government finds it profitable to hold the peg, and no devaluation occurs. This interval is usually referred to as the "ripe for attack" zone, to underline the possibility of a positive gain from short-selling.

• In the interval $(\bar{\theta},1]$ the "natural" exchange rate $f(\theta)$ is sufficiently close to the peg e^*, so that any profit resulting from a depreciation of the currency is offset by the transaction

---

[7] This assumption holds if v is "large" and t is "small".

[8] We assume that when the government is indifferent, he chooses to abandon the peg.

cost. Thus, in the reduced-form game, the action *do not attack* is strictly dominant, the peg is "stable" and no devaluation occurs.

The above tripartition of the state of fundamentals shows that there are two zones in which a unique equilibrium exists (attack/devalue in the first interval, and refrain from attacking/defend the peg in the third one), and one zone in which the game between speculators and the government entails multiple equilibria. In the last case, speculators' expectations are self-fulfilling. In fact, consider the interval $[\underline{\theta}, \bar{\theta}]$. The government observes the choices of his opponents and then decides whether to defend the peg. If speculators believe that the currency will depreciate, their best action is to attack the peg. Although the government might defend the peg successfully under a "limited" attack, when all agents sell the currency, the government is forced to devalue, thereby confirming agents' beliefs. On the other hand, if speculators "feel" that the peg is going to be maintained, their best action would be to refrain from attacking. This, in turn, allows the government to defend the peg successfully, confirming again speculators' beliefs.

The most important drawback of the existence of multiple equilibria is that they do not allow for precise predictions of the game's outcome. An interval of the fundamentals exists for which an attack is possible, but one cannot say whether or when the attack will happen. Note that the two equilibria in the "ripe for attack" zone (except for the unique value $\bar{\theta}$) are asymmetric: speculators can obtain a positive payoff by attacking the currency; refraining from attacking results in a null payoff.

The next two sections provide some extensions to this standard model by analyzing a framework in which speculators are uncertain about the true θ.

3. Incomplete information: the role of agents' expectations

In this section we assume that speculators do not know the true state of fundamentals and only have expectations in the form of a probability distribution over [0,1]. We also assume that this distribution is common knowledge and that it is absolutely continuous with "full support" over [0,1], and we denote its probability density function (p.d.f.) by η. Such modification of the original framework allows for a better understanding of the role of expectations in second generation currency crisis models.

Since θ is not known when agents choose their actions, there cannot be multiple equilibria in the sense of a complete information model (i.e. for a given value of θ). Hence, we can ask whether there are multiple equilibria for a given p.d.f. η over [0,1]. It is a simple task to verify that multiple equilibria are still possible;[9] nevertheless, by studying the conditions that must be fulfilled by η in order to have a model with multiple equilibria, we can gain some important insights into the effects of speculators' expectations.

The government's optimal strategy is the same function ψ as in the complete information game. Given ψ, we can determine the expected payoff of a generic speculator. It is enough to verify that:

• if speculator i refrains from attacking, she obtains a 0 payoff, no matter what the other speculators do;

• if speculator i attacks while all the other agents attack, her expected payoff is:

(1) $$\int_0^1 (e^* - f(\theta) - t)\eta(\theta)d\theta = e^* - E\left[f\left(\widetilde{\Theta}\right)\right] - t,$$

since, for any level of θ, an attack by all speculators forces the government to abandon the peg;[10]

• if a speculator i attacks while all the other agents refrain from attacking, her expected payoff is:

(2) $$\int_0^{\underline{\theta}} (e^* - f(\theta) - t)\eta(\theta)d\theta - \int_{\underline{\theta}}^1 t\eta(\theta)d\theta,$$

since the government abandons the peg in the interval $[0, \underline{\theta}]$ and maintains it in the interval $(\underline{\theta}, 1]$.

Let us denote integral (1) by $u(a_i, a_{-i})$ and expression (2) by $u(a_i, n_{-i})$. The strategy profile in which all agents attack the currency is an equilibrium *iff* $u(a_i, a_{-i}) \geq 0$; the strategy profile in which all agents refrain from attacking is an equilibrium *iff* $u(a_i, n_{-i}) \leq 0$. Let p

---

9 For instance, if η has support only over the subset $\left[\underline{\theta}, \overline{\theta}\right]$, agents know with certainty that θ is in the interval with multiple equilibria and they can coordinate their actions either on the "good" or on the "bad" equilibrium.

10 We have denoted by $\tilde{\Theta}$ the random variable which represents the distribution of the state of fundamentals and takes values over $[0, 1]$ with p.d.f. η.

be the probability that the state of fundamentals is not greater than $\underline{\theta}$. We can refer to p as the probability of an "unforced" currency depreciation because the government devalues even if $\alpha = 0$. The necessary and sufficient conditions for the *attack* equilibrium and for the *do not attack* equilibrium become, respectively:

(3) $$e^* - E\left[f\left(\widetilde{\Theta}\right)\right] - t \geq 0 \text{ and}$$

(4) $$e^* p - E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right] p - t \leq 0.$$

It is easy to show that, since $u(a_i, a_{-i}) \geq u(a_i, n_{-i})$,[11] speculators' choices of attacking the currency are strategic complements. Hence, since both $u(a_i, a_{-i}) \geq 0$ and $u(a_i, n_{-i}) \leq 0$ may hold for the same η three situations can be identified:

• if $u(a_i, n_{-i}) > 0$ (that implies $u(a_i, a_{-i}) > 0$), there is only one equilibrium: all speculators attack the currency and the government abandons the peg;

• if $u(a_i, a_{-i}) < 0$ (that implies $u(a_i, n_{-i}) < 0$), there is only one equilibrium: all speculators refrain from attacking the currency and the government either abandons or maintains the peg, depending on $\theta \leq \underline{\theta}$ or $\theta > \underline{\theta}$;

• if $u(a_i, a_{-i}) \geq 0$ and $u(a_i, n_{-i}) \leq 0$ there are multiple equilibria: agents may either attack the currency (and force a devaluation) or refrain from attacking (so that the peg is maintained, provided that $\theta > \underline{\theta}$).

We focus on the situation with multiple equilibria. After some simple algebra, it can be shown that there are multiple equilibria *iff*:

$$e^* \in \left[E\left[f\left(\widetilde{\Theta}\right)\right] + t, E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right] + t/p\right].$$

Let us denote the above interval by $E \equiv [e_1, e_2]$. We can verify that the condition $e^* \in E$ is a reasonable requirement for multiple equilibria. When the level of the peg e^* is "high" (i.e. $e^* > e_2$), speculators expect a large payoff from a successful attack; hence, the strategy profile where agents refrain from attacking is discarded and the unique equilibrium of

---

[11] Recall that, by hypothesis, $e^* \geq f(\theta)$ for all θ. Therefore, the condition $e^* \geq E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} > \underline{\theta}\right]$ is always satisfied.

the model entails massive speculative pressures. On the other hand, if e^* is "low" ($e^* < e_1$), speculators expect a negative payoff from a successful attack and the unique equilibrium of the model predicts that speculators do not attack the currency. For "intermediate" levels of e^*, both outcomes are equilibria of the game.

We can obtain a simple necessary condition for multiple equilibria, by requiring that E is not empty. One can show that $E \neq \emptyset$, *iff*:

(5)
$$p \leq \frac{t}{t + E\left[f\left(\widetilde{\Theta}\right)\right] - E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right]} \equiv s.$$

where, clearly, $s \in (0, 1)$.

Condition (5) can be used, together with conditions (3) and (4), to highlight the perturbations of the probability distribution over the fundamentals that remove the multiplicity of the equilibria. By looking at conditions (3) and (4), one realizes that changes in agents' expectations that alter the equilibrium structure involve changes in at least one of the following parameters: $E\left[f\left(\widetilde{\Theta}\right)\right]$, $E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right]$ or p. In general, when η changes, all these values may also change. However, to simplify the discussion and to better understand of the role of each of them, we consider perturbations of the p.d.f. that involve modifications of one parameter only.

Denote $E\left[f\left(\widetilde{\Theta}\right)\right] = \overline{e}$, $E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right] = \overline{e}_{\underline{\theta}}$ and recall the definitions of p and s; analogously, denote the corresponding parameters computed with a different p.d.f. η' by $\overline{e}'$, $\overline{e}'_{\underline{\theta}}$, p' and s'. Let us assume that an initial p.d.f. η entails multiple equilibria. Consider, first, an increase of $\overline{e}$. Specifically, suppose that speculators' beliefs change from η to a new p.d.f. η' on $[0, 1]$, such that $p' = p$, $\overline{e}'_{\underline{\theta}} = \overline{e}_{\underline{\theta}}$, and the expected exchange rate increases to a level $\overline{e}' > \overline{e}$ which makes $p' > s'$. Under the new beliefs η' there cannot be multiple equilibria and, checking that condition (3) does not hold, we find that the "good" equilibrium where agents do not attack is the game's unique equilibrium. This is not surprising since an increase in $\overline{e}$ corresponds to an improvement in the fundamentals of the economy and implies that the peg is believed to be closer to its "natural" level than under η. If the increase in $\overline{e}$ is sufficiently large, the expected gain to speculators from a successful attack (given by $e^* - \overline{e} - t$) is negative; the "bad" equilibrium is therefore discarded.

Second, let us consider a different perturbation of η, in which the new p.d.f. η' on $[0,1]$ is such that $p' = p$, $\overline{e}' = \overline{e}$, and the expected exchange rate conditioned on $\widetilde{\Theta} \leq \underline{\theta}$ decreases to a level $\overline{e}'_{\underline{\theta}} < \overline{e}_{\underline{\theta}}$, so that $p > s'$. Since condition (5) does not hold, there cannot be multiple equilibria. Checking that condition (4) is no longer fulfilled, we see that the game's unique equilibrium is the strategy profile in which all agents attack the currency. Although under the new beliefs η' the expected overvaluation of the peg with respect to the "natural" exchange rate does not change, the decrease in $\overline{e}_{\underline{\theta}}$ creates a better situation for agents: if the state of fundamentals is in the attack zone, speculators' payoffs from selling the currency increase. In particular, if $\overline{e}'_{\underline{\theta}}$ is sufficiently less than $\overline{e}_{\underline{\theta}}$, the utility from attacking when all others refrain, $u(a_i, n_{-i})$, becomes positive. Hence, with η' any agent expects a positive gain by selling the currency, even when she is the only speculator doing so. It follows that any agent has an incentive to attack, and the "bad" equilibrium becomes the game's unique equilibrium.

Lastly, we can consider a new p.d.f. η' on $[0,1]$ such that $\overline{e}' = \overline{e}$, $\overline{e}'_{\underline{\theta}} = \overline{e}_{\underline{\theta}}$, and the probability that the state of fundamentals is in the attack zone increases to $p' > s > p$. Similar to the previous case, under the new beliefs η' the game has a unique equilibrium in which all agents sell the currency, although the expected appreciation of the exchange rate has not changed. The increase in p, meaning that some probability has shifted towards the left tail of the distribution, makes $u(a_i, n_{-i}) \geq 0$. Hence, under η' each agent believes that she can achieve a positive payoff even if she is alone in attacking the currency; the "bad" equilibrium becomes the game's unique equilibrium.

The previous discussion shows that changes in speculators' expectations that do not imply a worsening in the perceived overvaluation of the peg may be sufficient to induce a speculative attack. In particular, a modification of speculators' beliefs such that the mean is preserved but the dispersion of the distribution increases may imply a switch from a multiple equilibria game to one with a unique "bad" equilibrium. In fact, an increase, say, in the variance of the distribution may imply a corresponding increase in p (which is the probability of the left tail of the distribution) and, by the mechanism highlighted above, it may trigger a speculative attack. This feature of the model can be used to shed some light on the aftermath of the Russian crisis, when many commentators pointed to an increase in agents' uncertainty as a likely explanation for the transmission of strong speculative pressures to several emerging market economies. In order to further highlight increased uncertainty as a possible cause of currency crises, we present a simple discrete example.

Assume that fundamentals can only take three values: $\theta \in \{\theta_1, \theta_2, \theta_3\}$ with $\theta_1 < \theta_2 < \theta_3$. Suppose that the parameters are such that: when $\theta = \theta_1$ the government devalues the currency even if no speculator attacks; when $\theta = \theta_2$ there are multiple equilibria; when $\theta = \theta_3$ no speculator attacks and the peg is maintained. Specifically, we need to assume:

$$\begin{aligned} e^* - f(\theta_3) - t &< 0, \\ e^* - f(\theta_2) - t &> 0, \\ v - c(0, \theta_1) &< 0, \\ v - c(0, \theta_2) &> 0. \end{aligned}$$

Let us keep the symbol η to denote the probability distribution over the state of fundamentals and denote (consistent with the previous notation) $p = Prob(\widetilde{\Theta} = \theta_1)$ and $q = Prob(\widetilde{\Theta} = \theta_3)$, with $p, q > 0$ and $p + q < 1$. The necessary and sufficient conditions to achieve an equilibrium with a speculative attack and an equilibrium without a speculative attack are conditions (3) and (4), respectively. The necessary condition for multiple equilibria (5) in this example takes the following form:

$$p \leq \frac{t}{t + E\left[f\left(\widetilde{\Theta}\right)\right] - f\left(\theta_1\right)} = s. \tag{6}$$

Consider an economy characterized by parameter levels such that the inequalities (3), (4) and (6) hold and where agents coordinate their choices so that they refrain from attacking ("good" equilibrium). If speculators revise their expectations to a p.d.f. η' with a higher probability of an "unforced" devaluation of the currency (so that (4) and (6) no longer hold), they trigger the attack that forces the government to abandon the peg. We have already shown that the updated p.d.f. η' can preserve the same mean as η. Indeed, in this simple example there are many probability density functions that preserve the same mean as η and have a higher p; all of them are characterized by an increase in the *variance* of the speculators' expectations. In fact, in order to maintain a constant mean, p and q must both increase. Hence, in this example the currency attack is triggered by an increase in the uncertainty over the state of fundamentals.

Finally, let us come back to the general case and suppose that f is an affine transformation. In Section 2, we determined a tripartition of the space of fundamentals relative to the complete information model. Now we can check whether a similar partition for the parameter space of the mean of the distribution over the fundamentals exists. It is easy to verify that the condition $E(\widetilde{\Theta}) \leq \overline{\theta}$ is necessary and sufficient for an equilibrium with a speculative attack, for any p.d.f. representing agents' beliefs. However, under a generic p.d.f., one cannot find a necessary and sufficient condition for the "good" equilibrium that depends only on the mean of the distribution.[12] Instead, one can find a sufficient condition: $E(\widetilde{\Theta}) > \overline{\theta}$ implies that the "good" strategy profile is the unique equilibrium of the game. However, "good" equilibria can also hold when $E(\widetilde{\Theta}) \leq \overline{\theta}$. From these results, a particular kind of source of instability follows. Suppose that $E(\widetilde{\Theta})$ is in the "ripe for attack zone" of the corresponding complete information game (i.e., $\underline{\theta} \leq E(\widetilde{\Theta}) \leq \overline{\theta}$). If p is sufficiently large or if $\overline{e}_{\underline{\theta}}$ is sufficiently small, the strategy profile with no attack is not an equilibrium; the only feasible outcome is the equilibrium with a devaluation of the currency. Therefore, even when the mean of the distribution over the fundamentals falls in a region that would yield multiple equilibria within a complete information model (and, thus, where agents could refrain from attacking the currency and allow the peg to be maintained), in the incomplete information case there might be a unique equilibrium with a speculative attack, depending on the degree of dispersion of the probability distribution representing speculators' beliefs.[13] In other words, when $E(\widetilde{\Theta}) \leq \overline{\theta}$ "good" equilibria tend to be not robust to an increase in uncertainty.

4. Public and private information

Morris and Shin (1998) have developed a different incomplete information version of the simple model outlined in Section 2. In a framework where agents do not know the true state of fundamentals and only have imperfect private information, Morris and Shin show that a unique equilibrium exists for each state of fundamentals. Their model highlights the importance of the removal of the *common knowledge* hypothesis to obtain the result. In fact, along the lines of Carlsson and van Damme (1993), the uniqueness of the equilibrium is not the consequence of

---

[12] In fact, condition (4) shows that the "good" equilibrium depends on parameters of the p.d.f. other than the mean. With special probability distributions, however, one can find conditions for the "good" equilibrium that depend only the mean.

[13] Note that for many classes of p.d.f. — when $\overline{e}$ is constant — both an increase in p and a decrease in $\overline{e}_{\underline{\theta}}$ are associated with an increase in the variance of the distribution.

uncertainty on the state of fundamentals *per se*; rather, it follows from each agent's uncertainty on the other players' actions that occurs in equilibrium, due to the impossibility of precisely establishing the information received by them. In other words, the uniqueness of the equilibria is not produced by removing the hypothesis of common knowledge of the fundamentals. Instead, the result is a consequence of the removal of the hypothesis of common knowledge of each player's action that takes place in equilibrium when players have private information.

In this section we compare Morris and Shin's results with our results achieved with a public information model; i.e., with a model in which each player has access to the same imperfect public signal. We show that in any equilibrium of this model the common knowledge of agents' actions is restored, although the lack of common knowledge of fundamentals is preserved. Consistent with Morris and Shin, we find that the public information model has multiple equilibria and that the borders of the multiple equilibria region are "close" to those of the "ripe for attack" zone of the complete information game.

Let us briefly recall the private information setup. With respect to the model presented in the previous section, Morris and Shin consider a specific prior probability distribution η, given by the uniform p.d.f. over $[0,1]$. They also assume that each agent i observes a signal x_i uniformly and independently (conditional to θ) drawn from the interval $[\theta-\epsilon,\theta+\epsilon]$, where ϵ is a small positive number.[14] The game is structured as follows: (a) Nature chooses the state of fundamentals θ according to a uniform p.d.f. over $[0,1]$; (b) speculators observe their signal x_i and decide simultaneously whether to attack the currency or to refrain from attacking; (c) the government, who knows the true state θ, observes the share of speculators attacking the currency and then decides whether to defend the peg or to abandon it, allowing a devaluation of the currency to its "natural" level $f(\theta)$.

Within this framework, it is possible to prove the following theorem:

Theorem 1 (Morris and Shin, 1998): There is a unique level θ^* of the fundamentals such that, in any equilibrium of the game with imperfect private

[14] Heinemann and Illing (1999) provide a generalization of Morris and Shin's setup: their model shows that the uniqueness of the equilibrium also holds with more general probability distributions.

information, the government abandons the currency peg *iff* $\theta \leq \theta^*$.

In order to understand why multiple equilibria are ruled out, a feature of the private information model has to be considered: it is never *common knowledge* that fundamentals are in a particular region. This can be explained by considering different *orders of knowledge*. Consider the "ripe for attack' zone $[\underline{\theta}, \bar{\theta}]$. For small values of ϵ and for some $\theta_0 \in \left(\underline{\theta}, \bar{\theta}\right)$, each player may receive a private signal that makes her sure that θ_0 is within the "ripe for attack" zone. Given θ_0, messages are distributed over $[\theta_0 - \epsilon, \theta_0 + \epsilon]$. Suppose that player 1 receives the message $x_1 = \theta_0 + \epsilon$. Such agent believes that $\theta \in [x_1 - \epsilon, x_1 + \epsilon]$. If this interval is inside $[\underline{\theta}, \bar{\theta}]$, agent 1 realizes that the state of fundamentals is in the "ripe for attack" zone. However, she cannot conclude that her opponents also know that θ is in that region. In fact, agent 1 knows that if $\theta = x_1 + \epsilon$ there may be an agent, say agent 2, who receives a message $x_2 = x_1 + 2\epsilon$. In this case, agent 1 knows that agent 2 believes that $\theta \in [x_2 - \epsilon, x_2 + \epsilon]$. In particular, agent 1 knows that agent 2 thinks that if $\theta = x_2 + \epsilon$, there is an agent, call her agent 3, who receives a message, $x_3 = x_2 + 2\epsilon$ (i.e. $x_3 = x_1 + 4\epsilon$). By iterating this argument, one realizes that it is never common knowledge that θ is in the "ripe for attack" zone.

This argument is relevant to understanding why multiple equilibria are discarded. When $\theta \in [\underline{\theta}, \bar{\theta}]$, computing the optimal action requires that agents know the others' actions. In the complete information framework, agents know that they can coordinate on *attack* or on *do not attack*. Since in equilibrium the actions chosen are common knowledge, both the "good" and the "bad" strategy profiles can be equilibria of the game. By contrast, in the private information model, it is assumed that in equilibrium there is common knowledge of *the strategy profile*, where a strategy for any agent i is a function $\varphi_i : X_i \longrightarrow \{\textit{attack}, \textit{do not attack}\}$, $X_i \equiv [0, 1]$. If there are two regions of X_i where the equilibrium strategy of agent i induces her to select two different actions,[15] there cannot be common knowledge of the action chosen by this agent because, as previously discussed, there is never common knowledge of the region where θ is (and, thus, of the region where x_i is). In computing the potential gains and losses that follow the choice of attacking, for any feasible message x_i the lack of common knowledge forces

[15] In the currency crisis game it is certain that agents act differently in different regions; in $[0, \underline{\theta})$, *attack* strictly dominates *do not attack*, whilst in $\left(\bar{\theta}, 1\right]$ *do not attack* strictly dominates *attack* (Section 2). Such regions are called *dominance solvable*. Morris *et al.* (1995) apply a similar argument to more general games (for which *dominance solvable* regions do not exist) by introducing the concept of *p-dominance*.

speculators to try to assess of the average behavior of the others. Such computations yield a unique optimal action for any message x_i and, therefore, a unique equilibrium.

We now consider a model where any agent receives the same public message $x \in X \equiv [0,1]$, which we assume to be uniformly drawn from the interval $[\theta - \delta, \theta + \delta]$. Such hypothesis maintains some uncertainty over θ but restores the common knowledge of speculators' actions. In fact, since a strategy for any agent i is a function $\varphi_i : X \longrightarrow \{$*attack*$,$ *do not attack*$\}$ and since in equilibrium there is common knowledge of the strategy profile $\{\varphi_i,\ i = 1, 2, ...\}$, common knowledge of x implies common knowledge of the actions $\varphi_i(x)$ chosen.[16] In the public information model, the following theorem holds:

Theorem 2: Let $\delta \leq \left(\bar{\theta} - \underline{\theta}\right)/4$. There exists a subset $M \equiv (m_1, m_2] \subset [0,1]$, with $m_1 \in (\underline{\theta}, \underline{\theta} + 2\delta)$ and $m_2 \in (\overline{\theta} - 2\delta, \overline{\theta} + 2\delta)$, such that if $\theta \in M$, the game with imperfect public information has multiple equilibria.

The proof of the theorem shows that the existence of multiple equilibria is entirely dependent on the assumption of common knowledge of the players' actions. We provide a sketch of the proof here, setting forth the more technical aspects in the Appendix. First of all, we determine the optimal strategy of the government in the last stage of the game. Then, we build two different strategy profiles and we verify that they are both equilibria of the game.

Let $\beta(\theta)$ be the smallest share of speculators that, by attacking, induce the government to abandon the peg. Of course, $\beta(\theta) = 0$ when $\theta \in [0, \underline{\theta}]$ and β is such that $c(\beta, \theta) - v = 0$ when $\theta \in (\underline{\theta}, 1]$. One can easily verify that: (i) $\beta \geq 0$; (ii) β is a continuous function of θ; (iii) $\beta' > 0$. The government chooses according to the following optimal rule ψ:[17]

$$\psi(\alpha, \theta) = \begin{cases} \textit{abandon}, \text{ if } \alpha \geq \beta(\theta) \\ \textit{defend}, \text{ if } \alpha < \beta(\theta) \end{cases} \quad \text{for any } \theta \in [0,1].$$

Given the optimal strategy of the government, we can solve the reduced-form game played by the speculators. Depending on the public message observed, we can identify three cases:

---

[16] In the case of *mixed strategies*, that we neglect, the knowledge of x implies knowledge of the *probability distribution* under which actions are chosen.

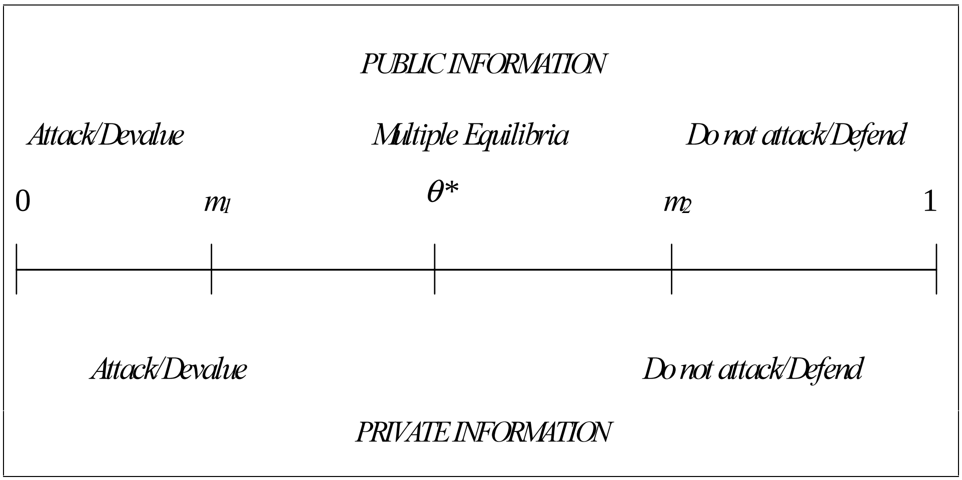
[17] This function coincides with the function ψ used in the previous sections.

- if $x < \underline{\theta} - \delta$, agents are certain that $\theta \in [0, \underline{\theta})$;
- if $x \in \left(\underline{\theta} + \delta, \overline{\theta} - \delta\right)$, agents are certain that $\theta \in (\underline{\theta}, \overline{\theta})$;
- if $x > \overline{\theta} + \delta$, agents are certain that $\theta \in \left(\overline{\theta}, 1\right]$.

We build two different equilibria. In the first equilibrium ($E1$) agents use a strategy φ_{E1} such that $\varphi_{E1}(x) =$ *attack* if $x \leq \underline{\theta} - \delta$, $\varphi_{E1}(x) =$ *do not attack* if $x \geq \underline{\theta} + \delta$, and $\varphi_{E1}(x)$ must be determined for $x \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$. In the second equilibrium ($E2$) agents use a strategy φ_{E2} such that $\varphi_{E2}(x) =$*attack* if $x \leq \overline{\theta} - \delta$, $\varphi_{E2}(x) =$ *do not attack* if $x \geq \overline{\theta} + \delta$ and $\varphi_{E2}(x)$ must be determined for $x \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$.

It is easy to verify that no speculator has an incentive to deviate from φ_{E1} when all the others are following the same strategy, at least for the values of φ_{E1} that we have specified. The same applies to φ_{E2}. The determination of $\varphi_{E1}(x)$ when $x \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$ and of $\varphi_{E2}(x)$ when $x \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$ is rather technical and the proof is presented separately in the Appendix. However, the intuition is straightforward. In the first equilibrium, the utility from attacking when $x = \underline{\theta} - \delta$ is strictly positive; it is strictly negative when $x = \underline{\theta} + \delta$. In the Appendix we show that there is a unique message x_1 such that, when received, the utility from attacking is equal to zero. Analogously, in the second equilibrium, the utility from attacking is strictly positive when $x = \overline{\theta} - \delta$, it is strictly negative when $x = \overline{\theta} + \delta$, and we prove in the Appendix that there is a unique message x_2 such that the utility from attacking is equal to zero. The two signals x_1 and x_2 are the two switching points of the agents' optimal cut-off strategies. From x_1 we obtain the unique level of the fundamentals, m_1, such that in the equilibrium $E1$ the currency depreciates *iff* $\theta \leq m_1$. Similarly, from x_2 we obtain the unique level of the fundamentals, m_2, such that in the equilibrium $E2$ the currency depreciates *iff* $\theta \leq m_2$. In this way we find an interval $M \in (m_1, m_2]$ where the currency may or may not be maintained, depending on whether agents coordinate at $E1$ or $E2$.

We can finally compare the results of the private and the public information model. By an inspection of the graphical argument proposed by Morris and Shin, one realizes that in general $\theta^* \in (\underline{\theta}, \overline{\theta} + 2\epsilon)$. However, it is easy to fix parameter values for which $\theta^* \in interior(M)$. The previous models have shown that if the true state of fundamentals is in (m_1, θ^*) and agents only have private information, a speculative attack is triggered that forces a currency devaluation. Instead, for the same levels of θ, if the true state of fundamentals is publicly revealed, multiple equilibria are restored and there is "hope" that the currency will not depreciate because agents may coordinate on the "good" equilibrium with no attack.[18] Now suppose that θ is in $(\theta^*, m_2]$. If agents only have private information there is no speculative attack; on the other hand, if θ is publicly revealed, a speculative attack may occur (because agents may coordinate on the "bad" equilibrium). Hence, providing public information turns out to be more advantageous to the government when fundamentals are "fairly bad" (i.e. in $(m_1, \theta^*]$) than when fundamentals are "fairly good" (i.e. in $(\theta^*, m_2]$)![19]



This paradox may be explained by a precise comparison of the private and the public information models, which takes into account the "amount of coordination" required to achieve the higher-payoff equilibrium and the "size" of this payoff. This comparison sheds light on the likelihood of the outcomes in the case of multiple equilibria. In the multiple equilibria region, the equilibrium with a coordinated currency attack always yields the highest payoff for speculators. Hence, in order to compare different equilibria for the same state of fundamentals, we can consider the "amount of coordination" required to achieve the highest payoff and the "size" of this payoff.

---

[18] This also holds if a noisy public signal is sent, as long as δ is sufficiently smaller then ε. In particular, it is easy to check that it holds if $\delta = 0$.

[19] In general, we can also say that providing public information seems to be more advantageous when fundamentals are "bad" (i.e. when $\theta < \theta^*$) than when fundamentals are "good" (i.e. when $\theta > \theta^*$). In fact, if $\theta^* \leq m_1$ ($\theta^* > m_2$), providing public information does not make any difference because the peg is abandoned (maintained) in any case. Note that the paradox exists as long as $\theta^* \in interior(M)$. In fact, if $\theta^* = m_2$ ($\theta^* = m_1$), providing public information is always (never) advantageous.

When fundamentals are "good", speculators holding only private information do not attack the currency because their expected gain from a successful attack is "small" and the "amount of coordination" required to get that payoff is "high" (the share of attackers needed to force the government to abandon the peg is high); this makes their expected payoff less than zero. The availability of public information eliminates the uncertainty over the others and offers speculators the opportunity to coordinate on the "bad" equilibrium and obtain also that small positive payoff. However, it does not seem very likely that speculators will succeed in achieving a high "amount of coordination" just to obtain a small payoff.

On the other hand, when fundamentals are "fairly bad", speculators' payoff from a successful attack increases and the "coordination effort" required to achieve that payoff decreases. Hence, speculators holding only private information attack the currency because, given that a small "coordination effort" is sufficient to get a high payoff, they think it is likely that the others will attack too. With public information they could refrain from attacking. However, this event does not seem very likely because it entails speculators wasting a large payoff that could be gained easily.[20]

This result can be compared with the comparative statics exercise in Heinemann and Illing (1999). Building on Morris and Shin's private information model, Heinemann and Illing show that a decline in ε makes θ^* decrease. According to the authors, a government could reduce ε by committing to a more transparent economic policy. Hence, they state that increased transparency in a government's economic policy reduces the likelihood of a currency attack. With respect to our model, this finding suggests that when a government's economic policy is transparent, the region (m_1, θ^*) - where the policy-maker would find it convenient that public information is released - is smaller. Thus, committing to a more transparent economic policy seems to reduce the benefits from a strategic use of public information.

An interesting extension of the present study would be a more detailed examination of the strategic use of public information. This analysis should take into account the possibility that the government finds it profitable to release biased information, and it is potentially complicated by the presence of multiple equilibria (that re-emerge as ε goes to zero or as precise public information are explicitly considered, as in our model). The exploration of this

[20] Note that a similar comparison can be made between the results of complete and private information models too.

issue is beyond the scope of the present paper. However, it could be an important development of the SGMs. It could also provide the correct theoretical tool for the evaluation of the benefits from adherence to programs such as the Special Data Dissemination Standard of the International Monetary Fund [see IMF(1999)].

5. Conclusions

The game-theoretic approach of SGMs has proven to be an important line of research for the investigation of the role of speculators' expectations and information in the onset of currency crises. A promising line of analysis is offered by the study of global games, initiated by Carlsson and van Damme (1993) and applied to speculative attacks by Fukao (1994) and to second generation currency crisis models by Morris and Shin (1998). Theoretically, global games show the importance of the hypothesis of common knowledge of agents' actions for multiple equilibria and give some insights into the likelihood of the equilibria of complete and public information games.

The results achieved in the paper are consistent with this theory. In fact, with public information, the reintroduction of the common knowledge of agents' actions that occurs in equilibrium (still maintaining some uncertainty over the fundamentals) leads us to show that a multiple equilibria zone exists. Moreover, a comparison of the results of the public and private information models highlights an interesting paradox: when agents have private information, the fact that the state of fundamentals is publicly revealed turns out to be more advantageous to the government when fundamentals are bad than when fundamentals are "good"! However, it is apparent that this happens because the equilibria of the public information model that are eliminated in the private information game depend on the occurrence of implausible events; e.g., obtaining a small payoff with a large amount of coordination or renouncing a big payoff that could have been achieved with a small amount of coordination.

Our example shows that it is easy to draw misleading conclusions from models with multiple equilibria, especially when consideration of the likelihood of outcomes is neglected. Hence, the paradox calls for an equilibrium selection procedure because, with multiple equilibria, game theory can be a "weak and uninformative theory" [Harsany and Selten (1988)] and it can also lead to implementing erroneous policies. Global games, which lead to a unique equilibrium in a wide class of models, can be a very powerful tool in this regard.

By focusing on speculators' expectations, we prove that mean-preserving changes in speculators' probability assessments of the state of fundamentals may be sufficient to drive agents to a unique "bad" equilibrium with a self-fulfilling currency attack. Hence, the model suggests an explanation for sudden shifts in speculators' behavior that trigger currency devaluations that do not seem to be justified by the fundamentals. In fact, modifications of agents' beliefs that induce speculative attacks can occur even without a worsening in the expected state of fundamentals. In particular, the model demonstrates that a crisis can be triggered by an increase in *uncertainty* over the fundamentals (as measured, for example, by the variance of the distribution) or, in general, by an increase in the subjective probability of an "unforced" currency devaluation.

Finally, by assuming an affine relationship between the "natural" exchange rate and the fundamentals, we prove that a mean-preserving change in beliefs that induces a shift to a unique equilibrium with a currency crisis may occur when the expected state of fundamentals is in the "ripe for attack" zone. Therefore, the model shows that the "good" equilibrium *is not robust* to changes in agents' beliefs for example due to an increase in uncertainty. Note that in a complete information model, when an economy's fundamentals fall the "ripe for attack" zone, the economy is *vulnerable* to a currency attack because speculators may or may not trigger a crisis. Our incomplete information model instead suggests that such an economy should be considered as *fragile* as it is in the "unstable" zone. In fact, for some (unbiased) agents' expectations, there may be a unique "bad" equilibrium with a speculative attack and a devaluation of the currency.

Appendix: Proof of Theorem 2

The proof of Theorem 2 can be completed by using "locally" – i.e. over the intervals $(\underline{\theta}-\delta,\underline{\theta}+\delta)$ and $\left(\overline{\theta}-\delta,\overline{\theta}+\delta\right)$ – the method applied by Morris and Shin in their private information model. However, our proof can be simplified since we are not interested in proving the uniqueness of the different equilibria for each state of fundamentals. Moreover, the public information framework allows us to eliminate any ambiguity in agents' beliefs; since agents receive exactly the same public message x, when they use the same decision rule they choose also the same actions. Hence, their belief about the share of attackers is always either 0 or 1 and, in equilibrium, coincides with the agents' actual behavior.[21] The proof is in five steps and makes use of a lemma that we prove separately in the fourth step.

1. Beliefs

For any given strategy profile of speculators, let $\pi(x)$ be the speculators' beliefs about the share of attackers when the public message is x. These beliefs are determined in equilibrium and must be consistent with speculators' equilibrium strategies. Given θ, the actual share of attackers depends on π (because of the consistency condition) and on the stochastic realization x. Hence, let $\alpha\left(\theta,\pi\right)$ be the expected share of attackers given θ and π. Since messages are uniformly distributed on $[\theta-\delta,\theta+\delta]$, we have:

$$\alpha(\theta,\pi)=\frac{1}{2\delta}\int\limits_{\theta-\delta}^{\theta+\delta}\pi(x)dx.$$

2. Expected payoff

Let $A\left(\pi\right)=\{\theta:\alpha(\theta,\pi)\geq\beta(\theta)\}$. When $\theta\in A\left(\pi\right)$, agents expect that the currency will be depreciated. Hence, the payoff for an agent who attacks when the state of fundamentals is θ and her belief is π may be written as:

$$h_a(\theta,\pi)=\begin{cases} e^*-f(\theta)-t, & \text{if } \theta\in A(\pi) \\ -t, & \text{if } \theta\notin A(\pi)\end{cases}.$$

[21] In a private information model, agents' actual choices can be diverse because agents can receive different messages. However, Morris and Shin assume that agents' beliefs about the "aggregate selling strategy" are always 0 or 1, even if, in equilibrium, the actual share can fall whithin the unit interval.

However, when agents decide whether to attack, they do not know θ and they only observe the public message x. Hence, the expected payoff from attacking the currency when they receive x and their belief is π is given by:

$$u_a(x,\pi) = \frac{1}{2\delta}\int_{x-\delta}^{x+\delta} h_a(\theta,\pi)d\theta = \frac{1}{2\delta}\int_{C_x}(e^* - f(\theta))d\theta - t,$$

where

$$C_x = A(\pi) \cap [x-\delta, x+\delta].$$

Speculators' expected payoff from not attacking the currency is always given by $u_n(x,\pi) = 0$.

3. Strategies

Let us consider a symmetric equilibrium (all speculators use the same strategy). Since speculators receive the same public message, in equilibrium we have $\pi(x) = 0 \Leftrightarrow u_a(x,\pi) \leq 0$ and $\pi(x) = 1 \Leftrightarrow u_a(x,\pi) > 0$.[22] We also consider equilibria where speculators' strategies are given by a cut-off rule of the following kind:[23]

$$\varphi(x) = \begin{cases} attack, & \text{if } x < k \\ do\ not\ attack, & \text{if } x \geq k \end{cases}.$$

These strategies imply that the belief function is an indicator function:

$$\pi(x) = I_k(x) = \begin{cases} 1, & \text{if } x < k \\ 0, & \text{if } x \geq k \end{cases}.$$

We now make use of a lemma demonstrated by Morris and Shin (1998). For ease of the reader, in the following step we report the lemma and its proof (corrected for a minor imperfection), with a notation consistent with the previous steps

4. Lemma: The function $u_a(k, I_k)$ is strictly decreasing and continuous in k.

---

22 We assume that speculators refrain from attacking whenever they are indifferent.

23 In the general framework of global games, Carlsson and van Damme (1993) prove that a cut-off rule is the unique optimal strategy.

Consider the function $\alpha(\theta, I_k)$, which represents the expected share of attackers when fundamentals are at level θ and speculators' beliefs are I_k. Since x is drawn from a uniform p.d.f., we have:

$$\alpha(\theta, I_k) = \begin{cases} 1, & \text{if } \theta \leq k - \delta \\ \frac{1}{2} - \frac{1}{2\delta}(\theta - k), & \text{if } k - \delta < \theta \leq k + \delta \\ 0, & \text{if } \theta < k + \delta \end{cases} .$$

Given k, let us define $\underline{\lambda}(k)$ as the minimum value of λ such that the following relation holds:

$$\alpha(k + \lambda, I_k) = \beta(k + \lambda). \tag{A1}$$

Recall that when $\alpha \geq \beta$ the currency depreciates. Observe that $\underline{\lambda}(k) = \delta$ if $k \leq \underline{\theta} - \delta$, and $-\delta < \underline{\lambda}(k) < \delta$ if $k > \underline{\theta} - \delta$.[24] In particular, in the latter case $\underline{\lambda}(k)$ solves:

$$\frac{1}{2} - \frac{\underline{\lambda}(k)}{2\delta} = \beta(k + \underline{\lambda}(k)). \tag{A2}$$

Since the government abandons the peg only if θ lies in the interval $[0, k + \underline{\lambda}(k)]$, the payoff function from attacking becomes:[25]

$$u_a(k, I_k) = \frac{1}{2\delta} \int_{k-\delta}^{k+\underline{\lambda}(k)} (e^* - f(\theta))d\theta - t. \tag{A3}$$

By hypothesis, $e^* - f(\theta)$ is strictly decreasing in θ. Therefore, to prove that $u_a(k, I_k)$ is strictly decreasing in k it is enough to show that $\underline{\lambda}(k)$ is non increasing.

Differentiating (A2) with respect to k yields:

$$\underline{\lambda}'(k) = -\frac{\beta'(\theta)}{\beta'(\theta) + 1/(2\delta)}.$$

---

[24] Note that if $k \geq \underline{\theta} - \delta$ there is a unique value of λ such that (A1) holds; this is not true if $k < \underline{\theta} - \delta$. Thus, here we depart from Morris and Shin's definitions, make them more accurate. In particular, we define $\underline{\lambda}(k)$ as " the *minimum* value of λ such that (A1) holds", so that we can deal with a function and not with a correspondence.

[25] It is indifferent to take the minimum or the maximum of λ such that (A1) holds because $f(\theta) = e^*$ between those values. Therefore, a different definition of $\underline{\lambda}$ would not change the value of the integral (A3).

Hence, $\underline{\lambda}'(k) \leq 0$, which is sufficient to prove the strict monotonicity of u_a. Finally, continuity follows from the fact that u_a is an integral in which the limits of integration are themselves continuous in k.

5. Equilibrium

Finally we can turn to the two equilibria $E1$ and $E2$ defined in Section 4. Consider the equilibrium $E1$ where agents attack if $x \leq \underline{\theta} - \delta$ and do not attack if $x \geq \underline{\theta} + \delta$. Of course, $u_a(\underline{\theta} - \delta, I_{\underline{\theta}-\delta}) > 0$ and $u_a(\underline{\theta} + \delta, I_{\underline{\theta}+\delta}) < 0$. Hence, from the lemma, it follows that there exists a unique value $x_1 \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$ such that $u_a(x_1, I_{x_1}) = 0$. Therefore, agents' optimal rule will be:

$$\varphi_{E1}(x) = \begin{cases} \textit{attack}, & \text{if } x < x_1 \\ \textit{do not attack}, & \text{if } x \geq x_1 \end{cases} .$$

With a simple graphic argument it is easy to show that there is a unique value of the fundamentals $m_1 \in (\underline{\theta}, \underline{\theta} + 2\delta)$ such that if $\theta \leq m_1$ the currency depreciates and if $\theta > m_1$ the peg is maintained.

Analogously, in the equilibrium $E2$ where agents attack if $x \leq \overline{\theta} - \delta$ and do not attack if $x \geq \overline{\theta} + \delta$, since $u_a(\overline{\theta} - \delta, I_{\overline{\theta}-\delta}) > 0$ and $u_a(\overline{\theta} + \delta, I_{\overline{\theta}+\delta}) < 0$, from the lemma it follows that there exists a unique value $x_2 \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$ such that $u_a(x_2, I_{x_2}) = 0$. Therefore, agents optimal rule is:

$$\varphi_{E2}(x) = \begin{cases} \textit{attack}, & \text{if } x < x_2 \\ \textit{do not attack}, & \text{if } x \geq x_2 \end{cases} .$$

Hence, we infer that there is a unique value of the fundamentals $m_2 \in (\underline{\theta} - 2\delta, \underline{\theta} + 2\delta)$ such that if $\theta \leq m_2$ the currency depreciates and if $\theta > m_2$ the peg is maintained.

Thus, we have found an interval $M \equiv (m_1, m_2]$ where the currency may or may not be maintained depending on whether agents coordinate on $E1$ or $E2$. The proof is completed by checking that if $\delta \leq \left(\overline{\theta} - \underline{\theta}\right)/4$, M is not empty.

References

Bank for International Settlements (1999), *A Review of Financial Markets Events in Autumn 1998*, Committee on the Global Financial System, October, Basel.

Carlsson, H. and E. van Damme (1993), "Global Games and Equilibrium Selection", *Econometrica*, Vol. 61, pp. 989-1018.

Corsetti, G., P. Pesenti and N. Roubini (1998), "What Caused the Asian Financial Crisis?", *NBER Working Paper,* No. 6833.

Flood, R. P. and P. Garber (1984), "Collapsing Exchange-Rate Regimes: Some Linear Examples", *Journal of International Economics*, Vol. 17, pp. 1-13.

Fukao, K. (1994), "Coordination Failures under Incomplete Information and Global Games", The Institute of Economic Research, Hitotsubashi University, *Discussion Paper Series A*, No. 299.

Harsany, J. C. and R. Selten (1988), *A General Theory of Equilibrium Selection in Games*, MIT Press Cambridge.

Heinemann, F. and G. Illing (1999), "Speculative Attacks: Unique Sunspot Equilibrium and Transparency", University of Frankfurt, *Working Paper*, No. 97.

Krugman, P. (1979), "A Model of Balance-of-Payment Crises", *Journal of Money, Credit and Banking*, Vol. 11, pp. 311-25.

Krugman, P. (1996), "Are Currency Crises Self-Fulfilling?", in B.S. Bernanke and J. J. Rotenberg (eds.), *NBER Macroeconomic Annual*, MIT Press, Cambridge.

International Monetary Fund (1999), *World Economic Outlook*, May, Washington D.C.

Morris, S., R. Rob and H. S. Shin (1995), "*p*-Dominance and Belief Potential", *Econometrica*, Vol. 63, pp. 145-57.

Morris, S. and H. S. Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Crises", *American Economic Review*, Vol. 88, pp. 587-97.

Obstfeld, M. (1986), "Rational and Self-Fulfilling Balance of Payment Crises", *American Economic Review*, Vol. 76, pp. 72-81.

Obstfeld, M. (1994), "The Logic of Currency Crises", *Cahiers Economiques et Monetaires*, Vol. 43, pp. 189-213.

Obstfeld, M. (1996), "Models of Currency Crises with Self-Fulfilling Features", *European Economic Review*, Vol. 40, pp. 1037-47.

Radelet, S. and J. Sachs (1998), "The Onset of the East Asian Financial Crises", *Harvard Institute for International Development*, mimeo.

Rubinstein, A. (1989), “The Electronic Mail Game: Strategic Behavior Under ‘Almost Common Knowledge”’, *American Economic Review*, Vol. 79, pp. 385-91.

RECENTLY PUBLISHED "TEMI" (*)

No. 367 — *An Investment-Function-Based Measure of Capacity Utilisation. Potential Output and Utilised Capacity in the Bank of Italy's Quarterly Model,* by G. PARIGI and S. SIVIERO (February 2000).

No. 368 — *Information Spillovers and Factor Adjustment,* by L. GUISO and F. SCHIVARDI (February 2000).

No. 369 — *Banking System, International Investors and Central Bank Policy in Emerging Markets,* by M. GIANNETTI (March 2000).

No. 370 — *Forecasting Industrial Production in the Euro Area,* by G. BODO, R. GOLINELLI and G. PARIGI (March 2000).

No. 371 — *The Seasonal Adjustment of the Harmonised Index of Consumer Prices for the Euro Area: a Comparison of Direct and Indirect Methods,* by R. CRISTADORO and R. SABBATINI (March 2000).

No. 372 — *Investment and Growth in Europe and in the United States in the Nineties*, by P. CASELLI, P. PAGANO and F. SCHIVARDI (March 2000).

No. 373 — *Tassazione e costo del lavoro nei paesi industriali, by* M. R. MARINO and R. RINALDI (June 2000).

No. 374 — *Strategic Monetary Policy with Non-Atomistic Wage-Setters*, *by* F. LIPPI (June 2000).

No. 375 — *Emu Fiscal Rules: is There a Gap?,* by F. BALASSONE and D. MONACELLI (June 2000).

No. 376 — *Do Better Institutions Mitigate Agency Problems? Evidence from Corporate Finance Choices,* by M. GIANNETTI (June 2000).

No. 377 — *The Italian Business Cycle: Coincident and Leading Indicators and Some Stylized Facts,* by F. ALTISSIMO, D. J. MARCHETTI and G. P. ONETO (October 2000).

No. 378 — *Stock Values and Fundamentals: Link or Irrationality?,* by F. FORNARI and M. PERICOLI (October 2000).

No. 379 — *Promise and Pitfalls in the Use of "Secondary" Data-Sets: Income Inequality in OECD Countries,* by A. B. ATKINSON and A. BRANDOLINI (October 2000).

No. 380 — *Bank Competition and Regulatory Reform: The Case of the Italian Banking Industry*, by P. ANGELINI and N. CETORELLI (October 2000).

No. 381 — *The Determinants of Cross-Border Bank Shareholdings: an Analysis with Bank-Level Data from OECD Countries*, by D. FOCARELLI and A. F. POZZOLO (October 2000).

No. 382 — *Endogenous Growth with Intertemporally Dependent Preferences*, by G. FERRAGUTO and P. PAGANO (October 2000).

No. 383 — *(Fractional) Beta Convergence*, by C. MICHELACCI and P. ZAFFARONI (October 2000).

No. 384 — *Will a Common European Monetary Policy Have Asymmetric Effects?*, by L. GUISO, A. K. KASHYAP, F. PANETTA and D. TERLIZZESE (October 2000).

No. 385 — *Testing for Stochastic Trends in Series with Structural Breaks*, by F. BUSETTI (October 2000).

No. 386 — *Revisiting the Case for a Populist Central Banke*r, by F. LIPPI (October 2000).

No. 387 — *The multimarket contacts theory: an application to italian banks,* by R. DE BONIS and A. FERRANDO (December 2000).

No. 388 — *La "credit view" in economia aperta: un'applicazione al caso italiano,* by P. CHIADES and L. GAMBACORTA (December 2000).

No. 389 — *The monetary trasmission mechanism: evidence from the industries of five OECD countries,* by L. DEDOLA and F. LIPPI (December 2000).

No. 390 — *Disuguaglianza dei redditi individuali e ruolo della famiglia in Italia,* by G. D'ALESSIO and L. F. SIGNORINI (December 2000).

(*) Requests for copies should be sent to:
Banca d'Italia - Servizio Studi - Divisione Biblioteca e pubblicazioni - Via Nazionale, 91 - 00184 Rome
(fax 0039 06 47922059). They are available on the Internet at www.bancaditalia.it