BANCA D'ITALIA
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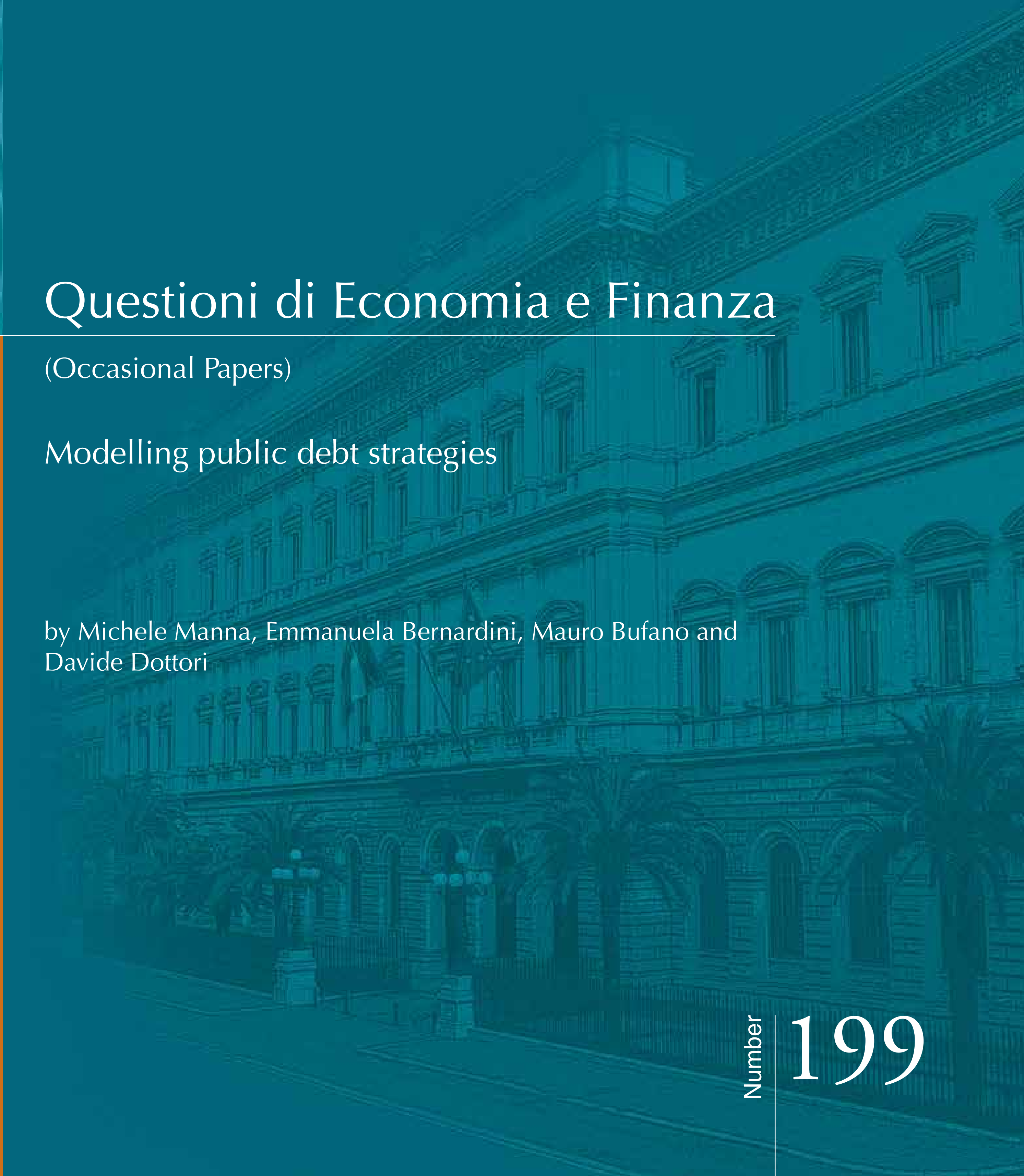

Number 199
September 2013

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The series is available online at www.bancaditalia.it .

ISSN 1972-6627 (print)
ISSN 1972-6643 (online)

Printed by the Printing and Publishing Division of the Bank of Italy

MODELLING PUBLIC DEBT STRATEGIES

by Michele Manna*, Emmanuela Bernardini*,
Mauro Bufano* and Davide Dottori*

Abstract

This paper puts forward a comprehensive framework to model medium-to-long term public debt refinancing strategies. Essentially the framework has two main building blocks. First, a large number of strategies are generated so as to determine a wide range of potential financing plans, regardless of whether they look conventional (close to current actual choices) or odd, provided they meet the Treasury's financing needs and legal constraints. Second, the performance of these viable strategies is measured in terms of current and future costs as well as various types of risk. As an add-on, through a panel model the framework accounts for the premium over current market rates that investors may demand in order to subscribe unusually large issues by the Treasury. All in all, this framework yields a frontier of efficient cost-risk outcomes. Moreover, it assesses how strategies perform when the interest rate forecasts relied on turn out to be wrong. Finally, it encompasses both a long-term perspective in debt management and a more tactical approach, allowing for time variant choices.

JEL Classification: D44, G11, H63.
Keywords: refinancing strategy, public debt, government auctions.

Contents

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1. Introduction [1]

The main objective of public debt management, as set out in IMF and World Bank (2001), is "to ensure that the government's financing needs and its payment obligations are met at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk". In principle, this goal could be broadened to include the implications of debt management for monetary policy and financial stability, in the light of the increasing awareness of the relationship between these fields (BIS, 2011).[2] In practice, however, most non-core goals of this type are either too elusive or too difficult to measure, or both.[3]

To pursue the stated goal of minimizing the cost over the medium-to-long term given an acceptable level of risk, the debt manager is asked to select a financing strategy – not only the overall amount issued in each unit of time but also its distribution among the various securities – while uncertainty clouds future interest rates and Treasury's financing needs. In doing so, he ought to bear in mind that the issuance process gives rise to a repeated game with investors subscribing the government bonds.[4] In addition, a number of "hard" and "soft" constraints restrict his room for manoeuvre.[5] Furthermore, the expansion of the public debt in many advanced economies has increased the volume of funds the manager must seek in the market (and also intensified public attention to issuance choices); at the same time, the globalization of financial markets has reduced the extent to which a sovereign issuer can rely on a captive constituency of domestic investors.

Against this background, modelling public debt management is gaining a foothold as a complement to first-hand market experience, and the related literature focusing on the main objective of cost-minimization is burgeoning.[6] In the model proposed by Bolder and Deelay (2011) refinancing strategies are defined in terms of proportions of each security in issue and are assumed to be constant over time. The horizon of the analysis is the long term (10 years). A similar model has been developed by the US Office of Debt Management, with the horizon set at 10 years and the optimal strategies among those simulated identified through the optimization of an objective function, subject to constraints that depend on a number of cost and risk metrics. To give an idea of the variety of choices tried by researchers: Renne and Sagnes (2008) compare financing strategies involving different shares of indexed debt while the macroeconomic variables are simulated using either a simple VAR or a Markov-switching VAR, and the horizon is the long term (10 years). Pick and Anthony (2006) develop a model of debt strategies in the very long run considering three maturity buckets and fixed issuance rules for each strategy. Alongside these works, we are also indebted to, among others, Leong

1 We would like to thank the participants in seminars held at the Bank of Italy, the UK Debt Management Office, the Bank of Canada/Ministry of Finance and the US Department of Treasury. We are indebted to Sergio Nicoletti Altimari, who challenged us to rethink the structure of the paper, and to Alice Chambers, who provided many valuable editorial suggestions. Of course, any remaining error is our fault. The views expressed in this article are those of the authors and do not involve the responsibility of the Bank of Italy.

2 For example, the debt manager could select a range of securities to be issued with a view to fostering the development of a sufficiently liquid bond market along the entire yield curve, from the very short end to extra-long maturities. In turn, this could assist the central bank's conduct of monetary policy.

3 This is not to mean that such additional goals are totally irrelevant. For example, one could argue that the choice of many debt managers to issue bonds with a maturity of 30 ore more years helps to complete the financial market.

4 Accordingly, the trade-off in costs and benefits of each action must be assessed considering the net present value of all future impacts.

5 Provisions of law fixing the maximum amount of net issue per calendar year and forbidding any overdraft by the Treasury with the central bank are examples of hard constraints, while a "soft constraint" could take the form, for instance, of pressure on debt managers to pre-announce their issuance calendar in order to foster orderly financial market conditions.

6 The "References and reading material for government debt management, Theory and practice", compiled by Storkey & Co, extends over 15 pages. Barro (1979 and 1999) is a standard source on why in a world with uncertainty the government should take its stance on the securities to be issued depending on an assessment of the possible states of the economy; see also Shin (2007).

(1999), Bergström et al. (2000, 2002), Bolder (2003, 2006, 2008), Berneschi et al. (2007), Larson and Lessard (2011) and Renne (2007).[7]

This paper seeks to advance debt modelling on several fronts.[8] First, no fixed weights are enforced across securities and over time, contrary to what appears to be the rule in the literature. In our set-up both the total supply of securities in each quarter – the fundamental unit of time of the model – and its distribution across the individual securities can vary. This adds flexibility to the model by making it possible to exploit movements of the yield curve, in a sort of tactical management of the "debt portfolio", while not impairing the long-term orientation of the resulting refinancing strategies. Furthermore, this flexibility yields a good number of potential solutions to the problem faced by the debt manager, and it is our intention to assess both "conventional" strategies (close to current actual choices) and "odd" ones, provided they meet the Treasury's financing needs and the provisions of law regarding government securities issuance.

Second, our model admits and quantifies levels of interest rates set at issuance which differ from the prevailing secondary market rates, when the supplied amount is large. To this end, we use a panel model to estimate the yield penalty charged by the market on large-volume issuance. To our knowledge, no similar attempt has been made in other models to quantitatively infer this penalty from data. Such a tool should probably be an ordinary component of any model of public debt; it becomes almost a must in a framework where no ceiling is exogenously set on the amount supplied for each and every security.

A flowchart of the model is reported on the next page (Figure 1). The model employs a suite of tools and algorithms to: i) generate a very large number of candidate strategies, using as initializing vector the actual issuances by the Italian Treasury, ii) filter the strategies that meet basic conditions and are thus deemed to be viable financing plans, and iii) introduce cluster techniques in order to select a subset of the strategies whose results are examined in greater detail. The strategies selected are scored according to the associated debt servicing cost and various measures of risk. For the derivation of future interest rates, we rely on two alternative approaches: a Cox-Ingersoll-Ross stochastic process (CIR) and a Vector AutoRegressive model (VAR), augmented with the Nelson and Siegel (NS) method for the yield curve construction. The choice here is to start out from two rather different approaches – one based entirely on market prices, the other taking into account the behaviour of key macro variables – that hopefully can cross-check each other.

The rest of the paper is organized as follows. Section 2 explains the process of generating strategies, Section 3 describes the evaluation of the strategies, Section 4 discussed the main results, and Section 5 concludes.

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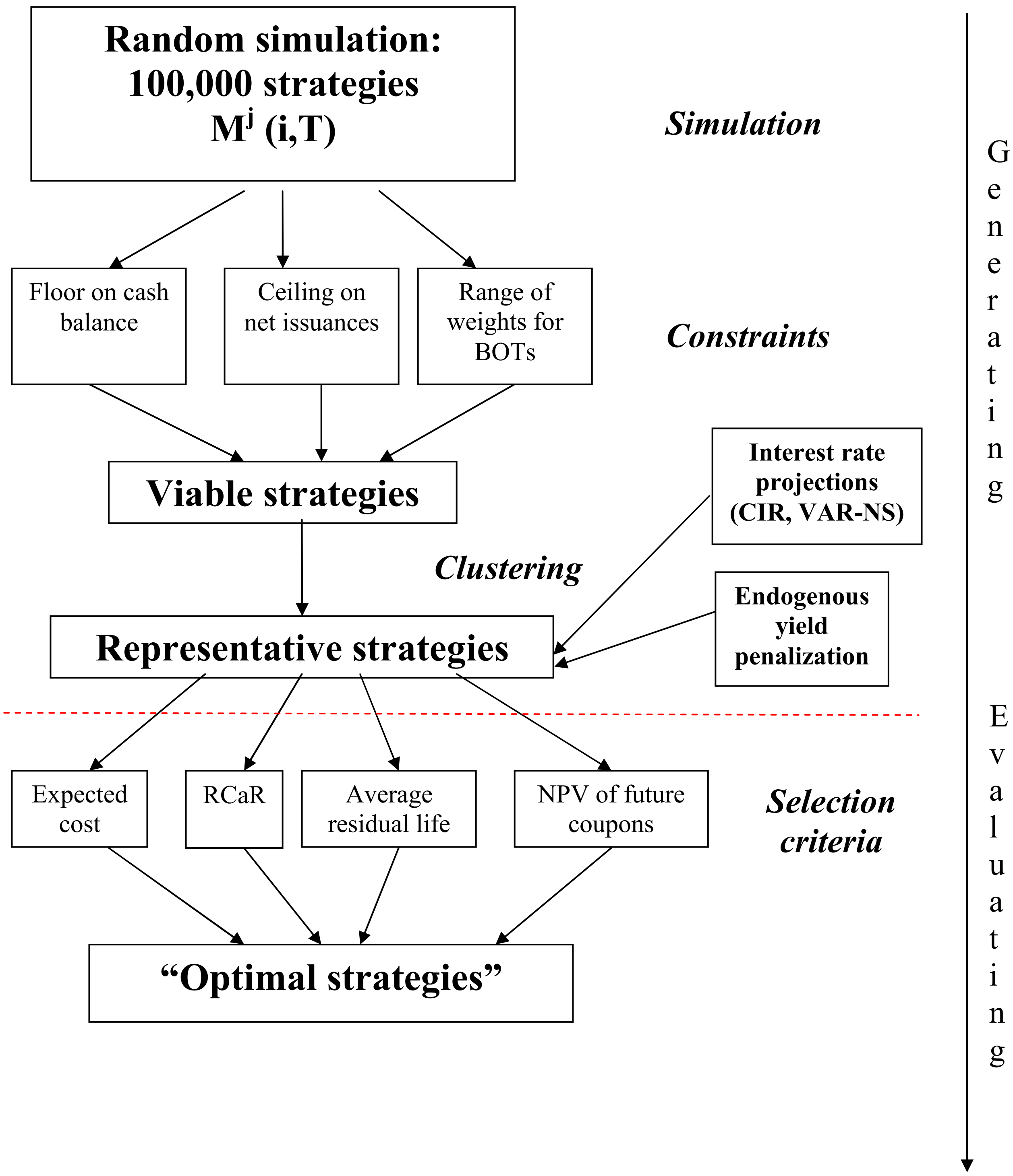
7 This is not to neglect papers which study specific issues in debt management: the optimal debt maturity, for example, is investigated by Angeletos (2002) and Buera and Nicolini (2004), the optimal composition of public debt by Missale (2001) and Missale and Bacchiocchi (2005). A different strand of literature tackles modelling public debt within an ALM framework (Velandia, 2002).

8 We do not claim that all the elements we are going to introduce are new to the literature; however, to the best of our knowledge no other paper considers all of them together.

Figure 1


Random simulation:
100,000 strategies
M^j (i,T)
Simulation
Floor on cash balance
Ceiling on net issuances
Range of weights for BOTs
Constraints
Viable strategies
Interest rate projections (CIR, VAR-NS)
Clustering
Endogenous yield penalization
Representative strategies
Expected cost
RCaR
Average residual life
NPV of future coupons
Selection criteria
"Optimal strategies"
Generating
Evaluating

2. Generating the strategies

2.1 *Definition of refinancing strategy*

In our set-up a refinancing strategy M^j is a n×(T+1) matrix filled in by the amounts of the gross supply, in billions of euros, of a number of key government securities (reported row-wise) over a number T+1 of periods (column-wise).[9] Within this horizon the debt manager is confronted with the financing of cash needs which fluctuate throughout the year and with his own front-loading preferences (if any). The frequency chosen in the model is the calendar quarter, to ensure sufficient granularity but without having to deal with too many data. The projection horizon ends with the fourth quarter of the third year after the baseline time 0, so that in general T may vary from 12 to 15.

The choice of the number of rows/securities of the matrix reflects a trade-off: the higher this number, the closer is the model to real-life conditions; the lower, the easier it is to handle several analytical and numerical questions. In determining the terms of this trade-off, the researcher should carefully assess two modelling issues: first, some securities could entail low volumes of issuance and would accordingly trigger close-to-zero-boundary plans; second, securities which display very similar risk/return profiles may lead to forms of multicollinearity. Hence, our choice was to lump together the actual securities issued by the Italian Treasury – which count nine basic categories and 19 more elementary types of securities – in seven buckets: 6- and 12-month BOTs; 3-, 5-, 10- and 20-year BTPs;[10] and 10-year inflation indexed BTPs.[11]

Table 1

Mapping of original securities into securities used in the model

(amounts in billions of euros)

Original security		Security included in the model	
Type	Amount issued in 2011 (1)	Type	Amount
3-month BOT	10	6-month BOT	109
6-month BOT	99		
12-month BOT	81	12-month BOT	81
24-month CTZ	28	3-year BTP	67
3-year BTP	39		
5-year BTP	35	5-year BTP	53
5-year CCTEU	3		
7-year CCTEU	15		
10-year BTP	46	10-year BTP	46
15-year BTP	12	20-year BTP (2)	15
30-year BTP	3		
5-year BTPI	6	10-year BTPI	13
10-year BTPI	6		
15-year BTPI	1		
30-year BTPI	> 0		
Total	384		384

(1) Based on the date of auction; supplementary auctions reserved to primary dealers ("specialists") are not included. (2) Fictitious bond.

9 The first column of the matrix shows issuance at the baseline time 0. The symbol 'M^j' refers to the matrix of strategies under the *j-th* replication (j = 1,..,100,000). When we wish to select the *t* column/quarter, we add a subscript: M_t^j (t = 0, …, T). Note that the content of the left-most column (t = 0) is fixed and we write M_0. When we wish to highlight the issuance of the *i-th* security, we use the symbol M_t [i], where i = 1,.., 7.

10 The 20-year BTP, a fictitious security, is meant to represent a (weighted) average between the actual 15- and 30-year BTPs, the former being more commonly used also for re-openings of off-the-run bonds.

11 The average maturity at issue of the reclassified portfolio is marginally different from that of the actual issuance in 2011 (3.9 years, against 3.6 of the original data).

2.2 *The algorithm for the generation and selection of strategies*

The matrix M^j is compiled using a tree algorithm where the direction to be taken at each node is dictated by the outcome of a uniform distribution. To see how the procedure works, let us denote with M_0 the 7×1 vector of actual offers made by the Treasury in the baseline "time 0".[12] At the first node, the supply of the *i-th* security in quarter 1 ($M_1[i]$) will differ from that supplied at time 0 ($M_0[i]$) by an amount equal to either $+2\Delta_i$, $+\Delta_i$, 0, $-\Delta_i$, $-2\Delta_i$ according to a number extracted from the uniform distribution.[13] Having thus set the supply at quarter 1, the process is repeated anew – again five options in terms of change and new extraction from the uniform distribution – in quarter 2, and so forth until quarter T. The whole process is then replicated 100,000 times for each security.

The magnitude of Δ_i is security-dependent: notably, it is set so as to preserve a rough comparability in the ratio of Δ_i to the issuance $M_0[i]$ of the *i-th* security at time 0 (Table 2).[14] In the end we opted for a more advanced approach where the Δ_i's not only change across securities but also over time: for each quarter they were adapted proportionally in order to get an overall expected gross supply in line with the Treasury's financing needs for that quarter.[15] A detailed presentation of the algebra is in Annex 1.

Table 2

Supply at time t_0 and step Δ

(billions of euro and percentages)

	BOTs		BTPs				BTPIs
	6- month	12- month	3- year	5-year	10-year	20-year*	10-year
$M_0[i]$**	28.0	27.0	17.0	13.0	11.0	4.0	3.0
Δ_i	3.0	3.0	1.5	1.5	1.5	1.0	1.0
$\Delta_i / M_0[i]$	11%	11%	9%	12%	14%	25%	33%

* Fictitious bond; the percentage reported under M_0 refers to the issuance of 15- and 30-year BTPs. ** This is the vector used to initialize the simulation and is based on the actual issuance in 2011.

One drawback of letting the refinancing strategies be drawn by a random number generator is that we get a number of plans which simply fail to meet Treasury financing needs, normative constraints or historical issuance policies. In effect, violating even one of these three constraints makes it necessary to ditch the plan. Let us look at them in some detail.

First, as laid down in EU and Italian laws, the Treasury cannot run overdrafts on its cash account with the central bank. In our model, where time is divided into quarters, we translate this constraint into a floor of €20 billion at the end of quarter so as to allow for within-the-quarter variability in the Treasury account balance (B_t^j) without infringing the no end-of-day overdraft rule.[16]

$$[2] \qquad B_t^j = B_{t-1}^j + M_t^j - L_t^j \geq 20,\ t = 1, .., T \qquad \forall\ t,j \qquad \text{(floor on cash balance)}$$

[12] See footnote 9. To render the whole process less dependent on the choices made in a single quarter, we compiled M_0 using average quarterly issuance in 2011 rather than referring only to the fourth quarter of that year.

[13] The outcomes +2Δ and –2Δ have been assigned a probability of 0.10 each, the two intermediate ones +Δ and –Δ a probability of 0.25 each, and the central outcome (no change) the residual 0.30.

[14] For the 20-year BTP and the 10-year inflation-indexed BTP an overriding objective was to keep Δ equal to at least €1 billion (per quarter).

[15] The reader should bear in mind that this element in the algorithm drives the overall supply and keeps it consistent with the Treasury financing needs, given by the sum of the quarterly deficit – broken down from yearly official targets using a standard X-12 Arima – and redemptions. No similar issue arises in the other models we examined, where the strategy is defined in terms of weights to be applied to a total gross offer fixed by the modeller.

[16] The size of this buffer was inferred from a preliminary analysis of the daily data of the Treasury's cash balance from 2008 to 2011.

where L_t^j is the Treasury's refinancing needs in quarter *t* under strategy *j*, defined as redemptions R_t^j plus deficit F_t (negative values of F_t denote a surplus) minus the available buffer on the Treasury's account balance with Bank of Italy at the end of quarter *t-1* in excess of the €20 billion floor:

$$[3] \qquad L_t^j \equiv R_t^j + F_t - \max\left(B_{t-1}^j - 20; 0\right) \qquad \forall\, t = 1, .., T; j = 1, .., 100{,}000$$

In [3], for the sake of simplicity, we omit some items of the deficit that do not cause changes in the balance (usually, transfers from the Treasury to non-central-government components of the broadly defined public sector).

Second, Italian law imposes a ceiling on net issues of government securities:

$$[4] \qquad \sum_{t \in yi} (M_t^j - R_t^j) \leq k_i \qquad i = 1,2,3, \forall\, j \qquad \text{(ceiling on net issuance)}$$

The ceilings k_i for each year of the simulation horizon are obtained by adding to the yearly targets for the budget deficit a narrow buffer, so to as give the debt manager some room for manoeuvre.

Finally, we set a range of 35% to 65% for the share of BOTs in total quarterly gross issuance, compared with the range of 48 % to 59% shown by the historical data from 2007 to 2011.[17] While the range of BOT shares implemented in the model is wider than that observed in recent years, allowing leeway in the algorithm to introduce odd strategies, it remains reasonably consistent with actual choices.[18] Bearing in mind that the two BOTs occupy the first two rows of the M^j matrix, the third constraint is:

$$[5] \qquad 0.35 < \left(\sum_{i=1}^{2} M_t^j[i]\right) / \left(\sum_{i=1}^{7} M_t^j[i]\right) < 0.65 \qquad \forall\, t,j \qquad \text{(range of weights for BOTs)}$$

In practice, only a handful of the starting strategies directly met all three criteria in each of the *T* quarters. So we did what the Treasury does, namely relying on the issuance of cash management instruments (BOTs with 3-month or broken maturity as well as commercial paper) when conditions warrant it. In our model, the issuance of such instruments is subject to a ceiling of €25 billion on their net outstanding stock at any point in time (see Box 1 in Annex 1) and to the condition that they are issued and expire within the same calendar year.

Once this degree of flexibility is embedded in the model, 3,700 strategies out of the 100,000 generated at the outset met all three criteria [2], [4] and [5]; accordingly, we call them the "viable strategies". Even within this subset, a thorough examination of each refinancing plan would be challenging, to put it mildly, for even the most capable debt manager. To complicate matters further, strategies quite often may differ from one other in relatively tiny details and one-by-one scrutiny would add limited information. For these reasons, we identify 60 clusters out of the sample of 3,700 and within each cluster we select a "representative strategy" using a criterion of least average Euclidean distance within the

[17] These percentages do not include non-regular issues of BOTs, such as quarterly and broken-maturity bills. Data for the 2007-2011 sample refer to the first and ninth deciles of quarterly shares.

[18] This range of solutions is set in order to introduce a sort of soft constraint in the model. In addition, it permits us to to circumvent a significant empirical issue. Had we not set such a range, there could have been feasible strategies based entirely on BOTs, considerably inflating the quarterly gross issuance of these instruments. In turn, consistently with our set-up, this would have required the estimation of a proportionally large yield penalty. As such extreme circumstances are not encountered in the actual records (a similar remark also applies to the opposite case of no BOTs), the data do not offer reliable guidance as to the level of the penalty, which would have been a matter of guesswork.

cluster itself (Everitt, 1993). To mitigate the risk of the outcome of this final selection being too dependent on initial seeds, we perform a two-step cluster analysis, applied first in the hierarchical and then in the non-hierarchical form (more details on the procedure are in Annex 2). Note that while the clustering techniques help us to focus on a limited number of choices, greatly facilitating evaluation of the results, nothing is lost of the initial richness of information, and when it is useful we revert back to the individual viable strategies.

2.3 A glance at 60 representative strategies

Figure 2 shows the median, minimum and maximum weight for each quarter and across the 60 representative strategies, of the 6- and 12-month BOTs taken together, the 3- to 10-year BTPs, the extra-long 20-year BTPs and the BTPIs; additional results are presented in Table 3. A few remarks are in order. First, the minimum and maximum weights are fairly volatile over time due to outlier strategies (a token example of what is referred to above as "odd" choices). Second, the weight of the two BOTs never fall below 40%, although in generating the first 100,000 strategies we allowed this weight to be as low as 35% (Section 2.2). This lower bound is accordingly not binding (while the 65% upper bound is), probably because strategies with too few bills are not able to accommodate changes in Treasury cash needs from quarter to quarter. Third, the model allows for diverse patterns over time across the different groups of securities. Fourth and last, as far as 20-year BTPs and BTPIs are concerned, the model accepts solutions where the issuance of these securities is entirely discontinued.

Figure 2

Weight of BOTs, 3-to-10 year BTPs, > 10-year BTPs and BTPIs in representative strategies

(per cent of total issuance; minimum, median and maximum weight across the 60 strategies)

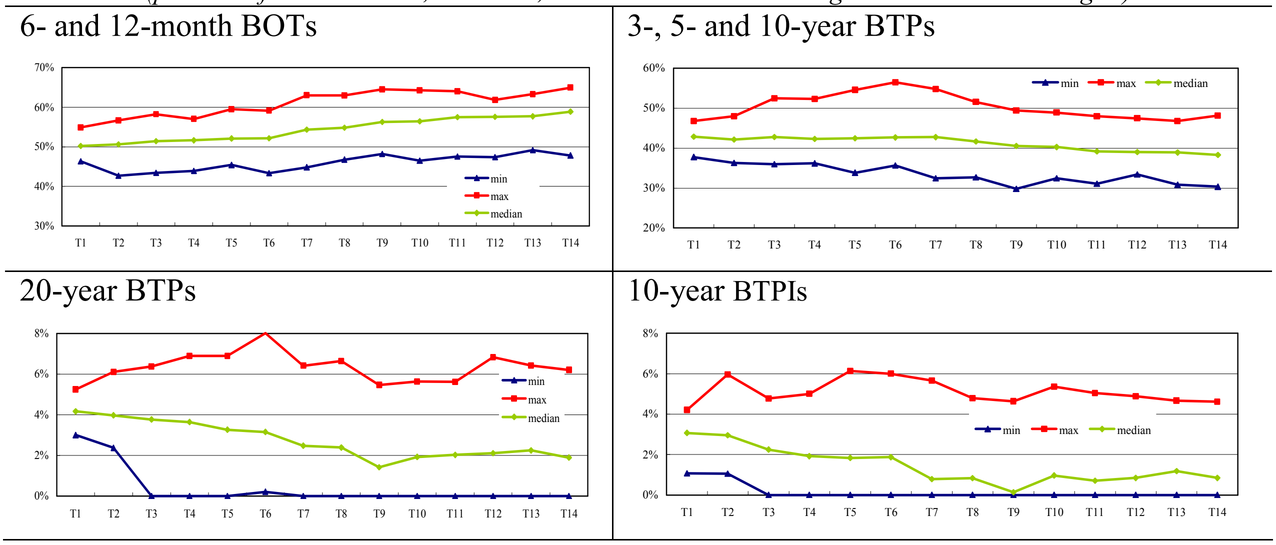


Table 3

Weight of securities in the simulated representative strategies

	6- and 12-m BOTs	3-, 5- and 10-y BTPs	20-year BTPs*	10-year BTPIs
		Model		
(a) Maximum	58.9 %	49.0 %	5.3 %	4.6 %
(b) Minimum	48.5 %	36.1 %	0.8 %	0.3 %
(a) – (b)	10.3 %	12.8 %	4.5 %	4.3 %
		Actual quarterly data, 2007-2011		
(c) 95th percentile	66.3 %	39.0 %	6.5 %	5.6 %
(d) 5th percentile	52.4 %	27.9 %	0 %	0.4 %
(c) – (d)	13.9 %	11.1 %	6.5 %	5.2 %

*Fictitious security; the actual data are based on the real 15- and 30-year BTPs

3. Evaluating the representative strategies

In principle, the most comprehensive score of a strategy ought to consider each of the 7×T gross amounts of issuance stored in matrix M^j (excluding, of course, the items in the first column from the left, which as explained above are fixed). Hence, the need arises for a limited number of summary statistics. We select five of them, of which two relate to the cost of serving the debt and the other three relate to various dimensions of risk, in keeping with the characterization of debt management as a portfolio problem where the debt manager aims to minimize the cost of servicing the debt, subject to predetermined levels of risk (Bolder and Deelay, 2011).

The reader should bear in mind that the algorithm works out 10,000 tries for each strategy, since this is the number of interest rates generated by both the CIR and VAR-NS for each node of the yield curve. Having a large number of interest expenses measures enables us to derive statistics on the average value of the indicator for each strategy – and in a subsequent step across the 60 representative strategies – as well as statistics on the dispersion around the average.

3.1 Scoring the results

In this section we describe the characteristics of the different measures used to score the various strategies in terms of cost and risk.

<u>The average cost of debt (COST)</u>. This is measured by adding up all coupon payments in the last calendar year of the simulation horizon (2015 in the current version of the model), according to a cash principle. Coupon bonds are issued at par and the coupon is set equal to the prevailing secondary market rate for the relevant maturity plus a penalty if the amount being auctioned off is large (see Section 3.3); coupons of the BTPIs are indexed to inflation using the Italian sovereign break-even inflation curve as at 3 July 2012, assuming a real coupon of 2.25%.[19] The discount at issue is treated as an implicit coupon in the zero-coupon securities BOTs and CTZs. The sum of actual and implicit cash outflows is re-scaled to the overall stock of public debt, to allow for those debt components in forms other than securities (coins and deposits, loans by monetary and financial institutions and other liabilities).[20] The results are shown as a percentage of IMF forecasts of Italy's GDP in 2015.

<u>Net present value of future coupons (NPV)</u>. This measures the future service of debt beyond 2015. Indeed, gross issuances of extra-long securities (say, the 20-year BTP) lengthen debt maturity, assuring a lower refinancing risk, but they imply the payment of coupons which are generally higher than for bonds with shorter maturities for a very long period. Therefore, the NPV measure aims to capture excessive future debt services due to strategies that insist on longer maturity issuances.

<u>Relative Cost-at-Risk</u> (RCaR). This signals the refinancing risk, namely the risk that the yield at issue of the securities will diverge from the central interest rate projection for the relevant maturity. The RCaR measures the difference between the cost of each given strategy, in 2015, at the 99th percentile of the distribution of interest expenses /GDP ratio distribution and the expected interest expenses/GDP ratio.

19 This is close to the weighted average of 2.27% measured on the securities of this type outstanding at 31 December 2011. Coupons are indexed to inflation by multiplying the real coupon by the indexation coefficient (the ratio between the level of the consumption inflation index at the coupon payment time and its level at the bond's issue date). The same applies to the amount to be reimbursed at maturity. If the coefficient were less than one (deflation), the coupons and principal would be paid based on the amount issued.

20 At the end of 2011 Italy's public sector debt amounted to €1,897 billion, of which €1,606 billion in the form of securities (Bank of Italy, 2012). Accurately modelling the cost of debt other than securities is anything but trivial and we simply assume that its average cost is the same as the average cost for the securities.

Residual life of outstanding debt (ROL). This is measured on the outstanding stock of securities at the end of 2015 and proxies the duration risk, namely the inertia with which changes in secondary market rates feed into the cost of servicing the overall stock of debt.[21] In addition, as long as the "average" residual life reasonably overlaps with the "mode" in debt maturity, this statistic is also informative on the maturity bucket of the yield curve whose shocks would have the largest impact on the cost of servicing the debt. Last but not least, it is an easily mnemonic and intuitive type of information, two elements which may explain its use in a broad range of contexts.

Maturing debt in the first year after the end of the horizon (MD). This captures the refinancing risk in the first year after the end of the simulated horizon (2016). We add it also because it is a simple way of capturing refinancing strategies after the horizon-end. In particular, it helps to flag those strategies with a relatively low measure of cost in 2015, which can be achieved through an increase in the share of bills (as long as the yield curve is positively sloped); in the short term this brings about a reduction in debt servicing *ceteris paribus*.

3.2 The models used to yield interest rate projections

Any model of public debt refinancing strategies is as good and reliable as the underlying projections on the interest rates are. On this point, our choice was to follow two approaches in parallel: a CIR (Cox, Ingersoll and Ross, 1985)[22] process and a combination of a standard VAR with the Nelson-Siegel model (VAR-NS, standard references for the VAR are Sims, 1980, Lütkepohl, 2006, Nelson and Siegel, 1987).[23] The aim is to implement a two-pillar approach, where CIR simulations are driven entirely by market data while in VAR-NS market developments are read through a few selected "fundamental" macroeconomic variables. The second approach could thus act as a cross-checking tool for the former, especially at business cycle turning points and in conditions of self-reinforcing trends in interest rates which could eventually deviate from the fundamentals themselves. On a more technical note, the CIR offers mean-reverting solutions toward a long-run equilibrium. As a result, if the current yield curve is rather sloped, in CIR long-run projections the yield curve tends to become flatter while this is not necessarily the case with the VAR-NS. We simulate 10,000 interest rate scenarios for each of these two approaches.

Given the standard CIR set-up:

$$[6] \qquad dr_t = \alpha(\mu - r_t)dt + \sqrt{r_t}\sigma dW_t$$

In our fit, $\alpha = 0.408$, $\mu = 0.070$ and $\sigma = 0.099$, based on market data as of 3 July 2012.[24] These values suggest a rather fast mean reverting process, as confirmed by forward rates (Figure A.1).

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[21] This relationship holds true only approximately since a small but not negligible portion of Italian public debt in the form of floating rate notes are indexed to 6-month Treasury bill rates or to the 6-month Euribor.

[22] Previous works on CIR for the Italian sovereign yield curve include those by Barone, Cuoco and Zautzik (1991), Berardi (1995), Dullman and Windfur (2000), Gentile and Renò (2005), Torosantucci et al. (2007).

[23] Another stream of literature (e.g. Exterkate et al., 2011; Brzoza-Brzezina and Kotlowsky, 2012) follows a different approach by modelling through a VAR the dynamics of the time-variant parameters of the NS model. We prefer to start from a macroeconomic VAR model, albeit a parsimonious one. In this way, we can forecast short- and long-term interest rates and derive the whole curve by applying the NS model. On yield curve modelling using a Nelson-Siegel technique, see also Diebold and Li (2006).

[24] We derived starting values of the parameters of [5] using a maximum likelihood estimation over the 6-month BOT rate from 2007 to 2012. Details on the estimation process and its Matlab implementation are in Kladivko (2007). The calibration of the CIR is carried out in cross-section mode on the zero coupon yield curve.

Some recent applications of VAR involving sovereign yields are Polito and Wickens (2011), Favero (2012) and De Santis (2012). An example of VAR in public debt management is Renne (2007), who adopts a parsimonious model based on a VAR(2) using four variables: the two interest rates (short and long), inflation and GDP, all on a quarterly basis. We build on that by adding two public finance variables such as the budget deficit and the debt level (more details on the dataset used are in Table A.1).

The estimation sample has a quarterly frequency and ranges from 1991Q1 to 2012Q2, in order to ensure a suitable data length. The short-term interest rate is taken at the 6-month maturity while the long-term one is at 10 years. Standard unit root tests show that, within our sample, the series of debt, GDP, deflator and both short- and long-term interest rates are I(1), accordingly they are taken in first differences, while the only borderline result is for the deficit series (Table A.2). Debt and net deficit proved to be seasonal and are adjusted using X-12 ARIMA (2011). All in all, the algebra of our VAR is

$$[7] \qquad z_t = \mu + A_1 z_{t-1} + A_2 z_{t-2} + \varepsilon_t$$

where z_t is the six-dimensional vector of transformed variables (first differences on I(1) variables with seasonal adjustment where appropriate), A_i are fixed coefficient matrices, μ is a fixed vector of intercept terms and ε_t is a white noise process. Since our focus is on forecasting, we select the optimal time lag based on the Final Prediction Error and Akaike Information Criteria, designed to minimize the forecast error variance. The lag order 2 is confirmed by a Likelihood Ratio sequential test. We produce iterated multi-period-ahead forecasts obtained by iterating forwards the one period ahead VAR.[25]

The two VAR projections on the interest rates at 6-months and 10-years are not enough to work out a fully-fledged refinancing strategy, since an entire yield curve is required. Hence, we combine the VAR forecasts with a Nelson Siegel model to gauge fits at the other maturities.

Figure 3 shows the spot yield curve at 3 July 2012 next to the projections referred to different horizons, from the end of 2012 to the end of 2015. Quite visibly, the more distant into the future the projection, the less sloped is the yield curve based on the CIR while the VAR-NS estimations tend to be more stable as the horizon lengthens. For the purposes of identifying optimal debt strategies, the difference in yield curve structures implies that while we adopt the CIR approach as a benchmark measure, the cross-checking through the VAR-NS suggests that there is scope for savings by "shopping along the yield curve" through some shortening of the maturity of new debt.

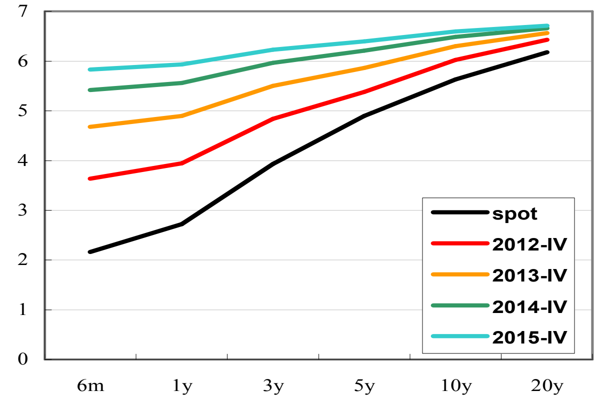
[25] Iterated forecasts are more efficient than direct forecasts if the one-period-ahead model is correctly specified (Marcellino, Stock and Watson, 2005).

Figure 3

Projections of interest rate curves at future dates

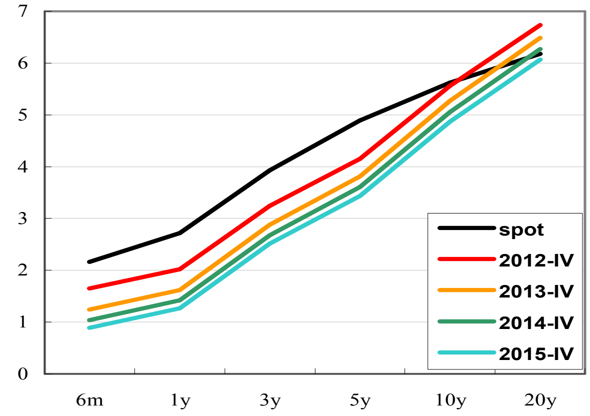
CIR

(percentage; central scenario)



VAR - Nelson Siegel

(percentage; central scenario)



3.3 The panel model to derive the penalty over the large issuances

One component of the model endogenously derives a penalty on the interest rates set at the auctions on top of the rates prevailing in the secondary market at the time of the auction itself. To this end, we fit a panel model to infer the extra yield requested by investors to underwrite larger-than-usual supplies of securities by the Treasury. The model is estimated using daily data on BTPs from January 2009 through March 2012 observed in the MTS.[26] The baseline version of the model employs the following variables: *highoff* is an interaction variable equal to the amount in auction if on day *t* security *i* is being auctioned off, and the amount auctioned is higher than a "norm" set equal to the average supply in the primary market; the one-period forward high-off variable is considered in order to take into account the effect of the announcement of the offer the day before the auction; *bidask* is a first control variable of bond's liquidity conditions, reflecting the eponymous spread; *spread* is a second control variable relating to the difference in yield between the benchmark 10-year BTP and its equivalent German Bund; *auction day* is a dummy whose role in the model owes to the theoretical and empirical literature on the so-called concession premium, according to which yields tend to rise with the occurrence of auctions; *yearstomaturity* is a variable that controls for the fact that securities with longer maturities show higher yields as long as the yield curve is upward sloping. Preliminary unit root tests signal the need to run the model on first differences. Our preferred fit is (robust standard errors are in parenthesis):

$$[8] \quad \Delta\,\text{yield}_{i,t} = \underset{(0.10)}{0.35}\,\frac{\Delta\,\text{highoff}_{i,t}}{100} + \underset{(0.10)}{0.25}\,\frac{\Delta\,\text{highoff}_{i,t+1}}{100} + \underset{(0.07)}{0.25}\,\Delta\,\text{bidask}_{i,t} + \underset{(0.51)}{1.20}\,\Delta\,\text{spread}_t$$

$$+ \underset{(0.01)}{0.02}\,\text{auctionday}_t - \underset{(0.004)}{0.03}\,\Delta\,\text{mts}_t + \underset{(0.0001)}{0.0001}\,\text{yearstomaturity}_{i,t} - \underset{(0.001)}{0.002}\,\text{coupon}_i$$

The estimated coefficients display the expected sign, at least when they are significantly different from zero. Since regressors are in logs, the results imply that doubling the amount

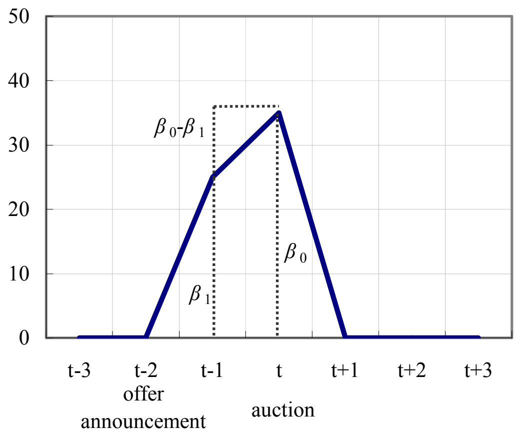
[26] We restricted the analysis to BTPs with at least one year of residual maturity. The resulting panel is unbalanced and spans over 60 bonds observed over a (maximum) of 826 time observations.

supplied (compared to the amount observed in our M_0 vector) is associated with a cost at issuance that is 35 bp higher than the relevant secondary market rate. All other things being equal, a supply that is double the "norm" drives the rate up by 25 bp already on the first day after the announcement. The next day, when the auction takes place, the interest rate rises by a further 10 bp, reaching a cumulated change of +35 bp. The day after the auction the extra supply effect vanishes and the interest rate goes back to the level that prevailed prior to the announcement (of course, the *ceteris paribus* assumption applies once more; Figure 4; more details are in Annex 4).

Figure 4

Cumulative impact

(basis points; assumed $highoff_{i,t}$ =100)



The cumulated impact of doubling the amount supplied compared to the "norm", (which in our fits is the average amount auctioned off in BTPs from January 2011 to March 2012) is 35 bp. At the time the offered amount used to be released two days before the auction, after markets close.

4. Main results

We report the results on the cost and risk indicators for our 60 representative strategies in four charts, where the cost of servicing the debt (as a % of GDP) is shown along the horizontal axis and in each chart a different measure of risk appears on the vertical axis (average residual life; relative CaR; debt maturing in the first year after the horizon; net present value of future coupons). The results are depicted for the CIR projection on interest rates. The reader should bear in mind that each dot stands for a representative strategy, which in turn represents a cluster of strategies. As a further element, we define as dominant those strategies which do not find a better match in terms of trade-off between cost and risk. Finally, we disentangle strong from weakly dominant strategies, where the former perform better than the others in all four charts while the latter do so only against one or more indicators but not all of them. In practice, strongly dominant strategies are typically located in the south-west corner of the scatter plot.

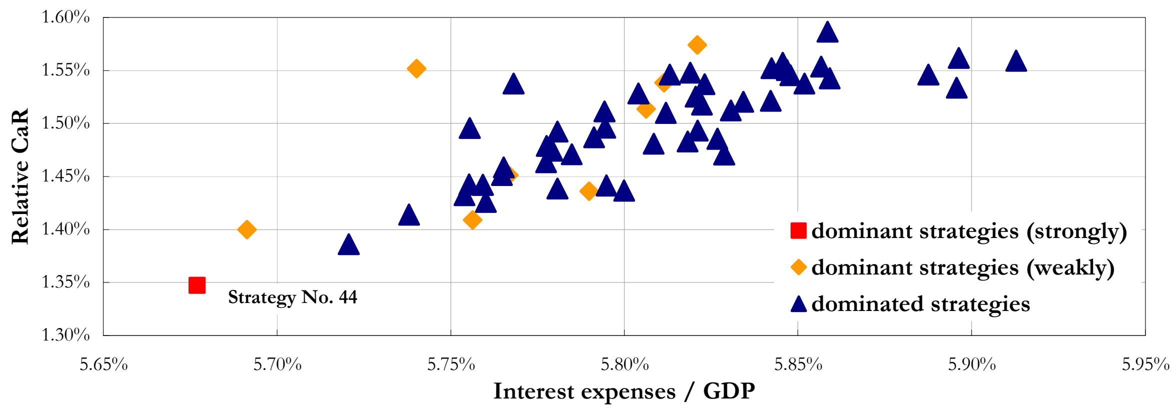
4.1 The results referred to the 60 representative strategies / clusters

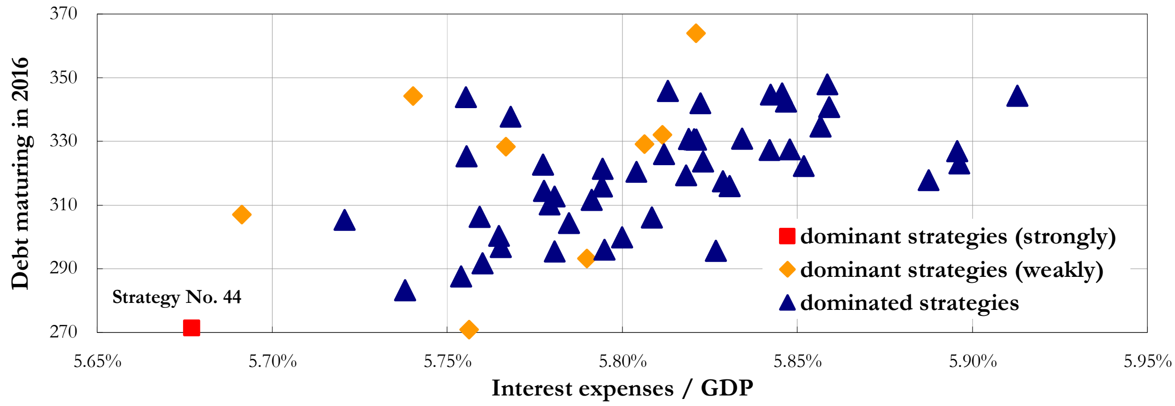
According to the interest rate scenarios simulated through the CIR model, strategy No. 44 emerges as *strongly dominant* in the sense defined above: given the level of cost associated with this strategy, no other one is even a close match in terms of riskiness (Figure 5). Moreover, this strategy emerges as having one of the highest average residual lives of outstanding debt at the end of the simulation period. Other strategies (yellow dots) are *weakly dominant*, in the sense that they dominate in some but not all the charts.

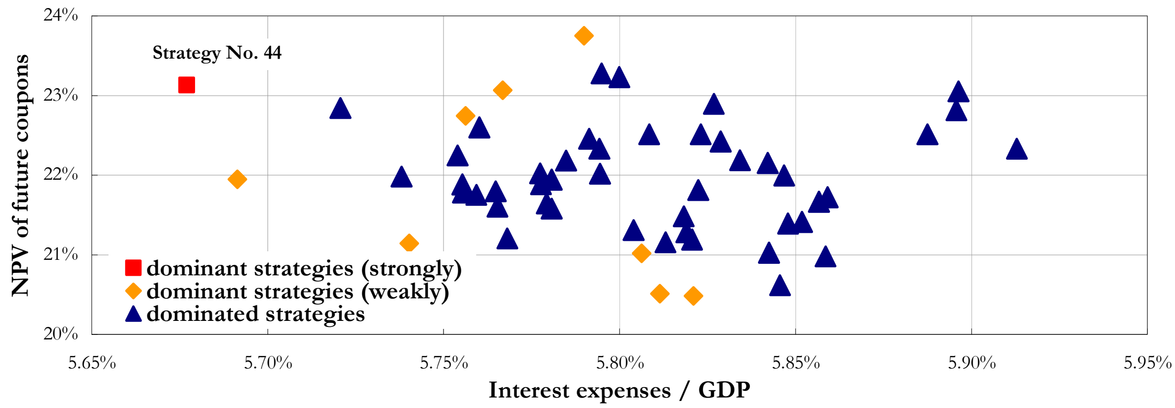
Figure 5

Debt strategies – results

(Interest expenses, ReCaR and Net Present Value of future coupons as a percent of GDP; average residual life in years; maturing debt in billions of euro)





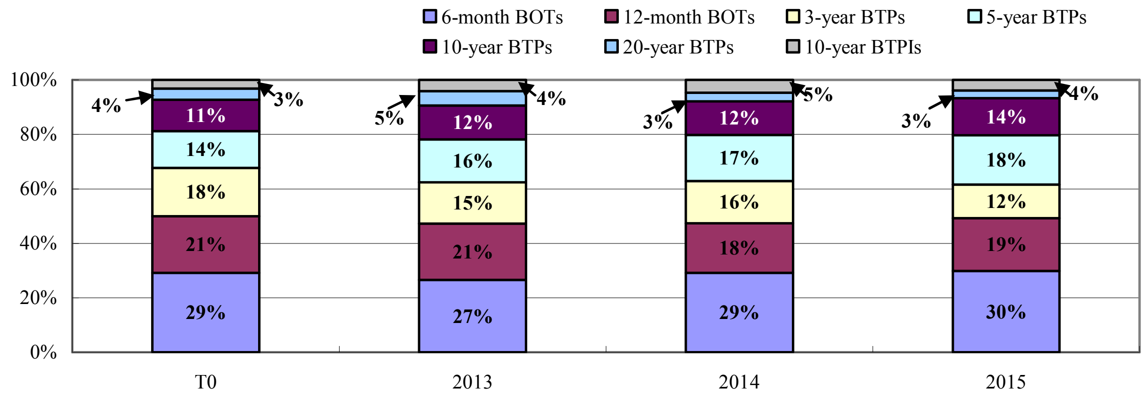


In the scenarios simulated using the CIR process, strategy No. 44 shows interest expenses equal to 5.68% of the GDP, a net present value of future coupons of 23.13% of GDP, a relative CaR equal to 1.35% of GDP and, finally, an average residual life of 6.2 years while debt maturing in 2016 amounts to €271 billion.

It is interesting to break down the shares of the individual securities over the horizon in this strategy, which under-weighs the 12-month BOT and 3-year BTP issuances, while it over-weighs 10-year BTPs and inflation linked securities (Figure 6). Extra-long BTP issuances are over-weighted in 2013, but decline in the last years.

Figure 6

Dominant strategy – yearly issues



When one turns to the VAR-NS forecasts, another strategy (No. 40) emerges as *strongly dominant* (Figure A.2). This has a residual life of 5.7 years and the debt maturing in 2016 is €344 billion, much higher than that for strategy No. 44.

The fact that in one instance strategy No. 40 prevails while in the other No. 44 tops our ranking underscores one simple fact of life: the best outcome is dependent on the interest rate projection and the latter is in turn unavoidably subject to errors. In other words, the debt manager will follow different paths depending on which interest rate projection he trusts most. Unfortunately, ex ante he can't be sure whether this trust is well placed and there is no way to solve this dilemma. What he can do, however, is assess the extent to which a given choice of strategy would underperform compared to the optimal one, if the chosen projection eventually proves to be wrong. To this end, we undertake an exercise where the strongly dominant strategy under the CIR set of projections is tried out under the VAR-NS projections and vice versa. The main results of such an exercise are shown in Table 5, where we assess the two winning strategies, Nos. 40 and 44, under both the CIR and the VAR-NS projections. Both strategies remain associated with much lesser-than-expected interest rate costs in 2015 under the VAR-NS; this is unsurprising given that under this approach short-term rates are predicted to remain relatively low in the years to come. However, and more importantly given the scope of the exercise, strategy No. 44, which is strongly dominant under CIR, performs fairly well even under VAR-NS (it would lose to the new strongly dominant strategy No. 40 only by 17 bp in terms of the cost-to-GDP in 2015) and the same applies to strategy No. 40 when this is assessed under CIR.[27]

[27] Results on additional robustness checks are available upon request.

Table 5

Dominant strategies – cost and risk statistics

(Interest expenses, ReCaR and Net Present Value of future coupons in percent of GDP; average residual life in years; maturing debt in billions of euro)

Indicator	Strategy No. 44 (CIR)	Strategy No. 44 (VAR-NS)	Strategy No. 40 (CIR)	Strategy No. 40 (VAR-NS)
Interest expenses/GDP (1)	5.68 %	4.94%	5.74%	4.77 %
NPV of future coupons/GDP	23.13 %	22.37%	21.14%	20.37 %
RCaR(1)	1.35 %	1.79%	1.55%	1.79 %

(1) Calculated in the last year.

4.2 Exploding the clusters

Further insights can be gathered by exploding the clusters referred to the strongly dominant strategies. Exploding the cluster offers a broader range of options, notably it identifies further scope for improvement compared with the single strategy that had been selected to represent the whole cluster. Solutions can be identified which allow for some shortening in duration to make strides in the shorter end of the curve and reduce the debt cost burden.

Let us focus here on cluster 44, as scored based on CIR projections.[28] In Figure 7, the red-coloured dot is the representative strategy of this cluster (this is the same dot commented on above in Figure 5), while the other dots are the residual population of strategies. Notably, we have identified two other strategies located south-east of the representative strategy itself using the colours yellow and blue. As a first, perhaps obvious outcome, the latter was selected based on the fact that it lies at the centre of the cluster (measured on Euclidean distances applied to data on volumes of issuance). This in turn may offer room for improvement moving towards the ‘periphery’ of the cluster, towards smaller costs, lower risks or both. Second, by drawing a line (in actual fact a curve) through the yellow and blue dots, a frontier of optimal solutions emerges, at least as far as the criteria of average residual life and relative CAR are concerned. Additional solutions emerge as optimal when the criteria of debt maturing in 2016 (the first post-end horizon calendar year) and NPV of future coupons are taken into consideration. Whether to rely more on the results offered by one or other indicator depends on what the debt manager considers the most relevant risk – be that market risk or refinancing risk – while where to stop along the frontier within each curve is ultimately related to its degree of risk aversion.

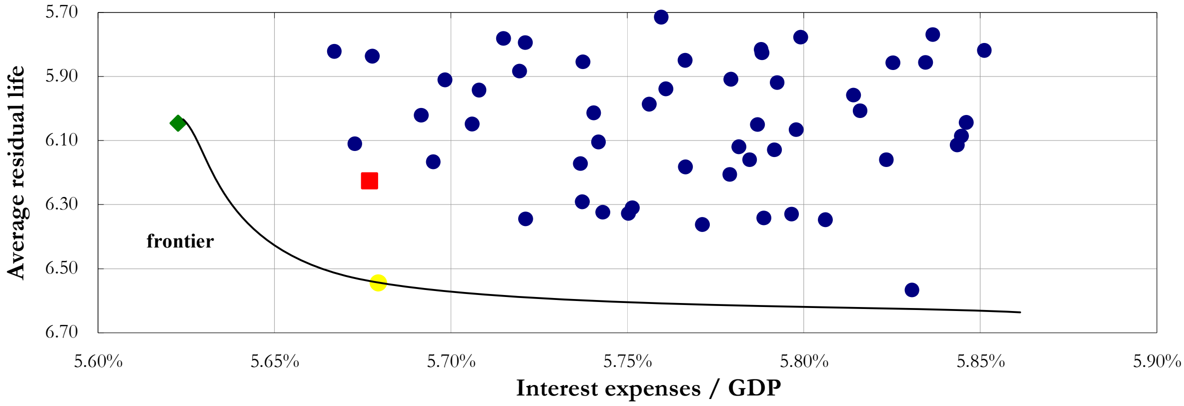
These insights ought to support an approach based on a rather large number of strategies, whereby focussing on a limited number of them (our ‘representative strategies’) is only a convenient tool for undertaking an initial screening but eventually all information (based on ‘feasible strategies’) is exploited.

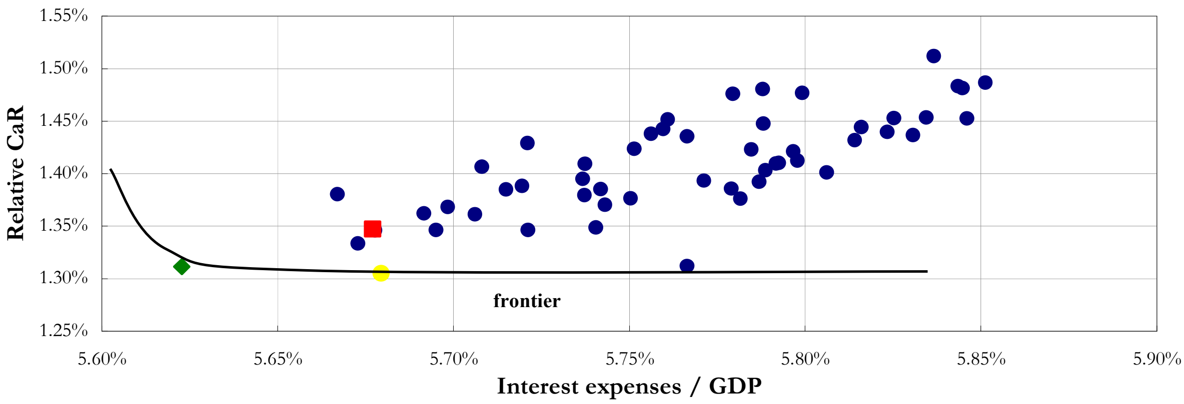
[28] The results for cluster 40 are in the appendix (Figure A.4)

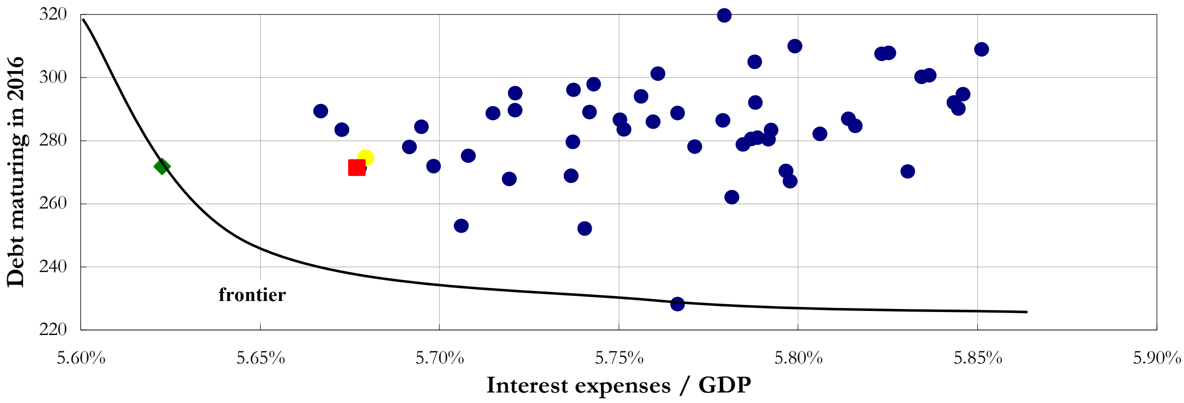
Figure 7

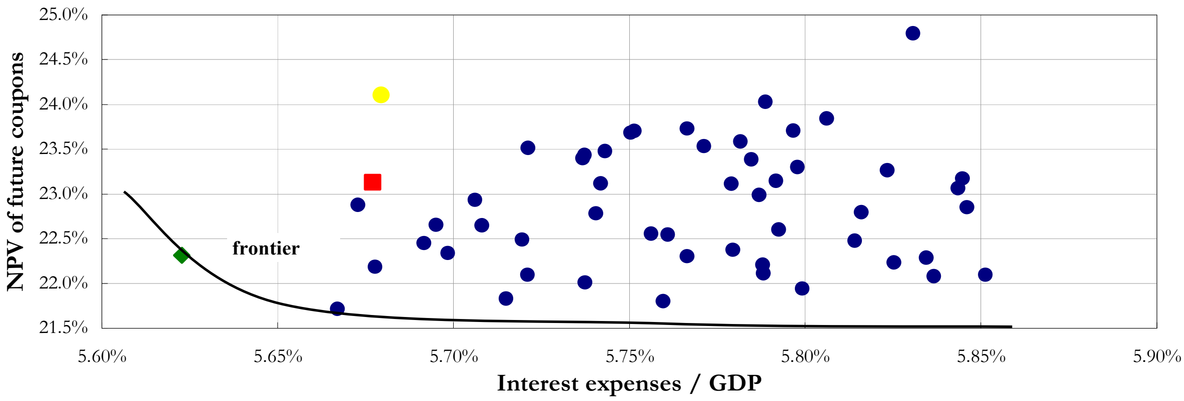
Results for strategies in cluster No. 44

(Interest expenses/GDP, ReCaR and Net Present Value of future coupons as a percent of GDP; average residual life in years; debt maturing in fourth year in billions of euro)









5. Concluding remarks

In the sixth year of a banking crisis that turned into a sovereign debt crisis, public debt managers have had plenty of confirmation of how challenging their task is. Extreme bouts of volatility have turned upside down, often in a matter of months if not weeks, the costs and benefits of issuing long rather than short. In this context, the concept of public debt refinancing strategies becomes even more central, in order to frame investors' expectations about future issuances. Notably, a refinancing strategy encompasses a choice as to the desired structure of issuances (short, medium or long term), the type of security (zero-coupon vs. coupon bonds and nominal vs. indexed bonds, just to mention two basic alternatives) and their maturity. From this perspective, it should emerge clearly that a refinancing strategy is a much more complex concept than merely an objective of average maturity of the outstanding debt.

At the same time, just like in a standard problem in portfolio asset management, pursuing the long-term goal should ideally remain associated with sufficient flexibility and tactical management in order to exploit major short-term opportunities as and when they arise. In the context of the public debt manager's task, this also means satisfying volatile investors' appetite as well as the varying financing needs of the Treasury.

This paper aims to feed this view of the world into the modelling of public debt refinancing strategies. The long-term orientation in assessing the refinancing plans is anchored by the fact that the cost of each strategy includes up to the last coupon of each bond outstanding at the end of our projection horizon. In practice, this means assessing the cost of debt up to 2035 in the current fit of the model. At the same time, as a relevant add-on compared to other published models in public debt management, we generate and study refinancing strategies where the amount to be issued for each of seven basic types of security is allowed to vary over time, in absolute and relative terms.

Some of the strategies examined in the paper are aligned to actual choices in terms of offer enacted by the Italian Treasury, while others represent less run-of-the-mill options. Regardless of whether the tested strategies are seen as odd or conventional, each one is viable in the sense that it meets both 'hard' constraints (the no-overdraft requirement on the account held by the Treasury with the Bank of Italy and the ceiling set annually by Italian law on the amount of net securities) and 'soft' ones (as a way to support the financial industry, the focus is on strategies that broadly span the entire yield curve).

It is important to acknowledge that the interest rates set at issuance may deviate from secondary market rates. Notably, a panel model on daily data on BTPs gauges the extent to which the interest rate on the security being auctioned off may surge, once a number of market factors are considered, if the offer of that security by the Treasury is well above a norm. In this way, rather than scratching in an exogenous way strategies which load 'too much' issuance onto one security, we penalize them endogenously since the model charges a higher-than-otherwise cost for the issuer in placing that load.

We highlight that any exercise of this type is as good as the forecasts on the underlying interest rates are. Alas, there is no way to be sure ex ante which is the best set of forecasts. In our case, we develop forecasts based on a CIR and a VAR model (the latter is integrated with the approach taken by Nelson and Siegel). Our hope is that since the two approaches are rather different – one draws only on market rates while the other uses some selected macro-variables – they ought to cross-check each other. We set no ranking between the two forecasting tools. Thus, there is an unavoidable modelling risk. As a basic level of control, what can be done, however, is to double check how far a given strategy is from the optimal choice when the former is picked based on a set of interest rate forecasts that eventually turn out to be wrong. To gauge the impact of this "error", we run the strategy being selected as optimal based on CIR interest rate forecasts assuming interest rates were eventually to follow what was predicted by the VAR-NS and vice versa we test the optimal VAR-NS strategy in a world with CIR rates. The exercise suggests that under these unfortunate circumstances, the cost of serving the debt could increase by one or two tenths of a percentage point of Italy's GDP (on an annual basis), an acceptable deviation given that the baseline cost could be in the order of four to five percentage points.

By way of concluding, we wish to express an even bolder ambition. While for several reasons the current version of the framework is run on Italy's data, much attention is devoted to the methodological aspects of generating strategies, filtering the viable ones and, if these are in large numbers, weeding out a smaller subset of strategies to be assessed in greater detail. Likewise, the exercise just described above hints at a simple approach to infer the modelling risk. Moreover, care was exercised in writing routines that allow for a relatively smooth updating of the model on a quarterly basis. To cut a long story short, we hope that the issues raised in this paper could attract the interest of a wider audience of researchers and practitioners, within and outside of Italy.

ADDITIONAL TABLES AND CHARTS

Table A.1

Dataset used to estimate the VAR model

Notation	Variable	Description	Units of measure and notes	Raw data Source
sr	Short-term interest rate	6-month BOTs gross compound allotment rate	Percentage. Average of monthly weighted averages	Bank of Italy
lr	Long-term interest rate	Gross yield of benchmark 10-year BTPs	Percentage. Period average	Bank of Italy
gdp	Real GDP	Gross domestic product – Expenditure Approach	Millions of euro; seasonally adjusted; GDP deflated	OECD
p	Price Index	GDP Deflator	Base year = 2005	OECD
b	Real net budget (deficit)	General Government Public net borrowing requirement net of settlement of past debt and privatization receipts	Deficit (-); Millions of euro[a]; sum of monthly data. Deflated and seasonally adjusted GDP	Bank of Italy
d	Debt	General Government gross debt	Millions of euro[a]; End of period data; seasonally adjusted	Bank of Italy

[a] For the years before 2001 the fixed euro-lira exchange rate of 1,936.27 is applied.

Table A.2

Augmented Dickey-Fuller unit root tests: summary

Variable	Test specifications (p-values) Constant	Constant and trend
lr	0.19	0.57
sr	0.42	0.36
gdp	0.39	0.95
d	0.02	0.09
b	0.06	0.13
p		0.96

Table A.3

Results of the VAR model

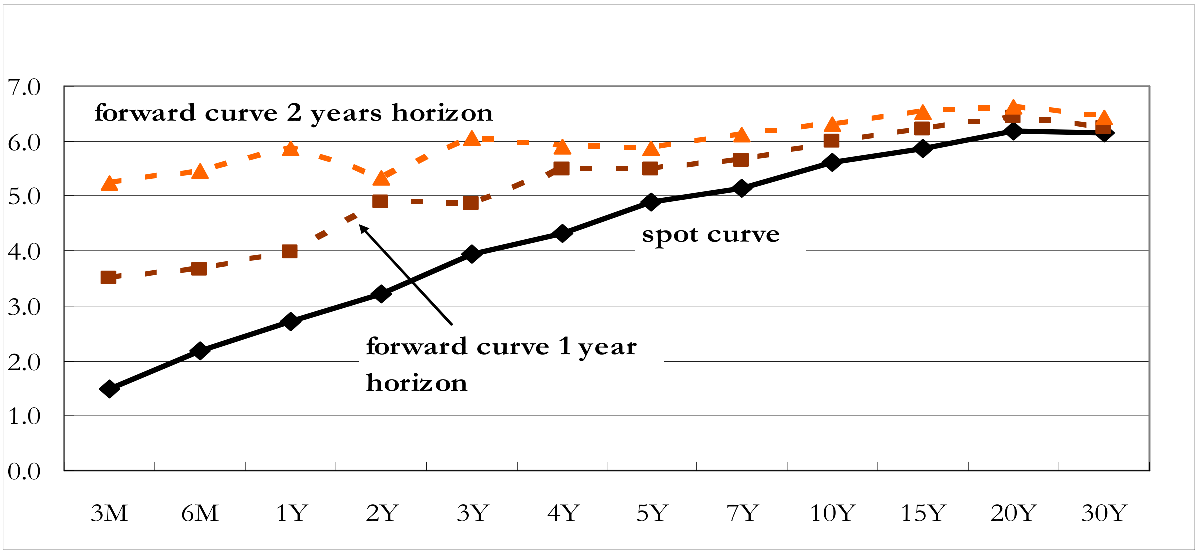
	$\Delta log(lr)_t$	$\Delta log(sr)_t$	$\Delta log(d)_t$	$log(b)_t$	$\Delta log(p)_t$	$\Delta log(gdp)_t$
$\Delta log(lr)_{t-1}$	0.477***	0.265	-0.003	-4.454*	0.007	-0.024
$\Delta log(lr)_{t-2}$	-0.089	-0.153	-0.014	1.668	0.006	0.011
$\Delta log(sr)_{t-1}$	-0.071	-0.339**	0.012	1.819**	-0.006	0.008
$\Delta log(sr)_{t-2}$	0.005	0.304*	0.014*	0.932	-0.002	-0.010
$\Delta log(d)_{t-1}$	1.342	-0.331	0.149	11.325	0.013	0.023
$\Delta log(d)_{t-2}$	0.039	2.628	0.176	17.581	0.141	0.061
$log(b)_{t-1}$	0.004	-0.014	0.002*	0.022	-0.0001	-0.0003
$log(b)_{t-2}$	-0.009	0.001	0.0002	0.122	0.001	-0.001
$\Delta log(p)_{t-1}$	-1.632	1.259	0.296	25.622	-0.254*	0.291**
$\Delta log(p)_{t-2}$	-0.862	1.468	0.321**	33.627*	0.177	0.222
$\Delta log(gdp)_{t-1}$	-0.056	10.523***	-0.448***	-0.300	0.275**	0.221
$\Delta log(gdp)_{t-2}$	2.360*	6.126	-0.353*	-46.123**	0.141	0.149
μ	0.051	0.036	-0.017	7.688***	5.02E-05	0.011
R-squared	0.2590	0.2922	0.3763	0.2718	0.2693	0.2180
Adj. R-squared	0.1320	0.1708	0.2694	0.1469	0.1441	0.0839
Sum sq. resids	0.3397	2.9960	0.0067	85.1338	0.0041	0.0049
S.E. equation	0.0697	0.2069	0.0098	1.1028	0.0077	0.0083
F-statistic	2.0393	2.4079	3.5202	2.1769	2.1502	1.6260

Figure A.1

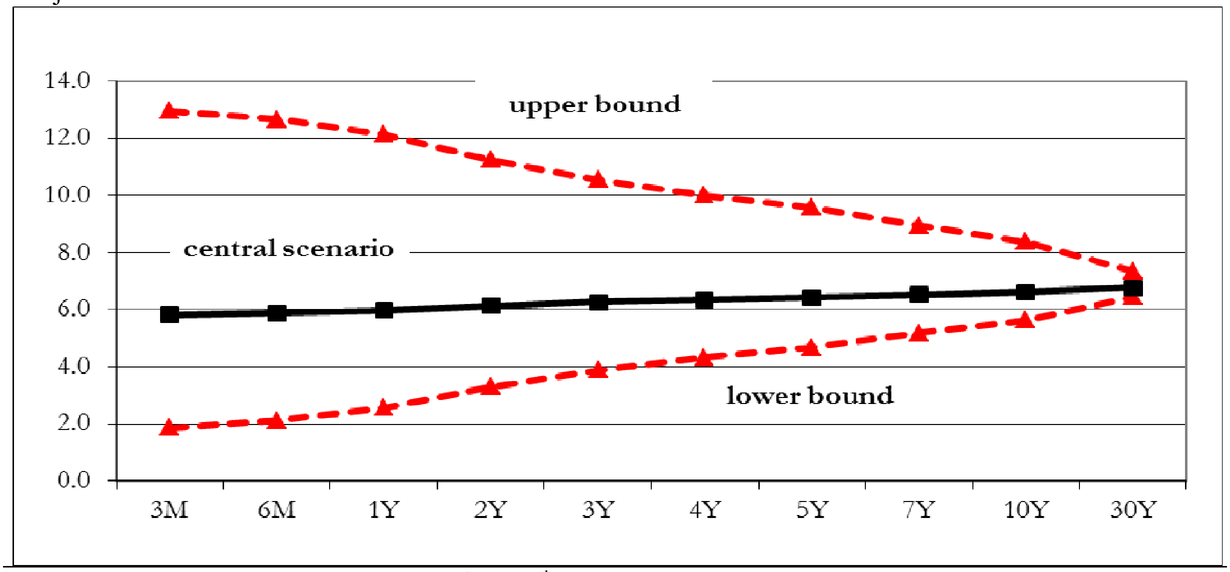
CIR projections of Italy's sovereign yield curve*

(percent)

Market data as of 3 July 2012



Projections referred to 31 December 2015



* Lower and upper bound are defined as the 1st and 99th percentile of the simulated interest rate scenario for each maturity.

Figure A.2

Debt strategies using VAR-NS interest rate projections

(Interest expenses, ReCaR and Net Present Value of future coupons in percent of GDP; average residual life in years; maturing debt in billions of euro)

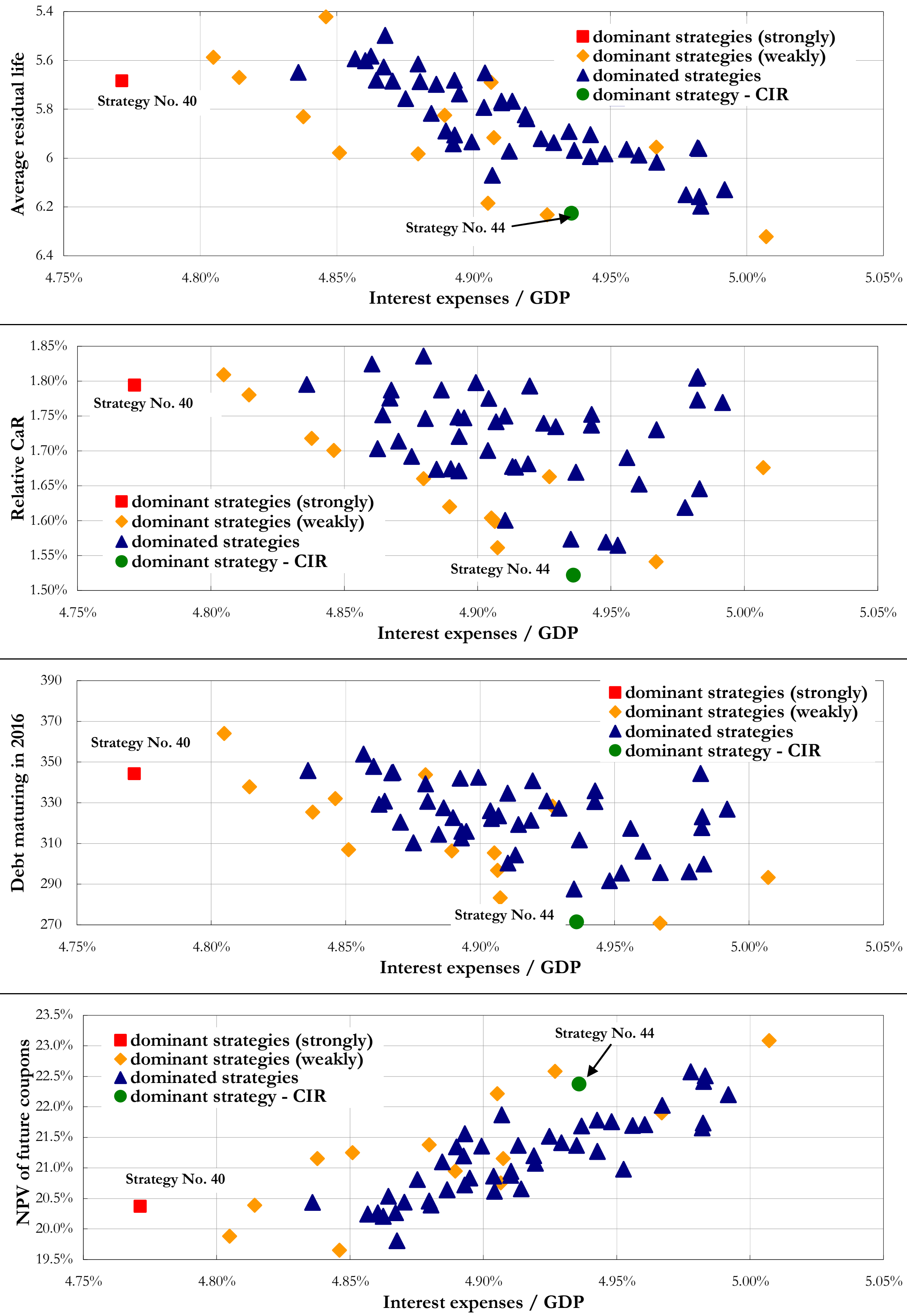


Figure A.3

Dominant strategy – yearly issues

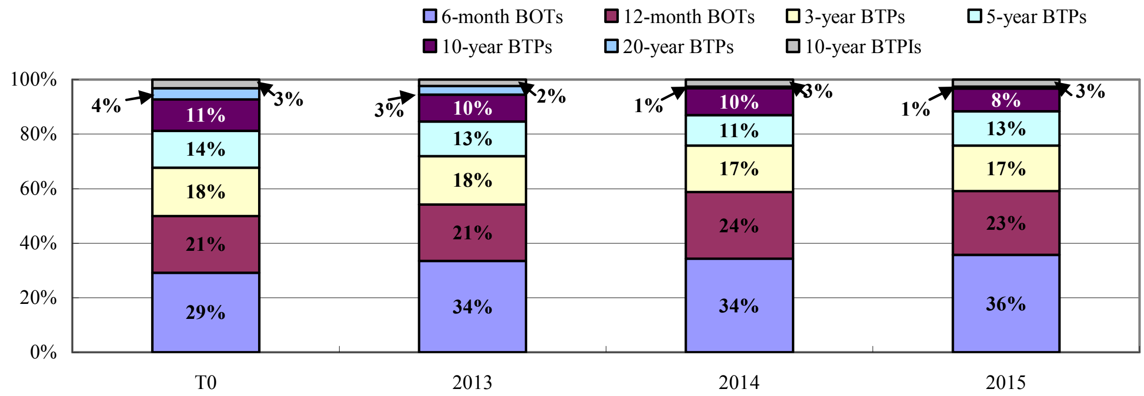
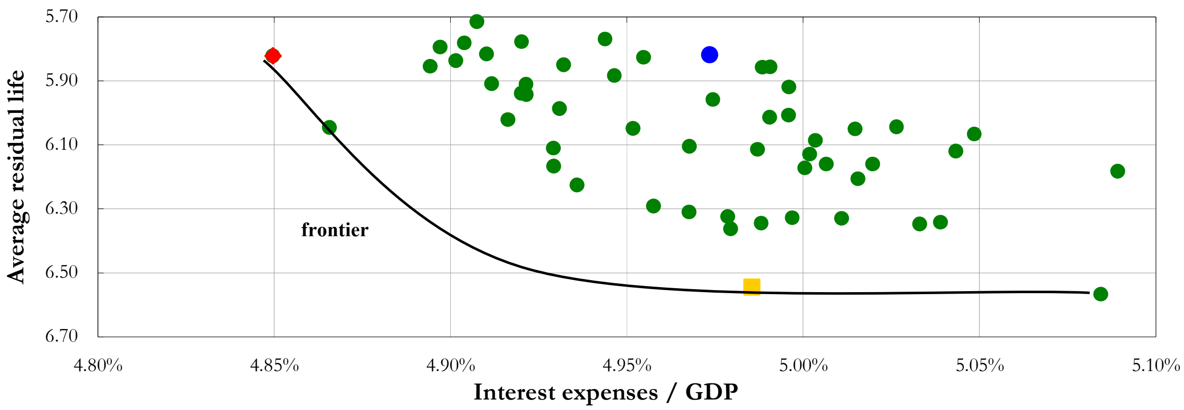
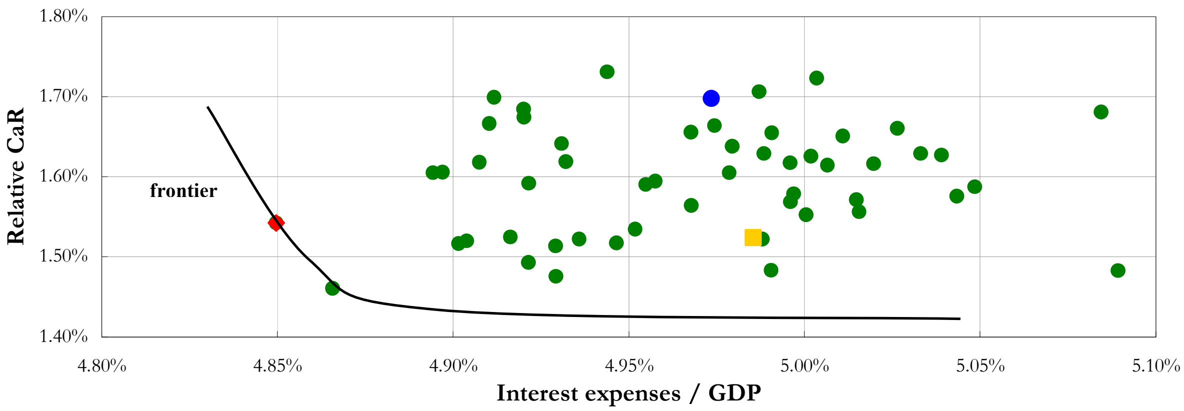


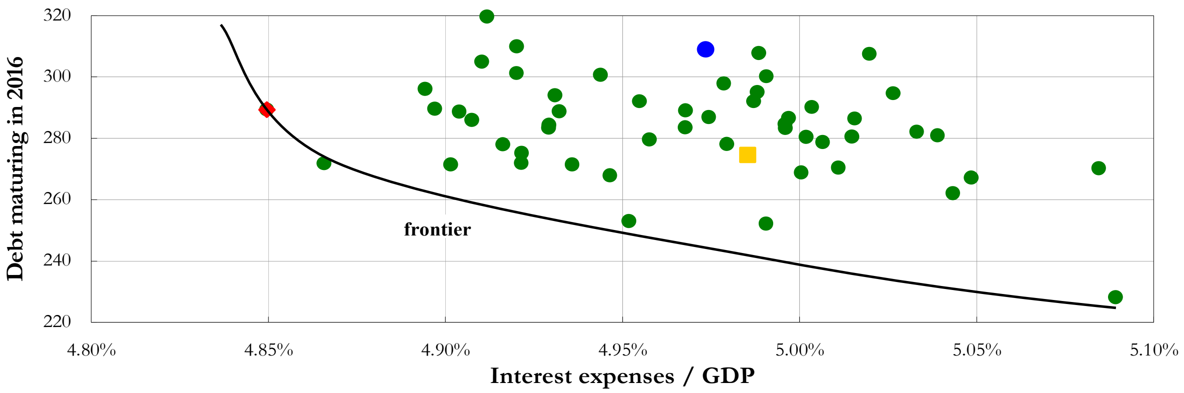
Figure A.4

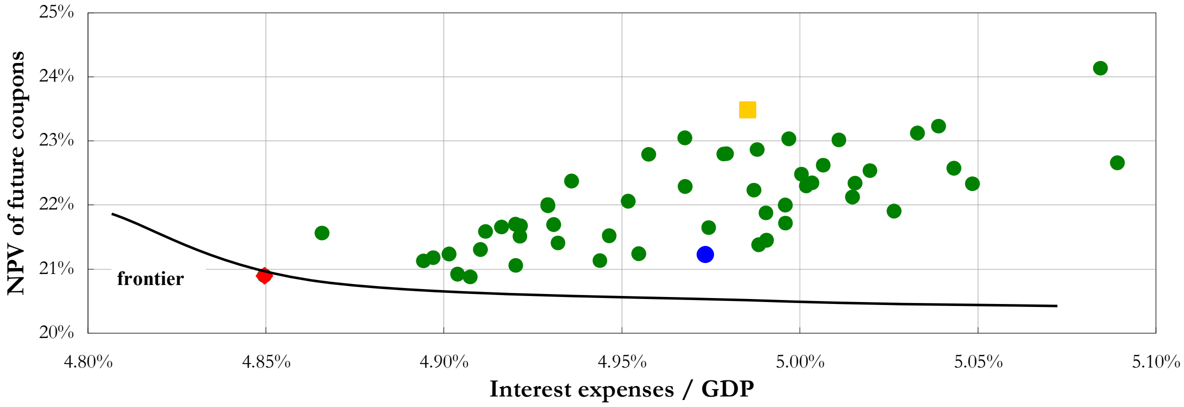
Results for strategies in cluster No. 40

(Interest expenses/GDP, ReCaR and Net Present Value of future coupons in percent of GDP; average residual life in years; debt maturing in fourth year in billions of euro)









Elaborating the 100,000 strategies

The algorithm is designed to provide the gross issuance for each security in each point of time ($M^j_t[i]$) for each of the 100,000 strategies.

It models the change from quarter *t-1* to quarter *t* in the gross issuance of the *i*-th security in the *j*-th strategy, which is defined by a random variable:

[A.1] $\tilde{\Delta}^j_{i,t} = \hat{M}^j_t[i] - M^j_{t-1}[i] \qquad \forall\ t, j, i = 1, .., 7$

where M_0 is the vector of actual issuances in 2011.

In the baseline set-up, we assign a value to $\tilde{\Delta}^j_{i,t}$ on the basis of a (pseudo)random number *n* drawn from a uniform distribution $\forall$ t, j, i[29]:

[A.2]

$$\tilde{\Delta}^j_{i,t} = \begin{cases} -2\Delta_i & \text{if} \quad n_{i,j,t} < 0.1 \\ -\Delta_i & \text{if} \quad 0.10 \le n_{i,j,t} < 0.35 \\ 0 & \text{if} \quad 0.35 \le n_{i,j,t} < 0.65 \\ +\Delta_i & \text{if} \quad 0.65 \le n_{i,j,t} < 0.90 \\ +2\Delta_i & \text{if} \quad n_{i,j,t} \ge 0.90 \end{cases}$$

where Δ_i is a given amount, different for each security, that is chosen according to the stock of each of the seven securities at time 0.

Three remarks are in order: first, in [A.2] the five outcomes are security-dependent but time-invariant; second, different probabilities are attached to the outcomes, so that the process [A.1]-[A.2] accepts as a solution the sequence $+2\Delta_i$ and $-2\Delta_i$ (or vice versa) in two following quarters even if the probability of this roller-coaster is only 2%; third, the expected value of this process is nil so that, on average, the gross issuance remains anchored to M_0 throughout the whole simulation horizon.

As a refinement of the basic version [A.2], we align the issuance in quarter *t* to the Treasury refinancing needs L^j_t in that given quarter. To do this we apply an enhanced version designed to give an expected value of net issues equal to the financing needs. The algebra is as follows

[A.3] if $L^j_t > M^j_{t-1}$ while if $L^j_t \le M^j_{t-1}$

$$\tilde{\Delta}^j_{i,t} = \begin{cases} \min[(\Delta_i + x^j_t), 2\Delta_i] \\ \min[\frac{1}{2}(\Delta_i + x^j_t), \Delta_i] \\ \Delta_i\left[\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right] \\ -\frac{1}{2}\Delta_i\left[1 - \left(\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right)\right] \\ -\Delta_i\left[1 - \left(\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right)\right] \end{cases} \qquad \tilde{\Delta}^j_{i,t} = \begin{cases} \Delta_i\left[1 - \left(\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right)\right] \\ \frac{1}{2}\Delta_i\left[1 - \left(\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right)\right] \\ -\Delta_i\left[\Phi(L^j_t - M^j_{t-1}) - \frac{1}{2}\right] \\ \max[-\frac{1}{2}(\Delta_i + x^j_t), -\Delta_i] \\ \Delta\max[-(\Delta_i + x^j_t), -2\Delta_i] \end{cases} \quad \text{with probability} \begin{cases} p_1 = 0.10 \\ p_2 = 0.25 \\ p_3 = 0.30 \\ p_4 = 0.25 \\ p_5 = 0.10 \end{cases}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a N(0,1).

Basically, we shift upwards the value taken by the five potential outcomes when the financing needs in *t* exceed the gross issuance in *t-1*, and thus the issuance in *t* needs to be augmented to reach par, and vice versa.

[29] Overall we extract 7×T×100,000 random numbers.

In order to avoid overly large variations in the supply of a security, involving unfeasible strategies, we set a floor ($-\Delta_i$ and $-2\Delta_i$) or cap ($+\Delta_i$ and $+2\Delta_i$) on the extreme values of the random variable $\widetilde{\Delta}_{i,t}^{j}$.

The results depend on the unknown quantity x (time- and strategy-dependent). This quantity is calculated such that M_{t-1}^{j} plus the expected value of $\widetilde{\Delta}_{t}^{j}$ equals the scalar L_{t}^{j}:

[A.4] $L_t^j = M_{t-1}^j + E(\widetilde{\Delta}_t^j)$

where $E(\widetilde{\Delta}_t^j) = p'\Delta_t^j \iota_7'$, p is a 5×1 vector filled in with the probabilities $p_1, p_2, .. p_5$; Δ_t^j is a 5×7 matrix which displays in Column 1 the five possible outcomes of [A.3] associated with the first security (the 6-month BOT), in Column 2 the corresponding outcomes for the second security (the 12-month BOT) and so on and so fourth; and, ι_7 is a 7×1 vector filled with all 1's.

The result for x is worked out by solving [A.4]:

[A.5] $$x = \frac{\mathrm{sign}\{L_t^j - M_{t-1}^j\} \times 2\left(L_t^j - M_{t-1}^j\right) - \mathrm{sign}\{L_t^j - M_{t-1}^j\} \times 2P'\Psi - p_k\Delta - 2p_m\Delta}{p_k + 2p_m}$$

where sign{.} is the logical operator which yields value +1 if the expression in curly brackets is greater than zero and -1 otherwise and Δ is the weighted average of Δ_i with weights given by the issued amount M_0.[30]

The values of P and Ψ depend on the relative position of L and M.
If in a point t in time $L_t^j > M_{t-1}^j$

$$P' = [p_3, p_4, p_5],$$

$$\Psi' = \left\{\Delta\left[\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right], -\frac{1}{2}\Delta\left[1 - \left(\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right)\right], -\Delta\left[1 - \left(\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right)\right]\right\},$$

$m = 1$ and $k = 2$,

while if $L_t^j \le M_{t-1}^j$

$$P' = [p_1, p_2, p_3],$$

$$\Psi' = \left\{\Delta\left[1 - \left(\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right)\right], \frac{1}{2}\Delta\left[1 - \left(\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right)\right], -\Delta\left[\Phi(L_t^j - M_{t-1}^j) - \frac{1}{2}\right]\right\},$$

$m = 5$ and $k = 4$.

One general remark is in order: [A.3] is construed so that the sign of the two most extreme outcomes are retained. When $L_t^j > M_{t-1}^j$ the fourth and fifth outcomes remain negative, even if they get closer to zero compared with the baseline set-up, while when $L_t^j \le M_{t-1}^j$ it is up to the first and second outcome to preserve the positive sign.

In practice, the model is flexible enough to accept financing plans where, even in a context of an overall increase in gross issuance, the Treasury may nevertheless decide to decrease the amount of one or a few specific securities.

[30] $$\Delta = \frac{\sum_{i=1}^{7} M_0[i]\Delta_i}{\sum_{i=1}^{7} M_0[i]}$$

Box 1

Introducing cash management instruments in the model

Cash management instruments (3-month and broken-maturity BOTs) are introduced in the model in accordance with the following rules:

- for each of the three years ($y1$, $y2$, $y3$) of the simulated horizon and each strategy j, we identify the quarter in which the amount of maturing debt is maximum: $R^j_{yi,max} = max\{R^j_t\}_{t \in yi}$;
- if that quarter happens to be the last of the calendar year, the algorithm stops here and no flexible BOTs are issued in year yi. Otherwise, the quarters following the one with highest values of reimbursements are ranked in descending order and let $R^j_{yi,min}$ be the lowest value of reimbursements in such quarters;
- if $R^j_{yi,max} - R^j_{yi,min} > Th$, then $R^{j,*}_{yi,min} = R^j_{t\,min} + Th$, where Th is the threshold in the outstanding stock of such instruments at any given point in time (€25 billion; this ceiling is derived from a preliminary analysis of the related time series). In practice, the model assumes that the cash management instruments are issued in the quarter where reimbursements are highest – to concur to their refinancing – and expire in the quarter where they are lowest;
- conversely, if $R^j_{yi,max} - R^j_{yi,min} \leq Th$, then an amount of cash management instruments equal to $R^j_{yi,max} - R^j_{yi,min}$ is scheduled to expire in the quarter where reimbursements are lowest while the residual is split evenly among the other quarters following the one where reimbursements were highest.

Annex 2

The cluster analysis as a tool to scan viable refinancing strategies

Cluster analysis allows us to identify groups of strategies that are similar according to a set of given attributes and a specific distance criterion, without making any a priori assumption as to the strategies themselves. In this paper the partition is undertaken using the K-means algorithm that belongs to the family of non-hierarchical clustering methods and operates through the minimization of the intra-cluster variance. This algorithm is reckoned to converge very fast, although it may not necessarily provide the global optimum because the solution depends on the starting points and some ill-choice in the randomly selected initial centroids can lead to poor results. Hence, to overcome this drawback, we apply a two-step procedure. First, we run a hierarchical cluster method based on the complete linkage,[31] to choose the initial centroids. Second, having extracted 60 clusters from the previous step, their centroids are used as starting points for the K-means algorithm to get the final classification. The measure of distance we choose, the Euclidean distance, is the same in both steps.

Notably, the hierarchical method groups the data in a sequential way according to a distance measure, but it does not allow units to move among different clusters once they are classified. Using our "two-step" cluster analysis we can look for the optimal solution by letting the units be free to change cluster.

As classification variables we select the gross issuance of all seven securities at time t = 5 and t = 12, that is M^j_5 and M^j_{12}. We purposely select the two quarters far enough from each other and neither too close to the origin, so as to hopefully rely on diversified sets of gross amounts in the columns.

The choice of 60 as number of the "representative clusters" means we can strike a balance between a parsimonious approach – which is especially useful in the graphical

[31] In the complete linkage clustering method the distance between groups is defined as that of the most distant pair of individuals, one from each group.

representations of the final results – and a solution which conveys enough variability across the clusters themselves.

Annex 3

The algebra of the indicators

In this appendix we report calculations made to derive the different measures of cost and risk described in section 3.1.

The cost of debt

$$\mathrm{COST}_{j,k} = \frac{\sum_{t=T-3}^{T}\sum_{i=1}^{7}\mathrm{CPN}_{j,i,k,t}}{\mathrm{OA}_{j,t=T}} \times \mathrm{Debt}/\mathrm{Gdp}_{t=T}$$

where:
COST$_{j,k}$ - the interest expenses/GDP ratio for the *j-th* strategy in the *k-th* interest rate scenario in the last year of the simulation horizon
CPN$_{i,j,k,t}$ - coupons paid for the *i-th* security, in the *j-th* strategy, in the *k-th* interest rate scenario and at the *t-th* quarter (in our case, as we consider the interest/GDP ratio at the last year of the horizon, we summed up interest expenses in the last year of our simulation).
OA$_{j,t=T}$ - outstanding amount of overall securities in the *j-th* strategy at the end of the simulation period.
Debt/Gdp$_{t=T}$ - debt/GDP ratio of Italy at the end of the simulation period.[32]
Hence, the expected debt cost for the *j-th* strategy (*COST*$_j$) is defined as the average interest expenses over 10 thousand interest rate scenarios.

Relative Cost-at-Risk (RCaR). See main text.

Residual life of outstanding debt (ROL)

$$\mathrm{ROL}_{j,t=T} = (\mathrm{TM'}_{j,t=T} \times \mathrm{SA}_{j,t=T}) / \mathrm{OA}_{j,t=T}$$

Where:
TM$_{i,t=T}$ – an n × 1 vector containing the time to maturity for each security outstanding at the end of the horizon (t=T).
SA$_{i,t=T}$ – an n ×1 vector containing the issued amount for each security outstanding at the end of the horizon (t=T).

Maturing debt in year y4 (MD). This indicator captures the refinancing risk in the first year after the end of the simulated horizon.

$$\mathrm{MD}_j = \sum_i \mathrm{SA}_{j,i,t=T}(\mathrm{TM}_{j,i,t=T} <= 1\mathrm{year})$$

Net present value of future coupons (NPV). This indicator measures the future service of debt beyond the given horizon.

$$\mathrm{NPV}_{j,k} = \frac{\sum_{t=T+1}^{\infty}\sum_{i=1}^{7}\mathrm{CPN}_{j,i,k,t}}{\mathrm{OA}_{j,t=T}} \times \mathrm{Debt}/\mathrm{Gdp}_{t=T}$$

[32] Forecasts of debt/GDP ratios in the years ahead are provided by the IMF in the World Economic Outlook Database, October 2012.

The panel model to fit the yield penalty

The panel model assesses the extent to which larger-than-usual issuances are penalized in terms of yields. The variables employed to measure this effect are reported in Table A.4.

As tests in Table A.5 show, the dependent variable behaves like a I(1) variable. In order to remove this non-stationarity we estimate the model in the first differences, instrumenting the endogenous regressors. The results are reported in Table A.6. The estimated coefficients of the control variables show the expected signs and the (uncentred) R^2 scores at 0.30; for what concerns us most, the extra supply is significant and positive, suggesting that large offers actually entail a cost in terms of yield (Column A). This key result holds true across a number of different specifications in the model. In Column B the (first differences of) bid-ask and the benchmark spreads, two variables which could be plagued by endogeneity, are instrumented using their second lags.[33] In Column C we add two other regressors: the residual life to maturity and the bond coupon.[34] In Columns (D) and (E) we consider a more substantial point. The amount supplied in auction is announced to the market two days before the auction, after 17:30 when MTS trades are closed.[35] Hence this piece of information may well be taken into account by the market also on the day before the auction. In order to test this hypothesis, we consider two distributed forward models where any extra offer is allowed to cause changes in yields even one day before the auction (Column D). An excessive supply has a significant positive effect on yields already the day after the announcement while the coefficient on the contemporaneous term increases and remains significant. On the other hand, as a cross-check, introducing a variable for the extra-supply two days forward should not – and in fact does not – add explanatory power (Column E), as the information on the amount to be supplied has not yet been released to the market at the time.

Table A.4

Dataset used to estimate the penalization in BTP auctions (1)

Yield	End-of-day (bid) yield on MTS
Bid-Ask Spread	End-of-day (log of) spread between ask price and bid price (times 100)
Spread	10-year-benchmark spread with Germany (basis points)
Auction	Dummy if an auction occurred in the day
Auction bond	Dummy if that bond was auctioned in the day
Offer	Amount supplied in auction (millions)
High Offer	Amount supplied in auction if above the average (millions)
Life to maturity	Number of years to maturity
MTS Volume	Total amount traded in MTS (millions)
Coupon	Bonds' coupon
Class	Bond class (BTP 3, 5, 10 years or longer)

Source: Derived from Bloomberg and Banca d'Italia data.
(1) All series are taken from end-of- day quotes, 1 January 2009 – 30 March 2012. A total of 60 BTPs is surveyed; BTPs are dropped when expiring within one year.

33 A one-lag only would not have overcome the endogeneity, were this at play, given that the model is fit on first differences.

34 The "coupon effect" is the impact of coupon level on the yield-to-maturity of coupon bonds with the same maturity (see Tuckman, 2002). Under an upward-sloping spot rate curve, a coupon bond has a lower yield than a zero coupon bond. The higher the coupon, the lower is the yield to maturity with respect to a zero coupon, *ceteris paribus*, and hence the lower tends to be the absolute variation in yields in basis points.

35 Since 2012 the offered amount has been announced three days before the auction, but this time span covers a tiny portion of auctions in our dataset.

Table A.5

Panel unit root test: Summary

Series: Yield
Sample: 1/02/2009 3/30/2012

Method	Statistic	Prob.	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t	-1.193	0.116	60	32604
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-3.392	0.000	60	32604
ADF - Fisher Chi-square	165.461	0.004	60	32604

Notes: Tests assume asymptotic normality; Exogenous variables: Individual effects; Automatic selection of maximum lags. Automatic lag length selection based on SIC: 0 to 11 Newey-West automatic bandwidth selection and Bartlett kernel.

Table A.6

Results of the panel model on the yield penalty

(dependent variable: $\Delta yield_{i,t}$)

	(A)		(B)		(C)		(D)		(E)	
Δ $highoff_{i,t} \times 100$	0.15	***	0.24	***	0.23	***	0.35	***	0.35	***
F.**Δ** $highoff_{i,t} \times 100$							0.25	**	0.24	*
F2.**Δ** $highoff_{i,t} \times 100$									-4.53E-5	
Δ $bidask_{i,t}$	0.1548	***	0.2501	***	0.2510	***	0.2506	***	0.2493	***
Δ $spread_{i,t}$	1.5576	***	1.2220	**	1.2095	**	1.1997	**	1.2278	**
$auction_t$	0.0061	***	0.0111		0.0149	*	0.0158	*	0.0136	
Δ mts_t	-0.0235	***	-0.0263	***	-0.0266	***	-0.0274	***	-0.0273	***
$yearstomaturity_{i,t}$	-		-		0.0001		0.0001		0.0001	
$coupon_i$	-		-		-0.0020	**	-0.0019	**	-0.0020	**
Number of obs	34,442		34,210		34,210		34,210		34,210	
R^2 (uncentered)	0.295		0.235		0.234		0.236		0.237	
Method	OLS		IV		IV		IV		IV	
Instrumented variables / instruments			$\Delta bidask_{i,t}$, $\Delta spread_{i,t}$ / L(2)$\Delta bidask_{i,t}$, L(2)$\Delta spread_{i,t}$							
Standard error	Robust		Robust		Robust		Robust		Robust	

Significance level: *** 1%; ** 5%; * 10%. The panel model is estimated on end-of-day MTS quotes from January 2011 to March 2012. The source of the MTS quotes is Bloomberg.

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