BANCA D'ITALIA

Temi di discussione

del Servizio Studi

Endogenous Growth with Intertemporally Dependent Preferences

by G. Ferraguto and P. Pagano

Number 382 - October 2000

ENDOGENOUS GROWTH WITH INTERTEMPORALLY DEPENDENT PREFERENCES

by Giuseppe Ferraguto* and Patrizio Pagano**

Abstract

This paper presents an endogenous growth model with intertemporally dependent preferences and “Ak” technology. We derive sufficient conditions for a balanced growth path to be an equilibrium, provide a full characterization of the equilibrium dynamics of the economy, and explore the implications of habit formation for the patterns of cross-country growth and convergence. Finally, we show that the alternative departure from the standard assumption of isoelastic preferences represented by the use of a Stone-Geary utility function can be interpreted as a special case of the model with habit formation. Our results highlight the importance of preferences in the dynamics of growth, a point neglected in most of the literature.

JEL classification : D91, O41, E32.

Keywords: preferences, habits, growth.

Contents

* Bocconi University, Istituto di Economia Politica.

** Bank of Italy, Research Department.

1. Introduction[1]

This paper presents an endogenous growth model that departs from the assumption of time-separable, constant intertemporal elasticity of substitution (CIES) preferences almost ubiquitous in the literature. The idea, shared with Ryder and Heal (1973), is rather that it is intuitively plausible to assume that past consumption choices and/or the social environment affect the utility an individual derives from consuming a given bundle of goods, we assume that the representative agent's instantaneous utility u is determined by comparing current consumption c to some reference stock, or standard, z, called alternatively "consumption experience", "habits", or "customary consumption", so that $u(c, z)$. With z taken to be a weighted average of past consumption levels, this choice leads to preferences that Ryder and Heal termed "intertemporally dependent".

These preferences represent a tractable departure from the hypothesis of a time-separable utility function, and have been used in a variety of different contexts. To mention just a few, Ryder and Heal (1973) and Boyer (1975, 1978) investigated their implications for the neoclassical optimal growth model, showing that they lead to a richer dynamic behavior of the main variables around an unchanged steady-state (the modified golden rule). Time non-separable preferences can help to reconcile rational choice theory with apparently irrational behavior (Becker and Murphy, 1983), to explain various time-series features of consumption data (Deatonl, 1992), and to shed light on open economy macroeconomic issues (Obstfeld, 1992; Mansoorian, 1993). Finally, time nonseparable preferences have more recently been used mainly in finance, often in the attempt to resolve the "equity premium puzzle". This growing literature includes contributions by Constantinides (1990), Boldrin et al. (1995), Campbell and Cochrane (1999), where instantaneous utility is assumed to be a power function of the difference between current consumption and habits, and by Abel (1990) and Gali (1994), where it is supposed to be a power function of the ratio between current consumption and the reference stock.

[1] We thank seminar participants at the Bank of Italy, Bocconi University, the 1999 SED Conference in Alghero, Italy (June 27-30, 1999) and the 7th Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics (May 24-26, 2000). Errors are our own. The opinions expressed in this paper do not necessarily reflect those of the Bank of Italy. E-mail: giuseppe.ferraguto@uni-bocconi.it, pagano.patrizio@insedia.interbusiness.it.

In this paper, we introduce the assumption of intertemporally-dependent preferences in an otherwise standard Ak growth model. While the two hypotheses of a linear production function and preferences belonging to the Ryder and Heal class make our setting closely akin to the one recently investigated by Carroll, Overland, and Weil (1997), our analysis differs from theirs in many, substantial respects.

In fact, while these authors assume from the outset a specific functional form for the instantaneous utility function (incidentally, one that — contrary to what they state — is not concave for the values of parameters they assume in their simulations), we work with a generic u, and provide sufficient conditions that this latter has to meet for a balanced growth path to qualify as an equilibrium when preferences are intertemporally dependent. This also allows us to unveil the difference between the "adjacent" and "distant" complementarity cases, and the ensuing dynamics, which are central to most of the literature on habit formation. Under adjacent complementarity, an increase in consumption experience induces the individual to want to increase current consumption, so that c and z will be positively related in equilibrium. The opposite is true when preferences are such that complementarity is distant. We choose to focus mainly on the case of adjacent complementarity, as we regard the addictive behavior it implies as more relevant in the one-sector framework we consider, where c has to be interpreted as consumption of a wide bundle of goods. Besides being theoretically plausible, this case seems also to be empirically relevant, as Fuhrer and Klein (1998) — who provide evidence suggesting that habit formation characterizes aggregate consumption behavior among most of the G-7 countries by testing a model that implies adjacent complementarity — have recently shown. Nevertheless, our analysis also encompasses the opposite case of complementarity, and all the results we present can be readily extended to consider the implications of this alternative behavioral assumption.

Furthermore, we adapt and extend a graphical device first introduced by Obstfeld (1992) in his analysis of a small open economy facing a constant world interest rate, to provide a pictorial representation of the equilibrium dynamics resulting from our growth model with intertemporally dependent preferences. This representation is, we believe, both simple and transparent, and helps to grasp in an intuitive way the somewhat tangled interactions among variables that set in under habit formation.

Finally, and more importantly, Carroll et al. mainly focus on the difference between what they term "inward-looking" and "outward-looking" cases. In the former, an individual's habits are accumulated by his own consumption; in the latter, they reflect aggregate per capita, or average, consumption choices. While this difference is interesting, the two cases lead to qualitatively similar transitional dynamics of the economy toward its balanced growth path. For this reason, focus on the case by now standard in the literature (of "inward", or "internal" habits), and — having provided a full characterization of equilibrium dynamics under habit formation — expand on the patterns of cross-country growth and convergence implied by intertemporally dependent preferences.

We show that the latter are consistent with two kinds of convergence of a country's per capita income to its long-run growth path: a convergence "from above", with growth and saving rates increasing over time and approaching asymptotically the constant value they will assume in balanced growth, and a convergence "from below", with growth and saving rates initially high, but then declining over time. The model predicts that the first type of convergence will be displayed by countries that start with a higher ratio of initial endowment of consumption experience to physical capital; on the other hand, convergence "from below" should be found in countries characterized by an initially lower ratio of consumption experience to physical capital. Under adjacent complementarity, the latter will initially save a lot, growing along the transition at higher rates, though declining towards a constant, steady-state level.

The available evidence suggests that both kinds of convergence actually occur. It also shows that there is no clear association between the starting level of per capita income in a country and the type of convergence it latter will display along the transition. Our model is consistent with this fact. For instance, a country that is "poor" at time zero could also have a level of reference consumption which is low in absolute terms, but high when compared with its endowment of physical capital — maybe because of the lower bound placed on z by subsistence consumption, or because its consumption standards are set by a comparison with richer countries with which it interacts closely. It will therefore converge "from above" in the very same way as a "rich" country that starts off better endowed with both customary consumption and physical capital, but has a similar initial habits-to-capital ratio.

These different patterns of convergence make the model consistent with a variety of outcomes in terms of time evolution of income differences among countries characterized by different initial conditions on the state variables: they include divergence, convergence, and leapfrogging. We also show that, although these different patterns of growth and convergence are consistent with the neoclassical model with an exogenous rate of technical progress, the latter has counterfactual implications that are absent in our setting.

We also compare the results of our model with those derived assuming a Stone-Geary instantaneous utility function — a different departure from the hypothesis of CIES preferences that, in a growth setting, has been proposed by Christiano (1989) and Rebelo (1992). We show that these preferences can be interpreted as belonging to the intertemporally dependent class, once one assumes that the stock of habits is constant over time. However, while in this case there is the implication that the rates of saving and growth must necessarily be increasing during the transition (so that, using the terminology introduced above, countries are predicted to converge "from above"), allowing the dynamics of habits to feed back to consumption and accumulation choices opens up the possibility of both kinds of convergence. Since the empirical evidence suggests there is no clear, common pattern with which countries converge to their long-run growth path, this leads us to prefer our model with habit formation over the alternative Stone-Geary specification of preferences.

It is worth emphasizing that our model does not intend to propose a novel explanation of long-run growth: as in the standard Ak model, this latter is driven by the absence of decreasing returns on physical capital. Rather, we think its main contribution lies in the analysis of the rich dynamics stemming from a plausible, and tractable, departure from the assumption of time-separable, isoelastic preferences. These dynamics should be superimposed on those implied by models that give a more realistic account of the production side of the economy, allow for the existence of barriers to the international diffusion and adoption of technology, take into account the role played by Governments and institutions and, more generally, the countless factors we deliberately neglect, but that which undoubtedly play an important role in the process of growth.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 introduces the concepts of equilibrium adopted and gives sufficient conditions for the existence of a steady-state balanced growth path. In Section 4, through a normalization of variables, we transform our original problem into one that involves only variables that take on constant values in balanced growth, and provide a full characterization of the equilibrium dynamics of the latter. In Section 5 we present a graphical device that helps determine the equilibrium dynamics of the economy starting from an arbitrary set of initial conditions on the state variables, and provide the economic intuition for the transitional dynamics implied by intertemporally dependent preferences. In Section 6 we describe the dynamics of the original variables implied by the solution of the transformed problem, and discuss the implications of our model in terms of patterns of cross-country growth and convergence. Section 7 presents two examples of instantaneous utility functions belonging to the intertemporally dependent class and compares our results with those derived from the assumption of Stone-Geary preferences. Section 8 concludes.

2. The model

We study a closed economy with an unbounded horizon, populated by infinitely living, identical individuals. The representative agent has preferences defined over his own consumption c, as well as on consumption experience, or habits, z, and maximizes the objective functional:

$$U(c,z) = \int_0^{\infty} e^{-\delta t} u(c(t),\, z(t))dt, \tag{1}$$

where u is the instantaneous felicity function, and δ the (positive and constant) rate of time preference.

As in Ryder and Heal (1973), we assume that consumption experience is a weighted average of the representative individual's past consumption levels c,

$$z(t) = z_0 e^{-\rho t} + \rho \int_0^{t} e^{-\rho(t-s)}\, c(s)\, ds, \tag{2}$$

where $\rho > 0$ is a constant that measures the rate of habit adjustment, and $z(0) = z_0 > 0$ is the exogenously inherited standard of living at the initial date. The larger is ρ, the higher the weight given to past consumption in determining the current level of consumption experience, and vice versa.

Differentiating equation (2) with respect to time, it follows that habits evolve according to

$$\dot{z}(t) = \rho[c(t) - z(t)]. \tag{3}$$

In the literature, one also finds the alternative specification

$$\dot{z}(t) = \rho c(t) - \alpha z(t),$$

that allows for a difference between the rate at which consumption accumulates habits and that at which the latter decay over time (α). Since it can be shown that our results are not qualitatively affected by the choice of setting $\alpha = \rho$, we opt for the simpler specification in (3). A more substantial modification would stem from the assumption that the representative agent's stock of habits is a weighted average of the aggregate per-capita (or average, or the Joneses') past consumption levels χ, rather than of his own past consumption c. The implications of this alternative assumption are studied by Carroll et al. (1997) and Ferraguto and Pagano (1999). Since the representative agent takes χ as given, although $\chi = c$ must hold in equilibrium, it introduces a "consumption externality" that breaks the equivalence between the centralized and competitive solutions of the model, with potential policy implications which are absent when habits evolve according to (3). Aside from this, the assumption that habits are accumulated by c, χ, or by a weighted average of the two does not change the steady-state growth rate of the economy, and does not lead to qualitatively different transitional dynamics to the balanced growth path that are the main object of the present paper.

We impose the following conditions on the instantaneous felicity function u, assumed to be twice continuously differentiable:

U1. $u_c > 0;\ \lim_{c \to 0} u_c = \infty,\ z \in (0, \infty);$

U2. $u_z \neq 0; u_{cz} \neq 0;$

U3. $u_{cc} < 0,\ \ u_{cc} u_{zz} - u_{cz}^2 \geq 0;$

U4. $u_c + (\frac{\rho}{\rho+g})\, u_z > 0 \;\; \forall g > 0.$

Assumption U2 restricts preferences to the intertemporally dependent class, and Assumption U3 amounts to the requirement of concavity of u in $(c, z),$ and strict concavity in c.

Finally, as will be shown in footnote 3 below, Assumption U4 (which, given Assumption U1, is always satisfied whenever u happens to be an increasing function of z) guarantees that a uniformly maintained increase in the level of consumption along a balanced growth path will increase utility.

There is only one good, which can be either consumed or invested, and whose output at each point in time is the result of the linear production function

$$y(t) = A\,k(t), \tag{4}$$

where y and k are per-capita output and capital, respectively, and A is a positive constant. We assume that individuals directly operate the economy's technology. Omitting from now on time indices whenever this choice does not risk confusion, it follows that the representative agent faces the budget constraint:

$$\dot{k} = Ak - c, \tag{5}$$

where, for simplicity, depreciation of physical capital has been assumed away — or incorporated in A. Constraint (5) captures the fact that, in the closed economy with no outside assets and identical individuals we are about to study, capital accumulation is the only possible use of savings.

Finally, in order to be able to retrieve the standard "Ak"-results as a special case of our model, we also assume:

T1. $A > \delta.$

3. Equilibrium

Given the above definitions and assumptions, we have the following definitions of competitive equilibrium and balanced-growth equilibrium, respectively.

DEFINITION 1. *A competitive equilibrium is a set of paths* $\{c(t), z(t), k(t)\}$ *that solve the maximization problem:*

$$\begin{aligned} \max \quad & U(c,z) \\ s.t. \quad & \dot{k} = Ak - c, \qquad && k(0) = k_0 > 0 \quad given, \\ & \dot{z} = \rho(c - z), \qquad && z(0) = z_0 > 0 \quad given. \end{aligned} \tag{P1}$$

DEFINITION 2. *A balanced, or steady-state, equilibrium is a solution* $\{c(t), z(t), k(t)\}$ *to the optimization problem (P1) so that* $c(t)$, $z(t)$ *and* $k(t)$ *grow at a constant rate* $g > 0$.

Equipped with these definitions, in Appendix 1 we prove the following

PROPOSITION 1. *Given Assumptions U1 through U4, and T1, a sufficient condition for constant, positive steady-state growth is homogeneity of degree* $\nu < 1$ *of the instantaneous felicity function* u. *The degree of homogeneity* ν *and the steady-state growth rate* g *of the economy will be related according to:*

$$g = \frac{A - \delta}{1 - \nu}. \tag{6}$$

That c, k and z will grow at a common rate g in a balanced equilibrium, as stated in Definition 2, can be readily verified by dividing the laws of motion of physical capital and habits by k and z respectively, and noting that the resulting growth rates of these variables will be constant if and only if the ratios c/k and c/z are also constant. As seems to be the rule in growth models where the utility function depends on a stock variable, Proposition 1 states that homogeneity of the instantaneous felicity function of the degree ν implicitly defined by equation (6) is a sufficient condition for a balanced growth path to qualify as an equilibrium[2]. In the proof of this Proposition given in Appendix 1, we also show that, when the economy evolves along this balanced path, one must have

$$g < A \tag{7}$$

for the transversality conditions associated with problem (P1) to be satisfied. In other words, the steady-state growth rate has to be less than the maximum “sustainable” rate that would be associated with zero consumption (see (5)).

Finally, it should be noted that, although Proposition 1 gives conditions under which a balanced path qualifies as an equilibrium, it does not imply that the economy will ever converge to it. As will become clear in the next Section, additional restrictions have to be placed on u to make sure that the economy asymptotically approaches a steady-state with constant, positive growth.

4. The transformed problem

Given the above results, from now on we shall assume an instantaneous utility function that is homogenous of degree $\nu < 1$. This assumption also allows us to reformulate problem (P1) in a way that greatly simplifies the analysis, and to give a graphical representation of the equilibrium evolution of the economy. To this end, following Caballé and Santos (1993), we introduce the normalized variables

$$\tilde{c}(t) = c(t)e^{-gt},$$
$$\tilde{z}(t) = z(t)e^{-gt},$$
$$\tilde{k}(t) = k(t)e^{-gt}.$$

---

[2] For an example of a model that resorts to the assumption of homogeneity of the instantaneous felicity function in a growth setting, see the one with endogenous leisure choice proposed by Rebelo (1991) along the lines of Heckman (1976). In that model, the momentary utility function depends on human capital; in our case, the stock variable on which istantaneous utility depends is consumption experience.

These new variables will remain constant along a balanced path, and $\{\tilde{c}^*, \tilde{z}^*, \tilde{k}^*\}$ will denote their steady-state, balanced growth values. They will also be referred to as "de-trended" variables, since the normalization factor e^{-gt} removes from the non-normalized ones the exponential growth trend that these latter will exhibit in a balanced equilibrium.

Next, we exploit the degree-ν homogeneity of u to transform (1) into a function of $(\tilde{c}, \tilde{z})$:

$$U(\tilde{c}, \tilde{z}) = \int_0^\infty e^{-\tilde{\delta}t} u(\tilde{c},\ \tilde{z})\, dt,$$

where

$$\tilde{\delta} = \delta - g\,\nu. \tag{8}$$

Writing the dynamic constraints in terms of de-trended consumption, habits, and physical capital, we are in a position to reformulate (P1) as follows:

$$\begin{aligned} \max \quad & U(\tilde{c}, \tilde{z}) & & \\ s.t. \quad & \dot{\tilde{k}} = (A-g)\tilde{k} - \tilde{c}, & & \tilde{k}_0 = k_0 > 0 \quad given, \\ & \dot{\tilde{z}} = \rho\tilde{c} - (\rho+g)\tilde{z}, & & \tilde{z}_0 = z_0 > 0 \quad given, \end{aligned} \tag{P1'}$$

and to write the corresponding current-value Hamiltonian function:

$$\tilde{H} = u\left(\tilde{c}, \tilde{z}\right) + \tilde{\lambda}\left[(A-g)\tilde{k} - \tilde{c}\right] + \tilde{\mu}\left[\rho\tilde{c} - (\rho+g)\tilde{z}\right].$$

The necessary conditions:

$$u_{\tilde{c}} + \rho\tilde{\mu} = \tilde{\lambda}, \tag{9}$$

$$\dot{\tilde{\lambda}} = (\tilde{\delta} - A + g)\tilde{\lambda}, \tag{10}$$

$$\dot{\tilde{\mu}} = (\tilde{\delta} + \rho + g)\tilde{\mu} - u_{\tilde{z}}, \tag{11}$$

are, with the laws of motion of $\tilde{k}$ and $\tilde{z}$, also sufficient for a maximum if the following transversality conditions are met:

$$\begin{aligned} &\lim_{t\to\infty} e^{-\tilde{\delta}t}\tilde{\lambda}(t)\tilde{k}(t) = 0, \\ &\lim_{t\to\infty} e^{-\tilde{\delta}t}\tilde{\mu}(t)\tilde{z}(t) = 0. \end{aligned} \tag{12}$$

While the co-state variable $\tilde{\lambda}$ is the shadow value of normalized capital, $\tilde{\mu}$, that — from (11) — can be written as

$$\tilde{\mu}(t) = \int_t^\infty e^{-(\tilde{\delta}+\rho+g)(s-t)}\, u_{\tilde{z}}(\tilde{c}(s), \tilde{z}(s))\, ds, \tag{13}$$

which is the shadow value of an additional unit of $\tilde{z}$. Condition (9) implies that, along an optimal path, at each time t the current marginal utility of consumption, plus the contribution of greater time-t consumption to the utility stream derived from future consumption experience — a contribution that is positive if $u_{\tilde{z}} > 0$, and negative in the opposite case — must be equal to the time-t shadow value of capital. We define the sum $(u_{\tilde{c}} + \rho\tilde{\mu})$ "the time-t full marginal benefit of $\tilde{c}$" to distinguish the present setting from the time-independent case, where the contribution of greater time-t consumption to the objective functional is given by the term $u_{\tilde{c}}$ only.

In the steady-state, $\tilde{\lambda}$ has to be constant. This requires

$$\tilde{\delta} = A - g,$$

a positive quantity by (7). Using (8), it follows that:

$$g = \frac{A - \delta}{1 - \nu},$$

an expression that gives the same steady-state growth rate of the economy derived in Proposition 1. From these results, it can immediately be verified that $\tilde{\lambda}$ will be constant throughout, at a level that we shall denote by $\tilde{\lambda}^*$ and whose expression will be derived below.

The differentiation of (9) with respect to time, using (10) and (11) and taking into account the laws of motion of $\tilde{z}$ and $\tilde{k}$, results in the following autonomous system of differential equations in $(\tilde{c}, \tilde{z}, \tilde{k})$:

$$\dot{\tilde{c}} = \left(\frac{1}{u_{\tilde{c}\tilde{c}}}\right) \cdot \left\{(A + \rho)(u_{\tilde{c}} - \tilde{\lambda}^*) + \rho u_{\tilde{z}} - u_{\tilde{c}\tilde{z}}[\rho\tilde{c} - (\rho + g)\tilde{z}]\right\}, \tag{14a}$$

$$\dot{\tilde{z}} = \rho\tilde{c} - (\rho + g)\tilde{z}, \tag{14b}$$

$$\dot{\tilde{k}} = (A - g)\tilde{k} - \tilde{c}. \tag{14c}$$

From the last two equations, the steady-state levels of the detrended habits and capital are:

$$\tilde{z}^* = \left(\frac{\rho}{\rho + g}\right) \cdot \tilde{c}^*, \tag{15a}$$

$$\tilde{k}^* = \left(\frac{1}{A - g}\right) \cdot \tilde{c}^*. \tag{15b}$$

To derive $\tilde{c}^*$, notice that (14a) implies that, in the steady-state,

$$u_{\tilde{c}}(\tilde{c}^*, \tilde{z}^*) + \frac{\rho}{A + \rho} u_{\tilde{z}}(\tilde{c}^*, \tilde{z}^*) = \tilde{\lambda}^*,$$

a positive quantity[3]. Given $(\tilde{z}^*/\tilde{c}^*) = \left(\frac{\rho}{\rho+g}\right)$, the homogeneity of degree $(\nu - 1)$ of $u_{\tilde{c}}$ and $u_{\tilde{z}}$ implies that the left hand side of the above equation is equal to $(\tilde{c}^*)^{\nu-1} \cdot \left[u_{\tilde{c}}(1, \frac{\rho}{\rho+g}) + (\frac{\rho}{A+\rho}) \cdot u_{\tilde{z}}(1, \frac{\rho}{\rho+g})\right]$, so that:

$$\tilde{c}^* = \left[\frac{\tilde{\lambda}^*}{u_{\tilde{c}}(1, \frac{\rho}{\rho+g}) + (\frac{\rho}{A+\rho}) \cdot u_{\tilde{z}}(1, \frac{\rho}{\rho+g})}\right]^{\frac{1}{\nu-1}}. \tag{15c}$$

We show in Appendix 2 that $\tilde{c}^*$, and therefore $\tilde{\lambda}^*, \tilde{z}^*$ and $\tilde{k}^*$, are uniquely pinned down by the need to satisfy the transversality conditions (12), given the initial conditions on the state variables. This result, and the assumptions placed on u, imply that the steady-state equilibrium just derived is unique[4].

To investigate the dynamic evolution of the economy and the stability properties of the steady-state just characterized, in Appendix 2 we linearize system (14) around the steady-state (15), and show that this latter is a saddlepoint provided that

$$j \equiv -\frac{(A+g+2\rho)u^*_{\tilde{c}\,\tilde{z}} + \rho u^*_{\tilde{z}\,\tilde{z}}}{u^*_{\tilde{c}\,\tilde{c}}} < \frac{(A+\rho)(\rho+g)}{\rho}, \tag{16}$$

where starred derivatives are evaluated at the steady-state.

The crucial role played by the sign and size of j in determining the dynamic evolution of consumption and habits in a model with intertemporally dependent preferences was first pointed out by Ryder and Heal (1973). In their terminology, one has "adjacent complementarity" — that is, complementarity between consumption at adjacent dates, a property of preferences that Becker and Murphy (1988) identify with addiction — if $j > 0$,

---

[3] This is also true when the marginal utility of habits happens to be negative. In this case, since $g < A$, assumption U4 implies that $u^*_{\tilde{c}} + (\frac{\rho}{A+\rho})u^*_{\tilde{z}} > u^*_{\tilde{c}} + (\frac{\rho}{\rho+g})u^*_{\tilde{z}} > 0$.

[4] Evaluated at this steady state, istantaneous utility is $u(\tilde{c}^*, \tilde{z}^*) = u(\tilde{c}^*, (\frac{\rho}{\rho+g})\tilde{c}^*)$. The derivative of this expression with respect to $\tilde{c}^*$ is $u^*_{\tilde{c}} + (\frac{\rho}{\rho+g})u^*_{\tilde{z}}$, which is positive by Assumption U4. To see what this latter implies, let us assume that the economy is on a balanced growth path, with consumption and habits growing over time at a constant, positive rate g, and consider the two sequences $\{c(t), z(t)\}_{t=s}^{\infty}$ and $\{c'(t), z'(t)\}_{t=s}^{\infty}$, with $c'(s) > c(s)$ and $\dot{c}(t)/c(t) = \dot{c}'(t)/c'(t) = g$, $t = s, ..., \infty$. Assumption U4 amounts to the (in our opinion, sensible) requirement that the second sequence will yield greater utility to the individual. Notice that this assumption is the generalization to a growth setting of the non-satiation condition in Ryder and Heal (1973, p.3).

and "distant complementarity" if $j < 0$. Notice that assumption U3 implies that j is always negative if $u_{\tilde{c}\tilde{z}} < 0$. To have $j > 0$, one needs $u_{\tilde{c}\tilde{z}} > 0$ and large enough. When this is the case, condition (16) places an upper bound on the degree of adjacent complementarity consistent with saddlepath stability of system (15). We show in Appendix 2 that values of j that violate condition (16) lead to instability, or to a violation of the hypothesis of concavity of u; both instances are ruled out by assumption in the present analysis, so that (16) always holds.

For this case, in the same Appendix we show that equilibrium normalized consumption, habits and physical capital evolve according to:

$$\tilde{c}(t) - \tilde{c}^* = [\tilde{c}_0 - \tilde{c}^*] \cdot e^{-\psi t}, \tag{17a}$$

$$\tilde{z}(t) - \tilde{z}^* = \omega_1 \cdot [\tilde{c}(t) - \tilde{c}^*], \tag{17b}$$

$$\tilde{k}(t) - \tilde{k}^* = \omega_2 \cdot [\tilde{c}(t) - \tilde{c}^*], \tag{17c}$$

where $-\psi$ is the negative, real characteristic root associated with the linearized version of system (14), and

$$\omega_1 \equiv \frac{\rho}{\rho + g - \psi}, \qquad \omega_2 \equiv \frac{1}{A - g + \psi}.$$

While $\omega_2 > 0$ always, in Appendix 2 we prove that ω_1 has the same sign as j. We also show that steady-state de-trended consumption is given by:

$$\tilde{c}^* = \frac{(A-g)(\rho+g)}{\psi(A+\rho)} \left[-\frac{1}{\omega_1}\, z_0 + \frac{1}{\omega_2}\, k_0 \right], \tag{18}$$

while $\tilde{z}^*$ and $\tilde{k}^*$, which are increasing in $\tilde{c}^*$, can be computed using (18) in equations (15a) and (15b).[5] Finally, the difference between optimal time-0 and steady-state normalized

[5] When complementarity is adjacent – $\omega_1,\ j > 0$ – the linearization imposes an upper bound on the value that the ratio of initial conditions may take on. Namely, and as is clear from (18), for an optimal program to exist, (z_0/k_0) has to be less than $(z_0/k_0)^{\max} = \rho[(A-g+\psi)/(\rho+g-\psi)]$. Values of (z_0/k_0) above this quantity imply so much consumption at time $t = 0$ that $\tilde{c}, \tilde{k}, \tilde{z}$ become zero in finite time. In terms of the diagrams we

consumption that appears in (17a) can be written as:

$$\tilde{c}_0 - \tilde{c}^* = \frac{(\rho+g)}{\psi(A+\rho)\omega_1\omega_2} \cdot [\, z_0 - \frac{\rho(A-g)}{(\rho+g)} k_0]. \tag{19}$$

Equations (17)-(19) imply the following facts about the equilibrium dynamics associated with the solution of the transformed problem (P1').

PROPOSITION 2. *In equilibrium:*

(i) *normalized consumption, habits and physical capital converge monotonically over time to the steady-state* $\widetilde{c}^*/\widetilde{z}^*$*;*

(ii) *the steady-state levels of the same variables are decreasing in* z_0 *under adjacent complementarity* $(j > 0)$*, and increasing in* z_0 *under distant complementarity* $(j < 0)$*; independently of the sign of* j*,* $\widetilde{c}^*, \widetilde{z}^*$ *and* $\widetilde{k}^*$ *increases with* k_0*;*

(iii) *when* $j > 0$*, normalized consumption increases (decreases) over time towards its steady-state level if* $\frac{z_0}{k_0} < (>) \frac{\rho(A-g)}{\rho+g}$*. The opposite conclusion holds when* $j < 0$*;*

(iv) *in the transition to the steady-state,* $\widetilde{c}$ *and* $\widetilde{k}$ *will always covary positively; normalized consumption and habits* $\widetilde{z}$ *will covary positively if* $j > 0$*, and negatively if* $j < 0$*.*

5. Picturing transitional dynamics

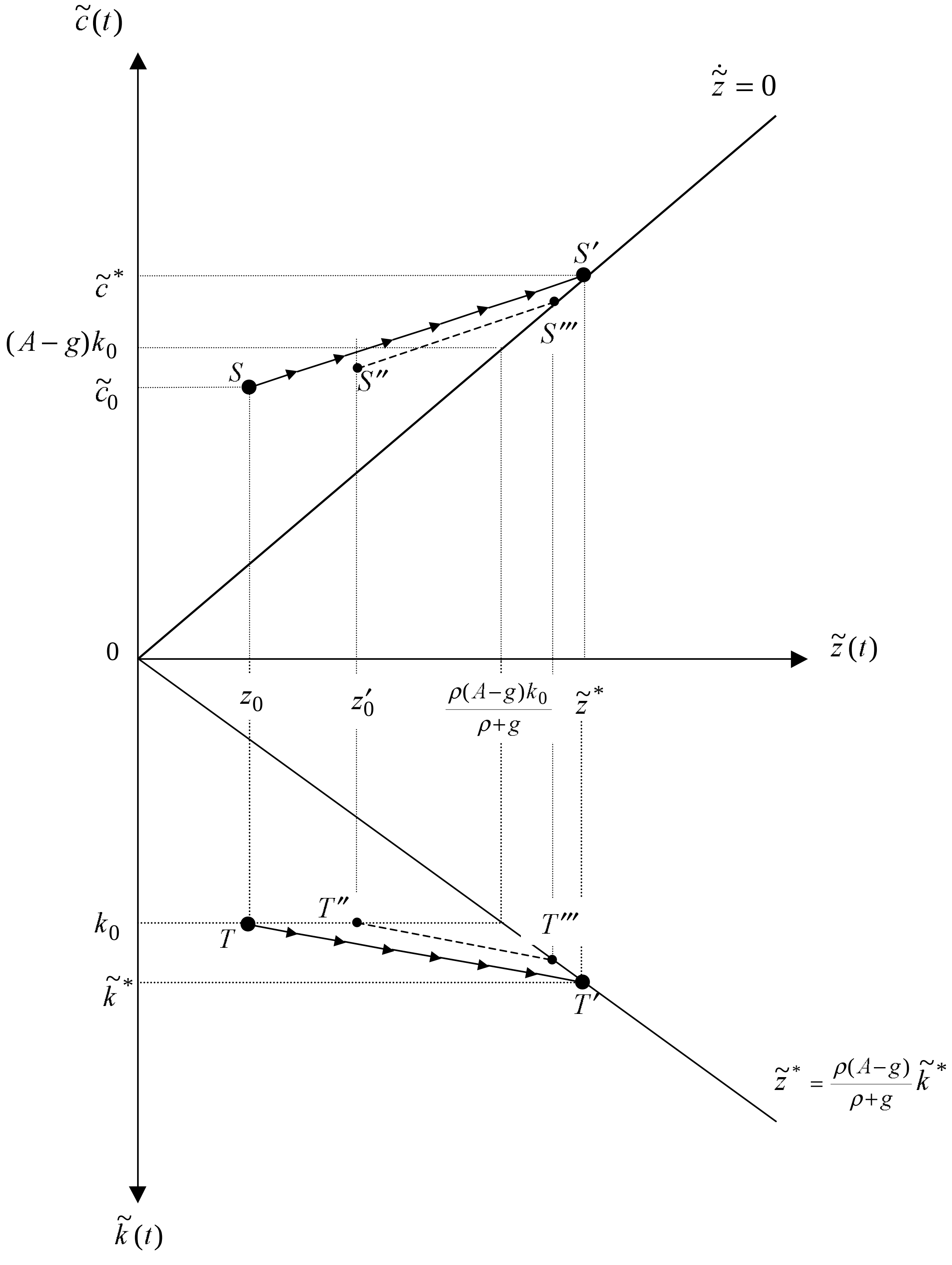
In this Section, we introduce a simple diagram to illustrate the transitional dynamics implied by our model and to provide the economic intuition for the results derived so far, and summarized in Proposition 2. Although we choose to focus on the case of adjacent complementarity, which we regard as most relevant, the same arguments can be used to give an account of the dynamic evolution of the variables in the model when complementarity is distant.

Assuming $j > 0$, in Figure 1 we draw four loci:

---

introduce in Section 4, for a given k_0 it is possible to identify the maximum stock of initial habits consistent with the existence of an equilibrium as the value of z_0 that, in Figure 2, generates a saddlepath SS' in the upper quadrant crossing the $\dot{\tilde{z}} = 0$ locus at $(\tilde{z}, \tilde{c}) = (0, 0)$.

Figure 1



$\tilde{c}(t)$
$\dot{\tilde{z}} = 0$
S'
$\tilde{c}^*$
$(A-g)k_0$
S'''
S
S''
$\tilde{c}_0$
0
$\tilde{z}(t)$
z_0
z_0'
$\frac{\rho(A-g)k_0}{\rho+g}$
$\tilde{z}^*$
T''
T'''
k_0
T
$\tilde{k}^*$
T'
$\tilde{z}^* = \frac{\rho(A-g)}{\rho+g}\tilde{k}^*$
$\tilde{k}(t)$

– the $(\dot{\tilde{z}} = 0)$-locus, which — see equation (14b) — is a straight line originating with slope $(\rho + g)/\rho > 1$ in the $(\tilde{z}, \tilde{c})$-plane; $\tilde{z}$ will be increasing over time above this locus, and decreasing below it;

– the stable saddle path in the same plane, obtained by combining equations (17a) and (17b); in the upper quadrant, it is the arrowed path labeled $SS^{'}$, with slope $(\rho + g - \psi)/\rho$, positive and smaller than the slope of the $\dot{\tilde{z}} = 0$ locus;

– the saddlepath in the $(\tilde{k}, \tilde{z})$-plane is obtained by combining equations (17b) and (17c) — the line $TT^{'}$ with slope $\frac{\rho(A-g+\psi)}{\rho+g-\psi}$;

– the relationship between steady-state levels of normalized habits and physical capital that is implied by (14a) and (14b) — the straight line originating in the lower quadrant, with slope $\frac{\rho(A-g)}{\rho+g}$ $(< \frac{\rho(A-g+\psi)}{\rho+g-\psi})$.

It should be noted that those shown in the figure are not standard phase diagrams. This is because the steady-state levels of $\tilde{c}$, $\tilde{z}$ and $\tilde{k}$ — and, with them, the location of the two saddlepaths $SS^{'}$ and $TT^{'}$ — depend on the set of initial conditions (z_0, k_0), as is clear from (17)-(19).

To understand how this graphical device helps determine the equilibrium dynamics for arbitrary initial conditions on the stock variables, let us assume that the economy starts off with the pair (z_0, k_0) given by point T in the lower quadrant of the figure. Notice that the assumed configuration of initial conditions is such that $(z_0/k_0) < \frac{\rho\,(A-g)}{\rho+g}$. The steady-state pair $(\tilde{k}^{*}, \tilde{z}^{*})$ — point $T^{'}$ — is found as the intersection between the line emanating from T with slope $\frac{\rho(A-g+\psi)}{\rho+g-\psi}$ and the steady-state locus $\tilde{z}^{*} = \frac{\rho(A-g)}{\rho+g}\tilde{k}^{*}$. Given the $\tilde{z}^{*}$ so determined, one uses the $(\dot{\tilde{z}} = 0)$-locus in the upper quadrant to find the pair $(\tilde{z}^{*}, \tilde{c}^{*})$ — point $S^{'}$. The saddlepath in the $(\tilde{z}, \tilde{c})$-plane is then the line going through $S^{'}$ with slope $(1/\omega_1) \equiv \frac{\rho+g-\psi}{\rho}$ (a positive quantity, under adjacent complementarity); finally, one determines the optimal time-0 choice of consumption, $\tilde{c}_0$, as the value of $\tilde{c}$ that, along this line, is associated with the assumed z_0.

From the figure, it is clear that, as stated in Proposition 1, a stable dynamics calls for levels of normalized consumption, habits, and physical capital to rise over time. In fact, given $(z_0/k_0) < \frac{\rho\,(A-g)}{\rho+g}$, optimal time-0 consumption is lower than the level $(A-g)k_0$ which — through (14c) — would yield $\dot{\tilde{k}}(0) = 0$, and point S is located above the $(\dot{\tilde{z}} = 0)$-locus. It follows that both normalized physical capital and habits will be increasing at time zero.

The next instant — which, for simplicity, we call $t = 1$ —, the economy will therefore start off with larger beginning-of-period $\tilde{k}$ and $\tilde{z}$. All other things being the same, a larger capital stock will exert a positive wealth effect on time-1 consumption, $\tilde{c}_1$. In addition, under adjacent complementarity the individual has a further incentive to raise his consumption level at time $t = 1$ because of the increase in the stock of habits. For both reasons, $\tilde{c}_1 > \tilde{c}_0$.

To understand why an increase in $\tilde{z}$ leads to an increase in the optimal choice of $\tilde{c}$, notice that (13) implies that what we have termed the "full marginal benefit" of current consumption can be written as:

$$u_{\tilde{c}}(\tilde{c}(t), \tilde{z}(t)) + \rho \int_t^{\infty} e^{-(A+\rho)(s-t)}\, u_{\tilde{z}}(\tilde{c}(s), \tilde{z}(s))\, ds. \tag{20}$$

Since, for $s > t$, one has

$$\tilde{z}(s) = \tilde{z}(t)e^{-(\rho+g)(s-t)} + \rho \int_t^{s} e^{-(\rho+g)(s-\tau)}\, c(\tau)\, d\tau,$$

the derivative with respect to $\tilde{z}(t)$ of (20) is[6]:

$$u_{\tilde{c}\tilde{z}}(\tilde{c}(t), \tilde{z}(t)) + \rho \int_t^{\infty} e^{-(A+g+2\rho)(s-t)}\, u_{\tilde{z}\tilde{z}}(\tilde{c}(s), \tilde{z}(s))\, ds. \tag{21}$$

Evaluated at the steady-state, (21) reduces to

$$u^*_{\tilde{c}\tilde{z}} + \frac{\rho}{A+g+2\rho}\, u^*_{\tilde{z}\tilde{z}} \equiv (-u^*_{\tilde{c}\tilde{c}}) \cdot (A+g+2\rho) \cdot j,$$

an expression which has the same sign as j. It follows that, in the local analysis of the equilibrium dynamics under adjacent complementarity we are carrying out, the full marginal benefit of $\tilde{c}$ will move in the same direction as $\tilde{z}$, and the individual has an incentive to increase

[6] This step involves the computation of the "Volterra derivative" of the functional in (20). For a definition of Volterra derivatives, see Ryder and Heal (1973), pp. 3-4.

$\tilde{c}$ when $\tilde{z}$ rises.

Having shown that, for the assumed configuration of initial conditions, $\tilde{c}_1 > \tilde{c}_0$, it is straightforward to verify that, at time $t = 1$, this higher level of consumption is still consistent with the accumulation of capital and habits, although at a slower rate than in the previous period. The same process is repeated the next instant and the economy converges over time to the steady-state ($S^{'}, T^{'}$) along the arrowed paths in the two quadrants.

Suppose now that the economy starts off with an unchanged level of physical capital, but with a $z_0^{\prime} > z_0$. If , as assumed in the figure, this increase in initial consumption experience is such that $(z_0^{\prime}/k_0)$ is still less than the critical level $\frac{\rho\,(A-g)}{\rho+g}$, we end up with the new saddlepaths given by the dashed lines labeled $S^{''}S^{'''}$ and $T^{''}T^{'''}$, and with lower steady-state levels of $\tilde{c}, \tilde{z}$ and $\tilde{k}$.

That the steady-state levels of the variables are decreasing in z_0 when $j > 0$ simply reflects the higher marginal benefit of consumption associated with higher initial habits. The individual will consume more at time 0, and will accumulate less capital[7]. This smaller accumulation will — via a wealth effect — cause a smaller increase in consumption, and therefore habits, during the transition to the steady-state, as well as lower levels of the variables in the new balanced growth equilibrium ($S^{'''}, T^{'''}$).

On the other hand, when z_0 is so large that $(z_0/k_0) > \frac{\rho\,(A-g)}{\rho+g}$, the whole dynamics is reversed. As shown in Figure 2, under adjacent complementarity the individual will choose to consume so much at time zero that $\tilde{k}$ will be decumulated ($\tilde{c}_0 > (A-g)k_0$). De-trended habits will decrease as well, since the economy starts off at point S, which is now below the $\dot{\tilde{z}} = 0$ locus: although the individual consumes a lot, the optimal initial choice of consumption — one that is consistent with the transversality condition on $\tilde{k}$ — does not add to consumption experience enough to compensate for the depreciation term $(\rho + g)z_0$, which is large because

---

[7] That $\tilde{c}_0$ is increasing in $\tilde{z}_0$ simply reflects the fact that consumption is increasing in habits under adjacent complementarity. It follows that, as shown in the Figure, point $S^{'}$ is located to the north-east of point S. This can be proved as follows. First, evaluate at time $t = 0$ the expressions for the two loci $TT^{\,'}$ and $SS^{'}$, obtaining $\tilde{z}_0 - \tilde{z}^* = \frac{\rho(A-g+\psi)}{(\rho+g-\psi)}(\tilde{k}_0 - \tilde{k}^*)$ and $\tilde{c}_0 - \tilde{c}^* = \frac{(\rho+g-\psi)}{\rho}(\tilde{z}_0 - \tilde{z}^*)$. Next, differentiate totally the first expression, setting $d\tilde{k}_0 = 0$ and $d\tilde{k}^* = \frac{(\rho+g)}{\rho(A-g)}d\tilde{z}^*$, to get $d\tilde{z}^* = -\frac{(A-g)(\rho+g-\psi)}{\psi(A+\rho)}d\tilde{z}_0$. Finally, differentiation of the second expression, using $d\tilde{c}^* = \frac{(\rho+g)}{\rho}d\tilde{z}^* = -\frac{(\rho+g)(A-g)(\rho+g-\psi)}{\rho\psi(A+\rho)}d\tilde{z}_0$, yields $d\tilde{c}_0/d\tilde{z}_0 = \frac{(\rho+g)(\rho+g-\psi)}{\rho(A+\rho)} > 0$. The same result can be derived, in a more straightforward fashion, using the explicit expresion for $\tilde{c}_0$ given by (A.2.12) in Appendix 2.

z_0 is large. In this case, normalized consumption, habits and physical capital will decrease over time toward their steady-state levels.

The same diagram can be used to determine the effects of changes in k_0 for a given z_0. For instance, and going back to Figure 1, an increase in k_0 would cause a parallel, downward shift of the TT' locus and an upward shift of the SS' locus, thus leading to an increase both in the initial optimal choice of $\tilde{c}$, and in the steady-state levels of the three variables on the axes. If the initial configuration of initial conditions is the one shown in Figure 2, an increase in k_0 such that the ratio (z_0/k_0) remains above the critical value $\frac{\rho\,(A-g)}{\rho+g}$ leads to qualitatively similar displacements of the TT' and the SS' loci, and, once again, to higher steady-state values of $\tilde{c}^*$, $\tilde{z}^*$, and $\tilde{k}^*$.

More generally, to assess the impact of simultaneous changes of k_0 and z_0, or the qualitative properties of the transitional dynamics of the variables starting from an arbitrary pair (z_0, k_0), all that matters is how the ratio (z_0/k_0) compares to $\frac{\rho\,(A-g)}{\rho+g}$ — or, in graphical terms, whether the point that denotes the initial conditions on the two stock variables in the lower quadrant of the Figure is located above or below the locus $\tilde{z}^* = \frac{\rho(A-g)}{\rho+g}\tilde{k}^*$. If, by accident, $(z_0/k_0) = \frac{\rho\,(A-g)}{\rho+g}$, the economy jumps immediately on the steady-state. In general, however, consumption, habits and physical capital converge to a balanced growth path increasing or decreasing over time, depending on whether $(z_0/k_0) \lessgtr \frac{\rho\,(A-g)}{\rho+g}$.

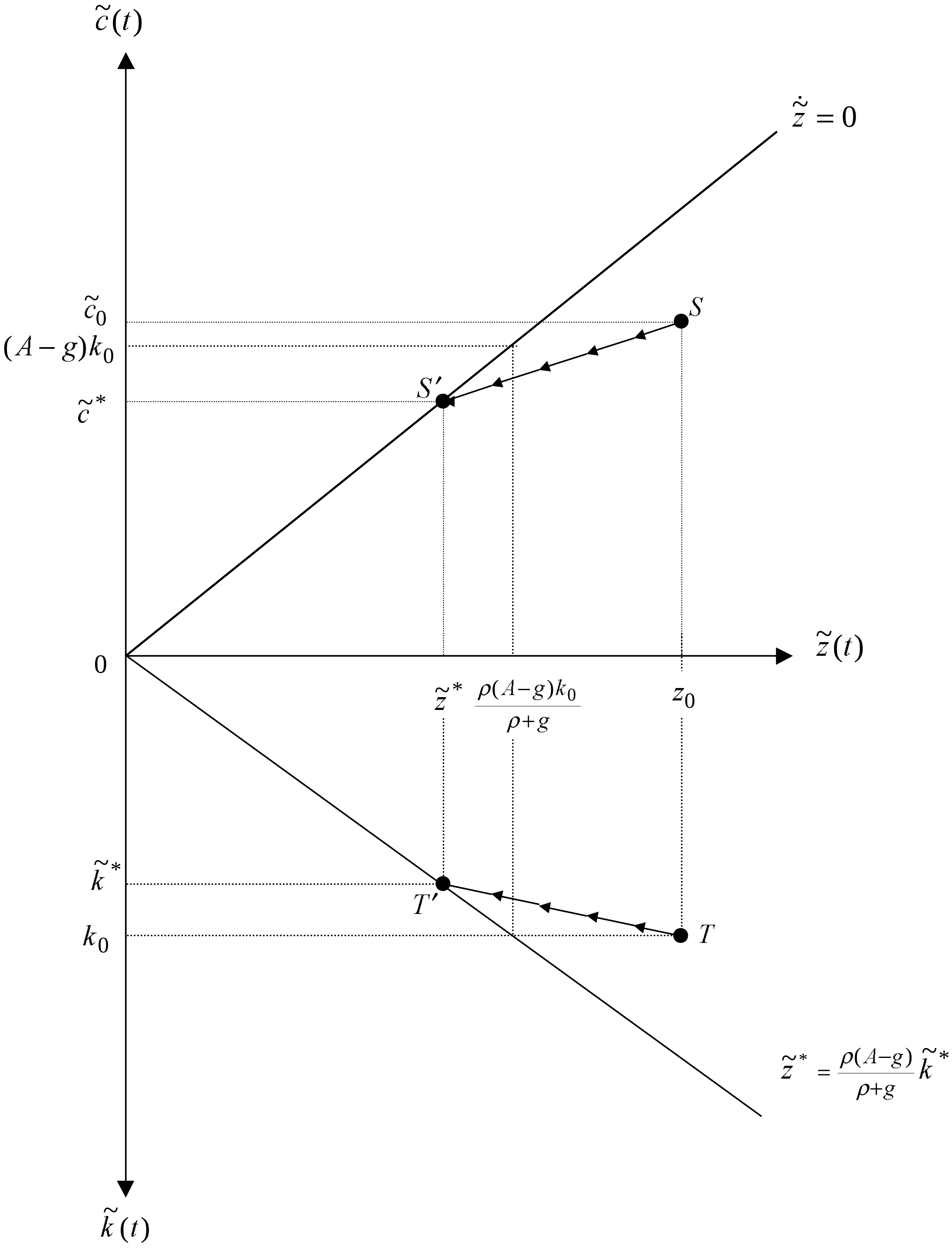
Finally, the results one gets under the standard assumption of time-separable preferences can be retrieved as a special case of our model.

To see this, first notice that, if $u_{\tilde{z}} = u_{\tilde{c}\tilde{z}} = 0$, so that Assumption U2 is violated, one has $\tilde{\mu} = 0,\ u_{\tilde{c}} = \tilde{\lambda}^* \forall t$. It follows that the right-hand side of equation (14a) is zero, and the saddlepaths $SS'$and TT' become flat at the levels of consumption $\tilde{c} = (A-g)k_0$ and capital $\tilde{k} = k_0$, respectively. Regardless of initial conditions, this implies that detrended consumption and physical capital will be constant over time, and that — as in the standard "Ak" model — c and k will always grow at the steady-state rate $g = \frac{A-\delta}{1-\ \nu}$. Given the usual time-separable, isoelastic instantaneous felicity function $\frac{c^{1-\sigma}}{1-\sigma}$, which is homogeneous of degree $\nu = 1-\sigma$, this is just the familiar growth rate $\frac{A-\delta}{\sigma}$.[8]

---

[8] If, for given k_0, $z_0 \neq \frac{\rho\,(A-g)}{\alpha+g}k_0$, there will be a transitional dynamics of the stock of habits; however, this will not affect consumption, capital, nor utility levels.

Figure 2



$\tilde{c}(t)$
$\dot{\tilde{z}} = 0$
$\tilde{c}_0$
$(A-g)k_0$
$\tilde{c}^*$
S
S′
0
$\tilde{z}(t)$
$\tilde{z}^*$
$\frac{\rho(A-g)k_0}{\rho+g}$
z_0
$\tilde{k}^*$
T′
k_0
T
$\tilde{z}^* = \frac{\rho(A-g)}{\rho+g}\tilde{k}^*$
$\tilde{k}(t)$

6. Dynamics in the original problem

The analysis in the previous Sections provides a full characterization of the equilibrium dynamics of what we have termed “normalized”, or “de-trended”, variables. In order to go from the latter to the behavior over time of “actual” consumption, habits, capital, and output, one has simply to remember that the generic variable x is related to its normalized counterpart $\tilde{x}$ according to $x = e^{gt}\tilde{x}$, or, in terms of growth rates:

$$\frac{\dot{x}}{x} = g + \frac{\dot{\tilde{x}}}{\tilde{x}} \, .$$

Since normalized variables converge monotonically over time to a steady-state where they take on constant values, the growth rate of the actual ones will converge asymptotically to g. In the transition, their growth rate will be above or below this value, depending on whether their de-trended counterparts converge to the steady-state increasing or decreasing over time — an information one can readily retrieve from Proposition 2, or the first row of Table 1.

In turn, it is possible to infer the behavior of the growth rate of per-capita output, g_y, by noticing that:

$$g_y \equiv \frac{\dot{y}}{y} = \frac{\dot{k}}{k} = g + \frac{\dot{\tilde{k}}}{\tilde{k}}. \tag{22}$$

Using these results, the last two rows of Table 1 summarize the transitional dynamics of g_y under adjacent complementarity. The growth rate of per-capita output is decreasing in (z_0/k_0), and can initially be negative for values of this ratio that are very high, while still being consistent with the upper bound mentioned in footnote 4.[9] For values of (z_0/k_0) below

[9] Since g_y asymptotically approaches $g > 0$, $g_y < 0$ is possible only during the first stages of transition. Furthermore, the possibility of a negative growth rate depends not just on the size of the ratio (z_0/k_0), but also – through j and ψ – on the characteristics of the instantaneous utility function. With reference to the two functional forms that will be introduced in Section 7 below, one can for instance show that $g_y < 0$ for some t is possible when u takes on the functional form in (23), while $g_y > 0$ $\forall t$ when u is given by (24).

(above) the threshold $\frac{\rho(A-g)}{(\rho+g)}$, g_y will be larger (smaller) than g, converging asymptotically to this constant, positive value.

Finally, defining the saving rate as:

$$s \equiv 1 - \frac{c}{Ak},$$

and noticing that $(c/k) = (\tilde{c}/\tilde{k}) = (A-g) - (\dot{\tilde{k}}/\tilde{k})$, from (22) one has:

$$s = \frac{g_y}{A}.$$

It follows that s will take on the constant value $s^* = (g/A) < 1$ in balanced growth, and that its transitional dynamics will be qualitatively identical to that of the rate of growth of per-capita output: the saving rate will be initially "high" — and decreasing over time — when $\frac{z_0}{k_0} < \frac{\rho(A-g)}{(\rho+g)}$, and relatively "low", but increasing toward its steady-state level, for the opposite configuration of initial conditions.

Using these results, in Figure 3 we compare the time path of the log of per capita output for various sets of initial conditions and identical values of the parameters in the model. In panel (a) we assume a given z_0, and consider four different initial conditions on the capital stock: $k_0^1 > k_0^2 > \frac{(\rho+g)}{\rho(A-g)} z_0 > k_0^3 > k_0^4$. In panel (b), we keep k_0 constant, and show the effect of various levels of initial consumption experience — with $z_0^1 < z_0^2 < \frac{\rho(A-g)}{(\rho+g)} k_0 < z_0^3 < z_0^4$ — on the equilibrium path of output.

If one interprets these diagrams as showing the time paths of per capita output for countries that differ only in terms of their initial endowment of physical capital *or* consumption experience, and defines convergence as the tendency for cross-country income differences to decrease over time, it is clear that the model with intertemporally dependent preferences predicts divergence. However, if one realistically allows for differences in *both* k_0 and z_0, the model is consistent with a wider range of possibilities: they include divergence, as well as convergence and leapfrogging.

Table 1

Transitional dynamics

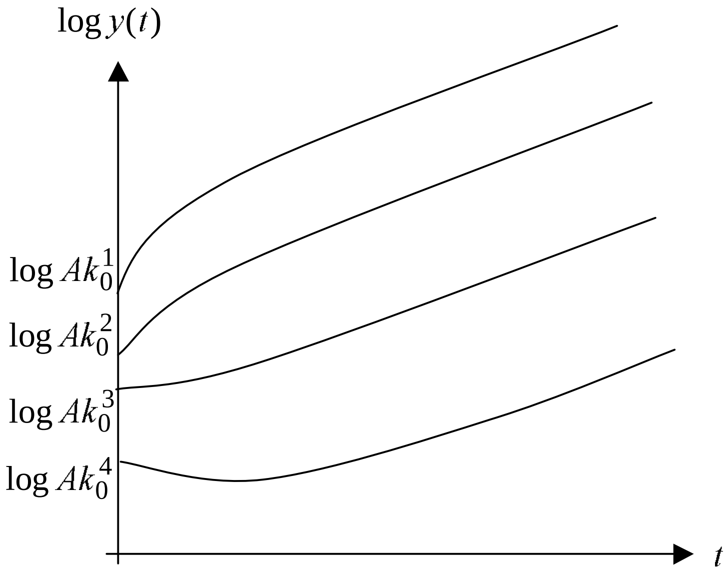
	$\frac{z_0}{k_0} < \frac{\rho(A-g)}{(\rho+g)}$	$\frac{z_0}{k_0} > \frac{\rho(A-g)}{(\rho+g)}$
$\dot{\tilde{c}}, \dot{\tilde{z}}, \dot{\tilde{k}}$	> 0	< 0
g_y	$> g$	$< g$
$\dot{g}_y$	< 0	> 0

To illustrate some of these possibilities, in panel (c) of Figure 3 we assume $k_0^1 > k_0^3 > k_0^2$ — so that, from the standpoint of time 0, country 1 is "rich", country 2 is relatively "poor", and country 3 "middle-income" — and differences in z_0 leading to $(z_0^3/k_0^3) < (z_0^2/k_0^2) < \frac{\rho(A-g)}{(\rho+g)} < (z_0^1/k_0^1)$. Although country 1 starts off with a lot of capital, it also has a value of z_0 so high that its ratio of initial conditions is above the critical level $\frac{\rho(A-g)}{(\rho+g)}$: it follows that its saving and growth rates will be initially low. On the other hand, country 2 is assumed to be better endowed with z than country 3: this difference in initial consumption experience more than offsets the difference in capital endowments, so that $(z_0^2/k_0^2) > (z_0^3/k_0^3)$.

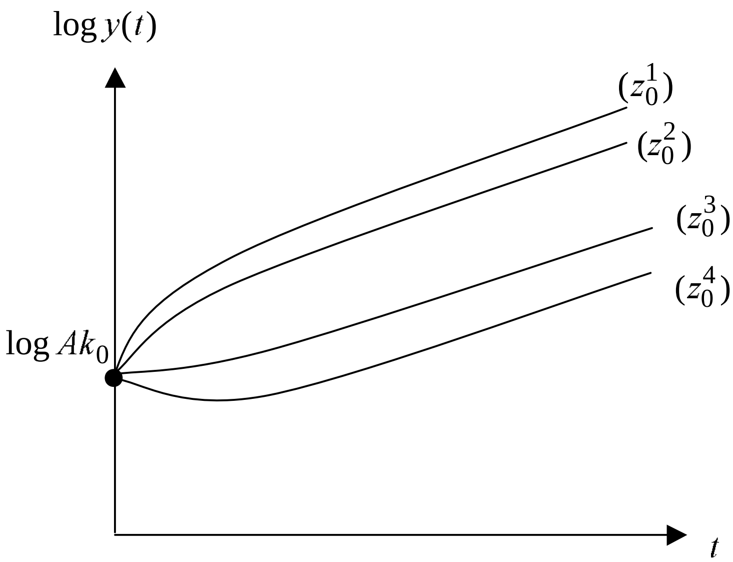
For the assumed configuration of initial conditions, the model predicts convergence between countries 1 and 2, and divergence between countries 2 and 3. Countries 1 and 3 will first converge, criss-cross, and then diverge.

This variety of possible outcomes clearly results from the fact that, depending on the size of (z_0/k_0), the model with intertemporally dependent preferences generates equilibrium paths for the log of per capita output that can be either convex or concave when plotted against time, with $\log y$ approaching its long-run path "from above" in the first case (as country 1 in panel (c) of Figure 3), and "from below" in the second (countries 2 and 3 in the same diagram). Figure 4 — where the figures on the time path of the log of real GDP per capita for the years 1950-92 are from the Summers and Heston data set — suggests the empirical occurrence of both kinds of convergence. The European countries in the first panel, as well as Switzerland in the third, seem to be converging to their steady-state "from below", with growth rates declining over time. Thailand, Indonesia and Korea in the second panel, but also Canada in the third, display what we have termed "convergence from above". In the last two panels, we also show two instances of criss-crossing.

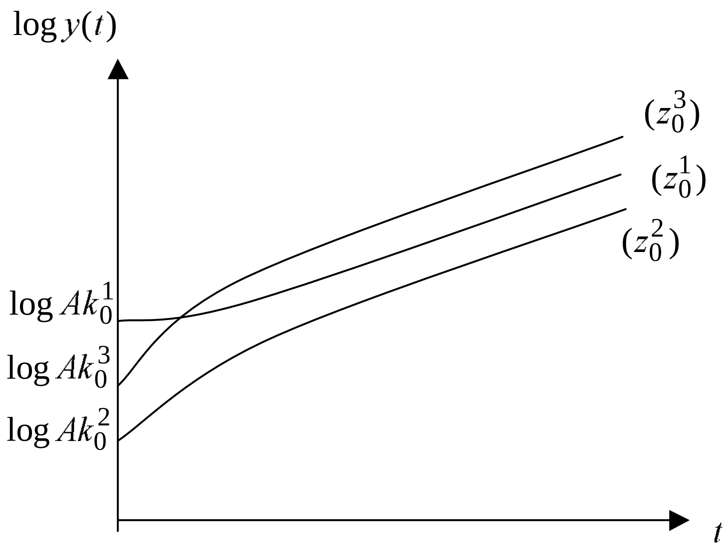
Figure 3



$(a) \quad k_0^1 > k_0^2 > \frac{(\rho+g)}{\rho(A-g)} z_0 > k_0^3 > k_0^4$



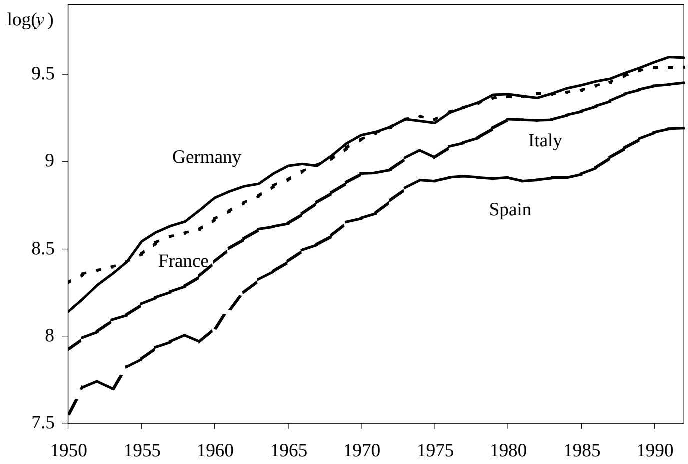
$(b) \quad z_0^1 < z_0^2 < \frac{\rho(A-g)}{(\rho+g)} k_0 < z_0^3 < z_0^4$

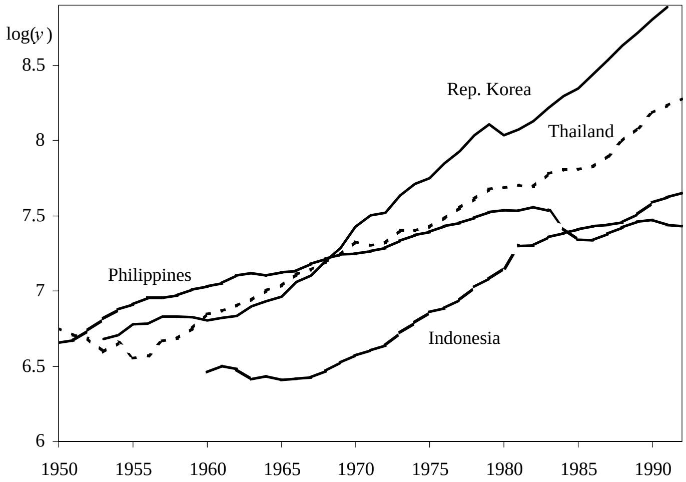


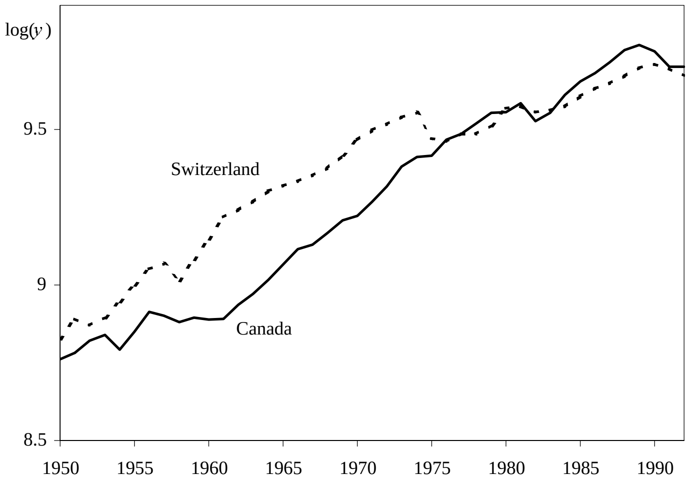
$(c) \quad z_0^1 / k_0^1 > \frac{\rho(A-g)}{(\rho+g)} > z_0^2 / k_0^2 > z_0^3 / k_0^3$

Figure 4

Logarithm of per capita GDP







It should be stressed that the patterns of cross-country growth and convergence in Figures 3 and 4 are also consistent with other models — for instance, the neoclassical model with time-separable preferences and an exogenous rate of technical progress (see Durlauf and Quah (1998), p.18). However, to generate cross-country differences in the position of the long-run paths of output, the neoclassical model has to assume differences in preferences or technology parameters, or in the initial level of technology. Furthermore, and assuming away such differences, it predicts that only countries that start off with a relatively high stock of capital — and therefore, only countries that are relatively "rich" at time zero — can display "convergence from above", a result that is not consistent with the evidence presented in Figure 4. On the contrary, intertemporal dependence implies that countries having access to the same technology and with identical preferences converge to different — albeit parallel — long-run paths just because of differences in the initial relative endowments of z and k. In addition, since it is the ratio between the initial conditions of these two variables that determines the transitional dynamics of the saving and growth rates, both kinds of convergence may be displayed by "rich" and "poor" countries alike. For instance, a country so poor at $t = 0$ as to have a stock of physical capital close to zero could also have a z_0 which is low in absolute terms, but high in relation to k_0. This could be the case because of the lower bound placed on z by the level of subsistence consumption, or — embracing the extended interpretation of the law of accumulation of habits mentioned in Section 2 — because its z also reflects consumption standards in other, richer countries with which it interacts due to their geographic or cultural proximity. Starting off with a relatively high (z_0/k_0), this country, no matter how "poor", is predicted to converge "from above" by our model.

Admittedly, this latter neglects a host of factors that surely play a major role in the explanation of actual growth performances. Nevertheless, we think it is remarkable that the simple, and tractable, modification of preferences studied in the present paper is able to generate dynamics that are not inconsistent with the empirical evidence. These dynamics should be superimposed on those implied by other models, which usually give a fuller and more realistic account of the role played by supply-side factors in the process of growth.

7. Two examples, and comparison with Stone-Geary preferences

In this Section we consider two specifications of preferences belonging to the intertemporally dependent class and compare the implications of our model with those associated with a different departure from the assumption of CIES preferences, proposed by Christiano (1989) and Rebelo (1992).

Consider first the utility function used by Abel (1990) and Carroll, Overland, and Weil (1997):

$$u(c,z) = \frac{c^{1-\sigma} z^{-\gamma(1-\sigma)}}{1-\sigma}, \tag{23}$$

with $\sigma > 0, \gamma < 0,\ \gamma(1-\sigma) + \sigma > 0$. For values of (σ, γ) satisfying these restrictions, (23) is homogeneous of degree $\nu = (1-\sigma)(1-\gamma) < 1$, and satisfies assumptions (U1)-(U4). The implied steady-state growth rate of the economy is $g = \frac{A-\delta}{\gamma(1-\sigma)+\sigma}$, which is positive for $A > \delta$.[10] Notice that when $\gamma = 0$, so that habits do not affect utility, we are back to the standard time-separable case, with CIES preferences. Setting $\delta = 0.05,\ \gamma = -0.5,\ \sigma = 0.5,$ $\rho = 0.1$ and $A = .0505$ (with A chosen to be consistent with a steady-state growth rate g of 2% per year, given the assumed $\delta, \gamma,$ and σ), it can be shown that $u_{cz} > 0,\ j = 0.165 > 0,$ so that u displays adjacent complementarity. These same parameter values imply a speed of convergence to the steady-state — as measured by ψ — of 2.7% per year, and a steady-state saving rate close to 0.4, which is reasonable given the broad concept of capital implied by the assumption of linear technology. Table 2 — in which we consider different values of ρ — shows that both the degree of adjacent complementarity and the speed of convergence to the steady-state increase with ρ.

---

[10] Carroll, Overland, and Weil (1997) assume a positive γ, and erroneously state that (23) is concave in (c,z) for $0 \leq \gamma < 1,\ \sigma \geq \frac{1}{1-\gamma}$. However, if one wants γ to be positive, it is easy to show that concavity requires $\gamma > 1,\quad \sigma \geq -\frac{\gamma}{1-\gamma}$. As a matter of fact, for the parameter values they use in their simulations (see their fn. 9, p.366), it turns out that $u_{zz} > 0,\ u_{cc}u_{zz} - u_{cz}^2 < 0,$ so that they work with a utility function which is not concave in (c,z) – and strictly convex in z.

More generally, given the functional form in (23), one cannot have concavity in (c,z), $\gamma > 0,$ and a positive and finite steady state rate of growth g for $A > \delta$. In fact, the conditions $\gamma > 1,\ \ \sigma \geq -\frac{\gamma}{1-\gamma}$ imply that concavity requires $\gamma(1-\sigma) + \sigma \leq 0$. Since the left-hand side of this inequality is $\nu - 1$, an expression that appears at the denominator of g, one must have $\gamma(1-\sigma)+\sigma < 0$ for g to be finite. However, $\gamma(1-\sigma)+\sigma < 0$ and $A > \delta$ imply a negative g. For this reason, in the text we assume $\gamma < 0$. Finally, notice that, although in this specific case the instantaneous utility function must be strictly concave if the economy has to have a positive and finite g, this is not true in general, as the next functional form we consider in the text – concave in (c,z), but nor strictly so – proves.

Table 2

Degree of adjacent complementarity and the speed of convergence for various values of ρ

ρ	0.1	0.3	0.5	0.7	0.9
j	0.165	0.339	0.518	0.698	0.878
ψ	0.027	0.088	0.150	0.213	0.275

As a second example, we consider the functional form:

$$u(c,z) = \frac{(c-z)^{1-\sigma}}{1-\sigma}, \qquad \text{for } c \geq z, \tag{24}$$
$$= -\infty \qquad \text{for } c < z,$$

with $\sigma > 0$ and $\neq 1$. This utility function — proposed by Constantinides (1990) in his attempt to solve the equity premium puzzle and used, among others, by Detemple and Zapatero (1991) — implies that only the excess of consumption over the standard of living is valued. Since $\lim_{c-z \to 0} u_c = \infty$ and $z_0 > 0$, $c_t > z_t > 0 \ \forall t$. In addition, $u_z, u_{cc}, u_{zz} < 0$, $u_{cc}u_{zz} - u_{cz}^2 = 0$, assumption U4 is met, and $j = A + g + \rho > 0$, so we are always in the adjacent complementarity case. Finally, given $1 - \nu = \sigma > 0$, the steady-state growth rate of the economy $g = \frac{A-\delta}{\sigma}$ is positive for $A > \delta$.

With this utility function, one has $\omega_1 = 1$, $\omega_2 = (1/A)$, $\psi = g$. It follows that $\tilde{k}$ evolves according to:

$$\tilde{k}_t - \tilde{k}^* = (k_0 - \tilde{k}^*)e^{-gt},$$

where:

$$\tilde{k}^* = \left[\frac{\rho + g}{g(A+\rho)}\right](Ak_0 - z_0),$$

and that the upper bound on the ratio of initial conditions mentioned in footnote 4 above now becomes $(z_0/k_0) < A$. Using $k_t = e^{gt}\tilde{k}_t$, the growth rate of per-capita output is:

$$g_y = \frac{\dot{k}}{k} = (k_t)^{-1}\left(\frac{\rho + g}{A+\rho}\right)(Ak_0 - z_0)e^{gt} > 0.$$

Notice that:

$$\dot{g}_y = (k_t)^{-2}\left(\frac{\rho + g}{A+\rho}\right)^2 (Ak_0 - z_0)\left[z_0 - \frac{\rho(A-g)}{\rho+g}k_0\right]e^{gt}, \tag{25}$$

which is negative for $(z_0/k_0) \in (0, \frac{\rho(A-g)}{\rho+g})$, and positive for $(z_0/k_0) \in (\frac{\rho(A-g)}{\rho+g}, A)$.

The first attractive property of the functional form under consideration is that these equilibrium paths — derived using (17a)-(17c), and therefore a local approximation around the steady-state — coincide with the global dynamics of the variables that one gets by solving system (14a)-(14c) directly .

The second concerns the fact that the results obtained with time-separable, Stone-Geary preferences used in a growth setting by Christiano (1989) and Rebelo (1992) are a special case of our model with instantaneous utility given by (24) and $\rho = 0$.

To appreciate this — and following Rebelo, who works with an Ak technology and whose model is therefore closest to ours —, consider the Stone-Geary utility function:

$$u(c) = \frac{(c-\bar{c})^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \neq 1,$$

where the positive constant $\bar{c}$ is the subsistence level of consumption, and $k_0 > \bar{k}$, $\bar{k} \equiv (\bar{c}/A)$. Since an Ak technology is assumed, $\bar{k}$ can be interpreted as the amount of capital needed to produce the subsistence level of consumption.

It is easy to show that, given this utility function, in equilibrium:

$$k_t = \bar{k} + (k_0 - \bar{k})e^{gt},$$

$$g_y = (k_t)^{-1} g(k_0 - \bar{k})e^{gt} > 0,$$

$$\dot{g}_y = (k_t)^{-2} g^2 (k_0 - \bar{k})e^{gt}\bar{k} > 0, \tag{26}$$

where $g = \frac{A-\delta}{\sigma}$. These are the same solutions that would be obtained assuming the functional form (24) and setting $\rho = 0$, so that $z_t = z_0 \equiv \bar{c}\ \forall t$, implying that customary consumption is just constant at the subsistence level $\bar{c}$. Comparing (25) and (26), it is clear that — bacause it allows a changing level of z — (24) is generally consistent with a wider range of possibilities in terms of transitional dynamics toward the steady-state growth path: while $\rho = 0$ (and therefore a Stone-Geary utility function) yields the implication that the growth rate of per-capita output and the saving rate must necessarily be increasing over time along the transition, they can be either increasing or decreasing when $\rho \neq 0$ and the dynamics of habits feed back to consumption and accumulation choices. Since the evidence documented in Figure 4 suggests that there is not a clear, common, pattern with which countries converge to their long run growth paths — with some countries seemingly converging "from above", and some "from below" —, in our opinion this makes the model with intertemporally dependent preferences more appealing than the departure from CIES preferences represented by the use of a Stone-Geary utility function with a constant reference level of consumption.

8. Concluding remarks

In this paper we studied an endogenous growth model with intertemporally dependent preferences and "Ak" technology. Working with a generic instantaneous utility function, we first provided sufficient conditions that this latter has to meet for a balanced growth path to qualify as an equilibrium when preferences are intertemporally dependent. We then provided a full characterization of the equilibrium dynamics of the economy, focusing on the situation of "inward-looking", or "internal" habits and unveiled the difference between the "adjacent" and "distant" complementarity cases, which is central to most of the literature on habit formation. We chose to focus mainly on the case of adjacent complementarity, so that c and z will be positively related in equilibrium, as we regarded the addictive behavior it implies as more relevant in the one-sector framework we considered.

Finally, we explored the implications of habit formation for the patterns of cross-country growth and convergence, showing that these latter are consistent with two kinds of convergence of a country's per capita income to its long-run growth path: a convergence "from above", with growth and saving rates increasing over time and asymptotically approaching the constant value they will take on in balanced growth, and a convergence "from below", with growth and saving rates initially high, but then declining over time. The model predicts that countries that will display the first type of convergence are those whose initial endowment of consumption experience in relation to that of physical capital is high; on the other hand, countries characterized by an initially relatively low ratio of consumption experience to physical capital should converge "from below". Under adjacent complementarity, the latter will initially save a lot, growing during the transition stage at rates which are high, but declining towards a constant, steady-state level.

Our model is consistent with available evidence that suggests of the empirical occurrence of both kinds of convergence and the absence of a clear association between a country's starting level of per capita income and the type of convergence it will display along the transition. For instance, a country that is "poor" at time zero could also have a level of reference consumption which is low in absolute terms, but high when compared with the endowment of physical capital — maybe because of the lower bound placed on z by subsistence consumption, or because its consumption standards are set by a comparison with richer countries with which it interacts closely. It will therefore converge "from above" in the very same way as a "rich" country that starts off better endowed with both customary consumption and physical capital, but having a similar initial habits to capital ratio. Although these different patterns of growth and convergence are consistent with the neoclassical model with an exogenous rate of technical progress, this latter has counterfactual implications that are absent in our setting.

Appendix 1

Proof of Proposition 1

The current-value Hamiltonian for problem (P1) is:

$$H = u(c,z) + \lambda(Ak - c) + \mu\rho(c - z),$$

where λ and μ are the co-state variables associated with k and z, respectively.

It follows that, among the necessary conditions for (P1), we have:

$$\begin{aligned} u_c + \rho\mu &= \lambda, && \text{(A.1.1)} \\ \dot{\lambda} &= (\delta - A)\lambda, && \text{(A.1.2)} \\ \dot{\mu} &= (\rho + \delta)\mu - u_z. && \text{(A.1.3)} \end{aligned}$$

By differentiating (A.1.1) with respect to time, and using (A.1.2)-(A.1.3), one gets:

$$u_{cc}\dot{c} + u_{cz}\dot{z} = (\delta + \rho)u_c - (A + \rho)\lambda + \rho u_z,$$

which can be rearranged as follows:

$$\frac{cu_{cc}(\dot{c}/c) + zu_{cz}(\dot{z}/z)}{u_c} = (\delta + \rho) - (A + \rho)\frac{\lambda}{u_c} + \rho\frac{u_z}{u_c}. \qquad \text{(A.1.4)}$$

To be an equilibrium, a balanced growth path must satisfy (A.1.4). Since c and z grow at the common rate g in balanced growth, in steady-state equilibrium (A.1.4) becomes:

$$g\left[\frac{cu_{cc} + zu_{cz}}{u_c}\right] = (\delta + \rho) - (A + \rho)\frac{\lambda}{u_c} + \rho\frac{u_z}{u_c}. \qquad \text{(A.1.5)}$$

Let us assume that u is homogeneous of degree ν in (c, z), so that u_c is homogeneous of degree $(\nu - 1)$ in the same variables. Euler's theorem then implies:

$$\left[\frac{cu_{cc} + zu_{cz}}{u_c}\right] = \nu - 1,$$

so that the left-hand side of (A.1.5) is a constant. For a balanced growth path to be an equilibrium, the right-hand side of (A.1.5) must also be constant. Since homogeneity of u implies that the term (u_c/u_z) is a function of the ratio (c/z) only — a constant in steady-state growth — this requires:

$$\frac{\dot{u}_c}{u_c} = \frac{\dot{\lambda}}{\lambda},$$

or, using (A.1.2) and evaluating at the steady-state the rate of change of the marginal utility of consumption,

$$g(\nu - 1) = \delta - A.$$

By rearranging, one obtains $\nu = \frac{\delta - A}{g} + 1$, or the equivalent expression for the relationship between g and the degree of homogeneity of u given by equation (6) in the text. Notice that the requirement of positive steady-state growth and Assumption T1 imply $\nu < 1$.

Finally, it is easy to verify that, in steady-state growth, $(\dot{\mu}/\mu) = (\dot{\lambda}/\lambda) = \delta - A$. It follows that, for a balanced growth path to satisfy the transversality conditions associated with problem (P1),

$$\begin{aligned}\lim_{t\to\infty} e^{-\delta t}\lambda_t k_t &= 0,\\ \lim_{t\to\infty} e^{-\delta t}\mu_t z_t &= 0,\end{aligned}$$

one must have $A > g$. We restrict the parameters in the model so as to make sure that this inequality always holds, implying that the balanced growth path just characterized satisfies all the necessary — and, given our assumptions, sufficient — conditions for an optimum.

Appendix 2

Derivation of equations (17a)-(17c)

Since $\tilde{k}$ does not enter (14a)- (14b), we begin our local analysis of the equilibrium dynamics associated with system (14) by focusing on the pair $(\tilde{c}, \tilde{z})$. The dynamics of $\tilde{k}$ follow recursively, through (14c).

By linearizing (14a)-(14b) around the steady-state, one gets:

$$\begin{bmatrix} \dot{\tilde{c}} \\ \dot{\tilde{z}} \end{bmatrix} = J \cdot \begin{bmatrix} \tilde{c} - \tilde{c}^* \\ \tilde{z} - \tilde{z}^* \end{bmatrix}, \tag{A.2.1}$$

where

$$J \equiv \begin{bmatrix} A+\rho & -j \\ \rho & -(\rho+g) \end{bmatrix},$$

and j is given by (16) in the text. The two roots of the characteristic equation associated with system (A.2.1) are:

$$\frac{(A-g)}{2} \pm \frac{\sqrt{\Delta}}{2}, \tag{A.2.2}$$

where

$$\begin{aligned} \Delta &\equiv (A-g)^2 + 4(A+\rho)(\rho+g) - 4\rho j \\ &= 4 \cdot \left\{ \left[\frac{(A+g)}{2} + \rho \right]^2 - \rho j \right\}. \end{aligned}$$

Using the definition of j:

$$\left[\frac{(A+g)}{2}+\rho\right]^2-\rho j=\left(\frac{1}{u^*_{\tilde{c}\tilde{c}}}\right)\cdot\left\{\left[\frac{(A+g)}{2}+\rho\right]^2u^*_{\tilde{c}\tilde{c}}+2\rho\left[\frac{(A+g)}{2}+\rho\right]u^*_{\tilde{c}\tilde{z}}+\rho^2u^*_{\tilde{z}\tilde{z}}\right\}\geq 0,$$

because the term in curly brackets is a quadratic form in $\left[\frac{(A+g)}{2}+\rho\right]$ and ρ, and the Hessian of u is negative semidefinite.

It follows that concavity of the instantaneous felicity function implies:

$$\rho j\leq\left[\frac{(A+g)}{2}+\rho\right]^2, \tag{A.2.3}$$

so that $\Delta\geq 0$, and the two roots in (A.2.2) are real.

Given this result, and the fact that $Trace(J)=A-g>0$, the sign of these roots can be determined on the basis of the sign of the determinant:

$$\mid J\mid=\rho j-(A+\rho)(\rho+g).$$

When $j<0$, the Jacobian determinant is negative and we have two real roots of opposite sign. To make sure that the system is also saddlepath stable in the case of adjacent complementarity $(j>0)$ on which we focus in the text, we assume:

$$\rho j<(A+\rho)(\rho+g),$$

which amounts to the restriction on j in (16). Notice that, given $(A+\ \rho)\ (\rho+g)<\left[\frac{(A+g)}{2}+\rho\right]^2$, any value of j consistent with saddlepath stability is also consistent with concavity of u, and the upper bound that this assumption imposes on j (see A.2.3).

We shall use ζ to denote the positive characteristic root associated with the linearized version of system (14), and, as stated in the text, use $-\psi$ to denote the negative one:

$$-\psi = \frac{(A-g)}{2} - \sqrt{\left[\frac{(A+g)}{2} + \rho\right]^2 - \rho j}. \tag{A.2.4}$$

Notice that, although j depends on the second partial derivatives of u evaluated at the steady-state $(\tilde{c}^*, \tilde{z}^*)$, homogeneity of u implies that these roots are not a function of the initial conditions (k_0, z_0). In fact,

$$\begin{aligned} j &= -\frac{(A+g+2\rho)u_{\tilde{c}\,\tilde{z}}(\tilde{c}^*, \tilde{z}^*) + \rho u_{\tilde{z}\,\tilde{z}}(\tilde{c}^*, \tilde{z}^*)}{u_{\tilde{c}\,\tilde{c}}(\tilde{c}^*, \tilde{z}^*)} \\ &= -\frac{(A+g+2\rho)(\tilde{c}^*)^{\nu-2}u_{\tilde{c}\,\tilde{z}}(1, \frac{\rho}{\rho+g}) + \rho(\tilde{c}^*)^{\nu-2}u_{\tilde{z}\,\tilde{z}}(1, \frac{\rho}{\rho+g})}{(\tilde{c}^*)^{\nu-2}u_{\tilde{c}\,\tilde{c}}(1, \frac{\rho}{\rho+g})} \\ &= -\frac{(A+g+2\rho)u_{\tilde{c}\,\tilde{z}}(1, \frac{\rho}{\rho+g}) + \rho u_{\tilde{z}\,\tilde{z}}(1, \frac{\rho}{\rho+g})}{u_{\tilde{c}\,\tilde{c}}(1, \frac{\rho}{\rho+g})}. \end{aligned}$$

It follows that changes in the initial conditions will determine parallel upward or downward shifts of the saddlepaths in Figures 1 and 2.

We also define the two constants $\omega_1 \equiv \frac{\rho}{\rho+g-\psi}$, $\omega_2 \equiv \frac{1}{A-g+\psi}$. While the second one is always positive, the sign of ω_1 depends on that of $(\rho + g - \psi)$, which is the same as the sign of j. To see this, notice that , using (A.2.4):

$$\rho + g - \psi = \left[\frac{(A+g)}{2} + \rho\right] - \sqrt{\left[\frac{(A+g)}{2} + \rho\right]^2 - \rho j},$$

where we know that the term under the radical is positive. It follows that $(\rho + g - \psi) \gtrless 0$ as $j \gtrless 0$.

These definitions and results imply that the general solution of system (A.2.1) can be written as follows:

$$\tilde{c}_t - \tilde{c}^* = \Omega_1 e^{-\psi t} + \Omega_2 e^{\zeta t}, \tag{A.2.5}$$

$$\tilde{z}_t - \tilde{z}^* = \omega_1 \Omega_1 e^{-\psi t} + \left(\frac{\rho}{\rho + g + \zeta}\right) \Omega_2 e^{\zeta t}, \tag{A.2.6}$$

where Ω_1 and Ω_2 are arbitrary constants, to be determined using the initial conditions on the state variables and the transversality conditions in (12). Using (A.2.5) in the linearized version of the law of motion of $\tilde{k}$ and solving the resulting first-order, non-autonomous differential equation, yields:

$$\begin{aligned} \tilde{k}_t - \tilde{k}^* = & \left[(k_0 - \tilde{k}^*) - \omega_2 \Omega_1\right] e^{(A-g)t} + \omega_2 \Omega_1 e^{-\psi t} + \\ & \left(\frac{\Omega_2}{g - A + \zeta}\right) (e^{(A-g)t} - e^{\zeta t}). \end{aligned} \tag{A.2.7}$$

Given $\tilde{\lambda}$ constant, the first transversality condition in (12) requires:

$$\begin{aligned} \lim_{t \to \infty} e^{-(A-g)t} \tilde{k}_t = & \lim_{t \to \infty} \{ e^{-(A-g)t} \tilde{k}^* + \left[(k_0 - \tilde{k}^*) - \omega_2 \Omega_1\right] + \\ & \omega_2 \Omega_1 e^{-(A-g+\psi)t} + \left(\frac{\Omega_2}{g - A + \zeta}\right) (1 - e^{[\zeta - (A-g)]t}) \} = 0. \end{aligned}$$

Since $\zeta - (A - g) = \psi > 0$, for the transversality condition on the capital stock to be met one must have:

$$\Omega_2 = 0, \tag{A.2.8}$$

$$\Omega_1 = \left(\frac{1}{\omega_2}\right) (k_0 - \tilde{k}^*), \tag{A.2.9}$$

which imply that, in equilibrium, (A.2.7) becomes:

$$\tilde{k}_t - \tilde{k}^* = (k_0 - \tilde{k}^*) e^{-\psi t}. \tag{A.2.10}$$

Now notice that, using (A.2.8) in (A.2.5)-(A.2.6), evaluating at time $t = 0$ the resulting expressions for $(\tilde{c}_t - \tilde{c}^*)$ and $(\tilde{z}_t - \tilde{z}^*)$ and, taking (A.2.9) into account, one obtains:

$$\tilde{c}_0 - \tilde{c}^* = \left(\frac{1}{\omega_1}\right)(z_0 - \tilde{z}^*) = \left(\frac{1}{\omega_2}\right)(k_0 - \tilde{k}^*). \tag{A.2.11}$$

By plugging the expressions for $\tilde{z}^*$and $\tilde{k}^*$ as a function of $\tilde{c}^*$ given by (15a)-(15b) into the second of these equalities, one gets:

$$\tilde{c}^* = \frac{(A-g)(\rho+g)}{\psi(A+a)}\left[-\frac{1}{\omega_1}\,z_0 + \frac{1}{\omega_2}\,k_0\right];$$

this result, used in the first equality in (A.2.11), yields:

$$\tilde{c}_0 = \frac{(\rho+g)}{(A+\rho)\omega_1}\,z_0 + \frac{(A-g)}{(A+\rho)\omega_2}\,k_0. \tag{A.2.12}$$

Finally, it is easy to verify that (A.2.5), (A.2.6) and (A.2.10) can be written as (17a)-(17c) in the text, with $\tilde{c}_0$ and $\tilde{c}^*$ taking on the values just derived. Since these equilibrium paths imply convergence to a steady-state in which all variables assume constant values, they also satisfy the second transversality condition in (12).

References

Abel, A. B. (1990), "Asset Prices Under Habit Formation and Catching Up with the Joneses", *American Economic Review Papers and Proceedings*, vol. 80, pp. 38-42.

Becker, G. S. and K. M. Murphy (1988), "A Theory of Rational Addiction", *Journal of Political Economy*, vol. 96, pp. 675-700.

Boldrin, M., L. Christiano and J. Fisher (1995), "Asset Pricing Lessons for Modelling Business Cycles", *NBER Working Paper*, no. 5662.

Boyer, M. (1975), "An Optimal Growth Model with Stationary Non-Additive Utilities", *Canadian Journal of Economics*, vol. 8, pp. 216-237.

— (1978), "A Habit Forming Optimal Growth Model", *International Economic Review*, vol. 19, pp. 585-609.

Caballé, J. and M.S. Santos (1993), "On Endogenous Growth with Physical and Human Capital", *Journal of Political Economy,* vol. 101, no. 6, pp. 1042-1067.

Campbell, J. and J. Cochrane (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior", *Journal of Political Economy,* vol. 107, no. 2, pp. 205-251.

Carroll, C., J. Overland and D. Weil (1997), "Comparison Utility in a Growth Model", *Journal of Economic Growth*, vol. 2, no. 4, pp.339-367.

Christiano, L. (1989), "Understanding Japan's Saving Rate: The Reconstruction Hypothesis", *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 2, pp. 10-25.

Constantinides, G. (1990). "Habit Formation: A Resolution of the Equity Premium Puzzle", *Journal of Political Economy*, vol. 98, pp. 519-543.

Deaton, A.S. (1992), "*Understanding Consumption*", Oxford University Press, New York.

Detemple, J.B. and F. Zapatero (1991), "Asset Prices in an Exchange Economy with Habit Formation", *Econometrica,* vol. 59, no. 6, pp. 1633-1657.

Durlauf, S.N., and D.T. Quah (1998), "The New Empirics of Economic Growth", *NBER Working Paper,* no. 6422.

Ferraguto, G. and P. Pagano (1999), "Growth Under Habit Formation and Keeping Up with the Joneses", mimeo, Bocconi University and Bank of Italy.

Fuhrer, C.J. and M.W. Klein (1998), "Risky Habits: On Risk Sharing, Habit Formation, and the Interpretation of International Consumption Correlations", *NBERWorkingPaper,* no. 6735.

Gali, J. (1994), “Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices”, *Journal of Money, Credit, and Banking*, vol. 26, no. 1, pp. 1-8.

Mansoorian, A. (1993), “Habit Persistence and the Harberger-Laursen-Metzler Effect in an Infinite Horizon Model”, *Journal of International Economics*, vol. 34, pp. 153-166.

Obstfeld, M. (1992), “International Adjustment with Habit-Forming Consumption: A Diagrammatic Exposition”, *Review of International Economics*, vol. 1, no. 1, pp. 32-48.

Rebelo, S. (1991), “Long-Run Policy Analysis and Long-Run Growth”, *Journal of Political Economy*, vol. 99, no. 3, pp. 500-521.

— (1992), “Growth in Open Economies”, *Carnegie-Rochester Conference Series on Public Policy*, vol. 36, pp. 5-46.

Ryder, H. E. Jr. and J. M. Heal (1973), “Optimal Growth with Intertemporally Dependent Preferences”, *Review of Economic Studies*, vol. 40, pp. 1-33.

RECENTLY PUBLISHED "TEMI" (*)

No. 358 — *The Impact of News on the Exchange Rate of the Lira and Long-Term Interest Rates,* by F. FORNARI, C. MONTICELLI, M. PERICOLI and M. TIVEGNA (October 1999).

No. 359 — *Does Market Transparency Matter? a Case Study,* by A. SCALIA and V. VACCA (October 1999).

No. 360 — *Costo e disponibilità del credito per le imprese nei distretti industriali,* by P. FINALDI RUSSO and P. ROSSI (December 1999).

No. 361 — *Why Do Banks Merge?,* by D. FOCARELLI, F. PANETTA and C. SALLEO (December 1999).

No. 362 — *Markup and the Business Cycle: Evidence from Italian Manufacturing Branches,* by D. J. MARCHETTI (December 1999).

No. 363 — *The Transmission of Monetary Policy Shocks in Italy, 1967-1997,* by E. GAIOTTI (December 1999).

No. 364 — *Rigidità nel mercato del lavoro, disoccupazione e crescita,* by F. SCHIVARDI (December 1999).

No. 365 — *Labor Markets and Monetary Union: A Strategic Analysis,* by A. CUKIERMAN and F. LIPPI (February 2000).

No. 366 — *On the Mechanics of Migration Decisions: Skill Complementarities and Endogenous Price Differentials,* by M. GIANNETTI (February 2000).

No. 367 — *An Investment-Function-Based Measure of Capacity Utilisation. Potential Output and Utilised Capacity in the Bank of Italy's Quarterly Model,* by G. PARIGI and S. SIVIERO (February 2000).

No. 368 — *Information Spillovers and Factor Adjustment,* by L. GUISO and F. SCHIVARDI (February 2000).

No. 369 — *Banking System, International Investors and Central Bank Policy in Emerging Markets,* by M. GIANNETTI (March 2000).

No. 370 — *Forecasting Industrial Production in the Euro Area,* by G. BODO, R. GOLINELLI and G. PARIGI (March 2000).

No. 371 — *The Seasonal Adjustment of the Harmonised Index of Consumer Prices for the Euro Area: a Comparison of Direct and Indirect Methods,* by R. CRISTADORO and R. SABBATINI (March 2000).

No. 372 — *Investment and Growth in Europe and in the United States in the Nineties,* by P. CASELLI, P. PAGANO and F. SCHIVARDI (March 2000).

No. 373 — *Tassazione e costo del lavoro nei paesi industriali, by* M. R. MARINO and R. RINALDI (June 2000).

No. 374 — *Strategic Monetary Policy with Non-Atomistic Wage-Setters, by* F. LIPPI (June 2000).

No. 375 — *Emu Fiscal Rules: is There a Gap?,* by F. BALASSONE and D. MONACELLI (June 2000).

No. 376 — *Do Better Institutions Mitigate Agency Problems? Evidence from Corporate Finance Choices,* by M. GIANNETTI (June 2000).

No. 377 — *The Italian Business Cycle: Coincident and Leading Indicators and Some Stylized Facts,* by F. ALTISSIMO, D. J. MARCHETTI and G. P. ONETO (October 2000).

No. 378 — *Stock Values and Fundamentals: Link or Irrationality?,* by F. FORNARI and M. PERICOLI (October 2000).

No. 379 — *Promise and Pitfalls in the Use of "Secondary" Data-Sets: Income Inequality in OECD Countries,* by A. B. ATKINSON and A. BRANDOLINI (October 2000).

No. 380 — *Bank Competition and Regulatory Reform: The Case of the Italian Banking Industry,* by P. ANGELINI and N. CETORELLI (October 2000).

No. 381 — *The Determinants of Cross-Border Bank Shareholdings: an Analysis with Bank-Level Data from OECD Countries,* by D. FOCARELLI and A. F. POZZOLO (October 2000).

(*) Requests for copies should be sent to:
Banca d'Italia – Servizio Studi – Divisione Biblioteca e pubblicazioni – Via Nazionale, 91 – 00184 Rome (fax 0039 06 47922059). They are available on the Internet at www.bancaditalia.it