What Is the Optimal Institutional Arrangement for a Monetary Union?

by Leonardo Gambacorta
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WHAT IS THE OPTIMAL INSTITUTIONAL ARRANGEMENT
FOR A MONETARY UNION?

by Leonardo Gambacorta *

Abstract

The aim of this paper is to design the optimal institutional arrangement for a monetary union. Using a two-country rational expectations model, the study analyses how the conservatism of the area-wide central bank and the penalty system for fiscal deviation (Stability and Growth Pact) should be designed with respect to different economic shocks. The optimal institutional arrangement is also dependent on who is the “leader” of the policy game. When national governments move first, the independent area-wide central bank can exercise greater discipline over national fiscal policies, making the Stability Pact unnecessary.

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1. Introduction

The economic policy scenario for the European monetary union, designed in the Maastricht Treaty, delegates monetary policy to an independent area-wide central bank, while fiscal instruments are wielded by national governments. Control of macroeconomic policies is therefore divided between authorities with different objectives. By statute, the European Central Bank’s main concern is controlling inflation, while output and employment are secondary targets. Conversely, national governments may seek to achieve full employment and maximise growth. These differences can cause a loss of co-ordination in macroeconomic policy making and create the potential for conflict not only on output demand control (Blake and Westaway, 1993; Rankin, 1995), but also on the reaction to supply shocks.

The institutional arrangement of the monetary union can be represented by i) the conservatism of the area-wide monetary authority and ii) the control system over national fiscal deficits. These must be chosen both to maintain the “sustainable convergence” among the economies (pre-condition achieved with the well-known Maastricht criteria) and to avoid the above-mentioned conflict between (national) fiscal and (common) monetary policies.

The latter aspect, which is considered in the Maastricht Treaty, led to the drafting of the “Stability and Growth Pact” (Dublin, December 1996), whose aim is to bring budget deficits under long-term durable control through a system of sanctions to be applied to any country that exceeds a public deficit/GDP ratio of three per cent.

This paper analyses the characteristics of the optimal institutional arrangement of a monetary union. In order to avoid inflation bias, which causes a problem of time

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2 U.S. macroeconomic policy in the eighties and German reunification in 1990 are two recent experiences which show the potential problems emerging in a system with an independent central bank. In particular, during the Reagan administration the Federal Reserve instituted a regime of tight monetary targets, while the Government pursued an expansionary fiscal policy by increasing defence spending and cutting taxes. German reunification was characterised by fiscal spending to assist the East German economy, which the Bundesbank countered by tightening monetary policy to fight inflation. Both experiences led to large budget deficits and high interest rates, penalising investment and generating a loss of future productive potential.
inconsistency, the area-wide central bank has to set its output objective at the natural level, which is lower than full employment due to market imperfections. On the other hand, national governments care principally about employment and are tempted to pursue an expansionary fiscal policy to increase output above the natural level. The independent monetary authority is able to win the “battle” over demand control but at the cost of a higher interest rate. There is a trade-off between low inflation, which is what delegation of control achieves, and a low interest rate, which benefits the long-run real economy.

The search for the optimal institutional arrangement is developed in two stages. In the first stage, the effects of the policy game on prices, output and the interest rate are analysed, taking into account all the possible shocks which may hit the economies. In the second stage, the solutions for the macroeconomic variables are utilised to determine the social loss function. This can be minimised with respect to the institutional variables to work out the optimal combination between the conservatism of the area-wide central bank and the penalty system for fiscal deviation discussed above.

Studying the Stackelberg equilibrium of the game, one of the main results of the paper is that the independent central bank is able to discipline the expansionary behaviour of the governments only if these are the “leader” in the policy game. When the area-wide central bank moves first, there is no disciplinary effect. These findings - consistent with the results obtained by Rankin (1995) and by Beetsma and Bovenberg (1998) in different analytical frameworks without stochastic shocks - differ from those obtained by the standard analysis in the traditional two-player game between the central bank and the public and depend on the introduction of the fiscal authorities as new players.

The remainder of the paper is organised as follows. The next section presents the analytical framework, based on a rational expectations model along the lines of Turnovsky and d’Orey (1989) and Monticelli (1993), which represents a monetary union between two symmetric countries. After discussing the institutional characteristics of the policy game represented by the loss functions of the players (Section 3), the characteristics of the non-cooperative Nash equilibrium are analysed (Section 4). Section 5 investigates the property of

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3 For detailed overview of EMU issues see, inter alia, Fratianni, von Hagen and Waller (1992), De Grauwe (1994a; 1994b) and Bayoumi, Eichengreen and von Hagen (1997).
the optimal institutional arrangement, while the consequences of one of the policymakers acting as a leader (Stackelberg equilibrium) are analysed in Section 6. The last section summarises the main conclusions.

2. The case of a monetary union between two symmetric economies

This section presents a rational expectations model in which two symmetric economies form a monetary union. The specification is log-linear and, in order to simplify the analytical form, all variables are expressed as deviations from their trend level. The economies are subject to real and nominal disturbances.

The system of equations is summarised in Table 1 where asterisks denote variables pertaining to the foreign country. The model has two assets (money and bonds) and a common traded good.

The first pair of equations relate to money market equilibrium. In particular, (1) represents a standard money demand function where, for simplicity, income elasticity is set to one, while (2) implies that the common money supply process at the area level is based on money targeting. It is worth noting that this rule determines the overall amount of money \( m^{*} = (m_{t} + m_{t}^{*})^{s} \), while its distribution between the two countries is endogenously determined.\(^4\)

The goods market is described by equations (4) and (5). The first relation represents output demand, which depends upon the other country’s output (via exports) and upon the price difference (measure of competitiveness). Output demand is also influenced by the real cost of financing for investment and consumption. It is worth noting that the nominal interest rate is unique. In fact, assuming the same default risk in the two countries and an irrevocably fixed exchange rate, the uncovered interest parity condition implies perfect substitutability between bonds at the area-level. Public expenditures \((x \text{ and } x^{*})\), controlled by national governments, can only be financed by direct taxation on income (for simplicity, assumed to be fixed and equal in the whole community) or by issuing bonds. Government expenditures

\(^4\) Given total wealth, the bond market is conveniently suppressed by Walras’ law using the private sector budget constraint.
### The case of a monetary union between two symmetric economies

<table>
<thead>
<tr>
<th>Home country</th>
<th>Foreign country</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Money market</strong></td>
<td></td>
</tr>
<tr>
<td>Money demand</td>
<td>( m_t^d - p_t = y_t - ai_t + u_{md} ) (( a &gt; 0 ))</td>
</tr>
<tr>
<td>Money supply</td>
<td>( (m_t + m_t^*)^s = m_t^{*s} )</td>
</tr>
<tr>
<td>Equilibrium condition</td>
<td>( m_t^d + m_t^s = (m_t + m_t^*)^s )</td>
</tr>
</tbody>
</table>

| **Output market** |                  |
| Aggregate demand  | \( y_t^d = by_t^* - f(i_t - E \ p_{t+1}^* + p_t^* + d(p_t^* - p_t^*), x_t^* + u_{yd}^* \) | \( y_t^* = by_t - f(i_t - E \ p_{t+1}^* + p_t^* + d(p_t^* - p_t^*), x_t^* + u_{yd}^* \) (4) |
| (\( 0 < b < 1, \ f > 0 \) e \( d > 0 \)) |                  |
| Aggregate supply | \( y_t^s = g(p_t - E \ p_{t-1}^* + u_{ys}^* \) | \( y_t^* = g(p_t^* - E \ p_{t-1}^* + u_{ys}^* \) (5) |
| Equilibrium condition | \( y_t^d + y_t^s = y_t^s + y_t^s \) (5') |                  |

| **Stochastic law of one price** | \( p_t = p_t^* + u_p \) (6) |
| **Time consistency condition** | \( E \ p_{t+1} = E \ p_{t+1}^* = 0 \) (7) |

Note: the meaning of all the symbols is reported at page 11.
List of symbols:

$a$ = semi-elasticity of demand for money with respect to the nominal interest rate

$b$ = elasticity of exports with respect to foreign real output (it also represents the degree of integration between the countries)

$d$ = elasticity of aggregate demand with respect to the price difference

$E_t$ = expectation operator, conditional on information at time $t$

$f$ = semi-elasticity of aggregate demand with respect to the real interest rate

$g$ = measure of the “price surprise” effect on real output

$i$ = nominal interest rate, expressed in units

$L$ = loss function

$m$ = nominal money supply at the area level, expressed in logs

$p$ = price of output expressed in logs

$u_{md}$ = stochastic disturbance in the demand for money

$u_p$ = stochastic disturbance in the law of one price (price-wedge)

$u_{sd}$ = stochastic disturbance in the aggregate demand

$u_{sy}$ = stochastic disturbance in the aggregate supply

$w_{ECB}$ = European Central Bank’s preference for price stability (degree of conservatism)

$w_S$ = society’s preference for price stability

$x$ = public expenditure, expressed in logs

$y$ = real output in logs, measured as a deviation from its natural rate level

$\hat{y}$ = full employment output level

$\hat{y}_{ECB}$ = European Central Bank output objective

$\beta$ = penalty for deviating fiscal policies (Stability Pact)

$\gamma$ = society’s preference for interest rate stability

cannot be covered by printing money. Since public expenditures are expressed as the deviation from their trend level while taxation is constant, $x$ and $x^*$ also represent public deficit deviations.\(^5\)

\(^5\) The hypothesis of fixed taxation has been adopted to simplify the analysis because the results of the study would not have changed if proportional taxation had been considered.
Equation (5) describes a supply function where deviations in output from its natural rate (normalised to zero) are a positive function of unanticipated movements in the price level. The microfoundation of the “price surprise” effect derives from nominal wage rigidity, due to the fact that wage bargaining takes place in advance of production (Friedman, 1968). The model is therefore characterised by an asymmetry in the setting of price expectations between workers and entrepreneurs. The workers (or the unions for them) formulate their price expectation before the entrepreneurs, who set their real interest rate expectation \( (i_t - E_{t+1} p_{t+1} + p_t) \) knowing the actual price level and working out price expectations for time \( t+1 \).

The two country blocks are linked by equation (6), which assumes perfect substitutability in the output market. The difference in prices depends only on the disturbance \( u_p \), which represents market imperfections (stochastic law of one price).\(^6\)

This wedge between the prices of the two countries is determined by a difference in the monopolistic power of firms and trade unions (Minsky and Ferri, 1984). In particular, the capacity of firms to influence prices (mark-up) depends on institutional factors (antitrust legislation) and demand elasticities, while the ability of trade unions to influence wages (and thus prices) depends on workers’ participation and the level of unemployment (see Layard, Nickell and Jackman, 1991). So, if homogeneous “anti-trust” legislation is assumed, this wedge in prices mainly reflects differences between national labour markets (unemployment levels) that give rise to different wage pressures on prices and demand.

All the stochastic variables are assumed to be independently distributed with zero mean and finite variance \( (u \sim \text{id}(0, \sigma_u)) \).

If the optimal policy rules chosen by public authorities \( (m', x \text{ and } x^*) \) are credible, given that the model is in deviation form all the expectations based on information at time \( t \) can be set to zero (see equation 7). However, although the rational expectations hypothesis assumes that agents know the model period by period, the institutional parameters are not necessarily

\(^6\) The simplifying assumption of an exogenous disturbance to the law of one price can be relaxed in a framework which explicitly recognises the difference between the goods produced in the two countries. In this case, the difference between the prices can be derived as a function of the asymmetric shocks hitting the two economies (Monticelli, 1993; Gambacorta, 1998).
constant over time and this may cause “price surprise” effects \( E_{t-1} p_t \), hereafter \( p_t^e \), could be different from zero).

The model described in Table 1 can be solved using five equations (the money market equilibrium, the output demand for the union, the stochastic law of one price and the two output supplies), which determine the five endogenous variables (the price levels \( p_t \) and \( p_t^e \), the real incomes \( y_t \) and \( y_t^e \) and the common interest rate on bonds \( i_t \)).

Given a rational expectations model, a classic “three-step” method has been used. The reduced form reported in Table 2 shows the following results, which are consistent with the existing literature.

If monetary and fiscal policies are perfectly anticipated, they only affect prices and the interest rate, while output only reacts to the shocks. Leaving aside disturbances and setting \( m'=m^e \), \( x=x^e \) and \( x^*=x^e \), it is possible to obtain:

\[
p = p^* = \frac{1}{2(1+a)} m^e + \frac{a}{2 f (1+a)} (x + x^*)
\]

\[
y = y^* = 0
\]

\[
i = -\frac{1}{2(1+a)} m^e + \frac{1}{2 f (1+a)} (x + x^*)
\]

In particular, government expenditure has a positive effect on prices and the interest rate with an intensity that is inversely related to “money channel” effectiveness \( f \). On the other hand, if the value of money demand semi-elasticity \( a \) is high, equilibrium prices tend to be higher with respect to the interest rate. Money supply has an effect of the opposite sign but the same intensity on prices and the interest rate which, as expected, maintains the real interest rate constant.

Given the perfect symmetry between the economies, money and output demand shocks enter the equations in additive form and, therefore, the effects of such disturbances are
Table 2

Reduced form of the model

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8) [ p_t = \frac{f}{\Delta} m^* + \frac{g(f + a\vartheta_1)}{(1 + a)\Delta} m^e + \frac{a}{\Delta} (x + x^<em>) + \frac{ag(f + a\vartheta_1)}{f(1 + a)\Delta} (x^e + x^{e</em>}) - \frac{f}{\Delta} (u_{md} + u_{md^<em>}) + \frac{a}{\Delta} (u_{yd} + u_{yd^</em>}) - \frac{(f + a\vartheta_1)}{\Delta} (u_{ys} + u_{ys^*}) + \frac{1}{2} u_p ]</td>
</tr>
<tr>
<td>(9) [ p_t^* = \frac{f}{\Delta} m^* + \frac{g(f + a\vartheta_1)}{(1 + a)\Delta} m^e + \frac{a}{\Delta} (x + x^<em>) + \frac{ag(f + a\vartheta_1)}{f(1 + a)\Delta} (x^e + x^{e</em>}) - \frac{f}{\Delta} (u_{md} + u_{md^<em>}) + \frac{a}{\Delta} (u_{yd} + u_{yd^</em>}) - \frac{(f + a\vartheta_1)}{\Delta} (u_{ys} + u_{ys^*}) - \frac{1}{2} u_p ]</td>
</tr>
<tr>
<td>(10) [ y_t = \frac{fg}{\Delta} (m^* - m^e) + \frac{ag}{\Delta} [(x - x^e) + (x^* - x^{e*})] - \frac{fg}{\Delta} (u_{md} + u_{md^<em>}) + \frac{ag}{\Delta} (u_{yd} + u_{yd^</em>}) + \frac{2f(1 + a) + g(f + a\vartheta_1)}{\Delta} u_{ys} - \frac{g(f + a\vartheta_1)}{\Delta} u_{ys^*} + \frac{g}{2} u_p ]</td>
</tr>
<tr>
<td>(11) [ y_t^* = \frac{fg}{\Delta} (m^* - m^e) + \frac{ag}{\Delta} [(x - x^e) + (x^* - x^{e*})] - \frac{fg}{\Delta} (u_{md} + u_{md^<em>}) + \frac{ag}{\Delta} (u_{yd} + u_{yd^</em>}) - \frac{g(f + a\vartheta_1)}{\Delta} u_{ys} + \frac{2f(1 + a) + g(f + a\vartheta_1)}{\Delta} u_{ys^*} - \frac{g}{2} u_p ]</td>
</tr>
<tr>
<td>(12) [ i_t = -\frac{f + g\vartheta_1}{\Delta} m^* - \frac{g(f - \vartheta_1)}{(1 + a)\Delta} m^e + \frac{1 + g}{\Delta} (x + x^<em>) - \frac{ag(f - \vartheta_1)}{f(1 + a)\Delta} (x^e + x^{e</em>}) - \frac{f + g\vartheta_1}{\Delta} (u_{md} + u_{md^<em>}) + \frac{1 + g}{\Delta} (u_{yd} + u_{yd^</em>}) + \frac{(f - \vartheta_1)}{\Delta} (u_{ys} + u_{ys^*}) ]</td>
</tr>
</tbody>
</table>

where:

\[ \Delta = 2(f + af + fg + ag\vartheta_1) > 0 \]

\[ 0 < \vartheta_1 = 1 - b < 1 \]

Note: the meaning of all the symbols is reported at page 11.
smaller if they are negatively correlated (the insulation is perfect if the shocks are opposite).\footnote{In a more general framework which considers structural asymmetries between countries, only shocks to the demand for money maintain an additive form, while aggregate demand disturbances have different effects on national endogenous variables. In fact, as pointed out by Monticelli (1998), when real shocks to aggregate demand prevail the difference in the slope of the \( AS \) curve is the key structural asymmetry that determines heterogeneous effects on national prices and incomes.}

Supply shocks and “price-wedge” disturbances have a different effect on the endogenous variables of the countries and therefore only when they are negligible does the stabilisation objective of each of the countries coincide with that of the monetary area as a whole.

3. Institutional characteristics of the policy game

The economic system described by the model has five players: two representative agents (one for each country), two national governments and an area-wide central bank.

The representative agents are assumed to care about three macroeconomic variables: output, inflation and the interest rate. Following the work of Hamada (1976), society’s preferences at the area-level are represented by a standard quadratic loss function:\footnote{Horowitz (1987) and Aizenman and Frenkel (1985) provide justifications for using quadratic loss functions as reasonable approximations to true measures of losses due to risk aversion or forgone consumer and/or producer surplus. However, as pointed out by Daniels and Vanhoose (1998), the main reason for the choice of a quadratic form in much of two-country model literature remains the gain of expositional simplicity.}

\[ L^S = (y - \bar{y})^2 + (y^* - \bar{y}^*)^2 + w_S (p^2 + p'^2) + \gamma (i - \bar{i})^2 \]

where penalty values have been normalised with respect to the weight of output. The symmetry of the model implies that full employment output \((\bar{y})\) is the same in the two countries. Moreover, due to the market imperfections discussed in Section 2 (monopolistic power of firms and trade unions), the latter is greater than natural output \((\hat{y} > y_N)\).

Since all variables are expressed as deviations from the trend, natural output and price averages can be set to zero \((y_N = 0, \bar{p} = 0 \text{ and } \bar{p}^* = 0)\). The last two conditions imply that the
second part of (13) represents the minimisation of national inflation rates (a common weight \( w_S \) is attached).  

The third part represents the fact that deviations of the interest rate from its optimal value have a cost for the society. As shown in Appendix I, solving for the precommitment solution of the game it is possible to prove that the optimal interest rate is equal to the trend value \( \hat{i} = 0 \) consistent with the “natural” long-run growth of the economy.  

Two main explanations for the inclusion of interest rate stability in the utility function can be identified in the existing literature. The first refers to theories of consumption decisions under uncertainty: interest rate instability has a negative effect on consumption, increasing the precautionary saving motive (Levhari and Srinivasan, 1969; Rothschild and Stiglitz, 1971; Selden, 1979; Sibley and Levhari, 1986; Epstein 1988; Epstein and Zin, 1989; Giovannini and Weil, 1989; Langlais, 1995). A similar effect can also be found in continuous time cash-in-advance models under uncertainty (Rebelo and Xie, 1997).

The second is given by the “financial instability hypothesis” approach (Fisher, 1933; Minsky, 1975, 1982; Kindleberger, 1978): a stable interest rate reduces the financial fragility of the economy and the negative consequences of the financial cycle. Moreover, interest rate smoothing discourages quick reversal of asset prices, reducing the variability of financial wealth (Goodhart, 1996).

Following Rogoff (1985), at the start of each period in each country the representative agent sets the nominal wage on the basis of the expected price level computed using the

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9 Given the presence of only one traded good, whose price can differ between the two countries only by a stochastic component (see equation 6), the simplifying hypothesis that the two inflation rates refer to GDP deflators has been adopted. In a more general model, with two separate output markets, it would be preferable to use consumer price indexes \( q = \lambda p + (1-\lambda) p^* \) and \( q^* = \lambda^* p + (1-\lambda^*) p \), which explicitly consider the proportion of income used to buy domestic goods (\( \lambda \) and \( \lambda^* \)) and foreign goods (\( 1-\lambda \) and \( 1-\lambda^* \)). This approach, however, tends to coincide with the simplified one when the two countries have the same degree of openness (\( \lambda = \lambda^* \)), a hypothesis that is likely to occur given their symmetric economic structures.

10 It is worth noting that, leaving aside the problems concerning the growth of the economy, this result also has a microfounded interpretation: the setting of the nominal interest rate to zero maximises the consumer surplus in the money market (Friedman, 1969). Using the real interest rate in the loss function would not have changed the results of the study. In fact, given the hypothesis of complete credibility of the behaviour of the central bank, the equilibrium of the economy is always characterised by the absence of inflation bias: this means that the nominal and real interest rate biases are equal.
information set of the previous period. By choosing \( p_t^e \) and \( p_t^e^* \) the public implicitly controls the position of the AS curves (see equations 5).

National governments use fiscal policies \((x \text{ and } x^*)\) to minimise the following loss functions (they control the IS curves):

\[
\min_x L^G = (y - \dot{y})^2 + \gamma(i - \dot{i})^2 + \beta x^2
\]

\[
\min_x L^G^* = (y^* - \dot{y})^2 + \gamma(i - \dot{i})^2 + \beta x^*^2
\]

In particular, each government cares only about two macroeconomic variables, while controlling inflation is delegated to the area-wide central bank in order to solve the time consistency problem. In the presence of democratic governments, the weights are taken to represent the average preferences of society (1 and \( \gamma \) are the same of equation 13).

The last part of equations (14) represents the penalty paid by each national government when fiscal policies deviate from their trend. Under the terms of the Stability and Growth Pact, a sanction is imposed on an offending country if it deviates from the threshold level. The parameter \( \beta \) in (14) represents the variable part of the sanction, proportional to the deviation of public expenditure from the trend value, which is assumed to be consistent with

\[\text{\textit{11 Using a more general loss function of the kind } } L^G = (y - \dot{y})^2 + \gamma(i - \dot{i})^2 + w_{GOV} p^2 + \beta x^2, \text{ with a lower weight given to inflation by the government than by the central bank (} w_{GOV} < w_{ECB}, \text{ would not have changed the result of the study. The main insight of Kydland and Prescott (1977) and Barro and Gordon (1983) is that if money is controlled by the government, the precommitment solution is not time consistent. In fact, it would be optimal for the government to create zero inflation but this equilibrium cannot be achieved since, if the expectation of zero inflation were built into wage contracts, the government would have an incentive to create inflation after contracts had been set, in order to get closer to its output target. In this situation, the best policy for the government is to delegate monetary control to a “conservative” central bank that is more concerned about inflation relative to output than society as a whole (Rogoff, 1985).}\]

\[\text{\textit{12 Specifically, the sanction is initially imposed as a non-interest bearing deposit with the European Commission, which can become a fine (composed of a fixed and a variable part) that could be as high as 0.5 per cent of the offending country’s GDP. The pact does allow for breaches of the limit in “exceptional circumstances” when a country is in recession. In fact, if annual GDP falls by 2 per cent or more there is an automatic exemption, while if the reduction is between 0.75 and 2 per cent the deviating country can appeal against the sanction.}}\]
the three per cent deficit/GDP ratio fixed in the Maastricht Treaty.  

The governing council of the area-wide central bank is assumed to be composed of two national members who care about national goals. Nevertheless, given the symmetry of the model, each member has the same decision power and an equal weight is attached to national objectives.

The independent area-wide central bank, as already pointed out, is assumed to care about inflation more than the society \( w_{ECB} > w_S \), while output deviations represent only a secondary target. Its loss function is minimised using money supply as a policy instrument (the central bank implicitly controls the \( LM \) curves for both countries):  

\[ L^{ECB} = w_{ECB} (p + p')^2 + (y + y' - 2 \gamma^{ECB})^2. \]

---

13 This hypothesis, adopted to simplify the model, can be generalised to the case of a rate of growth of public expenditure different from three per cent. For example, if the trend value of public expenditure is greater than the limit fixed in the Maastricht Treaty there is a deviation even if the detrended value is equal to zero. In this case, the loss function of national governments (14) should be modified in the following way:

\[ L^G = (y - \gamma)^2 + \gamma (i - \hat{i})^2 + \beta (x + \delta)^2, \]

where \( \delta > 0 \) represents the difference between the public expenditure trend and the three per cent limit. In the opposite case, when the trend is less than three per cent, \( \delta < 0 \) and there is a lower probability of deviation from the limit imposed in the Maastricht Treaty. Moreover, it is worth noting that the imposition of a proportional cost for deviation from a target value of fiscal expenditure is a simplification that does not completely fit the fact that the Stability Pact is aimed mainly at preventing fiscal policies from having an expansionary bias. The quadratic form of the loss function does not provide a way to discriminate between over- and undershooting the limit. In fact, equation (14) determines a loss even in the case of an extremely “tight” fiscal policy. In this case the quadratic form can be justified in two ways. The first refers to the “Political Business Cycle” theories (Nordhaus, 1975,1989): a fiscal policy not in line with the natural pattern of the economy (represented by the trend deviation) and the consequent welfare reduction have a negative impact on the electorate and cause the government to lose popularity. In this case, the \( \beta \) parameter does not represent the sanction imposed by the European Commission but rather the loss deriving from the lower probability of being reelected. The second justification is based on the fact that the medium term objective of “close to balance or in surplus” on the one hand allows automatic stabilisers to operate (Buti, Franco e Ongena, 1998) but, on the other may discourage countries from making active use of discretionary fiscal policies in response to large stochastic shocks if interpreted too strictly.

14 The meaning of “secondary target” in this context is not the same as that implied by Article 105 of the Maastricht Treaty. That article opens the way for the ECB to have secondary objectives but, if strictly interpreted, it would suggest a lexicographic preference ordering (with no substitution at the margin between the two objectives). For an analysis of policymakers’ revealed preferences and the output-inflation variability trade-off see Cecchetti and McConnell (1998).

15 It is worth noting that if the objectives of the central bank of the union were related to area-wide variables (with no weight attached to national goals), the loss function (15) should be written in the following way:

\[ \min_m L^{ECB} = w_{ECB} (p + p')^2 + (y + y' - 2 \gamma^{ECB})^2. \]

In particular, if the central bank’s main objective is price stabilisation it is not possible to consider the nominal interest rate as an additional objective in the loss function (15). In that case, in fact, it can be shown that the equilibrium of the policy game is characterised by inflation bias, which is not coherent with the time consistency condition (7). For a different interpretation of the future behaviour of the European Central Bank based on the seminal work of Taylor (1993), see Gerlach and Schnabel (1998) and Peersman and Smets (1998).
Equation (15) encompasses three kinds of institutional behaviour of the European Central Bank. In particular, setting \( w_{\text{ECB}} = \infty \) it is possible to obtain the loss function of an “ultraconservative” central bank that cares only about inflation (see equation 15a). Equation (15b) represents the case of a central bank that, as a secondary objective, wants to limit output deviations from the trend \( (\hat{y}_{\text{ECB}} = 0) \) in order to avoid severe recessions and explosive booms. Equation (15c) shows the case of a central bank that wants to bring unemployment below its natural level \( (\hat{y}_{\text{ECB}} = \hat{y}) \).

\[
\begin{align*}
(15a) & \quad \min_m L^{\text{ECB}} = (p^2 + p^*2) \\
(15b) & \quad \min_m L^{\text{ECB}} = w_{\text{ECB}}(p^2 + p^*2) + y^2 + y^*2 \\
(15c) & \quad \min_m L^{\text{ECB}} = w_{\text{ECB}}(p^2 + p^*2) + (y - \hat{y})^2 + (y^* - \hat{y})^2
\end{align*}
\]

Using the analytical framework discussed in Section 2, it is possible to describe a simple one-shot game between the above mentioned players (see Figure 1).\(^{16}\) Due to rigidity in wage determination, the public is the first mover. In fact, the two country representative agents have to form rational inflation expectations (\( p_t^e \) and \( p_t^{e*} \)) at the beginning of period \( t-1 \) and to embody them in nominal wage contracts which cannot be changed until \( t \). During the period the economies can be hit by stochastic shocks, which might be symmetric (\( u = u^* \)), asymmetric (\( u \neq 0 \) and \( u^* = 0 \)), idiosyncratic (\( u > u^* \)) or opposite (\( u = -u^* \)). The European Central Bank and the national governments simultaneously choose \( m', x \) and \( x^* \) at the end of the period taking into account the stochastic disturbances (Nash equilibrium).

---

\(^{16}\) Bini Smaghi and Del Giovane (1996) provide a simple way to take into account intertemporal aspects of monetary and fiscal policy issue by considering multiperiod problem in policymaking and coordination. In particular, the authors study a simple two-period version of a standard two-country model to evaluate policy criteria that might contribute to convergence among countries that join the European monetary union.
Characteristics of the one-shot policy game (Nash equilibrium)

4. Non-cooperative equilibrium

The reduced form reported in Table 2 is affected by expectations on policy instruments. Assuming that agents know the characteristics of the game and have no information on the disturbances, it is possible to work out their rational expectations. In Appendix II it is shown that:

\[
x^e = x^*e = \frac{fg[aw_{ECB}\hat{y} + \gamma \hat{y}_{ECB}(1 + g)]}{[\beta \Delta f + \gamma (1 + g)]w_{ECB}}
\]

\[
m^*e = \frac{2g[-\alpha ^2 \hat{y} w_{ECB} + \hat{y}_{ECB}[\beta \Delta f(1 + a) + \gamma (1 + g)]]}{[\beta \Delta f + \gamma (1 + g)]w_{ECB}}
\]

which give the solution for the endogenous variables reported in Table 3.

It is worth noting that substituting equations (16) in the price expectations formulas obtained with the “three-step” method, the result is \(p^e = p^*e = \frac{g\hat{y}_{ECB}}{w_{ECB}}\).

As expected, price expectations are zero only if the behaviour of the central banker is “ultraconservative” \((w_{ECB}=\infty\), see loss function 15a) or his secondary objective consists in limiting output deviations from the trend level \((\hat{y}_{ECB} = y_N = 0\), see equation 15b). On the other hand, if the central bank wants to bring unemployment below the natural level \((\hat{y}_{ECB} = \hat{y} > y_N = 0\), see loss function 15c) the public expects a permanent inflation bias with respect to the precommitment solution (see Appendix I). It is interesting to note that the
Table 3

**Solution of the model**

\[
(8') \quad p_t = \frac{f}{\Delta} m' + \frac{a}{\Delta} (x + x^*) - \frac{f}{\Delta} (u_{md} + u_{md^*}) + \frac{a}{\Delta} (u_{yd} + u_{yd^*}) - \left( \frac{f + a\bar{\vartheta}_1}{\Delta} \right) (u_{ys} + u_{ys^*}) + \frac{1}{2} u_p + \frac{2g^2 (f + a\bar{\vartheta}_1) \hat{y}_{ECB}}{\Delta w_{ECB}}
\]

\[
(9') \quad p_t^* = \frac{f}{\Delta} m' + \frac{a}{\Delta} (x + x^*) - \frac{f}{\Delta} (u_{md} + u_{md^*}) + \frac{a}{\Delta} (u_{yd} + u_{yd^*}) - \left( \frac{f + a\bar{\vartheta}_1}{\Delta} \right) (u_{ys} + u_{ys^*}) - \frac{1}{2} u_p + \frac{2g^2 (f + a\bar{\vartheta}_1) \hat{y}_{ECB}}{\Delta w_{ECB}}
\]

\[
(10') \quad y_t = \frac{f g}{\Delta} m' + \frac{a g}{\Delta} (x + x^*) - \frac{f g}{\Delta} (u_{md} + u_{md^*}) + \frac{a g}{\Delta} (u_{yd} + u_{yd^*}) + \frac{2f(1+a) + g(f + a\bar{\vartheta}_1)}{\Delta} u_{ys} - \frac{g(f + a\bar{\vartheta}_1)}{\Delta} u_{ys^*} + \frac{g}{2} u_p - \frac{2fg^2 (1+a) \hat{y}_{ECB}}{\Delta w_{ECB}}
\]

\[
(11') \quad y_t^* = \frac{f g}{\Delta} m' + \frac{a g}{\Delta} (x + x^*) - \frac{f g}{\Delta} (u_{md} + u_{md^*}) + \frac{a g}{\Delta} (u_{yd} + u_{yd^*}) - \frac{g(f + a\bar{\vartheta}_1)}{\Delta} u_{ys} + \frac{2f(1+a) + g(f + a\bar{\vartheta}_1)}{\Delta} u_{ys^*} - \frac{g}{2} u_p - \frac{2fg^2 (1+a) \hat{y}_{ECB}}{\Delta w_{ECB}}
\]

\[
(12') \quad i_t = \frac{f + g\bar{\vartheta}_1}{\Delta} m' + \frac{1 + g}{\Delta} (x + x^*) + \frac{f + g\bar{\vartheta}_1}{\Delta} (u_{md} + u_{md^*}) + \frac{1 + g}{\Delta} (u_{yd} + u_{yd^*}) + \frac{(f - \bar{\vartheta}_1)}{\Delta} (u_{ys} + u_{ys^*}) - \frac{2g^2 (f - \bar{\vartheta}_1) \hat{y}_{ECB}}{\Delta w_{ECB}}
\]

where:

\[\Delta = 2(f + af + fg + ag\bar{\vartheta}_1) > 0\]

\[0 < \bar{\vartheta}_1 = 1 - b - c(1-t) < 1\]

Note: the meaning of all the symbols is reported at page 11.
expected inflation bias is influenced neither by national governments output objectives ($\hat{y}$) nor by the Stability Pact ($\beta$).

In order to avoid the inflation bias, which causes a problem of time inconsistency, the area-wide central bank has to set its output target at the natural level. The attempt (at time $t$) to increase output to the full employment level determines a “price surprise” only until the next wage contract is negotiated. In fact, at time $t+1$, the public will incorporate the bias into their price expectations and this will cause an increase in the inflation trend (therefore, given the rational expectation hypothesis of this model, output deviation from the natural level is temporary and only due to rigidity in the wage determination mechanism). To sum up, the only way for the area-wide central bank to reduce unemployment below its natural level is to stimulate inflation.

If the independent central bank is assumed to care more about inflation than unemployment (the latter is only a secondary target), the loss function (15c), causing an increasing inflation bias, cannot represent its institutional task. Henceforth, the analysis will compare only the cases of an “ultraconservative” (see equation 15a) and a “partially conservative” (see equation 15b) central bank whose loss functions do not cause time consistency problems. Moreover, since equation (15a) is encompassed in equation (15b), the latter is adopted as the general case, while the results referring to the first loss function are obtained setting $w_{ECB}=\infty$.

The Nash equilibrium of the policy game is worked out by solving the following system:

\[
\begin{align*}
\min_x L^G &= (y - \hat{y})^2 + \gamma i^2 + \beta x^2 \\
\min_x L^G^* &= (y^* - \hat{y})^2 + \gamma i^2 + \beta x^{*2} \\
\min_m L^{ECB} &= w_{ECB} (p^2 + p^{*2}) + y^2 + y^{*2}
\end{align*}
\]

Substituting the solution of the model (see Table 3) in the above expressions, they turn out to be just a function of the instruments ($m'$, $x$ and $x^*$) and the shocks. Each public authority minimises its loss function by taking as given the actions of the other agents (Nash equilibrium). From the first order conditions of system (17) it is possible to obtain the
expression of the reaction functions of the players (see Appendix III). The point where these reaction functions intersect gives the Nash equilibrium:

\[ x^N = -\Phi_1 (u_{yd} + u_{yd*}) - \Phi_2 u_{ys} + \Phi_3 u_{ys*} - \Phi_4 u_p + \Phi_5 \]

\[ x^{*N} = -\Phi_1 (u_{yd} + u_{yd*}) + \Phi_3 u_{ys} - \Phi_2 u_{ys*} + \Phi_4 u_p + \Phi_5 \]

\[ m^{*N} = \Gamma_1 (u_{md} + u_{md*}) - \Gamma_2 (u_{yd} + u_{yd*}) + \Gamma_3 (u_{ys} + u_{ys*}) - \Gamma_4 \]

where:

\[ \Phi_1 = \frac{(1 + g) \gamma}{2[\beta \Delta f + \gamma (1 + g)]} \]

\[ \Phi_2 = \frac{\beta \Delta afg^3 + \beta \Delta \gamma (1 + g)(fg - \vartheta_1 w_{ECB}) + \frac{ag \gamma(1 + g)[g^2 + w_{ECB}]}{2\beta \Delta [\beta \Delta f + \gamma(1 + g)][g^2 + w_{ECB}]} - a^2 g \gamma(1 + g)}{2\beta \Delta f + \gamma(1 + g)} \]

\[ \Phi_3 = \frac{\beta \Delta afg^3 - \beta \Delta \gamma (1 + g)(fg - \vartheta_1 w_{ECB}) + \frac{ag \gamma(1 + g)[g^2 + w_{ECB}]}{2\beta \Delta [\beta \Delta f + \gamma(1 + g)][g^2 + w_{ECB}]}}{\beta \Delta f + \gamma(1 + g)} \]

\[ \Phi_4 = \frac{ag^2}{2\beta \Delta} \]

\[ \Phi_5 = \frac{a^2 g \gamma}{\beta \Delta f + \gamma(1 + g)} \]

\[ \Gamma_1 = 1 \]

\[ \Gamma_2 = \frac{a \beta \Delta}{\beta f + \gamma (1 + g)} \]

\[ \Gamma_3 = \frac{w_{ECB} \{[a^2 g + \gamma (1 + g)] - g \gamma (1 + g)\}}{\beta \Delta [\beta \Delta f + \gamma (1 + g)][g^2 + w_{ECB}]} \]

\[ \Gamma_4 = \frac{2a^2 g \gamma}{\beta \Delta f + \gamma (1 + g)} \]

The above expressions offer the following insights:

(i) fiscal policies do not react to money demand disturbances because the latter are completely offset by an equal variation in money supply (\( \Gamma_1 = 1 \));

(ii) if the economies are hit by aggregate demand shocks, fiscal and monetary policies react in the same direction. As expected, money supply and public expenditures are reduced in the case of a positive disturbance, and vice versa;
(iii) in the case of money and aggregate demand disturbances there is no difference between internal and external shocks. The reaction is also of the same nature if shocks are asymmetric ($u \neq 0$ and $u^* = 0$) and idiosyncratic ($u > u^*$). In the case of opposite disturbances ($u = -u^*$) there is no policy intervention;

(iv) the results are different if the economies are hit by a supply shock. In the case of an asymmetric supply disturbance the reaction of fiscal and monetary policies is uncertain, while if the shocks are symmetric both fiscal policies react negatively ($-\Phi_2 + \Phi_3 < 0$), while money supply injection is directly proportional to the conservatism of the central bank ($\Gamma_3 = \phi(w_{ECB})$). It is interesting to note that in the case of an opposite shock monetary policy does not change;

(v) price wedge shocks have an opposite effect on fiscal policies, whereas they do not influence money supply since the area-wide central bank cares about inflation in both countries with the same weight;

(vi) if the economies are not hit by any disturbances, fiscal policies are expansionary since national governments want to reach full employment. Conversely, the monetary authority counterbalances the effect on inflation by reducing money supply;

(vii) if fiscal authorities do not care about the interest rate ($\gamma = 0$), inspection of the formulas for the parameters $\Phi$ and $\Gamma$ reveals that: a) fiscal policies do not react at all to aggregate demand shocks, while the reaction of monetary policy would be larger; b) fiscal policies react only to idiosyncratic supply shocks ($u_{ys} \neq u_{ys^*}$) with opposite sign but, since $x$ and $x^*$ enter the equations of the endogenous variables in additive form (see Table 3), they cancel out;

(viii) in the special case $\beta = 0$ (while $\gamma > 0$), fiscal policies react to aggregate demand shocks with the same intensity ($x = x^* = -\left(u_{yd} + u_{yd^*}\right) / 2$). It is worth noting that if $\beta = \gamma = 0$, the Nash solution would not exist (the system of equations represented in Appendix III would become linearly dependent);
(ix) the parameter $w_{ECB}$ affects the reaction of fiscal and monetary policy to supply shocks only. This result depends upon the fact that the objectives of price and output stabilisation are in conflict only with regard to AS movements.

To analyse the consequences of the policies described above on the endogenous variables, the Nash solutions are substituted into the expressions reported in Table 3. The results are:

\begin{align*}
(18) & \quad p = -\frac{g}{2(g^2 + w_{ECB})}(u_{ys} + u_{ys^*}) + \frac{1}{2}u_p \\
(19) & \quad p^* = -\frac{g}{2(g^2 + w_{ECB})}(u_{ys} + u_{ys^*}) - \frac{1}{2}u_p \\
(20) & \quad y = \frac{g^2 + 2w_{ECB}}{2(g^2 + w_{ECB})}u_{ys} - \frac{g^2}{2(g^2 + w_{ECB})}u_{ys^*} + \frac{g}{2}u_p \\
(21) & \quad y^* = -\frac{g^2}{2(g^2 + w_{ECB})}u_{ys} + \frac{g^2 + 2w_{ECB}}{2(g^2 + w_{ECB})}u_{ys^*} - \frac{g}{2}u_p \\
(22) & \quad i = \frac{\beta \Delta}{2[\beta \Delta f + \gamma(1+g)]}(u_{yd} + u_{yd^*}) + \frac{\beta (fg - \vartheta_1 w_{ECB}) \Delta - agw_{ECB}}{2[\beta \Delta f + \gamma(1+g)](g^2 + w_{ECB})}(u_{ys} + u_{ys^*}) + \\
& \quad + \frac{ag\dot{y}}{\beta \Delta f + \gamma(1+g)}
\end{align*}

Money demand disturbances are completely insulated and have no effect on the endogenous variables. As far as aggregate demand shocks are concerned, their effects are limited to the interest rate, while prices and outputs remain unchanged with respect to their trend values.

Aggregate supply shocks have symmetric effects on prices and an asymmetric impact on output. In fact, a positive internal disturbance (for example an exogenous increase in productivity in the home economy) reduces prices in both countries, while it causes an increase in the level of internal output and a reduction abroad. If the $\text{AS}$ curve is steep (the value of $g$ is low) the effect on prices is greater than that on output, and vice versa. It is worth noting that, in the case of an “ultraconservative” central bank, price deviations are
cancelled, while output deviations are not insulated (each country completely suffers its internal supply shock but is unaffected by external disturbances). The effect on the interest rate depends on the value of the parameters. In particular, the absolute value of the variation of $i$ depends on the conservatism of the central bank: if this increases (decreases), interest rate deviations from its trend value tend to be higher (lower).

Price wedge shocks cause asymmetric effects both on prices and output and they are not influenced by $w_{ECB}$. In the case of output, the result is dependent on the slope of aggregate supply.

It is worth noting that if fiscal authorities did not care about the interest rate ($\gamma=0$), the solutions for prices and incomes would be unaffected (they do not depend on $\gamma$), while the effects on the nominal interest rate would be larger.

As expected, the equilibrium is characterised by the absence of inflation bias. On the other hand, output does not deviate from its trend level and full employment is not reached. This means that in the demand management “conflict” between monetary and fiscal authorities due to the delegation of control over inflation to a central bank which does not suffer from inflationary bias, the latter is able to win the battle. As consequence for the society, this conflict causes an interest rate bias that reduces public welfare.

If the area-wide central bank wished to achieve full employment ($\hat{y}_{ECB} = \hat{y}$), inflation and output bias would be the following:

---

17 As already pointed out, this conflict arises because both public authorities control aggregate demand (national governments set the IS curves, while the area-wide central bank controls $LM$). It is worth noting that in Alesina and Tabellini (1987), in a well-known attempt to model the coordination issue together with the inflationary bias problem, the demand management conflict is not considered because fiscal policy operates only through the supply side of the economy.

18 The interest rate bias tends to become larger in the case of interest rate pegging. In fact, if the area-wide central bank chooses a money supply function of the form $m' = m_t + m^*_t + ki$, (see Poole, 1970), the slope of $LM$ becomes $1/(a+k)$. Pure interest rate pegging is approximated by letting $k=\infty$, which provides a horizontal $LM$. With interest rate pegging, the effect on the outcome of the game is therefore the same as that obtained by allowing $a$, the interest semi-elasticity of money demand, to become larger. As pure interest rate setting is approached ($a\to\infty$) the interest rate bias reaches the maximum value $g\beta/(4\beta(f + \theta_1 g))$. Intuitively, this happens because interest rate pegging makes monetary policy more accommodating of changes in aggregate demand, inducing national governments to think that they can win the demand management conflict, even though they cannot.
Inflation bias = \[
\frac{2(fg^2(1+a) + w_{ECB} - g^2w_{ECB}(f + a\vartheta))}{\Delta w_{ECB}(g^2 + w_{ECB})} \]

Output bias = \[
\frac{2g(fg^2(1+a) + w_{ECB} - g^2w_{ECB}(f + a\vartheta)) - 1}{\Delta w_{ECB}(g^2 + w_{ECB})} \]

As already pointed out, output can rise above its natural level but this causes persistent inflation. Note that both biases are influenced by the conservatism of the central bank (for example \( w_{ECB} = \infty \) cancels the inflation bias but maximises the distance from the level of output that determines full employment). On the other hand, \( \beta \) (which represents the penalty set by the Stability Pact) does not enter the above equations, confirming the ineffectiveness of fiscal policies (the latter cannot counteract the effect of monetary policy on prices and output). In particular, the sanction imposed on fiscal deviation is only crucial in containing the effects on the interest rate (among equations 18-22, only the latter depends on \( \beta \)).

5. The optimal institutional arrangement

The solutions reported in equations (18) to (22) depend on the parameters of the model, shocks, the social weight attached to interest rate stabilisation (\( \gamma \)), the conservatism of the area-wide central bank (\( w_{ECB} \)) and the penalty established by the Stability Pact (\( \beta \)). While the former three are taken as given, the latter two (\( w_{ECB} \) and \( \beta \)) represent the institutional variables of the policy game and must be chosen by the area-wide authority in order to minimise the social loss.

Substituting the equations for prices, output and the interest rate into the public loss function (13), taking the expected value and rearranging, it is possible to obtain the following expression, which must be minimised:

\[
\min_{\beta, w_{ECB}} E(L^*) = \frac{\gamma(\beta\Delta)^2}{4[\beta\Delta f + \gamma(1+g)]^2} (\sigma_{u_{yd}}^2 + \sigma_{u_{yd}^*}^2) + \frac{(w_{\xi} + g^2)}{2} \sigma_{u_p}^2 + \\
\frac{w_{\xi}g^2}{2(g^2 + w_{ECB})^2} + \frac{(g^2 + 2w_{ECB})^2 + g^4}{4(g^2 + w_{ECB})^2} + \gamma[\beta (fg - \vartheta_1 w_{ECB}) \Delta - agw_{ECB}]^2 \]

\[
(\sigma_{u_{ys}}^2 + \sigma_{u_{ys}^*}^2) + \frac{\gamma(\Delta)^2}{[\beta\Delta f + \gamma(1+g)]^2} + 2\vartheta^2
\]
Given the complexity of the above expression, it is best to proceed in steps, considering each shock separately. In particular, if the economy is not hit by any disturbance the loss function becomes \( \min_{\beta} E(L^S) = \frac{\gamma(\beta \Delta)\gamma}{[\beta \Delta f + \gamma(1 + g)]^2} + 2\dot{y}^2. \) In this case the optimal choice regards only the penalty for national fiscal policy deviations, while the conservatism of the central bank has no influence. It is straightforward to note that the optimal solution is \( \tilde{\beta} = \infty. \) This means that it is optimal for the society to avoid any form of fiscal deviation since they are completely crowded out by monetary policy and can only cause an increase in the interest rate (see Section 4).

These considerations also are valid if the economies are hit by a price-wedge shock, since the coefficient \( \frac{(w_S + g^2)}{2} \) is not influenced by \( w_{ECB} \) or \( \beta. \)

In the case of aggregate demand disturbances, the loss function to minimise becomes \( E(L^S) = \frac{\gamma(\beta \Delta)^2}{4[\beta \Delta f + \gamma(1 + g)]^2} (\sigma^2_{u_{yd}} + \sigma^2_{u_{yd}^*}) + \frac{\gamma(\beta \Delta)^2}{[\beta \Delta f + \gamma(1 + g)]^2} + 2\dot{y}^2, \) which depends only upon \( \beta. \) From the first order condition \( \frac{\partial E(L^S)}{\partial \beta} = 0 \) it is possible to derive the optimal penalty \( \tilde{\beta} = \frac{4a^2g^2f\dot{y}^2}{\gamma(1 + g)\Delta(\sigma^2_{u_{yd}} + \sigma^2_{u_{yd}^*})}. \)

The analysis of this expression produces a number of interesting results. The optimal sanction is inversely related to output demand instability. If the latter tends to infinity no restriction should be imposed on fiscal budget deviations. This result derives from the fact that, in the case of aggregate demand shocks, as already pointed out, fiscal and monetary policies react in the same direction (albeit with a different intensity). In particular, fiscal policies help to stabilise the interest rate and the result \( \tilde{\beta} = 0 \) also can be obtained if the preference of the society to limit interest rate deviations tends to infinity (\( \gamma = \infty. \)). In general, the optimal penalty depends on the value of the parameters. In particular, \( \tilde{\beta} \) is directly
proportional to money and output demand elasticities with respect to the interest rate (respectively, $a$ and $f$) and inversely related to the slope of the AS curve ($1/g$).

In the case of aggregate supply shocks the optimal policy is determined by a combination of both $w_{ECB}$ and $\beta$. If the central bank is “ultraconservative” ($w_{ECB}=\infty$) it is possible to obtain an analytical solution for the optimal penalty

$$\tilde{\beta}_C = \frac{4a^2 g^2 f \hat{\delta}^2 + ag[agf - \gamma(1 + g)\theta_1](\sigma^2_{uys} + \sigma^2_{uys*})}{\Delta \theta_1 [\gamma(1 + g)\theta_1 - agf](\sigma^2_{uys} + \sigma^2_{uys*})},$$

whose sign is uncertain. This means that if the only objective of the monetary authority is to contain inflation, it could be optimal for the society to stimulate fiscal activism in response to shocks in order to limit the effect on the interest rate. This happens when the set of parameters is such that the public authorities react in the same direction with respect to supply side shocks (see Section 4). In fact, if the central bank is “ultraconservative”, monetary policy is extremely reactive to the disturbances and this causes not only an output gap (from its natural level), but also an interest rate bias. The latter can be reduced if fiscal policies are stimulated in order to counterbalance the effects of LM shifts with IS movements (the AD curve moves in the same direction but interest rate deviations are contained). In particular, if there is no expansionary bias on the part of fiscal authorities ($\hat{\delta}=0$) the formula becomes $\tilde{\beta}_C = -\frac{ag}{\Delta \theta_1}$, and it is socially optimal to encourage fiscal activism in response to supply shocks.

The general case that consists of choosing the optimal combination of both institutional policy variables ($w_{ECB}$ and $\beta$) is analysed using numerical methods where all the elasticities are set to one, $\sigma^2_{uys} = \sigma^2_{uys*} = 1$ and $\theta_1 = 1 - b = 0.3$.\textsuperscript{19} Figures 2 to 7 are composed of six graphs that show the variation of one parameter on the horizontal axis. In particular, the first row refers to the case of a “partially conservative” central bank whose loss function is given by equation (15b), while the second reports the

\textsuperscript{19} It is worth noting that even using different coefficients the simulations always confirm the conclusions. The analytical solution of the model (not reported because of the complexity of the expression but available from the author upon request) has been obtained from the system of equations formed by the two first order conditions of equation (23). In particular, the system has two solutions, of which only one represents a minimum.
Simulations on money demand elasticity
\( \sigma = 1; \, \sigma = 1 - b = 0.3; \, g = 1; \, \gamma = 1; \, w_S = 1 \)

Simulations on aggregate demand elasticity
\( \sigma = 1; \, \sigma = 1 - b = 0.3; \, g = 1; \, \gamma = 1; \, w_S = 1 \)

Simulations on the “price surprice” effect (reciprocal of the AS slope)
\( \sigma = 1; \, \sigma = 1 - b = 0.3; \, g = 1; \, \gamma = 1; \, w_S = 1 \)
Simulations on society's preference for inflation

\(a=1; \theta=1; b=0.3; g=I; \beta=I; \gamma=I; f=I\)

Figure 5

Simulations on society's preference for the interest rate

\(a=1; \theta=1; b=0.3; g=I; \beta=I; f=I; w_e=I\)

Figure 6

Simulations on the full employment output level

\(a=1; \theta=1; b=0.3; g=I; f=I; \gamma=I; w_e=I\)

Figure 7
result associated with an “ultraconservative” monetary authority. The first two graphic in the first row show the optimal values of the institutional arrangement for $\hat{w}_{ECB}$ and $\hat{\beta}$, while the third gives the overall optimal loss for the society ($L^S$). In the second row, given $w_{ECB}=\infty$, the first graph shows the optimal value of the penalty $\hat{\beta}_C$, while the second gives the loss $L^C$ associated with this policy combination ($\hat{w}_{ECB}=\infty$, $\hat{\beta}_C$). The last graph compares the two losses ($\hat{L}^S - \hat{L}^C$).

Simulations based on money demand semi-elasticity $a$ (Figure 2) show that if the latter increases (the $LM$ curve becomes flat, as in the Keynesian liquidity trap) the optimal institutional arrangement requires an increase (at decreasing rates) in the values of both $\hat{w}_{ECB}$ and $\hat{\beta}$. The degree of conservatism ($\hat{w}_{ECB}$) is always greater than the public preference ($w_S=1$). The optimal loss increases to a horizontal asymptote and then remains constant.

As already noted, given $w_{ECB}=\infty$, for certain values of the parameters (in this case for $a>0.5$) the value of $\hat{\beta}_C$ becomes negative. In this range $L^S_C$ decreases but always remains above $L^S$ (see last graph).

With low values of aggregate demand semi-elasticity $f$ (Figure 3) it is optimal to have a central bank that tends to have the same preference as the public ($\hat{w}_{ECB}=1$) and an infinite optimal sanction ($\hat{\beta}=\infty$). When $f$ increases (the slope of the $IS$ curve becomes flat, as in the neo-classical case), $\hat{w}_{ECB}$ initially rises (till $f=1$) then decreases slowly.

On the other hand, $\hat{\beta}$ tends to zero (as well known, a horizontal $IS$ represents the case of an ineffective fiscal policy). The second row shows similar characteristics to those discussed above for $a$.

Simulations based on $g$ (Figure 4), which represents the reciprocal of the slope of $AS$, depend on the absolute value of the slope of $AD$. The latter, given the values used for the simulation, is fixed at 0.3 (see the vertical asymptote in the second graph), which represents a case where output variations are bigger than those in prices (the slope of $AD$ is less than one). Therefore, if the product of the slope of $AS$ and that of $AD$ is greater than one (values of $g$ below 0.3) the effects of such variations are more contained than vice versa. Considering the
first row, values of $g<0.3$ produce a central bank that cares about inflation less than the society ($\tilde{w}_{ECB}<1$) and incentives for fiscal deviation ($\tilde{\beta}<0$) as an optimal institutional arrangement. This result depends on the fact that output deviations weigh more than inflation and it is optimal for the society to give more importance to containing the former. An opposite result is obtained for values of $g>0.3$ where the optimal institutional arrangement is characterised by a central bank which cares more than the society about inflation ($\tilde{w}_{ECB}>1$) and penalties for fiscal deviation ($\tilde{\beta}>0$). The optimal loss tends to decrease for low values of the slope of $AS$ (when $g$ tends toward infinity).

Simulations on society’s preference for inflation (Figure 5) show that the latter is always lower than the optimal conservatism of the area-wide central bank (the difference $\tilde{w}_{ECB}-w_S$ is a positive constant). On the other hand the optimal penalty $\tilde{\beta}$ increases exponentially with $w_S$. This means that if the preference of the society for inflation rises, the institutional arrangement has to penalise fiscal budget deviations (the interest rate bias is relatively less important). The optimal social loss ($\tilde{L}^S$) increases with $w_S$ and tends to reach a maximum value equal to the constant loss associated with an “ultraconservative” central bank ($\tilde{L}_C^S=2$). In fact, given $\tilde{w}_{ECB} = \infty$, $w_S$ does not influence either $\tilde{\beta}_C$ (always equal to -3) or $L_C^S$.

Considering society’s preference for the interest rate (Figure 6), the optimal degree of conservatism of the area-wide central bank shows a parabolic behaviour (increasing to a critical value and then decreasing). This happens because for $\gamma<1.7$ fiscal and monetary policies react in the same direction (the effect on the interest rate is limited), while the opposite is true for $\gamma>1.7$. In the latter case, it is optimal to contain the conservatism of the central bank ($LM$ movements are not counterbalanced by $IS$ shifts) in order to limit the effect on the interest rate. Moreover, the optimal penalty $\tilde{\beta}$ always decreases, reaching zero for $\gamma$ tending to infinity. The first graph in the second row of Figure 6 underscores the characteristics of the critical $\gamma=1.7$. The social loss in the case of an “ultraconservative” central bank ($\tilde{L}_C^S$) and the difference $\tilde{L}_C^S-\tilde{L}^S$ both increase with $\gamma$.

If the full employment output level diverges from the natural trend (Figure 7), both $\tilde{w}_{ECB}$ and $\beta$ rise (the first increases at a decreasing rate while the second behaves
exponentially). The first parameter has the task of offsetting the effects on prices, while the second reduces the destabilising movements of fiscal policies. The second row shows that, in this case as well, it is never optimal for the society to have an extremely conservative central bank (see last graph).

6. What are the consequences if one of the policymakers is a leader?

The equilibrium of the policy game described in Section 4 is based on the assumption that monetary and fiscal policies are determined simultaneously (Nash equilibrium). Assuming instead that one of the policymakers moves first, acting as a leader, it is possible to derive the Stackelberg equilibrium of the game.

If the area-wide central bank is the leader, it minimises its loss function (15) subject to being at the intersection of the reaction functions of the national governments (derived in Section 4). Clearly, in this case, the outcome is the same as in the Nash equilibrium: there is still a conflict over demand management policy, of which the outcome is always low inflation at the cost of a high interest rate bias. This result is different from that obtained by the standard analysis of the time-consistency inflationary bias problem (see among others Goodhart, 1993) where, if the independent central bank moves first, it is able to discipline the opposing player. As pointed out by Rankin (1995), studying the case of a closed economy without stochastic shocks, this result depends on the introduction of a third kind of player (the fiscal authority) in the traditional two-player game between the central bank and the public.20

The result of the game changes if both fiscal authorities are supposed to be leaders (given the symmetry of the model it is not realistic to consider only one government as leader). This description of fiscal-monetary interactions seems sensible considering that fiscal policies cannot be adjusted as quickly as monetary policy and that a particular choice of

---

20 This result is also due to the fact that the central bank’s primary objective is to stabilise inflation. In fact, if the nominal interest rate is introduced as an additional argument in the central bank’s loss function (a case which determines an inflation bias) the Nash equilibrium and the Stackelberg equilibrium in which the monetary authority acts as a leader no longer coincide. However, as already pointed out in note 15, this hypothesis is not coherent with the time consistency condition (7) and the lexicographic preference of the ECB.
government expenditure provides the fiscal authorities with a first-mover advantage (Debelle and Fischer, 1994; Beetsma and Bovenberg, 1998). This means that each fiscal authority minimises its loss function (14) subject to being on the central bank reaction function (see Appendix IV). In this institutional context, national governments have the opportunity to influence the central bank’s behaviour but, since short-term fiscal policy deviations only determine a higher interest rate with no effects on output, they have no incentive to deviate from the precommitment solution. In other words, the central bank reaction function acts as a disciplinary device for governments’ policies.

Substituting the Stackelberg solutions in the expressions reported in Table 3, prices and incomes remain the same as in the Nash equilibrium (equations 18 to 21 do not change), while the interest rate becomes:

\[
i = \frac{\beta f}{2\beta f^2 + \gamma} (u_{yd} + u_{yd^*}) + \frac{\beta f (f_B - \theta W_{ECB})}{(2\beta f^2 + \gamma)(\beta g^2 + W_{ECB})} (u_{ys} + u_{y^*_s})
\]

This outcome shows that if the fiscal authorities are leaders, a “disciplinary effect” does operate. In fact, there is no interest rate bias (if the economies are not hit by any disturbances, the precommitment solution is played, see Appendix IV) and aggregate demand shocks produce a smaller effect than in the Nash equilibrium \(\frac{\beta f}{2\beta f^2 + \gamma} < \frac{\beta \Delta}{2[\beta \Delta f + \gamma(1+g)]}\).

Following the procedure described in Section 5, it is also possible to analyse the optimal institutional arrangement in the case of fiscal authorities acting as leaders (see Appendix V). Since fiscal policies are self-disciplined, the result is that there is no need for the Stability Pact and the optimal value of the penalty is zero with respect to each kind of shock \(\hat{\beta} = 0\). Moreover, in the case of supply disturbances, it is optimal for the area-wide central bank to have the same preference as the public \(w_{ECB} = w_S\).

This finding is particularly interesting because it shows that, in the case of a monetary union, subsidiarity in fiscal policymaking may be conducive to both monetary and fiscal discipline only if governments are the leaders of the policy game and no sanction is imposed on excessive public expenditure. The relevance of this statement can be checked by comparing these results with those obtained considering the case of a centralised fiscal policy.
If there is co-ordination (or collusion) between the two national fiscal authorities it can be supposed that an area-wide fiscal committee (for example, Ecofin) determines the total amount of public expenditure ($x + x^*$) by minimising the sum of the two loss functions (14).

\[(14') \quad \min_{x+x^*} L^\text{ECOFIN} = (y - \hat{y})^2 + (y^* - \hat{y})^2 + \gamma(i - \hat{i})^2\]

In this case, there is no need for the Stability Pact ($\beta = 0$) and the public deficit of the whole area could be divided according to some fixed rule.

Using equations (14') and (15c) to determine Nash and Stackelberg equilibria, it is possible to prove that the solutions for prices and incomes do not change with respect to the case of a decentralised fiscal policy (equations 18 to 21 remain the same). By contrast, the effects on the interest rate are different. In particular, the Nash equilibrium and the Stackelberg equilibrium with the central bank acting as a leader determine the following interest rate

\[i = \frac{-ag\gamma ECB}{\gamma(1+\gamma)(g^2 + w^\text{ECB})} (u_{ys}^* + u_{ys^*}) + \frac{2ag\gamma}{\gamma(1+g)},\]

which denotes a lower stabilisation with respect to equation (22) in which $\beta$ is set to zero.

In other words, if no sanction is imposed on public deficits, the outcome of a co-ordinated fiscal policy is suboptimal with respect to the case of a decentralised one. This result is due to the fact that the area-wide fiscal committee’s response is more intense than national ones, which take into consideration the spillover effect of the fiscal policy of the other country (this can be also seen from inspection of national governments’ reaction functions in Appendix III, in which public expenditures appears as strategic substitutes).

Conversely, when the fiscal committee moves first the independent area-wide central bank perfectly disciplines fiscal policies and the solution for the interest rate becomes $i=0$. It is interesting to note that this result coincides with the case of a decentralised fiscal policy when no penalty is imposed on fiscal deviation.

7. Conclusions

This paper has analysed the optimal institutional arrangement in the case of a monetary union between two symmetric countries. The main conclusions can be summarised as follows.
In the absence of economic disturbances, it is not sensible to delegate demand control to both the national governments and the area-wide central bank. To avoid the inflation bias, which causes a problem of time inconsistency, the area-wide central bank has to set its output objective at the natural level which, due to market imperfections, is lower than the full employment level. Unless the fiscal authorities are the first mover in the policy game, the different final objectives always cause a conflict over demand management, which results in an excessively high interest rate. This means that when the monetary policy is delegated to the area-wide central bank inflationary bias is replaced by interest rate bias and social welfare will be higher only if the society values low inflation sufficiently more than a low and stable interest rate.

This result changes if the economies are subject to real and nominal disturbances. In this case it is optimal for the society to have a flexible system of control over fiscal policy deviations which depends on the nature of the shocks and an adaptable degree of conservatism of the area-wide central bank. The latter is crucial in the case of supply side shocks and it must be combined with the penalty system in order to reduce the consequences of the disturbances not only on prices but also on output and the interest rate. It is never optimal for the society to have an “ultraconservative” central bank because, in the case of supply shocks, each country fully suffers the impact of its internal disturbances on output.

In the Nash equilibrium, if fiscal authorities do not care about interest rate they end up behaving in a complementary way to the central bank. In this case the national government would intervene only to offset the output effects of the asymmetric portion of supply shocks, which a common monetary policy cannot offset. The stabilisation of output with respect to aggregate demand shocks and to area-wide supply disturbances would be left entirely to the monetary authority, regardless of its degree of conservatism. If fiscal authorities care about the interest rate, they will also intervene to moderate the interest rate effects of aggregate demand and supply shocks. This, however, will not modify the equilibrium value of prices and output, since the impact of fiscal policies will be offset by monetary policy. The reason why fiscal policies have a “residual” role in shock stabilisation is due to the sanction scheme. Indeed, if the use of public expenditure is not limited, IS shocks would be offset entirely by fiscal policies, which have a comparative advantage in influencing the output market.
The main message from this paper is twofold. First, a fixed penalty for fiscal deviation is not always optimal. In fact, even if the national governments attempt to reach full employment with short-term policies are ineffective, they have an impact on the interest rate which depends upon the nature of the institutional game and the shocks.

Second, only if governments are the leaders of the policy game does the independent area-wide central bank discipline national fiscal policies. In this case subsidiarity in fiscal policymaking may produce a welfare increase for the monetary union.

It goes without saying that further research should be directed towards this issue in order to complete the previous analysis. First, the interaction between the economies should include dynamic considerations of the fiscal deficit-public debt process, which is beyond the scope of this work. Second, it is necessary to extend the results of this study by taking into account structural asymmetries between EU countries and their consequences for monetary transmission processes.
Appendix I

The socially optimal interest rate

The socially optimal interest rate is worked out from the precommitment solution. The latter is obtained by (i) imposing perfectly anticipated policy behaviours \((m'=m^e, x=x^e\) and \(x^*=x^e)\) in the reduced form of the model reported in Table 2, (ii) minimising the authorities’ loss functions and (iii) solving the system of equations formed by the first order conditions.

The first step of the analysis is given by the following equations, discussed in Section 2, where economic disturbances have been set to zero.

\[ p = p^* = \frac{1}{2(1+a)}m^* + \frac{1}{2f(1+a)}(x+x^*) \]

\[ y = y^* = 0 \]

\[ i = -\frac{1}{2(1+a)}m^* + \frac{1}{2f(1+a)}(x+x^*) \]

Substituting the above equations into the loss functions (13) and (14), it is possible to obtain the following equations to be minimised:

\[ \min_x L^G = \hat{y}^2 + \gamma \left[ -\frac{1}{2(1+a)}m^* + \frac{1}{2f(1+a)}(x+x^*) - \hat{i} \right]^2 + \beta x^2 \]

\[ \min_{x^*} L^{G^*} = y^2 + \gamma \left[ -\frac{1}{2(1+a)}m^* + \frac{1}{2f(1+a)}(x+x^*) - \hat{i} \right]^2 + \beta x^{*2} \]

\[ \min_{m^*} L^{ECB} = 2w_{ECB} \left[ \frac{1}{2(1+a)}m^* + \frac{1}{2f(1+a)}(x+x^*) \right]^2 + 2\hat{y}_{ECB}^2 \]

The second step of the analysis consists in taking the first order conditions of the above expressions, which give the reaction functions of the public authorities:

\[ \frac{\partial L^G}{\partial x} = 0 \quad \Rightarrow \quad [4 \beta f^2 (1+a)^2] x + \gamma x^* - \gamma f \ m^* = 2f \gamma (1+a) \hat{i} \]

\[ \frac{\partial L^{G^*}}{\partial x^*} = 0 \quad \Rightarrow \quad \gamma x + [4 \beta f^2 (1+a)^2] \ x^* - \gamma f \ m^* = 2f \gamma (1+a) \hat{i} \]

\[ \frac{\partial L^{ECB}}{\partial m^*} = 0 \quad \Rightarrow \quad a \ x + a x^* + f \ m^* = 0 \]
Solving this system of equations with respect to the policy instruments, the precommitment solution is obtained:

\[
\begin{align*}
    x &= x^* = \frac{f_\gamma \hat{i}}{2\beta f^2(1+a) + \gamma} \\
    m' &= \frac{-2a\gamma \hat{i}}{2\beta f^2(1+a) + \gamma}
\end{align*}
\]  

(A1.2)

Substituting the precommitment solutions into the third equation of (A1.1) gives an implicit expression for the socially optimal nominal interest rate:

\[
\hat{i} = -\frac{1}{2(1+a)} \frac{-2a\gamma \hat{i}}{2\beta f^2(1+a) + \gamma} + \frac{1}{f(1+a)} \frac{f_\gamma \hat{i}}{2\beta f^2(1+a) + \gamma}
\]

whose unique solution is \( \hat{i} = 0 \). It is worth noting that, since output deviations are zero (see A1.1), the optimal value of the interest rate coincides with its trend value based on the growth of the factors of production and long-term structural policies. Every effort of the public authorities to increase output above its natural rate (\( \hat{y} \) and \( \hat{y}_{ECB} > y_N = 0 \)) is useless. In the absence of shocks, the optimal policy in the short term (given by equations A1.2) consists in doing nothing unexpected (\( m' = x = x^* = 0 \)) in order to avoid inflation (\( p = p^* = 0 \)).
Appendix II

Policy instrument expectations

Substituting the reduced form reported in Table 2 (in which expectations explicitly appear) into the loss functions and taking the derivative with respect to the instruments, it is possible to obtain:

\[
\frac{\partial L^G}{\partial x} = 0 \Rightarrow \frac{ag}{\Delta} \left[ \frac{fg}{\Delta} (m^* - m^{*e}) + \frac{ag}{\Delta} [(x - x^e) + (x^* - x^{*e}) + \text{shocks} - \hat{y}] + \right.
\]
\[
\left. \frac{\gamma(1+g)}{\Delta} \left[ \frac{f + g \vartheta_1}{\Delta} m^* - g(f - \vartheta_1) \right] m^{*e} + \frac{1+g}{\Delta} (x + x^*) + \right.
\]
\[
\left. + \frac{ag(f - \vartheta_1)}{f(1+a)\Delta} (x^e + x^{*e}) + \text{shocks} \right] + \beta x = 0
\]

\[
\frac{\partial L^G}{\partial x^e} = 0 \Rightarrow \frac{ag}{\Delta} \left[ \frac{fg}{\Delta} (m^* - m^{*e}) + \frac{ag}{\Delta} [(x - x^e) + (x^* - x^{*e}) + \text{shocks} - \hat{y}] + \right.
\]
\[
\left. \frac{\gamma(1+g)}{\Delta} \left[ \frac{f + g \vartheta_1}{\Delta} m^* - g(f - \vartheta_1) \right] m^{*e} + \frac{1+g}{\Delta} (x + x^*) + \right.
\]
\[
\left. + \frac{ag(f - \vartheta_1)}{f(1+a)\Delta} (x^e + x^{*e}) + \text{shocks} \right] + \beta x^e = 0
\]

\[
\frac{\partial L^{ECB}}{\partial m^e} = 0 \Rightarrow w_{ECB} \left[ \frac{f}{\Delta} m^* + \frac{g(f + a \vartheta_1)}{(1+a)\Delta} m^{*e} + \frac{a}{\Delta} (x + x^*) + \right.
\]
\[
\left. \frac{ag(f + a \vartheta_1)}{f(1+a)\Delta} (x^e + x^{*e}) + \text{shocks} \right] + \frac{fg}{\Delta} (m^* - m^{*e}) + \frac{ag}{\Delta} [(x - x^e) + (x^* - x^{*e}) + \text{shocks} - \hat{y}_{ECB}] = 0
\]

Taking expectations from these expressions and noting that shocks cannot be anticipated \(E(u_t)=0\), the following system is obtained:

\[
\begin{cases}
(1+g)\gamma x^e + 2\beta f(1+a) x^{*e} - \gamma (1+g) m^e = 2afg(1+a) \hat{y} \\
(1+g)\gamma x^e + [(1+g)\gamma x^{*e} + 2\beta f(1+a) x^{*e} - \gamma (1+g) m^e = 2afg(1+a) \hat{y} \\
aw_{ECB} x^e + aw_{ECB} x^{*e} + fw_{ECB} m^e = 2fg(1+a) \hat{y}_{ECB}
\end{cases}
\]

Solving this system with respect to policy instrument expectations gives:

\[
x^e = x^{*e} = \frac{fg[aw_{ECB} \hat{y} + \gamma \hat{y}_{ECB}(1+g)]}{[\beta f + \gamma (1+g)]w_{ECB}}
\]

\[
m^{*e} = \frac{2g[-a^2 \hat{y} w_{ECB} + \gamma \hat{y}_{ECB}] \beta f(1+a) + \gamma (1+g)]}{[\beta f + \gamma (1+g)]w_{ECB}}
\]
Appendix III

Nash equilibrium

The Nash equilibrium of the policy game can be obtained by solving the following system:

\[
\begin{align*}
\min_x L^G &= (y - \hat{y})^2 + \gamma i^2 + \beta x^2 \\
\min_x L^G' &= (y^* - \hat{y})^2 + \gamma i^2 + \beta x^{*2} \\
\min_m L^{ECB} &= w_{ECB}(p^2 + p^{*2}) + y^2 + y^{*2}
\end{align*}
\]

Substituting the solution of the model (see Table 3) into the above expressions, they turn out to be just a function of the instruments \((m', x \text{ and } x^*)\) and the different shocks. Each public authority minimises its loss function by taking as given the actions of the other agents (Nash equilibrium). The first order conditions of system (17) give the following reaction functions:

\[
\frac{\partial L^G}{\partial x} = 0 \Rightarrow [a^2 g^2 + \gamma(1 + g)^2 + \beta \Delta^2]x + [a^2 g^2 + \gamma(1 + g)^2]x^* + [afg^2 - \gamma(1 + g)(f + g \vartheta_1)]m' = \\
[afg^2 - \gamma(1 + g)(f + g \vartheta_1)](u_{md} + u_{md^*}) - [a^2 g^2 + \gamma(1 + g)^2](u_{yd} + u_{yd^*}) + \\
[-ag(2f + 2af + fg + ag \vartheta_1) + \gamma(1 + g)(f - \vartheta_1)]u_{ys} + \\
+ [ag^2(f + a \vartheta_1) - \gamma(1 + g)(f - \vartheta_1)]u_{ys^*} + \frac{ag^2 \Delta}{2}u_p + ag\Delta\hat{y}
\]

\[
\frac{\partial L^G'}{\partial x} = 0 \Rightarrow [a^2 g^2 + \gamma(1 + g)^2]x + [a^2 g^2 + \gamma(1 + g)^2 + \beta \Delta^2]x^* + [afg^2 - \gamma(1 + g)(f + g \vartheta_1)]m' = \\
[afg^2 - \gamma(1 + g)(f + g \vartheta_1)](u_{md} + u_{md^*}) - [a^2 g^2 + \gamma(1 + g)^2](u_{yd} + u_{yd^*}) + \\
+[ag^2(f + a \vartheta_1) - \gamma(1 + g)(f - \vartheta_1)]u_{ys} + \\
+[-ag(2f + 2af + fg + ag \vartheta_1) + \gamma(1 + g)(f - \vartheta_1)]u_{ys^*} + \frac{ag^2 \Delta}{2}u_p + ag\Delta\hat{y}
\]

\[
\frac{\partial L^{ECB}}{\partial m'} = 0 \Rightarrow a(w_{ECB} + g^2) x + a(w_{ECB} + g^2) x^* + f(w_{ECB} + g^2) m' = \\
f(w_{ECB} + g^2)(u_{md} + u_{md^*}) - a(w_{ECB} + g^2)(u_{yd} + u_{yd^*}) + \\
+[f + a \vartheta_1]w_{ECB} - fg(1 + a)](u_{ys} + u_{ys^*})
\]

The point where the reaction functions intersect represents the Nash equilibrium:
\[ \chi^N = -\Phi_1(u_{yd} + u_{yd^*}) - \Phi_2u_{ys} + \Phi_3u_{ys^*} - \Phi_4\ u_p + \Phi_5 \]

\[ \chi^{*N} = -\Phi_1(u_{yd} + u_{yd^*}) + \Phi_2u_{ys} - \Phi_3u_{ys^*} + \Phi_4\ u_p + \Phi_5 \]

\[ m^{*N} = \Gamma_1(u_{md} + u_{md^*}) - \Gamma_2(u_{yd} + u_{yd^*}) + \Gamma_3(u_{ys} + u_{ys^*}) - \Gamma_4 \]

where:

\[ \Phi_1 = \frac{(1 + g)\hat{\gamma}}{2[\beta \Delta f + \gamma(1 + g)]} \quad \Gamma_1 = 1 \]

\[ \Phi_2 = \frac{\beta \Delta afg^3 + \beta \Delta \gamma(1 + g)(f_g - \vartheta_1 w_{ECB}) + \beta \Delta afg^3 - \beta \Delta \gamma(1 + g)(f_g - \vartheta_1 w_{ECB}) + \beta \Delta afg^3 - \beta \Delta \gamma(1 + g)(f_g - \vartheta_1 w_{ECB})}{2\beta \Delta[\beta \Delta f + \gamma(1 + g)](g^2 + w_{ECB})} \quad \Gamma_2 = \frac{\beta \Delta f + \gamma(1 + g)}{\beta \Delta f + \gamma(1 + g)} \]

\[ \Phi_3 = \frac{a\gamma(1 + g)(g^2 + w_{ECB})}{\beta \Delta[\beta \Delta f + \gamma(1 + g)](g^2 + w_{ECB})} \quad \Gamma_3 = \frac{w_{ECB}[a^2 g + \gamma(1 + g)] - g \gamma(1 + g)}{w_{ECB}[a^2 g + \gamma(1 + g)] - g \gamma(1 + g)} \]

\[ \Phi_4 = \frac{ag^2}{2\beta \Delta} \quad \Gamma_4 = \frac{2a^2 g\hat{\gamma}}{\beta \Delta f + \gamma(1 + g)} \]

\[ \Phi_5 = \frac{af_g\hat{\gamma}}{\beta \Delta f + \gamma(1 + g)} \]
Appendix IV

Stackelberg equilibrium with national governments acting as leaders

The Stackelberg equilibrium of the policy game is worked out solving the following problem:

\[
\min_x L^G = (y - \hat{y})^2 + \gamma i^2 + \beta x^2
\]

\[
\min_{x^*} L^{G*} = (y^* - \hat{y})^2 + \gamma i^2 + \beta x^{*2}
\]

s.t.

\[
\frac{\partial L^{ECB}}{\partial m'} = 0 \Rightarrow a(w_{ECB} + g^2) x + a(w_{ECB} + g^2) x^* + f(w_{ECB} + g^2) m' =
\]

\[
f(w_{ECB} + g^2)(u_{md} + u_{md^*}) - a(w_{ECB} + g^2)(u_{yd} + u_{yd^*}) +
\]

\[
+ [(f + a\vartheta_1)w_{ECB} - f_0(1 + a)] (u_{ys} + u_{ys^*})
\]

Substituting the solution of the model (see Table 3) into the expressions to be minimised gives the result that they are just a function of the instruments (\(m', x\) and \(x^*\)) and the shocks. Each fiscal authority minimises its loss function by taking as given the reaction function of the area-wide central bank.

The Stackelberg equilibrium is given by the following solutions:

\[
x^S = x^{*S} = -\frac{\gamma}{2(2\beta f^2 + \gamma)}(u_{yd} + u_{yd^*}) - \frac{\gamma(fg - \vartheta_1 w_{ECB})}{2(2\beta f^2 + \gamma)(g^2 + w_{ECB})}(u_{ys} + u_{ys^*})
\]

\[
m^{S} = (u_{md} + u_{md^*}) - \frac{2a\beta f}{2\beta f^2 + \gamma}(u_{yd} + u_{yd^*}) +
\]

\[
+ \frac{2\beta f[w_{ECB}(f + a\vartheta_1) - fg(1 + a)] + \gamma(w_{ECB} - g)}{(2\beta f^2 + \gamma)(g^2 + w_{ECB})}(u_{ys} + u_{ys^*})
\]

In this case, if the economies are not hit by any disturbances, the precommitment solution (\(m' = x' = x^* = 0\)) is played. Moreover, note that the policy instruments do not react to price wedge shocks.
Appendix V

The optimal institutional arrangement when governments are leaders

Substituting the equations (18) - (21), representing prices and output, and the solution for the interest rate (equation 22') into the public loss function (13), taking the expected value and rearranging, it is possible to obtain the following expression to minimise with respect to the “institutional” parameters:

$$
\min_{\beta, w_{ECB}} E(L^S) = \frac{\gamma(\beta f)^2}{(2\beta f^2 + \gamma)^2} \left( \sigma^2_{u_{yd}} + \sigma^2_{u_{yd}^*} \right) + \frac{(w_S + g^2)}{2} \sigma^2_{u_p} + \\
+ \left[ \frac{w_S g^2}{2(g^2 + w_{ECB})^2} + \frac{(g^2 + 2w_{ECB})^2 + g^4}{4(g^2 + w_{ECB})^2} + \frac{\gamma(\beta f)^2}{(2\beta f^2 + \gamma)^2} \left( \sigma^2_{u_{ys}} + \sigma^2_{u_{ys}^*} \right) + 2\hat{y}^2 \\
\right]
$$

Given the complexity of the above expression it is best to proceed considering each case separately. In particular, if the economy is not hit by any disturbance the loss function becomes $E(L^S) = 2\hat{y}^2$, which is independent of $w_{ECB}$ and $\beta$. The same result is obtained if the economies are hit by a price-wedge shock, since the coefficient $\left( \frac{w_S + g^2}{2} \right)$ is not influenced by the variables which represent the institutional arrangement.

In the case of aggregate demand disturbances the loss function to minimise becomes

$$
\min_{\beta} E(L^S) = \frac{\gamma(\beta f)^2}{(2\beta f^2 + \gamma)^2} \left( \sigma^2_{u_{yd}} + \sigma^2_{u_{yd}^*} \right) + 2\hat{y}^2,
$$

which depends only upon $\beta$. From the first order condition $\left( \frac{\partial E(L^S)}{\partial \beta} = 0 \right)$ it is possible to derive an optimal penalty $\tilde{\beta} = 0$, which shows that the Stability Pact is not necessary.

In the case of aggregate supply shocks the optimal policy is determined by a combination of both $w_{ECB}$ and $\beta$. In fact, the expression to minimise is given by:
\[
\min_{\beta, w_{ECB}} E(L^S) = \left[ \frac{w_S g^2}{2(g^2 + w_{ECB})^2} + \frac{(g^2 + 2w_{ECB})^2 + g^4}{4(g^2 + w_{ECB})^2} + \frac{\gamma \beta g^2}{(2\beta f^2 + \gamma)^2 (g^2 + w_{ECB})^2} \right] (\sigma^2_{uys} + \sigma^2_{uys}^*) + 2\tilde{\gamma}^2
\]

The optimal solution, obtained from the system of equations formed by the two first order conditions, is \( \tilde{\beta} = 0 \) and \( \tilde{w}_{ECB} = w_S \). Once again, there is no need for the Stability Pact, while the area-wide central bank should have the same preference as the public as regards to inflation.
References


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