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(Fractional) Beta Convergence

by C. Michelacci and P. Zaffaroni

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by Claudio Michelacci* and Paolo Zaffaroni**

Abstract

Unit roots in output, an exponential 2 per cent rate of convergence and no change in the underlying dynamics of output seem to be three stylized facts that cannot go together. This paper extends the Solow-Swan growth model allowing for cross-sectional heterogeneity. In this framework, aggregate shocks might vanish at a hyperbolic rather than at an exponential rate. This implies that the level of output can exhibit long memory and that standard tests fail to reject the null of a unit root despite mean reversion. Exploiting secular time series properties of GDP, we conclude that traditional approaches to test for uniform (conditional and unconditional) convergence suit first step approximation. We show both theoretically and empirically how the uniform 2 per cent rate of convergence repeatedly found in the empirical literature is the outcome of an underlying parameter of fractional integration strictly between 1/2 and 1. This is consistent with both time series and cross-sectional evidence recently produced.

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* CEMFI.

**Bank of Italy, Research Department.

Contents

1 Introduction [1]

The debate on unit roots and stochastic trends has dominated macroeconometrics over the eighties. Since the seminal work of Nelson and Plosser (1982), this literature has noted how standard unit root tests have failed to reject the null of a unit root in output per capita. The nineties marked the revival of the empirics of growth and convergence. Conditional uniform convergence, Beta convergence, means that aggregate shocks are absorbed at an uniform exponential rate. Most empirical studies conclude that the output per capita of very different economies converges to its long run steady state value at a uniform exponential rate of 2 per cent per year, [see, for example, Barro (1991), Barro and Sala-I-Martin (1991), Barro and Sala-I-Martin (1995), Mankiw, Romer, and Weil (1992)]. These seem to be two of the most striking empirical regularities in modern empirical macroeconomics. More recently, Jones (1995b), has observed that in line with the standard exogenous growth Solow model, the trend of output per capita for OECD economies is fairly smooth over time and does not exhibit any persistent changes in the post World War era.

These three stylized facts seem to be inconsistent. On the one hand a

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unit root in output implies that shocks are permanent so that output does not exhibit mean reversion. On the other Beta convergence, henceforth β-convergence, implies that output converges to its steady state level at a rate that, even if very low, is positive and uniform across economies. The Jones invariance property implies that steady state output could well be represented by a smooth time dependent linear trend. If this is true, unit root tests and β-convergence are testing for the same hypothesis.

This paper starts from the observation that the 'size of the unit root' component in GDP (the long-run effect of a unit shock) is usually found to be very low, [see Cochrane (1988) , Campell and Mankiw (1987) and Lippi and Reichlin (1991)] and follows Quah (1995) in noting that cross-sectional and time series analysis can not arrive at different conclusions. In agreement with Diebold and Rudebusch (1989), and Rudebusch (1993) we propose a different explanation. Perhaps the speed with which aggregate shocks are absorbed is so low that standard unit root tests fail to reveal it[2]. This could actually be the case if GDP per capita exhibits long memory (Diebold and Rudebusch 1991). If we consider the standard Solow-Swan model and allow for cross sectional heterogeneity in the speed with which different units in the same countries adjust, then we intend to show that the dynamics of output can exhibit long memory. We can then test for both uniform conditional and unconditional convergence allowing for rate of convergence different from the exponential one. In this framework, we show how a 2 per cent rate of convergence superimposed as exponential and

[2]Diebold and Senhadji (1996) show that Rudebusch's (1993) approach produces evidence that distinctly favors trend-stationarity using long spans of annual data.

estimated over a time span that ranges from a minimum of 20 years to a maximum of 100 years correspond to a parameter of fractional integration that ranges from 0.51 to 0.99. This process is not covariance stationary but still mean reverting, so that standard unit root test are not likely to reject the null of non-stationarity, despite the fact that convergence takes place. Using GDP per capita data for OECD countries for the period 1885-1994, we will test this hypothesis. We conclude that it can not be rejected and, thus, convergence takes place at an hyperbolic very slow rate.

This paper intends to bring together two different branches of research. On the one hand, time series analysis has concluded that shocks tend to have a permanent effect on the level of output. On the other hand the literature on growth and convergence has concluded that countries converge to their long run steady state value at an exponential rate that is very low and uniform across countries. We note that the two literatures are inconsistent once we allow for the invariance property by Jones and we follow the standard exogenous growth Solow model in approximating the dynamics of long run GDP per capita with a linear trend. In line with Diebold and Senhadji (1996) we propose a theoretical solution and test it. We conclude that standard tests for convergence suit first step approximation despite the misspecification of the statistical model. In doing so we show that the parameters of fractional integration of different OECD countries, though of similar magnitude and smaller than one, are significantly different one from the other. This perhaps explains why time series tests of convergence based on cointegration reject the null of convergence even among OECD countries [see for example Quah (1992), Bernard and Durlauf (1995) and Bernard and Durlauf (1996)]. As they are, these tests are misspecified, as different variables can be cointegrated only if they exhibit the same order of integration.

Section 1 reviews the Solow-Swan model. Here we emphasize further why the three stylized facts cannot go together. Section 2 briefly reviews the theory of long memory processes and shows how the path of adjustment of output can exhibit long memory in a extension of the theoretical model. Further we show why standard unit root tests cannot reject the null of a unit root while a uniform 2 per cent rate of convergence can be found to be statistically significant. In section 3 we check for uniform conditional and unconditional convergence. Section 4 concludes.

2 Empirics of the Solow growth model and unit roots

Here the Solow growth model is reviewed, followed by consideration of the time series properties of the reduced form of the model.

Solow Growth Model

In the Solow model the rates of saving and technological progress are exogenous. There are two inputs, capital and labour. We assume a Cobb-Douglas production function, so production at time t is given by

$$Y_t = K_t^{\alpha}(A_t\, L_t)^{1-\alpha}, \qquad 0 < \alpha < 1.$$

The notation is standard: Y is output, K capital, L labour and A the level of technology. A is assumed to grow exogenously at rate g.

The model assumes that a constant fraction of output s is invested. Defining $\hat{k}$ and $\hat{y}$ as respectively the stock of capital and output per effective unit of labour, $\hat{k} = K/AL$ and $\hat{y} = Y/AL$, the evolution of $\hat{k}$ is governed by

$$\frac{d\,\hat{k}_t}{dt} = s\hat{k}_t^{\alpha} - (g+\delta)\hat{k}_t, \tag{1}$$

where δ is the depreciation rate. Equation (1) implies that $\hat{k}_t$ converges towards a steady state level $\hat{k}^*$ defined by

$$\hat{k}^* = (\frac{s}{g+\delta})^{\frac{1}{1-\alpha}}. \tag{2}$$

We can then consider a log-linear approximation of equation (1) around the steady state, so that

$$\begin{aligned} \frac{d[ln(\hat{y}_t)]}{dt} &= -\beta[ln(\hat{y}_t) - ln(\hat{y}_t^*)], \qquad (3) \\ &\text{with} \\ \beta &= (1-\alpha)(g+\delta), \end{aligned}$$

where $\hat{y}^* = (\hat{k}^*)^{\alpha}$. Discretizing equation (3) and indicating with y_t the log of output per capita, viz. $y_t = ln(Y_t/L_t)$ and by y_t^* the log of the level of output per capita in steady state we get

$$y_t - y_{t-1} = g + \beta y_{t-1}^* - \beta y_{t-1}, \qquad 0 < \beta < 1, \tag{4}$$

or equivalently

$$y_t - y_t^* = (1-\beta)[y_{t-1} - y_{t-1}^*]. \tag{5}$$

We now analyze the time series properties of both equations (4) and (5).

Time Series Properties

Equation (4) is the basic equation used to test β-convergence [see for example Barro (1991) , Barro and Sala-I-Martin (1991) and Mankiw, Romer, and Weil (1992)]. β-convergence applies if a poor economy tends to grow faster than a rich one. This happens if the estimate of the coefficient β in equation (4) is positive and significantly different from zero. If this is the case, aggregate shocks that have pushed the current level of output away from the steady state level will be absorbed at the exponential rate β, so that the dynamics of output will exhibit mean reversion. The standard approach to test this property consists of approximating $g + \beta y^*_{t-1}$ with some control or environmental variables like the investment rate, population growth, government expenditure and so on, and subsequently estimating the regression (4) and testing the significance of the coefficient β. In practice, empirical studies repeatedly find a 2 per cent coefficient, uniform across countries and significantly different from zero (Quah 1993).

A test of unit root, as for example the Dickey Fuller's test (Dickey and Fuller 1979), still uses an equation like (4) and tests for the coefficient β being significantly different from zero, where the term $g + \beta y^*_{t-1}$ is substituted by a smooth time dependent function. A value for the coefficient β not significantly different from zero is interpreted as a hint of the presence of a unit root in the underlying data generating process. If this is the case, a temporary shock has permanent effects on the level of output and the dynamics of output does not exhibit mean reversion towards the smooth trend. Since the seminal work of Nelson and Plosser (1982) these tests have not been able to reject the null of a unit root in GDP per capita, even if their low power is well recognized [see for example Diebold and Rudebusch (1991), Rudebusch (1993) and Diebold and Senhadji (1996)].

In general, the existence of a unit root in output is not in contradiction with β-convergence if we allow for the steady state level of output to be cointegrated with the current level of output. In this case aggregate shocks are still absorbed at an exponential rate despite the fact that output is integrated as implied by equation (5). However Jones (1995a, 1995b) has observed that the dynamics of aggregate output moves smoothly and independently of most of the controlled variable used for testing β-convergence. This is in line with the standard exogenous growth Solow model where the level of long run GDP per capita, y_t^*, is represented by the linear trend gt. If we take the data from Maddison (1995) for 16 OECD countries over the period 1885-1994 and plot the dynamics of per capita GDP versus a common linear trend among all the countries in the sample, we note that this simple common trend fits long run per capita GDP extremely well. This is shown in Figure 1 where we plotted each series together with a country specific linear trend and a common linear trend, obtained pooling the series of all 16 OECD countries in our sample. The former has been estimated with OLS, the latter with GLS. Particular informative is the GLS estimate of the common trend. GLS estimating procedure implies that the better the fit of the specific trend the greater the weight of this country in the determination of the common trend. In this case the US case performs much better than all the other countries. This shows up in the final outcome. In fact, the common GLS trend and the US OLS specific trend are almost undistinguishable (see Figure 1). Thus we can think of the US performance as representing the long run benchmark of the performances of all other countries. Nelson and Kang (1984) argue, however, that regressions of driftless integrated series against a time trend can result in the inappropriate inference that the trend is significant and, that it is a good description of the data as Durlauf and Phillips (1988) show. Instead, Jones (1995a) notes that a time trend calculated using data only from 1880 to 1929 forecasts the current level of GDP of the US economy extremely well. Following Diebold and Senhadji (1996) this is clearly incompatible with difference stationarity in aggregate output, as new information seems to be irrelevant for forecasting on very long horizons.

This suggests that, in accordance with the standard exogenous growth Solow model where the level of long run GDP per capita, y_t^*, is represented by the linear trend, gt, the dynamics of output in steady state follows a smooth trend. As a deterministic function cannot be cointegrated with a variable exhibiting a stochastic trend, it turns out that β-convergence and unit root tests are both checking for mean reversion towards a smooth time dependent trend. In a time series formulation we can say that β-convergence tests trend stationarity in output where the statistical model is given by an $AR(1)$ with a linear trend[3]. These simple considerations imply that testing β-convergence

[3]The nature of the problem is further complicated by the fact that growth theorists use panel data instead of time series. Recent results (e.g. Levin and Lin (1992)) show, however, that panel data dramatically increase the power of a unit root test as the cross sectional dimension increases.

is meaningless if we assume the Jones invariance property, together with the existence of a unit root in output[4]. As they stand, these three stylized facts cannot go together. Our claim is that the two equivalent tests are both checking for a superimposed rate of 'exponential' mean reversion.

If the rate of mean-reversion in (logged) GDP per capita or equivalently the rate of absorption of the shocks is hyperbolic (in a sense to be defined precisely below) instead of exponential, β-convergence would apply in the sense that poorer economies would grow faster and would converge towards their long run steady states, and standard unit root tests would fail to reject a unit root albeit not present [see for example Diebold and Rudebusch (1991)].

3 Theory of long memory and the Barro regression

In this section we briefly review the theory of long memory processes which allow the possibility of 'hyperbolic' mean reversion together with non-stationarity. We will then analyze why the Barro regression might be robust for rates of convergence different from the exponential one, delivering the right answer to the problem of convergence.

[4]For example, Den Haan (1995) notes that the slow speed of convergence observed in the data can be reconciled quantitatively with the neoclassical growth model, assuming either a capital share equal to around 0.8 or a sufficient amount of persistence in the stochastic process driving technological progress. In either cases, the 2 per cent rate of convergence is incompatible with aggregate output exhibiting a unit root (see his equation 3.4).

3.1 Theory of long memory processes

Unit roots describe only a small set of non-stationary processes. A class that groups together either covariance stationary processes and unit roots is given by 'strongly dependent' processes also known as 'long memory' or 'long range dependent' processes [see Robinson (1994) for a survey on the topic]. Usually only the second moment properties are considered in order to characterize such behaviour in terms of either that of the autocorrelation function at long lags, or that of the power spectrum near the zero frequency.

We shall assume that K denotes any positive constant (not necessarily the same) and $\sim$ asymptotic equivalence.

Definition 1

A real valued scalar discrete time process X_t is said to exhibit long memory in terms of the power spectrum (when it exists) with parameter $d > 0$ if

$$f(\lambda) \sim K\lambda^{-2d}, \qquad as \qquad \lambda \to 0^+.$$

In the non-stationary case ($d \geq 1/2$, see below) $f(\lambda)$ is not integrable and thus it is defined as a pseudo-spectrum.

The importance of this class of processes derives from smoothly bridging the gap between short memory (standard) stationary processes and unit roots in an environment that maintains a greater degree of continuity (Robinson 1994).

For the purpose, let us consider a parametric example. Let $\{y_t\}$ be a discrete time scalar time series , $t = 1, 2, \ldots$, suppose v_t is an unobservable covariance stationary sequence with spectral density that is bounded and bounded away from zero at the origin, such that for a real d

$$(1-L)^d y_t = v_t, \qquad t = 1, 2, \ldots \tag{6}$$

where L is the lag operator. If $d = 0$, then y_t is a standard or better short memory (covariance) stationary process with spectral density bounded away from zero (i.e. an $ARMA$ process), whereas y_t is a random walk if $d = 1$. The parameter d however does not need to be an integer.

In the following, we focus on the case in which y_t is a long memory process with parameter d positive, with $0 < d < 1$. In this case, when v_t is assumed to be a white noise process, the process y_t defined in (6) is called an $ARFIMA(0, d, 0)$ process and, more generally, when v_t is an (inverted) $ARMA(p, q)$ we obtain an $ARFIMA(p, d, q)$ process.

The power spectrum of the y_t process is given by

$$f_y(\lambda) = \mid 1 - e^{i\lambda} \mid^{-2d} f_v(\lambda) = (2sin(\lambda/2))^{-2d} f_v(\lambda)\,, -\pi \leq \lambda < \pi\,,$$

where $f_v(.)$ denotes the power spectrum of the v_t process. Thus from $sin(\omega)/\omega \sim 1$, as $\omega \rightarrow 0$, when $d > 0$ as $\lambda \rightarrow 0^+$ we obtain

$$f_y(\lambda) \sim 4^{-d} f_v(0) \lambda^{-2d}\,.$$

Whenever $d > 0$ the power spectrum is unbounded at the zero frequency, implying that the series y_t exhibits long memory. This class of processes has many important properties. When $0 < d < 1/2$, y_t has both finite variance and exhibits mean reversion. When $1/2 < d < 1$ the process has infinite variance but it still exhibits mean reversion. This process is not covariance stationary but 'less' non-stationary than a unit root process, so that standard unit root tests exhibit low power with respect to this alternative despite the presence of mean reversion (Diebold and Rudebusch 1991). When $d \geq 1$ the process has infinite variance and stops exhibiting mean reversion. In particular, a unit root process is obtained when $d = 1$. This represents a particular case of long memory process: a process with an infinite memory.

If $-1/2 < d$, (6) can be inverted so that

$$y_t = \sum_{i=0}^{\infty} \gamma_i v_{t-i}, \text{ with } \gamma_i = \prod_{k=1}^{i} \frac{k-1+d}{k}, \ i \geq 1, \ \gamma_0 = 1. \tag{7}$$

Using Stirling's approximation it follows that as $i \to \infty$

$$\gamma_i \sim K i^{d-1} . \tag{8}$$

This implies that effect of a shock v_{t-i}, i periods ahead, vanishes at a hyperbolic rather than exponential rate, exhibiting a high level of persistence, the higher the greater the parameter d. When $d = 1$, the unit root case arises where a shock arbitrarily far away in time exhibits permanent effects on the current level of y_t.

This persistence property reflects the characterization already given in the frequency domain. We have seen that a long memory process for $d > 0$ is defined by an unbounded spectrum at the origin. It is well accepted that the degree of persistence of a shock can be expressed by the 'level' of the spectral density at zero frequency (Cochrane 1988). The definition of long memory and the previous considerations suggest the 'slope' of the logged spectrum at the origin as an exact measure of persistence.[5] In fact, taking logs in both

[5] This concept is directly derived from a well known branch in the semi-nonparametric econometrics literature [(Robinson 1995), (Geweke and Porter Hudak 1983)].

terms in *Definition 1* one obtains, as $\lambda \to 0^+$,

$$ln\left(f(\lambda)\right) \sim K \, - \, 2dln(\lambda). \tag{9}$$

With respect to the scatterplot of the logged spectrum and $2ln(\lambda)$, the unit root case will be represented by a line with slope minus $\pi/4$ while the case $d < 1$ is represented by a flatter line. Obviously the greater the slope in absolute value, the greater the level of persistence. The idea expressed by (9) is at the core of the estimation procedure suggested by Geweke and Porter Hudak (1983) and formalized in Robinson (1995), and is briefly described in the Appendix.

3.2 Robustness of the Barro regression

This section tries to rationalize the finding of a significant estimate of the regression coefficient of β-convergence in (4).

At first, let us consider some back-of-the-envelope calculations. A 2 per cent rate of convergence superimposed as exponential over a time span of 20-110 years is almost observational equivalent to a parameter of fractional integration strictly between 1/2 and 1. In fact, bearing in mind the result in (8) a parameter of fractional integration, d, that resembles the 2 per cent exponential rate of decay after a k period ahead shock can be obtained solving the simple equation[6]

$$(0.98)^k = k^{d-1}. \tag{10}$$

[6]Of course this is just a very simple and approximate exercise, yet useful for understanding the background thoughts central to this paper.

In Table 1 below we report the solutions of this simple equation for values of k that range from 10 to 110. As most empirical studies have used samples that ranges from 20 to 100 years, we can consider an underlying parameter of fractional integration strictly between 1/2 and 1 as the driving force behind the 2 per cent rate of convergence found in the empirical literature on β-convergence.

Table 1	
d∼ 2% exp. rate	n. of obs.
0.912	10
0.865	20
0.821	30
0.781	40
0.742	50
0.703	60
0.667	70
0.631	80
0.596	90
0.561	100
0.527	110

Secondly, let us consider now the following theoretical result due to Sowell (1990, Theorem 4). Regressing a variable on its lagged value, the Student t of the coefficient behaves discontinuously when the process generating the variable is an $ARFIMA(0, d, 0)$ with $d > 0$. In fact, when $d = 1$, the asymptotic distribution of the Student t normalized at the value 1, becomes the well known Dickey Fuller distribution (Dickey and Fuller 1979) but, when $d \neq 1$, one obtains very different results, that is

$$t \rightarrow_p \begin{cases} \infty & , 1 < d < 3/2\,, \\ -\infty & , 1/2 < d < 1\,. \end{cases}$$

Let us now start to draw our conjecture.

If per capita GDP is well represented by a long memory process with parameter d, with $1/2 < d < 1$, thus displaying infinite variance and, most important, together with mean-reversion, fitting the Barro regression would tend to give a significantly negative Student t (actually converging to negative infinity in probability). Thus this simple inference gives exactly the same conclusion as the aforementioned regression (4) obtained in the literature when fitting an exponential rate of convergence.

Furthermore, the back-of-the-envelope calculations show that superimposing an exponential rate of decay over a long memory process with $1/2 < d < 1$ gives precisely the well established 2 per cent rate of β-convergence.

Finally, the property of long memory processes to nest the unit root case in a class that maintains a greater level of continuity rather than standard weak dependent processes motivates the empirical finding of systematically non significant unit root tests.

If our conjecture is right, we could say that the standard approach to test for β-convergence (Barro 1991), suits first step approximation despite the misspecification of the empirical model. This test tends to exhibit negative Student t in the case of mean reversion ($d < 1$), leaving nonetheless some margins of ambiguity in a particular case of lack of convergence, the unit root case ($d = 1$). On the other hand the Student t will diverge to plus infinity when $d > 1$ delivering the right answer to the issue of convergence.

At this stage, our conjecture still lacks two elements, one purely economic, and the other a conclusive statistical one. First of all we will show a possible source of the long memory feature of the data in a version of the Solow model augmented by cross sectional heterogeneity. In second place, there is the need of a rigorous time series analysis of the data to show that the logged per capita GDP is well represented by a mean-reverting long memory process with $1/2 < d < 1$. This is done in section 6.

4 The Solow model augmented by cross sectional heterogeneity

In this section we show how long memory could arise in the Solow growth model. Let us suppose that the economy is characterized by N units each behaving as in the standard Solow model outlined in section 2. That means that each of these units representing either different firms or sectors in the same economy are investing a fraction s_i of their output in the accumulation of capital [7]. If this is the case the dynamics of output, y_t^i, of each of these

[7] Theoretically this structure could arise either in a world with imperfect capital markets where human capital is used as a collateral or because of adjustment costs [see Barro and Sala-I-Martin (1995)]. We decided not to model directly these frictions here because of the space constraint. The assumption that each unit is evolving as an $AR(1)$ is a simplification that is not needed, to obtain the result, as long as there is an autoregressive polynomial (as will become clear below).

firm-sectors, with steady state output $y_t^{*^i}$, is governed by

$$y_t^i - y_t^{*^i} = (1-\beta_i)[y_{t-1}^i - y_{t-1}^{*^i}] + \epsilon_t^i + \eta_t, \qquad i = 1,\ldots,N,\ 0 < \beta_i < 1. \quad (11)$$

where ϵ_t^i, η_t represent respectively the idiosyncratic and aggregate shock assumed mutually uncorrelated white noise and β_i is equal to $(1-\alpha_i)(g_i+\delta_i)$. Here α_i, g_i and δ_i are respectively the unit's specific productivity of capital, the rate of technological progress and the depreciation rate. It follows that the variable $x_t^i = y_t^i - y_t^{*,i}$ behaves like a first-order autoregressive process.

If we indicate, respectively,

$$\begin{aligned} \bar{y}_t &= \frac{1}{N}\sum_{i=1}^{N} y_t^i\,, \\ \bar{y}_t^* &= \frac{1}{N}\sum_{i=1}^{N} y_t^{*^i}\,, \end{aligned}$$

the current and long run equilibrium aggregate output, one obtains that the amount of disequilibrium in the economy evolves as

$$\bar{y}_t - \bar{y}_t^* = \frac{1}{N}\sum_{i=1}^{N}(1-\beta_i)[y_{t-1}^i - y_{t-1}^{*,i} + \epsilon_t^i] + \eta_t\,. \quad (12)$$

Let us define $\bar{x}_t = \bar{y}_t - \bar{y}_t^*$. The above equation can behave very differently from equation (5) even if all the coefficients β_i are bounded between zero and one. We will show that under certain conditions on the cross sectional distribution of the coefficients β_i, $\bar{x}_t$ exhibits long memory. In fact if we assume that the aggregate η_t and idiosyncratic ϵ_t^i shocks are uncorrelated, we obtain the power spectrum $f_k(\lambda)$ of x_t^k equal to

$$f_k(\lambda) = \frac{var(\epsilon_t^k)}{2\pi \mid 1-(1-\beta_k)e^{i\lambda} \mid^2} + \frac{var(\eta_t)}{2\pi \mid 1-(1-\beta_k)e^{i\lambda} \mid^2}. \quad (13)$$

This implies that the power spectrum $\bar{f}(\lambda)\ \ , -\pi \leq \lambda < \pi$ of the aggregate $\bar{x}_t$ is equal to

$$\bar{f}(\lambda) = \bar{f}_1(\lambda) + \bar{f}_2(\lambda)\,, \tag{14}$$

where

$$\begin{aligned}
\bar{f}_1(\lambda) &= \frac{1}{N^2}\sum_{k=1}^{N} \frac{var(\epsilon_t^k)}{2\pi \mid 1-(1-\beta_k)e^{i\lambda} \mid^2}\,, \\
\bar{f}_2(\lambda) &= \frac{var(\eta_t)}{2\pi N^2} \mid \sum_{k=1}^{N} \frac{1}{(1-(1-\beta_k)e^{i\lambda})} \mid^2 \ .
\end{aligned}$$

If we assume that the coefficients β_k are independent drawings from a distribution $F(\beta)$ and that the $var(\epsilon_t^u)$ are drawn from another distribution independent of the first, we follow Robinson (1978) and Granger (1980) to obtain

$$\bar{f}(\lambda) \quad \simeq \frac{1}{2\pi N}(E[var(\epsilon_t^k)]) \int_{\mathcal{B}} \frac{1}{\mid 1-(1-\beta)e^{i\lambda} \mid^2} dF(\beta) + \tag{15}$$

$$\frac{var(\eta_t)}{2\pi} \mid \int_{\mathcal{B}} \frac{1}{(1-(1-\beta)e^{i\lambda})} dF(\beta) \mid^2\,, \tag{16}$$

where $\mathcal{B}$ denotes the support of the distribution $F(\beta)$ and $\simeq$ denotes that the relation holds approximately for large but finite N.

In general, long memory arises if the integral in (15), evaluated at $\lambda = 0$,

$$\int_{\mathcal{B}} \frac{1}{(1-(1-\beta))^2} dF(\beta) = E_F(1/\beta^2)\,, \tag{17}$$

diverges, where $E_F(.)$ denotes the expectation over the measure $F(.)$ In fact the second integral in (16), viz. $E_F(1/\beta)$, diverges under stronger conditions which imply the divergence of the former integral in (15) but not viceversa as we will make clear below.

We can establish necessary and sufficient conditions on the distribution function $F(.)$ such that the integral (17) is unbounded. In general we know (e.g. in Rudin (1973)) that the integral $\int_a^b h(t)dt$ for a continuous function $h(x)$ on an interval $[a,b)$ is unbounded, if $h(.)$ has at least the same order of infinity as $1/(b-x)^\alpha$, with $\alpha \geq 1$, when x is close to b, that is

$$1/(b-x)^\alpha \leq A\, h(x), \text{ as } \qquad x \to b^-,$$

for some finite constant $A > 0$. If we assume that the distribution function $F(\beta)$ is absolutely continuous having a density $f(\beta)$, the integrand function of (17) is given by $f(\beta)/\beta^2$. Thus a sufficient condition for $\bar{x}_t$ to exhibit long memory is simply given by

$$f(\beta) \geq K\beta, \quad \text{as } \beta \to 0^+ ,$$

for some positive constant K. Thus the density $f(\beta)$ might go to zero as $\beta \to 0$ but at a slower rate than β [8].

The main implication is that the aggregate process might display long memory even if the aggregating elements are stationary with probability one. The result is valid even if the aggregating elements are $ARMA$ processes. In this case, the condition to be satisfied is that, generally speaking, the probability of extracting a unit root in the autoregressive component should diminish slowly enough. Moreover, it is important to stress that the result

[8] Instead for the integral in (16), to diverge we need the stronger condition $f(\beta) \geq K$, as $\beta \to 0$ which clearly implies the former one. Moreover, the presence of the N and N^2 terms in (15) and (16) does not affect the result as we assume that the above arguments hold for a large but finite N.

does not depend in any way on the nature of the idiosyncratic and common shocks, given their stationarity, nor on the type of dependence between them. Mankiw, Romer, and Weil (1992) argue that the slow speed of convergence observed empirically can be reconciled quantitatively with the neoclassical growth model if the capital share is sufficiently big and around 0.8. This result, on the other hand, gives a different rational for the low rate of convergence found in the empirics of the Solow growth model based on aggregation of cross-sectional heterogeneous units [9].

Intuitively, long-range dependence means that shocks arbitrarily far away in time still exhibit some influence on the future dynamics of the process. Cross-sectional aggregation annihilates the Markovian property implicit in standard short memory processes, provided that there are some units with a sufficient amount of persistence. In this case, to keep track of the future dynamics of the aggregate system we must recover the past history of the units of the system if we want to know the relative distribution of disequilibria in the economy.

[9] As an example we can consider Granger (1980) formulation where the coefficients β_k are drawn (independently both of the idiosyncratic shocks, ϵ_t^i, and common shocks, η_t) from a $Beta(p,q)$ distribution. Thus we get that the integrand function (neglecting unimportant constant terms) is given by

$$\frac{(1-\beta)^{p-1}\beta^{q-1}}{\beta^2},$$

thus yielding the condition $q < 2$ which coincides with what Granger (1980) obtained by expanding the integral in terms of autocovariances. In fact, in this case, the aggregate process can be shown to display long memory with parameter $d = 1 - q/2$.

5 Generalizing the concept of β-convergence

In this version of the Solow model augmented by cross-sectional heterogeneity, it seems reasonable to propose the following definitions of β-convergence:

(i) An economy has no tendency to converge towards either its own or the common steady state if, after fitting either a country specific or a common (linear) trend respectively, the parameter of fractional integration d of the residuals is greater than or equal to 1 ($d \geq 1$). In the former case we say that there is no *conditional* convergence, and in the latter that there is no *unconditional* convergence.

(ii) The case of the Solow model without cross sectional aggregation is represented by the absence of long memory that is $d = 0$. In this case, if we want to recover the rate of convergence of the economy, we must find the roots of the characteristic equation and look for the greatest solution in absolute value.

(iii) *Uniform unconditional* convergence means that if we fit a common linear trend across all the units in the sample, then the residuals exhibit a similar parameter of fractional integration d.

(iv) *Uniform conditional* convergence means that if we fit a country's specific linear trend for all the units in the sample, then the residuals exhibit a similar parameter of fractional integration d.

We consider further evidence of the exponential 2 per cent rate of convergence, if we find a parameter of fractional integration strictly between 1/2 and 1 (c.f. section 3). In order to draw inference on the parameters of long memory of the series, we employ the semiparametric approach introduced by Geweke and Porter Hudak (1983). Rigorous analysis of this estimator is given in Robinson (1995) who established consistency and asymptotic normality of the estimator. The result has also been developed in a multivariate framework, a novel feature in this literature, representing an important property which is necessary in applying this estimator to a multicountry issue such as the question of convergence. Robinson (1995) results are valid without assuming any 'a priori' restrictions on the degree of dependence in the data. This allows for 'anti-persistence' ($-1/2 < d < 0$), 'weak' ($d = 0$) or 'long memory' ($0 < d < 1/2$), the only restriction on the parameter space being finite variance ($\mid d \mid < 1/2$). We defer to Robinson (1995) for a formal analysis of the log-periodogram estimator, describing the main features in the Appendix.

6 Empirical results

To motivate our conjecture that the per capita GDP is approximated by a long memory process, let us consider Figure 2. Interpreting the result according to *Definition 1*, Figure 2 shows that the periodogram displays a peak at the origin for each of the series in our sample. This is what Granger (1966) defines as the 'typical spectral shape of an economic variable' and is the main feature of a long memory process [10].

Figure 2 plots the logged periodogram against twice the logged frequency. As shown in section 3, the slope of the interpolating line expresses approximately the parameter d. The unit root case is represented by the line with slope $-\pi/4$. It is evident that the interpolating line is always flatter than the bisector, thus supporting the lack a unit root. Nevertheless, the slope appears still positive and more precisely between 1/2 and 1.

Table 2 reports the estimates based on the log-periodogram estimator [11]. Most of the parameters of fractional integration d are less than one even if with a high standard error.

As we are interested in the OECD countries as a group, we use an induced test based on the sequential Bonferroni approach[12]. We want to test for the existence of a number d_0 strictly less than 1, such that all the parameters of

[10] For an analysis of the behaviour of the periodogram for non-stationary processes see Hurvich and Ray (1994).

[11] Diebold and Rudebusch (1989) have used a similar estimator, valid in a univariate case only (Geweke and Porter Hudak 1983). The multivariate framework, the gains in efficiency and computability of the Robinson (1995) estimator motivate our choice in using the latter instead of the former, thus explaining the difference in the estimates of the parameter d for the US case obtained by Diebold and Rudebusch (1989). The appendix reviews the main features of the estimating procedure.

[12] This procedure yields a conservative yet consistent test (Gourieroux and Monfort 1989, Property 19.7). The exact test for a one-side multivariate hypothesis (Gourieroux and Monfort 1989, Chapter 21) is not implementable when the number of constraints is greater than two.

Table 2		
Log-periodogram estimates of d		
Country	Conditional	Unconditional
Belgium	0.52	0.55
Denmark	0.84	0.55
Finland	0.99	0.98
France	0.56	0.94
Germany	0.83	0.83
Italy	0.56	0.65
Netherlands	1.11	1.26
Norway	0.81	0.82
Sweden	0.58	1.30
Switzerland	1.03	0.84
U.K.	0.58	0.58
Australia	0.69	0.75
New Zealand	0.85	0.85
Canada	0.97	0.96
U.S.A.	0.57	0.46
Japan	0.61	0.92
Asymptotic S.E.	0.177	0.177
Wald test statistic	1.24e+16 (0.0)	1.62e+16 (0.0)

The estimation procedure is described in the Appendix.
The Wald test statistic is distributed as a χ^2
with 15 degrees of freedom under the hypothesis
$H_0 : d_1 = d_2 = \ldots = d_{16}$.
P-values are reported in parentheses.

fractional integration of the OECD countries in the sample are less than or equal to d_0. For an overall level of significance of 10 per cent, we examine the country with the highest ex post probability of rejecting the null hypothesis and set the significance level using the total number of countries examined.

The results of this procedure are reported in Figure 3. The horizontal line represents the 10 per cent critical value of the t-statistics such that the null hypothesis is rejected. The x-axis represents the coefficient d_0 considered under the null. The negatively sloped line shows the actual t-statistics calculated for different null hypotheses. Figure 3 shows that a non empty set of values of d_0, with lower bound strictly less than one, exists such that the null hypothesis that the parameter of fractional integration of all the OECD countries are less than d_0 cannot be rejected at the 10 per cent significance level. We also note that this set always lies above the value 1/2.

The empirical results can be summarized as follows:

(i) GDP per capita of all the countries in the sample exhibit long memory ($d > 0$). In our framework this suggests that the economy behaves as an aggregation of Solow models rather than as a Solow model itself.

(ii) The hypothesis that all the OECD countries are non-stationary and mean reverting ($1/2 < d < 1$) cannot be rejected using the induced test based on the Bonferroni procedure (Figure 3).

(iii) We found the 2 per cent rate of convergence in the form of a parameter of fractional integration strictly between 1/2 and 1.

(iv) The rates of convergence are very low and similar across countries even if the rate of convergence is not *uniform* as the null hypothesis that the coefficients of fractional integration are constant across countries is strongly rejected (see Table 2).

(v) As the order of integration of different OECD countries are different, time series tests of convergence based on cointegration are misspecified.

(vi) We conclude that there is unconditional convergence across OECD countries and the rates of convergence are pretty similar even if the test rejects the null of exact equality of the coefficients.

7 Conclusions

In this paper we place standard approaches to test for β-convergence in a more general framework. In order to do so we join different strands of literature, the aggregation theory of dynamic economic models, certain elements of the theory of long memory processes and the literature on the empirics of growth.

There is considerable evidence that the (de-trended) per capita GDP is well approximated at the low frequencies by a long memory process displaying non-stationarity together with mean reversion, stressing the importance of capturing in the very long run the true rate of convergence. We then find primitive conditions under which long memory arises naturally as the result of aggregating heterogeneous units in the same economy and we then apply it to an extension of the Solow-Swan model augmented by cross-sectional heterogeneity.

Finally we draw robust inference on the possibility of conditional and unconditional β-convergence among the OECD countries and as a result we support the conclusion of the well established 'Barro regression', and reconcile the time series with the cross sectional evidence.

Some questions still remain open. In particular, we stress that the degree of persistence is different among OECD countries. This begs the question of whether the underlying economic structure of OECD countries is different and requires further investigation into which country's specific economic mechanism makes long memory arise in the real world.

8 Appendix: the log-periodogram estimator

Following Robinson (1995), let us suppose that the time series under study is given by the G-dimensional real valued vector $Z_t = (Z_{1,t}, \ldots, Z_{G,t})'$. The $(g,h)th$ element of the spectral density matrix $f(\lambda)$ is denoted by $f_{gh}(\lambda)$. For (C_g, d_g) , $g = 1, ..., G$, satisfying $0 < C_g < \infty$ and $\mid d_g \mid < 1/2$ it is assumed that[13]

$$f_{gg}(\lambda) \sim C_g \lambda^{-2d_g} \quad \text{as } \lambda \to 0^+ .$$

This represents the only assumption, beside integrability to ensure stationarity, on the spectrum which motivates the semiparametric nature of the estimator of the $G + G$ parameters (C_g, d_g), $g = 1, \ldots, G$.

[13] Basically as in *Definition 1* for each component Z_{gt}.

The periodogram[14] for the g-th component $Z_{gt}, t = 1, \ldots, T$, with T being the sample size, is expressed by

$$I_g(\lambda) = \frac{1}{2\pi T} \mid \sum_{t=1}^{T} Z_{gt} e^{it\lambda} \mid^2 , \, g = 1, \ldots, G \, . \tag{18}$$

Defining the Fourier frequency $\lambda_j = \frac{2\pi j}{T}$ one has to define the log-periodogram

$$Y_{gk} = ln(I_g(\lambda_k)) \, , \, g = 1, \ldots, G \, , k = l+1, \ldots, m \, . \tag{19}$$

The positive integer m is the user-chosen bandwidth number and the positive integer l is the user-chosen trimming number[15]. In this context we need only say that the asymptotic results require that m and l both tend to infinity with T but more slowly together with $l/m \to 0$. Then defining the unobservable random variables U_{gk} by the following set of regressions

$$Y_{gk} = c_g - d_g(2ln\lambda_k) + U_{gk}, \;\; g = 1, \ldots, G, \;\; k = l+1, \ldots, m. \tag{20}$$

where $c_g = ln\, C_g + \psi(1)$ which involves the Digamma function $\psi(z) = (d/dz)ln\Gamma(z)$, with $\Gamma(.)$ being the Gamma function.

Then the OLS estimates of $c = (c_1, \ldots, c_G)'$ and $d = (d_1, \ldots, d_G)'$ are given by $\tilde{c}, \tilde{d}$

$$\begin{bmatrix} \tilde{c} \\ \tilde{d} \end{bmatrix} = vec(Y'X(X'X)^{-1}) \, ,$$

$$X = (X_{l+1}, \ldots, X_m)' \, , \, Y = (Y_1, \ldots, Y_G)' \, ,$$

$$X_k = (1, -2ln\lambda_k)' \, , \, Y_k = (Y_{g,l+1}, \ldots, Y_{g,m})' \, .$$

[14]Practically one will consider the periodogram at the Fourier frequencies only, thus making irrelevant the demeaning of the series by the sample mean (skipping the zero frequency).

[15]We refer to Robinson (1995) for a thorough discussion on the concepts and the roles played in the asymptotic theory by these two user-chosen numbers.

Expressing as usual the OLS residuals as

$$\tilde{U}_k = Y_k - \tilde{c} + \tilde{d}(2ln\lambda_k)\,,\; k = l+1,\ldots,m\,, \tag{21}$$

and the matrix of sample variances and covariances

$$\tilde{\Omega} = \frac{1}{m-l}\sum_{i=l}^{m}\tilde{U}_k\tilde{U}_k', \tag{22}$$

one gets that the OLS standard errors for $\tilde{d}_g$, $g = 1,\ldots,G$, are given by the square root of the $(G+g)$th diagonal element of the matrix $(Z'Z)^{-1}\otimes\tilde{\Omega}$.

This estimating procedure allows for cross-equation restrictions as, for example, that all (or some of) the G series are characterized by a common parameter of long memory, viz.

$$d_g = \delta, \qquad g = 1,\ldots,G\,,$$

or in matrix formulation

$$d = Q\delta,$$

where $Q = (1,1,\ldots,1)'$ is a $G\times 1$ vector and δ is a scalar representing the unknown common long memory parameter. Thus the GLS estimator $\hat{c}$ and $\hat{d}$ is given by

$$\begin{bmatrix}\hat{c}\\ \hat{\delta}\end{bmatrix} = \left\{\begin{bmatrix} I_G & 0\\ 0 & Q'\end{bmatrix}(X'X\otimes\tilde{\Omega}^{-1})\begin{bmatrix} I_G & 0\\ 0 & Q\end{bmatrix}\right\}^{-1}\begin{bmatrix} I_G & 0\\ 0 & Q'\end{bmatrix} vec(\tilde{\Omega}^{-1}Y'X)\,.$$

When there are no restriction we set $Q = I_G$ and we obtain again the OLS estimator[16].

[16] Also to obtain a consistent estimate of C_g $g = 1,\ldots G$, one has to consider the relation $C_g = exp(c_g - \psi(1))$.

Under certain regularity conditions (Robinson 1995) among which Gaussianity of the process Z_t the following asymptotic results are obtained, which allows to perform standard inference on the OLS and GLS estimators. For the OLS Robinson established

$$\begin{bmatrix} \frac{m^{1/2}}{ln\, n}(\tilde{c}-c) \\ 2m^{1/2}(\tilde{d}-d) \end{bmatrix} \rightarrow_d N\left(0, \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \otimes \Omega\right), \tag{23}$$

and for the GLS

$$\begin{bmatrix} \frac{m^{1/2}}{ln\, n}(\hat{c}-c) \\ 2m^{1/2}(\hat{d}-d) \end{bmatrix} \rightarrow_d N\left(0, \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \otimes Q(Q'\Omega^{-1}Q)^{-1}Q'\right). \tag{24}$$

One obtains a consistent estimate of Ω by using (22). Considering each $\tilde{d}_g$ individually the general result in (23) becomes

$$2m^{1/2}(\tilde{d}_g - d_g) \rightarrow_d N(0, \frac{\pi^2}{6}).$$

The results allow us to make use of all the regression theory. In particular one can build a Wald test for linear restriction expressed by

$$H_0: \qquad Pd = 0,$$

where P is a $H \times G$ matrix of rank $H < G$. The test statistic is given by

$$\tilde{d}'P'[(0,P)\left\{(X'X)^{-1} \otimes \tilde{\Omega}\right\}\begin{pmatrix} 0 \\ P' \end{pmatrix}]^{-1}P\tilde{d}, \tag{25}$$

which under H_0 is asymptotically distributed like a central χ^2 with H degrees of freedom.

Estimating Procedure - Firstly we detrend the data fitting both by a country specific and a common trend. The former has been estimated with

OLS, the latter with GLS. We then evaluate the order of integration of the residuals[17]. A preliminary analysis of the parameters $d_g\,, g = 1, \ldots, G$ gives estimated values greater than 1/2, thus out of the admissible region for the asymptotic results to be valid. If we first apply the first-difference operator to the data, the estimates would be totally independent of the type of β-convergence we are considering (conditional and unconditional). In fact, the periodogram evaluated at the Fourier frequencies is independent of any shift of location. For this reason we prefer to difference fractionally the data prior to estimation, by multiplying them by $(1 - L)^q$, $q = 0.5$. Obviously in doing so we have to approximate a series with a finite sum. Our choice of $q = 0.5$ reflects the trade-off between differentiating 'enough' (large q) to obtain estimates in the stationary region and minimizing the approximation from using a sum instead of a series (small q) [18]. To initialize the fractional filter $(1 - L)^q$ we use the first 10 observations in the sample.

Choice of Trimming and Bandwidth - In our application we choose

[17]It is reasonable to ask if the properties of the theoretical disturbances carry over to those residuals after detrending the data with either the country specific or the common trend [see i.e. Nelson and Kang (1981)]. There are good reasons to believe that they do so, once a semi-parametric frequency domain approach is undertaken. Nelson and Kang (1981) show that the regression of a driftless random walk against a time trend delivers residuals exhibiting a periodogram with a single peak at a period equal to 0.83 of sample size, thus asymptotically at frequency zero as one would expect. Put into words, the memory of the process is entirely reflected in the residuals.

[18]Even if not formally proved, we follow the empirical literature of long memory processes conjecturing that asymptotically this approximation becomes negligible. Further, the results are globally robust with respect to the choice of q .

a trimming coefficient equal to two ($l = 2$) so that we avoid the first periodogram ordinate. Unfortunately a complete theory for the optimal choice of the trimming and the bandwidth is still missing for this estimator but it seems that choosing a trimming bigger than one increases the performance of this estimator in finite samples (Hurvich and Ray 1994). Because of this reason we report estimates based on the same criterium as the one used by Diebold and Rudebusch (1989) for their univariate analysis, that is $m = T^{0.525}$ after checking for robustness under alternative bandwidths [19]. It is nevertheless important to point out that the empirical results are very robust to changes in the choice of the trimming and the bandwidth.

[19]We defer to Beran (1994) for a review on parametric and semi non-parametric estimation in a long memory framework.

Figure 1: The dashed and bold lines represent the country-specific (OLS) and common (GLS) trends, respectively. The solid line represents logged GDP.

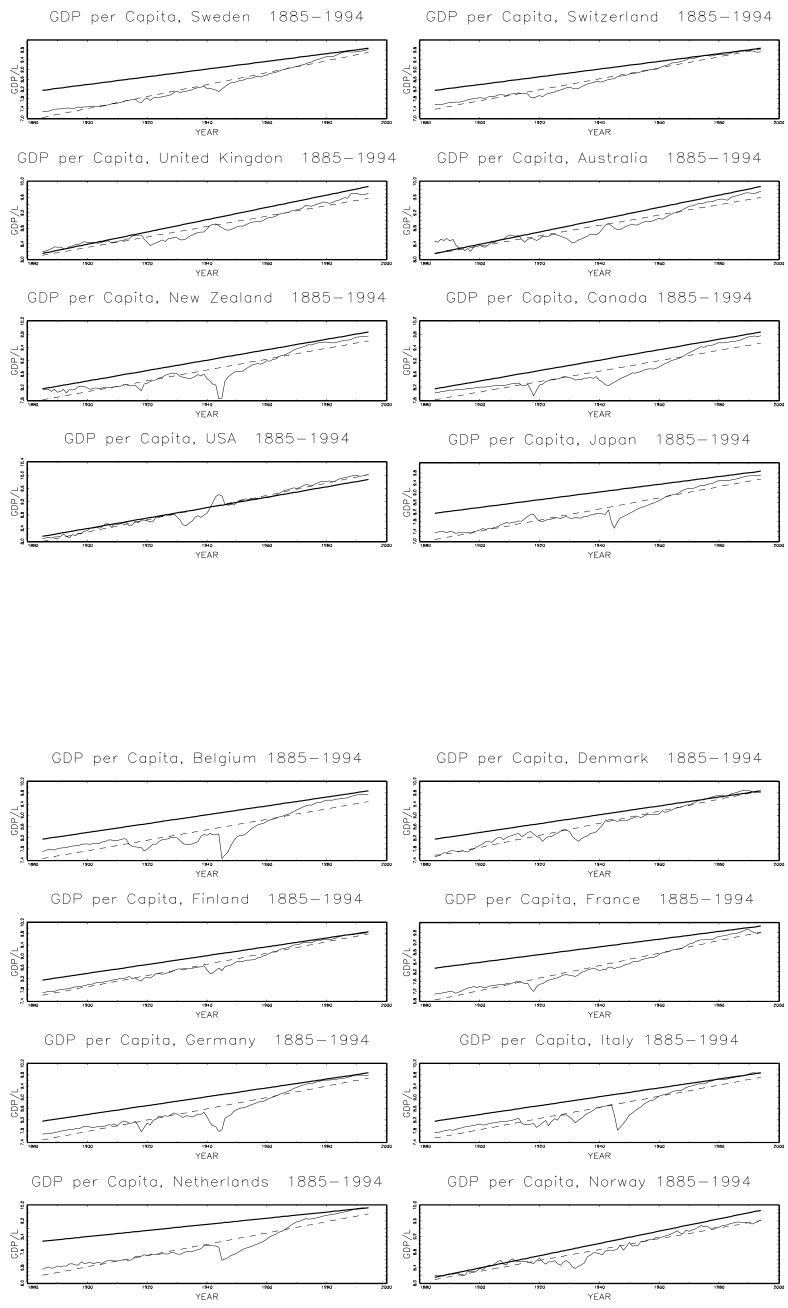
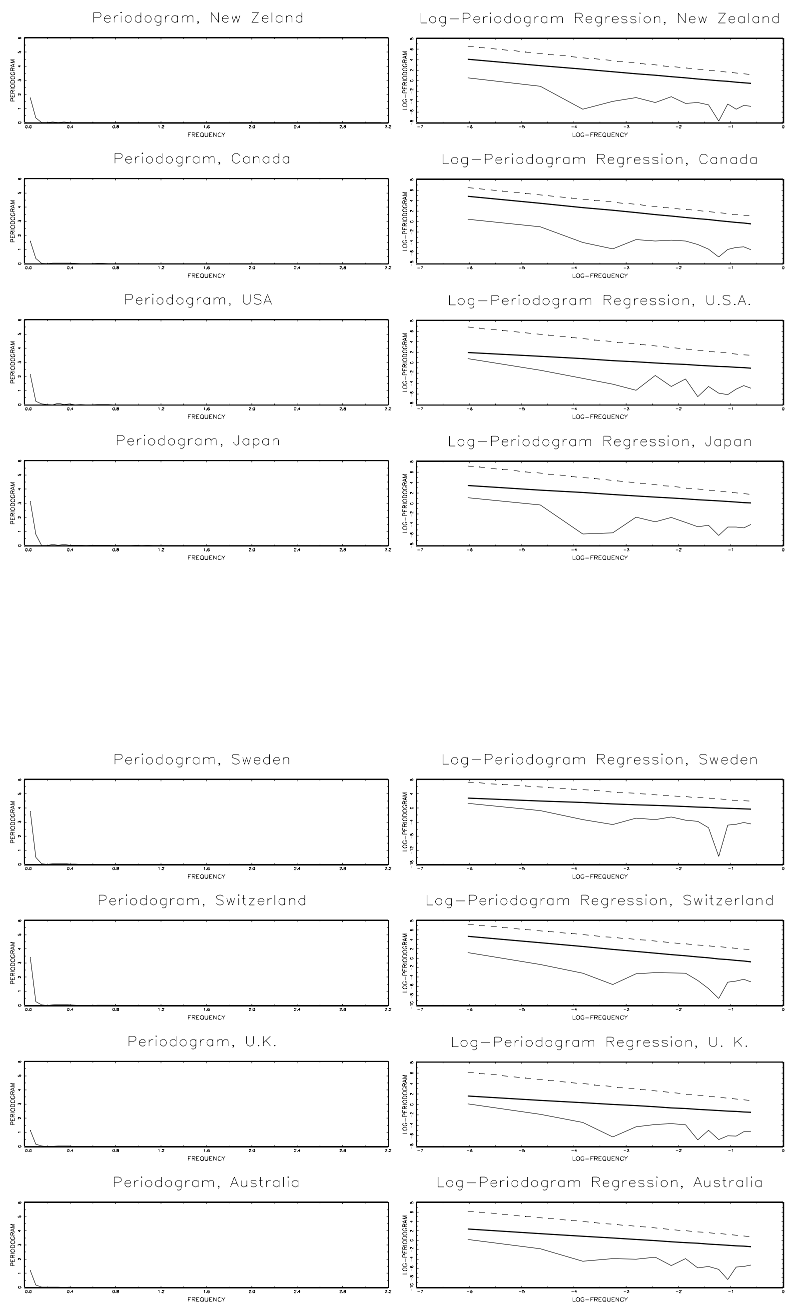
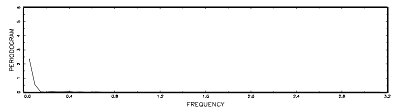


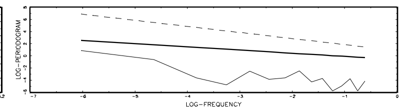
Figure 2: The left hand side column displays the periodogram of the logged GDP (1865-1994) for the 16 OECD countries considered here. The right hand side column displays three lines versus the logged frequency: the solid line represents the logged periodogram ordinates, the bold line represents the OLS interpolating line (cf. Table 2) while the dashed line represents the unit root case (slope $-\pi/4$). An interpolating line flatter than the bisector corresponds to a value of the long memory parameter d smaller than one.



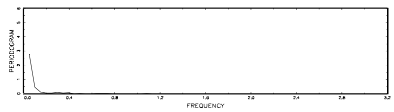
Periodogram, Belgium



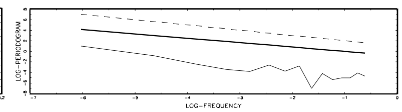
Log−Periodogram Regression, Belgium



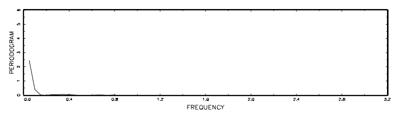
Periodogram, Denmark



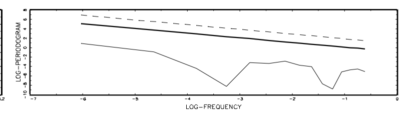
Log−Periodogram Regression, Denmark



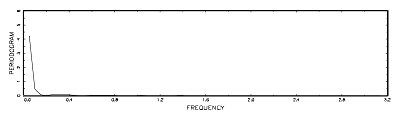
Periodogram, Finland



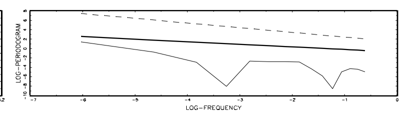
Log−Periodogram Regression, Finland



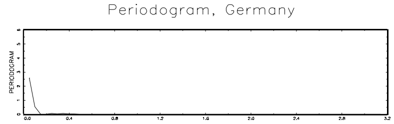
Periodogram, France



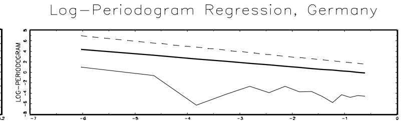
Log−Periodogram Regression, France



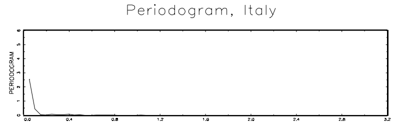
Periodogram, Germany



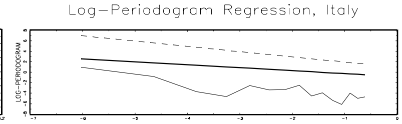
Log−Periodogram Regression, Germany



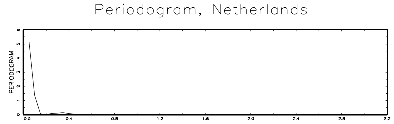
Periodogram, Italy



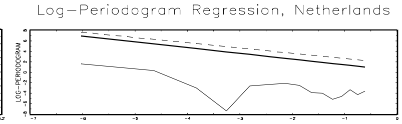
Log−Periodogram Regression, Italy



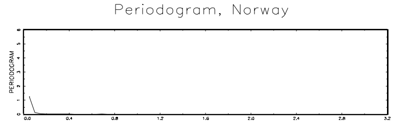
Periodogram, Netherlands



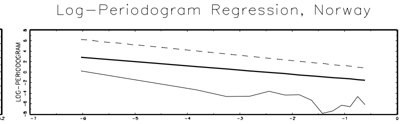
Log−Periodogram Regression, Netherlands



Periodogram, Norway



Log−Periodogram Regression, Norway



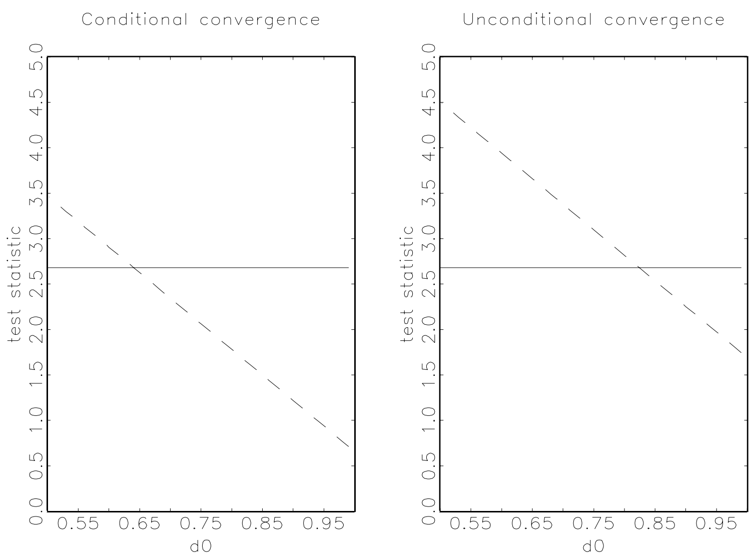


Figure 3: The test statistic for the null $H_0 \quad d_i \leq d_0$, $i = 1, \ldots 16$ is plotted for different values of d_0. The horizontal line represents the critical value for a 10 % significance level.

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